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The Visibility of Submerged Objects (Chapters 1 through 4)

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THE VISIBILITY OF SUBMERGED OBJECTS

(Chapters 1 through 4)

VISIBILITY LABORATORY

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

THE VISIBILITY OF
SUBMERGED OBJECTS

FINAL REPORT

Seibert Q. Duntley
research director

31 AUGUST 1952
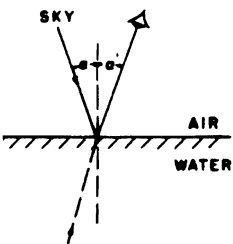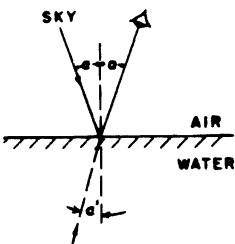
errata

THE VISIBILITY OF SUBMERGED OBJECTS

JUNE 16, 1953
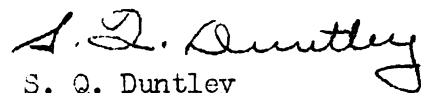
Page	Location	Reads	Should Read
Title Page	Line 7	report by the Office of Naval Research..	report was supported by the Office of Naval Research...
Title Page	Line 10	It was administered under Projects...	It was administered under M.I.T. Projects...
Foreword	Line 20	N5ore	N5ori
2	Figure 1		
3	Line 18	After suitable further amplication..	After suitable further amplification...
5	Eq. 1.1	$e-h^2\tan^2\phi$	$e-h^2\tan^2\phi$
11	Line 1	The slots is...	The slots in...
11	Line 26	from the depths B_{wz}.	from the depths B_{w2}
11	Line 27	may be written $(B_{wz+}$.	may be written $(B_{w2+}$...
11	Line 33	$\left[\text{See equations (1.8) and (1.13)}\right]$	$\left[\text{See equations (1.8) and (1.23)}\right]$
12	Line 10	term of equation (1.11),	term of equation (1.18),
12	Line 10 (add to)		add the following sentence after $/B_{w1} = C_R$. $\left[\text{See page 14 for discussion of } C_R\cdot\ C_R \text{ is also defined and discussed in Chapter 2. (See equations (2.31) and (2.32))}\right]$
12	Line 20	but since they appear only as ratio equations (1.22) and (1.23)...	but since they appear only as a ratio in equations (1.22) and (1.23), ...

Page	Location	Reads	Should Read
13	Line 3 (from bottom)	where the luminous density ...	The luminous density ...
14	Line 1	vertical, d_T is the ...	vertical, d_t is the ....
14	Line 8	equation (1.3)....	equation (1.25)....
14	Line 11	equation (1.4)...	equation (1.27)....
14	Line 21	Equation (1.28)...	Equation (1.27)....
14	Line 29	with equation (1.21):	with equation (1.31):
15	Line 1	Equation (1.24)....	Equation (1.32)....
15	Line 14		DELETE (1.32)
15	Line 16	In the fourth term of equation (1.33)....	In the fourth factor of equation (1.33)....
17	Eq. (2.4)	$K = \sqrt{\mu^2 - 2\mu B}$	$K = \sqrt{\mu^2 + 2\mu B}$
17	Eq. (2.5)	$-- \sqrt{\mu^2 - 2\mu B}$	$-- \sqrt{\mu^2 + 2\mu B}$
19	Eq. (2.8)	$\left[H_u = H_0 t_1 TR_\infty (1-RR..\right.$	$H_u = H_0 \, t_1 TR_\infty \left[(1-RR....\right.$
	Eq. (2.10)	$h = (1+R_\infty)H_\emptyset$	$h = (1+R_\infty)H_D$
	Eq. (2.11)	REPLACE ALL "k" 's WITH "K" 's.	
26	Line 1	DELETE "Such a plot is shown in Fig. 15."	
26	Line 20	Figure 15	Figure 14
28	Eq. (2.40)	N_{H1}	N_{H2}
	Line 16	A mat surface of...	A matte surface of ....
34	Line 15	When $\theta = -90$ degrees	When $\theta = +90$ degrees..
	Line 18	path (v_{-90})...	path (v_{+90})....
	Line 22	range (v_{-90})..	range (v_{+90})...

Page	Location	Reads	Should Read
35	Equations (4.4) (4.5) (4.6) (4.7) (4.8) (4.9) (4.10)	v_{-90}	v_{+90}
36	Equations (4.12) (4.13)	v_{-90}	v_{+90}

FOREWORD

The Visibility Laboratory has received requests for its
1952 report entitled "The Visibility of Submerged Objects."
Although original copies of this document are no longer
available, a limited supply of the first four chapters of the
report has been assembled for distribution. These chapters repre-
sent all of the material contained in the original report which
has direct bearing upon the visibility of submerged objects. The
missing three chapters, which deal with vision through the
atmosphere, are badly out of date and will be reprinted later after
revision.

S. Q. Duntley
Director
Visibility Laboratory

October 1960
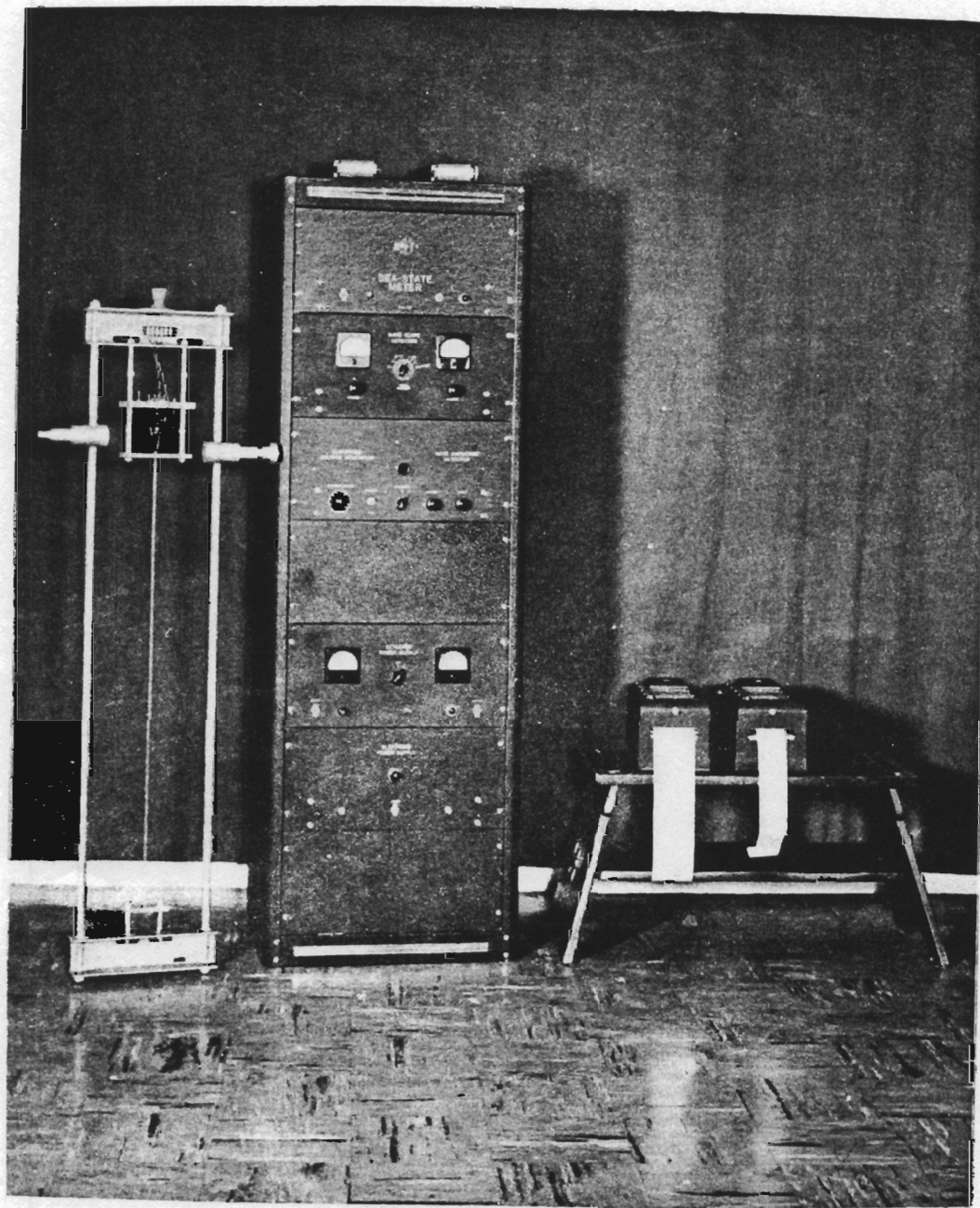
Frontispiece

The Sea-State Meter

· FOREWORD ·

The Visibility Laboratory was established at the Massachusetts Institute of Technology in 1948 by the Office of Naval Research by means of contract N5ori 07831. The initial problem undertaken by the laboratory concerned the physical factors which influence the visual detectability of objects submerged in the ocean. This research was an extension and a continuation of the investigations of the visibility of military targets carried on throughout World War II by the Optics and Camouflage Division of the National Defense Research Committee, as reported in Volume 2 of the Summary Technical Report of NDRC Division 16.

Support for the Visibility Laboratory at M.I.T. was provided first by ONR under contract N5ori 07831, then by the Bureau of Ships under NObs 50378, and finally by both agencies under separate contracts (N5ori 07864 and NObs 50378). The work under these contracts has been so interrelated and so interwoven that it is impossible to make an unambiguous separation of the work supported by each. In general, the Office of Naval Research has supported the basic aspects of the program and the Bureau of Ships has supported the instrumentation. This distinction however, can not be clearly drawn, all three of the contracts having supported both catagories of endeavor at different times. It has been necessary, therefore, to prepare a combined final report for the entire program.

Although this report is the final report under N5ore 07831, NObs 50378, and N5ori 07864 it is, in fact, only a progress report from the standpoint of the research program. The physical assets and some of the personnel of the Visibility Laboratory have been transferred to the Scripps Institution of Oceanography of the University of California, La Jolla, California, where the same research program is continuing under Bureau of Ships contract Nobs 43356. The Visibility Laboratory in California is being supported jointly by the Bureau of Ships, by the Office of Naval Research, and by the United States Air Force. In transferring to California, the Visibility Laboratory has been greatly expanded from the standpoint of sea, flight, and shop facilities, personnel, and scope of research topics. Unexpended funds from contract N5ori 07864 are being transferred to California for the specific purpose of completing a handbook dealing with the visibility of submerged objects. This handbook was one of the primary goals of the Laboratory at M.I.T.

Certain classified aspects of the work of the M.I.T. Visibility Laboratory have been reported separately and are, therefore, excluded from this report in order to avoid classification. Certain advanced theoretical work, not yet fully completed, has been separately reported and is not available for distribution at this time. Chapters 6 and 7 were written by the group leaders in charge of this work and are presented herein without editing and without attempt to collate and integrate this work into the preceeding chapters of the report. It is expected that subsequent reports to be produced by the Visibility Laboratory in California will provide a more finished product than can be presented at this time.

31 August 1952
S. Q. Duntley
Research Director

CHAPTER I
The Apparent Contrast of Submerged Objects

I. INTRODUCTION

During the spring of 1948, the Armed Forces-NRC Vision Committee received a request from Buships and C.N.O. for information concerning the visibility of submerged submarines. After due deliberation by the Subcommittee on Visibility and Atmospheric Optics, the Vision Committee recommended to the Office of Naval Research that a research project be initiated to explore the physical and visual factors which limit the detectibility of fully submerged submarines. (See Minutes of 21st meeting, pp. 81-88) This paper is the first part of the final report of the resulting research project.

The Minutes of the 23rd meeting of the Committee contain an interim report which describes exploratory studies of some of the physical factors that influence the visibility of submerged objects. (Min., 23rd meeting, pp. 123) That report was concerned primarily with the optical principles that govern the transmission of an image through water, and it set forth the experimental finding that the apparent contrast of any submerged object is exponentially attenuated with distance along any path of sight. Continued research on this topic has yielded further information, but renewed discussion of it will be postponed until the second portion of this report.

The interim report at the 23rd meeting also described photometric measurements of the apparent contrast of a submerged object as viewed both from above the water surface and from beneath the water surface. (See Figures 1 and 2, pp. 126, 23rd meeting) It was pointed out in Figure 2 of that paper, that these contrast data could be expressed as a contrast reduction factor which varies regularly with the state of the sea and with the altitude of the sun. The precise nature of the contrast reduction equation was not known nor were all of the physical factors apparent. It was stated, however, that major goals of further research in this field were to be the identification of the pertinent parameters, the development of experimental techniques for their measurement at sea, and the discovery of the law of contrast reduction. All three of these goals have been reached, and it now appears possible to predict the apparent contrast of a fully submerged submarine as seen by an aerial observer under virtually any circumstances.

The general nature of the problem is illustrated by Figure 1, in which a submerged object of reflectance R forms a contrast C_o with a water background of reflectance R_w. The magnitude of this contrast depends upon the altitude of the sun and the relative amount of sunlight in the daylight impinging on the surface of the water. Known optical principles can be used to predict the apparent contrast seen by the observer whenever the water surface is perfectly calm. If,

$$C_o = \frac{R - R_w}{R_w}$$
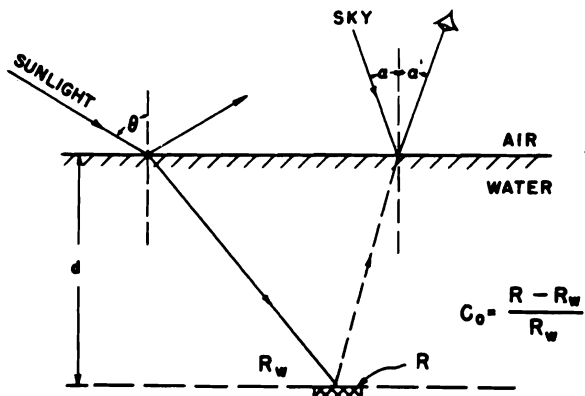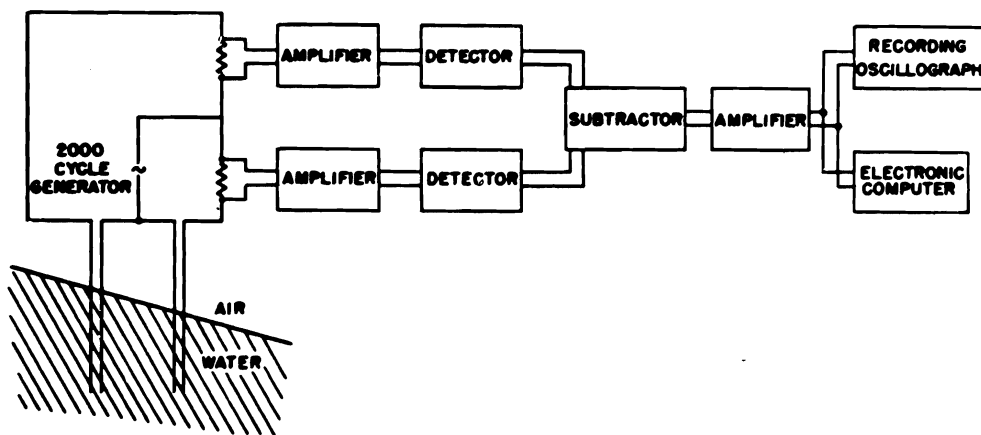
Figure 1

Figure 2

WIND VELOCITY: 5 KNOTS   13 DECEMBER 1949
                                                   7 P.M.

AMPLITUDE

TIME IN SECONDS

SLOPE   (WITH THE WIND)

SLOPE   (ACROSS THE WIND)

Figure 3

2
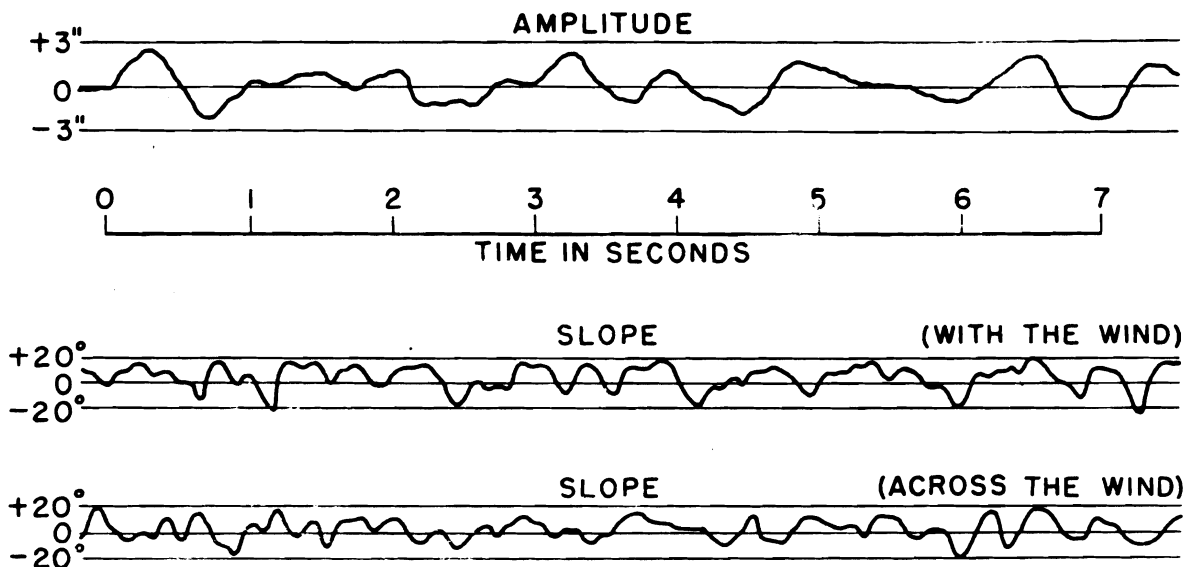
however, the water surface is roughened by wind, additional infor-
mation and new optical principles must be employed in the computa-
tion. The manner in which a wind-roughened water surface reduces
the apparent contrast of submerged objects will be discussed in the
following paragraphs.

2. THE SEA-STATE METER

The amplitude of water waves is of little concern optically; it is
the slope of the water surface which determines the refractive and
reflective effects. An electrical instrument for measuring wave
slopes has been devised and used in experiments both at Diamond Is-
land and at sea. The instrument, designed to indicate the conven-
tional (amplitude) sea-state as well as the water slope, has been
called a "sea-state meter". It is arranged to record the differ-
ence in water height at two closely adjacent points. Two pairs of
stainless steel wires are mounted as shown in Figure 2 and powered
by the same 2000 cycle alternator. After detection, the signals are
subtracted,thus producing a voltage proportional to the water slope.
After suitable further amplication, the signal is recorded by a
direct writing oscillograph (Brush, BL 202). The signal is also
sent through an electronic computer which performs a statistical
analysis of the data.

The apparatus shown in Figure 2 is used in duplicate so that or-
thogonal components of wave slope can be recorded simultaneously.
Provision has also been made for the simultaneous recording of wave
amplitude, and a typical record is shown in Figure 3. Slope cannot
be obtained by differentiation of the amplitude record with respect
to time, partly because the slope is a vector quantity and partly
because of dispersion. It will be noticed in Figure 3 that the slope
components exhibit a higher frequency than the amplitude record, but
that neither of the slope records can be obtained by differentiation
of the amplitude record.

After numerous water wave records had been studied, it was con-
cluded that the frequency of occurrence of a given slope always
closely approximates a normal distribution. This is illustrated by
Figure 4, which represents the first data obtained by the sea-state
meter (November 6, 1949). A slight asymmetry in the direction of
the wind can be seen by comparing the dotted (symmetrical) curve
with the solid (experimental) curve on the left side of the dis-
tribution. This indicates a slight preponderance of steeper slopes
in the direction away from the wind. These data were taken with
electrode spacing of approximately one inch between wire pairs.
More recent data taken with an effective electrode spacing of one
tenth of an inch, shows almost no "downwind" effect. This means
that the very tiny capillary wavelets, having wavelengths less than
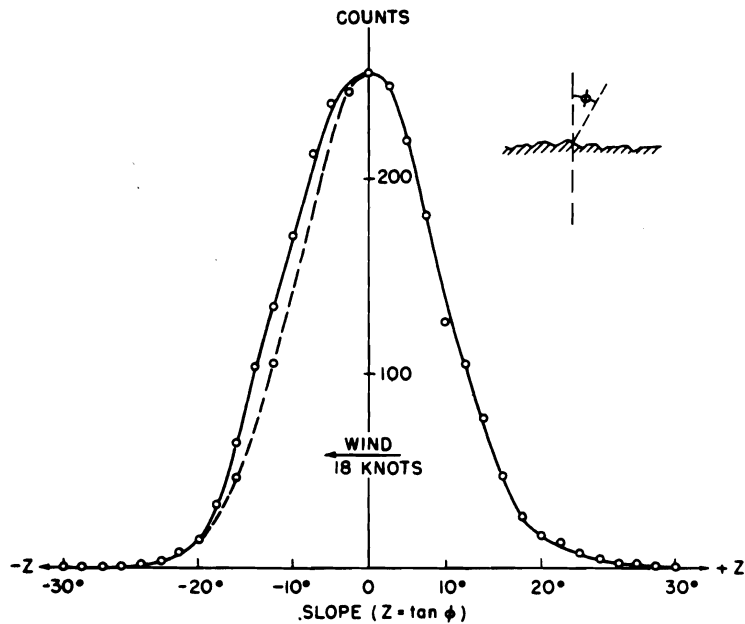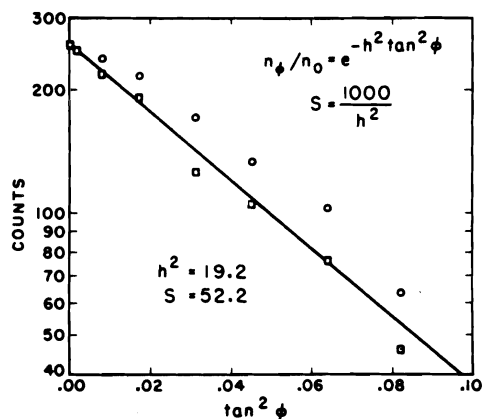an inch, contribute so many randomly distributed steep slopes that

Figure 4
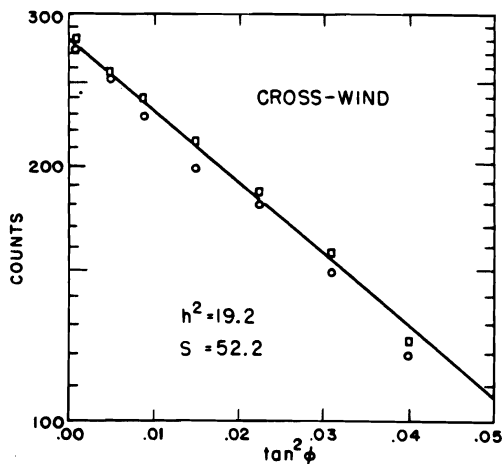

Figure 5

Figure 6

the asymmetry of the large gravity waves becomes negligible in their effect on the distribution of true slopes.

The closeness with which the slope measurements follow a normal distribution can be better judged by plotting on a logarithmic scale the relative number of counts against the square of the slope, as is done in Figure 5. The straight line in this figure represents the equation:

$$n_\phi / n_o = e^{-h^2 \tan^2 \phi} \qquad (1.1)$$

where $h^2 = 19.2.$

Figure 6 shows simultaneous data from electrodes mounted at right angles to those from which the foregoing data were taken. The straight line on this plot also represents equation (1.1) with $h^2 = 19.2$. It has been inferred from these plots, and from a large number of similar data, that equation (1.1) is a satisfactory representation of the probability of the occurrence of a slope component of given magnitude. The optical effects caused by water waves can be described, therefore, in terms of a single constant, h^2, which describes the spread of the probability distribution curve. It has been found convenient, however, to use the reciprocal of this quantity for describing the optical state of the sea because the reciprocal of h^2 provides a scale of numbers which runs from zero (for perfectly calm water) up to infinity. The symbol S and the definition $S = 1000/h^2$ has been adopted for "optical sea-state" and will be used throughout the remainder of this paper. The most commonly observed values of S fall in the range of 10 to 100.

The symmetrical two-dimensional nature of the probability curves just described is illustrated in Figure 7, which is an isometric projection of two identical probability curves at right angles. These curves may be regarded as traces on a surface of revolution. At any point on this surface, the altitude above the base plane is given by equation (1.1). The volume of any cylindrical element beneath this surface is a measure of the probability of the occurrence of a slope having magnitude between $\tan \phi - \frac{1}{2} d(\tan \phi)$ and $(\tan \phi) + \frac{1}{2} d (\tan \phi)$. The incremental fractional time $d\hat{t}_\phi$ during which slopes of this magnitude occur is then:

$$d\hat{t}_\phi = 2\pi \ \tan \phi \ C \ e^{-h^2 \tan^2 \phi} \ d(\tan \phi) \qquad (1.2)$$

Equation (1.2) will be used as the basis for deriving an expression for the time-averaged apparent contrast $\bar{C}$ of submerged objects.

5

$$ce^{-h^2 \tan^2 \phi}$$

Figure 7
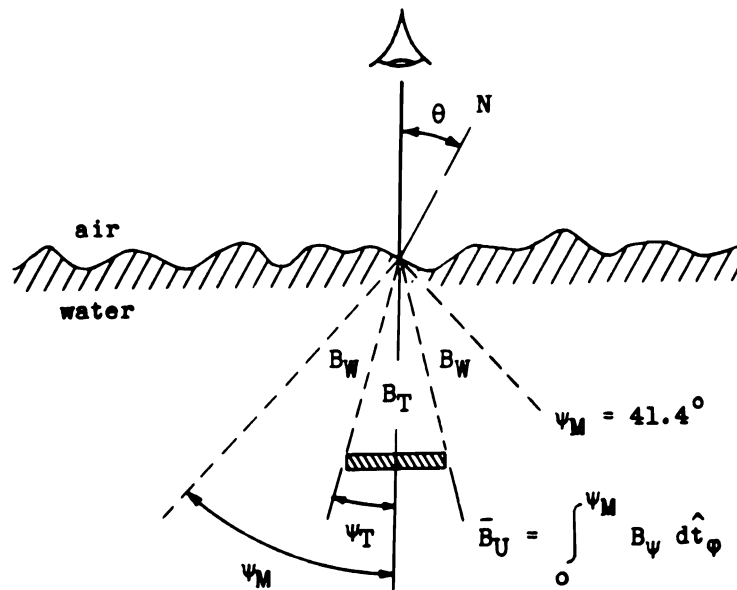

$$\Psi_M = 41.4°$$

$$\bar{B}_U = \int_o^{\Psi_M} B_\psi \, d\hat{t}_\varphi$$

Figure 8

6
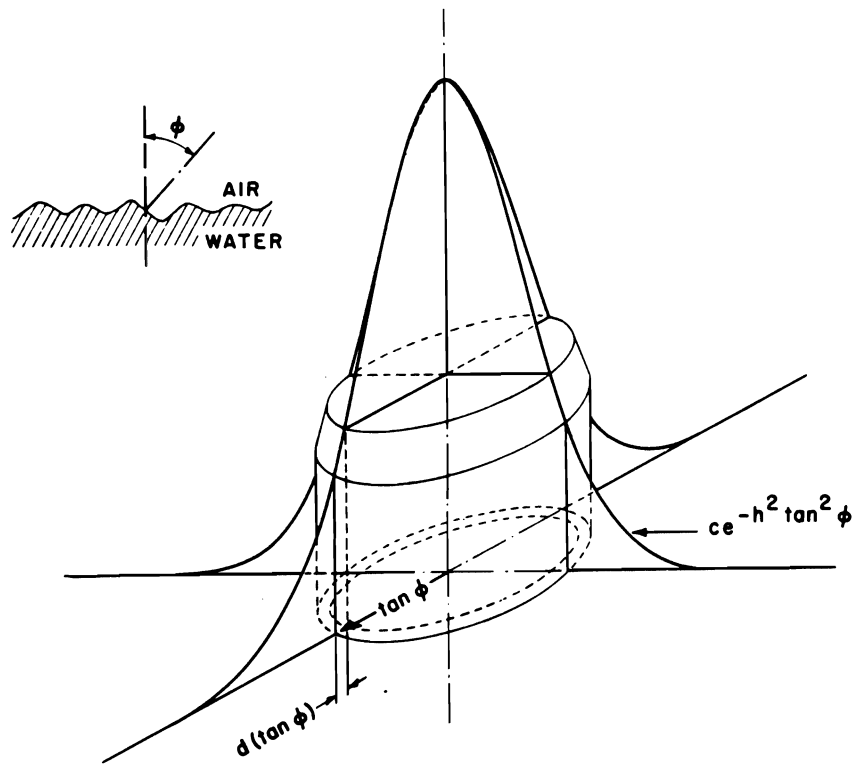
3. CONTRAST REDUCTION BY REFRACTION

Part of the reduction in contrast at the water surface is caused by time-varying refraction. An observer looking straight down at a wind-roughened water surface receives light from the depths directly beneath him only part of the time, because of the rapid and random changes in wave slope. His path of sight, deviated by refraction, sweeps out a solid cone having a half apex angle $\psi_M = \cos^{-1} 3/4 = 41.4$ degrees, as shown in Figure 8.

Let B_ψ represent the apparent luminance of the under-water field of view in Figure 8. The time-average upwelling apparent luminance $\bar{B}_u$ is then:

$$\bar{B}_u = \int_0^{\psi_M} B_\psi \, d\hat{t}_\phi \tag{1.3}$$

In the special but common case of a submerged circular target of angular subtense ψ_T and apparent luminance B_T seen against a uniformly luminous background of luminance B_w, the time-averaged apparent luminance of the center of the target $\bar{B}_T$ is:

$$\bar{B}_T = \int_0^{\psi_T} B_T \, d\hat{t}_\phi + \int_{\psi_T}^{\psi_M} B_w \, d\hat{t}_\phi \tag{1.4}$$

After integration:

$$\bar{B}_T = B_T \left(1 - e^{-1000 \, (\tan^2 \phi_T) \, /S} \right) + B_w \, e^{-1000 \, (\tan^2 \phi_T) \, /S} \tag{1.5}$$

Defining calm water apparent contrast C and time-averaged apparent contrast $\bar{C}$ as:

$$C = \frac{B_T - B_w}{B_w} \tag{1.6}$$

and

$$\bar{C} = \frac{\bar{B}_T - B_w}{B_w} \tag{1.7}$$

equations (1.5), (1.6), and (1.7) can be combined as follows:

$$\bar{C} = C \left(1 - e^{-1000 \, (\tan^2 \phi_T) \, /S} \right) \tag{1.8}$$

7
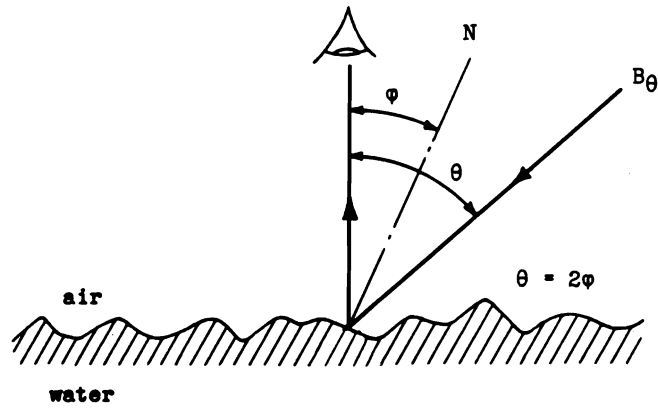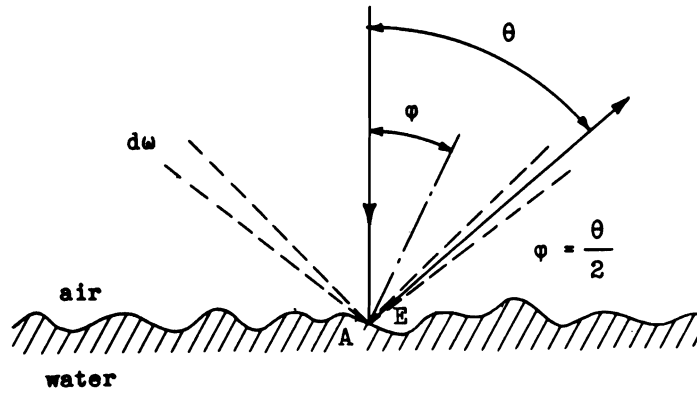
Figure 9

Figure 10

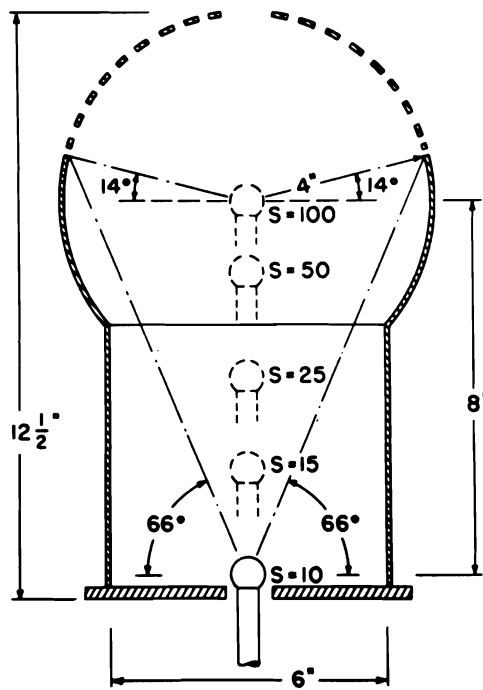**Figure 11**

Thus, the time-averaged apparent contrast of a submerged object is seen to depend upon the optical sea-state S and the angle subtended by the target at the surface of the water. For example, in the case of a target for which $\tan \phi_T = 0.1$ ($\psi_T = 4.3$ degrees) and barely ruffled water (S = 10), it will be seen from equation (1.8) that $\bar{C} = 0.63C$, whereas in water roughened by a 40 knot wind (S = 100) the time-averaged apparent contrast of the same target will be reduced to 0.095 of its calm-water value.

4. SKY REFLECTION

The surface of the sea acts as a moving mirror which always reflects light from some portion of the sky to the eye of an aerial observer. This reflected sky light further reduces the time-averaged apparent contrast of any submerged object. Equation (1.2) can be used as the basis for computing the time-averaged apparent luminance of the water surface $\bar{B}_r$. Referring to the geometry of Figure 9, it will be seen that

$$\bar{B}_r = \int_0^{\pi/4} r_\phi B_\theta \, d\hat{t}_\phi \qquad (1.9)$$

where r_ϕ is the reflectance of the water surface for light incident at an angle ϕ with the normal. Because of the complex nature of the luminance in the sky, equation (1.9) must be made two-dimensional.

The collection of sufficient data to permit the evaluation of the integral in equation (1.9) is impracticable. Fortunately, the following simple experimental approach can be made. Let equation (1.9) be modified as follows:

$$\bar{B}_r = \int_0^{\pi/4} r_\phi B_\theta \, d\hat{t}_\phi = s \, r_o \, B_o \qquad (1.10)$$

where $r_o B_o$ is the reflected luminance of the zenith sky and s is a "sky factor" defined by equation (1.10). It will be noted that s depends not only on the state of the sky as specified, by B_θ, but also on the state of the sea as specified by $d\hat{t}_\phi$. An experimental technique for the evaluation of s has been devised on the basis of the following reasoning:

An observer looking downward at a calm surface sees a reflected image of the zenith sky. Whenever the action of wind converts the water into a moving mirror the observer's path of sight sweeps the sky in exactly the same manner as would a beam of light projected downward from the position of the observer and reflected by the

moving water surface. This is illustrated by Figure 10. The distribution of time-averaged intensity of the water surface at point A has conical symmetry because of the symmetry of slopes as represented by Figure 7. The time-averaged flux $\overline{dF}_\theta$ within the conical solid angle $d\omega$ illustrated in Figure 10 is:

$$\overline{dF}_\theta = r_\phi \, E \, A \, \hat{dt}_\phi \qquad (1.11)$$

and the solid angle

$$d\omega = 2\pi \sin\theta \, d\theta \qquad (1.12)$$

The time-averaged intensity $\overline{I}_\theta$ is then:

$$\overline{I}_\theta = \frac{\overline{dF}_\theta}{d\omega} = \frac{r_\phi \, E \, A \, C \, e^{-1000 \, (\tan^2\phi) \, /S}}{2\pi(1 + \cos\theta)^2} \qquad (1.13)$$

When $\theta = 0$

$$\overline{I}_o = r_o \, E \, A \, C/8\pi \qquad (1.14)$$

Thus

$$\frac{\overline{I}_\theta}{\overline{I}_o} = \frac{4 \, r_\phi \, e^{-1000(\tan^2\phi) \, /S}}{r_o(1 + \cos\theta)^2} \qquad (1.15)$$

Equation (1.15) represents the time-averaged distribution of intensity of the water surface at point A when illuminated by a beam incident vertically downward. Let a photoelectric receptor be given a distribution of zonal sensitivity specified by equation (1.15). The response of this "sky collector" will then be proportional to the luminance of the entire sky (including the sun) weighted in accordance with the fractional time which the observer in Figure 9 sees a given portion of the sky. If the sky-collector is subsequently presented to a uniform (artificial) sky having the luminance of the real sky at the zenith, the ratio of its two readings will be the sky factor s in equation (1.10).

Such a sky-collector has been built and is illustrated in Figure 11. A matte-surfaced translucent sphere is secured to the top of a pipe having a photoelectric cell (not shown) fastened to its lower end. This spherical collector is mounted at the center of a hollow metal sphere containing curved meridianal slots so shaped that the collector is given the zonal sensitivity specified by equation (1.15). The outer sphere is rotated rapidly during measurement so that the sky is uniformly weighted in all azimuths.

10

The slots is the sky-collector sphere can be cut for only a single optical sea-state S, but it has been found that if this is done for S = 100 (rough water) the characteristics specified by equation (1.15) for calmer sea-states can be closely approximated by lowering the spherical collector relative to the outer sphere as shown in Figure 11.

Thus, the procedure for determining the sky factor s involves (1) the measurement of the optical sea-state S by means of the sea-state meter, (2) adjusting the sky collector to the appropriate value of S and noting its reading, and (3) covering the sky collector with an artificial sky of uniform luminance equal to that of the zenith B_o and noting a second reading. The ratio of the first and second sky collector readings is the sky factor s.

When the observer's path of sight is inclined at an angle α , as in Figure 1, the sky factor s_α can be measured by tipping the sky collector through the angle α in a direction away from the observer. Equation (1.10) should then be re-written as follows:

$$s_\alpha = \frac{{}_o\int^{\pi/4} r_\phi B_\alpha \, d\hat{t}_\phi}{r_\alpha B_\alpha}$$ (1.16)

where the denominator represents the reflected luminance of the sky when the water is calm and the numerator represents the time averaged reflected luminance of the sky when the sea is roughened by wind.

The time averaged apparent luminance of the surface of the sea is derived from two sources: surface-reflected skylight or sunlight $s_\alpha r_\alpha B_\alpha$; and luminance upwelling from the depths B_{wz} . Thus the apparent luminance of the sea may be written $(B_{wz} + s_\alpha r_\alpha B_\alpha)$.

If the luminance upwelling through the surface of the sea from the direction of the submerged object is B_2 , the time averaged apparent contrast of the submerged object is, by definition:

$$\overline{\overline{C}} = \frac{(B_2 + s_\alpha r_\alpha B_\alpha) - (B_{w2} + s_\alpha r_\alpha B_\alpha)}{(B_{w2} + s_\alpha r_\alpha B_\alpha)}$$ (1.17)

where $\overline{\overline{C}}$ denotes that no allowance has been made for contrast reduction by time-varying refraction. [See equations (1.8) and (1.13)] Equation (1.17) may be rewritten:

$$\overline{\overline{C}} = \left(\frac{B_2 - B_{w2}}{B_{w2}}\right) \left(\frac{B_{w2}}{B_{w2} + s_\alpha r_\alpha B_\alpha}\right)$$ (1.18)

11

Let the upwelling apparent luminance of the sea as seen from just beneath the surface be represented by B_{w1} , and let B_1 represent the upwelling apparent luminance from the direction of the target. Then:

$$B_2 = B_1 t_{a1} \qquad (1.19)$$

and

$$B_{w2} = B_{w1} t_{a1} \qquad (1.20)$$

where t_{a1} , is the Fresnel transmittance of the water surface.

If equations (1.19) and (1.20) are substituted in the first bracketed term of equation (1.11), the term reduces to $(B_1 - B_{w1})/B_{w1} \equiv C_R$. Thus, equation (1.18) may be written:

$$\bar{\bar{C}} = C_R \left(\frac{B_{w2}}{B_{w2} + s_a r_a B_a} \right) \qquad (1.21)$$

Convenience is served by rewriting equation (1.21) as follows:

$$\bar{\bar{C}} = C_R \left(1 + \frac{s_a r_a B_a}{B_{w2}} \right)^{-1} \qquad (1.22)$$

The effect of sky reflection on the time averaged apparent contrast of a submerged object may be allowed for by replacing C in equation (1.8) by $\bar{\bar{C}}$. Substituting equation (1.22) for $\bar{\bar{C}}$:

$$\bar{C} = C_R \left(1 + \frac{s_a r_a B_a}{B_{w2}} \right)^{-1} \left(1 - e^{-1000 (\tan^2 \phi_t / S)} \right) \qquad (1.23)$$

The quantities B_a and B_{w2} are absolute values of luminance throughout the foregoing derivation, but since they appear only as ratio equations (1.22) and (1.23) relative values on any scale will serve equally well. Experimental convenience is served if quantities $b_a = B_a/E$ and $b_{w2} = B_{w2}/E$ are substituted, where E is the illuminance on a horizontal surface at sea-level. After this substitution the right-hand member of equation (1.23) will be recognized as the contrast transmittance of the surface of the sea.

Before equation (1.23) can be used for the prediction of the apparent contrast of submerged objects, a relation is needed between the relative upwelling luminance of the sea b_{w2} and parameters which may be readily determined. Theoretical and experimental work not included in this paper has yielded the following equation:

$$b_{w2} = t_{a1} R_w \left(t_\theta \left(1 - y \right) + t_1 y \right) \Big/ \left(1 - r_s R_w \right) \qquad (1.24)$$

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where t_θ and t_1 are Fresnel transmittances for sunlight and sky-light respectively, y is the fraction contributed by skylight to the illuminance on a horizontal plane at sea-level, and r_s is the emergent reflectance of the water surface for the prevailing optical sea-state S. It will be seen from equations (1.1) and (1.16) that the apparent contrast of any submerged object will be greatly affected by the altitude of the sun, for this controls both t_θ and y. The value of y is also profoundly influenced by the presence of clouds and atmospheric haze; it represents a second datum on the state of the sky, independent and in addition to the value of s_α measured by the sky-state meter previously described.

5. CONTRAST ATTENUATION BY WATER

An experimental investigation of the reduction of contrast by water, conducted at Diamond Island during the summer of 1948, revealed (1) that luminous density decreases exponentially with depth; and, (2) that the apparent contrast of any object is exponentially attenuated with distance along any path of sight through water, the attenuation coefficient varying with the inclination of the path but not with the position of the sun. Further underwater experiments in 1949 showed that along horizontal paths of sight, the attenuation coefficient for contrast equals the attenuation coefficient for a directed beam of light, i.e. the contrast transmittance of a horizontal path equals the beam transmittance of the same water. This observation, coupled with the observation that no departures from exponential variation of apparent contrast are found even in the case of positive inherent contrasts greater than ten, supports the thesis of Middleton that all objects are detected only by rays which encounter no scattering processes between the object and the eye.

The experimental results just described have provided the basis for a new theory of the reduction of apparent luminance by water. This new theory set forth fully in Chapter 2 of this report, leads to the following relation between the apparent luminance B_R of a submerged object seen at distance R and the inherent luminance B_0 of the same object seen close aboard:

$$B_R = \frac{\sigma q_0}{\beta + k \cos a^1} \left\{ e^{+k(R\cos a^1 - d_t)} - e^{-(\beta R + k d_t)} \right\} + B_0\, e^{-\beta R} \qquad (1.25)$$

where the luminous density q_d at depth d is given by:

$$q_d = q_0\, e^{-kd} \qquad (1.26)$$

and where: σ is the scattering-rate coefficient for the path of

sight which is inclined at an angle α^1 with the vertical, d_T is the depth of the target, and β is the attenuation coefficient for the beam transmittance of the water. For horizontal paths of sight ($\alpha' = \pi/2$) equation(1.25)reduces to a form identical with Koschmieder's equation for the reduction of luminance along horizontal paths of sight through the atmosphere.

The apparent contrast of any submerged target against its deepwater background may be found from equation(1.3). The inherent contrast C_o is related to the apparent contrast C_R by the relation:

$$C_R = C_o \, e^{-\left(\beta + k \cos a^1\right) R} \tag{1.27}$$

It will be noted from equation(1.4)that the optical properties of water, as they affect contrast, may be described by means of the two constants β and k. It is fundamental that two independent constants are required to specify the properties of an optical material possessing absorption and scattering.

Conceptual and computational convenience is served by the introduction of a quantity to be called the hydrological range. This is the distance, measured along the path of sight, for which the contrast transmittance is two percent. Thus:

$$\beta = \ln 50 / v_{90} \tag{1.28}$$

Equation(1.28)can be written in terms of the hydrological range v_{α^1} as follows:

$$C_R = C_o \, e^{-\left(\ln 50 \right) R/v_{a^1}} \tag{1.29}$$

Combining(1.27),(1.28),and(1.29)and solving for v_{α^1} ;

$$v_{a^1} = \frac{v_{90}\ln 50}{\ln 50 + k v_{90}\cos a^1} \tag{1.30}$$

Recalling that R = d sec α' and that ln 50 = 3.912, the contrast transmittance (C_R/C_o) for the path of sight is seen to be:

$$C_R/C_o = e^{-3.912 \, d \, v_{a^1}^{-1} \, \sec a^1} \tag{1.31}$$

Combining equation(1.23)with equation(1.21):

$$\bar{C} = C_o \left(e^{-3.912 \, d \, v_{a^1}^{-1} \, \sec a^1} \right) \left(1 + \frac{s_a r_a B_a}{B_{w2}} \right)^{-1} \left(1 - e^{-1000 \, (\tan^2 \phi_T)/S} \right) \tag{1.32}$$

Equation (1.24) shows that the time averaged apparent contrast $\overline{C}$ of a submerged object is related to its inherent contrast C_o by three factors. The first of these factors relates to contrast attenuation produced by the water as the optical signal is transmitted from the object to the surface of the sea. The second factor specifies the contrast reduction caused by sky reflection at the water surface, and the third factor allows for the reduction of contrast due to time varying refractive effects caused by water waves.

6. CONTRAST REDUCTION BY THE ATMOSPHERE

If the observer is aloft, a fourth factor enters into the contrast reduction equation. This effect will be discussed at length in Chapters 6 and 7. The full contrast reduction equation may be written:

$$C = C_o \left(e^{-3.912 dv_{a^1}^{-1} \sec a^1} \right) \left(1 + \frac{s_a r_a B_a}{B_{W2}} \right)^{-1} \qquad (1.32)$$

$$\left(1 - e^{-1000 (\tan^2 \phi_T)/S} \right) \left(1 - J \, [1 - e^{+3.912\overline{R}/V_o}] \right)^{-1} \qquad (1.33)$$

In the fourth term of equation (1.33) the symbol J represents the sky-ground ratio, $\overline{R}$ is the optical slant range, and V_o is the meteorological range at sea-level. For definitions of these quantities, see "The Reduction of Apparent Contrast by the Atmosphere" (J. Opt. Soc. Am. 38, 179 (1948)), Chapters 5,6, and 7 of this report should also be consulted.

The visual detectability of submerged objects can be calculated by means of equation (1.33). It is expected that reports containing such calculations will result from the continued program of the Visibility Laboratory at its new headquarters in California.

CHAPTER II
Principles of Hydrological Optics

I. INTRODUCTION

Many factors combine to govern the visual detectability of sub-
merged objects, even when the observer is stationed beneath the
water surface. Detection in such a case requires that the luminance
or the color of the object differ sufficiently from that of its
background for the optical signal reaching the observer to exceed
his contrast threshold[1] despite attenuation by the intervening water.
It is the purpose of this paper to discuss the optical principles
which govern the generation of contrast by a submerged object and
the transmission of the resulting optical signal to any point within
the water. The discussion will be limited, however, to non-self-
luminous objects and to natural lighting conditions. Support for
these studies has been furnished by the Office of Naval Research and
by the Bureau of Ships, United States Navy.

In this paper it will be the basic plan to consider first, the pene-
tration of daylight to the submerged object; second, the generation
of contrast; and third, the attenuation of this contrast along the
path of sight from the object to the observer.

2. PENETRATION OF LIGHT INTO THE SEA

All natural bodies of water contain suspended particles which scat-
ter light. Natural waters may also contain dissolved matter which
absorbs light, and pure water itself exhibits absorption of impor-
tance within the visible spectrum. Because the magnitudes of the
scattering and absorbing effects differ from wavelength to wave-
length, it will be necessary to restrict the following discussion
to monochromatic light. In most cases of practical importance,
however, the hydrosol is sufficiently non-selective that the equa-
tions hereinafter developed for the monochromatic case have proved
to be useful in treating heterochromatic data.

Natural waters must be regarded as turbid media lighted from above
by the sun and sky. It will be useful to consider first the case of
diffuse illumination, i.e. light from a substantially uniform sky
with no direct sunlight. The water will be considered "infinitely
deep" in the sense that no radiometric quantities would be affected
if the water were deeper. Reflection will occur at the water sur-
face; its magnitude r_1 is given by the equation of Walsh:[2]

(1) Blackwell, H.R. "Contrast Thresholds for the Human Eye." J.O.S.A. 36,624 (1946)

(2) Walsh, Dept. Sci. Ind. Res. Illum. Res. Tech. Papers 2, 10 (1926)

$$r_1 = \frac{1}{2} + \frac{(n-1)(3n+1)}{6(n+1)^2} + \left\{ \frac{n^2(n^2-1)^2}{(n^2+1)^3} \right\} \ln \frac{n-1}{n+1} \qquad (2.1)$$

$$- \frac{2n^3(n^2+2n-1)}{(n^2+1)(n^4-1)} - \left\{ \frac{8n^4(n^4+1)}{(n^2+1)(n^4-1)^2} \right\} \ln n$$

where n is the refractive index of water. If n = 1.333, r_1 = 0.06638. The water surface may, therefore, be considered to have a non-selective transmittance t_1 = 1 − r_1 = 0.9336.

Beneath the water surface the down-welling diffused light is within a hydrosol , any layer of which can be described by reflectance R and transmittance T defined by equations (2.2) and (2.3).

$$R = \frac{B \sinh KX}{(\mu+B) \sinh KX + K \cosh KX} \qquad (2.2)$$

$$T = \frac{K}{(\mu+B) \sinh KX + K \cosh KX} \qquad (2.3)$$

where $\qquad K = \sqrt{\mu^2 - 2\mu B} \qquad (2.4)$

and x represents the thickness of the layer.

The origins of equations (2.2), and (2.3) and (2.4) were described in an earlier paper[3]. The symbol μ is the absorption coefficient, B the scattering coefficient for light scattered backward within a 2π solid angle, and X is the thickness of the layer. It is fundamental that a medium which possesses independent scattering and absorption mechanisms requires a minimum of two independent optical constants to specify its flux transmitting properties. Equations (2.2) and (2.3) relate only to the transfer of diffused flux through the water. It will be shown later in this paper that at least one additional optical constant is required before the image transmitting properties of the hydrosol can be specified.

If the sea is optically homogeneous (i.e. optically non-stratified), the reflectance R_∞ of an infinite depth is seen from equations (2.2) and (2.3) to be:

$$R_\infty = \frac{B}{\mu + B + \sqrt{\mu^2 - 2\mu B}} \qquad (2.5)$$

(3) Duntley, S.Q., The Optical Properties of Diffusing Materials. J.O.S.A. 32 61 (1942)

Dividing numerator and denominator of equation (2.5) by μ, R_∞ is seen to depend only upon the ratio of the back-scattering coefficient to the absorption coefficient:

$$R_\infty = \frac{(B/\mu)}{1 + (B/\mu) + \sqrt{1 - 2(B/\mu)}} \qquad (2.6)$$

Thus, the water of a silt-bearing river may have the appearance of coffee with cream when both the water and the coffee are optically infinite in thickness. The same liquids may, however, look quite unlike when poured into small tumblers. This observation illustrates that the reflectance R_∞ and the transparency of a turbid medium such as water are entirely independent, the former depending only upon the ratio of the scattering and absorption coefficients while the latter is controlled by their magnitudes. Typical values of R_∞ for water in the open ocean and in clear lakes range from 1 to 3 percent, but show no correlation with the transparency of these waters.

Penetration of light into the sea can be usefully described by specifying the volume density of radiant energy u at each depth. An equivalent quantity, more readily measurable, is the scalar irradiance h. By definition, h = (c/n) u where c is the velocity of light in free space and n is the refractive index of the water. Scalar irradiance can be measured in any light field by means of a small translucent hollow integrating sphere. The conventional irradiance on the inner wall of such a sphere, measurable by ordinary radiometric techniques, is proportional to the scalar irradiance. Scalar irradiance is, therefore,

$$h = \lim_{A \to 0} \frac{1}{A} \int H \, dA \qquad \text{where A is the area of a sphere.}$$

In the special case of the diffusely lighted sea, h is the sum of the irradiance on both sides of any horizontal plane. An expression for this sum will now be derived:

Consider any horizontal plane P at depth d beneath the surface of the sea. The layer of hydrosol between P and the surface of the water will have reflectance R and transmittance T as given by equations (2.2) and (2.3). Let the downward irradiance on the surface of the water be H_o. The downwelling irradiance H_D on the top surface of plane P would be $H_o t_1 T$ if no interreflections took place. An infinite series of interreflections occur, however, between the surface of the water and the layer of hydrosol above plane P. These increase H_D by a factor $(1 - r_2 R)^{-1}$, where r_2 is the emergent reflectance of water for diffused flux. Additional contributions to H_d arise from light which, having passed through the plane P, is reflected by the deeper water (R_∞). This light initiates an infinite series of interreflections between the hydrosol above and below plane P and further interreflections occur between the water surface and the hydrosol. Thus:

$$H_D = H_o t_1 T (1-r_2 R)^{-1} \sum_{j=0}^{\infty} \left[RR_\infty + r_2 R_\infty T^2 t_1 (1-r_2 R)^{-1} \right]^j$$

or $\qquad H_D = H_o t_1 T \left[(1-RR_\infty)(1-r_2 R) - r_2 T^2 t_1 R_\infty \right]^{-1}$ (2.7)

The upwelling irradiance H_u on the lower surface of plane P is also affected by interreflections. Infinite series of these arise from the same causes described above in connection with H_D. The complete expression for H_u may be written:

$$H_u = H_o t_1 T (1-r_2 R)^{-1} R_\infty \sum_{j=0}^{\infty} \left[RR_\infty + r_2 R_\infty T^2 t_1 (1-r_2 R)^{-1} \right]^j$$

or $\qquad \left[H_u = H_o t_1 T R_\infty \quad (1-RR_\infty)(1-r_2 R) - r_2 T^2 t_1 R_\infty \right]^{-1}$ (2.8)

Comparison of equations (2.7) and (2.8) shows that:

$$H_u = R_\infty H_D \qquad\qquad (2.9)$$

In the special case under discussion $h = H_D + H_u$. Thus;

$$h = (1+R_\infty) H_\phi \qquad\qquad (2.10)$$

If equations (2.7) and (2.10) are combined, and if R, T, and R are replaced by the functions of μ and B specified by equations (2.2), (2.3), and (2.5) respectively, the resulting equation can be written:

$$h = \frac{A e^{kd} + B e^{-kd}}{C e^{2kd} + D e^{-2kd} + E} \qquad\qquad (2.11)$$

where the constants A, B, C, D, and E are lengthy algebraic functions of B and μ , K is defined by equation (2.4), and d is the depth of plane P beneath the water surface.

At depths such that $d \gg K^{-1}$, equation (2.11) reduces to the simple exponential form:

$$h_2 = h_1 e^{-K(d_2 - d_1)} \qquad\qquad (2.12)$$

where the subscripts 1 and 2 denote any two appropriate depths.

No analytic discussion of scalar irradiance near the water surface $(d < K^{-1})$ will be attempted in this paper. Experimental evidence, shown in Figure 12, indicates that even near the surface h behaves in an essentially exponential manner. No significant difference in K was found between cloudy and sunny weather. It appears justifiable, therefore, throughout the remainder of this paper to represent the scalar irradiance by the relation:

$$h_d = h_o e^{-Kd} \qquad\qquad (2.13)$$

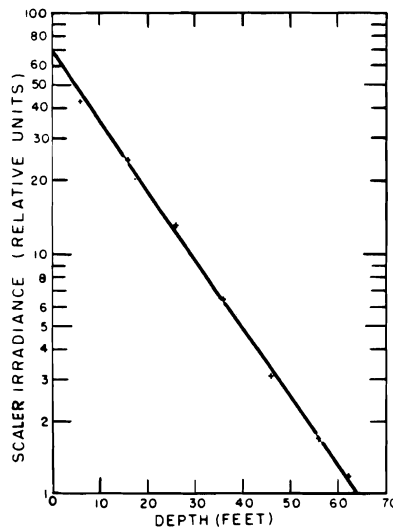where h_o is the scalar irradiance just beneath the water surface.

Figure 12

3. THE INHERENT CONTRAST OF SUBMERGED OBJECTS

Equation (2.9) indicates that the ratio of upwelling irradiance to downwelling irradiance is a constant, characteristic of the hydrosol but independent of depth, i.e. $H_u/H_D = R_\infty$. It follows that if a flat horizontal surface has a reflectance $R_o = R_\infty$, its apparent radiance $N_o = R_o H_D$ will match that (N_B) of the deep-water background beneath, since $N_B = H_u$. Since a surface is said to have zero inherent contrast for a downward-looking observer; the surface will be invisible regardless of its depth.

If N_o differs from N_B an inherent contrast C_o will exist. Let C_o be defined by the relation:

$$C_o = \frac{N_o - N_B}{N_B} \qquad (2.14)$$

Thus:
$$C_o = \frac{R_o H_D - H_U}{H_u} = \frac{R_o - H_u/H_D}{H_u/H_D} = \frac{R_o - R_\infty}{R_\infty} \qquad (2.15)$$

The inherent contrast of a submerged horizontal surface is shown by Eq. (2.15) to depend only upon its reflectance R_o and upon R_∞ for the hydrosol which surrounds it; <u>inherent contrast does not depend upon the depth of the object</u>.

If the observer does not observe the surface perpendicularly, Eq. (2.14) still defines the inherent contrast, but Eq. (2.15) does not apply because $N_B \neq H_u$. In such a case, data on the directional variation of N_B must be obtained by experiment or by calculation from Eq. (2.28) of this paper.

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If the reflecting surface is not horizontal, but is viewed perpendicular to its surface, inherent contrast will be given by:

$$C_o = \frac{R_o - H_1/H_2}{H_1/H_2} = \frac{R_o - H_1/H_2}{H_1/H_2} \qquad (2.16)$$

where H_2 is the irradiance on the surface facing the observer and H_1 is the irradiance which would reach the plane of the surface on the side facing away from the observer if no object were present. H_1/H_2 has been called the irradiance ratio. (See page 32 in Chapter III.) Any surface of reflectance R_o will produce zero inherent contrast at two azimuths. At other azimuths the contrast of the surface will assume either positive or negative values. A positive value of contrast indicates that the surface is more radiant than its background; negative contrast signifies a surface less radiant than its background.

4. THE REDUCTION OF APPARENT CONTRAST BY WATER

The apparent contrast seen by an observer at a distance R along a path of inclination ($\textcircled{H}$) as shown in Fig.13, is diminished by the
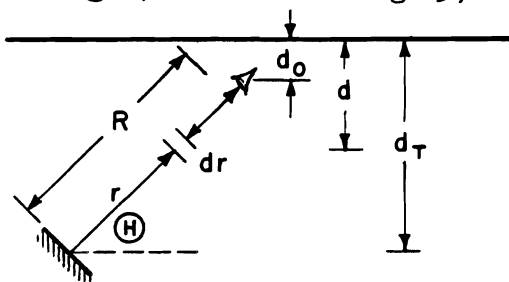

Figure 13

concurrent action of two optical processes: (1) light from the object is attenuated by scattering and by absorption; (2) daylight is scattered toward the observer throughout the entire length of the path of sight, thus producing a veil of light which lowers the apparent contrast of the object. At sufficient distance, the contrast falls below the observer's threshold and only the veiling light or space light can be seen.

The law of contrast reduction, to be derived in this section, will be developed from an expression for the apparent radiance N_R of an object of inherent radiance N_o located at a distance R from the observer, as in Fig.13. A derivation for the equation for N_R follows:

The apparent radiance N at a general distance r from the object will be altered by an amount dN in passing a further distance dr along the path of sight. The nature of this alteration is described by the following equation:

$$dN/dr = -\mu N - SN + dN^1/dr \qquad (2.17)$$

Eq. (2.17) shows that, in traversing an element of path of length dr, the apparent radiance N is diminished by absorption (μN), diminished by total scattering (SN), but augumented (dN^1/dr) by scattering of the daylight which permeates the water. In Eq. (2.17), μ is the absorption coefficient, S the total scattering coefficient, and N^1 is the radiance component resulting from scattered daylight. S is related to the backward-scattering coefficient B used in Section 2 of this paper by the relation S = B + F where F, the forward-scattering coefficient, relates to all of the light scattered forward with a 2π solid angle. For convenience, an attenuation coefficient β will be defined by the relation $\beta = \mu + S$.

The radiance increment dN^1 is not the same for each path increment dr, chiefly because deeper layers of the hydrosol are darker, but conceivably also because the permeating radiation is more diffuse at greater depths. However, experimental investigations of the spacial variation of dN^1/dr show it to be exponential along any path of sight and to depend upon depth in exactly the same manner as does the scalar irradiance. These experimental conclusions are based upon data taken in Lake Winnipesaukee, New Hampshire, and in gulf stream water off Key West, Florida; the desirability of tests in other waters is obvious.

Upon the basis of the above experimental findings, the term dN^1/dr may be replaced by Pe^{-Kd}, where P is a constant characteristic both of the path of sight and of its irradiance pattern; P has been called the path constant. K is given by Eq. (2.13).

Eq. (2.17) may now be written:

$$dN/dr = -\beta N + Pe^{-Kd} \qquad (2.18)$$

From the geometry of Fig. 13,

$$d = d_t - r \sin \Theta \qquad (2.19)$$

Eq. (2.18) may be combined with Eq. (2.19) and integrated between the limits N = N_0 and N = N_R, r = 0 and r = R, with the following results:

$$N_R = \frac{P}{K \sin \Theta + \beta} \left\{ e^{K(R \sin \Theta - d_T)} - e^{-(\beta R + Kd_T)} \right\} + N_0 e^{-\beta R} \qquad (2.20)$$

For the special case of a horizontal path of sight Eq. (2.20) reduces to:

$$N_R = \frac{Pe^{-Kd}}{\beta} \left\{ 1 - e^{-\beta R} \right\} + N_0 e^{-\beta R} \qquad (2.21)$$

Eq. (2.18) shows that, along any horizontal path of sight, one particular radiance level N_H is transmitted without change. Thus $dN_H/dr = 0$ when:

$$N_H = \frac{P e^{-Kd}}{\beta} \qquad (2.22)$$

N_H has been called the <u>equilibrium radiance</u> for the path of sight; its magnitude depends upon the direction of view and is the horizontal apparent radiance when no object is present. Combining Eq. (2.21) and (2.22):

$$N_R = N_H \left\{ 1 - e^{-\beta R} \right\} + N_o e^{-\beta R} \qquad (2.23)$$

Eq.(2.23) will be recognized as identical in form with Koschmieder's equation for the apparent radiance of an object seen along a horizontal path through an optically homogeneous atmosphere. It follows that all of the theorems derived therefrom concerning the transmission of contrasts along horizontal paths of sight through the atmosphere [4,5] apply equally well to the transmission of contrasts along horizontal paths of sight through water. For example, it is a principal consequence of Eq. (2.23) that radiance differences are exponentially attenuated, i.e.:

$$\Delta N_R = \Delta N_o e^{-\beta R} \qquad (2.24)$$

If the object is seen against a background of radiance N_H, the law of the attenuation of contrast may be found by dividing both sides of Eq. (2.24) by N_H. The apparent contrast at distance R is then:

$$C_R = C_o e^{-\beta R} \qquad (2.25)$$

Experimental verification of Eq. (2.25) has been obtained for both light and dark objects by means of the research barge shown in Fig.14 Photographic and photoelectric telephotometers have been used to measure the apparent luminance of targets mounted at five foot intervals along a horizontal track. A sample of the resulting data is shown in Fig.15 . By rotation of the barge about its mooring,
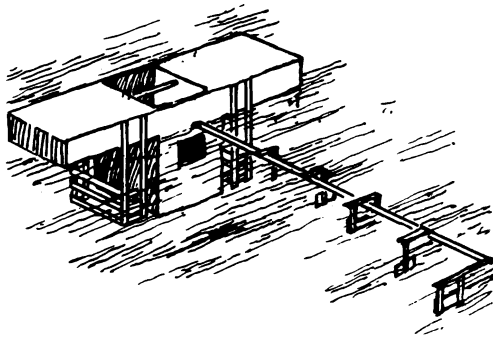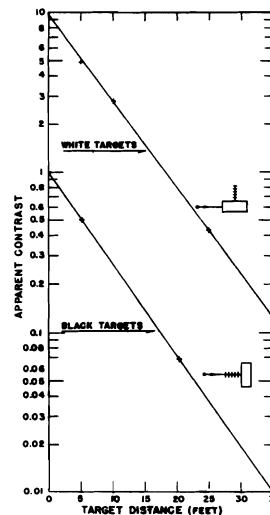

Figure 14

Figure 15

(4) Duntley, S.Q. "The Reduction of Apparent Contrast by the Atmosphere." J.O.S.A. 38,179 (1948)

(5) Middleton, W.E.K. "Vision Through the Atmosphere" (To be published soon by the Toronto Press.)

it has been verified that the attenuation coefficient β is a scalar quantity, independent of the azimuth of the path of sight relative to the sun.

No departures from equation (2.25) were observed, even with inherent contrasts greater than 10. This result and the absence of a "ground-glass plate" effect (see reference 4, page 23) strongly supports the hypothesis that the only light from the target which reaches the observer is light which traverses the intervening space without scattering. Scattered light from the target serves solely as a contribution to the scalar irradiance. The path constant term in equation (2.18) suffices, therefore, to take full and proper cognizance of all effects due to secondary scattering.

When the path of sight is not horizontal, Eq. (2.20) may be used to compute the background radiance N_{BO} seen by an observer at a depth d_o beneath the surface (See Fig. 13) and looking downward at an angle $-\Theta$ below the horizontal. Let N_{BT} represent the apparent radiance of the water background seen by an observer at depth d_T and looking downward at an angle $-\Theta$ below the horizontal. Eq. (2.20) may now be used to relate N_{BO} and N_{BT}:

From the geometry of Fig. 13, $(R \sin \Theta - d_T) = -d_o$, and Eq. (2.20) may be written:

$$N_R = \frac{P}{K \sin \Theta + \beta} \left\{ e^{-Kd_o} - e^{-(\beta R + Kd_T)} \right\} + N_o e^{-\beta R} \qquad (2.26)$$

To calculate N_{BT}, let N_o be replaced by zero, R by ∞, d_T by ∞, d_o by d_T, and N_R by N_{BT}. Then:

$$N_{BT} = Pe^{-Kd_T} \Big/ (K \sin \Theta + \beta) \qquad (2.27)$$

Similarly:
$$N_{BO} = Pe^{-Kd_o} \Big/ (K \sin \Theta + \beta) \qquad (2.28)$$

Combining Eq. (2.27) and Eq. (2.28):

$$N_{BO} = N_{BT} \, e^{+K(d_T - d_o)} \qquad (2.29)$$

It will be noted that Eq. (2.29) is independent of Θ.

The inherent contrast of the submerged object shown in Fig. 13 is:

$$C_o = \frac{N_o - N_{BT}}{N_{BT}} \qquad (2.30)$$

and the inherent contrast seen by the observer is:

$$C_R = \frac{N_R - N_{BO}}{N_{BO}} \qquad (2.31)$$

Combining Eq. (2.26) through Eq. (2.31):

$$C_R = C_O \, e^{-(\beta + K \sin \Theta) R} \qquad (2.32)$$

For the special case of horizontal path of sight ($\Theta = 0$) Eq. (2.32) reduces to Eq. (2.25). Along inclined paths of sight, however, the attenuation of contrast is governed by the two optical constants β and K. In the special case of a turbid medium possessing no absorption, $K = 0$ as shown by Eq. (2.4), and Eq. (2.32) reduces to Eq. (2.25) along any path of sight. This case is encountered in the atmosphere when absorption producing material such as smoke or dust is not present, but in all natural waters K cannot be neglected.

5. HYDROLOGICAL RANGE

The contrast transmitting properties of the atmosphere are usefully described in terms of <u>meteorological range</u>, that distance through an optically homogeneous atmosphere for which the contrast transmittance (C_R/C_O) equals two percent. The constant 0.0200 is arbitrary and its selection was of historical rather than of scientific origin. It has been found to be convenient to adopt a corresponding term to describe the contrast transmitting properties of water. Let <u>hydrological range</u> (v) be defined as that distance through optically homogeneous water for which the contrast transmittance is two percent. Substituting in Eq. (2.32):

$$C_R / C_O = 0.020 = e^{-(\beta + K \sin \Theta) v_\Theta} \qquad (2.33)$$

or

$$v_\Theta = \frac{\ln 50}{\beta + K \sin \Theta} \qquad (2.34)$$

Eq. (2.34) shows that the magnitude of the hydrological range depends upon the inclination of the path of sight, being minimum when the observer looks downward ($\Theta = \pi/2$) and maximum when the observer looks upward ($\Theta = -\pi/2$).

Along horizontal paths of sight ($\Theta = 0$), the horizontal hydrological range $v_O = \ln 50/\beta$. A quantity v_K may be used to describe the attenuation of scalar irradiance with depth. Thus $v_K = \ln 50/K$ If β and K are replaced by v_O and v_K, Eq. (2.34) becomes:

$$v_\Theta = \frac{v_O}{1 + \dfrac{v_O}{v_K} \sin \Theta} \qquad (2.35)$$

If Eq. (2.35) is plotted in polar coordinates a conic section is obtained. Since v_Θ must always be a finite distance, the horizontal hydrological range must always be less than the hydrological range for scalar irradiance, i.e. $v_O < v_K$. The polar plot of v_Θ

is, therefore, an ellipse of eccentricity v_O/v_K . Such a plot is shown in Fig.15 .

The foregoing inequality can also be proved with the aid of Eq.(2.4) and the relation $\beta = \mu + B + F$. Thus:

$$\frac{v_O}{v_K} = \frac{K}{\beta} = \frac{(\mu^2 - 2\mu B)^{1/2}}{\mu + B + F} = \frac{(1 - 2B/\mu)^{1/2}}{1 + B/\mu + F/\mu} \qquad (2.36)$$

Since v_O/v_K must be real, $(1 - 2B/\mu) > 0$, and since F, B, $\mu > 0$, $(1 - 2B/\mu)^{\frac{1}{2}} < 1$. Therefore $K < \beta$ and $v_O < v_K$.

The attenuation of apparent contrast along any path of sight through water, given by Eq. (2.32), may be expressed in terms of hydrological range by the equation:

$$C_R = C_O \, e^{-\ln 50 (R/v_\Theta)} \qquad (2.37)$$

6. BEAM TRANSMITTANCE

When a collimated beam of light is directed through otherwise dark water to a telephotometer having a sufficiently narrow field of view to exclude aureole effects, the attenuation of the apparent radiance is described by the differential equation:

$$dN/dr = -\mu^1 N + S^1 N \qquad (2.38)$$

If this equation is integrated between the limits N_O to N and zero to R:

$$N/N_O = e^{-(\mu^1 + S^1)R} = e^{-\beta^1 R} = e^{-\ln 50 (R/v^1)} \qquad (2.39)$$

Experiments at the research barge, shown in Fig.15, have shown that $v^1 = v_O$. The ratio N/N_O has been called the beam transmittance of the water, and one instrument for measuring N/N_O has been called a "hydrophotometer". The experimental evidence sighted above indicates that the beam transmittance and contrast transmittance (C/C_O) are equal. This finding strongly supports the hypothesis, set forth in Section 4 of this paper, that the only light from the target which reaches the observer is light which traverses the intervening space without scattering.

7. SECCHI DISK READING

A widely used method of measuring the clarity of ocean waters, proposed by Secchi, involves the lowering of a horizontal white disk into water until it disappears. The greatest depth at which the disk is visually detectable is called the Secchi disk reading for the water. A hooded, glass-bottomed boat or box must be used to eliminate contrast reduction due to wave action and sky reflection.

Secchi disk readings may be interpreted in terms of $V_\text{⊛}$ where ⊛ = 90 degrees provided the inherent contrast C_o of the disk, its area, and the adaptation level of the observer's eyes are known. Nomographic charts suitable for this purpose were prepared during World War II by the Tiffany Foundation under an O.S.R.D. contract.[6]

Eq. (2.15) shows that the inherent contrast C_o depends not only upon the reflectance of the disk <u>when submerged</u> (R_o^o), but also upon R_∞. Clearly, therefore, a Secchi disk reading unsupported by other data cannot be interpreted in terms of the more fundamental optical constants of a hydrosol.

A miminum of three independent measurements must be made in order to specify the optical properties of water. If, for example, V_o, V_K and R_∞ are measured directly, all of the optical constants described in this paper can be computed, except the path constant, (P), the irradiance ratio (H_1/H_2), and the background radiance N_{BO}. These polar-dependent quantities require additional (polar) data concerning the hydrosol and its lighting. The quantities P and N_{BO} are related by Eq. (2.28). The irradiance ratio (H_1/H_2) is of particular importance in most practical problems, because of its relation to the inherent contrast of the submerged object, as shown by Eq. (2.16). An automatic recording photoelectric photometer for obtaining polar curves of H_1/H_2 has been constructed.

8. WAVE ACTION

When a body of water is irradiated by the sun, the permeating sunlight, highly collimated near the surface, becomes progressively more diffuse at greater and greater depths. If wave structure exists, each wave facet initiates a plainly visible beam of sunlight. Thus, at shallow depths the water is filled with an intricate ensemble of dancing beams, but this pattern becomes rapidly less noticeable as depth is increased.

Beams of sunlight, like any collimated beams, have a hydrological range V_o. Scalar irradiance, however, has a hydrological range V_K which, as shown in Section 5 of this paper, always exceeds V_o. Thus, the beams of sunlight are attenuated more rapidly with depth than is the total radiant flux, and the spatial flux modulation due to wave action diminishes. For example, in a typical hydrosol for which $V_K = 2V_o$, the flux modulation at depth d = V_o is one seventh of the flux modulation just beneath the surface.

9. THE REFLECTANCE OF SUBMERGED SURFACES

The apparent reflectance of the submerged object, denoted by R_o in Eq. (2.15) and subsequent equations, is, by definition, the ratio of

6. Duntley, S. Q. "The Visibility of Distant Objects." J. O. S. A. <u>38</u>, 237 (1948)

the radiance of the object to the radiance of a perfectly reflecting surface similarly irradiated, i.e.:

$$R_O = \frac{N_O}{N_{H1}} \qquad (2.40)$$

Direct measurement of R_O is possible provided that the reflectometer is completely filled with water. Such equipment is rarely available. An alternative procedure based upon the use of a submergible telephotometer also requires uncommon equipment. The research barge shown in Fig.14 was used for most of the reflectance measurements made in connection with the studies described in this paper. A photoelectric telephotometer mounted within the barge looked along a horizontal path just above the track. A frame attached to the track about four feet from the barge held painted metal panels perpendicular to the axis of the telephotometer. Irradiance of the sample was at 45 degrees by means of a 200 watt incandescent lamp mounted within a blackened metal tube which served to minimize stray light in the water. A mat surface of opacified white plastic was used as a standard of reflectance. The absolute reflectance of this material in air was determined by conventional procedures.

Reflectance measurements with special water-filled equipment are not necessary. A technique has been evolved for measuring R_O with any conventional reflectometer which irradiates the sample at 45-degrees and views it normally. This technique involves wetting the sample and measuring it while the surface is covered by a thin film of water.

The light reflected from the surface of the sample will be given by Eq. (2.8) if t_1 is replaced by $t_{45} = 0.9719$ (computed for water of index 1.333 by means of Fresnel's equation), R_∞ is replaced by R_O, $T = 1$, and $R = 0$. This light will be diminished by a factor t_2 in escaping through the surface of the water film. The apparent reflectance w of the wet sample as measured by the reflectometer will, therefore, be:

$$w = \frac{t_2 H_O t_{45} R_O (1 - r_2 R_O)^{-1}}{H_O} \qquad (2.41)$$

The emergent transmittance (t_2) of the water film must be determined experimentally by means of direct measurement of w and R_O. Since, by definition, the emergent reflectance of the water film r_2 is related to t_2 by the relation $r_2 + t_2 = 1$, t_2 can be computed with the aid of Eq. (2.41). The best value is believed to be $t_2 = 0.580$. Thus, $r_2 = 0.420$.

Eq. (2.41) may be solved for R_O. Thus:

$$R_O = \frac{w}{r_2 w + t_2 t_{45}} \qquad (2.42)$$

Substituting the values given above for t_{45}, t_2, and r_z, Eq. (2.42) becomes:

$$R_O = \frac{w}{0.420w + 0.564} \qquad (2.43)$$

A plot of Eq. (2.43) is shown by the solid line in Fig.16 . Close agreement between measured values of R_O, shown by the plotted points, and the predictions of Eq. (2.43) was found, even for samples of moderate gloss.
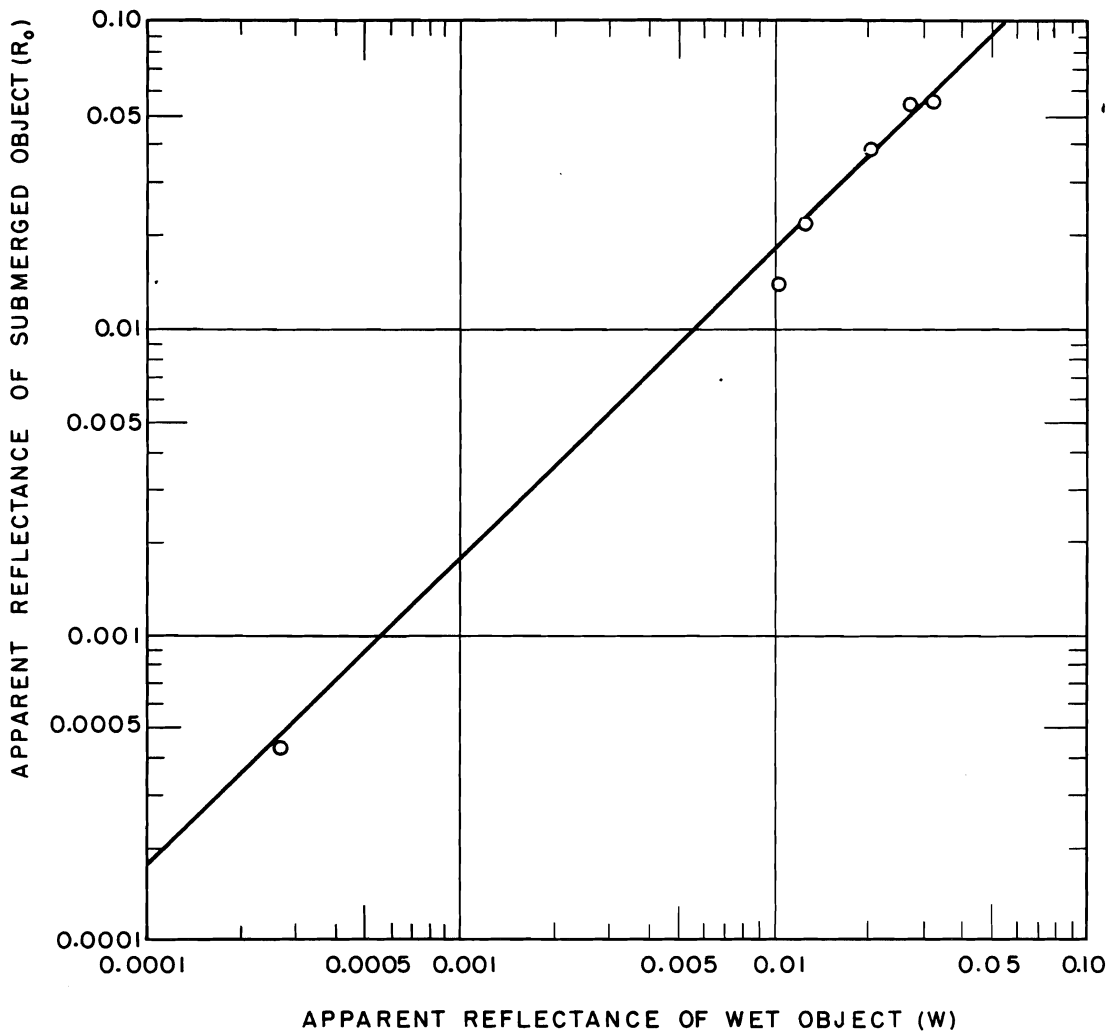

Figure 16

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CHAPTER III
The Inherent Contrast of Submerged Objects

The visual detectability of any object submerged in deep water depends upon its inherent contrast. This, in turn, depends both upon the reflectance of the object and upon the lighting conditions within the water. It is the purpose of this paper to discuss those special features of the distribution of light within the sea which govern the inherent contrast of submerged objects.

It will be useful to consider first the case of diffuse illumination at the surface of the sea from a substantially uniform sky. The water will be considered "infinitely deep" in the sense that no radiometric quantities at the target or at lesser depth would be affected if the water were deeper.

Consider any horizontal plane P at depth d beneath the surface of the sea. The layer of water between P and the surface will have reflectance R and transmittance T. Both R and T will depend upon the wavelength of the light used. The following discussion must be regarded, therefore, as strictly applicable to monochromatic light only, although it has been found in practice that the sea is sufficiently non-selective throughout the visible spectrum that equtions (3.3), (3.4), (3.5), and (3.6) of this paper are usefully applicable to photopically measured luminous quantities. Experiment, moreover, shows them to be applicable also on sunny days.

Let the downward irradiance on the surface of the water be H_o, and let t represent the transmittance of the water surface. The downwelling irradiance H_D on the top surface of plane P would be $H_o t_1 T$ if no interreflections took place. An infinite series of interreflections occur, however, between the surface of the water and the layer above the plane P. These increase H_D by a factor $(1-r_2R)^{-1}$, where r_2 is the "emergent reflectance" of water for diffused flux (see Duntley, S.Q. J. Opt. Soc. Am $\underline{32}$, 65 (1942)). Additional contributions to H arise from light which, having passed through the plane P, is reflected by the deeper water (R_∞). This light initiates an infinite series of interreflections between the waters above and below plane P and further interreflections occur between the water surface and the water. All of these interreflections may be summed as follows:

$$H_D = H_o t_1 T (1-r_2R)^{-1} \sum_{j=o}^{\infty} [RR_\infty + r_2 R_\infty T^2 t_1 (1-r_2R)^{-1}]^j$$

or $$H_D = H_o t_1 T \ [(1-RR_\infty)(1-r_2R) - r_2 T^2 t_1 R_\infty]^{-1} \qquad (3.1)$$

The upwelling irradiance H_u on the lower surface of plane P is also affected by interreflections. Infinite series of these arise from the same causes described above in connection with H_D. The complete expression for H_u may be written:

$$H_u = H_o t_1 T \ (1 - r_2 R)^{-1} R_\infty \sum_{j=0}^{\infty} [RR_\infty + r_2 R_\infty T^2 t_1 (1 - r_2 R)^{-1}]^j$$

or $\quad H_u = H_o t_1 T R_\infty [(1 - RR_\infty)(1 - r_2 R) - r_2 T^2 t_1 R_\infty]^{-1}$ (3.2)

Comparison of equations (3.1) and (3.2) shows that:

$$H_u = R_\infty H_D$$ (3.3)

Equation (3.3) indicates that the ratio of upwelling irradiance to downwelling irradiance is a constant, characteristic of the water, but independent of depth, i.e. $H_u/H_D = R_\infty$. It follows that if a flat matte horizontal surface has a reflectance $R_0 = R_\infty$, its apparent radiance $N_0 = R_o H_D$ will match that (N_B) of the deep-water background beneath, since $N_B = H_u$ in flux units. Such a surface is said to have zero inherent contrast for a downward-looking observer; the surface will be invisible regardless of its depth.

If N_o differs from N_B an inherent contrast C_0 will exist. Let C_0 be defined by the relation:

$$C_o = \frac{N_o - N_B}{N_B}$$ (3.4)

Thus: $\quad C_o = \dfrac{R_o H_D - H_u}{H_u} = \dfrac{R_o - H_u/H_d}{H_u/H_D}$

or $\quad\quad C_o = \dfrac{R_o - R_\infty}{R_\infty}$ (3.5)

The inherent contrast of a submerged matte horizontal surface is shown by equation (3.5) to depend only upon its reflectance R_o and upon R_∞ for the water; inherent contrast does not depend upon the depth of the object.

If the observer does not observe the surface perpendicularly, Equation (3.4) still defines the inherent contrast, but Equation (3.5) does not apply because $N_B \neq H_u$. In such a case, data on the directional variation of N_B must be obtained.

If the reflecting surface is not horizontal, but is viewed perpendicular to its surface, inherent contrast will be given by:

$$C_o = \frac{R_o - H_1/H_2}{H_1/H_2}$$ (3.6)

where H_2 is the irradiance on the surface facing the observer, and H_1 is the irradiance which would reach the plane of the surface on the side facing away from the observer if no object were present. H_1/H_2 has been called the "irradiance ratio". An analogous quantity E_1/E_2, called the "illuminance ratio" can usefully be employed to calculate inherent contrast by means of the approximate relation:

$$C_o = \frac{R_o - E_1/E_2}{E_1/E_2} \qquad (3.7)$$

An automatic recording photoelectric photometer for measuring illuminance ratio E_1/E_2 has been constructed and used at sea off Key West, Florida and off San Diego, California. Typical data obtained with this instrument are shown in Figure 17 .
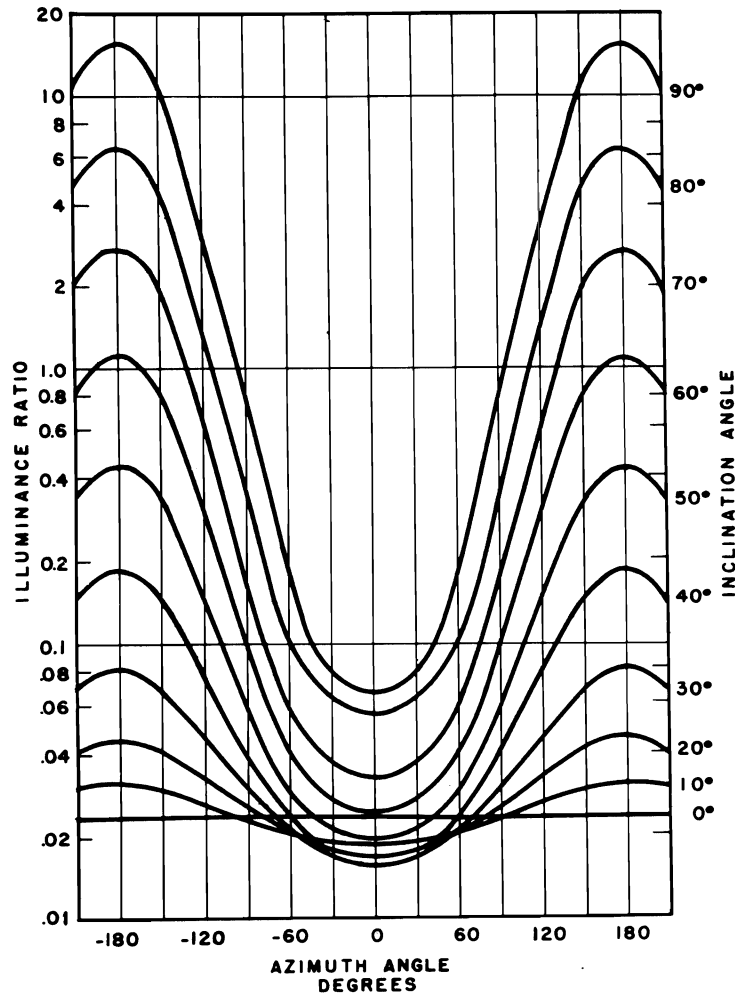

Figure 17

CHAPTER IV

A Water Clarity Meter

The fundamental objections raised by section 7 of Chapter 3 to the Secchi Disk method for measuring the clarity of ocean waters stimulated the design of a visual photometer free from the uncertainties inherent in the Secchi Disk method. Four of these instruments have been built and used at sea and at the Diamond Island Field Station.

The photometers, fabricated from stainless steel, have the approximate dimensions and shape shown in figure 18. They consist of two plane disks connected by a shaft. The upper disk slides on the
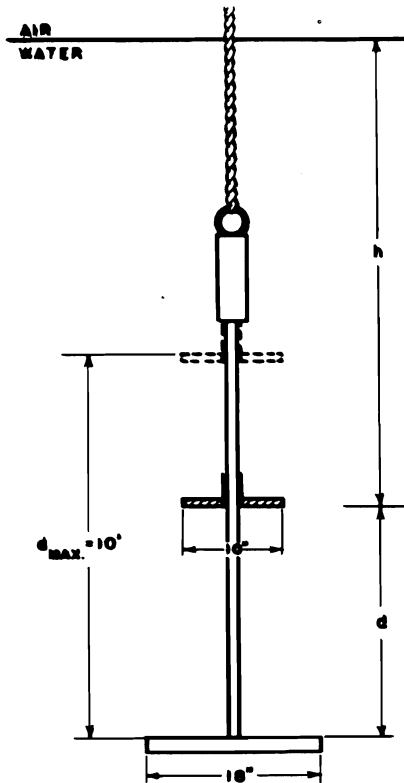

Figure 18

shaft so that it can be lifted above the lower disk by means of an auxiliary line. The photometer is to be suspended in the water and viewed from above. The upper (gray) disk is to be separated from the lower (white) disk until the two appear equally luminous. Once the disks are properly separated they will continue to match regardless of the depth to which the apparatus is submerged. The closer to the surface the photometer is brought, however, the easier will be the visual task of making the match. The separation of the disks, indicated by the displacement of the auxiliary line, is a direct measure of the clarity of the water and is convertible to hydrological range by a multiplying factor.

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The clarity of water can usefully be specified in terms of hydrological range (v). This is the distance, measured along the path of sight, at which the apparent contrast of any object seen against a deep-water background is reduced to two percent of its inherent value. Along a horizontal path of sight hydrological range (v_o) is related to the transmittance (T) of the water (as measured by a hydrophotometer) by the equation

$$T = e^{-3.912\,X/v_o} \qquad (4.1)$$

where X is the distance from the object to the observer and $3.912 =$ $\ln\frac{1}{.02} = \ln 50$. Along inclined paths of sight the hydrological range (v_θ) depends upon the angle (θ) between the line of sight and the horizontal in accordance with the following relation:

$$v_\theta = \frac{v_o \ln 50}{\ln 50 + K v_o \sin \theta} \qquad (4.2)$$

where k is the attenuation coefficient with depth of the volume density of luminous energy within the water. When $\theta = -90$ degrees the observer is looking straight downward. This special case is important to those interested in the visibility of submerged objects. The hydrological range along a downward vertical path (v_{-90}) is the basic datum needed for all submarine visibility calculations. From it, for example, can be calculated the disappearance depth of a small white disk, i.e., the "Secchi Disk" reading. Ordinarily the hydrological range (v_{-90}) will be 1/2 to 1/3 of the Secchi Disk reading, but the Secchi value will depend upon the state of the sea, the state of the sky, the adaptation level of the observer, the technique of observing, the reflectance of the disk, and the reflectance of the water.

The apparent contrast (C_1) of a submerged object of reflectance R_1 and depth h as seen by an observer looking vertically downward, Figure 19 , is given by

$$C_1 = \frac{R_1 - R_w}{R_w} \left(1 - e^{-1000\tan^2\phi_T/S} \right) \left(1 + \frac{S_o r_o b_z}{b_{w_1}} \right)^{-1} \left(e^{-3.912\,h/v_{-90}} \right) \qquad (4.3)$$
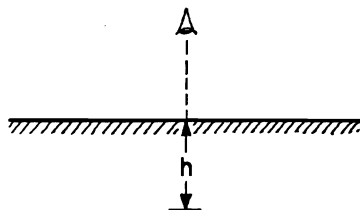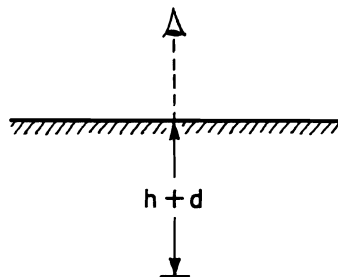

Figure 19   Figure 20

where R_w is the true reflectance of deep water and the remaining symbols refer to parameters which describe the state of the sea and the state of the sky. This equation allows for the effects of wave action and sky reflection. Although equation (4.3) is strictly true only for monochromatic light, it has been found to hold for white light as well.

The apparent contrast (C_2) of a submerged object of reflectance R_2 at depth h + d, Figure 20 , is given by

$$C_2 = \frac{R_2 - R_w}{R_w} \left(1 - e^{-1000 \tan^2 \phi \, \tau / S} \right) \left(1 + \frac{S_o r_o b_z}{b_{w_1}} \right)^{-1} \left(e^{-3.912(h+d)/v_{-90}} \right) \tag{4.4}$$

If equation (4.4) is divided by equation (4.3) all factors relating to sea-state, sky-state, wave structure, and sky reflectance divide out leaving:

$$\frac{C_2}{C_1} = \frac{R_2 - R_w}{R_1 - R_w} \, e^{-3.912 \, d/v_{-90}} \tag{4.5}$$

When the separation between the disks has been so adjusted that the two disks appear to match, the apparent contrasts C_1 and C_2 are equal. Thus equation (4.5) can be written:

$$\frac{R_1 - R_w}{R_2 - R_w} = e^{-3.912 d / v_{-90}} \tag{4.6}$$

The reflectance of ocean water (Rw) rarely exceeds 2 percent, the principal exceptions being regions of pollution near harbors. Since visual photometry of the type provided by the disks cannot be expected to provide precision as great as two percent, it is permissible to neglect Rw in equation (4.5) as long as R_1 and R_2 are much greater than Rw. If this is done equation (4.6) becomes:

$$\frac{R_1}{R_2} = e^{-3.912 d/v_{-90}} \tag{4.7}$$

Taking the logarithm of both sides of this equation:

$$\ln \frac{R_1}{R_2} = -3.912 \left(\frac{d}{v_{-90}} \right) \tag{4.8}$$

or $\qquad\qquad \ln \dfrac{1}{R_1} - \ln \dfrac{1}{R_2} = \ln 50 \left(\dfrac{d}{v_{-90}} \right) \tag{4.9}$

or $\qquad\qquad \log \dfrac{1}{R_1} - \log \dfrac{1}{R_2} = \log 50 \left(\dfrac{d}{v_{-90}} \right) \tag{4.10}$

It is convenient to speak of log 1/R as the "reflection density" of a painted surface, inasmuch as some commercial reflectometers are built to measure this quantity. Thus:

$$\log \frac{1}{R_1} - \log \frac{1}{R_2} = D_1 - D_2 = \Delta D \qquad (4.11)$$

Combining equations (4.10) and (4.11) and solving for hydrological range:

$$v_{-90} = \frac{\log 50}{D_1 - D_2} \, d \qquad (4.12)$$

or

$$v_{-90} = \frac{1.70}{\Delta D} \, d \qquad (4.13)$$

In equation (4.13) ΔD is the difference in reflection density between the surfaces of the two disks and d is their separation. The operator of the ocean clarity photometer need only multiply the value of d which he measures with the aid of the auxiliary line by a factor of $1.70/\Delta D$ in order to obtain the hydrological range.

Some of the water clarity photometers have been given a multiplying factor of 13.5, while others have a factor of 5.75. Since the maximum distance between disks is ten feet, some of the photometers can measure values of hydrological range up to 135 feet, while others provide an expanded scale for waters of lesser clarity. It is expected that water having hydrological range as great as 130 feet will be found in the Sargasso Sea and in the Mediterranean.