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Journal
Proceedings of the Annual Meeting of the Cognitive Science Society, 24(24)

ISSN
1069-7977

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Publication Date
2002

Peer reviewed

Detecting the Local Maximum: A Satisficing Heuristic

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Abstract

In a simulated yard sale task, participants were asked to sell a series of objects, each of which would attract three customers making a randomly determined offer. Participants were told to maximize the total "take" from the sale. The analysis of the data revealed that high-performing naive participants were using a strategy that made them relate the current event to the seemingly irrelevant preceding events. We argue that this strategy is consistent with Simon's (1982) notion of "satisficing heuristic", which accounted for both participants' limited computation capacity and the task environment.

Introduction

Intuitive predictions and probabilistic judgments are often used as tasks to evaluate people's performance in judgment and decision-making research, and a common scheme is to collect incorrect predictions and misjudgments by "setting up a 'trap' that subjects would fall into if they were using a particular heuristic" (W. Goldstein & Hogarth, 1996, p.26). In this type of research, predictions derived from probability theory are often used as an objective criterion, and violations of the normative models are labeled as biased or irrational. Tversky and Kahneman's "heuristics and biases" program has been the most influential in this field. They suggested that intuitive predictions and judgments are often mediated by a small number of distinctive mental operations, which they called *judgmental heuristics*. "These heuristics … are often useful but they sometimes lead to characteristic errors or biases" (Kahneman & Tversky, 1996, p.582). For example, people's tendency to use a small sample of preceding events to evaluate an overall process was attributed to a "representativeness" bias (Tversky & Kahneman, 1973). This bias has been used to account for many cognitive behaviors, such as the tendency to see streaks in random sequences (Gilovich, Vallone & Tversky, 1985), and failure to "acquire a proper notion of regression" (Tversky & Kahneman, 1973). In a recent study on gambling behaviors, Thaler & Johnson (1990) concluded that "current choices are often evaluated with the knowledge of the outcomes which have preceded them, (but) such knowledge can often be a handicap" (p.643).

However, the heuristics and biases research program has recently been controversial, partly because "biases" sometimes appear highly adaptive. Thus, Tweney & Doherty (1983) argued that confirmatory tendencies ("confirmation biases") can be adaptive when hypotheses are relatively new and untested. Further, in an extensive series of studies, Gigerenzer and his colleagues (e.g. 1991, 1994, Gigerenzer & Todd, 1999) found evidence which led them to strongly disagree with Kahneman and Tversky. They argued that many seemingly naïve "fast and frugal heuristics" are adaptive in an uncertain environment. Similarly, Kareev, et al., suggested that the limited capacity of working memory (hence the use of small samples) could actually help the early detection of covariation since small samples of correlated variables are highly skewed (Kareev, 1995; Kareev, Lieberman & Lev, 1997).

The present study followed Simon's (1982) notion of "bounded rationality", which takes into account both people's limited computation capacity, and the structure of task environments. Our findings suggest that under circumstances when the precise prediction derived from statistics or probability theory is not the only criterion, heuristics based on a small sample size can be valuable. With a ***satisficing*** strategy that only needs to "look for a satisfactory alternative" (Simon, 1982, p.295), naïve participants were able to effectively accomplish the goal of the task, based on the evaluation of a few preceding events.

Recognizing the Maximum of a Sequence

The statistical properties of sequential lists of evidence have long been of interest to mathematicians. The dowry problem (or the secretary problem) is a classic example in the dynamic programming literature, one analyzed by Cayley in 1875 (see Ferguson, 1989). As a mathematical problem, the dowry problem is difficult to solve, requiring advanced mathematical knowledge and problem solving ability. Obviously, few, if any, people are likely to work out the exact stopping point mathematically in an everyday life situation when a similar problem is encountered. Instead, without complicated calculations, a player might need to use "common sense" to make decisions. The present study adopted a simplified version of the problem – a simulated "yard-sale" task – to test how naïve people evaluate preceding events and make decisions when facing sequential events generated by an unknown process.

Participants were asked to sell a series of objects in a simulated yard sale. Each object attracted three potential buyers, each of whom came at a different time and made a different offer. It was explained that offers that were rejected would not return, so that the task was to guess which was the best offer, and to take it when available.

Imagine that a person is selling a used car, and that visitors with different offers come up in a random order. After 5 offers have been declined in a week, a visitor comes in with a price higher than any of the previous ones. Another 5 offers will probably take another week and by then this car must be sold. Whether to stop waiting and grab the currently available offer then depends on how satisfied the car owner feels about the current offer. The only information available to evaluate the current situation is the previous encounters. Probably, "common sense" would tell this car owner to take the offer now, because future offers might not get better.

This is in effect a ***satisficing heuristic*** (Simon, 1990), which is a strategy that only needs to "look for a satisfactory alternative" (Simon, 1982, p.295), as suggested by the notion of bounded rationality. The strategy also fits the category of fast and frugal heuristics suggested by ecological rationality, because it makes "a choice from a set of alternatives encountered sequentially when one does not know much about the possibilities ahead of time" (Gigerenzer & Todd, 1999, p.13).

We show that in at least one situation – when the random process that generates offers is independently and identically distributed – this satisficing strategy is ***optimal***. Let R_i denote the offer at time i, where $i = 0$ is the current offer, -1 is the previous one, +1 is the next one, and so on. Assume the car owner has encountered m R's (from R_{-1} to R_{-m}) and found that R_0 is the best one so far. If he actually chooses it, because R_0 now is the biggest number in a local sequence of $(m + 1)$ numbers, in the long run, the value of such R_0 has a good chance to be higher than the population mean. For a continuous distribution from 0 to 1, the expected value of such R_0 is $(m+1)/(m+2)$. Further, R_0 might just be a good stopping point because the potential gain from the following n offers after R_0 might not have a good chance to get better. To see this, let A denote the event that R_0 is higher than its previous m offers, and B denote the event that R_0 is higher than its following n offers. Then two prior probabilities can be described as

$$p(A) = p\ (R_0 > R_{-1}, \ldots, R_{-m}) = 1/(m+1)$$

$$p(B) = p\ (R_0 > R_1, \ldots, R_n) = 1/(n+1)$$

And the conditional probability can be calculated as

$$p(B|A) = p(AB)/p(A) = (m+1)/(m+n+1)$$

Note that, with a fixed n, $p(B|A)$ approaches to an asymptote of 1 as m increases. That is, with an appropriate m (after considering a certain number of offers), the car owner can make a better decision than a random guess. For example, when $m = 5$, $n = 5$, $p(B|A)$ is 6/11, and this favors selling. To take the message of $p(B|A)$ in another way, it has suggested a ***stopping point***, because the coming n offers do not have a good chance to get better.

Two Optimal Strategies for the Yard Sale Task

With the development above, we can easily determine the optimal strategy for the yard sale task. Suppose there is only one trial in the task (only one object for sale). Let P_1 denote the first offer, P_2 the second and P_3 the third. Before knowing any of the three offers, the prior probability for each offer to be the best is equal:

$$p(P_1 \text{ is best}) = p(P_2 \text{ is best}) = p(P_3 \text{ is best}) = 1/3$$

Note that knowing the exact value of P_1 does not change this probability. With a random guess, the chance of hitting any of the three possible prices is 1/3. However, if we skip P_1 and consider P_2, the conditional probability is no longer equal. If P_2 is higher than P_1, we should take it immediately because $p(P_2 > P_3 \mid P_2 > P_1) = 2/3$. Otherwise, we should take P_3. A pay-off matrix (Table 1) shows that the optimal strategy (Option B*) is to always skip P_1. If P_2 is better than P_1, accept P_2; if P_2 is worse, choose P_3. This strategy increases the chance of hitting the best offer to 1/2, with a 1/3 chance of hitting the middle price, and a 1/6 chance of hitting the lowest one. For convenience, we will refer to this strategy as the "**one deal strategy**".

Table 1: The pay-off matrix for the seller

Option	Rank orders of offers						Total
	LMH	*MHL*	*LHM*	*MLH*	*HLM*	*HML*	
A	-1	0	-1	0	1	1	0
B*	0	1	1	1	0	-1	2
C	1	-1	0	-1	-1	0	-2

Note: L is the lowest price, M the middle, H the highest. "*LMH*" means that the lowest price comes first, and so on.
Option A: always choose P_1 (random guess).
Option B*: choose P_2 if $P_2 > P_1$, otherwise choose P_3.
Option C: choose P_2 if $P_2 < P_1$, otherwise choose P_3
Gains: the seller gains –1 when hitting the lowest offer; 0 for the middle offer; 1 for the highest offer.

However, in a real-life situation, decisions are rarely made in temporal isolation. Thus, as in a common scheme in laboratory experimental settings, our yard sale task used repeated trials to collect multiple data points from each individual participant. This fact had a significant impact on the optimal strategy. Recall that the single deal strategy assumes that in each deal, the order in which three offers appear is completely independent from any other events, and requires that the first offer always be ignored. What if the first offer actually is the best one? With the information from the preceding trials, we can actually evaluate how good the first offer is. Calculating an optimal strategy for deals in a sequence is very complicated because it needs to specify a distribution of three offers for each deal. However, when distributions of offers in several deals within a local sequence are similar, as an approximation, the principles we presented above can be generalized. In our experiment, we set the basic price for each object to range from \$50 to \$100, with a maximum random fluctuation of ± \$16. Figure 1 shows the overall distribution of these offers.

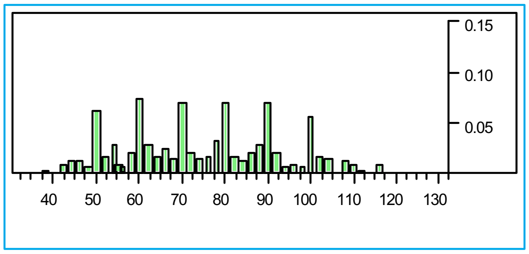

Figure 1 The distribution of offers. N = 5760, x-axis is price, y-axis is proportion.

The satisficing principle suggests that the offers for previous objects (in previous trials) can be used to predict whether you are getting a good offer for the next item. In other words, when you are considering a first offer for a table, if you can recall that the last several visitors, who were seeking other items, were not as generous as the current customer, you may want to sell the table right now. It seems quite against a researcher's intuition that a normative strategy would predict that previous offers for an umbrella will help to predict the current offers for a table, especially when one thinks that the umbrella deal is "over", and the two deals should be independent. The answer to this counter-intuitive puzzle is that the independence is only partial. While the order in which different offers come out for each deal is independent, the values of these offers, if they are in the same or similar distributions, regress to the population mean.

There are two ways to evaluate the first offer for a given trial. The "**local count strategy**" is based upon a count of the number of previous low offers. That is, if the current first offer is higher than a certain number of previous offers (for other items), take it. The optimal strategy depends on the specific distribution of the random offers and the payoff matrix. In our specific experimental setting, we used computer simulations and found that the best number of comparisons was 6. A modification of the local count strategy, the "**moving average strategy**" compares the current offer with the average of previous several offers. We reasoned that participants might not remember the exact values of the previous offer but might still have a vague memory of the overall average in a short local sequence. A logistic regression over simulated data showed that in our experiment setting, the value difference of the first offer for a given item from the mean of the previous 6 offers (for the other two items), is significant as a predictor of whether the first offer is the best among all three offers: $\chi 2$ (1, N = 1888) = 191.03, p<.01. That is, as this difference increases, this first offer is more likely to be the best of the three.

With this background, we were ready to find out whether participants are good at detecting good offers when they actually appear, and whether they use information from previous encounters to help their current decision-making.

Method

Participants were 15 undergraduate students from an introduction to psychology class at Bowling Green State University, none of whom had taken a course on game theory or probability theory. We refer to them as novice participants. One graduate student with extensive experience in judgment and decision-making and related research also participated, and will be referred as the expert participant.

The task was conducted using a self-paced computer program. Each participant completed 120 trials (the number of objects to be sold). One object was to be sold in each trial. Participants could take any of the three offers at the time it was available, but could not go back to an earlier declined offer. Once an offer was taken, offers thereafter were not presented. The third offer was forced if the first two were rejected by the participant, and this was the only case when participants knew exactly if they had hit the best out of three offers. After each trial, participants were given a confirmation that the object was sold at the price they selected. Participants' choices and their total earnings were recorded. An average experiment session lasted about 25 minutes.

Results

Overall Performance To evaluate participants' overall performance, we ran a simulation 5000 times using each of the three strategies: a random guess (randomly choosing one of the three offers), the "single deal strategy" and the "local count strategy". Each time the simulation sold 120 items using the actual selling list that was used in the experiment. In the local count strategy, the first offer for each item was compared with 6 previous offers (which were for the preceding items [1]). It was accepted when it was the highest in the comparison. Otherwise, it was declined and the single deal strategy was applied. Table 2 shows the simulation results and the actual participant data.

Table 2 Comparisons between human participants and 3 simulations

Group	N	Mean Score (95% confidence interval)	Std Dev.
Random Guess	5000	8889.8 ± 2.0	72.74
Single Deal Strategy	5000	9160.7 ± 2.1	75.54
Local count Strategy	5000	9277.7 ± 3.4	121.22
Human Participants	16	9196.0 ± 31.0	58.15

Note that all 16 participants received a score that was at least 1.5 SD above the mean of the random guess simulation.

[1] When an offer was taken before all offers were presented, the number of items whose offers were being compared may exceed 2.

Each participant's score was then compared to the result as if the single deal strategy had been applied to his/her actual selling list. Ten participants' scores were higher than the result of the single deal strategy. Using the standard deviation resulting from the single deal strategy simulation (75.54), four participants' scores were at least 1.5 SD above the score resulting from the single deal strategy. We will refer to these four as the "outstanding participants".

Strategy Use We looked at participants' choice patterns in regard to their consistency with the optimal strategies, at three steps when each offer was being considered. The following three choices are consistent with the optimal strategies (single deal or multiple deals):

C1. Accept the 1st offer if it is better than several previous offers (for other items).

C2. Decline the 1st offer, and accept the 2nd offer if it is better than the 1st one.

C3. Accept the 3rd offer if the 2nd is worse than the 1st.

C2 and C3 are equivalent to the single deal strategy, now separated into two parts. All three choices above are consistent with the local sequence strategy. Since choices at the 3rd offer were forced, whether participants' actual choices were consistent with the optimal strategies could be looked at whether they had met or violated the conditions at C1 and C2. Note that the single deal strategy actually forbids C1. Specifically, C1 can result from considering the count of the previous low offers (the local count strategy) or the value difference of the first offer compared to the mean of the previous offers (the local average strategy), and we tested them separately.

Of all 16 participants, only the expert participant found the single deal strategy, and followed C2 and C3 consistently. The 15 novice participants, by contrast, often violated either C2 or C3 or both. However, to a significant extent, their choices did follow C1. For each individual novice participant, we ran a logistic regression, using the value difference of the first offer from the mean of the previous 6 offers, to predict the participant's acceptances of the first offers. Of the 15 participants, 11 showed significant results at a 0.01 level. On the group level, the result is also significant: $\chi 2$ (1, N=1770) = 304.69, $p<.01$. This indicates that the novices were at least partly using the moving average strategy.

Since the one deal strategy is a subset of the local count strategy, we combined the 16 participants' reactions on all three offers to see if their behaviors were consistent with the local count strategy. Table 3.1 and Table 3.2 show that they did show such consistency when the previous 1 or 6 offers were compared to the current offer. That is, if the offer being considered was better than all of the previous 1 or 6 offers, participants were more likely to accept it. Otherwise, they were more likely to decline it. This finding was consistent with the local count strategy.

Table 3.1 Compared to previous 1 offer

	Worse	Better	Total
Decline	1773	952	2725
Accept	616	1424	2040
Total	2389	2376	4765

$\chi^2 = 580.177$, $p<.01$

Table 3.2 Compared to previous 6 offers

	Worse	Better	Total
Decline	2397	328	2725
Accept	1553	487	2040
Total	3950	815	4765

$\chi^2 = 114.140$, $p<.01$

All of the 4 "outstanding participants" were novice participants. However, they actually outperformed the expert participant and the one deal strategy. They were different from the other 11 non-expert participants in that their behaviors were consistent with one of the requirements of the one deal strategy (C2 and C3), although not both. Their gains on the first offers when these offers were the best had offset the losses from violations of the condition of C2.

Learning across Trials In a study of the Monty Hall dilemma, Granberg & Dorr found that participants showed signs of learning across trials under certain conditions. In our study, we also looked at whether there were systematic changes in participants' choices across trials. Specifically, we suspected that participants might have learned the specific distribution of random offers in earlier trials, so that, in later trials, they only needed to recognize "globally big numbers" instead of applying their heuristics independently and locally. For example, an offer of $116 might have been the best one for an item sold in an early trial. If participants had this number memorized, they might just pick an offer of $116 or higher in a later trial, no matter when this offer was presented (whether it was the 1st, 2nd, or 3rd offer). If this were the case, "big wins" might have been over-represented in terms of participants' uses of simple heuristics.

However, in our experimental setting, each item's 3 offers varied around its own basic price. Although these basic prices could be close, there was no way to tell that $116 was the best offer for item A only because it had been the best offer for item B. In other words, recognizing "big numbers" alone would not help in optimizing the total performance. In fact, when we partitioned each participant's 120 trials into 3 blocks with 40 trials each, we did not find any significant differences across blocks, indicating that learning was probably not important across trials.

Discussion

None of the 15 novice participants found and consistently used the one deal strategy. We reasoned that this was because finding and consistently applying this strategy required participants to use background knowledge in probability theory, and they simply did not have this training. Their consistency with the local sequence strategy explained why they had good performances. This does not suggest that they actually did the calculation and found the correct mathematical solution, because this would require even more knowledge and computational capacity, not to mention that it was within a short experiment session. However, as we suggested before, it is not necessary for a person to work out the correct mathematical proof to use the local sequence strategy. Such a strategy could arise from participants' everyday life experiences, from which they had learned a simple satisficing heuristic: "grab any good chance when you can".

Surprisingly, the outstanding performers actually outperformed the expert participant who found and consistently applied the one deal strategy. This was because the one deal strategy has to give up all opportunities of accepting the first offers when they were the best. One reason that prevented the expert participant from finding the local sequence strategy might have been that the everyday life heuristic had been "blocked" by his knowledge of probabilistic judgment research. This finding is very similar to Goldstein and Gigerenzer's (1999) "less-is-more" effect, that relative ignorance can actually benefit a decision maker. By isolating the previous encounters from the current decision-making situation, the expert participant had to search the infinite probability space again, and previous experience, either beyond or within the experiment task, could not help.

In their 1973 paper, Kahneman and Tversky suggested that "people do not acquire a proper notion of regression, … they do not expect regression in many situations where it is bound to occur", because "regression effects typically violate the intuition that the predicted outcome should be maximally representative of the input information". On the contrary, the finding in this study that participants' behaviors were consistent with the local sequence strategy, indicated that people do have good intuitions about such regression, and can also take advantage of it.

We argue that to evaluate naïve people's probabilistic judgment and decision-making, one has to take into account both people's limited computation capacity and the task environment. One obvious message of the task was that, if we had used the single deal strategy as the ***only*** criterion, we might have concluded that participants were being irrational, and would then have to face the puzzling evidence that they actually performed very well. Instead, the results suggest that the advantages of the satisficing principle are important and cannot be ignored. By using these strategies, people can benefit from their own experiences, even from a small sample of preceding events.

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