Title
PHASE SHIFT ANALYSIS OF HEAVY ION SCATTERING

Authors
Conzett, H.E.
Isoya, A.
Hadjimichael, E.

Publication Date
1963-04-01

UCRL-10769

University of California

Ernest O. Lawrence Radiation Laboratory

PHASE SHIFT ANALYSIS OF HEAVY ION SCATTERING

Berkeley, California

UNIVERSITY OF CALIFORNIA
Lawrence Radiation Laboratory
Berkeley, California

Contract No. W-7405-eng-48

PHASE SHIFT ANALYSIS OF HEAVY ION SCATTERING

H. E. Conzett, A. Isoya, and E. Hadjimichael

April, 1963

PHASE SHIFT ANALYSIS OF HEAVY ION SCATTERING*

H. E. Conzett, A. Isoya,† and E. Hadjimichael

Lawrence Radiation Laboratory
University of California
Berkeley, California

April, 1963

In this paper we want to emphasize the equivalence between phase shift and optical model analyses of the elastic scattering of strongly absorbed[1] particles, such as heavy ions. Then, we will point out some advantages in using the phase shift analyses. We include here all analyses[2] in which the partial-wave (complex) phase shifts are explicit parameters in the calculation and are not adjusted through an intermediary optical potential.

The differential cross-section for elastic scattering is

$$\sigma(\theta) = |f(\theta)|^2$$

with the scattering amplitude given by

$$f(\theta) = f_c(\theta) + \frac{i}{2k}\sum_{\ell=0}^{\infty} (2\ell+1)\, e^{2i\sigma_\ell} P_\ell(\cos\theta)\,(1 - A_\ell\, e^{2i\delta_\ell}), \qquad (1)$$

where $f_c(\theta)$ is the (point-charge) Coulomb scattering amplitude, $\sigma_\ell = \arg\Gamma(1+\ell+i\eta)$ with $\eta = \frac{Z_1 Z_2\, e^2}{\hbar v}$, A_ℓ is the amplitude of the <u>outgoing</u> ℓ^{th} partial wave, and δ_ℓ is its (real) nuclear phase shift. Thus, $0 \leq A_\ell \leq 1$, with $A_\ell = 1$ corresponding to no absorption and $A_\ell = 0$, to complete absorption. The optical model determines A_ℓ and δ_ℓ by solving the equation satisfied by the radial wave function f_ℓ.

$$\left[\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} + k^2 - \frac{2\mu}{\hbar^2} U(r)\right] f_\ell(r) = 0 \tag{2}$$

where $U(r) = V_c + V_{opt.}$. Comparison at $r \to \infty$ of f_ℓ with the solution of Eq. (2) for $V_{opt.} = 0$ gives A_ℓ and δ_ℓ. Strongly absorbed particles sample only the surface region of the nucleus, so one is justified in questioning the significance of using a parameterized optical potential to describe the scattering. The phase shift analysis parameterizes the A_ℓ and δ_ℓ directly. Thus, it is clear that the two analyses are equivalent, provided, of course, that the phase-shift parameterization is proper. We shall now examine that point.

McIntyre et al.[2] introduced the (arbitrary) forms

$$A_\ell = 1 - \left[1 + \exp \frac{\ell - \ell_A}{\Delta\ell_A}\right]^{-1}$$

$$\delta_\ell = \delta \left[1 + \exp \frac{\ell - \ell_\delta}{\Delta\ell_\delta}\right]^{-1} \tag{3}$$

in fitting α-particle scattering data, and these forms subsequently provided excellent fits to heavy ion results.[3] Because of the classical nature of the heavy-ion trajectories for $\eta \gg 1$ and $\lambda\!\!\!^{-} \ll R$,[4] one can make the correspondence between ℓ and r, the distance of closest approach for a particle of orbital angular momentum $[\ell(\ell+1)]^{1/2}\hbar$, through the equation

$$E = \frac{Z_1 Z_2 e^2}{r} + \frac{\hbar^2\, \ell(\ell+1)}{2\mu r} \tag{4}$$

which can be rearranged to

$$kr = \eta + [\eta^2 + \ell(\ell+1)]^{1/2} \tag{5}$$

where $k = (1/\hbar)\sqrt{2\mu E}$ and for $\ell \gg 1$ Eq. (5) can be approximated by

$$kr \simeq \eta + (\ell+1/2) \tag{6}$$

Thus, we can refer interchangeably to coordinate (r) space or orbital angular momentum (ℓ) space via Eq. (5), and we will indicate this correspondence by writing $A_\ell(r)$. Classically for particles penetrating an absorbing nucleus (in the Z direction)

$$A_\ell(r) = \exp\left[-\int_{-\infty}^{\infty} \sigma\rho(r')dz\right] \tag{7}$$

where $\rho(r')$ is the nuclear density function, $\bar{\sigma}$ is the average projectile-nucleon cross section, and $r' = [(r)^2 + Z^2]^{1/2}$. For a density function of the form

$$\rho(r') = \rho_0 \left(1 + \exp \frac{r'-R}{d}\right)^{-1}$$

it has been shown[5] that $A_\ell(r)$ takes a form which is essentially symmetric with respect to its half-value point, as in Eq. (3). An "interaction" radius determined from the half-value radius, R_a, of $A_\ell(r)$, will be called an absorption radius, since this terminology explicitly defines the method by which the determination was made. The corresponding radius parameter, r_a, is defined by $R_a = r_a (A_1^{1/3} + A_2^{1/2})$. One should note (Eq. (7)) that strong absorption will result in an absorption radius, R_a, substantially larger than R, the nuclear density radius, with a correspondingly smaller absorption surface-thickness, t. This is illustrated in Figure 1 where a typical optical potential radial function, f(r), is compared with an $A_\ell(r)$

determined from C^{12} + Ta scattering data at 124-MeV. We see that the substantial differences between $r_a = 1.45$ F and $r_0 = 1.30$ F and between $t = 0.36$ F and $a = 0.55$ F are explained.

We shall now examine the parameterization of δ_ℓ, at least in a qualitative manner. Figure 2 shows the radial wave function $f_\ell(r)/kr$ (for $V_c = 0$) with and without absorption. Without absorption, turning on the real potential $V_{opt} = -V$ results in the $\delta_\ell > 0$ given by $\delta_\ell \simeq kd$, where d is the distance the wave function has been "pulled in" as indicated by the scale change between cases (1) and (2). Figure 2(b) shows that strong absorption rapidly damps the wave function to zero inside the nucleus, resulting in a $\delta_\ell < 0$ as the wave function is effectively "pushed out" of the interior region. One sees that in the limit of complete absorption at the boundary, the wave function vanishes there just as for the scattering from an infinitely repulsive potential. Thus, except for ℓ values which correspond to the outermost regions of the nuclear density distribution, δ_ℓ will be negative. For ℓ values near ℓ_A the sign of δ_ℓ is determined by the competing effects of potential scattering versus absorption. As a further demonstration of these considerations, Fig. 3(B) compares the δ_ℓ of Eq. (3), determined from analysis of 48-MeV α + Pb scattering data, with these derived from an optical model analysis[7] of the same data. Through the region where A_ℓ goes from 0.9 to 0.1 (indicated by $4.4\Delta\ell_A$), the two curves are well matched. At lower values of ℓ, corresponding to trajectories penetrating to smaller radii, the two curves diverge rapidly. However, this makes little change in the scattering amplitude because $A_\ell \to 0$ in this region. Nevertheless, we have taken δ_ℓ of the form

$$\delta_\ell = \delta(2-e^{\chi'})e^{\chi} \quad \text{with} \quad \chi = \frac{L-\ell}{\Delta\ell_\delta} \; , \quad \chi' = \frac{L-\ell}{\Delta\ell'_\delta} \tag{8}$$

as having at least qualitative theoretical justification. Using this form[8] of δ_ℓ and the A_ℓ of Eq. (3), we have compared calculations with C^{12} + Ta scattering data to determine the phase-shift parameters.[9] The results are shown in Fig. 4 and 5. Even though previous fits to such data, using the δ_ℓ of Eq. (3), were excellent,[3] we feel that parameterized phase-shift analyses should be made of scattering data involving deuterons, He^3, α-particles, and other strongly absorbed particles; also, nucleon scattering data at higher energies can be included. A theoretically proper form of δ_ℓ then becomes necessary.

The advantages of the phase-shift analysis are the following:

1. The calculation is considerably simpler than that for an optical model analysis, since the numerical solution of Eq. (2) is bypassed.

2. The form of $A_\ell(r)$ gives a clear picture of the physical absorption of the incident particles. The optical model varies the parameters of $V_{opt} = V(R) + iW(R)$ to reproduce this physical fact since A_ℓ depends mainly on W, but, also, on V. This has resulted in some unwarranted comparisons between r_0 and r_a. Also, there has been some uncertainty concerning the choice of the radial form of the imaginary potential. With strong absorption, we believe the determined $A_\ell(r)$ curve is essentially unique; if so, the proper W(r) is one that reproduces that curve.

3. Eq. (1) for the scattering amplitude is, also, valid in the relativistic region as long as spin effects are unimportant.

FOOTNOTES AND REFERENCES

*Work was performed under the auspices of the U.S. Atomic Energy Commission.

†Present address: Department of Physics, Kyushu University, Fukuoka, Japan.

1. As used here, absorption encompasses all transitions out of the entrance channel.

2. J. S. Blair, Phys. Rev. 95, 1218 (1954), "sharp cutoff" model; J. A. McIntyre, K. H. Wang, and L. C. Becker, Phys. Rev. 117, 1337 (1960), modified "sharp cutoff" model.

3. J. Alster, and H. E. Conzett, Proceedings of the Second Conference on Reactions between Complex Nuclei, (John Wiley and Sons, Inc., New York, 1960); J. McIntyre, S. D. Baker, and K. H. Wang, Phys. Rev. 125, 584 (1962).

4. E. J. Williams, Rev. Mod. Phys. 17, 217 (1945).

5. L. R. B. Elton, Nucl. Phys. 23, 681 (1961)

6. J. Alster, Thesis, UCRL-9650 (1961) unpublished.

7. G. Igo, Phys. Rev. 115, 1665 (1959).

8. In analogy with the designation R as the optical potential (or nuclear density) radius, we use L as the value of ℓ for which $A_\ell = 1/2$, i.e., it is the absorption "radius" in ℓ-space and it appears as a parameter in the expressions for both A_ℓ and δ_ℓ.

9. Isoya et al., paper A9 this conference, report measurements of the inelastic contribution to $O^{16} + Ta^{181}$ elastic scattering data at 166-MeV. They suggest that heavy-ion elastic scattering data should be corrected, where necessary, before precise analyses of the data are justified.

FIGURE CAPTIONS

Figure 1. An optical potential $f(r)$ compared with $A_\ell(r)$. Since $r_0(A_1^{1/3} + A_2^{1/3})$ is the radius at which the optical potential is $\frac{V}{2}$, whereas $r_a(A_1^{1/3} + A_2^{1/3})$ is the radius at which absorption of incident particles is 75% (fractional absorption $= 1 - A_\ell^2 = 3/4$), an $r_0 = r_a$ would be purely fortuitous.

Figure 2(a). The radial wave function, $\frac{f_\ell(kr)}{kr}$, for no absorption (1) $U(r) = 0$ (2) $U(n) = V(r)$

2(b) The radial wave function for $V(r) = 0$ with absorption; i.e., equivalent to $U(r) = i\,W(r)$.

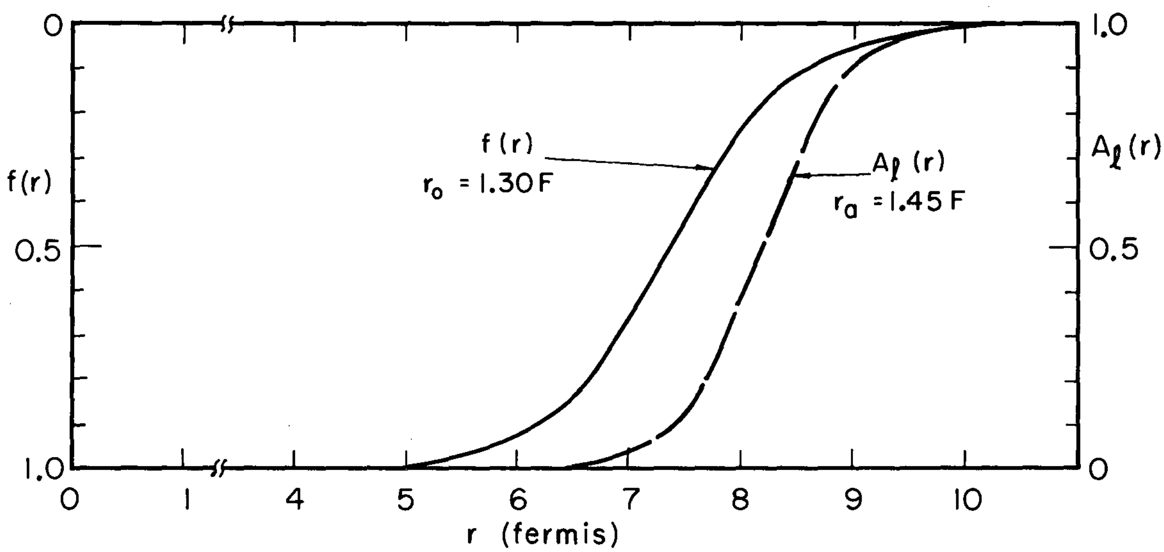
Figure 3. Parameterization of δ_ℓ.

Figure 4. Results of phase-shift analysis of C^{12} + Ta scattering at 124 MeV. The points are experimental data; the solid line represents the calculated results.

Figure 5. Curves of A_ℓ and δ_ℓ determined in fitting the data in Figure 4. The functional forms are

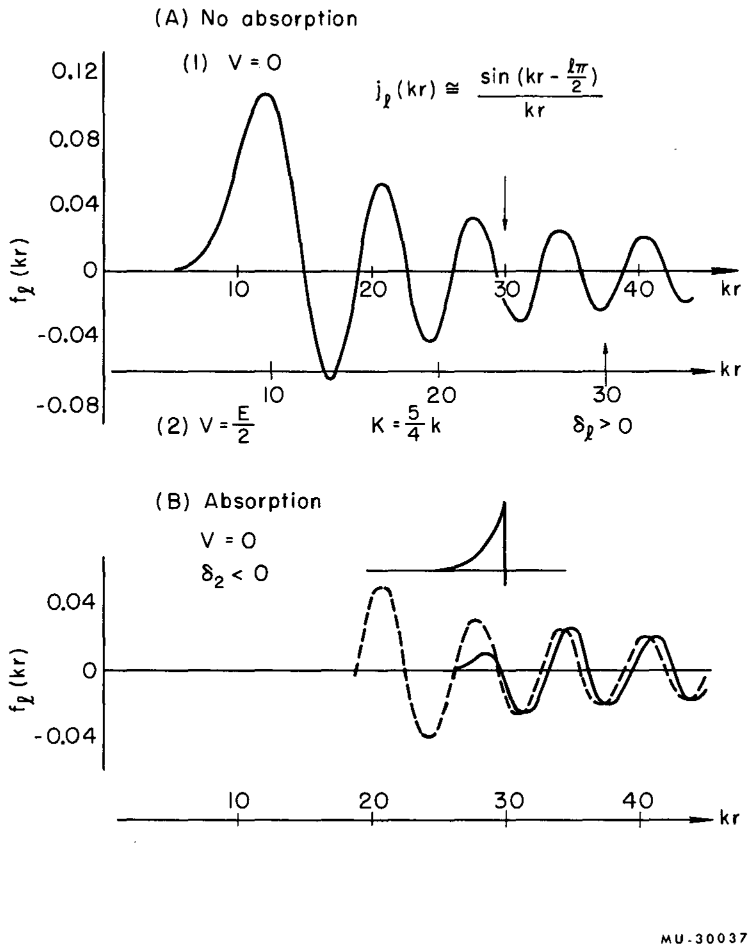
$$A_\ell = 1 - \left[1 + \exp\left(\frac{\ell - L}{\Delta\ell_A}\right)\right]^{-1}$$

$$\delta_\ell = \delta\left[2 - \exp\left(\frac{L - \ell}{\Delta\ell'_\delta}\right)\right]\exp\left(\frac{L - \ell}{\Delta\ell_\delta}\right)$$



MU-29413

Fig. 1



(A) No absorption
(I) V = 0
$j_\ell(kr) \cong \frac{\sin(kr - \frac{\ell\pi}{2})}{kr}$
0.12
0.08
0.04
0
-0.04
-0.08
f_ℓ (kr)
10
20
30
40
kr
(2) $V = \frac{E}{2}$
$K = \frac{5}{4}k$
$\delta_\ell > 0$
(B) Absorption
V = 0
$\delta_2 < 0$
MU-30037

Fig. 2

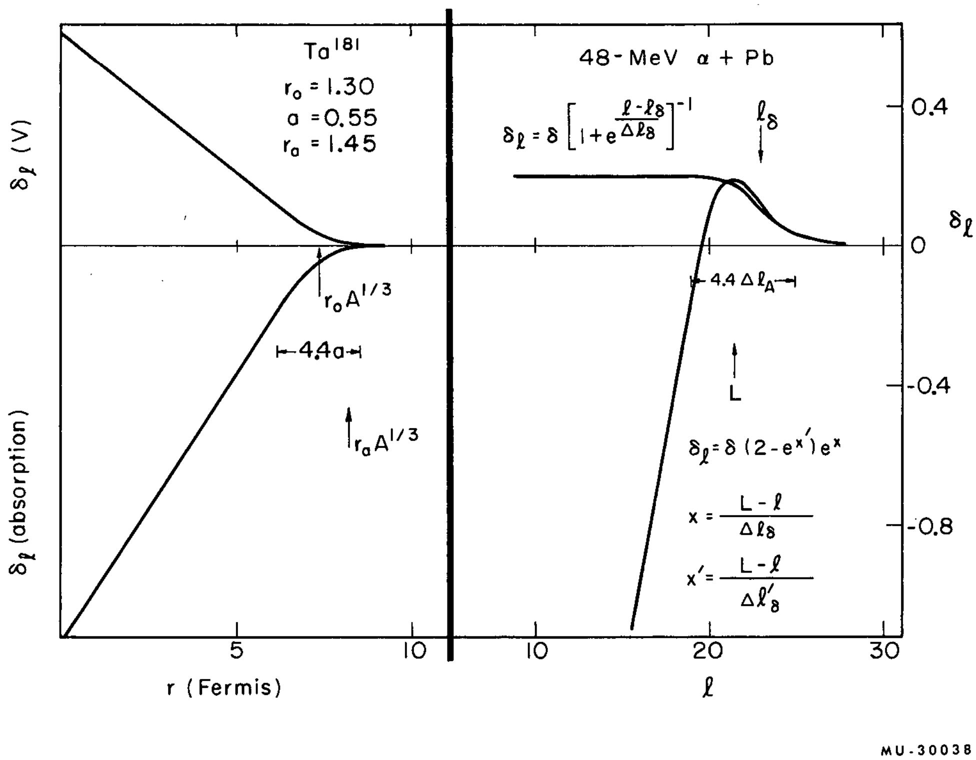

Ta181
r₀ = 1.30
a = 0.55
rₐ = 1.45
δℓ (V)
δℓ (absorption)
r₀A1/3
4.4a
rₐA1/3
r (Fermis)
5
10
48-MeV α + Pb
δℓ = δ [1 + e^((ℓ-ℓδ)/Δℓδ)]^-1
ℓδ
0.4
0
-0.4
-0.8
δℓ
4.4 ΔℓA
L
δℓ = δ (2-e^x′) e^x
x = (L - ℓ)/Δℓδ
x′ = (L - ℓ)/Δℓ′δ
10
20
30
ℓ
MU-30038

Fig. 3

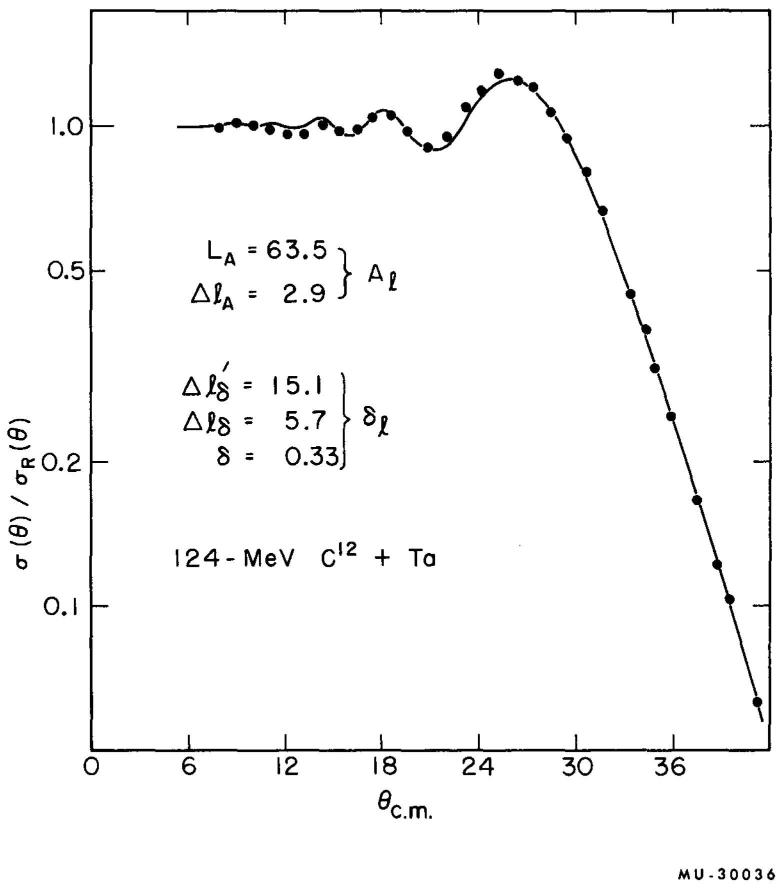

1.0
0.5
0.2
0.1
σ (θ) / σR (θ)
LA = 63.5
ΔℓA = 2.9
Aℓ
Δℓδ' = 15.1
Δℓδ = 5.7
δ = 0.33
δℓ
124-MeV C12 + Ta
0
6
12
18
24
30
36
θc.m.
MU-30036

Fig. 4

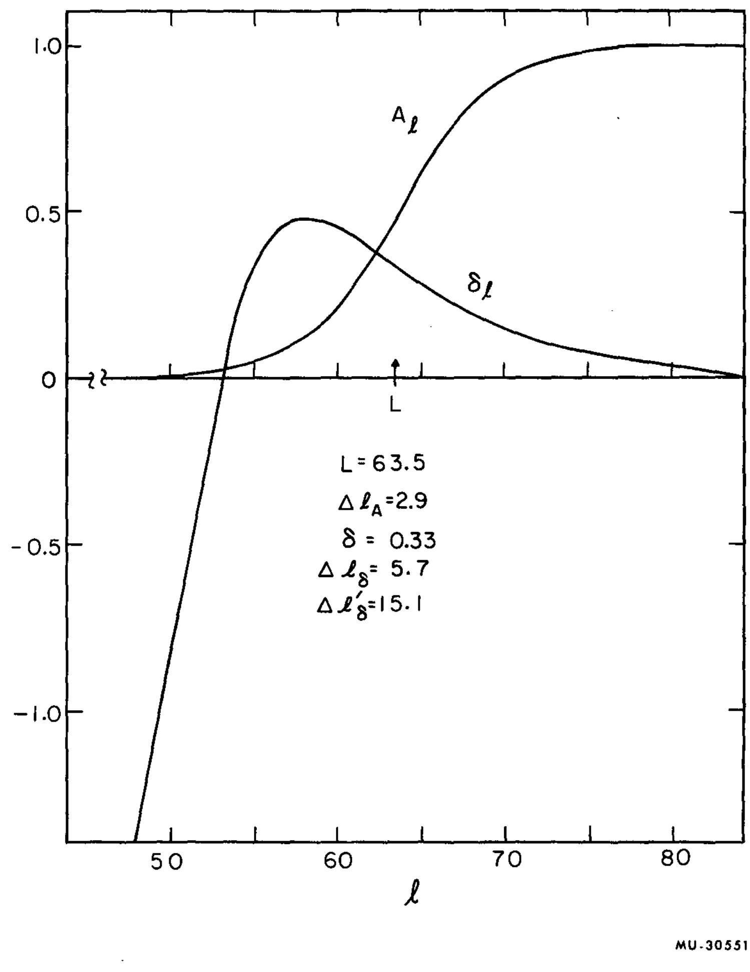

1.0
0.5
0
-0.5
-1.0
50
60
70
80
ℓ
A_ℓ
δ_ℓ
L
L = 63.5
$\Delta \ell_A$ = 2.9
δ = 0.33
$\Delta \ell_\delta$ = 5.7
$\Delta \ell'_\delta$ = 15.1
MU-30551

Fig. 5