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#### **Authors**

Große, Cornelia S.  
Renkl, Alexander

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# Example-Based Learning with Multiple Solution Methods: Effects on Learning Processes and Learning Outcomes

Cornelia S. Große (cornelia.grosse@uni-bremen.de)

University of Bremen, Pedagogy and Educational Sciences  
P. O. Box 330440, 28334 Bremen, Germany

Alexander Renkl (renkl@psychologie.uni-freiburg.de)

Department of Psychology, Educational Psychology  
Engelbergerstr. 41, 79085 Freiburg, Germany

## Abstract

Many mathematical problems can be solved by different solution methods. A previous study showed that learning with multiple solutions fosters learning outcomes. However, the number of solution methods and the number of representational codes were confounded. In order to separate the effects of learning with multiple solution methods from those resulting from learning with multiple representations, the present experiment was conducted. The participants ( $N = 53$ ) learned to solve probability problems in one of three groups (multiple solution methods with multiple representations vs. multiple solution methods sharing one representation vs. uniform solution method). The superiority of learning with multiple solutions could not be replicated; however, the results show that learning with multiple solutions which share one representation is more effective than learning with multiple solutions and multiple representations. Differences in the learning processes are described and discussed.

**Keywords:** learning with multiple solution methods; learning with worked examples.

## Introduction

Currently, learning with multiple representations and learning from multiple perspectives is a very prominent research area. In their *Cognitive Flexibility Theory*, Spiro and his colleagues (e.g., Spiro, Feltovich, Jacobson, & Coulson, 1991) argue that the consideration of different perspectives deepens understanding and fosters learning outcomes. Thus, they propose that learners should be given the opportunity to explore learning contents from different perspectives, for example, by using multiple representations or multiple approaches. Similarly, it can be argued that mathematics learning and deep understanding can be fostered by presenting learners different possible solution methods for solving a problem – though many people think that mathematical problems can always be solved with only one method, in most cases different methods can be applied.

A very effective instructional approach is learning with worked examples. However, worked examples only show one solution which stands in contrast to the "multiple solutions" approach. Taking this into consideration, it appeared worthwhile to explore the effectiveness of a combination of both approaches, that is, to test worked examples with different possible solution methods. In a study of Große and Renkl (2003), learners were given examples which demon-

strated two different solution methods. The results provided first evidence that this example feature fosters learning.

In the following, some models and findings relevant to the topic of multiple solutions are discussed, and research on learning from worked examples is briefly reviewed. Then, a study which analyzed learning with multiple solutions in detail is presented.

## Learning with Multiple Solutions

The following aspects underline the effectiveness of learning with multiple solution methods:

*Fostering understanding and improving flexibility.* It is assumed that the consideration of different solution methods can help the learners to apply them more flexibly and effectively. For example, Tabachneck, Koedinger, and Nathan (1994) found that in order to solve algebra problems a combination of strategies was more effective than the employment of only one strategy; according to their explanation, a combination of strategies helps to overcome disadvantages and weaknesses of single strategies. Spiro and his colleagues (e.g., Spiro et al., 1991) propose in their *Cognitive Flexibility Theory* that learning environments should be designed flexibly so that learners have the opportunity to consider learning contents from different perspectives which in turn deepens their understanding and fosters transfer performance. The theory accentuates the use of multiple approaches and multiple representations. As it can be argued that reflecting on multiple perspectives resembles the consideration of multiple solutions, it can be concluded that multiple solutions should foster learning outcomes.

*Individualization.* Some researchers (e.g., Sjuts, 2002) claim that many learners have preferences for specific solution methods. Thus, it is argued that performance can be enhanced by giving learners a variety of methods from which they can choose a strategy they comprehend well.

*Motivation.* According to Deci and Ryan (1993), three components are essential for motivation: experience of competence, autonomy, and social integration. Considering multiple solutions gives the learners degrees of freedom, which should have positive effects on autonomy and experienced competence and, consequently, on motivation.

Multiple solution methods in mathematics allow for the use of different representations. As Ainsworth, Bibby, and Wood (2002) point out, multiple representations can con-

tribute to the understanding of each single representation and can help to avoid misinterpretations. In addition, connecting multiple representations is suitable for gaining a deeper understanding of the learning contents, as complex interdependencies can be interpreted in new ways and generalization can be fostered. According to de Jong et al. (1998), multiple representations can have several functions: specific information can best be conveyed in a specific representation; a specified sequence of learning material is beneficial for learning; and expertise is often seen as the possession and coordinated use of multiple representations. Thus, it can be concluded that learning with multiple representations (or multiple solutions) may foster understanding. However, learning with multiple representations is also very challenging, as the learners not only have to understand each single representation, but they also have to integrate them in order to establish coherence. As learners rarely map different representations onto each other, the positive effects that were intended by the use of multiple representations often do not occur to the expected extent (cf. de Jong et al., 1998).

To sum up, it can be concluded that the consideration of multiple solution methods is very promising and challenging at the same time. As different solution methods can be presented by means of worked examples, the following section outlines some basic findings of this branch of research.

### Learning with Worked Examples

A worked example consists of a problem, the solution steps and the final solution itself. Learning with worked examples is primarily used in well-structured domains (e.g., mathematics) and means that multiple examples are given to students before they try to solve problems on their own. In comparison to learning by problem solving, learning with worked examples is very effective (for an overview see Atkinson, Derry, Renkl, & Wortham, 2000). The effectiveness of worked examples can be explained with the *Cognitive Load Theory* (e.g., Paas, Renkl, & Sweller, 2003). As worked examples relieve the learners from finding a solution on their own, extraneous cognitive load is reduced; thus, the learners can concentrate on understanding the solution and the underlying principles.

However, the employment of worked examples does not guarantee good learning results. Learning outcomes depend on design features (cf. Atkinson et al., 2000) and on the learner's activity. Chi et al. (1989) noted that worked examples typically do not include all of the reasons why a certain step in the solution is performed. They found that some learners attempted to establish a rationale for the solution steps by trying to explain the examples to themselves, and that these learners learned more than those who did not generate explanations – a phenomenon they termed *self-explanation effect*. A number of studies (e.g., Renkl, 1997) could replicate this finding of Chi et al. (1989).

### Worked Examples with Multiple Solutions

A combination of worked examples and multiple solutions might be very effective as it can be argued that the employ-

ment of worked examples reduces extraneous cognitive load. This enables the learners to use "free" cognitive capacity for the integration of multiple solution methods, which in turn may bring to bear the advantages of learning with multiple perspectives. Große and Renkl (2003) combined these two branches of research and tested the effectiveness of example-based learning with multiple solutions. It was explored whether the presentation of different solution methods fosters learning outcomes compared to the presentation of only one solution method, and whether learning is enhanced by the provision of textual instructional explanations or by prompting the learners to write down self-explanations. The participants ( $N = 170$ ) learned to solve combinatorics problems under six conditions that constituted a 2x3-factorial design (factor "multiple solutions": multiple vs. uniform; factor "instructional support": no support vs. prompting self-explanations vs. instructional explanations). In the learning phase, the learners were given two sets of worked examples; each set contained two examples which shared the same structure and many elements of the surface story. In the "multiple solutions" conditions, the two nearly identical examples of a set were solved using different solution methods (a diagram and an arithmetic solution); thus, the participants learned that more or less the same problem could be solved by using two different solution procedures. In the "uniform" conditions, both examples of an example set were solved with the same solution method (i.e., the two examples of the first set were solved using diagrams, and the two examples of the second set were solved arithmetically). Independently of the experimental condition, each solution method was presented twice to each learner (see table 1).

Table 1: Sequence of worked examples and solutions.

		<i>Multiple solutions</i>	<i>Uniform solution</i>
<i>Set 1</i>	<i>Example 1</i>	diagram	diagram
	<i>Example 2</i>	arithmetic	diagram
<i>Set 2</i>	<i>Example 3</i>	diagram	arithmetic
	<i>Example 4</i>	arithmetic	arithmetic

The learning outcomes were assessed through a post-test that required the flexible application of solution methods and the explication of advantages and disadvantages of different methods. The results showed that learning with multiple solutions was very effective, irrespective of any instructional help. Actually, no positive effect was found for instructional support.

However, the number of solution methods and the number of representations were confounded, as the participants either learned with multiple solutions which were realized by multiple representations (arithmetical *and* graphical), or they learned with only one solution method and thus with only one representation (arithmetical *or* graphical) in one example set. The present experiment was conducted in order to separate the effects of multiple solutions from the effects of multiple representations. In addition, it was not clear

whether the self-explanation prompts used by Große and Renkl (2003) focused on the "right" cognitive processes – possibly, other prompts would have had stronger effects on learning outcomes. Thus, in-depth analyses were conducted in order to identify the relevant processes in learning from multiple solutions.

## Research Questions

The following research questions were addressed:

(1) Can the effect be replicated that learning with multiple solutions is more effective than learning with one solution method? Is the number of representations of importance?

(2) In which way are learning processes, especially self-explanations, affected by the presentation of multiple solution methods with or without multiple representations?

## Methods

### Sample and Design

The participants of this study were 37 female and 16 male students of the University of Freiburg, Germany. The mean age was 23 years ( $M = 22.98$ ,  $SD = 3.34$ ). The study took place in individual sessions. A design with three groups was implemented. Participants in the group "multiple solution methods with multiple representations" ( $n = 18$ ) were presented different solution methods which were based on different representational codes (i.e., graphical *and* arithmetical). Learners in the group "multiple solution methods sharing one representation" ( $n = 17$ ) also received multiple solutions which, however, shared the same representation (i.e., graphical *or* arithmetical). In the third group "uniform solution method" ( $n = 18$ ), the participants were presented worked examples with only one solution method.

### Materials

Probability calculation was chosen as the learning domain. The learning materials contained four sets of problems which could be solved by four different solution methods (a diagram based on whole numbers, a diagram based on rational numbers, an arithmetic solution based on whole numbers, and an arithmetic solution based on rational numbers). Each set contained two structurally identical problems (e.g., "Three different balls are in an urn. Three times, you take a ball and put it back into the urn. What is the probability for taking a different ball each time?" and "Three different candies are in a glass. Three times, you take a candy and put it back into the glass. What is the probability for taking a different candy each time?"). Thus, eight examples were presented. The two examples of an example set shared not only the same underlying structure but also a number of surface features in order to make the correspondence salient to the learners. In the "multiple solutions" groups ("multiple solution methods with multiple representations" and "multiple solution methods sharing one representation"), each set of analogical problems was presented with two different solutions. In the case of multiple solutions with multiple repre-

sentations, different representations were used (i.e., the first problem was solved using a diagram, the second one was solved arithmetically); in the case of multiple solutions sharing one representation, both solutions used the same representation (for example, two very similar problems were solved using two different arithmetic solutions). The learners in the "multiple solutions" conditions should easily detect that different procedures were employed for more or less the same problem. In the "uniform solution method" group, both examples of a set were solved using the same method. Thereby, the number and the type of the presented worked-out solutions were held constant, that is, independently of the experimental condition, the eight worked examples demonstrated each of the four solution methods twice. Thus, differences between the experimental groups can not be attributed to the type or to the number of different worked-out solutions in the learning phase. The experimental variation focused on whether it was shown that the same problem can be solved by different methods.

### Procedure

First, the participants read a short text (488 words) which addressed basic principles of probability in order to enable them to understand the following worked examples. Afterwards, the learners worked on the pretest, which was followed by the learning phase (presentation of worked examples), where the experimental variation took place. The participants were told to verbalize everything that comes to mind while studying the worked examples, and to simultaneously relay each of their thoughts as they appear in their minds (according to the guidelines of Ericsson & Simon, 1993). This think-aloud procedure was trained with some brain-teasers. Due to technical problems, the thinking aloud data of 3 learners are missing. Finally, they worked on the post-test. The duration of the experiment was approximately 80 minutes ( $M = 80.58$ ,  $SD = 16.42$ ).

### Analysis of Example Processing

For the coding of example processing, the scheme of Renkl, Atkinson, and Große (2004; cf. also Renkl, 1997) was employed in an adapted version. The following categories were assessed:

(1) *Principle-based explanations*. The number of times that participants refer to the principles of probability was counted (e.g., "This is the multiplication rule"). In addition, this category was scored if participants verbalized principle-based elaborations concerning the construction of diagrams.

(2) *Noticing inter-example coherence*. This category assesses the extent to which coherence between examples is perceived. Each statement was coded in which the worked example presently being studied is related to an earlier one.

(3) *Comparisons and connections between solution methods*. This category was coded if the learners compared solution methods (e.g., with respect to advantages and disadvantages), or if they elaborated on connections and interrelations between solution methods.

(4) *No connection between solution methods.* This category was coded if learners stated that different solution methods did *not* share any connections or interrelations (e.g., "This arithmetic solution does not have anything to do with the diagram presented above").

(5) *Anticipative reasoning.* If learners elaborated on the next solution step in advance (i.e., without looking at the next worked step) this category was coded.

Each thinking-aloud protocol was coded independently by two raters. The interrater agreement was fair (kappa coefficient: .66). In cases of disagreement between raters, the final code was determined by discussion.

## Instruments

**Pretest: Assessment of prior knowledge.** A pretest containing 6 relatively simple problems examined the probability knowledge of the participants before the presentation of the worked examples. The maximum score was 6 points.

**Post-test: Assessment of learning outcomes.** The learning outcomes were measured with a post-test containing 12 problems. According to the number of required single answers they were awarded 1 to 3 points. *Computational skills* were assessed by 2 problems which were similar to the examples presented in the learning phase and 5 problems which required the application and adaptation of solution methods presented in the learning phase. A maximum score of 14 points could be achieved. *Conceptual skills* were tested by 5 items which asked the learners to give verbal elaborations on the correctness or universal validity of given solution methods. A maximum score of 10 points could be achieved.

## Results

Tables 2 and 3 show means (and standard deviations) of pretest and post-test scores and self-explanations in the experimental groups. Concerning the pretest, an ANOVA did not reveal significant group differences,  $F < 1$ .

Table 2: Means (and standard deviations) of the pretest and post-test scores in the experimental groups.

	<i>Multiple solutions with multiple representations</i>	<i>Multiple solutions sharing one representation</i>	<i>Uniform solution method</i>
<i>Pretest</i>	4.36 (1.26)	4.21 (1.21)	4.56 (1.19)
<i>Computational skills</i>	9.58 (2.01)	9.03 (3.28)	10.17 (3.33)
<i>Conceptual skills</i>	5.47 (1.16)	5.85 (1.18)	5.47 (1.19)

**Computational and conceptual skills.** In order to test the effects of the experimental variation and interaction effects with prior knowledge, ANOVAs with the factor "experimental group" and the factor "prior knowledge" (as *continu-*

*ous* variable) were computed. With respect to *computational skills*, a comparison of the "uniform" group with both "multiple" groups revealed neither a significant group difference nor a significant interaction with prior knowledge, both  $F < 1$ . As to be expected, prior knowledge substantially influenced computational skills,  $F(1, 49) = 19.92, p < .001, \eta^2 = .289$ . With respect to *conceptual skills*, a comparison of the "uniform" group with both "multiple" groups again revealed neither a significant group difference nor a significant interaction with prior knowledge, both  $F < 1$ . As to be expected, prior knowledge substantially influenced the acquisition of conceptual skills,  $F(1, 49) = 3.94, p = .053, \eta^2 = .074$ . Thus, the consideration of multiple solution methods did not foster learning outcomes.

Table 3: Means (and standard deviations) of the self-explanations in the experimental groups.

	<i>Multiple solutions with multiple representations</i>	<i>Multiple solutions sharing one representation</i>	<i>Uniform solution method</i>
<i>Principle-based explanations</i>	3.65 (3.32)	3.13 (2.55)	3.53 (3.43)
<i>Noticing coherence</i>	2.35 (2.09)	3.31 (3.18)	5.94 (4.15)
<i>Comparisons and connections</i>	2.24 (1.82)	1.31 (1.62)	2.00 (2.50)
<i>No connection between solutions</i>	.00 (.00)	.13 (.50)	.29 (.59)
<i>Anticipative reasoning</i>	.65 (.93)	.19 (.54)	1.29 (1.21)

In order to gain insight into the significance of the number of representations when learning with multiple solutions, ANOVAs comparing the two "multiple" groups were computed. Concerning *computational skills*, no significant group difference and no significant interaction with prior knowledge were found, both  $F < 1$ . As to be expected, a significant influence of prior knowledge was found,  $F(1, 31) = 10.19, p = .003, \eta^2 = .247$ . With respect to *conceptual skills*, the groups differed significantly,  $F(1, 31) = 4.57, p = .041, \eta^2 = .128$ . Multiple solutions sharing one representation were more effective than multiple solutions with multiple representations. In addition, a tendency was found that for learners with low prior knowledge, learning with only one representation was more effective, whereas learners with good prior knowledge profited from learning with multiple representations,  $F(1, 31) = 3.50, p = .071, \eta^2 = .102$ . As to be expected, the influence of prior knowledge was significant as well,  $F(1, 31) = 6.01, p = .020, \eta^2 = .162$ .

**Self-explanations.** Concerning principle-based self-explanations, no significant correlations with post-test measures were obtained. In addition, no significant group differences were found,  $F < 1$ , thus, multiple solutions did not influence

principle-based self-explanations. With respect to noticing inter-example coherence, significant group differences were found,  $F(2, 47) = 5.54, p = .007, \eta^2 = .191$ ; the learners in the "uniform" group stated similarities between examples more often than the learners in the other groups. Significant correlations with learning outcomes were only found in the group "multiple solutions with multiple representations", in which noticing coherence correlated with conceptual skills,  $r = .53, p = .028$ . No significant group differences were found with respect to comparisons and connections between solutions,  $F < 1$ . In the "uniform" group, comparing and connecting solution methods across example sets was associated with computational skills,  $r = .42, p = .091$ . In the "multiple" groups, no substantial correlations were found. Statements expressing that different solution methods did not share connections or interrelations did not occur with different frequencies,  $F(2, 47) = 1.88, p = .165$ . However, a comparison between the "uniform" group on the one hand and both "multiple" groups on the other hand revealed that learners in the "uniform" group tended to believe more often that solution methods did not have anything in common,  $F(1, 48) = 3.12, p = .084, \eta^2 = .061$ . A substantial correlation with learning outcomes was only found in the group "multiple solutions sharing one representation", as to be expected, computational skills correlated negatively with expressing that different solution methods did not share any commonalities,  $r = -.45, p = .083$ . With respect to anticipative reasoning, significant differences between the experimental groups were found,  $F(2, 47) = 5.75, p = .006, \eta^2 = .196$ ; the "uniform" learners anticipated more compared to the learners in the other groups. In the "uniform" group, anticipative reasoning correlated positively with conceptual skills,  $r = .42, p = .096$ . Contrary to expectation, in the group "multiple solutions sharing one representation", anticipative reasoning correlated negatively with conceptual skills,  $r = -.45, p = .080$ , thus, in this group, anticipations were not associated with good learning outcomes.

In summary, learning with multiple solutions seems to foster the insight in the interrelations between different solution methods. However, it reduces the awareness of similarities between problem types. In addition, when learning with multiple solutions, the number of spontaneous problem solving activities (anticipations) is reduced.

## Discussion

Despite the promising results of Große and Renkl (2003), the superiority of learning with multiple solutions could not be replicated. However, in order to overcome the weakness of this study, where the number of presented solution methods and the number of representations were confounded, each participant – irrespective of the experimental condition – learned four different solution methods. Strictly speaking, the experimental variation only took place with respect to the order of the presentation of the solution methods; therefore, the experimental variation was rather "small".

The variation of the number of representations in one example set led to differences with respect to conceptual

skills; multiple solutions which shared one representation led to better learning outcomes than multiple solutions with multiple representations. However, for learners with good prior knowledge, the employment of multiple solutions *and* multiple representations seems to be beneficial. Possibly, for learners with low prior knowledge, instructional support with respect to the integration of multiple representations could lower the demands and foster learning outcomes.

In this experiment, solution methods which shared one representation differed with respect to their mathematical basis, whereas solution methods which used different representations shared the same mathematical basis. Thus, it can be concluded that possibly not only the number of representations is of importance but also the degree of mathematical dissimilarity: Learning with multiple solutions may be especially effective when, from a mathematical point of view, the presented solution methods differ to a great extent.

Learning with multiple solutions caused some interesting changes in the learning processes. The "uniform" learners anticipated substantially more compared to the learners in the "multiple" groups and this was associated with good learning results with regard to conceptual skills. Maybe learning with multiple solutions is so demanding that learners do not try to anticipate solutions on their own. In the group "multiple solutions sharing one representation", anticipative reasoning correlated negatively with conceptual skills. It is possible that a strong tendency to anticipate solutions may implicate that the presented solutions – which looked quite similar as they were based on the same representation – were not processed attentively by the learners, which may have led to lower learning outcomes.

The "uniform" learners stated similarities between problems more often than the "multiple" learners. It is likely that the uniform solutions were easier to process which may have given the learners more opportunities to think about similarities between problem types; in addition, solving isomorphic problems using the same solution method may have helped the learners to find commonalities between problems. When learning with multiple solutions and multiple representations, noticing inter-example coherence seems to be crucial for the acquisition of conceptual skills. This may be explained by the fact that in this group, inter-example coherence was the least salient, thus, in order to acquire general problem-solving schemata, an active elaboration on commonalities may be very important. Possibly, learning outcomes can be further enhanced by asking the learners to compare the presented examples attentively.

Against our intention, in the "uniform" group, the "multiplicity" of the solution methods was recognized in some cases, and comparing different solution methods (across example sets) correlated with learning outcomes with respect to computational skills. Thus, the consideration of multiple solutions seems to be effective even if the solution methods are not contrasted explicitly. These findings can explain the result that no substantial superiority of the "multiple" groups was found with respect to learning outcomes.

In order to reach good learning outcomes, it seems to be important that interrelations between solution methods are *not* neglected. Especially in the group "multiple solutions sharing one representation", where different solution methods looked similar, neglecting interrelations between them was associated with lower learning outcomes with respect to computational skills. Statements expressing that different solution methods have nothing in common tended to occur more often in the "uniform" group compared to the "multiple" groups; thus, the presentation of multiple solution methods seems to be suitable in order to foster insights in commonalities and analogies between different solution methods. Possibly, learning outcomes could be further enhanced if the learners are explicitly prompted to compare and integrate the presented solution methods.

In the following, some hypotheses on factors that enhance learning outcomes from multiple solutions are outlined. These hypotheses should be addressed in further research:

(1) Providing instructional support with respect to the integration of multiple solutions (or multiple representations, respectively) may enhance learning outcomes.

(2) Certain combinations of solution procedures may be more helpful than others. For example, a combination of informal methods with sophisticated ones may especially enhance learning outcomes, as the informal methods may help the learners to better comprehend the more sophisticated ones.

(3) Many students think that mathematical problems can have only one solution (Schoenfeld, 1992). Thus, the effectiveness of learning with multiple solutions may be enhanced if learners are informed that there are several different correct approaches to solving a problem. Only then, may students be willing to "accept" different methods. Beyond this, results in the long term may differ from those in the short term. It is likely that students have to get used to considering multiple solutions before substantially profiting from them.

(4) Presenting different solution methods by means of worked examples is not the only possibility; in contrast, it may also be very effective to encourage the learners to develop them on their own.

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