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ABSTRACT

The equation of state of nuclear matter at finite temperature and density is calculated in a relativistic field theory of interacting baryons and mesons. At subnuclear densities there exists a liquid-gas phase transition with a critical point at T = 17.3 MeV, n = 0.052 fm. We estimate the critical beam energy in symmetric head-on nucleus-nucleus collisions for which the exploding nuclear matter passes through the critical point. The dynamics of phase separation is studied in the extreme limits of no dissipation and maximum dissipation.

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I. Introduction

The existence of a liquid-gas phase transition in nuclear matter has been predicted for decades. However, there has been a great surge of interest recently in this phase transition due to the possibility of observing its effects in high precision high energy proton-nucleus or medium energy nucleus- nucleus collisions. (See, for example, refs. 1-7. For a review see ref. 8). Our purposes in this short paper are twofold: (1) For central collisions of massive nuclei there ought to exist a critical beam energy such that the exploding nuclear matter passes through the critical point. Whether or not passage through the critical point leaves an observable signal is still a much debated question.[4,6-10] However, it is certainly important to know *where* one should be looking! We make a best estimate of the critical beam energy, as well as reasonable upper and lower limits. (2) It is virtually impossible for the exploding nuclear matter in medium and high energy nucleus-nucleus collisions to avoid the liquid-gas phase separation boundary, unless, of course, the nuclear matter goes out of thermal equilibrium and free-streams before it reaches the boundary.[5] Thus it is essential to understand the dynamics of the phase separation. We study phase separation in the extreme limits of no dissipation and maximum dissipation assuming phase coexistence, that is, no supercooling or superheating and the like.

II. Relativistic Nuclear Field Theory

We calculate the properties of isospin symmetric nuclear matter by solving, in mean field approximation, the field equations that are derived from a relativistically invariant Lagrangian which describe the interaction of baryon and meson fields. These hadronic fields are regarded as effective, composite fields of the elementary quarks and gluons which are convenient for the description of hadronic matter at temperatures and densities below the deconfinement phase transition. The coupling constants are regarded as effective couplings that are determined by the bulk properties of nuclear matter, although, in practice, they do not differ very much from their values in vacuum. This approach was proposed long ago[11] and has enjoyed a recent revival because of its success in describing not only bulk nuclear properties but also a large number of single-particle properties.[12,13]

It will turn out that the density and temperature ranges of relevance to our study are $n /20 < n < 3n_0$ and $T < 60$ MeV, where $n_0 = 0.145$ fm is normal nuclear matter density. Therefore, it is sufficient to include the nucleon and delta fields, the scalar (σ) and vector (ω) fields which develop nonzero mean values, as well as the other mesons such as the pion, eta, rho, kaon, etc. Interactions are included as follows: the nucleons and deltas have Yukawa interactions with the scalar and vector fields, and the scalar field has cubic and quartic self-interactions. The coupling constants in the theory are determined by requiring that cold nuclear matter saturates at $n_0 = 0.145$ fm with a binding energy of 16 MeV and a compressibility of $K = 280$ MeV.

The details of how the equation of state is calculated in the mean field approximation have been discussed many times before and so we do not repeat them here.[12-15] As an application, which will be useful later, we solve the relativistic Rankine-Hugoniot-Taub shock equations for collisions of infinite slabs of nuclear matter.[16] This is usually taken as a rough estimate of the maximum density to be achieved and of the entropy to be generated in central collisions of very massive nuclei. In Fig. 1 we plot the compression n/n_0, the temperature T and the entropy per baryon S versus the laboratory beam energy. These initial conditions will be of interest when we consider the subsequent expansion of the nuclear matter into the region of the liquid-gas phase instability.

III. Critical Beam Energy

The relativistic nuclear field theory discussed above naturally gives rise to a nuclear liquid-gas phase transition. The phase diagram is shown in Fig. 2. The outer envelope is the Maxwell curve. A line at fixed T intersects this curve at two points. The high density point is the liquid and the low density point is the gas. They are in kinetic equilibrium (equal temperatures, $T_L = T_G$), in chemical equilibrium (equal chemical potentials, $\mu_L = \mu_G$), and in mechanical equilibrium (equal pressures, $P_L = P_G$).[17] The critical point is at $T_c = 17.3$ MeV, $n_c = 0.052$ fm. For $T > T_c$ there is no distinction between liquid and gas, there is only nuclear vapor.

The middle envelope, the one that touches the critical point, is the isothermal spinodal. Along this spinodal the isothermal speed of sound, u_T, vanishes. Inside this spinodal $u_T^2 < 0$ and so neither uniform liquid nor uniform gas is stable.[17] Between the isothermal spinodal and the Maxwell curve nuclear matter in a single phase is metastable. On the high density side liquid nuclear matter is metastable, while on the low density side gaseous nuclear matter is metastable.[17]

The inner envelope is the adiabatic spinodal within which the isentropic speed of sound is imaginary, $u_S^2 < 0$, in addition to $u_T^2 < 0$.

In a central collision between massive nuclei at medium or high energy the nuclear matter will be heated and compressed shortly after impact. Subsequently the matter cools and decompresses. As T and n decrease the matter will eventually reach the Maxwell curve. Phase separation begins in one of two ways depending on the circumstances. If the nuclear matter reaches the Maxwell curve on the high density side ($n > n_c$, $T < T_c$) then bubbles begin to form in the liquid. If the nuclear matter reaches the Maxwell curve on the low density side ($n < n_c$, $T < T_c$) then droplets begin to form in the gas.

There are several proposed observables to signal the passage of the expanding nuclear matter through the critical point.[1-6] Without entering into the discussion of whether or not it is possible to observe the passage experimentally, we would still like to know what beam energy gives rise to the initial conditions necessary for the subsequent expansion through the critical point. We will make some estimates based on energy and entropy considerations.

In order for the nuclear matter to pass through the critical point there must be sufficient energy. The excitation energy (above the energy of cold nuclear matter) at the critical point is 26 MeV per baryon. In general as the system expands internal energy is converted to collective energy of motion. Therefore the minimum beam energy in the center of mass frame must be 26 MeV per nucleon. This would be the case if thermal equilibrium was first attained at density n. The lower bound on the beam energy in the laboratory frame is thus 105 MeV per nucleon.

Assuming that the colliding nuclei are massive enough that they stop in the center of mass frame an upper limit on the beam energy may be found from entropy conservation. The argument goes as follows: The entropy per baryon at the critical point is $S = 2.8$. The maximum attainable density for a given beam energy is given by the solution of the relativistic Rankine-Hugoniot-Taub shock equations. If the colliding nuclei thermalize later, at a lower density, the entropy will be higher. Since the laws of thermodynamics forbid a decrease of entropy during the expansion, this means that the shock equations provide an upper bound on the beam energy to achieve the critical point entropy S. From Fig. 1 we see that it takes a laboratory beam energy of 540 MeV per nucleon to generate 2.8 units of entropy.

Thus we have placed bounds on the critical beam energy; it must be between 105 and 540 MeV per nucleon in the laboratory frame. Actually we can make an estimate of its most likely value. A number of studies in detailed dynamical models[19-21] indicate that finite nuclei should thermalize at a somewhat lower density than that given by the shock equations, and that some entropy is generated during the expansion by dissipative forces.[8] Altogether the entropy per baryon may be enhanced over that indicated in Fig. 1 by 20%. Thus our best estimate for the critical beam energy is 330 MeV per nucleon.

IV. Trajectories in Phase Space

What happens when the expanding nuclear matter hits the Maxwell curve depends on the relative magnitudes of the nucleation rate and the expansion rate. If the nucleation rate is relatively large then the system will expand in phase equilibrium with the relative proportions of gas and liquid determined by the temperature T and the mean density n. This is the scenario we shall assume. If the nucleation rate is relatively small then phase equilibrium will not be achieved, resulting in substantial superheating or supercooling. It may even be that the system enters the zone of isothermal instability ($u_T^2 < 0$) or even the zone of isentropic instability ($u_S^2 < 0$).[7] If this be the case then phase separation begins via spinodal decomposition and not nucleation.[18] Clearly it is important to estimate the nucleation rates but that is a topic beyond the scope of this brief work. We will assume that the nucleation rates are relatively large so that the nuclear matter expands in phase equilibrium.[5]

At fixed $T < T_c$ liquid and gas can be in phase equilibrium with densities $n_L(T)$ and $n_G(T)$. For densities n such that $n_G(T) < n < n_L(T)$ the system consists of a mixture of liquid and gas with the above mentioned densities. The phase abundances are defined as follows: $\alpha_G(T)$ and $\alpha_L(T)$ are the fractions of the baryon number contained in the gas and liquid phases with $\alpha_L + \alpha_G = 1$, and $\lambda_G(T)$ and $\lambda_G(T)$ are the fractions of the volume occupied by the gas and liquid phases with $\lambda_G + \lambda_L = 1$.

These are two extreme scenarios for how the system expands through the region of phase equilibrium. In the limit of maximum dissipation no additional internal energy is converted to collective motion of expansion; the nuclear matter simply coasts. This is an isoergic expansion. In the limit of no dissipation the maximum allowable fraction of internal energy is converted to collective motion of expansion; the system expands as rapidly as possible. This is an adiabatic expansion.

Sample expansion trajectories for these scenarios are illustrated in Fig. 2. The adiabatic and the isoergic expansions begin at the Maxwell curve with T = 10, 15 and 17 MeV. The system expands with fixed entropy per baryon S = 1.25, 2.02, 2.56 and 3.04 in the former instance, and with fixed energy per baryon E* = 6, 16, 23 and 29 MeV in the latter instance. In the T-n plane these two limiting scenarios give qualitatively similar trajectories.

The fraction of the volume which is occupied by the gas phase is plotted in Fig. 3. If the system starts in the liquid phase then λ_G begins at zero, whereas if the system starts in the gas phase then λ_G begins at unity. These two scenarios also give qualitatively similar dependences of λ_G on T. Note that in all cases the gas phase occupies nearly all the volume as the system cools.

The fraction of baryon number which is carried by the gas phase is plotted in Fig. 4. Now we do see a qualitative difference between the scenarios. At late times a is increasing for the isoergic expansion but a is decreasing for the adiabatic expansion. This is not unreasonable. The isoergic expansion generates entropy at the maximum allowable rate. Since the specific entropy of the gas phase is higher than the specific entropy of the liquid phase[5] it is most efficient to make as much gas phase as possible. This difference in the time dependence of the baryon fraction a carried by the gas phase may lead to observable consequences in the fragment mass spectra.[4,8]

V. Conclusion

We have studied the nuclear liquid-gas phase transition using a relativistic quantum field theory for baryons and mesons. For central collisions between massive nuclei we place lower and upper bounds of 105 and 540 MeV per nucleon on the laboratory energy necessary for the nuclear matter to expand through the critical point. Our best estimate is 330 MeV per nucleon. We have also studied the dynamics of expansion in the region of mixed phase in the opposite limits of maximum and minimum dissipation. A greater percentage of the baryon number is carried by the gas phase in the isoergic expansion than in the adiabatic expansion.

Clearly our study, and others, point to the need for accurate calculations of the nucleation rates and spinodal decomposition times to better understand what actually happens when nuclear matter expands through the region of phase coexistence in heavy ion collisions.

Acknowledgments

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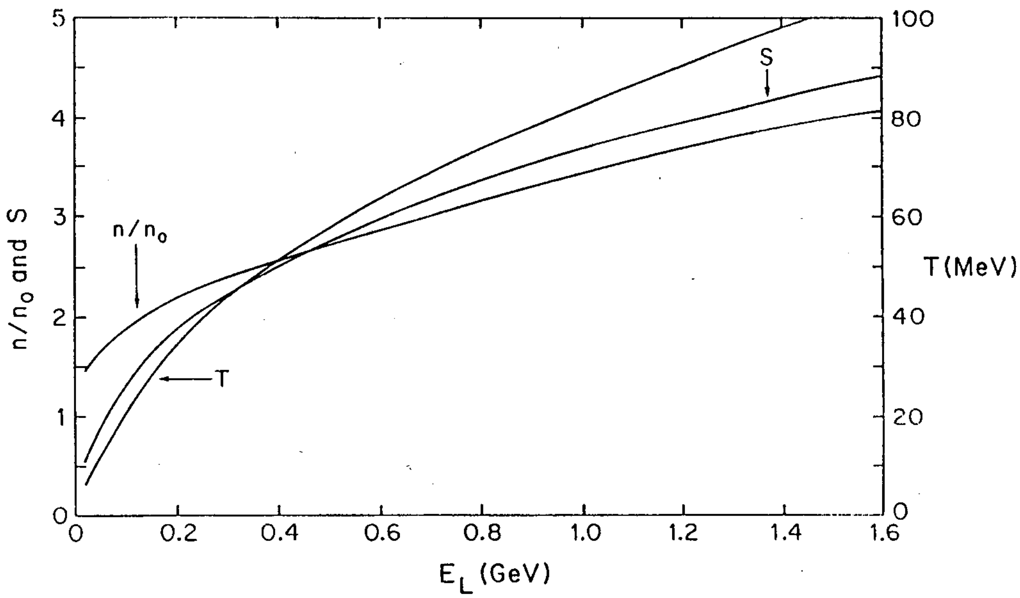

Fig. 1. Compression n/n_0, temperature T and entropy per baryon generated by the Rankine-Hugoniot-Taub shock equations for the collisions of infinite slabs of nuclear matter, as functions of the laboratory bombarding energy per nucleon.

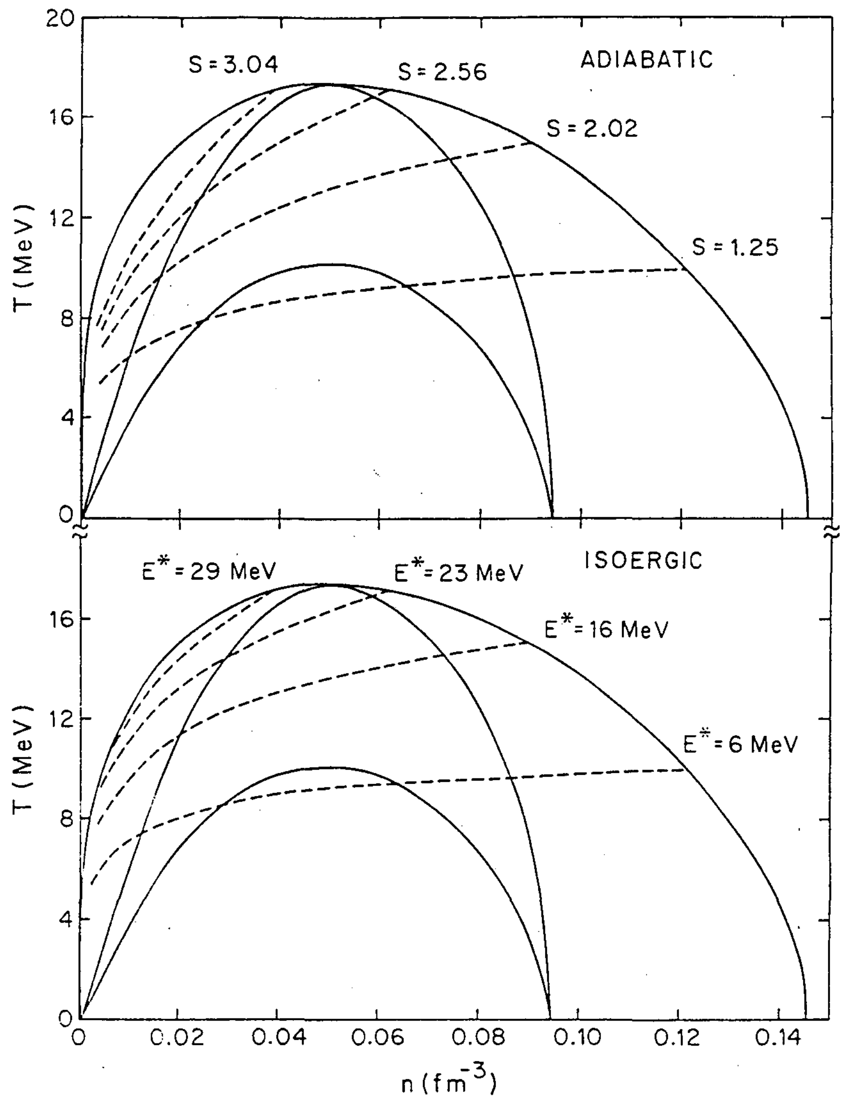

Fig. 2. The outer curve is the Maxwell construction for phase coexistence, the middle curve is the isothermal spinodal and the inner curve is the isentropic spinodal. Trajectories are shown for the case of constant entropy expansion (top) and constant energy expansion (bottom).

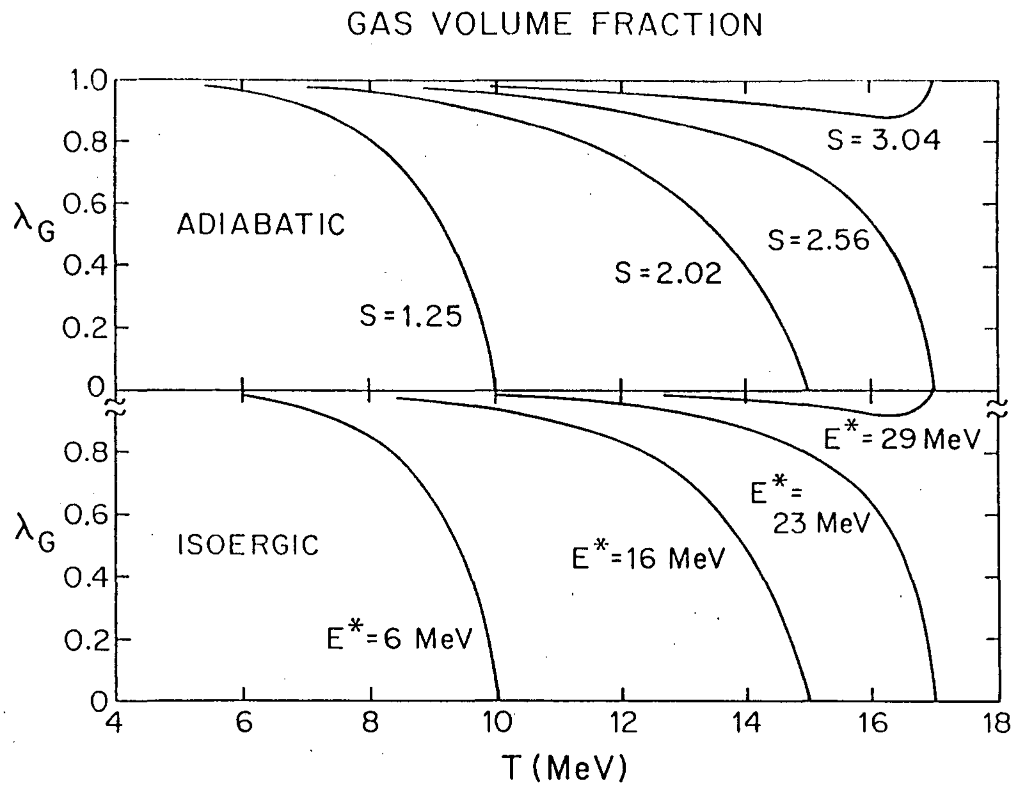

Fig. 3. The fraction of volume occupied by the gas phase for the trajectories shown in Fig. 2.

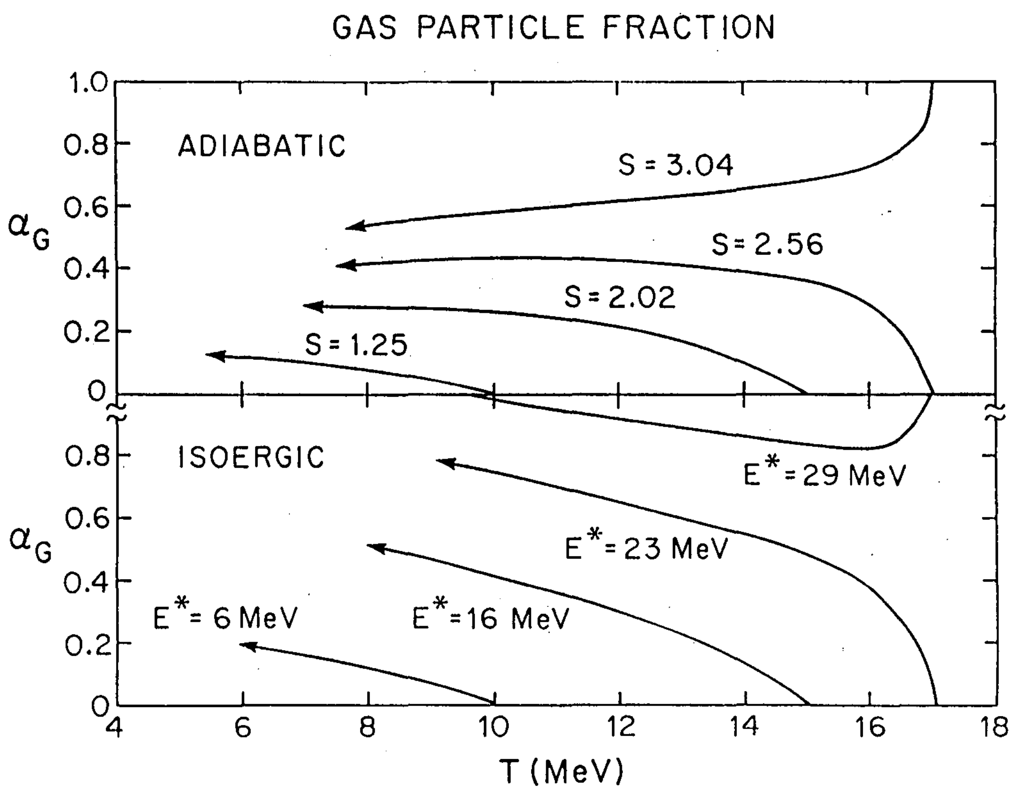

Fig. 4. The fraction of baryon number carried by the gas phase for the trajectories shown in Fig. 2.

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