### Title

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### Authors

Brower, Richard C.
Harte, John.



### Publication Date

1967-06-05

UCRL-17613

# University of California

# Ernest O. Lawrence Radiation Laboratory

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AEC Contract No. W-7405-eng-48

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ABSTRACT

A kinematical constraint on the energy dependence of infinitely rising Regge trajectories is established. Under quite general conditions it is shown that asymptotically $\alpha(s) \leq C\, s^{\frac{1}{2}}$, provided that certain channels couple to high-spin Regge recurrences. The implications of this result for dynamical models of ever-rising trajectories and for the decay rates of resonances are discussed in the light of experimental indications that $\alpha(s) \sim Cs$.

# I. INTRODUCTION

The fascinating possibility that Regge trajectories rise indefinitely is currently a subject of much interest.[1] In a recent report, S. Chu and C. Tan[2] consider a "simple bootstrap model" for infinitely rising $\pi$ and N trajectories and find the asymptotic solution $\mathrm{Re}\alpha(s) \sim Cs^{\frac{1}{2}} = CE$. They consider channels with two high-spin recurrences and maximize the total spin, consistent with the demand that the channel's threshold is equal to the mass of the composite particle and that the orbital angular momentum is small. On the other hand, the known high spin resonances[3], and recent fits of high energy scattering data with direct channel resonances[4] are consistent with trajectories rising quadratically in E.

Here it is shown that the restriction to $\alpha(E) \leq O(E)$ results from assumptions which are considerably weaker than those of Chu and Tan. In fact, for any number of Regge trajectories, $\alpha_i(E)$, coupling to channels with an arbitrary number of particles, conservation of energy and angular momentum, alone, lead to the asymptotic form $\alpha_i(E) \sim C_i E^{a_i}$ with $a_i \leq 1$, if certain assumptions are made about the composition of the communicating channels. The demonstration is general enough to completely characterize the channels which must couple to high-spin recurrences, if trajectories are to rise more rapidly than linearly in the energy. As a consequence we find that important dynamical assertions can be made if trajectories do rise quadratically in E.

In Sect. II we present a general theorem for a $s^{\frac{1}{2}}$ bound on trajectories, while in Sect. III we discuss the dynamical consequences of this theorem.

## II. GENERAL THEOREM

Consider the conservation of energy and angular momentum for a resonance of mass E and spin $\alpha_i(E)$ on the i-th trajectory. Throughout the paper $\alpha$ and E will refer to the real parts of the spin and mass of the particles. If, as we assume here, Im $\alpha$ and $\Gamma$ do not increase as fast as $\alpha$ and E, respectively, then we may safely ignore them.

This initial resonance on the i-th trajectory communicates with a multiparticle channel composed of $n_f$ particles from each of N trajectories (f = 1,...,N). In this channel the $\nu$-th particle ($\nu = 1,\ldots,n_f$) on the f-th trajectory has mass $m_{if}^{(\nu)}$ and spin $\alpha_f(m_{if}^{(\nu)})$. With the usual definition of the Q-value, conservation of energy for this channel can be written,

$$\sum_{f=1}^{N} \sum_{\nu=1}^{n_f} m_{if}^{(\nu)} + Q_i = E. \tag{2.1}$$

Conservation of angular momentum requires the initial particle to have spin intermediate between the extremes of the fully aligned configuration of spins $\alpha_f$ and total orbital angular momentum $\ell_i$, and the minimum of the unaligned configurations. In our notation this is

$$\ell_i + \sum_{f=1}^{N} \sum_{\nu=1}^{n_f} \alpha_f(m_{if}^{(\nu)}) \geq \alpha_i(E) \geq \min \left| \ell_i' + \sum_{f=1}^{N} \sum_{\nu=1}^{n_f} (\pm)\, \alpha_f(m_{if}^{(\nu)}) \right| \tag{2.2}$$

Also it is convenient to define fractional masses $X_{if}^{(\nu)}$ of the constituents in this channel by the relation,

$$X_{if}^{(\nu)} = m_{if}^{(\nu)}/E. \tag{2.3}$$

Now the theorem can be stated. For N infinitely rising trajectories $\alpha_i$ with asymptotic power behavior $C_i E^{a_i}$, assume that each trajectory has <u>at</u> <u>least</u> <u>one</u> communicating channel satisfying the following conditions, as $E \to \infty$,

i. $\ell_i/E^a \to 0$, where $a = \max\{a_i\}$

ii. $Q_i \geq 0$, or $|Q_i|/E \to 0$.

iii. All $X_{if}^{(\nu)} < 1$, for $a_i = a_f = a$.

Then it follows that all $a_i \leq 1$.

For this theorem to hold each trajectory must have <u>one</u> channel which satisfies these mild conditions on $\ell_i$, $Q_i$ and $X_{if}^{(\nu)}$. However, the strength of the general theorem lies precisely in not having to specify the set of channels involved.[5] Indeed, by letting the X's vary from zero to some fraction less than one, the actual composition of the channels involved may vary as E goes to infinity.

This theorem merely states the kinematical fact that if $a_i > 1$, conservation of energy and angular momentum will not allow the channels satisfying conditions (i) - (iii) to exist. The strength of the coupling to these channels never enters the theorem, precisely because kinematics alone decouples all such channels for $a_i > 1$. On the other hand, the claim that the condition (i) - (iii) are physically realized for strongly coupled channels is certainly a dynamical assertion. In Section III we will discuss both the plausibility of this assertion and the dynamical implications if the assertion is false and trajectories violate our $s^{\frac{1}{2}}$ bound.

Now let us give the proof that the conditions (i) - (iii) are in contradiction with $a_i > 1$. Conservation of energy and angular momentum take the form

$$\sum_{f=1}^{N} \sum_{\nu=1}^{n_f} X_{if}^{(\nu)} \leq 1 + \delta, \quad \text{where} \quad \delta \to 0 \quad \text{as} \quad E \to \infty \tag{2.4}$$

$$\ell_i/E^a + \sum_{f=1}^{N} \sum_{\nu=1}^{n_f} (X_{if}^{(\nu)})^{a_f} (E^{a_f}/E^a)\, C_f \geqslant (E^{a_i}/E)\, C_i \tag{2.5}$$

In the last expression all the terms drop out in the limit $E \to \infty$ except for those with $a_f = a$. Suppose the $a_f$'s are in decreasing order such that $a_f = a$ for $f = 1,\cdots,N_0$. Then

$$\sum_{f=1}^{N_0} M_{if}\, C_f > 1, \quad M_{if} = \sum_{\nu=1}^{n_f} (X_{if}^{(\nu)})^a \tag{2.6}$$

where $i = 1,\cdots,N_0$.

The theorem of Frobenius[6] states that for any non-negative square matrix there is a maximum positive real eigenvalue, $r$, and it is bounded by the row sums

$$\min \sum_{f=1}^{N_0} M_{if} \leq r \leq \text{Max} \sum_{f=1}^{N_0} M_{if} \tag{2.7}$$

Since Eq. (2.6) requires[7] that $r \geq 1$ and since all the row sums are less than one for $a > 1$ and $X_{if}^{(\nu)} < 1$ (see Appendix), there is a contradiction and we have proved $a_i \leq a \leq 1$.

This proof can easily be extended to the case of infinite numbers of particles and trajectories. The production of an infinite number of soft particles (finite mass) each with a finite spin and finite orbital angular momentum will still allow the proof to hold. Their contribution to Eq. (2.5) is bounded by terms linear in E because the number of soft particles is bounded by $E/m_\pi$, by conservation of energy. Next the possibility of an infinite number of particles with increasing spin must be considered. For example, pion recurrences may be produced in quantity $k \propto E/\mu(E)$ with mass $\mu(E) = \mu_0 E^\gamma$. For $\gamma$ between 0 and 1 both the mass and number of pions increases with E. Conservation of angular momentum gives

$$kC_\pi \, (\mu(E))^a \geq CE^a \tag{2.8}$$

which implies $1 - \gamma + \gamma a \geq a$ for large E, or $a \leq 1$.

In order to extend the proof to the case of an infinite number of particles in a more general way, let us suppose that there is still a maximum power $a_f = a$ and that there is a maximum coefficient $C_f = C_m$ for this power $(f = 1, \cdots, N_0(E))$. Now, every term on the left of the inequality (2.5) for the leading trajectory can be replaced by $C_m(X_{if}^{(\nu)}E)^a$, since for large enough E this replacement only increases the inequality. This implies

$$\sum_{f=1}^{N(E)} \sum_{\nu=1}^{n_f(E)} (X_{mf}^{(\nu)})^a \geq 1, \quad \text{as} \quad E \to \infty \tag{2.9}$$

We refer the reader to the Appendix for the demonstration that Eq. (2.5) is in contradiction with $a > 1$.[8]

## III. DYNAMICAL CONSIDERATIONS

To consider whether the physical trajectories should actually obey our theorem, we must consider if channels satisfying the conditions (i) - (iii) are necessary dynamically for the production of the high spin resonances. If the interaction has a finite range, the strongly coupled channels have $\ell_{max} \sim kR \lesssim O(E)$. Hence, for these channels, one expects condition (i) to hold for $a > 1$. In most models of composite particles the resonant channels $(Q > 0)$ and the channels with moderate binding energy $(0 \lesssim -Q < E)$ are important so condition (ii) also seems reasonable.

But condition (iii) is more difficult to support. In fact models have been proposed[9] in which quasi-two-body channels with one high-spin resonance are important. It will be interesting to see if these models produce trajectories that violate the $s^{\frac{1}{2}}$ bound. In any case, violation of the third condition appears, on the surface, to be the most likely way for our theorem to break down. In view of the present experimental situation which suggests a power law, $\alpha_i \sim CE^2$, it is interesting to consider in detail the consequences of such channels dominating the dynamics.

First consider the case of such an asymmetrical channel for the decay of a high spin resonance of mass $M_1$ and spin $\alpha_1$ into a near-by resonance of mass $M_2$ and spin $\alpha_2$ plus small mass particles. As $X_{12} \to 1$, all the other fractional masses are forced to zero. Conservation of spin requires

$$C_2(X_{12})^a \geq C_1 \geq C_2(X_{12})^a \quad . \tag{3.1}$$

For $X_{12} = ( 1 - \frac{Q}{E} ) < 1$, the resonance is decaying into one other high spin resonance on a different trajectory to the left $(C_1 < C_2)$ on the Chew-Frautschi plot. As $X_{12} \to 1$, the slopes of the two trajectories must become equal $(C_1 = C_2)$. This "gradual decay mode" is characterized by

$$1 - X_{12} = \frac{\Delta M}{E} = \frac{M_1 - M_2}{M_1} \to 0, \quad \text{as } E \to \infty \tag{3.2}$$

and it occurs between a trajectory and itself or one of equal slope.

Consider further, decay processes involving trajectories which are linear in s and have equal slopes, as suggested by experiments. Because the C's are all equal, it is evident from the matrix inequality (2.6) and energy conservation that every row sum must be unity. Hence for every decay process, the decay mode is a "gradual decay mode" characterized by Eq. (3.2).

Moreover for trajctories quadratic in E, the orbital angular momentum must increase linearly.

$$\ell \geqslant \alpha_1(M_1) - \alpha_2(M_2) \approx \frac{\partial\alpha}{\partial E} \Delta M + \alpha_1(0) - \alpha_2(0) \tag{3.3}$$

Since the pion is the minimum mass that can be emmitted by the resonant system, and since the slopes $C_i$ of the trajectories (with the possible exception of the Pomeranchuk) appear to be approximately

$1 (\mathrm{BeV})^{-2}$, one has

$$\ell \gtrsim 0.3\ \mathrm{E/BeV} + \alpha_1(0) - \alpha_2(0) \qquad (3.4)$$

Smaller orbital angular momentum is allowed, if the decay product lies on a trajectory to the left.

To summarize, decays are expected to take place in cascades through resonances of gradually decreasing masses. To keep the orbital angular momentum small, the cascade will have a tendency to feed into higher lying trajectories. Eventually high orbital angular momentum will be unavoidable as the cascade reaches the leading trajectory.

The width of the high spin resonance on trajectories linear in $s$ will be strongly reduced by centrifugal barrier effects resulting from high orbital momentum. For symmetric two body decays (equal masses) one has

$$\ell \gtrsim \frac{1}{2}\, \mathrm{CE}^2 \ . \qquad (3.5)$$

Even at $E \approx 3$ BeV, one is being forced into high angular momentum. This leaves only the asymmetric decay channels discussed above. Hence the orbital angular momentum may be expected to gradually quench <u>all</u> the resonant channels of the high-spin recurrences.

If these resonant channels are dynamically responsible for maintaining the ever-rising trajectories, then the trajectories would eventually have to increase no faster than linearly in the energy. Referring to Eq. (3.4), we see that this orbital effect may become important near $E \approx 10$ BeV. and we might expect $\alpha(s)$ when plotted against $s$, to begin curving over to a $s^{\frac{1}{2}}$ behavior in this energy region. If this phenomenon does not occur and trajectories continue to rise linearly in $s$, then it appears very likely that the bound channels will be playing a dominant role in dynamically maintaining the rising trajectories as the resonant channels are gradually quenched.

In conclusion, the dynamics of trajectories that violate the $s^{\frac{1}{2}}$ bound will be quite distinct from the dynamics of those that respect it. Quite different models will be suited to the study of either alternative.

## ACKNOWLEDGMENTS

We would like to thank Professor G. F. Chew for many helpful discussions and suggestions. Also we wish to thank Professor S. Mandlestam and Dr. J. Finklestein for their suggestions. We thank Professor G. F. Chew for the hospitality extended to us at the Lawrence Radiation Laboratory.

# APPENDIX

The following inequality is necessary for the theorem for either the finite or the infinite case. (In the finite case K(E) remains finite, $N = N_0$ and the result is trivial.)

$$\lim_{E\to\infty} \sum_{k=1}^{K(E)} (X_k)^a < 1 \qquad (A.1)$$

In $X_{if}^{(\nu)}$ the fixed index $i$ has been surpressed, and the indices $f$ and $\nu$ replaced by $k$ running from 1 to

$K(E) = \sum_{f=1}^{N(E)} n_f$, where K(E) is the number of particles in the channel communicating with the initial resonance on trajectory $\alpha_i$.

For $a > 1$ and $X_k < 1$, the above inequality follows from conservation of energy in the form

$$\lim_{E\to\infty} \sum_{k=1}^{K(E)} X_k = A < 1 + \delta, \quad \delta \text{ is arbitrarily small.} \qquad (A.2)$$

Subtracting (A.1) from (A.2), we have

$$\Delta = \lim_{E\to\infty} \sum_{k=1}^{K(E)} (X_k - X_k{}^a) = \lim_{E\to\infty} \sum_{k=1}^{K(E)} X_k(1 - X_k{}^{a-1}). \qquad (A.3)$$

Since $\mathrm{Max}\left\{X_k\right\} < 1 - \epsilon$, we have

$$\Delta \geqslant \lim_{E\to\infty} \sum X_k \left(1 - (1 - \epsilon)^{a-1}\right).$$

Choosing $\epsilon$ small enough, we have

$$\Delta \geq A \epsilon (a - 1).$$

Now pick $\delta < A \epsilon (a - 1)$ so that $\lim_{E \to \infty} \sum_{k=1}^{K(E)} (X_k)^a < 1.$ q.e.d.

## FOOTNOTES AND REFERENCES

* This work was done under the auspices of the U. S. Atomic Energy Commission.

† AEC Postdoctoral Fellow under contract No. AT(11-1)-34.

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2. S. Chu and C. Tan, The High Energy Behavior of Regge Trajectories in Bootstrap Model, Lawrence Radiation Laboratory Report UCRL-17511, April 1967 (unpublished).

3. A. Rosenfeld, et. al. Rev. Mod. Phys. 39, 1 (1967).

4. V. Barger and D. Cline, Phys. Rev. 155, 1792 (1967).

5. Of course, the general theorem may be applied to the channels considered by Chu and Tan. In this case, we find that $a_\pi = a_N \equiv a \leq 1$ and $C_\pi / C_N \leq 2(\frac{1}{2})^a$. Moreover, if the equality sign in Eq. (3.3) is taken and $\frac{Q}{E} \to 0$, then $a = 1$ and $C_\pi = C_N$. The full results of Chu and Tan are obtained without their procedure of maximizing the spin.

6. F. R. Gantmacher, The Theory of Matrices, Vol. II, Chelsea Publishing Co., New York, N.Y.

7. A. M. Ostrowski, Positive Matrices and Functional Analysis in Recent Advances in Matrix Theory, H. Schneider, Ed. (Univ. of Wisconsin Press, 1964.)

8. Here, condition (iii) should be replaced by the condition that $X_{mf}^{(\nu)} < 1$ for all f. Clearly the finite case can also be proved this way with this stronger assumption. On the other hand it is not evident that the full set of inequalities can be satisfied for $a \leq 1$ without further restrictions being imposed. In any case, neither method of proof exploits all the information implicit in angular momentum conservation. Full use of the inequalities will give restrictions on the C's, a few examples of which are included in the text.

9. P. A. Carruthers, Phys. Rev. 154, 1399 (1967).