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**WORKING PAPER NO. 92-210**

**MORE ON THE EFFICIENCY OF THE MARKET  
FOR SINGLE FAMILY HOMES: DEFAULT**

By

**JOHN M. QUIGLEY  
ROBERT VAN ORDER**

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MORE ON THE EFFICIENCY OF THE MARKET  
FOR SINGLE FAMILY HOMES: DEFAULT

by

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## Abstract

This paper provides additional evidence on the efficiency of the housing market by investigating the default behavior of individual homeowners. We estimate a model of "ruthless" default (*i.e.*, one in which transactions costs, reputation costs and moving costs play no role) and analyze its implications -- the relationship between equity and default, the timing of default, its dependence upon initial conditions, and the severity of losses. Absent transactions costs and market imperfections, economic theory makes well-defined predictions about these various outcomes.

The empirical analysis is based upon two particularly rich bodies of micro data: one indicating the default and loss experience of all mortgages purchased by the Federal Home Mortgage Corporation (Freddie Mac); and a large sample of all repeat sales of single family houses whose mortgages were purchased by Freddie Mac since 1976.

## I. INTRODUCTION

Within the past few years, there has been increasing debate about the "efficiency" of the housing market. Recently, Case and Shiller [1989] argued on a priori grounds that, in a market dominated by individuals trading in the houses in which they live, there is "good reason" to expect that the market for single family homes is less efficient than the market for financial assets. They then conducted rather extensive empirical analyses of price changes in the housing market (Case and Shiller [1989, 1990]), concluding that despite transactions costs, carrying costs, and tax considerations, profitable trading rules apparently exist -- at least for those free to time the purchase of single family homes.

This paper presents some evidence on a very different aspect of the efficiency of the single family housing market, concentrating on the default behavior of homeowners with conventional mortgage obligations. Again, there may be good reason to believe that transactions and reputation costs, moving costs and capital constraints make exercise of the default option on mortgage contracts by homeowners less ruthless than the exercise of equivalent put options by investors in "frictionless" financial markets. We provide evidence on the extent to which homeowner behavior can be

characterized as "ruthless," or narrowly wealth maximizing in a world without transactions costs.

Our tests are based upon two hitherto unexploited data sources: a rich micro data set indicating default and loss severity experience from the Federal Home Mortgage Corporation (Freddie Mac); and a large sample of repeat sales of single family houses whose mortgages were purchased by Freddie Mac.<sup>1</sup> These data allow us to estimate the relationship between homeowner equity and default behavior. First, we test explicitly for ruthless default behavior by homeowners. The results we report are quite consistent with predictions about behavior in the absence of transactions costs. We then subject the model to closer scrutiny, investigating predictions about the timing of default and the dependence of default on initial mortgage terms. It is less clear that homeowner behavior can be characterized as ruthless from these perspectives. Finally, we investigate the severity of losses on defaulted properties. Absent transactions costs and market imperfections, economic theory makes well defined predictions about these various outcomes. These rich bodies of data test provide a rather pure of the predictions. The "ruthless" model does less well in these latter tests.

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<sup>1</sup> The default data are described more fully in Quigley and Van Order [1991]; the sales data are described in Abraham and Schauman [1990].

Section II below lays out the general issue. Sections III and IV present the empirical analysis; conclusions are summarized in Section V.

## II. RUTHLESS DEFAULT AND OPTION MODELS

It is by now widely accepted that a fruitful way of analyzing home mortgages is to view them as ordinary debt instruments with specific options attached to them; these options can be analyzed with modern contingent claims models. To default on a mortgage is to exercise a put option; the defaulter sells his house back to the lender in exchange for eliminating the mortgage obligation. To prepay a mortgage is to exercise a call option; the borrower exchanges the unpaid balance on the debt instrument for a release from further obligation.<sup>2</sup> Absent transactions costs, default and prepayment are purely financial matters, which, in the spirit of Modigliani-Miller, can be priced properly and will have no effects on real behavior (except, perhaps, through subsequent wealth variations).

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<sup>2</sup> Analogously, caps and floors on adjustable-rate mortgages and other attributes of these debt instruments can be formulated as options. Dunn and McConnell [1983], Buser and Hendershott [1989], Brennan and Schwartz [1985], among other, apply recent contingent claims models of the prepayment option to pricing mortgages. Cunningham and Hendershott [1984] focus specifically on pricing the default option. Kau et al [1986, 1991] analyze both options simultaneously.

The contingent claims approach, while based on complicated arbitrage models and requiring the solution of a messy partial differential equation, leads to a familiar and quite practical result. The value of a financial claim is the risk-adjusted expected present value of the net income from the claim, where the expected present value calculation takes account of all of the possible options involved.<sup>3</sup> The model describes exactly how the risk adjustments should be made; it also describes the optimal exercise of the various options.

Well-informed borrowers in a perfectly competitive market will exercise options when they can thereby increase their wealth. Absent either transactions costs or reputation costs which reduce credit ratings, these individuals can increase their wealth by defaulting when the market value of the mortgage exceeds the value of the house. Similarly, by prepaying when market value exceeds par, they can increase wealth by refinancing. Note that the value of the mortgage exceeds the present value of the remaining payment stream because the mortgage claim includes both the options to prepay and also to default at some subsequent date. Thus, even if the market value of the house is less than the present value

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<sup>3</sup> This expected present value interpretation follows from lemma 4 in Cox, Ingersol, and Ross [1985].

of future mortgage payments (*i.e.*, the default option is "in the money"), it may not be optimal to exercise the option.<sup>4</sup>

The problem of determining when to exercise an option requires specifying the underlying state variables and parameters that determine the price of any security and then deducing the rule for exercise that maximizes borrower wealth. For residential mortgages, the key state variables are interest rates and house values. The value of a mortgage  $M(i,V,t,T)$  depends upon a vector of relevant interest rates,  $i$ , property value,  $V$ , the age of the mortgage,  $t$ , the remaining time to maturity,  $T$ , and various parameters. A standard arbitrage argument is sufficient to derive an equilibrium condition for  $M$  (a second order partial differential equation), specifying that the expected return on the security (that is, the coupon return plus capital gains) must equal the risk-free rate of return plus a risk adjustment. This condition applies to any claim that is contingent on the underlying state variables; again it has the interpretation that the value of the mortgage equals the risk-adjusted expected present value of its net cash flows.

To simplify matters and to isolate the default option, assume that interest rates are non-stochastic (so that the only source of risk is house price volatility). Assume that

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<sup>4</sup> Many of the complications in contingent claims models revolve around this point, which makes optimal exercise "forward-looking."

house price changes are continuous with an instantaneous mean  $\mu$  (which need not be constant) and a constant percentage standard deviation  $\sigma$ . Let  $\rho$  be the imputed rent payout ("dividend") rate. The arbitrage model implies that the value of the mortgage  $M$  satisfies

$$(1) \quad (1/2)M^2\sigma^2(\partial^2M/\partial V^2) + M(i-\rho)(\partial M/\partial V) + (\partial M/\partial t) + c(t) = rM,$$

where  $i$  is the instantaneous interest rate and  $c(t)$  is the coupon payment on the mortgage. (This follows almost directly from Black and Scholes [1973]). Note that the expected appreciation rate of traded assets (in this case, houses) does not appear, nor does the risk premium for holding the asset. If the underlying state variables are traded assets, then arbitrage leads to a risk-neutral interpretation of the price of a contingent claim on an asset relative to the price of that asset. The value of the option is the expected present value of the outcome, where prices are projected to grow at a mean rate of  $i-\rho$  (and variance  $\sigma^2T$ ) and are discounted at the risk-free rate. This is equivalent to assuming risk neutrality (See Smith [1976] for a discussion).

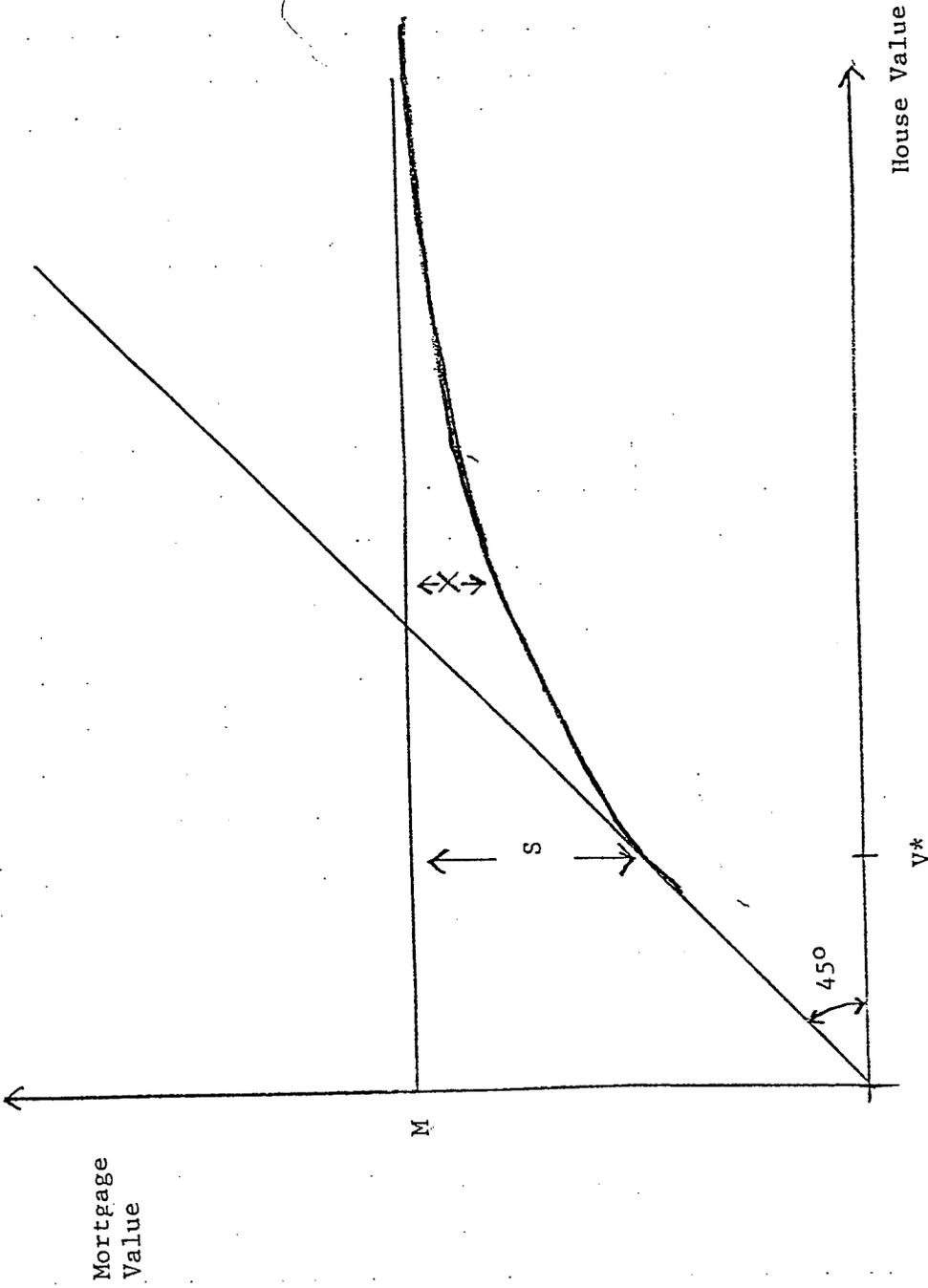
An infinite number of functions satisfy (1) (depending on boundary conditions), which reflects the infinite number of ways that coupon plus capital gain can equal the required expected return. By incorporating the optimal call and put

strategies, the function appropriate for a particular mortgage can be determined.

If there are no costs to default other than losing the house, the optimal default "strategy," given  $t$ , is characterized simply by the house value  $V_t^*$ , at which default takes place. The optimal  $V_t^*$  minimizes the value of the mortgage (this maximizes the borrower's net worth), subject to the condition that  $V_t^*$  equal the value of the remaining balance when the option is exercised.

Figure 1 (adapted from Quigley and Van Order [1990]), illustrates the optimal strategy. This strategy is represented by the lowest curve which satisfies equation (1) and is not above the 45 degree line (where the remaining balance equals the value of the house). If the solution is an interior one, it is represented by the tangency depicted in the figure. The curve must also be below the horizontal line  $M$ , which gives the value of a riskless mortgage. It approaches  $M$  asymptotically as  $V$  increases. The tangency determines  $V_t^*$ , the default "strategy." The entire curve gives the market relationship between mortgage values and interest rates. The distance  $X$  between  $M$  and the mortgage value is the value of the default option, the premium for insurance that a competitive mortgage insurer would charge. At  $V_t^*$  the distance  $S (=X)$  represents the extent to which the option must be in the money before default. It is also the

FIGURE 1: Optimal Default



amount lost by the lender or mortgage insurer (absent transactions costs) from selling the house after foreclosure.

The virtue of the contingent claim model is its simplicity. The default option is exercised at  $V_t^*$ , which depends only on the variables in (1) and on the boundary and tangency conditions. The equilibrium condition has the property that the mean price change of any traded asset as well as the risk premium are irrelevant in pricing the option or in exercising it. Thus, circumstances under which default occurs depend only on  $i$ ,  $\rho$ ,  $\sigma$ ,  $c$ ,  $M$  and  $V$ ; they are independent of the original house price, expected price appreciation, the original loan-to-value ratio, LTV, and the historic path of prices. Loss severities (measured by  $S$  in Figure 1) depend on these same things.

The model also has implications about default frequencies. These are more complicated than those about loss severities. This is because, although the value of the default option is independent of expected inflation, estimating the probability of exercise also requires estimating expected inflation.

Kau et al [1991] have recently constructed a careful analytical model of the optimal default frequency implied by the frictionless model.<sup>5</sup> Their analysis shows that it is

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<sup>5</sup> The forward looking aspects of the option pricing problem mean that the solution to the differential equation is

typically optimal to wait until the default option is well into the money before actually defaulting. Indeed, they present an example where  $S$  is more than 10 percent of the mortgage balance at optimal exercise. They also simulate cumulative expected default rates by LTV, given a variety of initial parameters. These simulations of optimal behavior are not substantially different from casual empiricism about observed default frequencies. Introducing transactions costs appears to make default frequencies implausibly low. Thus, the authors conclude that research which rejects the frictionless model, simply because people with negative equity do not default frequently, is misleading.

The following example provides some intuition about the magnitude of  $S$ . Assume interest rates are constant, the borrower takes a mortgage priced at par and the coupon rate prices the credit risk properly, given initial LTV. The cost of defaulting on the mortgage arises solely from the loss of the house; if the borrower defaults she will immediately buy the same house, and there are no changes in the underlying parameters. Under these circumstances the cost of exercising the default option can be viewed as the downpayment necessary to buy the house again, financed with a mortgage having the same coupon rate. The benefit of defaulting is the difference

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solved numerically by working backwards from the terminal conditions.

between the mortgage balance, B and the value of the house, V. If L is the initial LTV, then it is optimal to default when

$$(2) \quad (1-L)V = B-V$$

or

$$(3) \quad V/B = 1/(2-L)$$

For instance for  $L = 0.8$ , then  $V/B$  is 0.83, i.e., exercise is not until the mortgage is 17% into the money; when  $L = 0.9$ ,  $V/B$  is 0.91. Obviously, exercise will change if the mortgage is not at par, if coupon does not reflect risk, and with the age of the mortgage. But this example does indicate that  $S$  is probably not very close to zero. These examples also suggest, as is discussed below, that  $S$  will be smaller for high LTV loans, although holding coupon constant  $S$  will be independent of mutual LTV.

We explicitly test the frictionless or "ruthless" model by analyzing the predictions discussed above. We begin by estimating a hazard model which specifies default as a function of the extent to which the option is "in the money." The parameters of the model can be used to simulate default frequencies. We can thus test whether simulated default behavior differs from ruthless behavior -- in terms of variations in initial LTV and variations over time. We then analyze loss severities and test other propositions implied by the theory.

### III. HOUSING EQUITY AND DEFAULT BEHAVIOR

Our empirical model of default is based upon the behavior of a random sample of the holders of mortgage contracts issued between 1976 and 1980 and bought by Freddie Mac. The statistical analysis is based upon a simple random sample of about five percent of these mortgages -- all fixed rate, level payment, fully amortized loans, most with thirty year terms. For each mortgage we observe the year of origination, the housing value at origination (the purchase price of the property), the contractual terms, and the region in which the property is located.

We estimate hazard models of default,<sup>6</sup> where  $H(d_t)$ , the instantaneous default hazard at age  $t$ , is:

$$(4) \quad H(d_t) = \alpha_t \exp[\sum \beta_i Y_i + \gamma E_t]$$

In this formulation the  $Y$ 's are fixed covariates, dummy variables indicating the year of mortgage origination, and  $E_t$  is a time-varying covariate reflecting homeowner equity when the mortgage is at age  $t$ . As the model is specified, the hazard is not proportional to  $t$ , but knowledge of the time profile of  $E$  determines the relative change in the hazard.

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<sup>6</sup> A fully developed model would analyze both prepayment and default simultaneously (as in Foster and Van Order [1985]) because the two decisions can be interrelated -- refinancing of mortgage debt is more difficult without positive equity.

Thus the parameters  $\beta_i$  and  $\gamma$  can be estimated by maximizing the likelihood function without reference to the parameters governing the baseline hazard  $\alpha_t$  (See Kalbfleisch and Prentice, [1980]).

At any age of the mortgage  $t$ , the equity of the mortgage holder  $E_t$  is :

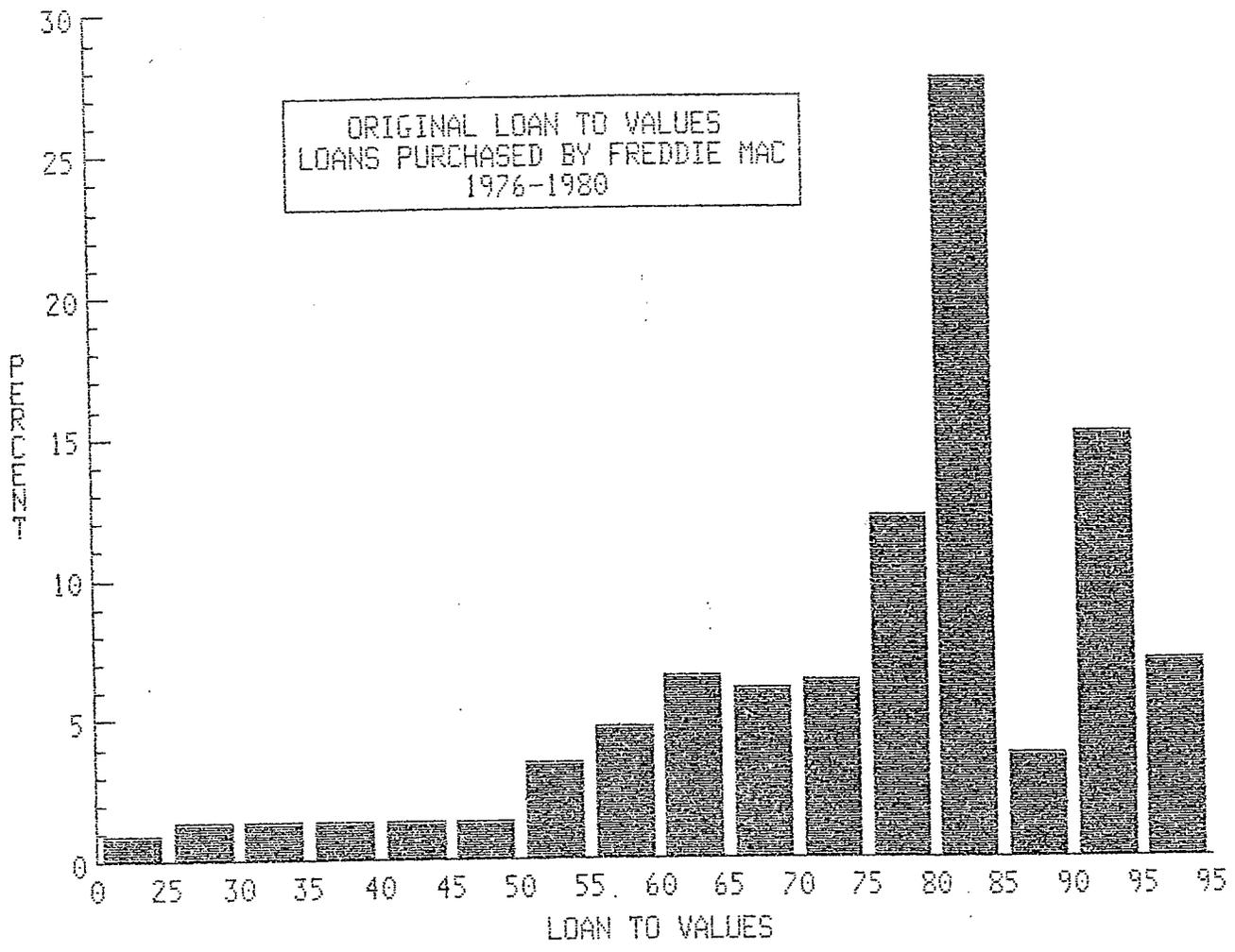
$$(5) \quad E_t = V_{\tau+t} - D_t \\ = V_{\tau+t} - V_{\tau} L f(N, t, \omega) \quad ,$$

where the  $V_{\tau+t}$  is the current value of the house and  $D_t$  is the outstanding debt at age  $t$ . This unpaid balance depends upon the value of the house at the year of purchase  $\tau$ ,  $V_{\tau}$ , and the loan-to-value ratio at origination,  $L$ .  $f(N, t, \omega)$  is the outstanding fraction after  $t$  periods, on a fully amortized level payment loan written for  $N$  periods at contract rate  $\omega$ :

$$(6) \quad f(N, t, \omega) = \frac{1 - 1/(1+\omega)^{N-t}}{1 - 1/(1+\omega)^N}$$

As equation (5) indicates, values of the key state variable are strongly affected by  $L$ , the initial loan to value ratio, as well as the course of housing prices after purchase. Figure 2 presents the distribution of  $L$  for mortgages purchased by Freddie Mac during this period. The mode is a mortgage loan for 80 percent of the purchase price of a property, but there is considerable variation in these ratios.

FIGURE 2



A substantial fraction of loans were for 70 percent or less of market value, and there were some loans for as much as 95 percent of value.

We do observe the purchase price of each house  $V_{\tau}$ , but we do not observe the subsequent course of price variation for individual houses in the sample. We do, however, have access to the prices of about 200,000 properties whose mortgages were purchased by Freddie Mac at least twice during the period 1970-1989. These data are sufficient to estimate, rather precisely, a quarterly weighted repeat sales (WRS) price index for each of five U.S. regions, using the methodology proposed by Case and Shiller [1987].

These indexes and the methodology which underlies them are discussed by Abraham and Schauman [1990].<sup>7</sup> Figure 3

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7 These price indices are estimated according to the three stage regression procedure outlined in the appendix to Case and Shiller's 1987 paper, but they incorporate one slight extension. The model assumes that logarithm the housing price  $P_{it}$  in each region is given by

$$P_{it} = I_t + H_{it} + N_{it} , \quad (N-1)$$

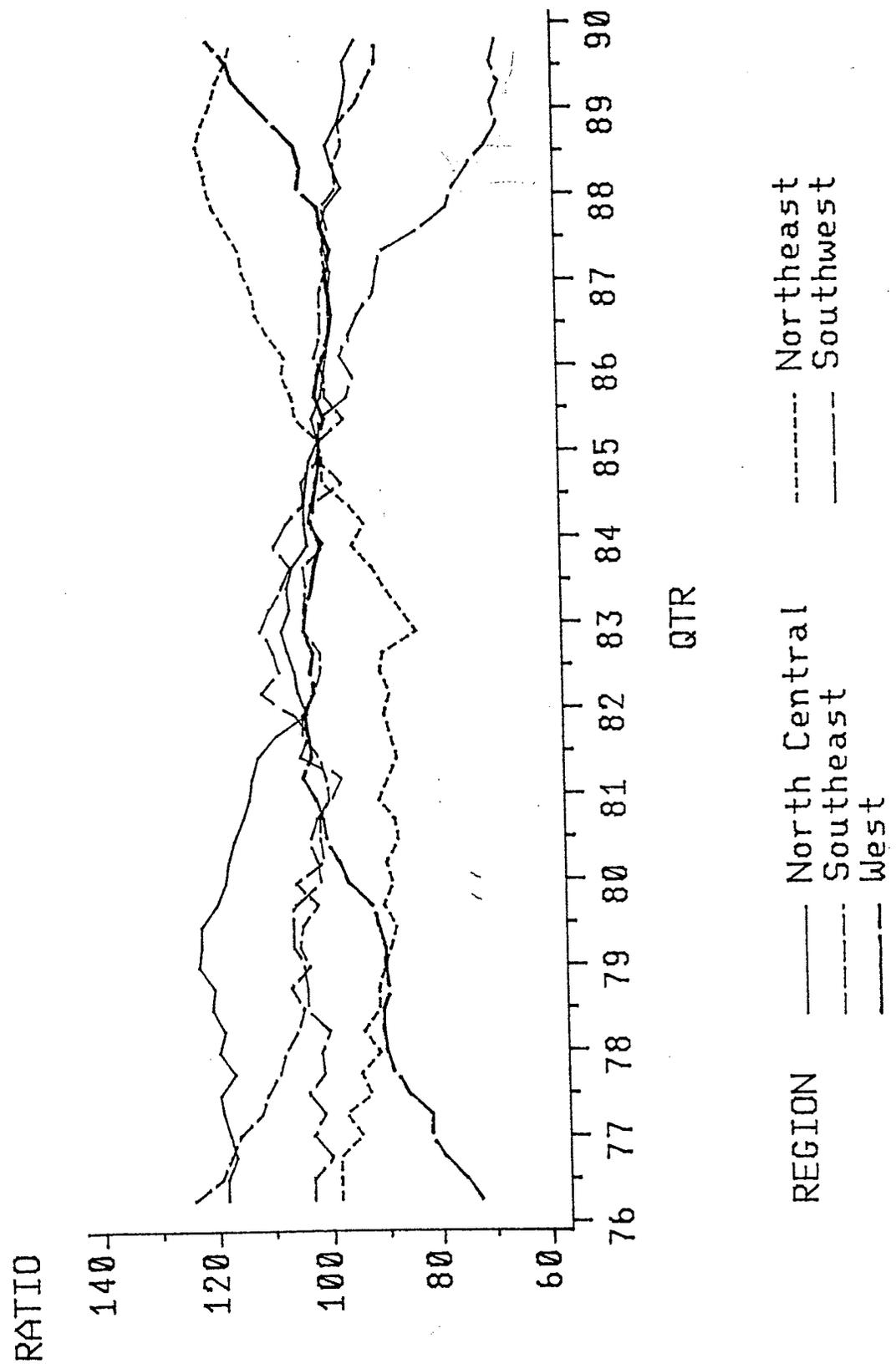
where  $I_t$  is the log of the price level,  $H_{it}$  is a Gaussian random walk (i.e.,  $E[H_{i\tau} - H_{it}] = 0$ ;  $E[H_{i\tau} - H_{it}]^2 = A[\tau-t] + B[\tau-t]^2$ , and  $N_{it}$  is white noise (i.e.,  $E[H_{it}] = 0$ ;  $E[N_{it}]^2 = C$ ). The first stage is the regression of the difference in log sale prices, for multiple sales of the same property, upon a set of dummy variables with values of zero for all quarters except those in which the two sales occurred:

$$P_{i\tau} - P_{it} = g(\tau, t) . \quad (N-2)$$

FIGURE 3

# Regional Price Indices

relative to a national housing price index



Source: Freddie Mac

summarizes the course of the price indices relative to the national average for the period 1976-1989. The figure reveals substantial regional variation about the national price trend.

#### A. A Crude Test of the Model

If we assume that all the houses in our sample appreciate at the average for the region as a whole,

$$(7) \quad V_{\tau+t} = V_{\tau} I_{r,\tau,\tau+t} \quad ,$$

where  $I_{r,\tau,\tau+t}$  is the proportionate change in the WRS price index for region  $r$  between  $\tau$  and  $\tau+t$ , then

$$(8) \quad E_t(\tau, L, N, \omega, r) = V_{\tau} I_{r,\tau,\tau+t} - V_{\tau} Lf(N, t, \omega)$$

can be calculated from sample information. In the empirical analysis, we compute  $E_t$  quarterly, and we also observe individual mortgage defaults and hence hazards quarterly.

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The second stage is a weighted regression of the squared residuals upon an intercept, the elapsed time between sales, and its square, yielding estimates of  $A$ ,  $B$ , and  $C$ :

$$(P_{it} - \hat{P}_{it})^2 = A [\tau-t] + B[\tau-t]^2 + C \quad . \quad (N-3)$$

The third stage is a re-estimation of the stage one regression by generalized least squares (GLS) using the fitted values in the second stage as GLS weights. The incorporation of the square of elapsed time between sales in the second stage, not considered by Case and Shiller, reflects the expectation that the variance of prices does not increase at the same rate forever.

Table 1 presents coefficient estimates for these simple nonproportional hazard models.<sup>8</sup> Columns 1 and 2 present coefficients using the dollar value of expected equity as an independent variable; columns 3 and 4 report the results using equity as a fraction of current housing value as an independent variable.

As indicated by the pattern of dummy variables in Table 1, *ceteris paribus*, successive origination years had higher default rates. One explanation, consistent with the ruthless default model, emphasizes the effect of the interest rate cycle. Borrowers who took out low rate mortgages early in the period saw the market values of their liabilities fall over time, as interest rates rose, so that economic equity was larger than the book value of equity, reducing default behavior.<sup>9</sup>

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8 One consequence of the reliance upon average housing prices in equation (7) is the measurement error thereby introduced into the variable representing individual homeowner equity. *Ceteris paribus*, we should expect a higher (lower) probability of default for those whose housing price appreciation is below (above) the average in any region. By ignoring the dispersion of housing prices around the regional average in equation (6), we ignore the fact that positive equity for the average homeowner in a given region will coexist alongside negative equity and higher default risk for some homeowners.

9 An alternative explanation, not consistent with the ruthless model, emphasizes the rise in the cash flow costs of housing relative to incomes in the late 1970's (Housing prices increased faster than incomes, and mortgage interest rates rose substantially). Hence, those who took out fixed rate mortgages earlier in the period were less likely to have had difficulty making repayments after the recession began in the early 1980s -- simply because their mortgage payments were relatively low. Finally, for reasons indicated in note

TABLE 1

Hazard Models of Mortgage Default  
(9,229 observations)

$$H(d_t) = \alpha_t \exp[\sum \beta_i Y_i + \gamma (E_t)]$$

Parameter	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>
Origin Year Dummies, $\beta_i$				
1976	-2.199 (7.37)		-1.417 (4.46)	
1977	-1.669 (8.43)		-0.878 (3.99)	
1978	-1.165 (7.51)		-0.718 (4.37)	
1979	-0.076 (6.01)		-0.826 (5.33)	
Equity, $\gamma$				
E (in thousands)	-0.076 (13.54)	-0.093 (16.39)		
E Ratio (E/V)			-9.352 (16.58)	-10.235 (22.26)
$\chi^2$	451.9	243.6	632.2	511.7

Note: Asymptotic t ratios in parentheses.

When these temporal effects are not accounted for separately, the variables measuring equity are even more important. In any case, the coefficients of the variables measuring equity are highly significant and rather large in magnitude. For example, a decline in equity of ten thousand dollars multiplies the probability of default by at least  $\exp[.76]$  or by about 214 percent. A decline in the equity ratio by 0.05 increases the conditional default rate by about 160 percent.

Not surprisingly, the option model passes the simple test: equity clearly matters a lot in the calculus of default.

#### **B. A More Refined Test: The Distribution of Equity**

The large sample of repeat sales which underlies the regional price indices can also be used to estimate the variance in individual house prices. Indeed, the procedure used to compute the WRS price indices provides a direct estimate of price dispersion. The course of individual housing prices is specified as a random walk, with variance

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6, over time those mortgages that remain outstanding may tend to have less equity than average. For loans originated early in the period, this selectivity is not likely to be large (since the subsequent course of mortgage interest rates exceeded the coupon rates for these mortgages). Mortgages originated later in the period did eventually experience interest rates lower than their coupons. This is also consistent with monotonically increasing dummy variables for origination year.

increasing with the elapsed time after purchase, though generally at a decreasing rate.

As indicated in note 7, the WRS procedure yields an estimate of the expected value of each individual house price over time, as well as the variance in that estimate as a function of the elapsed time after purchase.

From the central limit theorem, we can estimate the distribution of values for houses in region  $r$  purchased at time  $t$  and observed at time  $(t+\tau)$ . Since  $D$  is nonstochastic, we can also estimate the distribution of homeowner equity. For each observation in the sample, we can estimate the probability that homeowner equity falls within any arbitrary range.

Table 2 presents estimates of the relationship between default hazards and the probability distribution of homeowner equity.<sup>10</sup> Columns 1 and 2 present coefficients using the probability of negative equity as an independent variable; columns 3 through 6 use portions of the equity distribution.

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<sup>10</sup> Since  $D$  is nonstochastic, the distribution of the equity-debt ratio ( $E/D$ ) can be computed as a simple linear transformation of  $V$ , i.e., as  $(V-D)/D$ . In the estimation of the hazard models, the mean and variance of  $E/D$  is computed for each house for each quarter. The independent variable in the hazard model is the probability that  $E/D$  falls in the range  $(a,b)$ , computed by integration of the normal distribution within the bounds  $(a,b)$ . This is quite computation intensive. The entries in Table 2 present the probability statement in terms of the more conventional equity ratio,  $E/V$ .

TABLE 2

Hazard Models of Mortgage Default  
(9,229 observations)

$$H(d_t) = \alpha_t \exp[\sum \beta_i Y_i + \gamma \text{prob}(E_t)]$$

<u>Parameter</u>	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>	<u>6</u>
Origin year Dummies, $\beta_i$						
1976	-3.331 (11.33)		-2.581 (8.47)		-2.898 (9.58)	
1977	-2.385 (12.32)		-1.685 (8.06)		-1.930 (9.28)	
1978	-1.368 (8.87)		-0.893 (5.42)		-1.015 (6.15)	
1979	-0.794 (5.10)		-0.780 (5.03)		-0.712 (4.55)	
Probability, $\gamma$						
(E/V) $\leq$ -0.20					17.315 (6.85)	24.967 (10.24)
-0.20 < (E/V) $\leq$ -0.10					15.882 (2.76)	14.917 (2.55)
-0.10 < (E/V) $\leq$ 0					0.196 (7.13)	3.558 (8.68)
(E/V) $\leq$ 0	4.378 (8.38)	4.355 (7.02)	4.320 (6.98)	5.060 (8.17)		
0 < (E/V) $\leq$ +0.15			3.356 (5.55)	4.767 (8.32)		
+0.15 < (E/V) $\leq$ +0.30			1.007 (6.98)	0.042 (0.06)		
$\chi^2$	372.0	46.7	619.5	483.2	573.1	419.0

Note: Asymptotic t ratios in parentheses.

These models indicate an even more powerful relationship between homeowner equity and default probabilities. Columns 1 and 2 imply that a homeowner with negative equity is more than 81 times (*i.e.*,  $\exp[4.4]$ ) as likely to exercise the option as a homeowner with positive equity. Columns 3 and 4 again indicate that negative equity is strongly associated with higher default rates, but that low positive levels of equity are also associated with increased default probabilities.

The coefficients in column 3, for example, imply that households with a 15 to 30 percent equity stake in their houses are about 2.7 times as likely to default as those with larger equity stakes. Households with a zero to 15 percent equity stake are about 29 times as likely to default as those with at least 30 percent equity. Finally those with negative equity are more than 75 times as likely to default. The coefficients in column 4 are even more extreme.

The coefficients in columns 5 and 6 disaggregate negative equity into classes. Quite clearly, for equity ratios more negative than -0.1, default is essentially complete and "instantaneous" (but, with quarterly data, an instant is three months). Again, for small negative equity ratios (less than 0.1 in absolute terms), the probabilities of default are significantly larger. In column 5 the estimate is an increased default probability of 22 percent. In column 6, the estimate is very much larger indeed.

These results are quite consistent with the ruthless default model: higher probabilities of default for moderately negative equities (where the option has value) and instantaneous default for highly negative equities.

Some caution is required, however, in interpreting the default responses. The model imposes a particular exponential structure on the equity-default relationship. Moreover, the sample does not include many observations where the probability of negative equity is at all close to one. For these reasons, we have simulated default responses using various assumptions about the mean and variance of house price changes; we compare the responses with several versions of ruthless model.

### C. Simulations

Figure 4 uses the estimates in Table 1, column 2 to simulate cumulative default rates over time as a function of the initial loan-to-value ratio, assuming the average pattern of housing price appreciation observed during the 1976-1989 period.<sup>11</sup> Given the rapid build up in equity as real housing prices increased during the sample period, variations in

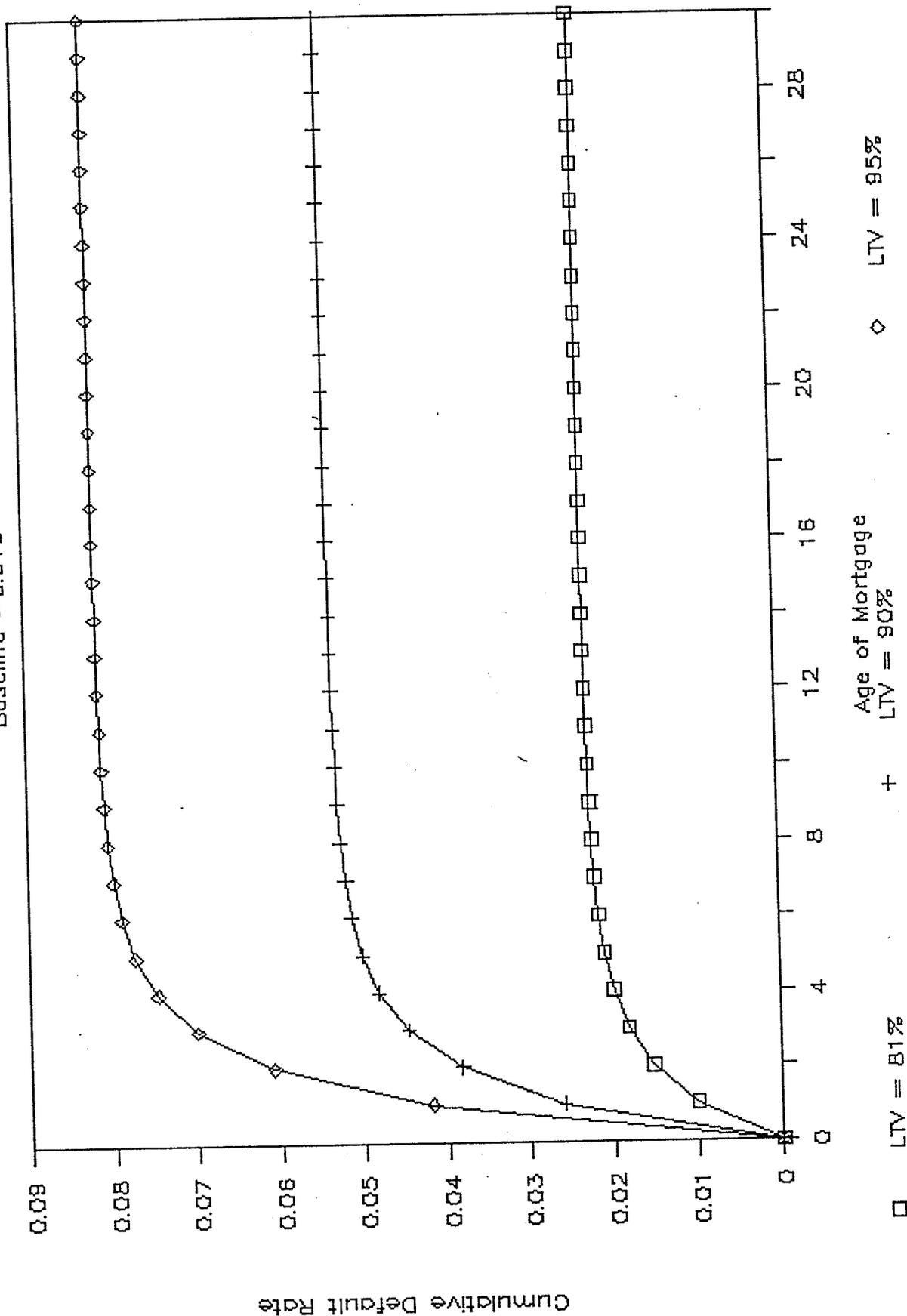
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<sup>11</sup> The simulations are conducted using yearly calculations assuming a 30 year level payment fully amortized loan at an interest rate of ten percent. The simulations also assume that the baseline hazard for an 80 percent loan-to-value loan is 0.1 percent per year, consistent with Freddie Mac default history.

FIGURE 4

# Cumulative Defaults by LTV

Baseline = 0.010



cumulative defaults arise principally from default behavior in the first few years.

Table 3 presents additional simulations emphasizing the stochastic nature of house prices. We compare four default models. The first, "Ruthless" Model I, is based upon the rigorously derived model of Kau et al [1991]. The second is a simpler specification which assumes immediate default if the option is 10 percent in the money.<sup>12</sup> The third, a "behavioral" model, is based upon the hazard rate estimates in Table 2, column 6 assuming a baseline hazard rate of 0.1 percent per year. The fourth is another "behavioral" model, based upon coefficients estimated from mortgage pools by Foster and Van Order [1985].<sup>13</sup>

Panel A of the table reports simulations using a baseline housing price appreciation rate of 3.5 percent and a standard deviation of ten percent. In Panel B, the underlying house price appreciation rate is increased to 4.5 percent. In Panel C, the standard deviation of house prices is increased from 10 to 15 percent.

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12 The results are qualitatively similar for other variants of the ruthless model, for example one with immediate default at  $(E/V) \leq -.2$ .

13 In each of the latter three models, we abstract from prepayment, assuming a constant ten percent per year prepayment rate.

TABLE 3

**Cumulative Defaults Rates by LTV for  
Behavioral and Ruthless Default Models\***

	Loan to Value Ratio		Default Ratio
	<u>80</u>	<u>95</u>	<u>95 vs. 80</u>
A. $\mu=3.5\%$ , $\sigma=10\%$			
Ruthless Model I	0.8%	10.5%	13.1
Ruthless Model II	1.8	11.3	6.3
Behavioral Model I	1.1	3.1	2.8
Behavioral Model II	4.8	13.3	2.8
B. $\mu=4.5\%$ , $\sigma=10\%$			
Ruthless Model I	-	-	-
Ruthless Model II	1.2	7.7	6.4
Behavioral Model I	0.9	1.9	2.1
Behavioral Model II	3.5	10.8	3.1
C. $\mu=3.5\%$ , $\sigma=15\%$			
Ruthless Model I	2.9	19.1	6.6
Ruthless Model II	12.2	28.0	2.3
Behavioral Model I	5.3	19.1	3.6
Behavioral Model II	10.1	18.6	1.8

## Notes:

\*Default responses for Ruthless Model I are computed from Kau et al [1991], Table 1. Ruthless Model II assumes instantaneous default at  $(E/V) \leq -.1$ . Behavioral Model I utilizes coefficients reported in Table 2, column 6 with a baseline hazard rate of 0.1%. Behavioral Model II utilizes coefficients estimated by Foster and Van Order [1985]. Table entries are mean defaults and ratios for 1000 replications of a 30 year random walk in housing prices with mean increasing by  $\mu$  percent per year and with standard deviation of  $\sigma$  percent of the mean price.

- indicates not reported.

The results reported in Panels A and B are quite consistent. When price increases are larger, as in Panel B, defaults rates are lower in all cases. However, the ratio of high LTV defaults to low LTV defaults is much larger for either of the ruthless models than for either of the behaviorally estimated models (where the ratio is 2 or 3). When the standard deviation of house prices is increased, in Panel C, defaults again increase. Behavioral Model I, utilizing the results in Table 2, does not produce a compression in default ratios, but the other two models do.

The general conclusion from these simulations is that the ratio of high LTV to low LTV default rates based on actual experience is smaller than predicted by a ruthless default model. This finding is also consistent with the unadjusted raw default data reported in the Appendix.<sup>14</sup>

#### D. Timing

Absent transactions costs, the ruthless model has implications for the timing of defaults. Put simply: for any price generating process it takes longer on average (*i.e.*, it takes more draws from the price distribution) for a low LTV loan to get into the money than for a high LTV loan. The

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<sup>14</sup> Appendix Table A1 presents actual unadjusted default experience on comparable Freddie Mac mortgages. These data are consistent with the qualitative properties of the ruthless model but again, the ratio of high LTV to low LTV defaults is not as large as predicted by the ruthless model.

hazard rate, as a function of time, will exhibit a single peak. Defaults are about zero after origination (because there is virtually no chance that the option will be in the money after a few draws) and revert to about zero later (as housing price increases make negative equity increasingly improbable). The peak will be earlier for higher LTV loans.

To illustrate, suppose house prices are lognormally distributed with mean  $P_t$  and variance  $\sigma^2 t$ . In this case,<sup>15</sup> the probability that a house has LTV above some critical level  $c$  is  $F[(P_t - \log\{c\})/\sigma t^{1/2}]$  where  $F$  is the cumulative normal. Assuming that  $c$  is constant (i.e., that the loan is perpetual rather than self-amortizing) and differentiating reveals that the probability that LTV is less than  $c$  peaks when  $t = [\log\{c\}]/P$ . For given price appreciation, the peak in the hazard rate is later for lower LTVs. For  $p=0.035$ ,  $t$  is about 5 years for an 85 percent LTV loan,  $c=0.85$ , and is about 10 years for  $c=0.70$ . The precise analysis of Kau et al [1991] quantifies and qualifies this relationship and indicates that average time to default (i.e., expected duration conditional on default) varies inversely with LTV.

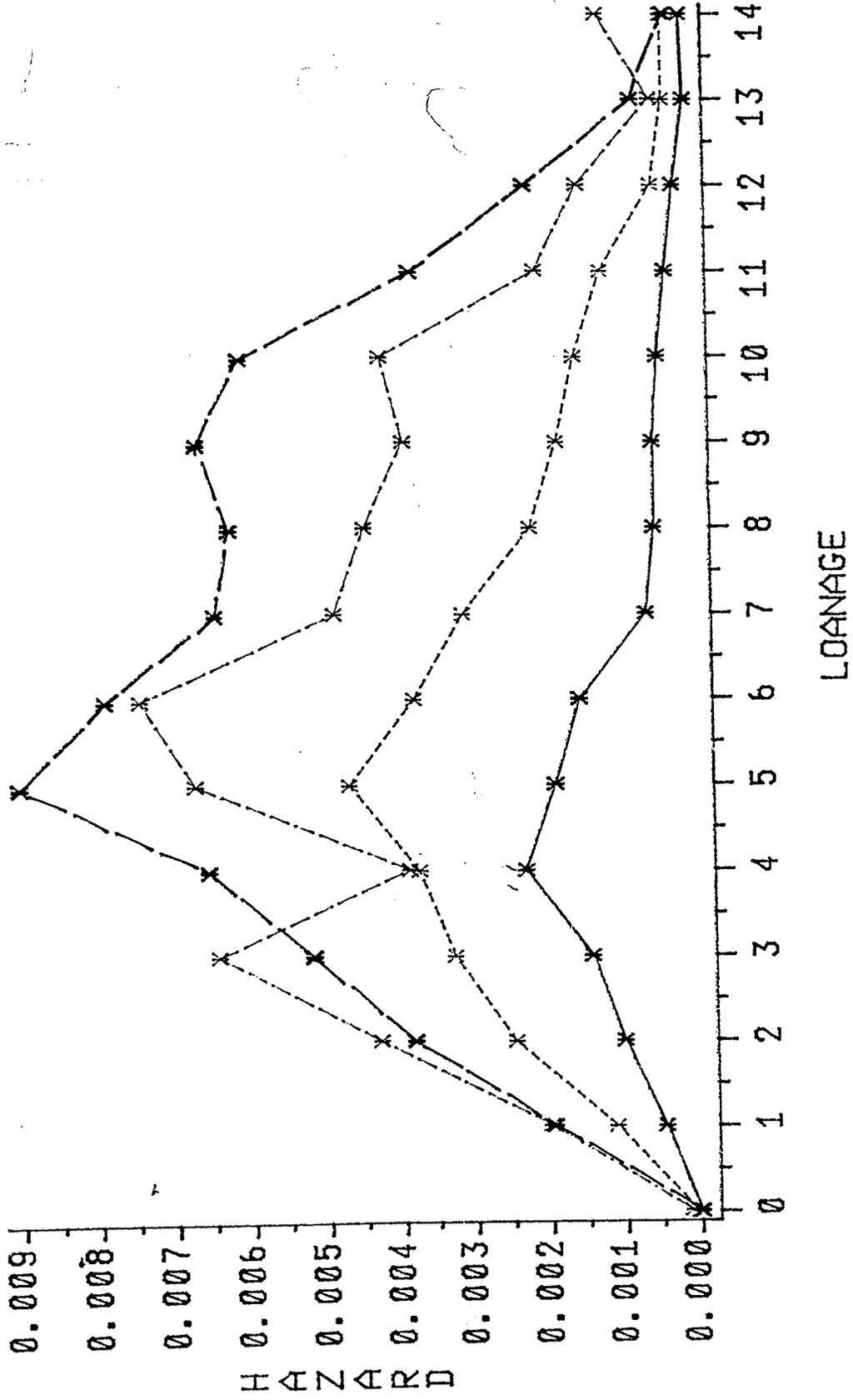
Figure 5 presents simple hazard functions by LTV, computed from unadjusted Freddie Mac data aggregated over origination years. In contrast to the predictions of the

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<sup>15</sup> Note that in our estimation (reported in footnote 7), house prices are assumed to be lognormally distributed with variance a function of  $t$  and  $t^2$ .

FIGURE 5

# MEAN HAZARD RATE LTV CLASSES 80, 90, 94, AND 95 PERCENT



ruthless model, the timing of the peak hazard to quite similar for all LTV categories. This behavior is inconsistent with the ruthless model, but is hardly definitive.

#### IV. LOSS SEVERITY AND OPTIMAL EXERCISE

Severity rates should depend only on items in (1) and in the boundary conditions. Important factors are the age of the mortgage, interest rates and the mortgage coupon. Coupon rate matters relative to current interest rates. If the coupon rate is high relative to current rates, the value of the mortgage exceeds par, which lowers the value of keeping the option alive. This implies more rapid exercise of the option and therefore lower severity rates. Similarly, the older is the mortgage (and therefore the closer it is to maturity) the less important is future option value, implying quicker exercise and lower loss severity.

In summary, the ruthless model of default implies four propositions about severity rates:

1. *Ceteris paribus*, severity should be independent of initial LTV. However, high LTV loans almost always have insurance if they are purchased by Freddie Mac. The cost of insurance increases the effective coupon rate to the borrower for high LTV loans, but not the mortgage coupon rate measured in these data. Thus for this data set, severity should fall as LTV increases.

2. *Ceteris paribus*, severity should be the same in regions with high default frequencies as in regions with low frequencies.
3. Severity should decrease with the age of the mortgage.
4. Severity should decrease as coupon rate minus the current interest rate increases.

Table 4 tabulates loss severity for all Freddie Mac defaults on loans purchased within two years of origination.<sup>16</sup> Loss severity, gross of any insurance payments, is measured by the difference between the mortgage balance and the value of the house for defaulted loans. House values are measured in two ways. The first is based on an appraisal at the time a defaulted property is acquired by Freddie Mac. The second is the actual sale price when (about a year later) the house is sold from the Freddie Mac inventory. Neither is a perfect measure of the extent to which the option was in the money when the borrower chose to default. Nonetheless, there is no reason to believe there is a systematic bias by LTV, coupon rate, interest rate, or age.

Panel A presents loss severity as a fraction of loan balance for all loans defaulted from 1975-1990. The first column indicates the losses based on appraisal data at acquisition while the second column uses eventual selling

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<sup>16</sup> Seasoned loans acquired by Freddie Mac were eliminated to avoid potential selectivity biases.

TABLE 4

Loss Severity by LTV as a Percent of Mortgage Balance\*  
1975-1990

<u>Original LTV</u>	<u>Loss I</u>	<u>Loss II</u>
A. All Loans		
51-60	-7.0%	2.0%
61-70	-0.3	8.7
71-75	0.9	10.0
76-80	2.1	11.7
81-90	5.2	15.1
91-95	14.6	24.3
B. Texas Loans		
51-60	-1.5%	6.9%
61-70	13.1	21.6
71-75	13.8	21.7
76-80	16.7	24.5
81-90	18.2	26.8
91-95	23.0	32.0

## Notes:

\* Average losses on all defaulted loans, 1975-1990, excluding defaults on seasoned loans purchased by Freddie Mac.

Loss I is computed as mortgage balance minus appraised value at acquisition.

Loss II is computed as mortgage balance minus actual sales price at time of sale.

Source: Freddie Mac.

prices. As expected, actual losses consistently exceeded appraised losses, by about 8 to 10 percent. In both columns, however, there is a strong effect of LTV. High LTV loans have much higher severity rates. This is not consistent with the first prediction of the ruthless model.

Panel B of the table presents similar calculations for Texas defaults during the same time period. The LTV effect remains, though it appears to be much smaller. However, the Texas losses are substantially higher. This is not consistent with the second prediction of the frictionless model.<sup>17</sup>

Of course, these static comparisons require holding other things constant. To control better for other factors, we regressed individual severity rates using actual (not appraised) losses on LTV categories, a dummy variable for Texas loans, dummy variables for origination years, the age of the mortgage and coupon-rate-minus-current-mortgage rates.

Table 5 summarizes these regressions. All four predictions are rejected in the results reported in column 1. The effects of LTV and of Texas loans are the same as those

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<sup>17</sup> Institutional differences, such as homestead provisions and state laws requiring delays in enforcing eviction, may cause average loss rates to vary among states. In general, however, Texas provides fewer protections against eviction than any other state. Thus based on institutional differences alone, Texas loss rates should be lower than elsewhere. See Clauretje and Herzog [1989].

TABLE 5

Regression Models of Actual Loss Severity in Percent  
(20,459 observations)

<u>Variable</u>	<u>1*</u>	<u>2*</u>	<u>3</u>	<u>4</u>
LTV 51-60 (dummy)	1.985 (0.48)	-1.356 (0.35)	1.538 (0.36)	1.434 (0.33)
LTV 61-70 (dummy)	9.383 (2.60)	6.227 (1.79)	8.680 (2.31)	8.547 (2.27)
LTV 71-75 (dummy)	11.712 (3.29)	8.758 (2.55)	10.326 (2.78)	10.200 (2.74)
LTV 76-80 (dummy)	14.068 (4.07)	11.271 (3.39)	12.683 (3.52)	12.550 (3.49)
LTV 81-90 (dummy)	17.948 (5.21)	14.884 (4.49)	16.053 (4.47)	15.931 (4.44)
LTV >90 (dummy)	24.543 (7.10)	21.414 (6.47)	25.661 (7.15)	26.571 (7.13)
Age of mortgage (thousand days)	6.331 (21.56)	9.005 (11.29)	10.302 (5.16)	10.401 (5.23)
Age of mortgage squared (x 10 <sup>7</sup> )		6.875 (3.88)		
Coupon minus current rate	103.132 (5.83)	169.190 (8.92)	-25.38 (1.62)	
Coupon minus current rate squared (x 10 <sup>-2</sup> )		47.279 (8.03)		
Texas (dummy)	13.134 (28.00)	12.858 (6.47)		
Intercept	-19.561 (2.21)	-13.713 (1.91)	-1.461 (0.41)	-1.317 (0.37)
R <sup>2</sup>	0.528	0.528	0.491	0.490

## Notes:

\*Regression also includes dummy variables for each year of origination, 1975-1989.

t ratios are reported in parentheses.

reported in Table 4. Age has a positive effect, as does the interest rate differential.

These tests are not definitive. We cannot control for  $\sigma$  and  $\rho$  directly, and it is possible that they are correlated with the explanatory variables.<sup>18</sup> The simple linear model does not capture all the nonlinearities implicit in the option model. In column 2, we add quadratic terms for the age of the mortgage and for coupon-minus-current interest rate. Again, all four predictions are rejected. The result for coupon minus current interest rate is, however, fragile and disappears when the specification is altered slightly as in column 3. In contrast, the effect of LTV appears to be quite robust to changes in specification.

#### IV. CONCLUSION

This paper investigates the home mortgage default behavior of households, using micro data on household choice and a rich body of data on individual housing prices. The results provide rather powerful evidence that homeowner equity is an important determinant of default decisions and that negative equity triggers default behavior. "In the money" options are exercised, and the probability of default approaches one rapidly at moderate values of negative equity.

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<sup>18</sup> For instance  $\sigma$  might be high in Texas causing severity to be higher -- though whether this could explain a 13 percentage point difference is unclear.

Of particular importance is the empirical finding that at low levels of negative equity the option is not exercised immediately (confirming that the option itself has value), but (at higher levels of negative equity (above about .1 in absolute magnitude) default is essentially instantaneous.

Whether a really "ruthless" zero-transaction cost model fully explains default is a more difficult question. Our analysis does suggest that the ruthless model is qualitatively consistent with observed default data. Nonetheless, there are discrepancies and complications:

First, there is some rather weak evidence that the ruthless model overstates the spread between default frequencies for high LTV and low LTV loans. Second, the ruthless model overstates the variations over time in the peaks in default rates for different initial LTVs. Empirically, the peaks in the average default rates over time are more similar for various initial LTVs than is predicted by the ruthless default model. Third, loss severities increase significantly as a function of initial LTV, contrary to the theory.

Transactions costs by themselves do not explain these discrepancies (Indeed, transactions costs alone imply improbably low default rates). However, coupled with a random, rather than a deterministic, term of the mortgage, they appear to be consistent with observed behavior.

Homeowners move for various random reasons, and the holders of non assumable mortgages pay off at par when they move. A particularly important random move is one "forced" because, for exogenous reasons, homeowners get into "trouble" (they lose a job, get a divorce, etc.). Under these circumstances, the prospective term of the mortgage is very short, and the value of keeping the option alive may be negligible. This alone could explain the higher default rates observed for 80% LTV loans relative to 95% loans.<sup>19</sup>

Any randomness in the term of mortgage, by itself, leads to higher default rates than predicted by the frictionless model. Transactions costs, in the form of reputation costs to default, together with randomness in mortgage terms are consistent with higher defaults on low LTV loans and lower defaults on high LTV loans.

Transaction costs may also explain the relationship between loss severity and LTV. Suppose, in the extreme, that homeowners always default if they have been in "trouble" for a fixed period (because they are then cash-flow constrained) and

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<sup>19</sup> One way in which future research could test the importance of these random terms is by examining both assumable and non assumable mortgages. Random terms are only relevant for non assumable mortgages (If a mortgage is assumable and is worth keeping "alive," it will be kept alive, in the ruthless model, by the new housebuyer.) The Freddie Mac data do not distinguish between assumable and non assumable loans, and during this sample period most conventional loans were not assumable.

also have negative equity (that is, transactions costs are sufficient to limit "ruthless" default). High LTV loans will be more likely to have negative equity during any fixed period, which is consistent with a positive default relationship, and loss severities will also be larger for high LTV loans. If the period of time that borrowers are "in trouble" before defaulting is randomly distributed and independent of initial LTV, then we should expect the time profile of hazard rates by LTV to be independent of LTV, as depicted in Figure 5. Similarly, severities will be higher in depressed areas, like Texas, because prices will fall further during the "in trouble" period.

None of this really implies that transactions costs and other imperfections are disproportionately large in the housing market. Indeed, as Case and Shiller observe "There is little hope of proving definitively whether the housing market is [or is] not efficient." (p.135) Our evidence suggests that in these respects, at least, the housing market is not too different from other markets, presumed to operate reasonably efficiently, though not with textbook perfection.

APPENDIX TABLE A1

Default Rates By LTV and Origination Year for Freddie Mac Loans

Actual Cumulative Default Rates Through 1990\*

Origination Year	LTV Categories			
	0-75%	76-80%	81-85%	91-95%
1975	0.04%	0.11%	0.32%	0.94%
1976	0.06	0.11	0.42	0.92
1977	0.09	0.22	0.66	1.89
1978	0.25	0.63	1.22	4.30
1979	0.50	1.25	1.75	7.42
1980	0.81	2.87	3.08	10.43
1981	1.00	4.81	4.99	12.04
1982	0.78	3.40	3.21	12.69
1983	0.33	1.43	2.74	8.21
1984	0.20	0.54	1.20	5.23
1985	0.10	0.49	0.67	3.48
1986	0.06	0.32	0.44	2.16
1987	0.03	0.17	0.21	0.75
1988	0.03	0.06	0.07	0.26
1989	0.02	0.02	0.00	0.07
Average	0.29	1.10	1.40	4.72

Note:

\* Including only 30 year, fixed rate, single family mortgages.

Source: Freddie Mac

## References

- Abraham, Jesse and Bill Shauman, "Evidence on House Prices from FHLMC Repeat Sales," Federal Home Loan Mortgage Corporation, May 1990, mimeo.
- Black, F. and M. Scholes, "The Pricing of Options and Corporate Liabilities," *Journal of Political Economy*, Vol. 81, 1973, pp. 637-659.
- Buser, S.A. and P. Hendershott, "Pricing Default Free Mortgages," *Housing Finance Review*, Vol. 3, 1984.
- Brennan M. and E.S. Schwartz, "Determinants of GNMA Mortgage Prices," *Journal of AREUEA*, Vol. 13, 1985, pp. 209-228.
- Campbell T., and J. Dietrich, "The Determinants of Default on Insured Conventional Residential Mortgage Loans," *Journal of Finance*, December 1983, pp. 1569-1580.
- Clauretje, T., and T. N. Herzog, "How State Laws Affect Foreclosure Costs," *Secondary Mortgage Markets*, Vol. 6, Spring 1989, pp. 25-28.
- Cox, J., J. Ingersol, and S. Ross, "An Intertemporal General Equilibrium Model of Asset Prices," *Econometrica*, Vol. 53, 1985, pp. 363-384.
- Cunningham, D. and Patric Hendershott, "Pricing FHA Default Insurance," *Housing Finance Review*, Vol. 3, #4, 1984, pp. 342-373.
- Dunn, K. and J. McConnell, "Valuation of GNMA Mortgage Backed Securities," *Journal of Finance*, Vol. 32, 1983, pp. 613-623.
- Foster, Chester and Robert Van Order, "FHA Terminations: A Prelude to Rational Mortgage Pricing," *Journal of AREUEA*, Vol. 13, 1985, pp. 273-291.
- Green, Jerry and John B. Shoven, "The Effect of Interest Rates on Mortgage Prepayments," *Journal of Money, Credit and Banking*, Vol. 18, February 1986, pp. 41-50.
- Jones, Lawrence D., "Deficiency Judgments and the Exercise of the Default Option in Home Mortgage Loans," Faculty of Commerce and Business Administration, University of British Columbia, June 1991, mimeo.

Kalbfleisch, J.D. and R.L. Prentice, **The Statistical Analysis of Failure Time Data**, New York: John Wiley and Sons, 1980.

Kau J., D. Keenan, W. Muller III, and J. Eppensor, "Option Theory and Fixed-Rate Mortgages," University of Georgia, 1986, mimeo.

Kau, James B., Donald C. Keenan and Taewon Kim, "Default Probabilities for Mortgages," University of Georgia, 1991, mimeo.

Quigley, John M., "Interest Rate Variations, Mortgage Prepayments and Household Mobility, **Review of Economics and Statistics**, Vol. LXIX, #4, November 1987, pp. 636-643.

Quigley, John M. and Robert Van Order, "Efficiency in the Mortgage Market: The Borrower's Perspective," **Journal of AREUEA**, Fall 1990.

Quigley, John M. and Robert Van Order, "Defaults on Mortgage Obligation and Capital Requirements for U.S. Savings Institutions," **Journal of Public Economics**, Vol. 44, 1991, pp. 353-369.

Smith, Clifford W., "Option Pricing," **Journal of Financial Economics**, Vol. 3, Jan-Mar 1976, pp. 616-621.