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TRANSITION CURVES OF 330 MEV BREMSSTRAHLUNG

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January 24, 1952

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Radiation Laboratory, Department of Physics
University of California, Berkeley, California

January 24, 1952

ABSTRACT

Transition curves produced by 300 Mev bremsstrahlung from the Berkeley synchrotron have been measured in carbon, aluminum, copper and lead. The measurements have been made with a thing ionization chamber "immersed" in the materials under study. The results are in agreement with expected shower behavior. An important part of this investigation is to provide a suitable means for standardizing the synchrotron beam. This is accomplished in two ways.

1. The area of the shower curves gives a good measure of the beam energy.
2. Analysis of transition curves with thin converters permits separation of Compton and pair electron ionization. The pair ionization can be compared with theory and the primary energy deduced.

Agreement between these methods is very good. As a result of these measurements absolute cross section measurements in the synchrotron beam become possible.

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I INTRODUCTION

Since the discovery of electron-gamma-ray cascade showers in the cosmic radiation by Blackett and Occhialini[1] numerous investigations on the properties of these showers have been carried out. All such experiments fall essentially into three classes.

1. Experiments in which the progress of a shower is traced visually by counting the numbers of particles at a given depth between converters.[2]
2. Experiments in which the ionization is measured as a function of depth.[3]
3. Experiments studying the radial extent of the cascade radiation.

Cascade theory in its present form[4] gives expressions for the functions $P(E,t)$ and $\gamma(E,t)$ which give the probabilities of finding an electron or a gamma-ray, respectively, of energy E at a depth t.

Evaluation of these functions depends on initial "boundary conditions," i.e., on the primary energy spectrum of the initiating particles or particle. Experiments of type (1) as enumerated above give a direct count of the numbers of particles; however, the primary energy can only be inferred from the track count and the "critical energy"[5] of the electrons in the material studied.

Ionization experiments on the other hand constitute a more direct measurement of shower energy but their interpretation in terms of number of particles at a given depth is more tenuous. This is particularly true in the case of heavy elements for which the large scattering angles[5] of the low energy electrons make the ionization corresponding to a given particle uncertain. When such an experiment is carried out in the cosmic radiation the primary event cannot be identified. A more definite initial condition for a shower can be established by studying the transition from the equilibrium distribution in a light material to a heavy material, as has been done by Vernov and Vavilov[6] and others.

In this experiment the method of ionization measurement in an effectively infinite medium has been used to study the longitudinal, and also to some extent the transverse, behavior of a shower. A definite initial condition is established by using as the initial radiation the bremsstrahlung of the Berkeley synchrotron with 330 Mev quantum limit. Transition curves in matter, initiated by accelerator produced radiation, have been studied previously only at energies where cascade

effects are essentially negligible.[7] Showers initiated by synchrotron bremsstrahlung are not exactly equivalent to showers initiated by a single electron owing to the fact that (a) only 15 percent of the incident electron energy is lost by radiation in the synchrotron target, and (b) the target material, platinum, is not the same as the converter in which the shower propagation was studied (Pb, Cu, Al, and C). The synchrotron bremsstrahlung spectrum is, on the other hand, not the ideal thin target spectrum because of finite target thickness. This modifies somewhat the primary gamma-ray energy distribution by lessening the steepness of the spectrum near the quantum limit.

In addition to serving as a contribution to the experimental material bearing on cascade theory this work serves the practical purpose of providing a primary standard of beam energy of the synchrotron radiation. The magnitude of the ionization during the first part of the gamma-ray initiated shower can be interpreted simply in terms of the cross sections for pair production in the field of the nucleus, triplet production in the field of the electrons, and Compton electron production. Let these cross sections be $\phi_{pair}(k)$, $\phi_{trip}(k)$, and $\phi_C(k)$, respectively, and let the energy contained in the primary x-ray beam be given by

$$U = \int N_k k dk, \qquad (1)$$

where $N_k dk$ is the number of primary quanta in the energy interval between k and k + dk. The cross sections per atom will have the form

$$\phi_{pair} + \phi_{trip} = (Z^2 + Z)\phi_{pair}^{\circ} \text{ and } \phi_C = Z\phi_C^{\circ}, \qquad (2)$$

where the ϕ° are functions essentially independent of the atomic number Z. If N_e is the number of electrons per unit area in a convertor, then the ionization $I(N_e)$ produced in an ionization chamber of thickness g by secondaries from the converter of a primary beam of energy U is given by

$$I(N_e) = N_e g I_o \left\{ \int (kN_k)\phi_C^{\circ}(k)\frac{dk}{k} + 2(Z+1)\int (kN_k)\phi_{pair}^{\circ}(k)\frac{dk}{k} \right\}, \quad (3)$$

where I_o is the ionization per unit length in air of an electron near minimum ionization. The variation of ionization with energy, the small variation of ϕ° with atomic number, and the variation of pair production in the field of an electron, will be considered later in the more detailed calculations. The Z-dependence of the beginning of the shower curve, where cascade processes are as yet negligible, in combination with (1) and (3) gives, therefore, a measurement of U. Such an evaluation will depend on the theoretical expressions for kN_k and ϕ_{pair}°, and therefore for large values of Z will show up deviations from the (Born approximation) theory previously found by Lawson[8] and others.[9] This deviation has in fact been observed.

The Compton integral in Eq. (3) as it stands is divergent. Actually it has a definite finite value due to the self-absorption of the Compton electrons in the converter. In addition to this, the Compton integral would be modified by the fact that the low energy spectrum N_k is affected by the quartz vacuum chamber walls. Theoretical evaluation of the first term is therefore not very fruitful.

Two or more equations of the type (3) can be obtained by measuring the ionization behind converters of different Z, but of the same N_e preferably since any background ionization due to electron contamination of the beam will be a function of N_e essentially independent of Z, and can thus be considered with the first term. Corresponding to different methods of solving two simultaneous equations, there are two ways to evaluate the beam energy U. The first term representing the background and Compton contributions to the total ionization may be solved for and then subtracted from the total ionization to give the purely pair and triplet contribution. From this the beam energy can be determined since the second term of Eq. (3) can be evaluated theoretically with U as a parameter. A second method of solution is to solve the two equations directly for U by eliminating the first term between them. Columns I and II of Table III correspond to these two methods.

An independent way of arriving at the energy of the primary beam is from the area under the shower curve. Let I(t) be the observed ionization at a depth t of converter. If (-dE/dt) is the stopping power for electrons near minimum ionization in the material under study, then the total beam energy is closely given by

$$U = \int_0^\infty (-dE/dt)[I(t)/I_o g]dt, \qquad (4)$$

since all energy is lost eventually by ionization. It is of course assumed here that the converters and ion chamber used contain the total radial extent of the shower.

The degree of agreement of the energy values calculated from Eqs. (3) and (4) for various values of Z furnishes a valuable check on the internal consistency of the data as well as on the behavior of the pair cross section as a function of Z.

These two methods serve to determine the total energy in a 330 Mev x-ray beam or an effective number of quanta defined by

$$Q = U/k_{max} = \int_0^1 k N_k d(k/k_{max}) \qquad (5)$$

where k_{max} is the energy of the quantum limit. Q has the useful property that it is the constant of proportionality in the common approximation of the bremsstrahlung spectrum

$$N_k dk = Q \frac{dk}{k} . \qquad (6)$$

It is also advantageous to introduce Q rather than U into equation (3). This analysis of the shower curves thus provides a basis for the determination of absolute photo cross sections in the synchrotron beam. These methods are considerably simpler and more reliable than the use of a graphite lined chamber as carried out by Lax;[10] as proposed in his work both the Compton and pair effects must be calculated in detail, and also specific calculations of the behavior of the electrons as to scattering, etc., are necessary. Studying the Z-dependence of the ionization for converters of a small fraction of a radiation length thickness eliminates either complication.

The accuracy of the methods using Eq. (3) decreases with k_{max} since the fraction of ionization contributed by pairs becomes

small for k_{max} less than about 25 Mev. For k_{max} greater than about 100 Mev these methods are more convenient than that based upon Eq. 4 since the pair contribution is large and measurement of the whole shower requires more time and an inconveniently large volume of absorber.

A remark might be made here concerning the usual manner in which the intensity of the x-ray beam from a high energy electron accelerator is specified. It is customary to give a value of a certain number of roentgens per hour at a definite distance from the target. This quantity is measured usually with a small cylindrical ionization chamber of wall material of given composition. As has been pointed out by Lax[10] such a method gives easily interpretable results if the electron range is small compared with the wall thickness, which in turn is small compared to a radiation length. This condition can be met at small electron energies (up to 1 to 2 Mev), but is not applicable at higher energies. It would be more desirable to specify the energy flux since this would avoid all ambiguity.[11]

II APPARATUS

The progress of the shower produced by the synchrotron x-ray beam was observed by inserting a thin integrating ion chamber in plane parallel geomerty into an effectively infinite slab of the converter under study at a distance, t, from the front face of the converter. The material both from the front and rear of the chamber contributes greatly to the observed ionization, as was pointed out recently by Vernov and Vavilov.[6] A back scatterer is therefor necessary to account for the total energy in the shower.

In principle the slit into which the ion chamber is inserted would need to be infinitely thin in order that the chamber measure a quantity corresponding to the ionization at a given depth in the converter, and also that no energy be lost by side leakage across the edges of the chamber. The latter point is particularly important for heavy materials, since a shower initiated in a heavy element will contain a large number of electrons traveling at large angles to the initial beam direction. It was therefore found necessary to extrapolate the data to effectively zero chamber thickness.

The location of the apparatus with respect to the synchrotron is shown in Fig. 1. The beam of x-rays originated in the synchrotron target, which was a strip of platinum 0.020 inch thick located within the vacuum chamber. For external experimental use, the beam passes through the wall of the vacuum chamber made of fused quartz

approximately 3/8 inch thick. At the position of the collimator the beam has a diameter of 7/8 inch, measured at the half-intensity circle. The intensity at 2 inches from the beam axis is 2 percent of the intensity at the beam axis. The lead collimator block is 6 inches thick, 8 inches wide, 12 inches high and has approximately a 1/2 inch diameter tapered collimating hole through its center which is coaxial with the beam.

The shower intensity in the converter is measured by comparing the integrated ionization current of the ion chamber inserted into the thin slot in the converter with the reading of a monitor chamber located beyond the collimator (see Fig. 1).

At the position occupied by the monitor ionization chamber, the x-ray beam diameter is approximately 1 inch; the sensitive volume of the monitor chamber is 6 inches in diameter. Adequate beam monitoring intensity was obtained from a sensitive volume of 1 inch depth. Both the monitor and detector ionization chambers contain air and are vented to the atmosphere. Details of the monitor chamber are shown in Fig. 2. The cylindrical walls are constructed of Lucite. All foils are of 0.00035 inch aluminum. The active volume of the chamber is bounded by foils, in the planes bb' and dd', which were applied to the Lucite with polystyrene dope and stretched slightly. The collecting foil is in the plane cc' and is completely surrounded by the active volume. A grounded electrostatic shield surrounds the entire chamber and serves incidentally as a dust shield. It was also applied in the above manner to the

surfaces aa' and ee' of the outer rings which serve as high voltage insulation from the foils bb' and dd', but over surfaces ae and a'e' this shield was not allowed to touch the Lucite, thus minimizing leakage to gound. The diameter of the monitor chamber is small enough so that the foils remain tight when supported only by the Lucite. In passing through the monitor, the beam traverses 0.00175 inch of aluminum.

The soft x-ray background in the vicinity of the synchrotron is appreciable, with a harder component apparently originating at the injector. A wall of lead bricks shields both the ionization chambers from direct radiation coming from the region of the injector, while a 1/8 inch thick lead hood placed over the monitor, extending 30 inches from the monitor in both directions along the beam, shields it from the general soft x-ray background.

The construction of the detector chamber is basically the same as that of the monitor (see Fig. 2); however, no outer rings aa'b'b and dd'e'e are used. The converter plates (see Fig. 1) serve in lieu of electrostatic shield foils. The sensitive volume, 11 inches in diameter and 1/2 inch deep, is bounded by 0.00035 inch aluminum foils which were applied to the surfaces bb' and dd' with polystyrene dope and stretched slightly. The diameter of the chamber is large enough so that a mesh of 0.003 inch stainless steel wire, spaced two inches apart, is required to support the outer foils bb' and dd'.

The centers of the ionization chambers were positioned to

within 1/8 inch of the beam axis by means of a transit located coaxially with the beam. The line of sight of the transit was adjusted and checked photographically so that it was within 1/16 inch of the beam axis at all points where the apparatus was placed.

The collimator was provided with a peepsight insert which allowed it to be checked photographically for coaxial alignment with the beam.

The ionization current of each chamber was collected on a low leakage, polystyrene insulated capacitor, C (see Fig. 3). The voltage on the condenser was measured by a null method. The principle of the method is to maintain the condenser terminal connected to the ion collecting electrode at ground potential by sliding back the other terminal by a voltage exactly measurable by a potentiometer. This system also makes any correction for lead capacities, etc., unnecessary. The zero voltage condition of the collector can be checked by an electronic voltmeter which serves only as a null indicator.

The complete circuit is shown in Fig. 3. The electronic voltmeter is disconnected from the collector electrode during the run (in order to avoid possible grid rectification of high transient pulses, such as might be produced by voltages induced by microphonic pulses of the ion chamber in the noise field of the synchrotron). After the run the collector is slid back to zero and the slideback voltage is measured. The grid current of the voltmeter is 10^{-15} amp. so that the charge loss during measurement is negligible. Leakage

resistance between the high voltage foil and the collecting foil of the chamber was held above 10^{15} ohms without benefit of guard rings. Less than 1/2 percent of the minimum voltage utilized for an ionization measurement was attributable to leakage. The condenser used was a 0.01μf condenser and the voltage collected of the order of 1 volt.

The collecting voltage P applied to the detector chamber was varied, and it was found that with fields between 800 and 1500 volts per inch the ionization readings for a given set of conditions remained a constant within the experimental error. 1200 volts per inch collecting field was used in both monitor and detector chambers.

III EXPERIMENTAL PROCEDURE

The block of converter was formed of closely stacked plates of the element under study, located both ahead and behind the ion chamber. The x-ray beam, after passing through the monitor, fell normally on the stack and the shower electrons were observed by the detector ionization chamber situated in a slot of width a and depth t in the stack. In all cases, as shown in Fig. 4, only a few inches of backscattering converter were required behind the chamber to serve in place of the remainder of the ideal infinite block. The ionization due to the backscatterer, essentially at shower maximum, is expressed as a fraction of the total ionization by the following: Pb, 0.41; Cu, 0.20; Al, 0.09; C, 0.04. The curves in Fig. 4 are taken approximately at the depth t corresponding to the shower maximum in each element.

When there is no converter before the detector chamber, allowing the beam to fall through the chamber directly onto the backscatterer, no detectable ionization due to the presence of the backscatterer is observed. This indicates that shower electrons arising in the backscatterer from the pure x-ray beam are created and scattered at depths which are too great to allow them to be scattered through the ionization chamber in the backward direction in appreciable numbers.

The carbon and aluminum plates were ten inches square and the copper and lead plates were 12 inches by 14 inches. These sizes

were ample to contain the shower diameter[12] in the absence of an air gap. A very rough check of shower diameter was made using Eastman Type K x-ray film placed in the stack of plates at various points, and in all elements used no indication was found of the presence of radiation near the lateral borders of the stack of plates.

The elements studied were obtained from stock. The carbon was graphite, type C-18, of nominal density 1.58. The 2S aluminum used was found to contain less than 0.5 percent impurities. The copper and lead used were of commercial grade.

It was found to be unfeasible to construct a detector chamber of large enough diameter to intercept all the ionization from shower electrons in the slot, even though the showers were initiated by an x-ray beam less than 2 inches in diameter. Appreciable fractions of the electrons were backscattered into the slot at angles approaching 90 degrees with respect to the shower axis, especially in the case of a lead converter. It was thus necessary to observe the total ionization, at a given depth t, cooresponding to several slot widths from 1.5 inches to 0.75 inch, and then to extrapolate the ionization values to zero slot width. The active volume of the chamber was kept in contact with the backscatterer plates at all slot widths. This extrapolated value for the total ionization was assumed to be the same value which one would find in the ideal case discussed above. The curves in Figs. 5 and 6 show the magnitude of the necessary extrapolation for the lead and copper converters. In the case of aluminum, a tendency for the ionization values to approach a plateau

with decreasing slot width was observed, and the extrapolations were made accordingly. No extrapolation was found to be necessary in the case of a carbon converter. The complete shower curves and the results derived from them are based upon these extrapolated data.

The small energy discrepancies associated with the area of the lead curve (Section V) may be due to the difficulty of extrapolating in the correct manner. Also in lead some of the electrons have such low energies that the range in the chamber itself cannot be considered entirely negligible, nor is it necessarily true that all electrons are at minimum ionization. Small discrepancies in the energy balance of the heavy elements are thus expected. The construction of a chamber thinner than 1/2 inch was not considered compatible with reasonable accuracy, since flexibility of the window foils would result in untenable variations in the sensitive volume.

The x-ray beam was contaminated by electrons arising from the wall of the fused quartz vacuum chamber and from the air through which the beam passed before striking the apparatus. This background contamination was of small intensity, compared with the maximum shower intensity observed, principally due to the fact that the quartz vacuum chamber walls are at a point where the synchrotron magnetic field has a value of approximately one-half of the value at the orbit. The fraction of a radiation length of air traversed by the beam is very small. The detector chamber is largely shielded against this background by the converter plates, except at very small values of t, at which the contamination was adequately treated as a background. The intensity of this contamination background was proportional to the beam intensity

so that the monitor chamber, which keeps only a proportional check on the beam intensity, was not affected.

An ionization value observed under any conditions was taken to be the ratio of the detector condenser voltage to the corresponding monitor condenser voltage, both voltages resulting from charge collection during a single run of approximately 1 minute duration. This ratio was extrapolated in the manner discussed in the preceding paragraphs to arrive at the value for total ionization (Figs. 7 and 8). Two successive runs were made for every slot width $\underline{a}$ at each depth $\underline{t}$ in the converter. The internal consistency is about 1.5 percent for points within a factor of 50 in intensity of the shower peak; for points farther down the curve the consistency drops to 10 percent. Long time changes in background were checked and found to be negligible.

IV RESULTS ON SHOWER CURVES

The data for the normalized shower curves in Figs. 7 and 8 were obtained for the elements lead, copper, aluminum, and carbon. On repeating a run it was found that while the shape of the curve was unaltered, the values of the ordinates may have changed slightly. It is believed that these changes were due in part to differences in the electron contamination of the x-ray beam at the times of the experimental runs, since the geometry was not precisely reproduced for all runs.

The ordinates are proportional to the ratios of the voltages measured on the integrating condenser of the detector to the corresponding voltages measured for the monitor. The changes in absolute value discussed above required that the ordinates be normalized to some standard in order that the curves for the different elements might be compared. This was done by obtaining the ratio of the ordinate for the experimental point nearest the peak of a shower curve to that of the peak point for the copper curve for each of the elements. Each ratio was obtained once with the exception of the C/Cu ratio which was later checked, the two values agreeing to within 0.3 percent. This agreement is fortuitous since for successive runs for the same point the mean ordinate was reproducible to only about 1.5 percent on the average. In reducing the data, all points were normalized to correspond to the value 100 for the peak of the lead curve.

It is seen that the zero points for all the curves do not coincide. This is probably due to differences in electron contamination of the beam incident upon the converter. This means that the initial portions of the curves are slightly in error. This is not significant however, since the slopes are so great at this point that the maximum difference would correspond to a converter thickness of only about 0.1 g/cm^2.

After the synchrotron beam has passed through a sufficiently great thickness of converter, it should be attenuated along the remainder of its path in such a manner that the absorption coefficient has a constant and minimum value. The curves of Figs. 7 and 8 show that the required thicknesses have been approached for lead and copper. The converter thickness of aluminum or carbon is not great enough to give this condition, but it is large enough so that absorption coefficients determined from the curves can be used without serious error in the integration of the area of the shower curves as described later in Section VI.

The dashed lines in Fig. 7 are lines having slopes which correspond to the minimum absorption coefficient for gamma-rays given by Heitler.[13] The pair contribution to the lead absorption coefficient has been decreased by 10 percent to take account of Lawson's[8] values for the lead pair production cross section. These lines are plotted for comparison with the experimental data.

V SEPARATION OF PAIR AND COMPTON CONTRIBUTIONS IN THE IONIZATION

As has been shown in Section I, the Z-dependence of the ionization for small values of N_e can be used to separate the Compton and pair contributions to the total ionization. Let us rewrite Eq. (3), putting it into a form suited for easy numerical evaluation and taking into account several factors which affect the total ionization observed. In the notation of Section I

$$I(N_e) = N_e I_o \, g \left\{ \int (kN_k)\phi_C^o \frac{dk}{k} + \frac{2}{Z} \int \left[(\phi_{pair} + \phi_{trip}) \left(\frac{I(k/2)}{I_o} \right) (kN_k) \right] \frac{dk}{k} \right\} \tag{7}$$

where N_e/Z is the number of atoms per square centimeter in the converter. Rewriting,

$$I(N_e) = N_e I_o \, g \left\{ \int (kN_k)\phi_C^o \frac{dk}{k} + 2(Z+1) \int_o^{k_{max}} \left[\left(\frac{\phi_{pair}}{Z^2} \right) \left(\frac{Z + \frac{Z\phi_{trip}}{\phi_{pair}}}{Z+1} \right) \left(\frac{I(k/2)}{I_o} \right) (kN_k) \right] \frac{dk/k_{max}}{k/k_{max}} \right\}. \tag{8}$$

Upon substituting

$$\alpha = \frac{Z\,\phi_{trip}}{\phi_{pair}}$$

and

$$d\left(\log \frac{k}{k_{max}}\right) = \frac{dk/k_{max}}{k/k_{max}}$$

Eq. (8) becomes

$$I(N_e) = N_e I_\theta g\left\{ \int (kN_k)\phi_C^\circ \frac{dk}{k} + 2(Z+1)\int_{-\infty}^{\circ}\left[\left(\frac{\phi_{pair}}{Z^2}\right)\left(\frac{Z+\alpha}{Z+1}\right)\left(\frac{I(k/2)}{I_o}\right)(kN_k)\right] d\left(\ln\frac{k}{k_{max}}\right)\right\}. \quad (9)$$

The individual terms in the pair integral have been written to be slowly varying functions of Z and k. The term ϕ_{pair}/Z^2 has only a small Z-dependence, arising from the variation of the screening correction[17] as a function of Z. Fig. (13) is a plot of ϕ_{pair}. The calculations of this paper have taken account of the screening correction and also, in the case of lead, of the 10 percent correction to the cross section obtained by Lawson.[8]

According to the calculations of K. M. Watson[14] and of A. Borsellino,[15] α has a value from 0 to 0.9 in the range of interest, but the effective average value is 0.49, and hence the factor $(Z+\alpha)/(Z+1)$ is only a small correction. It has been assumed that only two of the three triplet members are observed since one of the electrons always has negligible energy.[14] The curve of Fig. 14 shows the k-dependence of α.

I(k/2) represents the ionization per unit length in air of a pair electron of energy (k/2), it being assumed that each pair electron receives half of the quantum energy in all cases. Since $I(k/2)/I_o$ is a correction factor close to unity, it is not necessary to consider the detailed division of energy among the pair members.

In the calculations any variation with incident particle energy of the average energy required to produce an ion pair has been neglected. Therefore $I(k/2)/I_o$ has been taken to be equivalent to $\left(\frac{dE}{dt}\right)_E = k/2 \Big/ \left(\frac{dE}{dt}\right)_{\text{min.ion.}}$. The latter quantity can be calculated from the Bethe-Bloch expression for the electron energy loss[16] or can be obtained with sufficient accuracy from Fig. 1 of Ref. 5. The inclusion of this factor increases the value of the integral about 20 percent for 330 Mev bremsstrahlung. The bremsstrahlung spectrum kN_k corrected for the finite target thickness has been kindly calculated for a single electron of 330 Mev energy incident upon the synchrotron target by W. Aron; the spectrum is shown in Fig. 9. The integral in the second term of Eq. (9) can thus be evaluated numerically as a ratio to the number of "effective quanta," Q, in the beam. Let us designate this ratio by $R(Z)$, i.e., let

$$R(Z) = \frac{\int_{-\infty}^{\circ} \left\{ \left(\frac{\phi_{pair}}{Z^2} \right) \left(\frac{Z + \alpha}{Z + 1} \right) \frac{I(k/2)}{I_o} (kN_k) \right\} d\left(\ln \frac{k}{k_{max}} \right)}{\int_{o}^{1} (kN_k) d(k/k_{max})} \qquad (10)$$

This quantity represents the cross section for pair production (normalized to $Z = 1$) averaged over the primary bremsstrahlung spectrum and weighted according to the variation with electron energy of the detection efficiency of the particular method used to detect the electrons. In evaluating $R(Z)$, for kN_k the spectrum for a single incident electron may be used since the constant of proportionality

between the total beam intensity and the intensity due to a single electron is contained in both numerator and denominator. $R(Z)$ is tabulated in Table I. Eq. (7) thus finally becomes

$$I(N_e) = N_e I_o \, g\left\{ \int (kN_k)\phi_C^o \frac{dk}{k} + 2(Z+1)Q\, R\,(Z)\right\} \qquad (11)$$

Figure 10 shows the initial parts of the shower curves (taken without backscatterer) plotted as a function of the number of electrons per unit area of converter. In addition to the experimental curves a curve is plotted labeled "Compton and Background." This curve is obtained by subtracting the proper fraction, computed from Eq. (11) as described in Section I, of the difference between the aluminum and the carbon curve from the carbon curve; this fraction is unity if α in Eq. (6) is unity and if ϕ_{pair}/Z^2 is independent of Z. It is not feasible here to compare the Compton curve with theory, since it is difficult to separate from the background, and also since the Compton contribution cannot be evaluated without consideration of the converter self-absorption. After subtraction of the "Compton and Background" curve, the curves presumably representing the pair ionization are obtained (Fig. 11).

If cascade effects were negligible, the curves of Fig. 11 should be straight lines. This is not the case and the observed curvature is a measure of the loss of the pair electrons by radiation and of primary absorption. For thicknesses of converter which are a small fraction of a radiation length, the slope will decay with distance

esentially exponentially with a characteristic decay distance of the order of a radiation length. The slopes of the curves of Fig. 11 conform to this interpretation; points computed under the assumption that the slope falls exponentially from its observed initial value are plotted on the curves on Fig. 11. The initial slopes thus represent the pair contribution to the total ionization, and according to Eq. (6) should be a linear function of $(Z + 1)$, after correction for the variation of $R(Z)$ with Z has been made. This correction has been applied and the resultant slopes are plotted against $(Z + 1)$ in Fig. 12; it is seen that the agreement is good. As has been mentioned above, in agreement with the deviations observed by Lawson[8] for the pair cross sections in heavy elements for 88 Mev radiation, and by Walker[9] at 17.5 Mev, a 10 percent correction has been applied to $R(Z)$ in the case of lead. The point is also plotted without the correction, with significantly poorer agreement.

VI STANDARDIZATION OF THE SYNCHROTRON BEAM

The separation of the pair and Compton contributions described above makes it possible to obtain a direct measurement of the total beam energy, using the theoretically calculated values $R(Z)$ given in Table I. It will be recalled that $(Z^2 + Z)R(Z)$ is the cross section for the production of pair and "triplet" electrons when averaged over the primary bremsstrahlung spectrum. Column I of Table III lists the values calculated for the energy flux from the separated pair produced ionization in the limit of zero converter thickness.

It is perhaps more convenient to standardize the beam by a "two element" method based on the second of the two ways discussed in Section I of solving the Eqs. (11). Consider the measurement of the ionization I_1 and I_2 behind two converters of atomic numbers Z_1 and Z_2 of equal numbers (N_e) of electrons per square centimeter. N_e must not be great enough to allow cascade effects to enter appreciably. It follows from Eq. (11) that the primary energy and the number of quanta (see Eq. (5)) is given by

$$Q = \frac{U}{k_{max}} = \frac{1}{2N_e g I_o} \frac{I_2(N_e) - I_1(N_e)}{(Z_2 + 1)R(Z_2) - (Z_1 + 1)R(Z_1)} . \qquad (12)$$

This equation assumes that the converter thickness (N_e) has been chosen to be small enough that no correction is necessary for the radiation effect noticeable in Fig. 11. Table II gives a typical tabulation of the calculated energy as a function of converter thickness,

which indicates the effect of losses by radiation and primary absorption. Table II was computed directly from the experimental results plotted in Fig. 11. The data are smoothed, however, by assuming an exponential decay of the slope. This table can be used for a decision on the maximum converter thickness permissible and for a derivation of the corrections necessary for extrapolation to zero thickness. For the greatest accuracy several thicknesses of each element should be used and the ratio $I(N_e)/N_e$ obtained for each element by extrapolating to zero thickness should be substituted into Eq. (12). As a further alternate means of beam standardization the beam energy has been evaluated using the areas under the shower curves (Figs. 7 and 8) and Eq. (4). The areas were evaluated numerically except for the exponential tail of the curves; the tail area was evaluated analytically using the asymptotic absorption coefficients. This latter method is considered to be less accurate, for reasons discussed above. In particular, for heavy elements appreciable deviations are expected.

Table III shows the beam energy evaluated for typical operation of the Berkeley synchrotron. The value I_o used is 62.5 ion pairs/cm. This value corresponds to the theoretical energy loss for fast electrons divided by 32 volts.

The values of beam energy as entered in Table III are not of equal reliability. In the measurement by shower curve area (column III) the values observed in the heavy elements are of lesser reliability owing to the large extrapolation to zero chamber thickness

and the low energy and range of the ionizing electrons. The two element method (column II) will lead to the best results for pairs of most dissimilar Z, since errors in the background and Compton subtraction are than less significant. It is gratifying to note that the low Z shower data and large differential Z "converter pair" data are in good agreement.

The method of standardization based on the pair cross sections (using Eq. (8)) can be executed quite rapidly and is being used for the calibration of chambers serving as secondary standards.

This work, including the method used, was proposed by Professor E. M. McMillan. He has guided the work throughout its progress and in particular is responsible for the method of separating pair and Compton effects. The calculation of the primary spectrum, pair cross sections, and the stopping powers used in these calculations was made by W. Aron.

The crew of the Berkeley synchrotron contributed greatly in carrying out this experiment by their efficient operation of the machine during the bombardments.

TABLE I

The cross section for pair production, $R(Z)$, in cm^2.

k_{max} \ Z	6	13	29	82
330 Mev	2.54×10^{-26}	2.55×10^{-26}	2.50×10^{-26}	2.20×10^{-26}
242 Mev[18]			2.16	1.90
161 Mev[18]			1.76	1.59
46 Mev[19]	0.657	0.673	0.666	0.607

TABLE II

Normalized values of the energy flux in the collimated beam measured by the "two-element" method, in units of 10^8 Mev/sec.[a]

Converter pair / N_e/N_o	Pb-Cu	Pb-Al	Pb-C	Cu-Al	Cu-C	Al-C
0.0	102.3	102.2	101.0	102.0	98.5	90.7
0.025	101.0	101.0	99.9	101.1	97.8	90.5
0.05	99.7	99.8	98.8	100.1	97.0	90.2
0.075	98.5	98.6	97.7	99.1	96.3	90.0
0.1	97.2	97.5	96.6	98.2	95.5	89.6
0.2	92.3	92.9	92.4	94.4	92.6	88.6
0.3	87.7	88.5	88.4	90.8	89.8	87.6
0.4	83.2	84.3	84.5	87.2	87.0	86.5
0.5	79.0	80.3	80.8	83.8	84.3	85.5

a Various pairs of converters are used. The energy flow is tabulated against the thickness of the converter measured in terms of electrons/cm.^{2} N_o = Avogadro's number. The variability of the entries indicates the effect of primary absorption and of radiation. This table serves as a guide to select proper converter thicknesses and also a means to convert to zero converter thickness. This table is normalized to a reference intensity on the beam axis of about 1400 R/hr behind 1/8 in. of lead 1 meter from the target. This intensity was estimated from the reading of a large Zeus meter using the effective Zeus meter area and the measured value of 1.6 for the ratio of central intensity to average intensity in the beam emerging from a 1 inch collimator. This reference performance corresponds to about one-fourth of the usual operating intensity.

TABLE III

Beam energy flow through a one inch collimator distant 55 inches from the synchrotron target in units of 10^8 Mev/sec., or $10^8/330$ quanta/sec., as computed by the various methods outlined. This tabulation is normalized to an axial beam intensity of 1400 R/hr at 1 meter from the synchrotron target, which is the same reference intensity used in Table II.

Method / Material	I	II	III
	Pair production using Fig. 11	Pair production using two converters of equal electron surface density in the limit of zero converter thickness (see Table II)	Shower curve area
C	90.8		100.4
Al	90.7		103.0
Cu	96.7		94.6
Pb	100.0		84.6
Pb-Cu		102.3	
Pb-Al		102.2	
Pb-C		101.0	
Cu-Al		102.0	
Cu-C		98.5	
Al-C		90.7	

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FIGURE CAPTIONS

Fig. 1 Schematic arrangement of the apparatus with respect to the synchrotron. The collimator, absorbers, and ionization chambers are coaxial with the beam. The lead wall shields the apparatus from x-rays originating at the injector. The slot width, a, is variable.

Fig. 2 Cross section (side view) of the monitor ionization chamber.

Fig. 3 Schematic diagram of the ionization chamber and associated circuits. The voltage, V, is measured with a potentiometer and decade voltage divider.

Fig. 4 The observed total ionization, in arbitrary units, plotted against the thickness of the backscatterer. The thickness of the converter corresponds approximately to the maximum of the shower curve for that element (see Figs. 7 and 8). The cross on each curve indicates, for that element, the thickness of backscatterer used to obtain the data for the corresponding shower curve in Fig. 7 or 8. Converter thickness ahead of the chamber is indicated on each curve.

Fig. 5 Extrapolation curves for lead. The total ionization, in arbitrary units, measured by the detector chamber is plotted against the distance, a, in inches, between the converter and the backscatterer, for the converter thicknesses (g/cm^2) noted at the right of the curves. The curve for each converter thickness is extrapolated to $a = 0$, and these limiting

Fig. 5 values are assumed to give the total ionization within an infinitesimally thin slot in an infinite block of converter. The ordinates obtained by extrapolation are plotted in Figs. 7 and 8.

Fig. 6 Extrapolation curves for copper. The total ionization, in arbitrary units, measured by the detector chamber is plotted against the distance, a, in inches, between the converter and the backscatterer, for the converter thicknesses (g/cm^2) noted at the right of the curves. The curve for each converter thickness is extrapolated to $a = 0$, and these limiting values are assumed to give the total ionization within an infinitesimally thin slot in an infinite block of converter. The ordinates obtained by extrapolation are plotted in Figs. 7 and 8.

Fig. 7 The total ionization, in arbitrary units, in an effectively infinite block of converter, plotted against the depth (g/cm^2) in the converter. The dashed lines have slopes calculated from the minimum absorption coefficient given by Heitler (reference 13), and are plotted for comparison with the data. The discrepancies in the slopes are considered to be within the experimental errors. The ordinates are extrapolated values obtained as in Figs. 5 and 6.

Fig. 8 The total ionization, in arbitrary units, in an effectively infinite block of converter, plotted against the depth (radiation lengths) in the converter. The ordinates are extrapolated values obtained as in Figs. 5 and 6.

Fig. 9 The theoretical bremsstrahlung energy distribution kN_k for the Berkeley synchrotron, calculated for a single electron of energy 330 Mev incident normally upon a platinum target 0.020 inch in thickness. The area under this curve divided by k_{max} is the target x-ray efficiency.

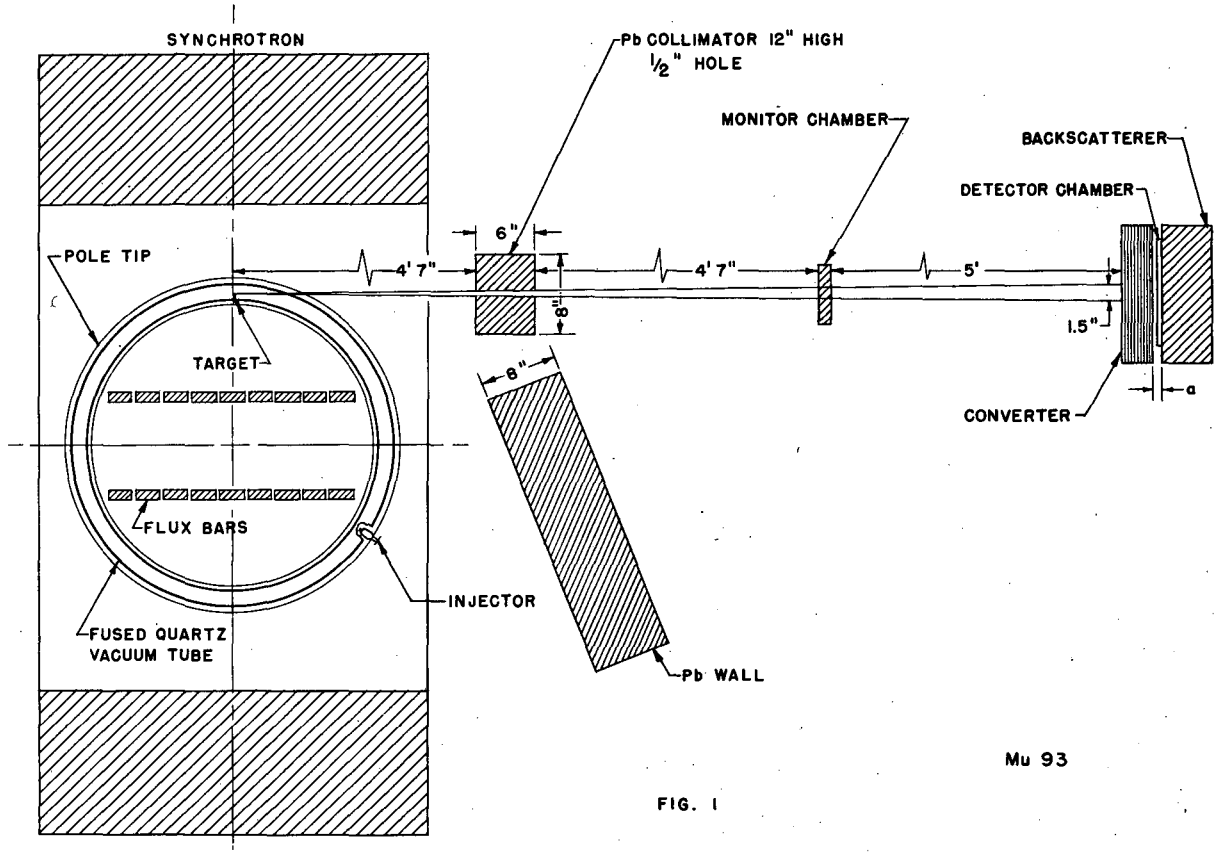
Fig. 10 The total ionization, in arbitrary units, observed by a thin ionization chamber plotted against an abscissa proportional to the number of electrons per square centimeter of the converter. No backscatterer was present and no extrapolation to zero ionization chamber thickness was found necessary. Contributions to the total ionization due to Compton recoils and background are given by the lowest curve which is independent of Z.

Fig. 11 The pair ionization, in arbitrary units, plotted against an abscissa proportional to the number of electrons per square centimeter of the converter. These curves are obtained from the curves of Fig. 10 by subtracting the Compton plus background curve from the total ionization curves.

Fig. 12 The initial slopes of the curves in Fig. 11, divided by the function R (Z) (as tabulated in Table I), plotted against $(Z + 1)$. A point (x) is also shown indicating the deviation if the Lawson correction (reference 8) is not used in computing R(Z). The units are arbitrary.

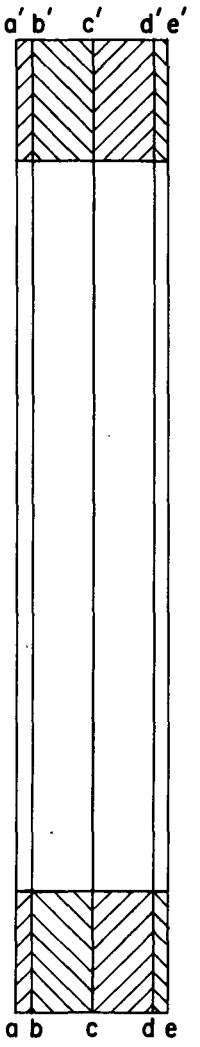
Fig. 13 ϕ_{pair} (k). The unlabeled curve includes no screening correction. The screening correction has been made in the other curves. See Ref. 13 for the definition of $\bar{\phi}$.

Fig. 14 α(k). Taken from Reference 15.


SYNCHROTRON
Pb COLLIMATOR 12" HIGH
1/2" HOLE
MONITOR CHAMBER
BACKSCATTERER
DETECTOR CHAMBER
POLE TIP
6"
4' 7"
4' 7"
5'
8"
1.5"
TARGET
8"
FLUX BARS
INJECTOR
FUSED QUARTZ
VACUUM TUBE
Pb WALL
CONVERTER
a

Mu 93

FIG. 1



CROSS SECTION OF IONIZATION CHAMBER

FIG. 2

Mu·94

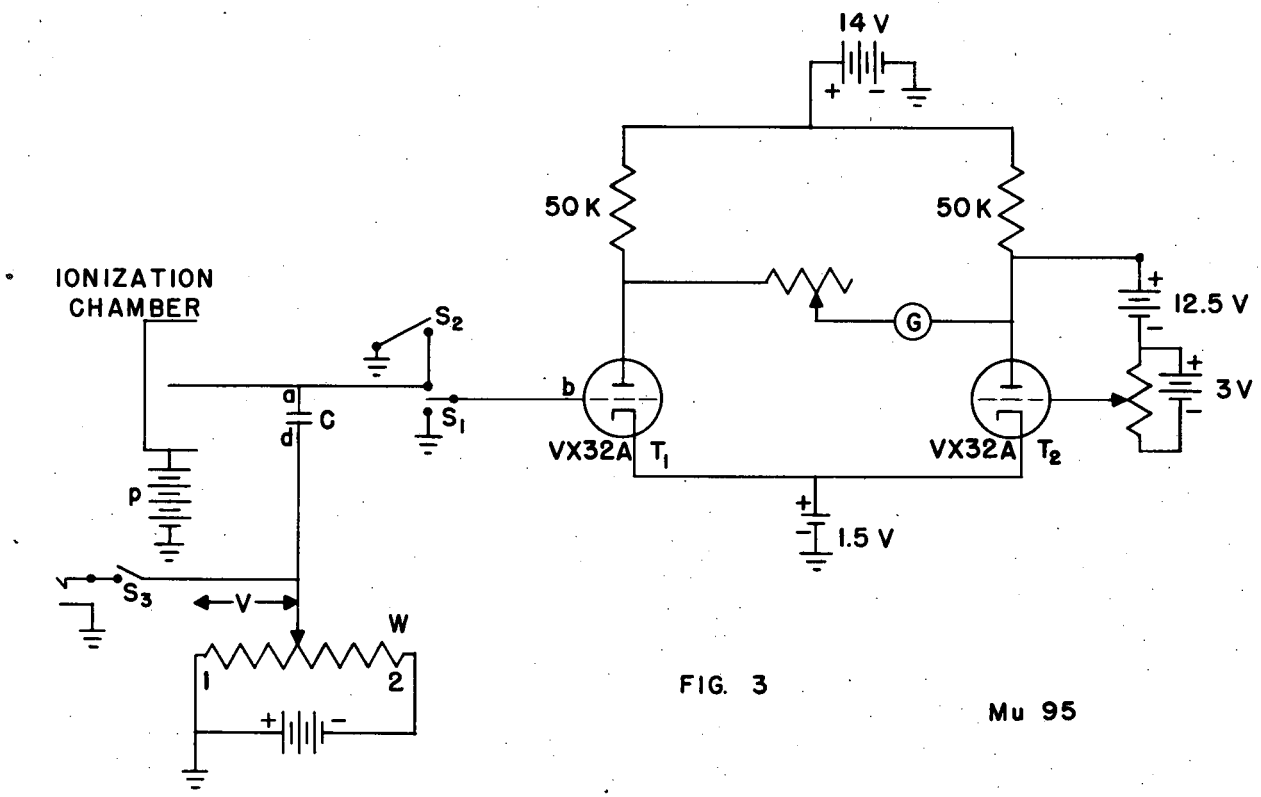

14 V
50 K
50 K
IONIZATION
CHAMBER
S2
G
12.5 V
3 V
a
C
d
S1
b
VX32A T1
VX32A T2
p
1.5 V
S3
V
W
1
2

FIG. 3

Mu 95

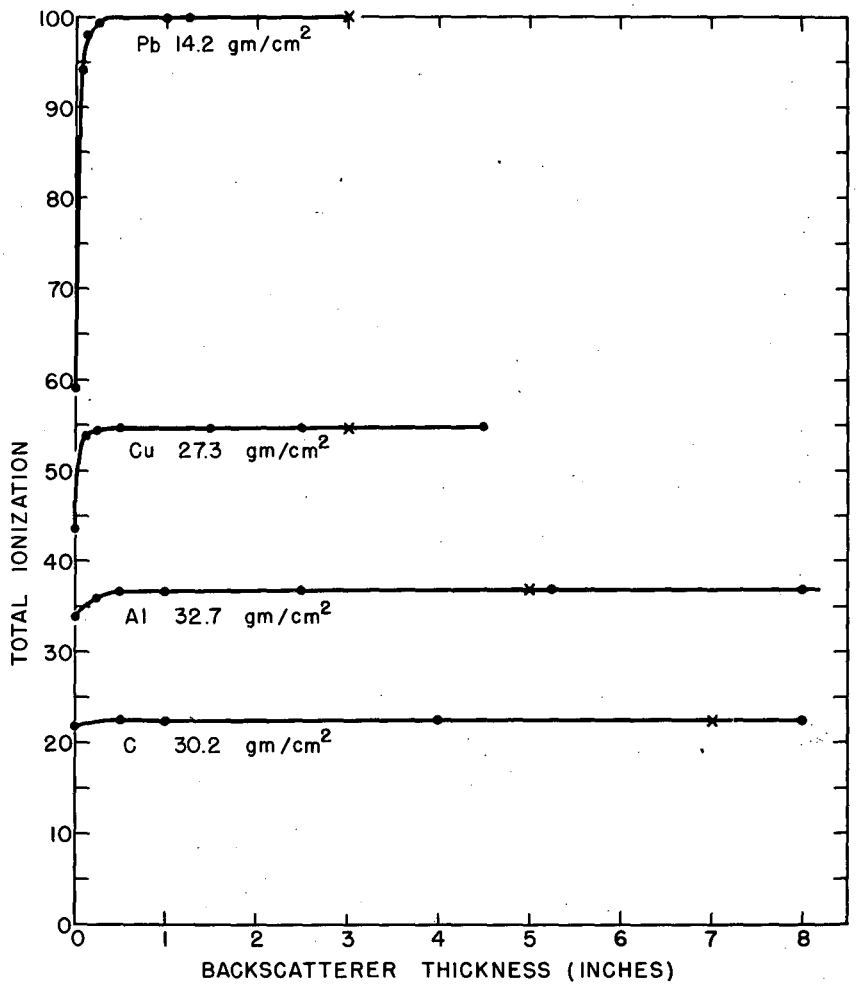

Pb 14.2 gm/cm²
Cu 27.3 gm/cm²
Al 32.7 gm/cm²
C 30.2 gm/cm²
TOTAL IONIZATION
BACKSCATTERER THICKNESS (INCHES)
0
10
20
30
40
50
60
70
80
90
100
0
1
2
3
4
5
6
7
8

FIG. 4

Mu 96

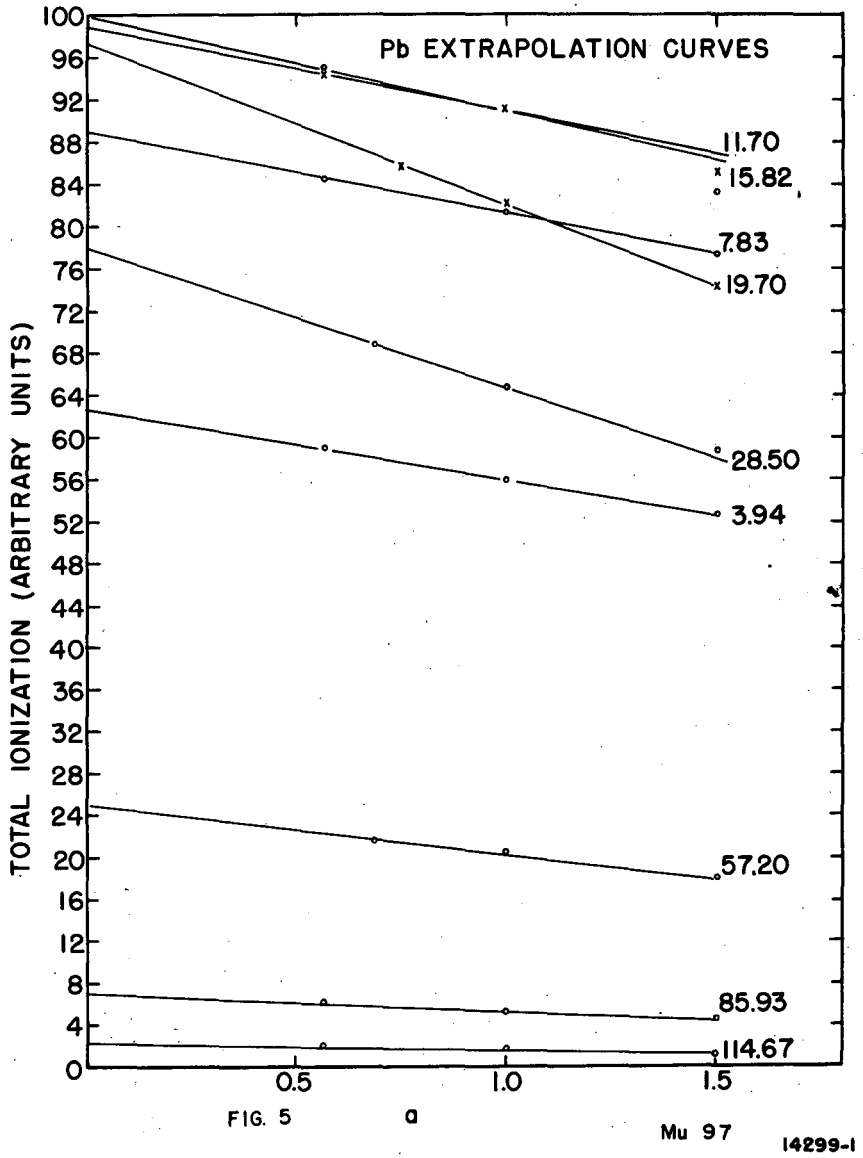

Pb EXTRAPOLATION CURVES
TOTAL IONIZATION (ARBITRARY UNITS)
100
96
92
88
84
80
76
72
68
64
60
56
52
48
44
40
36
32
28
24
20
16
12
8
4
0
0.5
1.0
1.5
11.70
15.82
7.83
19.70
28.50
3.94
57.20
85.93
114.67
Mu 97
14299-1

FIG. 5 a

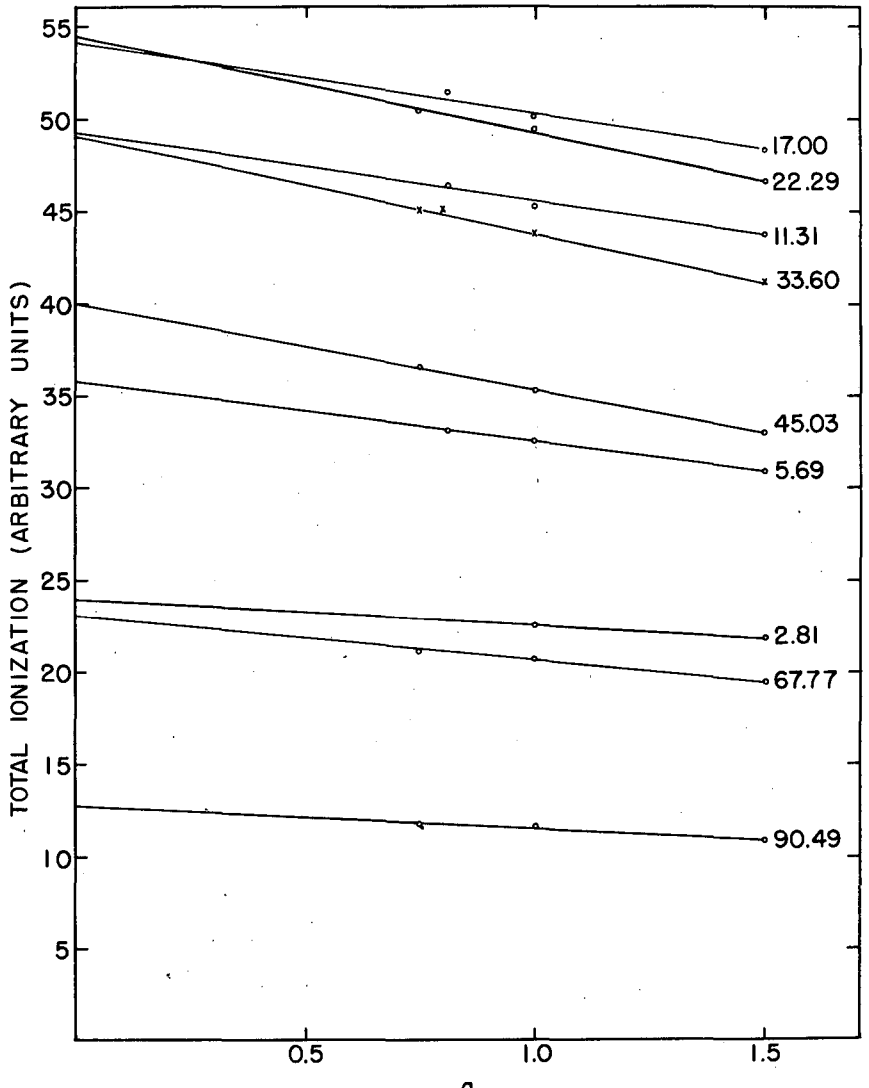

TOTAL IONIZATION (ARBITRARY UNITS)
55
50
45
40
35
30
25
20
15
10
5
0.5
1.0
1.5
a
17.00
22.29
11.31
33.60
45.03
5.69
2.81
67.77
90.49

FIG. 6 Mu 98

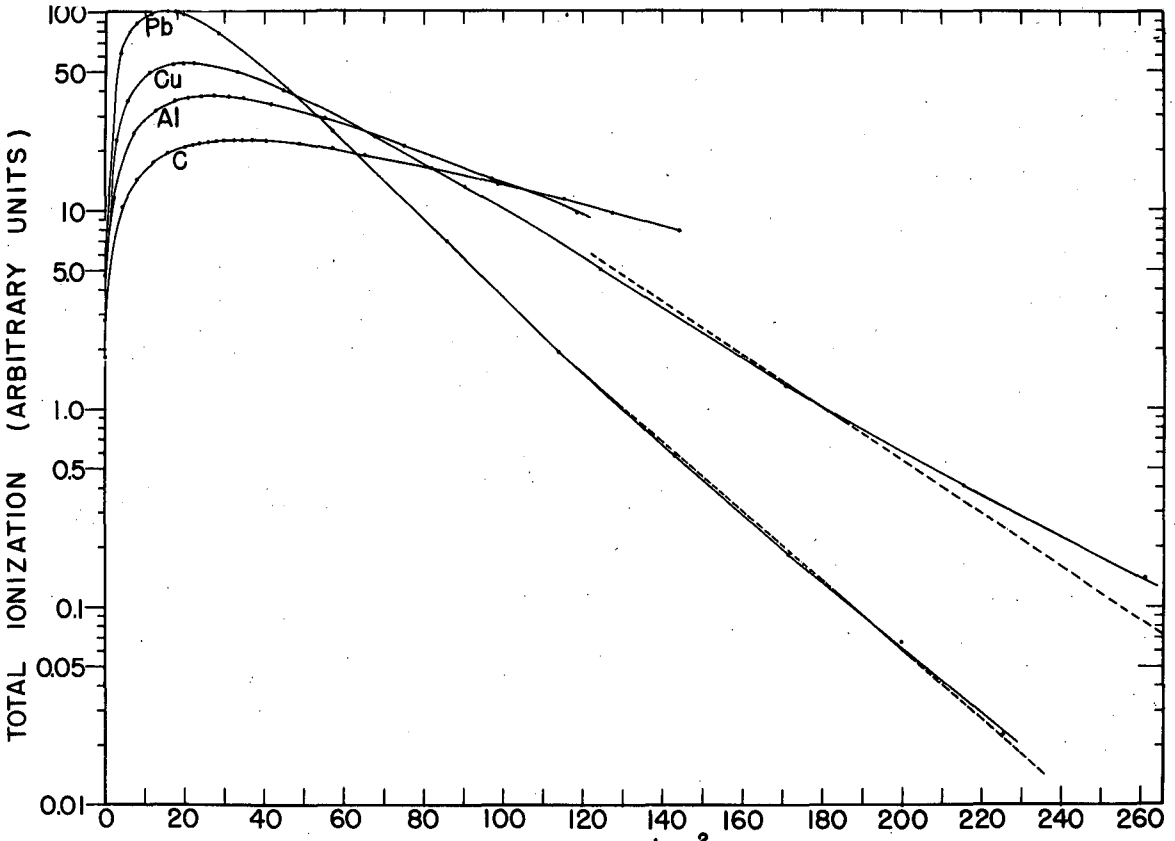

100
50
10
5.0
1.0
0.5
0.1
0.05
0.01
Pb
Cu
Al
C
TOTAL IONIZATION (ARBITRARY UNITS)
0
20
40
60
80
100
120
140
160
180
200
220
240
260
gm / cm²
Mu 99

FIG. 7

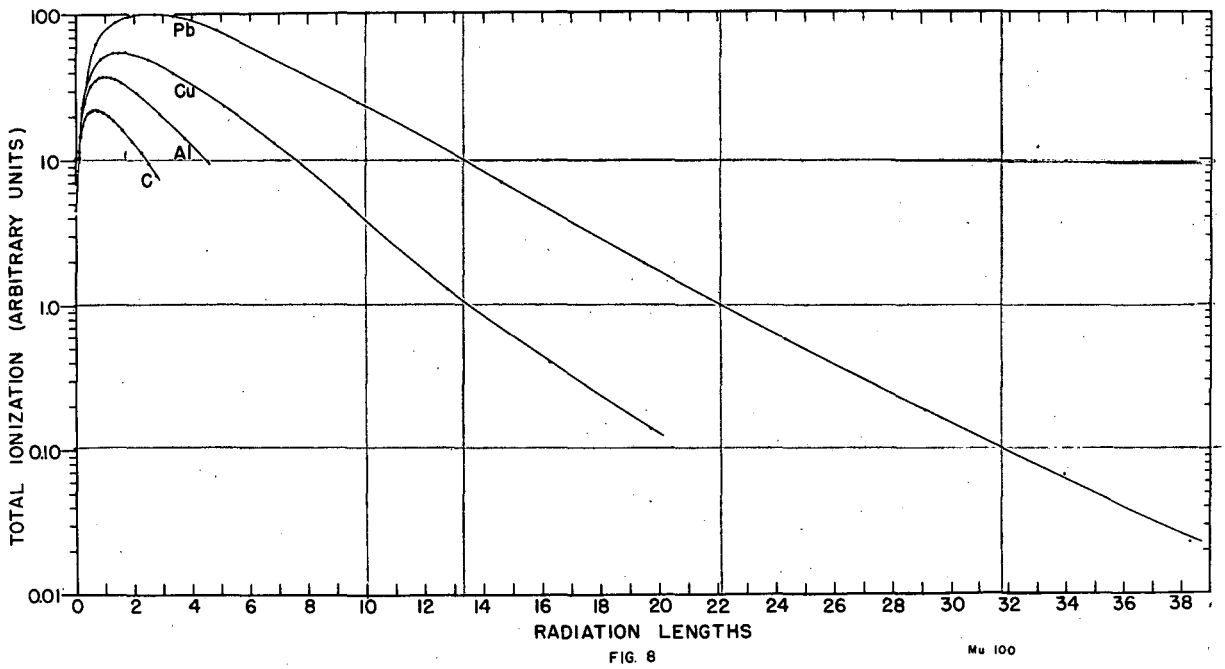


FIG. 8

Mu 100

14587

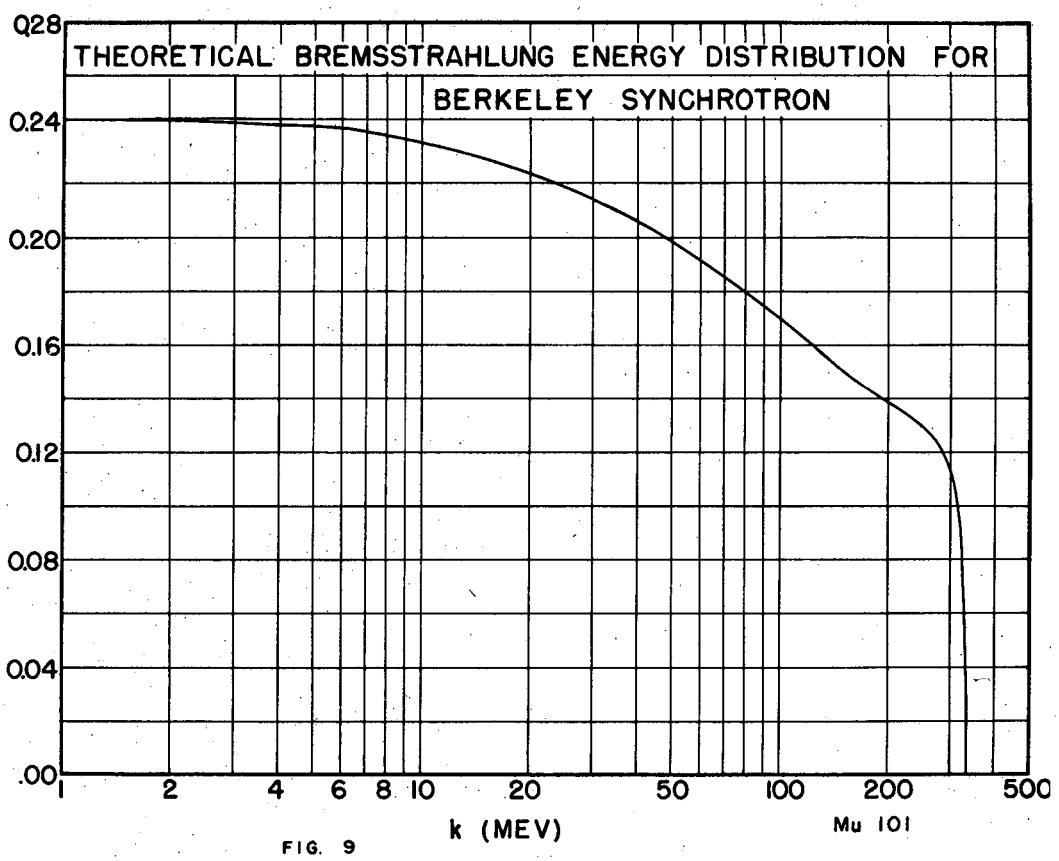

THEORETICAL BREMSSTRAHLUNG ENERGY DISTRIBUTION FOR
BERKELEY SYNCHROTRON
Q28
0.24
0.20
0.16
0.12
0.08
0.04
.00
1
2
4
6
8
10
20
50
100
200
500
k (MEV)
Mu 101

FIG. 9

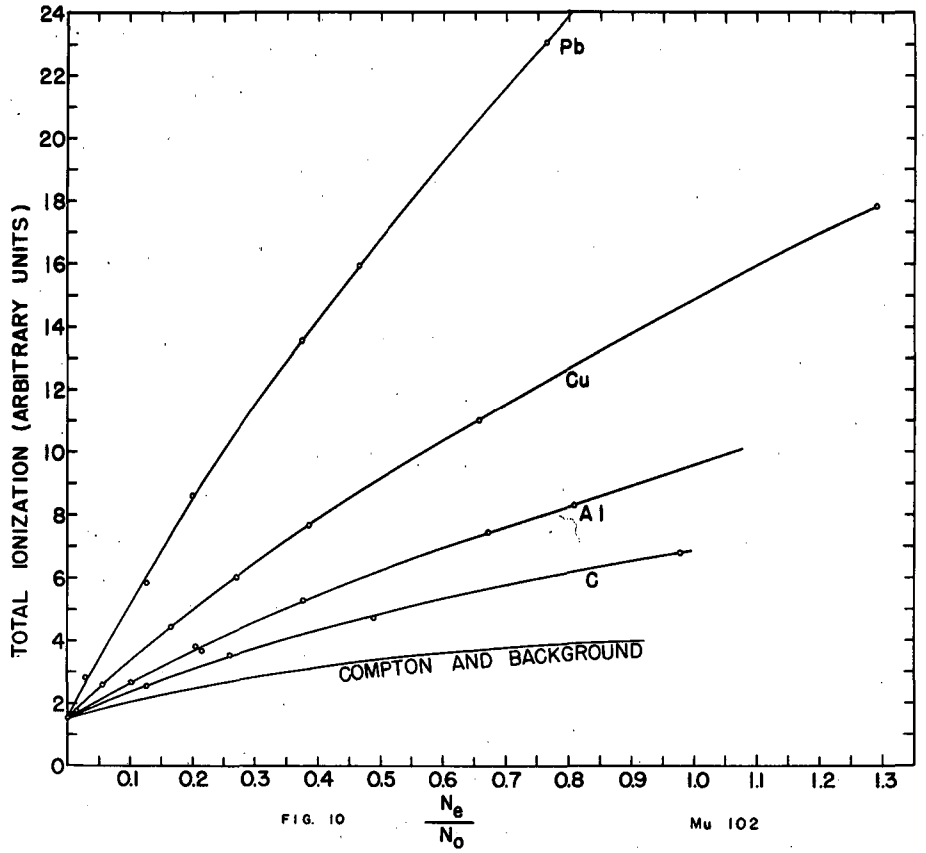

TOTAL IONIZATION (ARBITRARY UNITS)
24
22
20
18
16
14
12
10
8
6
4
2
0
Pb
Cu
Al
C
COMPTON AND BACKGROUND
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2
1.3
$\frac{N_e}{N_0}$

FIG. 10

Mu 102

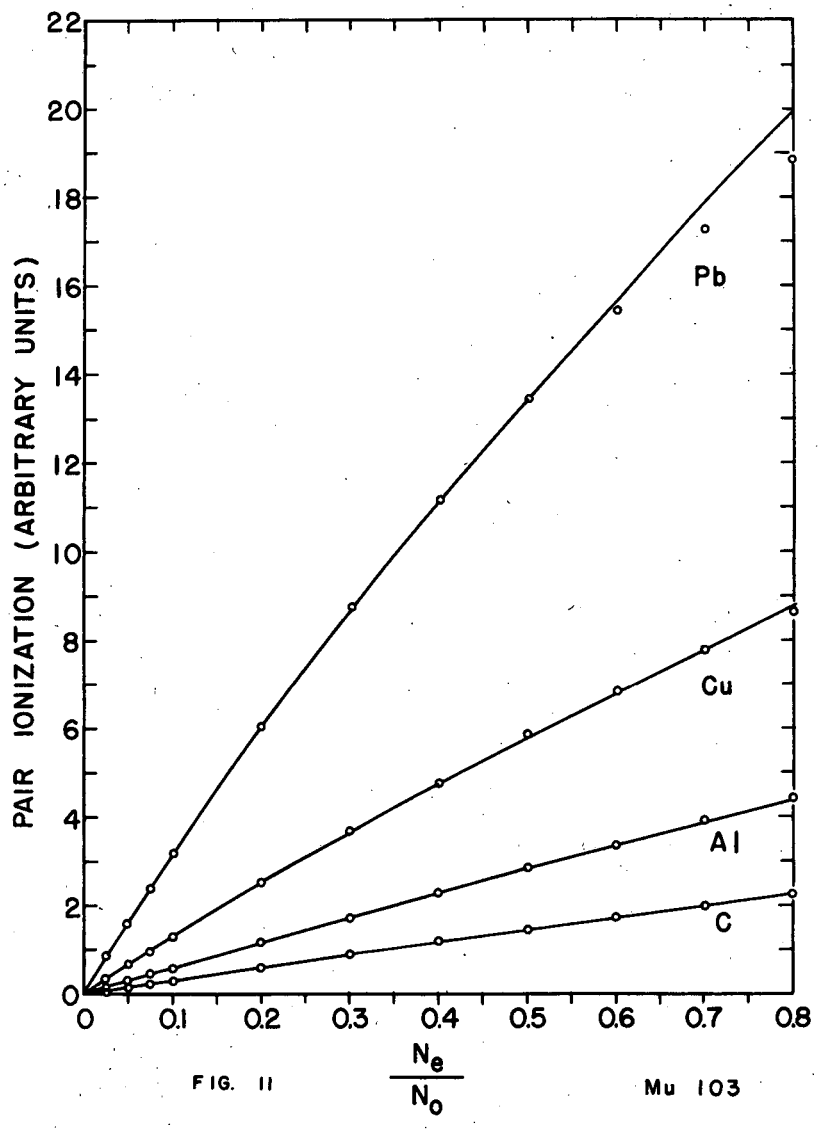


FIG. 11

Mu 103

14667-1

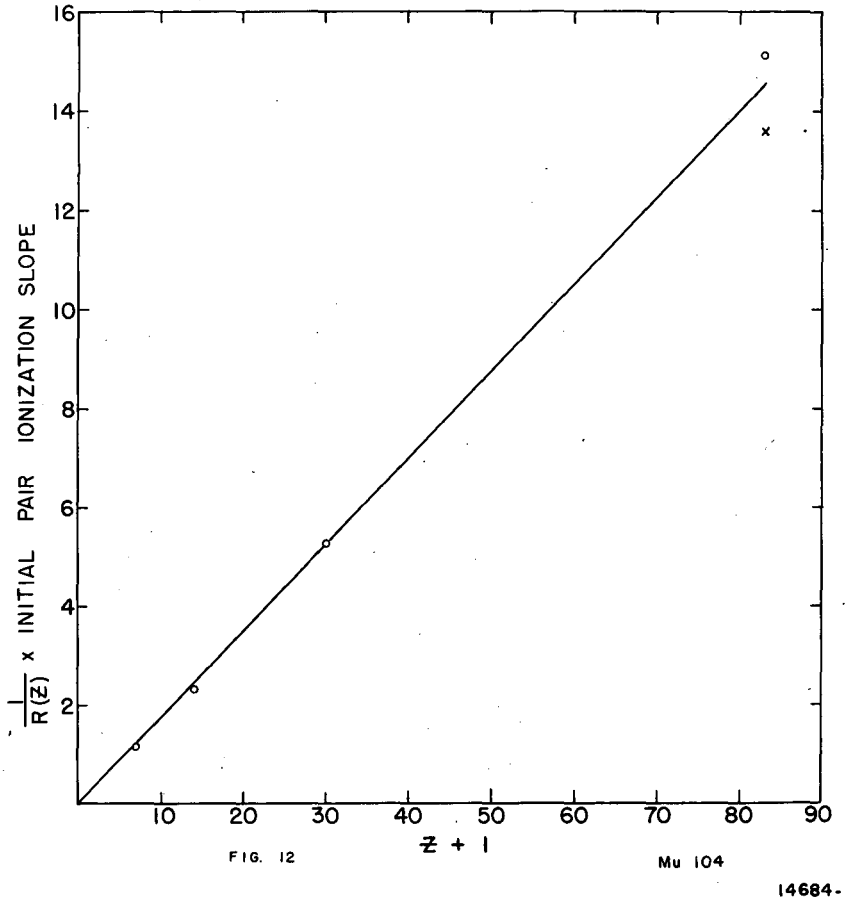

16
14
12
10
8
6
4
2
$\frac{1}{R(\bar{z})}$ x INITIAL PAIR IONIZATION SLOPE
10
20
30
40
50
60
70
80
90
$\bar{z}$ + 1
Mu 104
14684-

FIG. 12

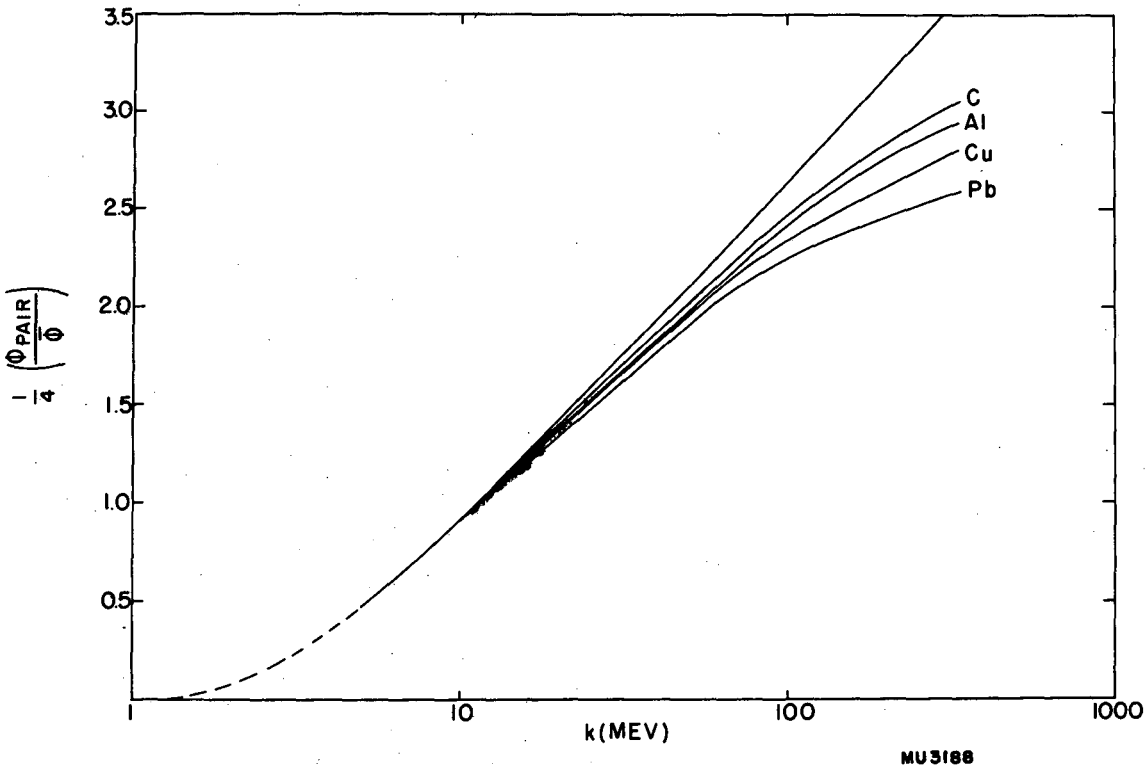


Fig. 13

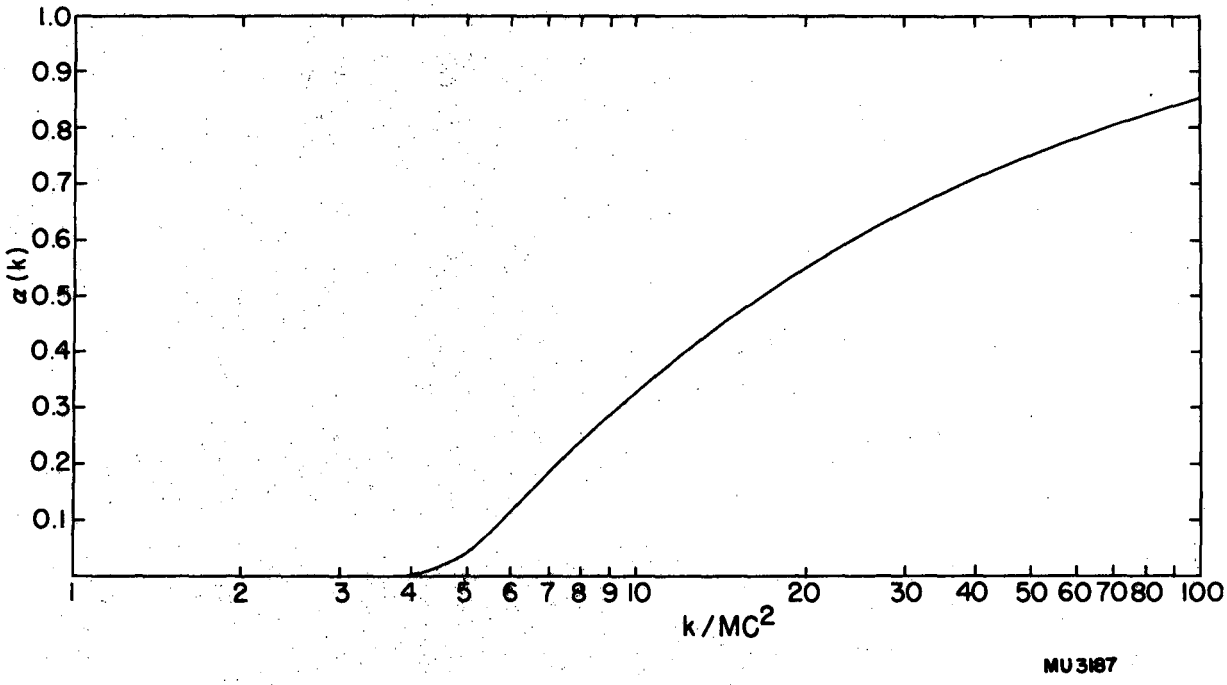

1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
α(k)
1
2
3
4
5
6
7
8
9
10
20
30
40
50
60
70
80
100
k/MC²
MU 3187

Fig. 14