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Fermion Geometry and the Renormalization of the Standard Model Effective Field Theory

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Abstract: The geometry of field space governs on-shell scattering amplitudes. We formulate a geometric description of effective field theories which extends previous results for scalars and gauge fields to fermions. The field-space geometry reorganizes and simplifies the computation of quantum loop corrections. Using this geometric framework, we calculate the fermion loop contributions to the renormalization group equations for bosonic operators in the Standard Model Effective Field Theory up to mass dimension eight.

Contents

1 Introduction

Effective field theory (EFT) is a quantum field theory with a systematic expansion in a hierarchy of scales. There is an inherent redundancy in the Lagrangian description of effective field theories: fields can be redefined while leaving physical observables unchanged [1–4]. The freedom of applying field redefinitions sometimes obscures the underlying simplicity in physical EFT observables. Field redefinitions can be viewed as coordinate changes on a field-space manifold. Since S-matrix elements are invariant under field redefinitions, they are invariant under coordinate changes of the field-space manifold, and only depend on geometric quantities. The connection between field redefinitions and geometry is well-established for scalar field redefinitions of the form $\phi^I \to \phi'^I(\phi)$ [5–7]. This geometric picture has several practical consequences, e.g., that scattering amplitudes for scalars are given in terms of the Riemann curvature and covariant derivatives [8], and that the scattering amplitudes satisfy a geometric soft theorem [9]. The geometric underpinning of EFTs has seen recent interest [10–22], and the geometric framework has been generalized in several directions to include particles with spin and field redefinitions with derivatives [23–27].

The field-space geometry simplifies calculations. For instance, the renormalization group equations (RGEs) for a theory of scalars [11] or a theory with scalars and gauge bosons [28] depend on the curvature in field space. This reorganization of terms into compact geometric objects is particularly useful when the number of effective operators is large. This is the case for the Standard Model Effective Field Theory (SMEFT), which augments the Standard Model with higher-dimensional operators. The SMEFT RGEs have been calculated for operators with mass dimension six [29–31], and partial results are available for operators with mass dimension eight [28, 32–35]. The field-space geometry has been employed to calculate parts of the bosonic RGEs up to mass dimension eight [28]. In this work, we extend this approach by including the fermion geometry in the calculation of the SMEFT RGEs, and use this to compute one-loop fermionic corrections to bosonic operators to mass dimension eight.

The paper is organized as follows. In sec. 2 we define the geometry for the combined scalar-fermion field space. In sec. 3, we show that scattering amplitudes depend only on geometric quantities, i.e., the curvature in field space and covariant derivatives. In sec. 4 we use this field-space geometry to derive a general formula for the fermionic one-loop contribution to the bosonic RGEs. We then apply this formula to the SMEFT in sec. 5 and the Low-energy Effective Field Theory (LEFT) below the electroweak scale in sec. 6. We conclude in sec. 7. Appendix A lists the operator basis for the SMEFT and the RGE results for the SMEFT are listed in app. B. Useful identities for selfdual dipoles are summarized in app. C, while the extension of the RGE formula to include Majorana mass and dipole terms is described in app. D, which are needed for computing the RGEs in the LEFT [36, 37].

2 Field-space geometry

We start with a theory of scalars, gauge bosons, and fermions, including interactions with at most two derivatives and two fermion fields, ignoring CP-violating operators in the bosonic sector for simplicity. The generalization to Lagrangians with higher derivatives is discussed in refs. [24, 25, 27]. The general Lagrangian takes the form (we use the notation of refs. [11, 26])

$$
\begin{aligned}
\mathcal{L} =& \frac{1}{2} h_{IJ}(\phi)(D_\mu\phi)^I(D^\mu\phi)^J - V(\phi) - \frac{1}{4} g_{AB}(\phi) F^A_{\mu\nu} F^{B\mu\nu} \\
&+ \frac{1}{2} i k_{\bar{p}r}(\phi) \left(\bar{\psi}^{\bar{p}} \gamma^\mu \overleftrightarrow{D}_\mu \psi^r \right) + i\omega_{\bar{p}rI}(\phi)(D_\mu\phi)^I \bar{\psi}^{\bar{p}} \gamma^\mu \psi^r - \bar{\psi}^{\bar{p}} \mathcal{M}_{\bar{p}r}(\phi)\psi^r + \bar{\psi}^{\bar{p}} \sigma_{\mu\nu} \mathcal{T}^{\mu\nu}_{\bar{p}r}(\phi, F)\psi^r \,,
\end{aligned}
\tag{2.1}
$$

where $\bar{\psi}^{\bar{p}} \gamma^\mu \overleftrightarrow{D}_\mu \psi^r = \bar{\psi}^{\bar{p}} \gamma^\mu \left(D_\mu \psi^r\right) - \left(D_\mu \bar{\psi}^{\bar{p}}\right) \gamma^\mu \psi^r$. Here $I, J, K, \ldots$ are scalar indices, $A, B, C, \ldots$ are gauge field indices, and $p, \bar{p}, r, \bar{r}, \ldots$ are fermion indices. All quantities $h_{IJ}(\phi)$, $V(\phi)$, $g_{AB}(\phi)$, $k_{\bar{p}r}(\phi)$, $\omega_{\bar{p}rI}(\phi)$, and $\mathcal{M}_{\bar{p}r}(\phi)$ are functions of the scalar fields, while $\mathcal{T}^{\mu\nu}_{\bar{p}r}(\phi, F)$ depends on both the scalar fields and the field strength. The field strength and covariant derivatives are

$$
F^B_{\mu\nu} = \partial_\mu A^B_\nu - \partial_\nu A^B_\mu - f^B{}_{CD} A^C_\mu A^D_\nu \,, \tag{2.2a}
$$

$$
(D_\mu\phi)^I = \partial_\mu \phi^I + A^B_\mu t^I_B(\phi) \,, \tag{2.2b}
$$

$$
(D_\mu\psi)^p = \partial_\mu \psi^p + A^B_\mu t^p_{B,s} \psi^s \,. \tag{2.2c}
$$

Note that the gauge couplings have been absorbed into the definitions of $f^A{}_{BC}$, and the gauge couplings and i into t^I_B and $t^p_{B,s}$ [26]. As discussed in ref. [11], the gauge symmetry is a geometric symmetry of the scalar manifold, and $t^I_B(\phi)$ are Killing vectors,

$$
\nabla_J t_{IB}(\phi) + \nabla_I t_{JB}(\phi) = 0 \,, \tag{2.3}
$$

where the covariant derivative uses the metric connection of h_{IJ} defined below in eq. (2.10), and $t_{IB}(\phi) = h_{IJ} t^J_B(\phi)$ is the Killing vector with the index lowered using the metric. The Killing vectors satisfy the Lie algebra

$$
[t_A, t_B] = f^C{}_{AB}\, t_C, \tag{2.4}
$$

where the left-hand side is the Lie bracket of the vectors t^I_A. The fermionic generators $t^p_{B,s}$ satisfy the same algebra up to a sign, where the left-hand side is now matrix multiplication.

2.1 Field redefinitions

Now consider a redefinition of the scalar field, $\phi^I \to \phi'^I(\phi)$. We will consider only point transformations, i.e., field redefinitions not involving derivatives. The scalar field transforms as a coordinate under this field redefinition, while $(D_\mu\phi)^I$ transforms as a vector,

$$
(D_\mu\phi)^I \to \left(\frac{\partial \phi'^I}{\partial \phi^J}\right) (D_\mu\phi)^J \,, \tag{2.5}
$$

and $h_{IJ}(\phi)$ transforms as a tensor,

$$
h_{IJ} \to \left(\frac{\partial \phi^K}{\partial \phi'^I}\right) \left(\frac{\partial \phi^L}{\partial \phi'^J}\right) h_{KL} \,. \tag{2.6}
$$

From this we identify $h_{IJ}(\phi)$ as the metric in the scalar field space, since it has the transformation properties of a metric under coordinate changes.

We want to extend the notion of a field-space geometry to include the fermions. The field-space geometry for gauge fields was discussed in refs. [24, 26, 28], so we drop them in the following discussion for simplicity. In the end, the full field-space geometry will include all scalars, fermions, and gauge bosons.

The scalar kinetic term involves the scalar field-space metric $h_{IJ}(\phi)$. The gauge kinetic term prefactor $g_{AB}(\phi)$ enters in the metric for the gauge-scalar field space [24, 26, 28]. Thus, it is natural to expect that the prefactor of the fermion kinetic term, $k_{\bar{p}r}(\phi)$, will play a central role in the field-space geometry for fermions. Under a redefinition of the fermion field that depends on the scalar field,

$$\psi^p \to R^p{}_s(\phi)\psi^s \,, \tag{2.7}$$

the fermion kinetic term prefactor $k_{\bar{p}r}(\phi)$ mixes with $\omega_{\bar{p}rI}(\phi)$ as

$$k_{\bar{p}r} \to \left[(R^\dagger)^{-1} k R^{-1}\right]_{\bar{p}r} \,, \tag{2.8}$$

$$\omega_{\bar{p}rI} \to \left[(R^\dagger)^{-1} \omega_I R^{-1}\right]_{\bar{p}r} + \frac{1}{2}\left[(R^\dagger)^{-1} k (\partial_I R^{-1})\right]_{\bar{p}r} - \frac{1}{2}\left[(\partial_I (R^\dagger)^{-1}) k R^{-1}\right]_{\bar{p}r} \,, \tag{2.9}$$

where $\partial_I \equiv \partial/\partial\phi^I$. If the fermionic indices $\bar{p}$ and r are regarded as indices in a local Cartan frame, and eq. (2.7) as a frame redefinition, $k_{\bar{p}r}$ transforms like the metric and $\omega_{\bar{p}rI}$ like the Cartan connection. This means that both $k_{\bar{p}r}$ and $\omega_{\bar{p}rI}$ are needed in the fermion geometry.

2.2 Curvature

Before discussing the geometric construction for the fermions, we start with the scalar sector, defined by the metric h_{IJ}. From this field-space metric, we can calculate the Christoffel symbol, field-space covariant derivatives, and the Riemann curvature. To distinguish the different geometric objects, all quantities in the combined scalar-fermion field space are denoted with a bar, while no bar is used for the scalar sector.

The Christoffel symbol is given by

$$\Gamma^I_{JK} = \frac{1}{2} h^{IL} \left(h_{LK,J} + h_{JL,K} - h_{JK,L}\right) \,, \tag{2.10}$$

where $h_{IJ,K} = \partial_K h_{IJ}$. This Christoffel symbol is the connection used in the field-space covariant derivative ∇_I. The Riemann curvature is

$$R^I{}_{JKL} = \partial_K \Gamma^I_{LJ} - \partial_L \Gamma^I_{KJ} + \Gamma^I_{KN}\Gamma^N_{LJ} - \Gamma^I_{LN}\Gamma^N_{KJ} \,. \tag{2.11}$$

The curvature is a function of the scalar field ϕ. It is often useful to evaluate the curvature at the vacuum expectation value (VEV), $\langle \phi^I \rangle = v^I$. In particular, the curvature and field-space covariant derivatives appearing in scattering amplitudes and soft theorems are all evaluated at the VEV [9].

We now combine the space of scalar and fermion fields, and define geometric quantities in this combined space. Our construction for the scalar-fermion field space is similar to those found in supersymmetric nonlinear sigma models [38] and in refs. [23, 39]. The usefulness of the following definitions will be evident in that scattering amplitudes and renormalization group equations depend on geometric quantities in this combined space.

The first difference from the scalar case above is that we must include the fermions as coordinates on the manifold. The natural extension of a Riemannian manifold which allows for Grassmann coordinates is called a *supermanifold* [40, 41]. This is not limited to theories with supersymmetry, and our main application in this work is the SMEFT (which is not supersymmetric). All geometric quantities in the scalar-fermion field space inherit properties from the underlying supermanifold. In essence, this means that the ordering of terms and operations like differentiation need to be handled with care, since some of the coordinates are anticommuting.

Similar to the scalar case, we will evaluate expressions at the VEV. When we evaluate expressions at the VEV, we assume that the VEVs for the fermions vanish. Note that even though the fermions have vanishing VEV, derivatives with respect to the fermion field can still be nonzero, so the VEV is only taken after evaluating all derivatives.

We start with the metric for the scalar-fermion field space evaluated at the VEV,

$$\bar{g}_{ab}\left(\langle\phi\rangle = v, \langle\psi\rangle = 0\right) = \begin{pmatrix} h_{IJ} & 0 & 0 \\ 0 & 0 & k_{\bar{r}p} \\ 0 & -k_{\bar{p}r} & 0 \end{pmatrix}, \tag{2.12}$$

where we group the scalars and fermions into the multiplet

$$\Phi^a = \begin{pmatrix} \phi^I \\ \psi^p \\ \bar{\psi}^{\bar{p}} \end{pmatrix}. \tag{2.13}$$

Lower-case Latin letters from the beginning of the alphabet run over both scalar indices $I, J, K, \ldots$ and fermion indices $p, \bar{p}, r, \bar{r}, \ldots$ Since the metric is evaluated at the VEV, where by assumption the fermions vanish, this metric does not contain enough information to derive all descendant geometric quantities. This is because we need to evaluate derivatives of the metric with respect to the fermion fields. However, starting from an ansatz and requiring that the metric transforms as a tensor under field redefinitions, we uniquely fix the metric. The combined scalar-fermion metric—not evaluated at the VEV—is

$$\bar{g}_{ab}(\phi,\psi) = \begin{pmatrix} h_{IJ} & -\left(\frac{1}{2}k_{\bar{s}r,I} - \omega_{\bar{s}rI}\right)\bar{\psi}^{\bar{s}} & \left(\frac{1}{2}k_{\bar{r}s,I} + \omega_{\bar{r}sI}\right)\psi^{s} \\ \left(\frac{1}{2}k_{\bar{s}p,J} - \omega_{\bar{s}pJ}\right)\bar{\psi}^{\bar{s}} & 0 & k_{\bar{r}p} \\ -\left(\frac{1}{2}k_{\bar{p}s,J} + \omega_{\bar{p}sJ}\right)\psi^{s} & -k_{\bar{p}r} & 0 \end{pmatrix}. \tag{2.14}$$

From this metric we derive the Christoffel symbol and Riemann curvature. In contrast to the definition in eq. (2.10) for the scalar sector, the Christoffel symbol on a supermanifold differs in that various sign factors appear in the expression, which depend on the Grassmann nature of the components. Instead of letting the reader combat this plethora of sign factors, we explicitly provide the various components of the Christoffel symbol, which are

$$\bar{\Gamma}^{I}_{JK} = \Gamma^{I}_{JK}\,,$$

$$\begin{aligned}\bar{\Gamma}^p_{Is} =& \bar{\Gamma}^p_{sI} = k^{p\bar{r}}\left(\frac{1}{2}k_{\bar{r}s,I} + \omega_{\bar{r}sI}\right) , \\ \bar{\Gamma}^{\bar{p}}_{I\bar{s}} =& \bar{\Gamma}^{\bar{p}}_{\bar{s}I} = \left(\frac{1}{2}k_{\bar{s}r,I} - \omega_{\bar{s}rI}\right) k^{r\bar{p}} ,\end{aligned} \tag{2.15}$$

and all other components are zero when evaluated at the VEV. The field-space covariant derivative $\bar{\nabla}_a$ uses these connections. These connections coincide with the connections in ref. [23], although the routes to obtain them differ. As anticipated from the discussion about field redefinitions, both $k_{\bar{p}r}$ and $\omega_{\bar{p}rI}$ make their appearances in the connections for the field-space geometry.

The Riemann curvature is

$$\bar{R}_{\bar{p}rIJ} = \omega_{\bar{p}rJ,I} - \left(\frac{1}{2}k_{\bar{p}s,I} - \omega_{\bar{p}sI}\right) k^{s\bar{t}} \left(\frac{1}{2}k_{\bar{t}r,J} + \omega_{\bar{t}rJ}\right) - (I \leftrightarrow J) , \tag{2.16}$$

again evaluated at the VEV. As expected, the Riemann curvature satisfies all the usual symmetry and Bianchi identities appropriate for a curvature on a supermanifold [42]. For instance, the curvature with scalar indices $\bar{R}_{IJKL}$ is antisymmetric in IJ and antisymmetric in KL. The mixed curvature $\bar{R}_{\bar{p}rIJ}$ in eq. (2.16) is antisymmetric in IJ, but it is symmetric in $\bar{p}r$, because $\bar{p}$ and r are fermionic indices and there is an additional minus sign from their exchange.

3 Scattering amplitudes

The geometric quantities defined above enter in scattering amplitudes. For simplicity, we turn off the couplings to the gauge fields along with the scalar potential $V(\phi)$ and the fermion mass matrix $\mathcal{M}(\phi)$ in the Lagrangian in eq. (2.1). We are then only left with the operators in the combined scalar-fermion field-space connections in eq. (2.15). Similar to the scalar case, a vielbein derived from the fermion metric appears in the interpolation between the fermion field and the scattering state [9]. The vielbein is the factor for each external leg that appears in the LSZ reduction formula for the S-matrix. The indices of the geometric objects in the scattering amplitudes are all contracted with these vielbeins, since the scattering amplitude is defined for external states, and not external fields. For ease of notation, we use the same indices for the quantities in the scattering amplitudes and in the Lagrangian. For the 4-point and 5-point amplitudes below, all momenta are taken to be incoming.

The $\psi^p\phi^I \to \psi^{\bar{r}}\phi^J$ scattering amplitude is

$$\mathcal{A}_{pI\bar{r}J} = (\bar{u}_{\bar{r}}\not{p}_I u_p)\bar{R}_{\bar{r}pJI} , \tag{3.1}$$

and the $\psi^p\phi^I \to \psi^{\bar{r}}\phi^J\phi^K$ scattering amplitude is

$$\mathcal{A}_{pI\bar{r}JK} = (\bar{u}_{\bar{r}}\not{p}_J u_p)\bar{\nabla}_K\bar{R}_{\bar{r}pIJ} + (\bar{u}_{\bar{r}}\not{p}_K u_p)\bar{\nabla}_J\bar{R}_{\bar{r}pIK} , \tag{3.2}$$

where the curvature $\bar{R}$ is defined in eq. (2.16) and

$$\bar{\nabla}_K\bar{R}_{\bar{r}pIJ} = \bar{R}_{\bar{r}pIJ,K} - \bar{\Gamma}^{\bar{s}}_{\bar{r}K}\bar{R}_{\bar{s}pIJ} - \bar{\Gamma}^s_{pK}\bar{R}_{\bar{r}sIJ} - \bar{\Gamma}^L_{IK}\bar{R}_{\bar{r}pLJ} - \bar{\Gamma}^L_{JK}\bar{R}_{\bar{r}pIL} . \tag{3.3}$$

The scattering amplitudes take remarkably compact forms when expressed in terms of the Riemann curvature and covariant derivatives of the Riemann curvature.

Turning back on the scalar potential and fermion mass matrix, but keeping all particles massless, the scattering amplitudes become

$$\begin{aligned}\mathcal{A}_{pI\bar{r}J} =&(\bar{u}_{\bar{r}}\not{\phi}_I u_p)\left(\bar{R}_{\bar{r}pJI} + k^{s\bar{t}}\left(\frac{\mathcal{M}_{\bar{r}s;I}\mathcal{M}_{\bar{t}p;J}}{s_{\bar{r}I}} - \frac{\mathcal{M}_{\bar{r}s;J}\mathcal{M}_{\bar{t}p;I}}{s_{pI}}\right)\right)\\ &- (\bar{u}_{\bar{r}}u_p)\left(\mathcal{M}_{\bar{r}p;IJ} - h^{LK}\frac{\mathcal{M}_{\bar{r}p;L}V_{;IJK}}{s_{IJ}}\right),\end{aligned} \tag{3.4}$$

and

$$\begin{aligned}\mathcal{A}_{pI\bar{r}JK} =& (\bar{u}_{\bar{r}}\not{\phi}_J u_p)\bar{\nabla}_K\bar{R}_{\bar{r}pIJ} + (\bar{u}_{\bar{r}}\not{\phi}_K u_p)\bar{\nabla}_J\bar{R}_{\bar{r}pIK}\\ &+ k^{s\bar{t}}\left\{\frac{\mathcal{M}_{\bar{t}p;J}}{s_{pJ}}\bar{R}_{\bar{r}sIK}(\bar{u}_{\bar{r}}\not{\phi}_K\not{\phi}_J u_p) + \frac{\mathcal{M}_{\bar{r}s;J}}{s_{\bar{r}J}}\bar{R}_{\bar{t}pKI}(\bar{u}_{\bar{r}}\not{\phi}_J\not{\phi}_K u_p) + (IJK)\right\}\\ &+ \left\{\frac{k^{s\bar{t}}k^{n\bar{o}}}{s_{pJ}s_{\bar{r}I}}\mathcal{M}_{\bar{r}n;I}\mathcal{M}_{\bar{o}s;K}\mathcal{M}_{\bar{t}p;J}(\bar{u}_{\bar{r}}\not{\phi}_I\not{\phi}_J u_p) + (I\leftrightarrow J\leftrightarrow K)\right\}\\ &+ \left\{(\bar{u}_{\bar{r}}\not{\phi}_K u_p)\frac{1}{2}\left[-\frac{V_{;IJM}}{s_{IJ}}h^{ML}\left[\bar{R}_{\bar{r}pLK} + k^{s\bar{t}}\left(\frac{\mathcal{M}_{\bar{r}s;K}\mathcal{M}_{\bar{t}p;L}}{s_{\bar{r}K}} - \frac{\mathcal{M}_{\bar{r}s;L}\mathcal{M}_{\bar{t}p;K}}{s_{pK}}\right)\right]\right.\right.\\ &\left.\left.+ k^{s\bar{t}}\left(\frac{\mathcal{M}_{\bar{r}s;K}\mathcal{M}_{\bar{t}p;IJ}}{s_{\bar{r}K}} - \frac{\mathcal{M}_{\bar{r}s;IJ}\mathcal{M}_{\bar{t}p;K}}{s_{pK}}\right)\right] + (I\leftrightarrow J\leftrightarrow K)\right\}\\ &+ (\bar{u}_{\bar{r}}u_p)\left\{-\mathcal{M}_{\bar{r}p;KJI} + \frac{\mathcal{M}_{\bar{r}p;L}}{s_{p\bar{r}}}h^{LM}\bar{R}_{KJIM}(s_{IK} - s_{JI}) + \frac{\mathcal{M}_{\bar{r}p;M}}{s_{p\bar{r}}}h^{ML}V_{;KJIL}\right.\\ &\left.+\left[\frac{1}{2}\frac{V_{;IJM}}{s_{IJ}}h^{ML}\left(\mathcal{M}_{\bar{r}p;KL} - \frac{\mathcal{M}_{\bar{r}p;N}}{s_{p\bar{r}}}h^{NO}V_{;OKL}\right) + (I\leftrightarrow J\leftrightarrow K)\right]\right\}.\end{aligned} \tag{3.5}$$

Here $s_{ab} = (p_a + p_b)^2$, where p_a is the incoming momentum on the particle line with index a. The covariant derivatives of V are $V_{;IJK} \equiv \nabla_K\nabla_J\nabla_I V$ and $V_{;IJKL} \equiv \nabla_L\nabla_K\nabla_J\nabla_I V$. $V_{;IJK}$ is completely symmetric in its indices if evaluated at an extremum of the potential, where $\nabla_I V = 0$. In the massless limit, the mass matrix $\nabla_I\nabla_J V = 0$ and $\mathcal{M}_{\bar{r}p} = 0$, and $V_{;IJKL}$ and $\mathcal{M}_{\bar{r}p;IJ}$ are also completely symmetric. The notation (IJK) means we sum over the three cyclic permutations in I, J, K whereas $(I \leftrightarrow J \leftrightarrow K)$ means we sum over the six permutations in I, J, K.

All couplings in the scattering amplitude are grouped in field-space covariant combinations. These results further strengthen the claim that scattering amplitudes are composed of geometric data [8–10, 14, 24, 26, 28].

4 Renormalization

The geometric formulation also simplifies the computation of the renormalization counterterms and anomalous dimensions for an EFT. We will calculate the divergent one-loop

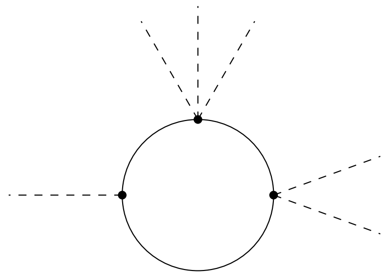

Figure 1: One-loop correction to the action. The solid line is the fermion loop, and the dashed lines represent external scalar or gauge fields. There can be an arbitrary number of vertices, each of which has two fermion lines.

terms from the second variation of the action by following the procedure of ref. [43]. See also refs. [12, 44–47] for similar functional approaches. We focus on the fermionic contributions to bosonic operators, including nontrivial metric contributions. This should be combined with the analogous geometric calculations in refs. [11, 28] for the contributions to the bosonic operators from scalar, gauge, and mixed scalar-gauge loops.

The one-loop divergences are given by computing loop graphs with vertices given by the second variation of the action (see fig. 1). The second fermionic variation of the action is computed by writing the fermion field ψ as $\psi \to \psi_B + \chi$, where ψ_B is a background (external) field, and χ is a quantum field which is integrated over. The one-loop graph in fig. 1 involves the order χ^2 part of the Lagrangian. The general Lagrangian bilinear in χ that we consider is

$$\delta_{\bar{\chi}\chi} S = \int \mathrm{d}^4 x \left\{ \frac{1}{2} i k_{\bar{p}r} \left(\bar{\chi}^{\bar{p}} \gamma^\mu \overleftrightarrow{\mathcal{D}}_\mu \chi^r \right) - \bar{\chi}^{\bar{p}} \mathcal{M}_{\bar{p}r} \chi^r + \bar{\chi}^{\bar{p}} \sigma_{\mu\nu} \mathcal{T}^{\mu\nu}_{\bar{p}r} \chi^r \right\}, \tag{4.1}$$

where the fermion fluctuation combines left- and right-handed degrees of freedom as

$$\chi = \begin{pmatrix} \chi_L \\ \chi_R \end{pmatrix} . \tag{4.2}$$

The covariant derivative is $\mathcal{D}_\mu = \partial_\mu \mathbb{1} + \omega_\mu$. The metric, mass, and dipole terms[1] are written as

$$k = \begin{pmatrix} \kappa_L & 0 \\ 0 & \kappa_R \end{pmatrix} , \qquad \mathcal{M} = \begin{pmatrix} 0 & M \\ M^\dagger & 0 \end{pmatrix} , \qquad \mathcal{T}^{\mu\nu} = \begin{pmatrix} 0 & T^{\mu\nu} \\ T^{\mu\nu\dagger} & 0 \end{pmatrix} , \tag{4.3}$$

which are functions of the external fields (possibly including derivatives). We do not include fermion bilinears with additional derivatives acting on the fermion fields in eq. (4.1). Such terms are not needed for analyzing the original Lagrangian in eq. (2.1). The dipole $T^{\mu\nu}$ can be taken to be selfdual; several relations arising from this condition are given in appendix C, and were used to simplify the results. In cases where Majorana mass or Majorana dipole

[1] We refer to $\mathcal{T}^{\mu\nu}$ as the dipole term.

terms are present, as in the LEFT [36, 37], one can use the more general form for the matrices presented in appendix D. Previous calculations [43] did not include dipole couplings, which are present in a general EFT, and are needed for both the SMEFT and the LEFT.

The infinite bosonic part of the one-loop functional integral in $4-2\epsilon$ dimensions is[2]

$$\begin{aligned}\Delta S =&\frac{1}{32\pi^2\epsilon}\int \mathrm{d}^4x\, \bigg\{\frac{1}{3}\mathrm{Tr}\left[\mathcal{Y}_{\mu\nu}\mathcal{Y}^{\mu\nu}\right] + \mathrm{Tr}\left[(\mathcal{D}_\mu\mathcal{M})(\mathcal{D}^\mu\mathcal{M}) - (\mathcal{M}\mathcal{M})^2\right] \\ &- \frac{16}{3}\mathrm{Tr}[(\mathcal{D}_\mu\mathcal{T}^{\mu\alpha})(\mathcal{D}_\nu\mathcal{T}^{\nu\alpha}) - (\mathcal{T}^{\mu\nu}\mathcal{T}^{\alpha\beta})^2] \\ &- 4i\mathrm{Tr}[\mathcal{Y}_{\mu\nu}(\mathcal{M}\mathcal{T}^{\mu\nu} + \mathcal{T}^{\mu\nu}\mathcal{M})] - 8\mathrm{Tr}(\mathcal{M}\mathcal{T}^{\mu\nu})^2\bigg\}\,,\end{aligned} \tag{4.4}$$

where

$$[\mathcal{Y}_{\mu\nu}]^p{}_r = [\mathcal{D}_\mu,\mathcal{D}_\nu]^p{}_r = \bar{R}^p{}_{rIJ}(D_\mu\phi)^I(D_\nu\phi)^J + \left(\bar{\nabla}_r t^p_A\right) F^A_{\mu\nu}\,, \tag{4.5}$$

$$(\mathcal{D}_\mu\mathcal{M})^p{}_r = k^{p\bar{t}}(\mathcal{D}_\mu\mathcal{M}_{\bar{t}r}) = k^{p\bar{t}}\left[D_\mu\mathcal{M}_{\bar{t}r} - \bar{\Gamma}^{\bar{s}}_{I\bar{t}}(D_\mu\phi)^I\mathcal{M}_{\bar{s}r} - \bar{\Gamma}^s_{Ir}(D_\mu\phi)^I\mathcal{M}_{\bar{t}s}\right]\,, \tag{4.6}$$

$$(\mathcal{M}\mathcal{M})^p{}_r = k^{p\bar{t}}\mathcal{M}_{\bar{t}q}k^{q\bar{s}}\mathcal{M}_{\bar{s}r}\,, \tag{4.7}$$

$$(\mathcal{D}_\mu\mathcal{T}^{\alpha\beta})^p{}_r = k^{p\bar{t}}(\mathcal{D}_\mu\mathcal{T}^{\alpha\beta}_{\bar{t}r}) = k^{p\bar{t}}\left[D_\mu\mathcal{T}^{\alpha\beta}_{\bar{t}r} - \bar{\Gamma}^{\bar{s}}_{I\bar{t}}(D_\mu\phi)^I\mathcal{T}^{\alpha\beta}_{\bar{s}r} - \bar{\Gamma}^s_{Ir}(D_\mu\phi)^I\mathcal{T}^{\alpha\beta}_{\bar{t}s}\right]\,, \tag{4.8}$$

$$(\mathcal{T}^{\mu\nu}\mathcal{T}^{\alpha\beta})^p{}_r = k^{p\bar{t}}\mathcal{T}^{\mu\nu}_{\bar{t}q}k^{q\bar{s}}\mathcal{T}^{\alpha\beta}_{\bar{s}r}\,. \tag{4.9}$$

The connections and curvature are defined in eqs. (2.15) and (2.16), while the covariant derivative of the Killing vector is $\bar{\nabla}_r t^p_A = t^p_{A,r} + \bar{\Gamma}^p_{Ir}t^I_A$. The final expression involves the field-strength tensor computed from the commutator of two fermion covariant derivatives, as well as covariant derivatives of the mass and dipole matrices. The field-strength tensor $\mathcal{Y}_{\mu\nu}$ has the same formula as for the scalar case [11] and the scalar-gauge case [28], with the curvature tensor now having two fermionic and two bosonic indices, rather than four bosonic indices. From the above expressions we can calculate the one-loop anomalous dimensions of the effective operators in the Lagrangian. In the next section we apply the general formula in eq. (4.4) to the SMEFT, for both dimension-six and dimension-eight operators.

5 Standard Model Effective Field Theory

The results in the previous sections hold for a general effective field theory. In this section, we will apply these results to the SMEFT.

The Standard Model Lagrangian is

$$\mathcal{L} = -\frac{1}{4}F^A_{\mu\nu}F^{A\,\mu\nu} + (D_\mu H)^\dagger(D^\mu H) - \lambda\left(H^\dagger H - \frac{1}{2}v^2\right)^2 + \delta_{\bar{p}r}i\bar{\psi}^{\bar{p}}\gamma^\mu D_\mu\psi^r - \bar{\psi}^{\bar{p}}\mathcal{M}_{\mathrm{SM},\bar{p}r}\psi^r\,, \tag{5.1}$$

where we have grouped all gauge fields from the gauge group $SU(3)_c\otimes SU(2)_L\otimes U(1)_Y$ into the multiplet

$$A^B_\mu = \begin{pmatrix} G^{\mathscr{B}}_\mu \\ W^b_\mu \\ B_\mu\end{pmatrix}\,, \qquad F^B_{\mu\nu} = \partial_\mu A^B_\nu - \partial_\nu A^B_\mu - f^B{}_{CD}A^C_\mu A^D_\nu\,. \tag{5.2}$$

[2]The counterterm Lagrangian is the negative of this expression.

Similarly, all fermions are grouped into the multiplet

$$\psi^p = \begin{pmatrix} \ell_L^p \\ q_L^p \\ e_R^p \\ u_R^p \\ d_R^p \end{pmatrix} , \tag{5.3}$$

where the index p runs over all flavor indices and gauge group indices. The Yukawa terms are all grouped into one term,

$$\bar{\psi}^{\bar{p}} \mathcal{M}_{\mathrm{SM},\bar{p}r} \psi^r = [Y_e]^\dagger_{\bar{p}r} (\bar{L}_{\bar{p}} e_r H) + [Y_u]^\dagger_{\bar{p}r} (\bar{Q}_{\bar{p}} u_r \tilde{H}) + [Y_d]^\dagger_{\bar{p}r} (\bar{Q}_{\bar{p}} d_r H) + \mathrm{h.c.} \tag{5.4}$$

The fermion metric and dipole are trivial in the Standard Model,

$$k_{\mathrm{SM},\bar{p}r} = \delta_{\bar{p}r} , \qquad \mathcal{T}^{\mu\nu}_{\mathrm{SM}} = 0 . \tag{5.5}$$

The SMEFT extends the Standard Model by adding a complete, non-redundant set of higher-dimensional operators built from the fields of the Standard Model. For our purposes, we focus on the operators with fermions at mass dimension six and eight that contribute to eq. (2.1). These operators are listed in table 2 and table 4, respectively. The dimension-six operators are all in the Warsaw basis [48], and the dimension-eight operators coincide with the operator basis in ref. [49],[3] with two exceptions: the operators ${}^8Q^{(2)}_{l^2H^4D}$ and ${}^8Q^{(3)}_{l^2H^4D}$ in table 4 are linear combinations of two operators in ref. [49], and similarly for the corresponding operators with quarks. Also, our definition of ${}^8Q_{udH^4D}$ is a factor of 2 smaller than the definition in ref. [49].

We will illustrate how the operators in the SMEFT can be grouped into the scalar and dipole terms previously defined. We will focus on the terms involving the right-handed electron fields for simplicity; all other components are extracted analogously. The mass matrix is

$$M_{\bar{p}r} \supset [Y_e]^\dagger_{\bar{p}r} H - {}^6C_{\underset{\bar{p}r}{leH^3}} H (H^\dagger H) - {}^8C_{\underset{\bar{p}r}{leH^5}} H (H^\dagger H)^2 , \tag{5.6}$$

where in addition to the Yukawa couplings, we also have dimension-six and dimension-eight contributions. We have included a left superscript 6 or 8 to make clear which contributions are dimension six or dimension eight. This coincides with the grouping of terms introduced in ref. [14]. Similarly, the dipole term with the $B_{\mu\nu}$ field strength up to mass dimension eight is

$$T^{\mu\nu}_{\bar{p}r} \supset {}^6C_{\underset{\bar{p}r}{leBH}} H \frac{1}{2} \left(B^{\mu\nu} - i\tilde{B}^{\mu\nu} \right) + {}^8C_{\underset{\bar{p}r}{leBH^3}} H (H^\dagger H) \frac{1}{2} \left(B^{\mu\nu} - i\tilde{B}^{\mu\nu} \right) . \tag{5.7}$$

Note that the dipole is explicitly selfdual; this is required to use eq. (4.4).

We apply the formalism developed above to the SMEFT. To illustrate the method, we include SMEFT operators that contribute to the mass, dipole, and fermion covariant derivative. The SMEFT dimension-six operators of type $\psi^2 H^3$ and dimension-eight operators of

[3] An operator basis at mass dimension eight was also constructed in ref. [50].

type $\psi^2 H^5$ are included in the mass matrix, dimension-six operators of type $\psi^2 XH$ and dimension-eight operators of type $\psi^2 XH^3$ in the dipole matrix, and dimension-six operators of type $\psi^2 H^2 D$ and dimension-eight operators of type $\psi^2 H^4 D$ in the fermion covariant derivative.

There are additional dimension-eight SMEFT operators which can also be included — $\psi^2 X^2 H$ and $\psi^2 H^3 D^2$ with no fermion derivatives could be included in the mass, $\psi^2 X^2 H$, $\psi^2 XHD^2$ with no fermion derivatives, and $\psi^2 H^3 D^2$ could be included in the dipole, and $\bar{R}RXH^2D$ and $\bar{L}LXH^2D$ could be added to the fermion covariant derivative. These do not introduce any new features in the calculation. However, they complicate the final RGE, and so will be discussed elsewhere.

Four-fermion operators can also be included in our analysis using Fierz identities. In a four-fermion operator, two fermion fields are treated as background fields, and two as quantum fields. Writing $\psi = \psi_B + \chi$ in terms of background and quantum fields, we get

$$\begin{aligned}(\bar{\psi}_1 \Gamma \psi_2)(\bar{\psi}_3 \Gamma \psi_4) \to & (\bar{\psi}_{1,B} \Gamma \psi_{2,B})(\bar{\chi}_3 \Gamma \chi_4) + (\bar{\psi}_{1,B} \Gamma \chi_2)(\bar{\psi}_{3,B} \Gamma \chi_4) + (\bar{\psi}_{1,B} \Gamma \chi_2)(\bar{\chi}_3 \Gamma \psi_{4,B}) \\ & + (\bar{\chi}_1 \Gamma \psi_{2,B})(\psi_{3,B} \Gamma \chi_4) + (\bar{\chi}_1 \Gamma \psi_{2,B})(\chi_3 \Gamma \psi_{4,B}) + (\bar{\chi}_1 \Gamma \chi_2)(\psi_{3,B} \Gamma \psi_{4,B}) \,.\end{aligned} \tag{5.8}$$

The first and last terms on the right-hand side are included in the quadratic fermion Lagrangian in eq. (4.1) in either the mass, tensor, or covariant derivative depending on whether $\Gamma = 1, \sigma^{\mu\nu}, \gamma^\mu$. The coefficient $(\bar{\psi}_{1,B} \Gamma \psi_{2,B})$ is an external bosonic source, since it contains two fermion fields. The other terms can be treated the same way, by Fierzing so that the two quantum fields are in a single fermion bilinear.[4] These terms will also be discussed elsewhere.

We proceed to calculate the renormalization group equations for bosonic operators coming from a fermion loop in the SMEFT to mass dimension eight. This means that we can directly apply the general formula in eq. (4.4) to the SMEFT operators. As in ref. [28], evaluating the expressions in eq. (4.4) leads to operators which are not necessarily in the chosen operator basis. They can be converted to the standard basis by integration-by-parts relations and by field redefinitions. Many of these relations were worked out in ref. [28], so we can reuse them here. When the smoke clears, we are only left with operators in our basis, and we can read off the RGE from their coefficients. The renormalization group equations for bosonic SMEFT operators from fermionic operators to mass dimension eight are listed in appendix B.

The RGE with dipole operator insertions exhibit the holomorphic structure in ref. [51, 52]. As shown in eq. (5.7), the dipole operators project the field strength into the selfdual component, which corresponds to a gauge boson in the helicity eigenstate. This helicity structure is preserved as long as we do not use equation of motion on the field strength.[5] When the field strength remains untouched, the RGE under a suitable basis choice should only depend on the coefficient of the dipole operator but not on its conjugate. To illustrate

[4] Using Fierz identities is sufficient for tree-level amplitudes and one-loop renormalization. However, since the Fierz identities are valid only in four dimension, we need a different approach to describe the one-loop finite terms and higher-loop effects from four-fermion operators.

[5] The equation of motion is the same for selfdual and anti-selfdual fields since $D_\mu \tilde{F}^{\mu\nu} = 0$.

this, consider the two operators of the type B^2H^2. We can recast them into a helicity basis via

$$ {}^6Q^{(\pm)}_{B^2H^2} = \frac{1}{2}\left({}^6Q^{(1)}_{B^2H^2} \mp i\,{}^6Q^{(2)}_{B^2H^2}\right) \tag{5.9} $$

such that ${}^6Q^{(+)}_{B^2H^2}$ contains only selfdual fields and ${}^6Q^{(-)}_{B^2H^2}$ only contains anti-selfdual ones. The RGE in this basis reads

$$ \begin{aligned} {}^6\dot{C}^{(+)}_{B^2H^2} =& {}^6\dot{C}^{(1)}_{B^2H^2} + i\,{}^6\dot{C}^{(2)}_{B^2H^2} = -2g_1(\tau_4 - i\tilde{\tau}_4) \\ =& -4g_1 \mathrm{Tr}\left[(y_e+y_\ell)Y_e\,{}^6C_{\ell eBH} + (y_d+y_q)N_cY_d\,{}^6C_{qdBH} + (y_u+y_q)N_cY_u\,{}^6C_{quBH}\right], \end{aligned} \tag{5.10} $$

and ${}^6\dot{C}^{(-)}_{B^2H^2}$ is given by the complex conjugate. We can see that ${}^6\dot{C}^{(+)}_{B^2H^2}$ only depends on the coefficients of the dipole operators but not their conjugate as required by the holomorphy. We check that all the RGE with dipole operator insertions obey this property as long as the field strength is not evaluated using its equation of motion.

6 Low-energy Effective Field Theory

The effective field theory below the electroweak scale is the Low-energy Effective Field Theory, or LEFT, which is obtained from the SMEFT after integrating out all fields with masses on the order of the electroweak scale [36, 37, 53, 54]. This includes the Higgs field. Since the low-energy theory has no scalar fields, many of the previous geometric considerations are less important. For instance, the metric is trivial,

$$ k_{\mathrm{LEFT},\bar{p}r} = \delta_{\bar{p}r}\,. \tag{6.1} $$

However, we can still apply the general formula in eq. (4.4) to derive the RGE coming from double insertions of dipole operators, $T^{\mu\nu}$, which are dimension five in the LEFT. This serves as an independent cross-check on parts of the RGE results in ref. [37]. The relevant dipole operators in the LEFT are

$$ \sum_{\psi=e,u,d}\left(L_{\underset{rs}{\psi\gamma}}(\bar{e}_{Lp}\sigma^{\mu\nu}e_{Rr})F_{\mu\nu} + \mathrm{h.c.}\right) + \left(L_{\underset{rs}{\nu\gamma}}(\nu^T_{Lp}C\sigma^{\mu\nu}\nu_{Lr})F_{\mu\nu} + \mathrm{h.c.}\right), \tag{6.2} $$

which contribute to the RGE for the four-fermion operator

$$ L^{V,LL}_{\underset{prst}{ee}}(\bar{e}_{Lp}\gamma^\mu e_{Lr})(\bar{e}_{Ls}\gamma_\mu e_{Lt})\,. \tag{6.3} $$

We obtain using eq. (4.4) that

$$ \dot{L}^{V,LL}_{\underset{prst}{ee}} \supset e^2q_e^2\delta_{pr}\delta_{st}\zeta_e\,, \tag{6.4} $$

where

$$ \zeta_e = \frac{8}{3}\mathrm{Tr}\left[2L_{\nu\gamma}L^\dagger_{\nu\gamma} + L_{e\gamma}L^\dagger_{e\gamma} + N_cL_{u\gamma}L^\dagger_{u\gamma} + N_cL_{d\gamma}L^\dagger_{d\gamma}\right]. \tag{6.5} $$

These terms arise from the quadratic terms in $T^{\mu\nu}$ in eq. (4.4), after applying a field redefinition. We also find similar results for the RGE of other four-fermion operators in the LEFT, in agreement with ref. [37].

7 Conclusion

We have formulated a field-space geometry for scalars and fermions. This geometric picture simplifies calculations and makes the covariance of scattering amplitudes under field redefinitions manifest by expressing the scattering of particles in terms of the Riemann curvature in field space and covariant derivatives. We then used the geometry to calculate the fermionic contributions to the bosonic RGEs in the SMEFT to mass dimension eight. The results are listed in app. B.

The RGEs for fermionic operators were not the focus in this work. However, there is no obstacle in computing them using the geometric tools developed in this paper. We will return to the evaluation of these RGEs in the future.

The field-space geometry considered in this paper is linked to field redefinitions with no derivatives. Scattering amplitudes are insensitive to a larger class of field redefinitions including derivatives. Some work has been done to incorporate these general field redefinitions into the geometric language [24, 25, 27]. However, a complete description of the field-space geometry with fermions and higher-derivative field redefinitions is still lacking. We leave this for future work.

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A Operators

Here we list the operator basis used in this work. The bosonic SMEFT operators at dimension six are in Table 1, while the dimension-six operators with a fermion bilinear are given in Table 2. The Warsaw basis [48] also contains four-fermion operators, but we do not include them in this work. The SMEFT operators at dimension eight are similarly split into bosonic operators in Table 3 and fermion bilinear operators in Table 4. Our convention for these operators aligns with the operator basis in ref. [49], with a few exceptions noted in the main text.

$\boldsymbol{X^3}$

Q_G	${}^6Q_{G^3}$	$f^{\mathscr{A}\mathscr{B}\mathscr{C}} G_\mu^{\mathscr{A}\nu} G_\nu^{\mathscr{B}\rho} G_\rho^{\mathscr{C}\mu}$
Q_W	${}^6Q_{W^3}$	$\epsilon^{abc} W_\mu^{a\,\nu} W_\nu^{b\,\rho} W_\rho^{c\,\mu}$

$\boldsymbol{H^4D^2}$

$Q_{H\Box}$	${}^6Q_{H^4\Box}$	$(H^\dagger H)\Box(H^\dagger H)$
Q_{HD}	${}^6Q_{H^4D^2}$	$\left(D^\mu H^\dagger H\right)\left(H^\dagger D_\mu H\right)$

$\boldsymbol{H^6}$

Q_H	${}^6Q_{H^6}$	$(H^\dagger H)^3$

$\boldsymbol{X^2H^2}$

Q_{HG}	${}^6Q^{(1)}_{G^2H^2}$	$(H^\dagger H)\, G^{\mathscr{A}}_{\mu\nu} G^{\mathscr{A}\mu\nu}$
$Q_{H\tilde{G}}$	${}^6Q^{(2)}_{G^2H^2}$	$(H^\dagger H)\, G^{\mathscr{A}}_{\mu\nu} \tilde{G}^{\mathscr{A}\mu\nu}$
Q_{HW}	${}^6Q^{(1)}_{W^2H^2}$	$(H^\dagger H)\, W^a_{\mu\nu} W^{a\mu\nu}$
$Q_{H\tilde{W}}$	${}^6Q^{(2)}_{W^2H^2}$	$(H^\dagger H)\, W^a_{\mu\nu} \tilde{W}^{a\mu\nu}$
Q_{HB}	${}^6Q^{(1)}_{B^2H^2}$	$(H^\dagger H)\, B_{\mu\nu} B^{\mu\nu}$
$Q_{H\tilde{B}}$	${}^6Q^{(2)}_{B^2H^2}$	$(H^\dagger H)\, B_{\mu\nu} \tilde{B}^{\mu\nu}$
Q_{HWB}	${}^6Q^{(1)}_{WBH^2}$	$(H^\dagger \tau^a H)\, W^a_{\mu\nu} B^{\mu\nu}$
$Q_{H\tilde{W}B}$	${}^6Q^{(2)}_{WBH^2}$	$(H^\dagger \tau^a H)\, \tilde{W}^a_{\mu\nu} B^{\mu\nu}$

Table 1: Bosonic dimension-six operators in the SMEFT. The first column is the notation of ref. [48], and the second column is the notation used in this paper.

$\boldsymbol{\psi^2H^3}$ + h.c.

Q_{eH}	${}^6Q_{\ell eH^3}$	$(H^\dagger H)(\bar{\ell}_p e_r H)$
Q_{uH}	${}^6Q_{quH^3}$	$(H^\dagger H)(\bar{q}_p u_r \tilde{H})$
Q_{dH}	${}^6Q_{qdH^3}$	$(H^\dagger H)(\bar{q}_p d_r H)$

$\boldsymbol{\psi^2XH}$ + h.c.

Q_{eW}	${}^6Q_{\ell eWH}$	$(\bar{\ell}_p \sigma^{\mu\nu} \tau^a e_r H) W^a_{\mu\nu}$
Q_{eB}	${}^6Q_{\ell eBH}$	$(\bar{\ell}_p \sigma^{\mu\nu} e_r H) B_{\mu\nu}$
Q_{uG}	${}^6Q_{quGH}$	$(\bar{q}_p \sigma^{\mu\nu} T^{\mathscr{A}} u_r \tilde{H}) G^{\mathscr{A}}_{\mu\nu}$
Q_{uW}	${}^6Q_{quWH}$	$(\bar{q}_p \sigma^{\mu\nu} \tau^a u_r \tilde{H}) W^a_{\mu\nu}$
Q_{uB}	${}^6Q_{quBH}$	$(\bar{q}_p \sigma^{\mu\nu} u_r \tilde{H}) B_{\mu\nu}$
Q_{dG}	${}^6Q_{qdGH}$	$(\bar{q}_p \sigma^{\mu\nu} T^{\mathscr{A}} d_r H) G^{\mathscr{A}}_{\mu\nu}$
Q_{dW}	${}^6Q_{qdWH}$	$(\bar{q}_p \sigma^{\mu\nu} \tau^a d_r H) W^a_{\mu\nu}$
Q_{dB}	${}^6Q_{qdBH}$	$(\bar{q}_p \sigma^{\mu\nu} d_r H) B_{\mu\nu}$

$\boldsymbol{\psi^2H^2D}$

$Q^{(1)}_{H\ell}$	${}^6Q^{(1)}_{\ell^2H^2D}$	$(\bar{\ell}_p \gamma^\mu \ell_r)\,(H^\dagger i\overleftrightarrow{D}_\mu H)$
$Q^{(3)}_{H\ell}$	${}^6Q^{(3)}_{\ell^2H^2D}$	$(\bar{\ell}_p \gamma^\mu \tau^a \ell_r)\,(H^\dagger i\overleftrightarrow{D}^a_\mu H)$
Q_{He}	${}^6Q_{e^2H^2D}$	$(\bar{e}_p \gamma^\mu e_r)\,(H^\dagger i\overleftrightarrow{D}_\mu H)$
$Q^{(1)}_{Hq}$	${}^6Q^{(1)}_{q^2H^2D}$	$(\bar{q}_p \gamma^\mu q_r)\,(H^\dagger i\overleftrightarrow{D}_\mu H)$
$Q^{(3)}_{Hq}$	${}^6Q^{(3)}_{q^2H^2D}$	$(\bar{q}_p \gamma^\mu \tau^a q_r)\,(H^\dagger i\overleftrightarrow{D}^a_\mu H)$
Q_{Hu}	${}^6Q_{u^2H^2D}$	$(\bar{u}_p \gamma^\mu u_r)\,(H^\dagger i\overleftrightarrow{D}_\mu H)$
Q_{Hd}	${}^6Q_{d^2H^2D}$	$(\bar{d}_p \gamma^\mu d_r)\,(H^\dagger i\overleftrightarrow{D}_\mu H)$
Q_{Hud} + h.c.	${}^6Q_{udH^2D}$	$(\bar{u}_p \gamma^\mu d_r)\,(\tilde{H}^\dagger i D_\mu H)$

Table 2: Fermionic dimension-six operators in the SMEFT (not including four-fermion operators). The first column is the notation of ref. [48], and the second column is the notation used in this paper.

H^8

${}^8Q_{H^8}$	$(H^\dagger H)^4$

H^6D^2

${}^8Q^{(1)}_{H^6D^2}$	$(H^\dagger H)^2(D_\mu H^\dagger D^\mu H)$
${}^8Q^{(2)}_{H^6D^2}$	$(H^\dagger H)(H^\dagger \tau^I H)(D_\mu H^\dagger \tau^I D^\mu H)$

H^4D^4

${}^8Q^{(1)}_{H^4D^4}$	$(D_\mu H^\dagger D_\nu H)(D^\nu H^\dagger D^\mu H)$
${}^8Q^{(2)}_{H^4D^4}$	$(D_\mu H^\dagger D_\nu H)(D^\mu H^\dagger D^\nu H)$
${}^8Q^{(3)}_{H^4D^4}$	$(D^\mu H^\dagger D_\mu H)(D^\nu H^\dagger D_\nu H)$

X^3H^2

${}^8Q^{(1)}_{G^3H^2}$	$f^{\mathscr{A}\mathscr{B}\mathscr{C}}(H^\dagger H)G_\mu^{\mathscr{A}\nu}G_\nu^{\mathscr{B}\rho}G_\rho^{\mathscr{C}\mu}$
${}^8Q^{(1)}_{W^3H^2}$	$\epsilon^{abc}(H^\dagger H)W_\mu^{a\,\nu}W_\nu^{b\,\rho}W_\rho^{c\,\mu}$
${}^8Q^{(1)}_{W^2BH^2}$	$\epsilon^{abc}(H^\dagger \tau^a H)B_\mu{}^\nu W_\nu^{b\,\rho}W_\rho^{c\,\mu}$

X^2H^4

${}^8Q^{(1)}_{G^2H^4}$	$(H^\dagger H)^2 G^{\mathscr{A}}_{\mu\nu}G^{\mathscr{A}\mu\nu}$
${}^8Q^{(2)}_{G^2H^4}$	$(H^\dagger H)^2 G^{\mathscr{A}}_{\mu\nu}\tilde{G}^{\mathscr{A}\mu\nu}$
${}^8Q^{(1)}_{W^2H^4}$	$(H^\dagger H)^2 W^a_{\mu\nu}W^{a\,\mu\nu}$
${}^8Q^{(2)}_{W^2H^4}$	$(H^\dagger H)^2 W^a_{\mu\nu}\tilde{W}^{a\,\mu\nu}$
${}^8Q^{(3)}_{W^2H^4}$	$(H^\dagger \tau^a H)(H^\dagger \tau^b H)W^a_{\mu\nu}W^{b\,\mu\nu}$
${}^8Q^{(4)}_{W^2H^4}$	$(H^\dagger \tau^a H)(H^\dagger \tau^b H)W^a_{\mu\nu}\tilde{W}^{b\,\mu\nu}$
${}^8Q^{(1)}_{WBH^4}$	$(H^\dagger H)(H^\dagger \tau^a H)W^a_{\mu\nu}B^{\mu\nu}$
${}^8Q^{(2)}_{WBH^4}$	$(H^\dagger H)(H^\dagger \tau^a H)\tilde{W}^a_{\mu\nu}B^{\mu\nu}$
${}^8Q^{(1)}_{B^2H^4}$	$(H^\dagger H)^2 B_{\mu\nu}B^{\mu\nu}$
${}^8Q^{(2)}_{B^2H^4}$	$(H^\dagger H)^2 B_{\mu\nu}\tilde{B}^{\mu\nu}$

XH^4D^2

${}^8Q^{(1)}_{WH^4D^2}$	$i(H^\dagger H)(D^\mu H^\dagger \tau^a D^\nu H)W^a_{\mu\nu}$
${}^8Q^{(2)}_{WH^4D^2}$	$i(H^\dagger H)(D^\mu H^\dagger \tau^a D^\nu H)\tilde{W}^a_{\mu\nu}$
${}^8Q^{(3)}_{WH^4D^2}$	$i\epsilon^{abc}(H^\dagger \tau^a H)(D^\mu H^\dagger \tau^b D^\nu H)W^c_{\mu\nu}$
${}^8Q^{(4)}_{WH^4D^2}$	$i\epsilon^{abc}(H^\dagger \tau^a H)(D^\mu H^\dagger \tau^b D^\nu H)\tilde{W}^c_{\mu\nu}$
${}^8Q^{(1)}_{BH^4D^2}$	$i(H^\dagger H)(D^\mu H^\dagger D^\nu H)B_{\mu\nu}$
${}^8Q^{(2)}_{BH^4D^2}$	$i(H^\dagger H)(D^\mu H^\dagger D^\nu H)\tilde{B}_{\mu\nu}$

$X^2H^2D^2$

${}^8Q^{(1)}_{G^2H^2D^2}$	$(D^\mu H^\dagger D^\nu H)G^{\mathscr{A}}_{\mu\rho}G^{\mathscr{A}\rho}_\nu$
${}^8Q^{(2)}_{G^2H^2D^2}$	$(D^\mu H^\dagger D_\mu H)G^{\mathscr{A}}_{\nu\rho}G^{\mathscr{A}\nu\rho}$
${}^8Q^{(1)}_{W^2H^2D^2}$	$(D^\mu H^\dagger D^\nu H)W^a_{\mu\rho}W^{a\,\rho}_\nu$
${}^8Q^{(2)}_{W^2H^2D^2}$	$(D^\mu H^\dagger D_\mu H)W^a_{\nu\rho}W^{a\,\nu\rho}$
${}^8Q^{(4)}_{W^2H^2D^2}$	$i\epsilon^{abc}(D^\mu H^\dagger \tau^a D^\nu H)W^b_{\mu\rho}W^{c\,\rho}_\nu$
${}^8Q^{(5)}_{W^2H^2D^2}$	$\epsilon^{abc}(D^\mu H^\dagger \tau^a D^\nu H)$
	$\times(W^b_{\mu\rho}\tilde{W}^{c\,\rho}_\nu - \tilde{W}^b_{\mu\rho}W^{c\,\rho}_\nu)$
${}^8Q^{(1)}_{WBH^2D^2}$	$(D^\mu H^\dagger \tau^a D_\mu H)B_{\nu\rho}W^{a\,\nu\rho}$
${}^8Q^{(2)}_{WBH^2D^2}$	$(D^\mu H^\dagger \tau^a D_\mu H)B_{\nu\rho}\tilde{W}^{a\,\nu\rho}$
${}^8Q^{(3)}_{WBH^2D^2}$	$i(D^\mu H^\dagger \tau^a D^\nu H)$
	$\times(B_{\mu\rho}W^{a\,\rho}_\nu - B_{\nu\rho}W^{a\,\rho}_\mu)$
${}^8Q^{(4)}_{WBH^2D^2}$	$(D^\mu H^\dagger \tau^a D^\nu H)$
	$\times(B_{\mu\rho}W^{a\,\rho}_\nu + B_{\nu\rho}W^{a\,\rho}_\mu)$
${}^8Q^{(6)}_{WBH^2D^2}$	$(D^\mu H^\dagger \tau^a D^\nu H)$
	$\times(B_{\mu\rho}\tilde{W}^{a\,\rho}_\nu + B_{\nu\rho}\tilde{W}^{a\,\rho}_\mu)$
${}^8Q^{(1)}_{B^2H^2D^2}$	$(D^\mu H^\dagger D^\nu H)B_{\mu\rho}B_\nu{}^\rho$
${}^8Q^{(2)}_{B^2H^2D^2}$	$(D^\mu H^\dagger D_\mu H)B_{\nu\rho}B^{\nu\rho}$

Table 3: Bosonic dimension-eight operators in the SMEFT. The XH^4D^2 operators have a factor of i relative to ref. [49] to make them hermitian. There are also X^4 operators which have not been listed.

$\psi^2 H^5$ + h.c.

${}^8Q_{\ell e H^5}$	$(H^\dagger H)^2(\bar{\ell}_p e_r H)$
${}^8Q_{quH^5}$	$(H^\dagger H)^2(\bar{q}_p u_r \tilde{H})$
${}^8Q_{qdH^5}$	$(H^\dagger H)^2(\bar{q}_p d_r H)$

$\psi^2 X H^3$ + h.c.

${}^8Q^{(1)}_{\ell eWH^3}$	$(\bar{\ell}_p\sigma^{\mu\nu}\tau^a e_r H)(H^\dagger H)W^a_{\mu\nu}$
${}^8Q^{(2)}_{\ell eWH^3}$	$(\bar{\ell}_p\sigma^{\mu\nu} e_r H)(H^\dagger\tau^a H)W^a_{\mu\nu}$
${}^8Q_{\ell eBH^3}$	$(\bar{\ell}_p\sigma^{\mu\nu} e_r H)(H^\dagger H)B_{\mu\nu}$
${}^8Q_{quGH^3}$	$(\bar{q}_p\sigma^{\mu\nu}T^{\mathscr{A}} u_r \tilde{H})(H^\dagger H)G^{\mathscr{A}}_{\mu\nu}$
${}^8Q^{(1)}_{quWH^3}$	$(\bar{q}_p\sigma^{\mu\nu}\tau^a u_r \tilde{H})(H^\dagger H)W^a_{\mu\nu}$
${}^8Q^{(2)}_{quWH^3}$	$(\bar{q}_p\sigma^{\mu\nu} u_r \tilde{H})(H^\dagger\tau^a H)W^a_{\mu\nu}$
${}^8Q_{quBH^3}$	$(\bar{q}_p\sigma^{\mu\nu} u_r \tilde{H})(H^\dagger H)B_{\mu\nu}$
${}^8Q_{qdGH^3}$	$(\bar{q}_p\sigma^{\mu\nu}T^{\mathscr{A}} d_r H)(H^\dagger H)G^{\mathscr{A}}_{\mu\nu}$
${}^8Q^{(1)}_{qdWH^3}$	$(\bar{q}_p\sigma^{\mu\nu}\tau^a d_r H)(H^\dagger H)W^a_{\mu\nu}$
${}^8Q^{(2)}_{qdWH^3}$	$(\bar{q}_p\sigma^{\mu\nu} d_r H)(H^\dagger\tau^a H)W^a_{\mu\nu}$
${}^8Q_{qdBH^3}$	$(\bar{q}_p\sigma^{\mu\nu} d_r H)(H^\dagger H)B_{\mu\nu}$

$\psi^2 H^4 D$

${}^8Q^{(1)}_{\ell^2H^4D}$	$(\bar{\ell}_p\gamma^\mu\ell_r)\,(H^\dagger i\overleftrightarrow{D}_\mu H)(H^\dagger H)$
${}^8Q^{(2)}_{\ell^2H^4D}$	$(\bar{\ell}_p\gamma^\mu\tau^a\ell_r)\,(H^\dagger i\overleftrightarrow{D}_\mu H)(H^\dagger\tau^a H)$
${}^8Q^{(3)}_{\ell^2H^4D}$	$(\bar{\ell}_p\gamma^\mu\tau^a\ell_r)\,(H^\dagger i\overleftrightarrow{D}^a_\mu H)(H^\dagger H)$
${}^8Q^{(4)}_{\ell^2H^4D}$	$\epsilon_{abc}(\bar{\ell}_p\gamma^\mu\tau^a\ell_r)\,(H^\dagger i\overleftrightarrow{D}^b_\mu H)(H^\dagger\tau^c H)$
${}^8Q_{e^2H^4D}$	$(\bar{e}_p\gamma^\mu e_r)\,(H^\dagger i\overleftrightarrow{D}_\mu H)(H^\dagger H)$
${}^8Q^{(1)}_{q^2H^4D}$	$(\bar{q}_p\gamma^\mu q_r)\,(H^\dagger i\overleftrightarrow{D}_\mu H)(H^\dagger H)$
${}^8Q^{(2)}_{q^2H^4D}$	$(\bar{q}_p\gamma^\mu\tau^a q_r)\,(H^\dagger i\overleftrightarrow{D}_\mu H)(H^\dagger\tau^a H)$
${}^8Q^{(3)}_{q^2H^4D}$	$(\bar{q}_p\gamma^\mu\tau^a q_r)\,(H^\dagger i\overleftrightarrow{D}^a_\mu H)(H^\dagger H)$
${}^8Q^{(4)}_{q^2H^4D}$	$\epsilon_{abc}(\bar{q}_p\gamma^\mu\tau^a q_r)\,(H^\dagger i\overleftrightarrow{D}^b_\mu H)(H^\dagger\tau^c H)$
${}^8Q_{u^2H^4D}$	$(\bar{u}_p\gamma^\mu u_r)\,(H^\dagger i\overleftrightarrow{D}_\mu H)(H^\dagger H)$
${}^8Q_{d^2H^4D}$	$(\bar{d}_p\gamma^\mu d_r)\,(H^\dagger i\overleftrightarrow{D}_\mu H)(H^\dagger H)$
${}^8Q_{udH^4D}$ + h.c.	$(\bar{u}_p\gamma^\mu d_r)\,(\tilde{H}^\dagger iD_\mu H)(H^\dagger H)$

Table 4: Fermionic dimension-eight operators in the SMEFT (not including four-fermion operators or other operators not appearing in the initial Lagrangian). This notation coincides for the most part with the notation used in ref. [49].

B Renormalization Group Evolution in the SMEFT to Dimension Eight

We now list the renormalization group equations for bosonic operators renormalized by fermionic operators in the SMEFT up to dimension eight. There is partial overlap with the results in refs. [32–34], and the terms common to both calculations agree.[6]

B.1 Definitions

Here we list some combinations of couplings that enter in the RGE results below. We group then depending on which term in eq. (4.4) they originate from.

[6]The results in refs. [32, 33] are given by the replacement $g_i \to -g_i$ for gauge couplings, $Y_i \leftrightarrow Y_i^\dagger$ for Yukawa couplings, and ${}^8C_{udH^4D} \to 2\,{}^8C_{udH^4D}$ because of the different conventions used.

$\mathrm{Tr}\left[\mathcal{Y}_{\mu\nu}\mathcal{Y}^{\mu\nu}\right]$**:**

$$\kappa_1 = \left[y_e\ {}^6C_{e^2\underset{tt}{H^2}D} + 2y_\ell\ {}^6C^{(1)}_{\ell^2\underset{tt}{H^2}D} + N_c y_u\ {}^6C_{u^2\underset{tt}{H^2}D} + N_c y_d\ {}^6C_{d^2\underset{tt}{H^2}D} + 2N_c y_q\ {}^6C^{(1)}_{q^2\underset{tt}{H^2}D}\right], \tag{B.1}$$

$$\kappa_1^{(8)} = \left[y_e\ {}^8C_{e^2\underset{tt}{H^4}D} + 2y_\ell\ {}^8C^{(1)}_{\ell^2\underset{tt}{H^4}D} + N_c y_u\ {}^8C_{u^2\underset{tt}{H^4}D} + N_c y_d\ {}^8C_{d^2\underset{tt}{H^4}D} + 2N_c y_q\ {}^8C^{(1)}_{q^2\underset{tt}{H^4}D}\right], \tag{B.2}$$

$$\kappa_2 = \left[{}^6C^{(3)}_{\ell^2\underset{tt}{H^2}D} + N_c\ {}^6C^{(3)}_{q^2\underset{tt}{H^2}D}\right], \tag{B.3}$$

$$\kappa_2^{(8)} = \left[{}^8C^{(3)}_{\ell^2\underset{tt}{H^4}D} + N_c\ {}^8C^{(3)}_{q^2\underset{tt}{H^4}D}\right], \tag{B.4}$$

$$\kappa_3 = \left[{}^8C^{(2)}_{\ell^2\underset{tt}{H^4}D} + N_c\ {}^8C^{(2)}_{q^2\underset{tt}{H^4}D}\right], \tag{B.5}$$

$$\begin{aligned}\kappa_4 =& \mathrm{Tr}\left[\left({}^6C_{e^2H^2D}\right)^2 + 2\left({}^6C^{(1)}_{\ell^2H^2D}\right)^2 + N_c\left({}^6C_{u^2H^2D}\right)^2 + N_c\left({}^6C_{d^2H^2D}\right)^2\right.\\ &\left.+2N_c\left({}^6C^{(1)}_{q^2H^2D}\right)^2\right],\end{aligned} \tag{B.6}$$

$$\kappa_5 = \mathrm{Tr}\left[2\left({}^6C^{(3)}_{\ell^2H^2D}\right)^2 + 2N_c\left({}^6C^{(3)}_{q^2H^2D}\right)^2\right], \tag{B.7}$$

$$\kappa_6 = \mathrm{Tr}\left[2N_c\left({}^6C_{udH^2D}\right)\left({}^6C^\dagger_{udH^2D}\right)\right]. \tag{B.8}$$

$\mathrm{Tr}\left[(\mathcal{D}_\mu\mathcal{M})(\mathcal{D}^\mu\mathcal{M})\right]$:

For the terms involving the two Yukawa couplings, we define

$$\gamma_H^{(Y)} = \mathrm{Tr}\left[Y_e^\dagger Y_e + N_c Y_u^\dagger Y_u + N_c Y_d^\dagger Y_d\right]. \tag{B.9}$$

$$\kappa_7 = \mathrm{Tr}\left[Y_e\ {}^6C_{\ell eH^3} + N_c Y_u\ {}^6C_{quH^3} + N_c Y_d\ {}^6C_{qdH^3} + \text{h.c.}\right], \tag{B.10}$$

$$\kappa_7^{(8)} = \mathrm{Tr}\left[Y_e\ {}^8C_{\ell eH^5} + N_c Y_u\ {}^8C_{quH^5} + N_c Y_d\ {}^8C_{qdH^5} + \text{h.c.}\right], \tag{B.11}$$

$$\kappa_8 = \mathrm{Tr}\left[{}^6C_{\ell eH^3}\ {}^6C^\dagger_{\ell eH^3} + N_c\ {}^6C_{quH^3}\ {}^6C^\dagger_{quH^3} + N_c\ {}^6C_{qdH^3}\ {}^6C^\dagger_{qdH^3}\right], \tag{B.12}$$

$$\begin{aligned}\kappa_9 =& \mathrm{Tr}\left[-Y_eY_e^\dagger\,{}^6C_{e^2H^2D} + Y_e^\dagger Y_e\,{}^6C^{(1)}_{\ell^2H^2D} - N_cY_dY_d^\dagger\,{}^6C_{d^2H^2D} + N_cY_d^\dagger Y_d\,{}^6C^{(1)}_{q^2H^2D}\right.\\ &\left.+N_cY_uY_u^\dagger\,{}^6C_{u^2H^2D} - N_cY_u^\dagger Y_u\,{}^6C^{(1)}_{q^2H^2D}\right],\end{aligned} \tag{B.13}$$

$$\begin{aligned}\kappa_9^{(8)} =& \mathrm{Tr}\left[-Y_eY_e^\dagger\,{}^8C_{e^2H^4D} + Y_e^\dagger Y_e\,{}^8C^{(1)}_{\ell^2H^4D} - N_cY_dY_d^\dagger\,{}^8C_{d^2H^4D} + N_cY_d^\dagger Y_d\,{}^8C^{(1)}_{q^2H^4D}\right.\\ &\left.+N_cY_uY_u^\dagger\,{}^8C_{u^2H^4D} - N_cY_u^\dagger Y_u\,{}^8C^{(1)}_{q^2H^4D}\right],\end{aligned} \tag{B.14}$$

$$\kappa_{10} = \mathrm{Tr}\left[Y_e^\dagger Y_e\,{}^6C^{(3)}_{\ell^2H^2D} + N_c\left(Y_d^\dagger Y_d + Y_u^\dagger Y_u\right){}^6C^{(3)}_{q^2H^2D}\right], \tag{B.15}$$

$$\kappa_{10}^{(8)} = \mathrm{Tr}\left[Y_e^\dagger Y_e\,{}^8C^{(3)}_{\ell^2H^4D} + N_c\left(Y_d^\dagger Y_d + Y_u^\dagger Y_u\right){}^8C^{(3)}_{q^2H^4D}\right], \tag{B.16}$$

$$\kappa_{11} = \text{Tr}\left[-N_c Y_d Y_u^{\dagger}\,{}^6C_{udH^2D} - N_c Y_u Y_d^{\dagger}\,{}^6C^{\dagger}_{udH^2D}\right]\,, \tag{B.17}$$

$$\kappa_{11}^{(8)} = \text{Tr}\left[-N_c Y_d Y_u^{\dagger}\,{}^8C_{udH^4D} - N_c Y_u Y_d^{\dagger}\,{}^8C^{\dagger}_{udH^4D}\right]\,, \tag{B.18}$$

$$\kappa_{12} = \text{Tr}\left[Y_e^{\dagger} Y_e\,{}^8C^{(2)}_{\ell^2H^4D} + N_c\left(Y_d^{\dagger}Y_d + Y_u^{\dagger}Y_u\right){}^8C^{(2)}_{q^2H^4D}\right]\,, \tag{B.19}$$

$$\begin{aligned}\kappa_{13} = \text{Tr}\Big[&2Y_e\,{}^6C^{(1)}_{\ell^2H^2D}Y_e^{\dagger}\,{}^6C_{e^2H^2D} - Y_eY_e^{\dagger}\,{}^6C_{e^2H^2D}\,{}^6C_{e^2H^2D} - Y_e^{\dagger}Y_e\,{}^6C^{(1)}_{\ell^2H^2D}\,{}^6C^{(1)}_{\ell^2H^2D}\\ &+2N_cY_d\,{}^6C^{(1)}_{q^2H^2D}Y_d^{\dagger}\,{}^6C_{d^2H^2D} - N_cY_dY_d^{\dagger}\,{}^6C_{d^2H^2D}\,{}^6C_{d^2H^2D} - N_cY_d^{\dagger}Y_d\,{}^6C^{(1)}_{q^2H^2D}\,{}^6C^{(1)}_{q^2H^2D}\\ &+2N_cY_u\,{}^6C^{(1)}_{q^2H^2D}Y_u^{\dagger}\,{}^6C_{u^2H^2D} - N_cY_uY_u^{\dagger}\,{}^6C_{u^2H^2D}\,{}^6C_{u^2H^2D} - N_cY_u^{\dagger}Y_u\,{}^6C^{(1)}_{q^2H^2D}\,{}^6C^{(1)}_{q^2H^2D}\Big]\,,\end{aligned} \tag{B.20}$$

$$\begin{aligned}\kappa_{14} = \text{Tr}\Big[&2Y_e\,{}^6C^{(3)}_{\ell^2H^2D}Y_e^{\dagger}\,{}^6C_{e^2H^2D} - Y_e^{\dagger}Y_e\left({}^6C^{(1)}_{\ell^2H^2D}\,{}^6C^{(3)}_{\ell^2H^2D} + {}^6C^{(3)}_{\ell^2H^2D}\,{}^6C^{(1)}_{\ell^2H^2D}\right)\\ &+2N_cY_d\,{}^6C^{(3)}_{q^2H^2D}Y_d^{\dagger}\,{}^6C_{d^2H^2D} - N_cY_d^{\dagger}Y_d\left({}^6C^{(1)}_{q^2H^2D}\,{}^6C^{(3)}_{q^2H^2D} + {}^6C^{(3)}_{q^2H^2D}\,{}^6C^{(1)}_{q^2H^2D}\right)\\ &-2N_cY_u\,{}^6C^{(3)}_{q^2H^2D}Y_u^{\dagger}\,{}^6C_{u^2H^2D} + N_cY_u^{\dagger}Y_u\left({}^6C^{(1)}_{q^2H^2D}\,{}^6C^{(3)}_{q^2H^2D} + {}^6C^{(3)}_{q^2H^2D}\,{}^6C^{(1)}_{q^2H^2D}\right)\Big]\,,\end{aligned} \tag{B.21}$$

$$\kappa_{15} = \text{Tr}\left[-Y_e^{\dagger}Y_e\,{}^6C^{(3)}_{\ell^2H^2D}\,{}^6C^{(3)}_{\ell^2H^2D} - N_cY_d^{\dagger}Y_d\,{}^6C^{(3)}_{q^2H^2D}\,{}^6C^{(3)}_{q^2H^2D} - N_cY_u^{\dagger}Y_u\,{}^6C^{(3)}_{q^2H^2D}\,{}^6C^{(3)}_{q^2H^2D}\right]\,, \tag{B.22}$$

$$\begin{aligned}\kappa_{16} = \text{Tr}\Big[&-\left(Y_e\,{}^6C_{\ell eH^3} + {}^6C^{\dagger}_{\ell eH^3}Y_e^{\dagger}\right){}^6C_{e^2H^2D} + \left(Y_e^{\dagger}\,{}^6C^{\dagger}_{\ell eH^3} + {}^6C_{\ell eH^3}Y_e\right){}^6C^{(1)}_{\ell^2H^2D}\\ &-N_c\left(Y_d\,{}^6C_{qdH^3} + {}^6C^{\dagger}_{qdH^3}Y_d^{\dagger}\right){}^6C_{d^2H^2D} + N_c\left(Y_d^{\dagger}\,{}^6C^{\dagger}_{qdH^3} + {}^6C_{qdH^3}Y_d\right){}^6C^{(1)}_{q^2H^2D}\\ &+N_c\left(Y_u\,{}^6C_{quH^3} + {}^6C^{\dagger}_{quH^3}Y_u^{\dagger}\right){}^6C_{u^2H^2D} - N_c\left(Y_u^{\dagger}\,{}^6C^{\dagger}_{quH^3} + {}^6C_{quH^3}Y_u\right){}^6C^{(1)}_{q^2H^2D}\Big]\,,\end{aligned} \tag{B.23}$$

$$\begin{aligned}\kappa_{17} = \text{Tr}\Big[&\left(Y_e^{\dagger}\,{}^6C^{\dagger}_{\ell eH^3} + {}^6C_{\ell eH^3}Y_e\right){}^6C^{(3)}_{\ell^2H^2D}\\ &+N_c\left(Y_d^{\dagger}\,{}^6C^{\dagger}_{qdH^3} + {}^6C_{qdH^3}Y_d\right){}^6C^{(3)}_{q^2H^2D} + N_c\left(Y_u^{\dagger}\,{}^6C^{\dagger}_{quH^3} + {}^6C_{quH^3}Y_u\right){}^6C^{(3)}_{q^2H^2D}\Big]\,,\end{aligned} \tag{B.24}$$

$$\kappa_{18} = \text{Tr}\left[N_c\left(Y_d\,{}^6C_{quH^3} + {}^6C^{\dagger}_{qdH^3}Y_u^{\dagger}\right){}^6C_{udH^2D} + \text{h.c.}\right]\,, \tag{B.25}$$

$$\kappa_{19} = \text{Tr}\left[N_c\left(Y_uY_u^{\dagger}\right){}^6C_{udH^2D}\,{}^6C^{\dagger}_{udH^2D} + N_c\left(Y_dY_d^{\dagger}\right){}^6C^{\dagger}_{udH^2D}\,{}^6C_{udH^2D}\right]\,, \tag{B.26}$$

$$\kappa_{20} = \text{Tr}\left[N_c\left(Y_d\,{}^6C^{(3)}_{q^2H^2D}Y_u^{\dagger}\right){}^6C_{udH^2D} + N_c\left(Y_u\,{}^6C^{(3)}_{q^2H^2D}Y_d^{\dagger}\right){}^6C^{\dagger}_{udH^2D}\right]\,. \tag{B.27}$$

$\text{Tr}\left[(\mathcal{M}\mathcal{M})^2\right]$:

For the terms involving the four Yukawa couplings, we define

$$\kappa_{21} = \text{Tr}\left[Y_eY_e^{\dagger}Y_e\;{}^6C_{\ell eH^3} + N_cY_uY_u^{\dagger}Y_u\;{}^6C_{quH^3} + N_cY_dY_d^{\dagger}Y_d\;{}^6C_{qdH^3} + \text{h.c.}\right]\,, \tag{B.28}$$

$$\kappa_{21}^{(8)} = \text{Tr}\left[Y_eY_e^{\dagger}Y_e\;{}^8C_{\ell eH^5} + N_cY_uY_u^{\dagger}Y_u\;{}^8C_{quH^5} + N_cY_dY_d^{\dagger}Y_d\;{}^8C_{qdH^5} + \text{h.c.}\right]\,, \tag{B.29}$$

$$\begin{aligned}\kappa_{22} = \text{Tr}\Big[&Y_e\;{}^6C_{\ell eH^3}Y_e\;{}^6C_{\ell eH^3} + N_cY_u\;{}^6C_{quH^3}Y_u\;{}^6C_{quH^3} + N_cY_d\;{}^6C_{qdH^3}Y_d\;{}^6C_{qdH^3}\\ &+Y_e\;{}^6C_{\ell eH^3}\;{}^6C^{\dagger}_{\ell eH^3}Y_e^{\dagger} + N_cY_u\;{}^6C_{quH^3}\;{}^6C^{\dagger}_{quH^3}Y_u^{\dagger} + N_cY_d\;{}^6C_{qdH^3}\;{}^6C^{\dagger}_{qdH^3}Y_d^{\dagger}\end{aligned}$$

$$+Y_e^\dagger\,{}^6C_{\ell eH^3}^\dagger\,{}^6C_{\ell eH^3}Y_e+N_cY_u^\dagger\,{}^6C_{quH^3}^\dagger\,{}^6C_{quH^3}Y_u+N_cY_d^\dagger\,{}^6C_{qdH^3}^\dagger\,{}^6C_{qdH^3}Y_d+\text{h.c.}\Big]\,. \tag{B.30}$$

$\text{Tr}[(\mathcal{D}_\mu\mathcal{T}^{\mu\alpha})(\mathcal{D}_\nu\mathcal{T}^{\nu\alpha})]$:

For the dipole terms, we use

$$\tau_0=\frac{1}{2}\text{Tr}\left[{}^6C_{qdGH}{}^6C_{qdGH}^\dagger+{}^6C_{quGH}{}^6C_{quGH}^\dagger\right]\,, \tag{B.31}$$

$$\tau_1=\text{Tr}\left[{}^6C_{\ell eBH}{}^6C_{\ell eBH}^\dagger+N_c{}^6C_{qdBH}{}^6C_{qdBH}^\dagger+N_c{}^6C_{quBH}{}^6C_{quBH}^\dagger\right]\,, \tag{B.32}$$

$$\tau_2=\frac{1}{2}\text{Tr}\left[{}^6C_{\ell eBH}{}^6C_{\ell eWH}^\dagger+N_c{}^6C_{qdBH}{}^6C_{qdWH}^\dagger+N_c{}^6C_{quBH}{}^6C_{quWH}^\dagger+\text{h.c.}\right]\,, \tag{B.33}$$

$$\tilde\tau_2=\frac{1}{2}\text{Tr}\left[i{}^6C_{\ell eBH}{}^6C_{\ell eWH}^\dagger+iN_c{}^6C_{qdBH}{}^6C_{qdWH}^\dagger+iN_c{}^6C_{quBH}{}^6C_{quWH}^\dagger+\text{h.c.}\right]\,, \tag{B.34}$$

$$\tau_2'=\frac{1}{2}\text{Tr}\left[{}^6C_{\ell eBH}{}^6C_{\ell eWH}^\dagger+N_c{}^6C_{qdBH}{}^6C_{qdWH}^\dagger-N_c{}^6C_{quBH}{}^6C_{quWH}^\dagger+\text{h.c.}\right]\,, \tag{B.35}$$

$$\tilde\tau_2'=\frac{1}{2}\text{Tr}\left[i{}^6C_{\ell eBH}{}^6C_{\ell eWH}^\dagger+iN_c{}^6C_{qdBH}{}^6C_{qdWH}^\dagger-iN_c{}^6C_{quBH}{}^6C_{quWH}^\dagger+\text{h.c.}\right]\,, \tag{B.36}$$

$$\tau_3=\text{Tr}\left[{}^6C_{\ell eWH}{}^6C_{\ell eWH}^\dagger+N_c{}^6C_{qdWH}{}^6C_{qdWH}^\dagger+N_c{}^6C_{quWH}{}^6C_{quWH}^\dagger\right]\,, \tag{B.37}$$

$$\tau_3'=\text{Tr}\left[{}^6C_{\ell eWH}{}^6C_{\ell eWH}^\dagger+N_c{}^6C_{qdWH}{}^6C_{qdWH}^\dagger-N_c{}^6C_{quWH}{}^6C_{quWH}^\dagger\right]\,. \tag{B.38}$$

$i\text{Tr}[\mathcal{Y}_{\mu\nu}(\mathcal{M}\mathcal{T}^{\mu\nu}+\mathcal{T}^{\mu\nu}\mathcal{M})]$:

For the dipole terms which are linear in the Yukawa coupling, we use

$$\tau_4=\text{Tr}\left[(y_e+y_\ell)Y_e{}^6C_{\ell eBH}+(y_d+y_q)N_cY_d{}^6C_{qdBH}+(y_u+y_q)N_cY_u{}^6C_{quBH}+\text{h.c.}\right]\,, \tag{B.39}$$

$$\begin{aligned}\tau_4^{(8)}=\text{Tr}\Big[&(y_e+y_\ell)(Y_e{}^8C_{\ell eBH^3}-{}^6C_{\ell eH^3}^\dagger{}^6C_{\ell eBH})+(y_d+y_q)N_c(Y_d{}^8C_{qdBH^3}-{}^6C_{qdH^3}^\dagger{}^6C_{qdBH})\\&+(y_u+y_q)N_c(Y_u{}^8C_{quBH^3}-{}^6C_{quH^3}^\dagger{}^6C_{quBH})+\text{h.c.}\Big]\,,\end{aligned} \tag{B.40}$$

$$\tilde\tau_4=\text{Tr}\left[i(y_e+y_\ell)Y_e{}^6C_{\ell eBH}+i(y_d+y_q)N_cY_d{}^6C_{qdBH}+i(y_u+y_q)N_cY_u{}^6C_{quBH}+\text{h.c.}\right]\,, \tag{B.41}$$

$$\begin{aligned}\tilde\tau_4^{(8)}=\text{Tr}\Big[&i(y_e+y_\ell)(Y_e{}^8C_{\ell eBH^3}-{}^6C_{\ell eH^3}^\dagger{}^6C_{\ell eBH})+i(y_d+y_q)N_c(Y_d{}^8C_{qdBH^3}-{}^6C_{qdH^3}^\dagger{}^6C_{qdBH})\\&+i(y_u+y_q)N_c(Y_u{}^8C_{quBH^3}-{}^6C_{quH^3}^\dagger{}^6C_{quBH})+\text{h.c.}\Big]\,,\end{aligned} \tag{B.42}$$

$$\tau_5=\text{Tr}\left[(y_e+y_\ell)Y_e{}^6C_{\ell eWH}+(y_d+y_q)N_cY_d{}^6C_{qdWH}-(y_u+y_q)N_cY_u{}^6C_{quWH}+\text{h.c.}\right]\,, \tag{B.43}$$

$$\begin{aligned}\tau_5^{(8)}=\text{Tr}\Big[&(y_e+y_\ell)(Y_e{}^8C^{(1)}_{\ell eWH^3}+Y_e{}^8C^{(2)}_{\ell eWH^3}-{}^6C_{\ell eH^3}^\dagger{}^6C_{\ell eWH})\\&+(y_d+y_q)N_c(Y_d{}^8C^{(1)}_{qdWH^3}+Y_d{}^8C^{(2)}_{qdWH^3}-{}^6C_{qdH^3}^\dagger{}^6C_{qdWH})\\&-(y_u+y_q)N_c(Y_u{}^8C^{(1)}_{quWH^3}-Y_u{}^8C^{(2)}_{quWH^3}-{}^6C_{quH^3}^\dagger{}^6C_{quWH})+\text{h.c.}\Big]\,,\end{aligned} \tag{B.44}$$

$$\tilde\tau_5=\text{Tr}\left[i(y_e+y_\ell)Y_e{}^6C_{\ell eWH}+i(y_d+y_q)N_cY_d{}^6C_{qdWH}-i(y_u+y_q)N_cY_u{}^6C_{quWH}+\text{h.c.}\right]\,, \tag{B.45}$$

$$
\begin{aligned}
\tilde{\tau}_5^{(8)} =&\mathrm{Tr}\Big[i(y_e+y_\ell)(Y_e{}^8C^{(1)}_{\ell eWH^3}+Y_e{}^8C^{(2)}_{\ell eWH^3}-{}^6C^\dagger_{\ell eH^3}{}^6C_{\ell eWH})\\
&+i(y_d+y_q)N_c(Y_d{}^8C^{(1)}_{qdWH^3}+Y_d{}^8C^{(2)}_{qdWH^3}-{}^6C^\dagger_{qdH^3}{}^6C_{qdWH})\\
&-i(y_u+y_q)N_c(Y_u{}^8C^{(1)}_{quWH^3}-Y_u{}^8C^{(2)}_{quWH^3}-{}^6C^\dagger_{quH^3}{}^6C_{quWH})+\mathrm{h.c.}\Big]\,, \qquad \text{(B.46)}\\
\tau_6 =&\mathrm{Tr}\Big[Y_e{}^6C_{\ell eBH}+N_cY_d{}^6C_{qdBH}-N_cY_u{}^6C_{quBH}+\mathrm{h.c.}\Big]\,, \qquad \text{(B.47)}\\
\tau_6^{(8)} =&\mathrm{Tr}\Big[(Y_e{}^8C_{\ell eBH^3}-{}^6C^\dagger_{\ell eH^3}{}^6C_{\ell eBH})+N_c(Y_d{}^8C_{qdBH^3}-{}^6C^\dagger_{qdH^3}{}^6C_{qdBH})\\
&-N_c(Y_u{}^8C_{quBH^3}-{}^6C_{quH^3}{}^6C_{quBH})+\mathrm{h.c.}\Big]\,, \qquad \text{(B.48)}\\
\tilde{\tau}_6 =&\mathrm{Tr}\Big[iY_e{}^6C_{\ell eBH}+iN_cY_d{}^6C_{qdBH}-iN_cY_u{}^6C_{quBH}+\mathrm{h.c.}\Big]\,, \qquad \text{(B.49)}\\
\tilde{\tau}_6^{(8)} =&\mathrm{Tr}\Big[i(Y_e{}^8C_{\ell eBH^3}-{}^6C^\dagger_{\ell eH^3}{}^6C_{\ell eBH})+iN_c(Y_d{}^8C_{qdBH^3}-{}^6C^\dagger_{qdH^3}{}^6C_{qdBH})\\
&-iN_c(Y_u{}^8C_{quBH^3}-{}^6C^\dagger_{quH^3}{}^6C_{quBH})+\mathrm{h.c.}\Big]\,, \qquad \text{(B.50)}\\
\tau_7 =&\mathrm{Tr}\Big[Y_e{}^6C_{\ell eWH}+N_cY_d{}^6C_{qdWH}+N_cY_u{}^6C_{quWH}+\mathrm{h.c.}\Big]\,, \qquad \text{(B.51)}\\
\tau_7^{(8)} =&\mathrm{Tr}\Big[(Y_e{}^8C^{(1)}_{\ell eWH^3}-{}^6C^\dagger_{\ell eH^3}{}^6C_{\ell eWH})+N_c(Y_d{}^8C^{(1)}_{qdWH^3}-{}^6C^\dagger_{qdH^3}{}^6C_{qdWH})\\
&+N_c(Y_u{}^8C^{(1)}_{quWH^3}-{}^6C^\dagger_{quH^3}{}^6C_{quWH})+\mathrm{h.c.}\Big]\,, \qquad \text{(B.52)}\\
\tilde{\tau}_7 =&\mathrm{Tr}\Big[iY_e{}^6C_{\ell eWH}+iN_cY_d{}^6C_{qdWH}+iN_cY_u{}^6C_{quWH}+\mathrm{h.c.}\Big]\,, \qquad \text{(B.53)}\\
\tilde{\tau}_7^{(8)} =&\mathrm{Tr}\Big[i(Y_e{}^8C^{(1)}_{\ell eWH^3}-{}^6C^\dagger_{\ell eH^3}{}^6C_{\ell eWH})+iN_c(Y_d{}^8C^{(1)}_{qdWH^3}-{}^6C^\dagger_{qdH^3}{}^6C_{qdWH})\\
&+iN_c(Y_u{}^8C^{(1)}_{quWH^3}-{}^6C^\dagger_{quH^3}{}^6C_{quWH})+\mathrm{h.c.}\Big]\,, \qquad \text{(B.54)}\\
\tau_8 =&\mathrm{Tr}\Big[Y_e{}^8C^{(2)}_{\ell eWH^3}+N_cY_d{}^8C^{(2)}_{qdWH^2}-N_cY_u{}^8C^{(2)}_{quWH^3}+\mathrm{h.c.}\Big]\,, \qquad \text{(B.55)}\\
\tilde{\tau}_8 =&\mathrm{Tr}\Big[iY_e{}^8C^{(2)}_{\ell eWH^3}+iN_cY_d{}^8C^{(2)}_{qdWH^2}-iN_cY_u{}^8C^{(2)}_{quWH^3}+\mathrm{h.c.}\Big]\,, \qquad \text{(B.56)}\\
\tau_9 =&\mathrm{Tr}\Big[Y_d{}^6C_{qdGH}+Y_u{}^6C_{quGH}+\mathrm{h.c.}\Big]\,, \qquad \text{(B.57)}\\
\tau_9^{(8)} =&\mathrm{Tr}\Big[Y_d{}^8C_{qdGH^3}-{}^6C^\dagger_{qdH^3}{}^6C_{qdGH}+Y_u{}^8C_{quGH^3}-{}^6C^\dagger_{quH^3}{}^6C_{quGH}+\mathrm{h.c.}\Big]\,, \qquad \text{(B.58)}\\
\tilde{\tau}_9 =&\mathrm{Tr}\Big[iY_d{}^6C_{qdGH}+iY_u{}^6C_{quGH}+\mathrm{h.c.}\Big]\,, \qquad \text{(B.59)}\\
\tilde{\tau}_9^{(8)} =&\mathrm{Tr}\Big[iY_d{}^8C_{qdGH^3}-i{}^6C^\dagger_{qdH^3}{}^6C_{qdGH}+iY_u{}^8C_{quGH^3}-i{}^6C^\dagger_{quH^3}{}^6C_{quGH}+\mathrm{h.c.}\Big]\,. \qquad \text{(B.60)}
\end{aligned}
$$

We also need

$$
\begin{aligned}
\tau_{10} =&\mathrm{Tr}\Big[{}^6C^{(1)}_{e^2H^2D}Y_e{}^6C_{\ell eBH}+{}^6C^{(1)}_{\ell^2H^2D}{}^6C_{\ell eBH}Y_e+N_c{}^6C^{(1)}_{d^2H^2D}Y_d{}^6C_{qdBH}\\
&+N_c{}^6C^{(1)}_{u^2H^2D}Y_u{}^6C_{quBH}+N_c{}^6C^{(1)}_{q^2H^2D}({}^6C_{qdBH}Y_d+{}^6C_{quBH}Y_u)+\mathrm{h.c.}\Big]\,, \qquad \text{(B.61)}\\
\tilde{\tau}_{10} =&\mathrm{Tr}\Big[i{}^6C^{(1)}_{e^2H^2D}Y_e{}^6C_{\ell eBH}+i{}^6C^{(1)}_{\ell^2H^2D}{}^6C_{\ell eBH}Y_e+iN_c{}^6C^{(1)}_{d^2H^2D}Y_d{}^6C_{qdBH}\\
&+iN_c{}^6C^{(1)}_{u^2H^2D}Y_u{}^6C_{quBH}+iN_c{}^6C^{(1)}_{q^2H^2D}({}^6C_{qdBH}Y_d+{}^6C_{quBH}Y_u)+\mathrm{h.c.}\Big]\,, \qquad \text{(B.62)}\\
\tau_{11} =&\mathrm{Tr}\Big[{}^6C^{(1)}_{e^2H^2D}Y_e{}^6C_{\ell eWH}+{}^6C^{(1)}_{\ell^2H^2D}{}^6C_{\ell eWH}Y_e+N_c{}^6C^{(1)}_{d^2H^2D}Y_d{}^6C_{qdWH}
\end{aligned}
$$

$$-N_c{}^6C^{(1)}_{u^2H^2D}Y_u{}^6C_{quWH}+N_c{}^6C^{(1)}_{q^2H^2D}({}^6C_{qdWH}Y_d-{}^6C_{quWH}Y_u)+\text{h.c.}\Big]\ , \quad \text{(B.63)}$$

$$\begin{aligned}\tilde{\tau}_{11}=&\text{Tr}\Big[i{}^6C^{(1)}_{e^2H^2D}Y_e{}^6C_{\ell eWH}+i{}^6C^{(1)}_{\ell^2H^2D}{}^6C_{\ell eWH}Y_e+iN_c{}^6C^{(1)}_{d^2H^2D}Y_d{}^6C_{qdWH}\\&-iN_c{}^6C^{(1)}_{u^2H^2D}Y_u{}^6C_{quWH}+iN_c{}^6C^{(1)}_{q^2H^2D}({}^6C_{qdWH}Y_d-{}^6C_{quWH}Y_u)+\text{h.c.}\Big]\ ,\end{aligned} \quad \text{(B.64)}$$

$$\tau_{12}=\text{Tr}\Big[{}^6C^{(3)}_{\ell^2H^2D}{}^6C_{\ell eBH}Y_e+N_c{}^6C^{(3)}_{q^2H^2D}({}^6C_{qdBH}Y_d-{}^6C_{quBH}Y_u)+\text{h.c.}\Big]\ , \quad \text{(B.65)}$$

$$\tilde{\tau}_{12}=\text{Tr}\Big[i{}^6C^{(3)}_{\ell^2H^2D}{}^6C_{\ell eBH}Y_e+iN_c{}^6C^{(3)}_{q^2H^2D}({}^6C_{qdBH}Y_d-{}^6C_{quBH}Y_u)+\text{h.c.}\Big]\ , \quad \text{(B.66)}$$

$$\tau_{13}=\text{Tr}\Big[{}^6C^{(3)}_{\ell^2H^2D}{}^6C_{\ell eWH}Y_e+N_c{}^6C^{(3)}_{q^2H^2D}({}^6C_{qdWH}Y_d+{}^6C_{quWH}Y_u)+\text{h.c.}\Big]\ , \quad \text{(B.67)}$$

$$\tilde{\tau}_{13}=\text{Tr}\Big[i{}^6C^{(3)}_{\ell^2H^2D}{}^6C_{\ell eWH}Y_e+iN_c{}^6C^{(3)}_{q^2H^2D}({}^6C_{qdWH}Y_d+{}^6C_{quWH}Y_u)+\text{h.c.}\Big]\ , \quad \text{(B.68)}$$

$$\tau'_{13}=\text{Tr}\Big[{}^6C^{(3)}_{\ell^2H^2D}{}^6C_{\ell eWH}Y_e+N_c{}^6C^{(3)}_{q^2H^2D}({}^6C_{qdWH}Y_d-{}^6C_{quWH}Y_u)+\text{h.c.}\Big]\ , \quad \text{(B.69)}$$

$$\tilde{\tau}'_{13}=\text{Tr}\Big[i{}^6C^{(3)}_{\ell^2H^2D}{}^6C_{\ell eWH}Y_e+iN_c{}^6C^{(3)}_{q^2H^2D}({}^6C_{qdWH}Y_d-{}^6C_{quWH}Y_u)+\text{h.c.}\Big]\ , \quad \text{(B.70)}$$

$$\tau_{14}=\text{Tr}\Big[N_c{}^6C_{udH^2D}(Y_d{}^6C_{quWH}+{}^6C^\dagger_{qdWH}Y_u^\dagger)+\text{h.c.}\Big]\ , \quad \text{(B.71)}$$

$$\tilde{\tau}_{14}=\text{Tr}\Big[iN_c{}^6C_{udH^2D}(Y_d{}^6C_{quWH}-{}^6C^\dagger_{qdWH}Y_u^\dagger)+\text{h.c.}\Big]\ , \quad \text{(B.72)}$$

$$\tau'_{14}=\text{Tr}\Big[N_c{}^6C_{udH^2D}(Y_d{}^6C_{quWH}-{}^6C^\dagger_{qdWH}Y_u^\dagger)+\text{h.c.}\Big]\ , \quad \text{(B.73)}$$

$$\tilde{\tau}'_{14}=\text{Tr}\Big[iN_c{}^6C_{udH^2D}(Y_d{}^6C_{quWH}+{}^6C^\dagger_{qdWH}Y_u^\dagger)+\text{h.c.}\Big]\ . \quad \text{(B.74)}$$

$\text{Tr}[(\mathcal{M}\mathcal{T}^{\mu\nu})^2]$:

For the dipole terms which are quadratic in the Yukawa coupling, we use

$$\tau_{15}=\text{Tr}\Big[Y_e{}^6C_{\ell eBH}Y_e{}^6C_{\ell eBH}+N_cY_d{}^6C_{qdBH}Y_d{}^6C_{qdBH}+N_cY_u{}^6C_{quBH}Y_u{}^6C_{quBH}+\text{h.c.}\Big]\ , \quad \text{(B.75)}$$

$$\tilde{\tau}_{15}=\text{Tr}\Big[iY_e{}^6C_{\ell eBH}Y_e{}^6C_{\ell eBH}+iN_cY_d{}^6C_{qdBH}Y_d{}^6C_{qdBH}+iN_cY_u{}^6C_{quBH}Y_u{}^6C_{quBH}+\text{h.c.}\Big]\ , \quad \text{(B.76)}$$

$$\tau_{16}=\text{Tr}\Big[Y_e{}^6C_{\ell eBH}Y_e{}^6C_{\ell eWH}+N_cY_d{}^6C_{qdBH}Y_d{}^6C_{qdWH}-N_cY_u{}^6C_{quBH}Y_u{}^6C_{quWH}+\text{h.c.}\Big]\ , \quad \text{(B.77)}$$

$$\tilde{\tau}_{16}=\text{Tr}\Big[iY_e{}^6C_{\ell eBH}Y_e{}^6C_{\ell eWH}+iN_cY_d{}^6C_{qdBH}Y_d{}^6C_{qdWH}-iN_cY_u{}^6C_{quBH}Y_u{}^6C_{quWH}+\text{h.c.}\Big]\ , \quad \text{(B.78)}$$

$$\tau_{17}=\text{Tr}\Big[Y_e{}^6C_{\ell eWH}Y_e{}^6C_{\ell eWH}+N_cY_d{}^6C_{qdWH}Y_d{}^6C_{qdWH}+N_cY_u{}^6C_{quWH}Y_u{}^6C_{quWH}+\text{h.c.}\Big]\ , \quad \text{(B.79)}$$

$$\tilde{\tau}_{17}=\text{Tr}\Big[iY_e{}^6C_{\ell eWH}Y_e{}^6C_{\ell eWH}+iN_cY_d{}^6C_{qdWH}Y_d{}^6C_{qdWH}+iN_cY_u{}^6C_{quWH}Y_u{}^6C_{quWH}+\text{h.c.}\Big]\ , \quad \text{(B.80)}$$

$$\tau_{18}=\text{Tr}\Big[N_cY_d{}^6C_{quWH}Y_u{}^6C_{qdGH}+\text{h.c.}\Big]\ , \quad \text{(B.81)}$$

$$\tilde{\tau}_{18}=\text{Tr}\Big[iN_cY_d{}^6C_{quWH}Y_u{}^6C_{qdGH}+\text{h.c.}\Big]\ , \quad \text{(B.82)}$$

$$\tau_{19} = \text{Tr}\left[Y_d {}^6C_{qdGH} Y_d {}^6C_{qdGH} + Y_u {}^6C_{quGH} Y_u {}^6C_{quGH} + \text{h.c.}\right], \tag{B.83}$$

$$\tilde{\tau}_{19} = \text{Tr}\left[iY_d {}^6C_{qdGH} Y_d {}^6C_{qdGH} + iY_u {}^6C_{quGH} Y_u {}^6C_{quGH} + \text{h.c.}\right]. \tag{B.84}$$

B.2 Field Anomalous Dimensions

The fermionic contributions to the wavefunction factors are

$$\gamma_H = \gamma_H^{(Y)}, \tag{B.85}$$

$$\gamma_G = \frac{4n_g}{3} g_3^2, \tag{B.86}$$

$$\gamma_W = \frac{4n_g}{3} g_2^2, \tag{B.87}$$

$$\gamma_B = \frac{20n_g}{9} g_1^2. \tag{B.88}$$

This comes in addition to the bosonic contributions to the wavefunction factors. We have not listed the contribution to the RGEs coming from the wavefunction factors below, but it is straightforward to add them. For the RGE for any coefficient C, the right-hand side should have a term $C(\sum_i \gamma_i)$ where the sum is over all fields in the operator corresponding to C, weighted by the multiplicity of the field.

B.3 Dimension 0

The RGE for the cosmological constant is

$$\dot{\Lambda} = 0. \tag{B.89}$$

B.4 Dimension 2

The RGE for the Higgs mass is

$$\dot{m}_H^2 = 0. \tag{B.90}$$

B.5 Dimension 4

The RGE for the Higgs self-coupling is

$$\dot{\lambda} = m_H^2 \left[\frac{4}{3} g_2^2 \kappa_2 + \kappa_7 - 4\kappa_{10} - 2\kappa_{11}\right]. \tag{B.91}$$

B.6 Dimension 6

The RGEs for the dimension-six coefficients in the SMEFT Lagrangian are listed below. The dimension-eight contributions are all of order m_H^2/M^4 in the SMEFT power counting. The dimension-six contributions agree with refs. [29–31].

B.6.1 H^6

The RGE for the H^6 coupling is

$$\begin{aligned}{}^6\dot{C}_{H^6} =&\frac{16}{3}\lambda g_2^2\kappa_2 - 4\lambda(-\kappa_7 + 4\kappa_{10} + 2\kappa_{11}) - 4\kappa_{21} + m_H^2\left\{\left(\frac{1}{3}g_1^2\ {}^6C_{H^4D^2} + \frac{2}{3}g_1 g_2\ {}^6C_{WBH^2}\right)\kappa_1\right.\\ &+\left(-\frac{16}{3}g_2^2\ {}^6C_{H^4\Box} + g_2^2\ {}^6C_{H^4D^2} + \frac{4}{3}g_1 g_2\ {}^6C_{WBH^2}\right)\kappa_2\\ &+\left(4\ {}^6C_{H^4\Box} - \frac{1}{2}\ {}^6C_{H^4D^2}\right)(-\kappa_7 + 4\kappa_{10} + 2\kappa_{11}) - \kappa_7^{(8)} + 2\kappa_8 + \frac{1}{3}g_1^2\kappa_1^{(8)} + \frac{1}{3}g_2^2\kappa_2^{(8)}\\ &+\frac{1}{3}g_2^2\kappa_3 + \frac{1}{3}g_2^2\kappa_4 + \frac{1}{3}(2g_1^2 - g_2^2)\kappa_5 + \frac{1}{12}(-g_1^2 + g_2^2)\kappa_6 + \kappa_{13} - \kappa_9^{(8)} + \kappa_{16} - \kappa_{10}^{(8)}\\ &\left.-\kappa_{12} + \kappa_{14} + \kappa_{17} + \kappa_{15} + g_1 g_2\tau_2' + \frac{2}{3}g_2^2\tau_3 - g_2\tau_{11} - 2g_1\tau_{12} - \frac{1}{2}g_2\tau_{14}\right\}. \end{aligned} \tag{B.92}$$

B.6.2 H^4D^2

The RGEs for the H^4D^2 couplings are

$$ {}^6\dot{C}_{H^4\Box} =\frac{2}{3}g_1^2\kappa_1 + 2g_2^2\kappa_2 - 2\kappa_9 - 6\kappa_{10} - 2\kappa_{11}\,, \tag{B.93}$$

$$ {}^6\dot{C}_{H^4D^2} =\frac{8}{3}g_1^2\kappa_1 - 8\kappa_9 + 4\kappa_{11}\,. \tag{B.94}$$

B.6.3 X^2H^2

The RGEs for the X^2H^2 couplings are

$$ {}^6\dot{C}^{(1)}_{G^2H^2} =-2g_3\tau_9\,, \tag{B.95}$$

$$ {}^6\dot{C}^{(2)}_{G^2H^2} =2g_3\tilde{\tau}_9\,, \tag{B.96}$$

$$ {}^6\dot{C}^{(1)}_{W^2H^2} =-g_2\tau_7\,, \tag{B.97}$$

$$ {}^6\dot{C}^{(2)}_{W^2H^2} =g_2\tilde{\tau}_7\,, \tag{B.98}$$

$$ {}^6\dot{C}^{(1)}_{B^2H^2} =-2g_1\tau_4\,, \tag{B.99}$$

$$ {}^6\dot{C}^{(2)}_{B^2H^2} =2g_1\tilde{\tau}_4\,, \tag{B.100}$$

$$ {}^6\dot{C}^{(1)}_{WBH^2} =-2g_1\tau_5 - g_2\tau_6\,, \tag{B.101}$$

$$ {}^6\dot{C}^{(2)}_{WBH^2} =2g_1\tilde{\tau}_5 + g_2\tilde{\tau}_6\,. \tag{B.102}$$

B.7 Dimension 8

The dimension-eight RGEs for the dimension-eight coefficients in the SMEFT Lagrangian are listed below. The contributions are all of order $1/M^4$ in the SMEFT power counting.

B.7.1 H^8

The RGE for the H^8 coupling is

$$\begin{aligned}{}^{8}\dot{C}_{H^8} =&\lambda\left(-\frac{4}{3}g_1^2\,{}^{6}C_{H^4D^2}-\frac{8}{3}g_1g_2\,{}^{6}C_{WBH^2}\right)\kappa_1\\&+\left(-8g_2^2\,{}^{6}C_{H^6}+\lambda\left(\frac{64}{3}g_2^2\,{}^{6}C_{H^4\Box}-4g_2^2\,{}^{6}C_{H^4D^2}-\frac{16}{3}g_1g_2\,{}^{6}C_{WBH^2}\right)\right)\kappa_2\\&+\left(6\,{}^{6}C_{H^6}-16\lambda\,{}^{6}C_{H^4\Box}+2\lambda\,{}^{6}C_{H^4D^2}\right)(-\kappa_7+4\kappa_{10}+2\kappa_{11})\\&-\frac{4}{3}\lambda g_1^2\kappa_1^{(8)}-\frac{4}{3}\lambda g_2^2\kappa_2^{(8)}-\frac{4}{3}\lambda g_2^2\kappa_3-\frac{4}{3}\lambda g_2^2\kappa_4-\frac{4}{3}\lambda(2g_1^2-g_2^2)\kappa_5+\frac{1}{3}\lambda(g_1^2-g_2^2)\kappa_6\\&+4\lambda\kappa_7^{(8)}-8\lambda\kappa_8+4\lambda\kappa_9^{(8)}+4\lambda\kappa_{10}^{(8)}+4\lambda\kappa_{12}-4\lambda\kappa_{13}-4\lambda\kappa_{14}-4\lambda\kappa_{15}-4\lambda\kappa_{16}\\&-4\lambda\kappa_{17}-4\kappa_{21}^{(8)}+2\kappa_{22}-4\lambda g_1g_2\tau_2'-\frac{8}{3}\lambda g_2^2\tau_3+4\lambda g_2\tau_{11}+8\lambda g_1\tau_{12}+2\lambda g_2\tau_{14}\,.\end{aligned}\tag{B.103}$$

B.7.2 H^6D^2

The RGEs for the H^6D^2 couplings are

$$\begin{aligned}{}^{8}\dot{C}^{(1)}_{H^6D^2} =&\left(2g_1^2\,{}^{6}C_{H^4D^2}+\frac{16}{3}g_1g_2\,{}^{6}C_{WBH^2}\right)\kappa_1\\&+\left(-\frac{32}{3}g_2^2\,{}^{6}C_{H^4\Box}+\frac{2}{3}g_2^2\,{}^{6}C_{H^4D^2}+8g_1g_2\,{}^{6}C_{WBH^2}\right)\kappa_2\\&+\left(8\,{}^{6}C_{H^4\Box}+\,{}^{6}C_{H^4D^2}\right)(-\kappa_7+4\kappa_{10}+2\kappa_{11})\\&+2g_1^2\kappa_1^{(8)}+\frac{10}{3}g_2^2\kappa_2^{(8)}+2g_2^2\kappa_3+\frac{8}{3}g_2^2\kappa_4+\left(4g_1^2-\frac{10}{3}g_2^2\right)\kappa_5+\frac{1}{2}(-g_1^2+2g_2^2)\kappa_6\\&+2\kappa_8-6\kappa_9^{(8)}-10\kappa_{10}^{(8)}-2\kappa_{11}^{(8)}-6\kappa_{12}+6\kappa_{13}+6\kappa_{14}+10\kappa_{15}+6\kappa_{16}+10\kappa_{17}\\&-2\kappa_{18}-\kappa_{19}+4\kappa_{20}+\frac{20}{3}g_1g_2\tau_2'+\frac{20}{3}g_2^2\tau_3-8g_2\tau_{11}-12g_1\tau_{12}-6g_2\tau_{14}\,,\end{aligned}\tag{B.104}$$

$$\begin{aligned}{}^{8}\dot{C}^{(2)}_{H^6D^2} =&\left(\frac{4}{3}g_1^2\,{}^{6}C_{H^4D^2}+\frac{4}{3}g_1g_2\,{}^{6}C_{WBH^2}\right)\kappa_1+\left(-\frac{4}{3}g_2^2\,{}^{6}C_{H^4D^2}+\frac{16}{3}g_1g_2\,{}^{6}C_{WBH^2}\right)\kappa_2\\&+\left(2\,{}^{6}C_{H^4D^2}\right)(-\kappa_7+4\kappa_{10}+2\kappa_{11})+\frac{4}{3}g_1^2\kappa_1^{(8)}+\frac{4}{3}g_2^2\kappa_3+\frac{2}{3}g_2^2\kappa_4+\frac{8}{3}g_1^2\kappa_5\\&-\frac{1}{6}(2g_1^2+g_2^2)\kappa_6-4\kappa_9^{(8)}+2\kappa_{11}^{(8)}-4\kappa_{12}+4\kappa_{13}+4\kappa_{14}+4\kappa_{16}\\&+2\kappa_{18}+\kappa_{19}-4\kappa_{20}+\frac{10}{3}g_1g_2\tau_2'-2g_2\tau_{11}-8g_1\tau_{12}+g_2\tau_{14}\,.\end{aligned}\tag{B.105}$$

B.7.3 H^4D^4

The RGEs for the H^4D^4 couplings are

$$ {}^{8}\dot{C}^{(1)}_{H^4D^4} = \frac{8}{3}\kappa_4 - \frac{8}{3}\kappa_5 - \frac{4}{3}\kappa_6\,, \tag{B.106} $$

$$ {}^{8}\dot{C}^{(2)}_{H^4D^4} = -\frac{8}{3}\kappa_4 - \frac{8}{3}\kappa_5\,, \tag{B.107} $$

$$ {}^{8}\dot{C}^{(3)}_{H^4D^4} = \frac{16}{3}\kappa_5 + \frac{4}{3}\kappa_6\,. \tag{B.108} $$

B.7.4 X^2H^4

The RGEs for the X^2H^4 coefficients are

$$ {}^{8}\dot{C}^{(1)}_{G^2H^4} = \left(-\frac{8}{3}g_2^2\,{}^{6}C^{(1)}_{G^2H^2}\right)\kappa_2 + \left(2\,{}^{6}C^{(1)}_{G^2H^2}\right)(-\kappa_7 + 4\kappa_{10} + 2\kappa_{11}) - 2g_3\tau_9^{(8)} + 2\tau_{19}\,, \tag{B.109} $$

$$ {}^{8}\dot{C}^{(2)}_{G^2H^4} = \left(-\frac{8}{3}g_2^2\,{}^{6}C^{(2)}_{G^2H^2}\right)\kappa_2 + \left(2\,{}^{6}C^{(2)}_{G^2H^2}\right)(-\kappa_7 + 4\kappa_{10} + 2\kappa_{11}) + 2g_3\tilde{\tau}_9^{(8)} - 2\tilde{\tau}_{19}\,, \tag{B.110} $$

$$ \begin{aligned} {}^{8}\dot{C}^{(1)}_{W^2H^4} = &\left(-\frac{2}{3}g_1g_2\,{}^{6}C^{(1)}_{WBH^2}\right)\kappa_1 + \left(2\,{}^{6}C^{(1)}_{W^2H^2}\right)(-\kappa_7 + 4\kappa_{10} + 2\kappa_{11}) - \frac{1}{3}g_2^2\kappa_4 + g_2^2\kappa_5 \\ &- \frac{1}{6}g_2^2\kappa_6 - \frac{1}{3}(g_1g_2\tau_2' + 2g_2^2\tau_3) - g_2\tau_7^{(8)} + g_2\tau_{11} - 2g_2\tau_{13} + \frac{1}{2}g_2\tau_{14} + 8\tau_{18}\,, \end{aligned} \tag{B.111} $$

$$ \begin{aligned} {}^{8}\dot{C}^{(2)}_{W^2H^4} = &\left(-\frac{2}{3}g_1g_2\,{}^{6}C^{(2)}_{WBH^2}\right)\kappa_1 + \left(2\,{}^{6}C^{(2)}_{W^2H^2}\right)(-\kappa_7 + 4\kappa_{10} + 2\kappa_{11}) \\ &- \frac{1}{3}g_1g_2\tilde{\tau}_2' + g_2\tilde{\tau}_7^{(8)} - g_2\tilde{\tau}_{11} + 2g_2\tilde{\tau}_{13} - \frac{1}{2}g_2\tilde{\tau}_{14} - 8\tilde{\tau}_{18}\,, \end{aligned} \tag{B.112} $$

$$ {}^{8}\dot{C}^{(3)}_{W^2H^4} = \left(\frac{2}{3}g_1g_2\,{}^{6}C^{(1)}_{WBH^2}\right)\kappa_1 + \frac{1}{3}g_1g_2\tau_2' - g_2\tau_8 - g_2\tau_{11} + \frac{1}{2}g_2\tau_{14} + 4\tau_{17} - 8\tau_{18}\,, \tag{B.113} $$

$$ {}^{8}\dot{C}^{(4)}_{W^2H^4} = \left(\frac{2}{3}g_1g_2\,{}^{6}C^{(2)}_{WBH^2}\right)\kappa_1 + \frac{1}{3}g_1g_2\tilde{\tau}_2' + g_2\tilde{\tau}_8 + g_2\tilde{\tau}_{11} - \frac{1}{2}g_2\tilde{\tau}_{14} - 4\tilde{\tau}_{17} + 8\tilde{\tau}_{18}\,, \tag{B.114} $$

$$ {}^{8}\dot{C}^{(1)}_{B^2H^4} = \left(\frac{8}{3}g_1^2\,{}^{6}C^{(1)}_{B^2H^2}\right)\kappa_1 + \left(-\frac{8}{3}g_2^2\,{}^{6}C^{(1)}_{B^2H^2} - \frac{4}{3}g_1g_2\,{}^{6}C^{(1)}_{WBH^2}\right)\kappa_2 $$

$$+\left(2\,{}^{6}C^{(1)}_{B^2H^2}\right)(-\kappa_7+4\kappa_{10}+2\kappa_{11})+\frac{1}{3}g_1^2\kappa_4-g_1^2\kappa_5+\frac{1}{6}g_1^2\kappa_6$$
$$+\frac{2}{3}(g_1^2\tau_1-g_1g_2\tau_2')-2g_1\tau_4^{(8)}-2g_1\tau_{10}+2g_1\tau_{12}+4\tau_{15}\,, \tag{B.115}$$

$${}^{8}\dot{C}^{(2)}_{B^2H^4}=\left(\frac{8}{3}g_1^2\,{}^{6}C^{(2)}_{B^2H^2}\right)\kappa_1+\left(-\frac{8}{3}g_2^2\,{}^{6}C^{(2)}_{B^2H^2}-\frac{4}{3}g_1g_2\,{}^{6}C^{(2)}_{WBH^2}\right)\kappa_2$$
$$+\left(2\,{}^{6}C^{(2)}_{B^2H^2}\right)(-\kappa_7+4\kappa_{10}+2\kappa_{11})+\frac{2}{3}g_1g_2\tilde{\tau}_2'+2g_1\tilde{\tau}_4^{(8)}+2g_1\tilde{\tau}_{10}-2g_1\tilde{\tau}_{12}-4\tilde{\tau}_{15}\,, \tag{B.116}$$

$${}^{8}\dot{C}^{(1)}_{WBH^4}=\left(\frac{8}{3}g_1g_2\,{}^{6}C^{(1)}_{B^2H^2}\right)\kappa_1+\left(\frac{8}{3}g_1g_2\,{}^{6}C^{(1)}_{W^2H^2}-4g_2^2\,{}^{6}C^{(1)}_{WBH^2}\right)\kappa_2$$
$$+\left(2\,{}^{6}C^{(1)}_{WBH^2}\right)(-\kappa_7+4\kappa_{10}+2\kappa_{11})$$
$$+\frac{2}{3}(g_1g_2\tau_1-g_2^2\tau_2'-g_1g_2\tau_3)-2g_1\tau_5^{(8)}-g_2\tau_6^{(8)}-2g_2\tau_{10}+2g_2\tau_{12}$$
$$-2g_1\tau_{13}+8\tau_{16}+g_1\tau_{14}\,, \tag{B.117}$$

$${}^{8}\dot{C}^{(2)}_{WBH^4}=\left(\frac{8}{3}g_1g_2\,{}^{6}C^{(2)}_{B^2H^2}\right)\kappa_1+\left(\frac{8}{3}g_1g_2\,{}^{6}C^{(2)}_{W^2H^2}-4g_2^2\,{}^{6}C^{(2)}_{WBH^2}\right)\kappa_2$$
$$+\left(2\,{}^{6}C^{(2)}_{WBH^2}\right)(-\kappa_7+4\kappa_{10}+2\kappa_{11})$$
$$+\frac{2}{3}g_2^2\tilde{\tau}_2'+2g_1\tilde{\tau}_5^{(8)}+g_2\tilde{\tau}_6^{(8)}+2g_2\tilde{\tau}_{10}-2g_2\tilde{\tau}_{12}+2g_1\tilde{\tau}_{13}-8\tilde{\tau}_{16}-g_1\tilde{\tau}_{14}\,. \tag{B.118}$$

B.7.5 XH^4D^2

The RGEs for the XH^4D^2 couplings are

$${}^{8}\dot{C}^{(1)}_{BH^4D^2}=\left(-\frac{32}{3}g_1\,{}^{6}C^{(1)}_{B^2H^2}\right)\kappa_1+\left(\frac{48}{3}g_2\,{}^{6}C^{(1)}_{WBH^2}\right)\kappa_2-\frac{8}{3}g_1\kappa_4+8g_1\kappa_5-\frac{4}{3}g_1\kappa_6$$
$$-\frac{8}{3}(g_1\tau_1-3g_2\tau_2')+8\tau_{10}-24\tau_{12}\,, \tag{B.119}$$

$${}^{8}\dot{C}^{(2)}_{BH^4D^2}=\left(-\frac{32}{3}g_1\,{}^{6}C^{(2)}_{B^2H^2}\right)\kappa_1+\left(\frac{48}{3}g_2\,{}^{6}C^{(2)}_{WBH^2}\right)\kappa_2-8g_2\tilde{\tau}_2'-8\tilde{\tau}_{10}+24\tilde{\tau}_{12}\,, \tag{B.120}$$

$${}^{8}\dot{C}^{(1)}_{WH^4D^2}=\left(\frac{16}{3}g_1\,{}^{6}C^{(1)}_{B^2H^2}\right)\kappa_1+\left(-\frac{32}{3}g_2\,{}^{6}C^{(1)}_{W^2H^2}\right)\kappa_2+\frac{8}{3}g_2\kappa_4-8g_2\kappa_5+\frac{4}{3}g_2\kappa_6$$
$$+\frac{8}{3}(g_1\tau_2'+3g_2\tau_3)-8\tau_{11}+8\tau_{13}-8\tau_{14}\,, \tag{B.121}$$

$${}^{8}\dot{C}^{(2)}_{WH^4D^2}=\left(\frac{16}{3}g_1\,{}^{6}C^{(2)}_{B^2H^2}\right)\kappa_1+\left(-\frac{32}{3}g_2\,{}^{6}C^{(2)}_{W^2H^2}\right)\kappa_2+\frac{8}{3}g_1\tilde{\tau}_2'+8\tilde{\tau}_{11}-8\tilde{\tau}_{13}+8\tilde{\tau}_{14}\,, \tag{B.122}$$

$${}^{8}\dot{C}^{(3)}_{WH^4D^2}=\frac{8}{3}g_1\tilde{\tau}_2+8\tilde{\tau}_{13}'+4\tilde{\tau}_{14}'\,, \tag{B.123}$$

$${}^{8}\dot{C}^{(4)}_{WH^4D^2}=-\frac{8}{3}g_1\tau_2+8\tau_{13}'+4\tau_{14}'\,. \tag{B.124}$$

B.7.6 $X^2H^2D^2$

The RGEs for the $X^2H^2D^2$ couplings are

$$ {}^{8}\dot{C}^{(1)}_{G^2H^2D^2} = \frac{16}{3}\tau_0 \,, \tag{B.125} $$

$$ {}^{8}\dot{C}^{(2)}_{G^2H^2D^2} = -\frac{4}{3}\tau_0 \,, \tag{B.126} $$

$$ {}^{8}\dot{C}^{(1)}_{W^2H^2D^2} = \frac{16}{3}\tau_3 \,, \tag{B.127} $$

$$ {}^{8}\dot{C}^{(2)}_{W^2H^2D^2} = -\frac{4}{3}\tau_3 \,, \tag{B.128} $$

$$ {}^{8}\dot{C}^{(4)}_{W^2H^2D^2} = 0 \,, \tag{B.129} $$

$$ {}^{8}\dot{C}^{(5)}_{W^2H^2D^2} = \frac{8}{3}\tau_3' \,, \tag{B.130} $$

$$ {}^{8}\dot{C}^{(1)}_{WBH^2D^2} = -\frac{8}{3}\tau_2' \,, \tag{B.131} $$

$$ {}^{8}\dot{C}^{(2)}_{WBH^2D^2} = -\frac{8}{3}\tilde{\tau}_2' \,, \tag{B.132} $$

$$ {}^{8}\dot{C}^{(3)}_{WBH^2D^2} = 0 \,, \tag{B.133} $$

$$ {}^{8}\dot{C}^{(4)}_{WBH^2D^2} = \frac{16}{3}\tau_2' \,, \tag{B.134} $$

$$ {}^{8}\dot{C}^{(6)}_{WBH^2D^2} = \frac{16}{3}\tilde{\tau}_2' \,, \tag{B.135} $$

$$ {}^{8}\dot{C}^{(1)}_{B^2H^2D^2} = \frac{16}{3}\tau_1 \,, \tag{B.136} $$

$$ {}^{8}\dot{C}^{(2)}_{B^2H^2D^2} = -\frac{4}{3}\tau_1 \,. \tag{B.137} $$

C Self-duality relations for the dipole

We use the convention $\epsilon_{0123} = +1$. The duality relation $\epsilon_{\mu\nu\rho\sigma}\sigma^{\rho\sigma} = 2i\sigma_{\mu\nu}\gamma_5$ gives

$$\frac{i}{2}\epsilon_{\mu\nu\rho\sigma}\sigma^{\rho\sigma}P_R = -\sigma_{\mu\nu}P_R\,, \qquad \frac{i}{2}\epsilon_{\mu\nu\rho\sigma}\sigma^{\rho\sigma}P_L = \sigma_{\mu\nu}P_L\,. \tag{C.1}$$

The dual of a tensor $T^{\mu\nu}$ is defined by

$$\widetilde{T}_{\mu\nu} \equiv \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}T^{\rho\sigma}, \qquad \widetilde{\widetilde{T}}_{\mu\nu} = -T_{\mu\nu}\,. \tag{C.2}$$

A tensor will be called selfdual if $\tilde{T}^{\mu\nu} = iT^{\mu\nu}$, and anti-selfdual if $\tilde{T}^{\mu\nu} = -iT^{\mu\nu}$. For example, the selfdual part of the electromagnetic field-strength tensor is $F_{\mu\nu} - i\widetilde{F}_{\mu\nu}$, and the anti-selfdual part is $F_{\mu\nu}+i\widetilde{F}_{\mu\nu}$. If $T^{\mu\nu}$ is selfdual, $T^{\dagger\mu\nu}$ is anti-selfdual. Eq. (C.1) relates the chirality of fermion dipole bilinears to their duality property,

$$\overline{\psi}\,T^{\mu\nu}\sigma_{\mu\nu}P_R\psi = -i\overline{\psi}\,\widetilde{T}^{\mu\nu}\sigma_{\mu\nu}P_R\psi, \tag{C.3}$$

so $\overline{\psi}\,T^{\mu\nu}\sigma_{\mu\nu}P_R\psi$ vanishes if $T^{\mu\nu}$ is anti-selfdual, i.e., $T^{\mu\nu}$ can be chosen to be selfdual. One useful implication of this self-duality is that the chiral projectors can be dropped for tensor bilinears if $T^{\mu\nu}$ is selfdual,

$$\bar{\psi}\sigma_{\mu\nu}T^{\mu\nu}\psi = \bar{\psi}\sigma_{\mu\nu}T^{\mu\nu}P_R\psi\,, \qquad \bar{\psi}\sigma_{\mu\nu}T^{\mu\nu\dagger}\psi = \bar{\psi}\sigma_{\mu\nu}T^{\mu\nu\dagger}P_L\psi\,. \tag{C.4}$$

There are a few relations that can be derived from this self-duality condition,

$$T^{\mu\nu}T^\dagger_{\mu\nu} = 0\,, \tag{C.5}$$
$$T_\mu{}^\alpha T^\dagger_{\nu\alpha} = T_\nu{}^\alpha T^\dagger_{\mu\alpha}\,, \tag{C.6}$$
$$T_\mu{}^\alpha T_{\nu\alpha} = \frac{1}{2}g_{\mu\nu}T_{\alpha\beta}T^{\alpha\beta} - T_\nu{}^\alpha T_{\mu\alpha}\,, \tag{C.7}$$
$$4\,T^{\mu\nu}T^\dagger_{\nu\alpha}T^{\alpha\beta}T^\dagger_{\beta\mu} = T_{\mu\nu}T^\dagger_{\alpha\beta}T^{\mu\nu}T^{\dagger\alpha\beta}\,. \tag{C.8}$$

These relations have been used to derive eq. (4.4).

D Majorana masses and dipoles

If Majorana mass or dipole terms are present in the Lagrangian, they can be included in the formalism by promoting the field in eq. (4.2) to

$$\chi = \begin{pmatrix} \chi_L \\ (\chi_R)^c \\ (\chi_L)^c \\ \chi_R \end{pmatrix}, \tag{D.1}$$

where $(\chi_R)^c = C(\overline{\chi_R})^\intercal$ is left-handed, with $C = i\gamma^2\gamma^0$ the charge conjugation matrix. In the SMEFT, the fields in eq. (D.1) are $\chi_L = \{\ell, q\}$, $(\chi_R)^c = \{e^c, u^c, d^c\}$, $(\chi_L)^c = \{\ell^c, q^c\}$,

$\chi_R = \{e, u, d\}$. In this new basis where the degrees of freedom are essentially doubled, the metric in the Lagrangian eq. (4.1) takes the 4×4 block form

$$k = \frac{1}{2}\begin{pmatrix} \kappa & 0 \\ 0 & \kappa^\intercal \end{pmatrix}, \qquad \kappa = \begin{pmatrix} \kappa_L & 0 \\ 0 & \kappa_R^* \end{pmatrix}. \tag{D.2}$$

The field strength has the block form

$$\mathcal{Y}_{\mu\nu} = \begin{pmatrix} Y_L^{\mu\nu} & 0 & 0 & 0 \\ 0 & Y_R^{\mu\nu *} & 0 & 0 \\ 0 & 0 & Y_L^{\mu\nu *} & 0 \\ 0 & 0 & 0 & Y_R^{\mu\nu} \end{pmatrix}, \tag{D.3}$$

with $Y_{L(R)}^{\mu\nu} = [\mathcal{D}_{L(R)}^\mu, \mathcal{D}_{L(R)}^\nu]$. The field strength tensors have the same anti-hermitian property as the gauge field, i.e., $\kappa_L^{-1} Y_L^{\mu\nu\dagger} = -Y_L^{\mu\nu}\kappa_L^{-1}$ and similarly for $Y_R^{\mu\nu}$. The scalar $\mathcal{M}$ and tensor $\mathcal{T}^{\mu\nu}$ objects retain the same global structure as in eq. (4.3) where M and $T^{\mu\nu}$ are now 2×2 submatrices

$$M = \frac{1}{2}\begin{pmatrix} \mu_L & m_D \\ m_D^\intercal & \mu_R^* \end{pmatrix}, \qquad T^{\mu\nu} = \frac{1}{2}\begin{pmatrix} t_L^{\mu\nu} & t_D^{\mu\nu} \\ -t_D^{\mu\nu\intercal} & -t_R^{\mu\nu *} \end{pmatrix}, \tag{D.4}$$

where we have defined $\mu_{L(R)}$ as the symmetric Majorana mass matrices and $t_{L(R)}^{\mu\nu}$ as the antisymmetric Majorana dipole matrices for the left-(right-)handed fields. The Dirac mass matrix is denoted as m_D and the Dirac dipole matrix as $t_D^{\mu\nu}$. The one-loop divergence formula in eq. (4.4) can be used with the above modifications, with the prefactor $1/(32\pi^2\epsilon)$ replaced by $1/(64\pi^2\epsilon)$, since each term in the trace is now counted twice, because of the doubled set of fields.

References

[1] J. S. R. Chisholm, *Change of variables in quantum field theories*, Nucl. Phys. **26** (1961) 469.

[2] S. Kamefuchi, L. O'Raifeartaigh and A. Salam, *Change of variables and equivalence theorems in quantum field theories*, Nucl. Phys. **28** (1961) 529.

[3] H. D. Politzer, *Power Corrections at Short Distances*, Nucl. Phys. B **172** (1980) 349.

[4] C. Arzt, *Reduced effective Lagrangians*, Phys. Lett. B **342** (1995) 189 [hep-ph/9304230].

[5] K. Meetz, *Realization of chiral symmetry in a curved isospin space*, J. Math. Phys. **10** (1969) 589.

[6] J. Honerkamp and K. Meetz, *Chiral-invariant perturbation theory*, Phys. Rev. D **3** (1971) 1996.

[7] J. Honerkamp, *Chiral multiloops*, Nucl. Phys. B **36** (1972) 130.

[8] D. V. Volkov, *Phenomenological lagrangians*, Sov. J. Particles Nucl. **4** (1973) 1.

[9] C. Cheung, A. Helset and J. Parra-Martinez, *Geometric soft theorems*, JHEP **04** (2022) 011 [2111.03045].

[10] R. Alonso, E. E. Jenkins and A. V. Manohar, *A Geometric Formulation of Higgs Effective Field Theory: Measuring the Curvature of Scalar Field Space*, *Phys. Lett. B* **754** (2016) 335 [1511.00724].

[11] R. Alonso, E. E. Jenkins and A. V. Manohar, *Geometry of the Scalar Sector*, *JHEP* **08** (2016) 101 [1605.03602].

[12] R. Alonso, K. Kanshin and S. Saa, *Renormalization group evolution of Higgs effective field theory*, *Phys. Rev. D* **97** (2018) 035010 [1710.06848].

[13] A. Helset, M. Paraskevas and M. Trott, *Gauge fixing the Standard Model Effective Field Theory*, *Phys. Rev. Lett.* **120** (2018) 251801 [1803.08001].

[14] A. Helset, A. Martin and M. Trott, *The Geometric Standard Model Effective Field Theory*, *JHEP* **03** (2020) 163 [2001.01453].

[15] C. Hays, A. Helset, A. Martin and M. Trott, *Exact SMEFT formulation and expansion to* $\mathcal{O}(v^4/\Lambda^4)$, *JHEP* **11** (2020) 087 [2007.00565].

[16] T. Cohen, N. Craig, X. Lu and D. Sutherland, *Is SMEFT Enough?*, *JHEP* **03** (2021) 237 [2008.08597].

[17] T. Corbett, A. Helset, A. Martin and M. Trott, *EWPD in the SMEFT to dimension eight*, *JHEP* **06** (2021) 076 [2102.02819].

[18] T. Corbett, A. Martin and M. Trott, *Consistent higher order* $\sigma\,(\mathcal{G}\mathcal{G} \to h)$, $\Gamma\,(h \to \mathcal{G}\mathcal{G})$ *and* $\Gamma\,(h \to \gamma\gamma)$ *in geoSMEFT*, *JHEP* **12** (2021) 147 [2107.07470].

[19] T. Cohen, N. Craig, X. Lu and D. Sutherland, *Unitarity violation and the geometry of Higgs EFTs*, *JHEP* **12** (2021) 003 [2108.03240].

[20] J. Talbert, *The geometric* ν*SMEFT: operators and connections*, *JHEP* **01** (2023) 069 [2208.11139].

[21] A. Martin and M. Trott, *More accurate* $\sigma(\mathcal{G}\,\mathcal{G} \to h)$, $\Gamma(h \to \mathcal{G}\,\mathcal{G}, \mathcal{A}\mathcal{A}, \bar{\Psi}\Psi)$ *and Higgs width results via the geoSMEFT*, 2305.05879.

[22] V. Gattus and A. Pilaftsis, *Minimal Supergeometric Quantum Field Theories*, 2307.01126.

[23] K. Finn, S. Karamitsos and A. Pilaftsis, *Frame covariant formalism for fermionic theories*, *Eur. Phys. J. C* **81** (2021) 572 [2006.05831].

[24] C. Cheung, A. Helset and J. Parra-Martinez, *Geometry-kinematics duality*, *Phys. Rev. D* **106** (2022) 045016 [2202.06972].

[25] T. Cohen, N. Craig, X. Lu and D. Sutherland, *On-Shell Covariance of Quantum Field Theory Amplitudes*, *Phys. Rev. Lett.* **130** (2023) 041603 [2202.06965].

[26] A. Helset, E. E. Jenkins and A. V. Manohar, *Geometry in scattering amplitudes*, *Phys. Rev. D* **106** (2022) 116018 [2210.08000].

[27] N. Craig, Y.-T. Lee, X. Lu and D. Sutherland, *Effective Field Theories as Lagrange Spaces*, 2305.09722.

[28] A. Helset, E. E. Jenkins and A. V. Manohar, *Renormalization of the Standard Model Effective Field Theory from geometry*, *JHEP* **02** (2023) 063 [2212.03253].

[29] E. E. Jenkins, A. V. Manohar and M. Trott, *Renormalization Group Evolution of the Standard Model Dimension Six Operators I: Formalism and* λ *Dependence*, *JHEP* **10** (2013) 087 [1308.2627].

[30] E. E. Jenkins, A. V. Manohar and M. Trott, *Renormalization Group Evolution of the Standard Model Dimension Six Operators II: Yukawa Dependence*, *JHEP* **01** (2014) 035 [1310.4838].

[31] R. Alonso, E. E. Jenkins, A. V. Manohar and M. Trott, *Renormalization Group Evolution of the Standard Model Dimension Six Operators III: Gauge Coupling Dependence and Phenomenology*, *JHEP* **04** (2014) 159 [1312.2014].

[32] M. Chala, G. Guedes, M. Ramos and J. Santiago, *Towards the renormalisation of the Standard Model effective field theory to dimension eight: Bosonic interactions I*, *SciPost Phys.* **11** (2021) 065 [2106.05291v3].

[33] S. Das Bakshi, M. Chala, A. Díaz-Carmona and G. Guedes, *Towards the renormalisation of the Standard Model effective field theory to dimension eight: bosonic interactions II*, *Eur. Phys. J. Plus* **137** (2022) 973 [2205.03301].

[34] M. Accettulli Huber and S. De Angelis, *Standard Model EFTs via on-shell methods*, *JHEP* **11** (2021) 221 [2108.03669].

[35] S. Das Bakshi and A. Díaz-Carmona, *Renormalisation of SMEFT bosonic interactions up to dimension eight by LNV operators*, *JHEP* **06** (2023) 123 [2301.07151].

[36] E. E. Jenkins, A. V. Manohar and P. Stoffer, *Low-Energy Effective Field Theory below the Electroweak Scale: Operators and Matching*, *JHEP* **03** (2018) 016 [1709.04486].

[37] E. E. Jenkins, A. V. Manohar and P. Stoffer, *Low-Energy Effective Field Theory below the Electroweak Scale: Anomalous Dimensions*, *JHEP* **01** (2018) 084 [1711.05270].

[38] L. Alvarez-Gaume and D. Z. Freedman, *Geometrical Structure and Ultraviolet Finiteness in the Supersymmetric Sigma Model*, *Commun. Math. Phys.* **80** (1981) 443.

[39] R. Nagai, M. Tanabashi, K. Tsumura and Y. Uchida, *Scalar and fermion on-shell amplitudes in generalized Higgs effective field theory*, *Phys. Rev. D* **104** (2021) 015001 [2102.08519].

[40] B. S. DeWitt, *Supermanifolds*, Cambridge Monographs on Mathematical Physics. Cambridge Univ. Press, Cambridge, UK, 5, 2012, 10.1017/CBO9780511564000.

[41] A. Rogers, *Supermanifolds: theory and applications*. World Scientific, 2007.

[42] R. L. Arnowitt and P. Nath, *Riemannian Geometry in Spaces with Grassmann Coordinates*, *Gen. Rel. Grav.* **7** (1976) 89.

[43] G. 't Hooft, *An algorithm for the poles at dimension four in the dimensional regularization procedure*, *Nucl. Phys. B* **62** (1973) 444.

[44] H. Neufeld, J. Gasser and G. Ecker, *The one loop functional as a Berezinian*, *Phys. Lett. B* **438** (1998) 106 [hep-ph/9806436].

[45] B. Henning, X. Lu and H. Murayama, *One-loop Matching and Running with Covariant Derivative Expansion*, *JHEP* **01** (2018) 123 [1604.01019].

[46] G. Buchalla, O. Cata, A. Celis, M. Knecht and C. Krause, *Complete One-Loop Renormalization of the Higgs-Electroweak Chiral Lagrangian*, *Nucl. Phys. B* **928** (2018) 93 [1710.06412].

[47] G. Buchalla, A. Celis, C. Krause and J.-N. Toelstede, *Master Formula for One-Loop Renormalization of Bosonic SMEFT Operators*, 1904.07840.

[48] B. Grzadkowski, M. Iskrzynski, M. Misiak and J. Rosiek, *Dimension-Six Terms in the Standard Model Lagrangian*, *JHEP* **10** (2010) 085 [1008.4884].

[49] C. W. Murphy, *Dimension-8 operators in the Standard Model Effective Field Theory*, *JHEP* **10** (2020) 174 [2005.00059].

[50] H.-L. Li, Z. Ren, J. Shu, M.-L. Xiao, J.-H. Yu and Y.-H. Zheng, *Complete set of dimension-eight operators in the standard model effective field theory*, *Phys. Rev. D* **104** (2021) 015026 [2005.00008].

[51] R. Alonso, E. E. Jenkins and A. V. Manohar, *Holomorphy without Supersymmetry in the Standard Model Effective Field Theory*, *Phys. Lett. B* **739** (2014) 95 [1409.0868].

[52] C. Cheung and C.-H. Shen, *Nonrenormalization Theorems without Supersymmetry*, *Phys. Rev. Lett.* **115** (2015) 071601 [1505.01844].

[53] E. Fermi, *Trends to a Theory of beta Radiation. (In Italian)*, *Nuovo Cim.* **11** (1934) 1.

[54] W. Dekens and P. Stoffer, *Low-energy effective field theory below the electroweak scale: matching at one loop*, *JHEP* **10** (2019) 197 [1908.05295].