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Authors

Soniat, Marielle
Tesfaye, Meron
Brooks, Daniel
et al.

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1 Predictive simulation of non-steady-state transport of gases 2 through rubbery polymer membranes

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4 Marielle Soniat,^{a,b} Meron Tesfaye^{c,d}, Daniel Brooks,^e Boris Merinov,^e William A. Goddard, III,^e
5 Adam Z. Weber^{a,c} and Frances A. Houle^{*a,b}

6^a Joint Center for Artificial Photosynthesis, Lawrence Berkeley National Laboratory, Berkeley,
7 CA 94720

8^b Chemical Sciences Division, Lawrence Berkeley National Laboratory, Berkeley, CA 94720

9^c Energy Storage and Distributed Resources Division, Lawrence Berkeley National Laboratory,
10 Berkeley, CA 94720

11^d Department of Chemical and Biomolecular Engineering, University of California, Berkeley, CA
12 94720

13^e Materials and Process Simulation Center (MSC), Beckman Institute, California Institute of
14 Technology, Pasadena, CA, 91125

15

16* Author to whom correspondence should be addressed. fahoule@lbl.gov, (510) 495-8135.

17

18

19Abstract

20A multiscale, physically-based, reaction-diffusion kinetics model is developed for non-steady-
21state transport of simple gases through a rubbery polymer. Experimental data from the literature,
22new measurements of non-steady-state permeation and a molecular dynamics simulation of a
23gas-polymer sticking probability for a typical system are used to construct and validate the model
24framework. Using no adjustable parameters, the model successfully reproduces time-dependent
25experimental data for two distinct systems: (1) O₂ quenching of a phosphorescent dye embedded
26in poly(n-butyl(amino) thionylphosphazene), and (2) O₂, N₂, CH₄ and CO₂ transport through
27poly(dimethyl siloxane). The calculations show that in the pre-steady-state regime, permeation is
28only correctly described if the sorbed gas concentration in the polymer is dynamically
29determined by the rise in pressure. The framework is used to predict selectivity targets for two
30applications involving rubbery membranes: CO₂ capture from air and blocking of methane cross-
31over in an aged solar fuels device.

32

33

34**Keywords:** rubbery polymers; reaction-diffusion modeling; gas transport

35 Nomenclature: List of symbols

Symbol	Meaning	Units
A	area	m^2
$B + 1$	ratio of intensities	--
dx	distance in x	m
D	diffusivity	m^2/s
h	Planck constant	J s
I_0	intensity of light emission in absence of O_2	-- *
I_{eq}	intensity of light emission at a constant $[O_2]$	-- *
$I(t)$	time-dependent light intensity	-- *
J	flux through membrane	$mol/(m^2 s)$
k_B	Boltzmann constant	J/K
k_{ads}	pseudo-first order rate constant for adsorption	s^{-1}
k_{bub}	zeroth order rate constant for CH_4 to enter gas phase	$mol/(L s)$
k_{coll}	rate constant for collision of gases with the surface	s^{-1}
k_{des}	rate constant for desorption from a surface	s^{-1}
k_{ex}	pseudo-first order rate constant for excitation of PtOEP	s^{-1}
k_{hyd}	zeroth order rate constant for CH_4 to enter gas phase	$mol/(L s)$
k_{nr}	rate constant for radiative decay of PtOEP	s^{-1}
k_{prod}	zeroth order rate constant for production of CH_4	$mol/(L s)$
k_q	rate constant for quenching of PtOEP by O_2	$L/(mol s)$
k_r	rate constant for emission of photons from PtOEP	s^{-1}
k_{TTA}	rate constant for TTA	$L/(mol s)$
l	thickness of the membrane	m
m_w	molecular mass	kg/mol
n	amount of gas	mol
n_{max}	maximum amount of gas	mol
N_{Av}	Avogadro's number	molecules/mol
P_m	permeability of the membrane	$(mol m)/(m^2 s Pa)$ for gas phase m^2/s for liquid phase
p_{ds}	downstream pressure	Pa
p_{O_2}	external partial pressure of O_2	Pa
p_{up}	upstream pressure	Pa
Δp	difference between upstream and downstream pressures	Pa
r_{sun}	ratio of current sunlight level to maximum sunlight level	---
R	universal gas constant	$J/(mol K)$
R_{AB}	encounter distance	m
S	solubility	$mol/(L Pa)$ for gas phase $(mol/L)/(mol/L)$ for liquid phase
t	time	s

T	temperature	K
V	volume	m^3
V_{ds}	downstream volume	m^3
x	position	m
Z	collision frequency	s^{-1}
μ	sticking probability	--
ν_L	frequency of exciting light	m
π	pi	--
ρ	mass density	kg/m^3
σ_a	absorption cross section	m^2
τ	lifetime of 3PtOEP in the presence of a constant $[O_2]$	s
τ_0	lifetime of 3PtOEP in the absence of O_2	s
ϕ_P	phosphorescence quantum yield	--
χ	rate	mol/(L s)
ω_L	energy density	J/m^2
$[gas_{(p)}]$	concentration of a gas within the polymer	mol/L
$[O_{2(p)}]_{eq}$	concentration of O_2 within the polymer at equilibrium	mol/L
$[O_{2(p)}](x,t)$	concentration of O_2 within the polymer as a function of position and time	mol/L
$[PtOEP]$	total concentration of PtOEP	mol/L
$[^3PtOEP]_{init}$	initial concentration of excited-state PtOEP	mol/L

36* All reported intensities are normalized and so unitless.

38 I. Introduction

39 Permeant transport through a polymer membrane is driven by differences in pressure,
40 concentration, temperature, and, if it is a polyelectrolyte, the electrical potential across it.¹ The
41 most common model is the solution-diffusion model for steady-state, which is defined in terms
42 of the permeability, P_m . The permeability is the product of the Henry's Law solubility, S , and
43 diffusivity, D , i.e. $P_m = S \times D$.² Models with a high level of detail concerning the physical nature
44 of uptake and permeation are not generally available. Such models are valuable for their
45 predictive character and their ability to provide scientific insight. This work builds a foundation
46 for the development and use of these more detailed models in the study of transport through
47 polymers.

48 Polymeric membrane materials fall into two basic classes, highly permeable rubbers with low
49 glass transition temperatures (T_g) and weakly permeable glasses with high T_g . The solution-
50 diffusion model applies to both. The physical processes that take place in the polymer bulk do
51 not depend on whether the permeant was initially present as a component of a liquid or in the gas
52 phase. The details of the process of uptake into the membrane, on the other hand, are likely to
53 depend on permeant phase. The solution-diffusion model assumes that no matter what the source
54 of the permeant, its rate of uptake at the polymer interface is much faster than diffusion through
55 the polymer bulk,³ and therefore is not rate limiting. Molecular dynamics simulations and
56 kinetics models using parameterized schemes have provided important insights to the detailed
57 nature of uptake and permeation for ion-conducting, phase-separated perfluorosulfonic-acid
58 polymers.^{4,5} This level of physical detail, however, is not available for most membrane systems.

59 Steady-state permeation as described by the solution-diffusion model is relevant to many
60 applications using polymeric membranes such as purification and electrolyzer systems, but not to
61 those in which permeant concentrations and other operating conditions such as temperature vary
62 significantly with time. Examples are gas sensor and intermittent gas separations systems,
63 electrolyzers using electricity from renewable sources, or photoelectrochemical systems that use
64 sunlight to convert components of air and water into fuels. The membrane composition and
65 structure required for each of these applications varies, and at our current level of understanding
66 the physical details governing time-dependent membrane performance cannot be assumed to be
67 as simple to capture as they are by the solution-diffusion model. To begin to develop a

68description of membrane behaviors far from steady state, we focus in this study on one extreme –
69inert gases permeating a rubbery polymer. Our basic model is constructed in a manner that
70enables it to be extended to more complex situations such as mixed-matrix membranes, metal
71foam separators, permeant-induced ageing, permeation of mixtures whose transport properties
72through the membrane are not independent of each other, and permeation under variable
73temperature conditions. The ability to model such situations predictively allows membrane
74architectures well-suited for time-varying applications to be developed.

75 The rubbery, non-interacting regime applies to membranes for gas sensors and certain gas
76separators. We consider time-dependent permeation in two configurations: (1) reversible
77permeation of gases into a gas-sensor system, and (2) pre-steady-state transport through a gas
78separator. Time-dependent data for a sensor system have been reported in the literature.⁶⁻⁸ For the
79present work, we examine a comprehensive data set describing quenching of phosphorescence of
80a dye in a series of rubbery polymers by O₂ and have used them to develop and validate a basic
81model framework. Time-dependent gas-separator data obtained under well-controlled conditions
82are scarce, so we have measured non-steady-state permeation of gases over a range of pressures
83through a rubbery polymer, polydimethylsiloxane (PDMS) for this work. The gases - O₂, N₂,
84CO₂, and CH₄ - are selected to span a range of gas solubilities and nature of interaction with the
85polymer chains. We have examined aspects of uptake by using molecular dynamics simulations
86to estimate the fraction of CO₂-PDMS collisions that result in adsorption or absorption. Using
87these data, a multiscale, reaction-diffusion model framework has been constructed that
88successfully reproduces both sensor and separator data with no adjustable parameters, and
89reveals several aspects of the permeation process that are kinetically significant. Because it is
90predictive, the model is useful to examine functional systems. As an example, we describe
91simulation results for use of rubbery membranes to separate gas mixtures, and product crossover
92between cathode and anode chambers during a diurnal cycle of a solar-driven
93photoelectrochemical CO₂ reduction system. The latter case is applicable to a system in which
94the membrane has become rubbery due to prolonged exposure to CO₂.

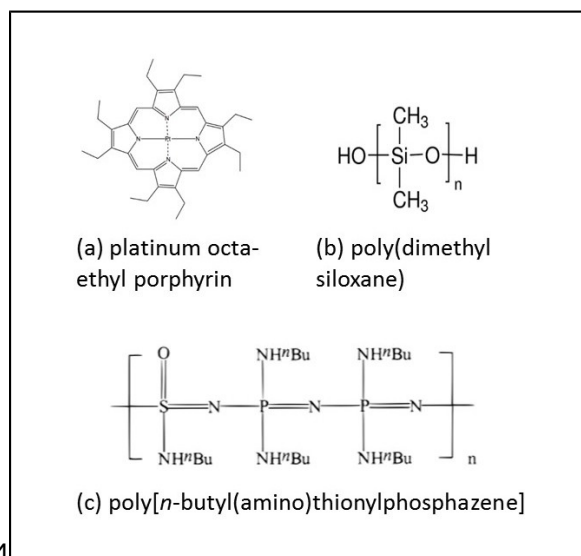
95 II. Methods and Materials

96 Three separate methodologies have been combined in the present study: stochastic reaction-
 97 diffusion simulations, molecular-dynamics simulations, and gas-permeation measurements. The
 98 materials set considered in this work is shown in Figure 1.

A99 Reaction-diffusion simulations

100 The goal of this work is to develop a predictive model whose simulation outputs can be
 101 compared directly to experimental data. This requires that all simulation inputs (system
 102 geometry, rate constants and concentrations of all reactants and products) be measured

103



104

105 Figure 1. Chemical structures of the phosphorescent dye (a) platinum octa-ethyl porphyrin
 106 (PtOEP), and the polymers (b) poly(dimethyl siloxane) (PDMS) and (c) poly[n-
 107 butyl(amino)thionylphosphazene] (C4PTP) investigated in this work.

108

109 experimentally, calculated by molecular dynamics simulations, or derived from established
 110 theory. The simulation reaction-diffusion code used in this study incorporates stochastic
 111 methods, a type of kinetic Monte Carlo that produces an accurate timebase.⁹⁻¹⁰ It provides a
 112 rigorously accurate solution to the master equation for Markov systems, and is a useful
 113 alternative to coupled differential equation integrators for complex materials systems. It is
 114 particularly well-suited for systems that require detail at both nanoscale and macroscale
 115 dimensions, and involve large dynamic ranges in rates. The open access code package is

116Kinetiscope.¹¹ Details on its computational methodology and applications to various types of
117condensed organic phase reaction-diffusion systems are described in previous publications.¹²⁻¹⁷
118The basic framework used in this work is a 1-dimensional reaction-diffusion system comprising
119a stack of micro-scale compartments to represent the polymer bulk, and source and receiver
120compartments separated by 1-nm-thick interfacial regions from the membrane. The construction
121of the reaction-diffusion model is described in Section III.

2.2 Molecular Dynamics Simulations

123 In most continuum models, gas uptake and desorption at the surface of a polymer membrane
124are considered to be instantaneous, with bulk transport being the controlling factor in permeation
125rate. However, to build a predictive model, it is necessary to use physically-derived rate
126constants for all processes. The dynamics of gas-rubbery polymer collisions are not well studied,
127so we have selected CO₂ among the gases used in this work, N₂, O₂, CH₄, and CO₂, for a
128thorough investigation of the uptake process using molecular dynamics (MD) simulations. All of
129the gases are weakly interacting with the polymers they permeate, so we assume that the sticking
130coefficient obtained from the study of CO₂ can be applied to all the gases studied in this work.

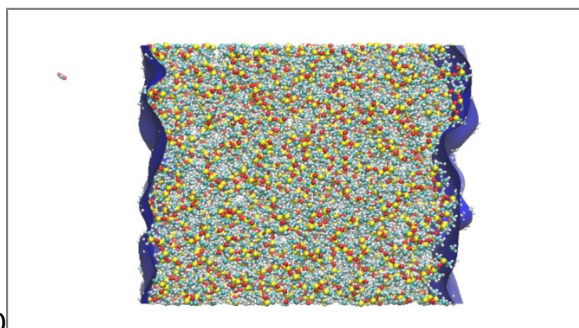
131 Simulations are performed using the Desmond MD simulation package¹⁸⁻²⁰ and the OPLS-
1322005 force field.²¹ A time-step of 1 fs is used for short-range interactions and a 3 fs time-step is
133used for long-ranged interactions. Long-ranged electrostatics are computed using the Ewald
134summation. A short-ranged Coulomb cutoff of 9 Å is used. Center of mass motion is removed at
135each time step in the adsorption simulations.

136 The initial PDMS structure is generated using standard protocols in polymer construction.
137The initial low-density ($\rho = 0.0245 \text{ kg/m}^3$) structure is created using an amorphous builder. This
138polymer structure has 25 chains of PDMS of 100-monomer length, for a total of 25,053 atoms.
139To ensure that there are no overlapping atoms in the structure, 100 steps of energy minimization
140and 10 ps of dynamics in the NVT ensemble²² at 10 K are performed (using a time constant of
1410.1 ps for the thermostat). The density of the structure is increased by running 500 ps of
142dynamics in the NPT ensemble using the algorithm of Martyna, Tobias, and Klein (MTK) with a
1431 ps time constant for the barostat.²³ To ensure entanglement of the polymer chains, a Scaled
144Effective Solvent (SES)²⁴ equilibration step is performed in which long-ranged van der Waals
145and Coulomb interactions scaled to 20% of their original values, and dynamics are run for 2000

146ps in the NVT ensemble with a Nosé-Hoover thermostat. Finally, with van der Waals and
 147electrostatic interactions at their full strength, energy minimization is performed for 300 ps and
 148the lattice parameters of the structure are again relaxed with 200 ps of NPT dynamics.

149 The above procedure results in a roughly 70-Å thick slab of PDMS created with dimensions
 150of $6.79 \times 6.79 \times 6.79 \text{ nm}^3$. This procedure results in a bulk density of $\approx 0.985 \text{ kg/m}^3$, which is
 151above the experimental reference value of 0.970 kg/m^3 ,²⁵ but below the experimental sample
 152densities of 1.06 to 1.08 kg/m^3 obtained in this study (see Section II.C.). To create a PDMS
 153surface, the length of the cell is increased by 200 Å in the x-direction to generate a region of
 154empty space. All polymer chains are kept intact. The surface is then equilibrated for 3000 ps in
 155the NVT ensemble using the Berendsen thermostat at 300 K. The density near the surface is
 156reduced to $\approx 0.94 \text{ kg/m}^3$ due to surface roughness. The instantaneous surface is designated using
 157the Gaussian smoothing method of Willard and Chandler,²⁶ which is described more fully in the
 158SI Section 1. The final, equilibrated structure and its instantaneous surface are shown in Figure 2.

159



160

161Figure 2. The structure of poly(dimethyl siloxane) (PDMS) in the molecular dynamics
 162simulations. Hydrogen atoms are shown in white, carbon in light blue, oxygen in red and silicon
 163in yellow. The instantaneous interface is shown in dark blue. The CO₂ molecule (upper left hand
 164corner of the image) is sent towards the surface of the PDMS polymer structure for an adsorption
 165simulation.

166

167 CO₂ absorption, adsorption, and desorption events are tracked using a procedure based on the
 168molecular adsorption studies of Julin *et al.*²⁷⁻²⁸ A CO₂ molecule is introduced at a distance of
 169approximately 15 Å from the surface and is assigned a velocity from the Maxwell-Boltzmann
 170distribution at 300 K, with the constraint that the x-component of the velocity vector lies within a
 17145-degree cone normal to the surface. After 100 ps of NVE simulation, the outcome (adsorption,

absorption, desorption) is recorded based on the position of the CO₂ molecule relative to the surface region, which is defined as points within 4 Å, *i.e.* twice the van der Waals radius, of the instantaneous surface. Justification of the 4-Å cutoff is given in the results section.

175

Experimental permeation measurements

Complete time-dependent permeation curves for several gases through PDMS were obtained in this work. To our knowledge such data have not been previously reported, and they are needed for accurate model construction and validation. Additional details are presented in the SI Sections 2 and 3.

Sample Preparation. PDMS samples are prepared using standard procedures. The PDMS base and a proprietary crosslinker (Sylgard 184, Dow Corning Corp., Auburn, MI) are mixed in a 10:1 ratio by weight. The PDMS solution is degassed to remove air bubbles and cast onto silicon wafers to the desired thickness *via* the spin-coating method. The polymer films are stored in deionized water until use. Prior to measurement, each sample is thoroughly dried and degassed. The thickness of the dry films is in the range of 163 to 165 μm, as measured by a micrometer (Mitutoyo, Aurora, IL). A helium gas displacement pycnometer (AccuPyc II 1340 from Micromeritics Instrument Corp., Norcross, GA) is used to measure the density, which ranges from 1.06 to 1.08 kg/m³. The glass transition temperature is 149 ± 0.3 K, measured using differential scanning calorimetry (DSC800 Perkin Elmer, Waltham, MA).

Gas Permeation. A custom-built system is used for constant-volume gas-permeation experiments. A schematic is available in Reference ²⁹. Prior to the permeation experiment, gas is evacuated from the entire apparatus, a check for leaks is performed, and the measured permeabilities are corrected for any small pressure drifts. Upstream pressures of 119 to 835 kPa are tested. The upstream pressure is maintained at least two orders of magnitude greater than the downstream pressure during the measurements so that the pressure difference across membrane is effectively constant. The upstream pressure is monitored using a Honeywell STJE (Honeywell, Morris Plains, NJ) pressure transducer, which has a range of -2 to 7000 kPa and an accuracy of ±100 kPa.³⁰ The downstream pressure is monitored using a Baratron MKS (MKS Instruments, San Jose, CA) pressure transducer, which has a range of 0.1 to 1000 Pa and an accuracy of ±7 Pa.³¹ The temperature of the system is maintained at 308 K in a water bath using an Alpha

202immersion thermostat (Lauda, Delran, NJ). All gases, N₂, O₂, CH₄, and CO₂, (Praxair, Danbury,
203CT) are at least 99.9% pure and are used as received.

204 After the gas permeation process has reached steady state, the gas permeability of the
205membrane, P_m , is calculated from the gas flux, J , through membrane of thickness, l , due to a
206difference in partial pressure, Δp , across the membrane:

$$P_m = \frac{Jl}{p_{up} - p_{ds}} = \frac{V_{ds}l}{\Delta p ART} \left[\left(\frac{dp_{ds}}{dt} \right)_{SS} - \left(\frac{dp_{ds}}{dt} \right)_{leak} \right] \quad (1)$$

207where V_{ds} is the downstream collection volume, A is the membrane area, R is the universal gas
208constant, T is the temperature, p_{ds} refers to the downstream pressure, p_{us} refers to the upstream
209pressure, and $(dp_{ds}/dt)_{SS}$ is the steady-state pressure rise, which is corrected for the leak rate
210 $(dp_{ds}/dt)_{leak}$. At steady state, $P_m = D \times S$.

211III. Development of a kinetic model for O₂ sorption and phosphorescent dye 212 quenching in a sensor system

213 Detailed reaction-diffusion models for non-steady state transport in polymer membranes
214 have not been reported previously, and development of a predictive description requires
215 multiple independent types of time-dependent experimental measurements to validate it.^{12, 14}
216 There are few of these in the literature for any polymer type, however, a particularly useful
217 set for rubbery polymers has been reported by Jayarajah and coworkers.⁶⁻⁷ In this section, we
218 describe how we developed the model for their measurements, quenching of a
219 phosphorescent dye in a rubbery polymer by oxygen for a sensor application,⁸ and compared
220 predictions to experimental observations. The polymer used is poly(n-butyl (amino) thionyl
221 phosphazene) (C4PTP). This polymer has a low Tg of 257 K and was studied at a thickness
222 of 163 μm . The dye platinum octa-ethyl porphyrin (PtOEP) is dispersed in the polymer, and
223 quenching of its laser-excited phosphorescence acts as an indicator of the internal
224 concentration of O₂ gas. The structures of the C4PTP monomer and of PtOEP are shown in
225 Figure 1 and the reaction mechanism and rate coefficients are summarized in Table 1.

226

227

	units	PS	Ref.	C4PTP	Ref.
$D(\text{O}_2)$	m ² /s	1×10^{-11}	³² (a)	4.0×10^{-10}	7
$S(\text{O}_2)$	mol/(L Pa)	8.26×10^{-8}	³² (a)	1.18×10^{-8}	7

$[O_{2(p)}]_{eq}$	mol/L	1.73×10^{-3}	this work	2.40×10^{-4}	this work
$[PtOEP]$	mol/L	0.0731	³³	3.1×10^{-4}	⁷
$[^3PtOEP]_{init}$	mol/L	5.85×10^{-4}	this work	2.48×10^{-6}	this work
ϕ_P		0.54	³³	0.54	this work
τ_0	μs	60	³³	103	⁷
k_r	s^{-1}	8.33×10^3	³³	3.9×10^3	this work
k_{nr}	s^{-1}	8.30×10^3	³³	5.8×10^3	this work
k_{TTA}	$M^{-1} s^{-1}$	4×10^7	³³	4×10^7	this work
k_q	$M^{-1} s^{-1}$	1.82×10^7	this work (b)	3×10^9	this work
k_{ex}	s^{-1}	-----		50	this work

228(a) for amorphous, atactic polystyrene

229(b) calculated from the reported pseudo-first order rate constant $3.20 \times 10^4 s^{-1}$ reported in Ref. ³³
230 and the expected concentration of O_2 in the polymer.

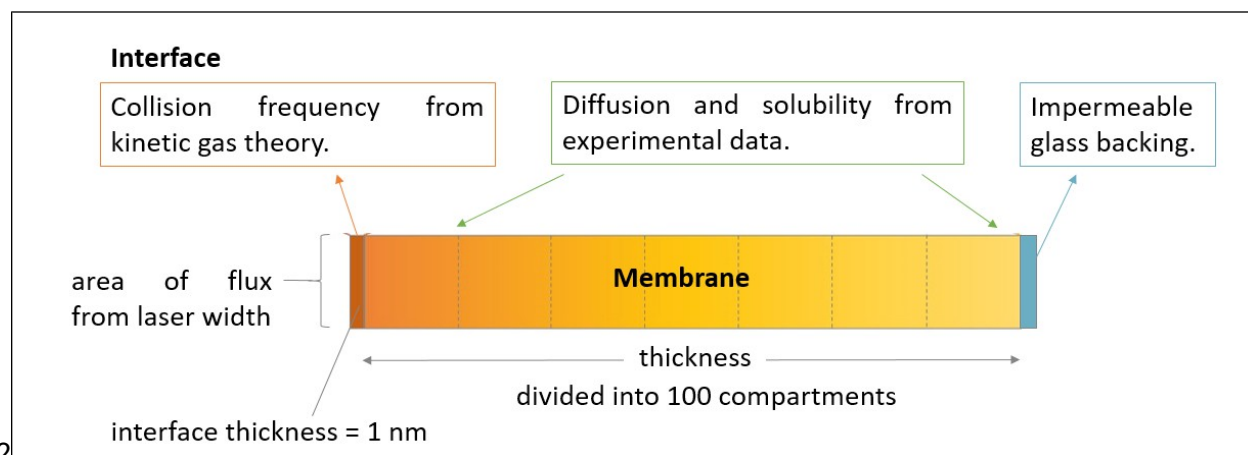
231 Table 1. Simulation inputs for phosphorescence quenching.

232

233A. System Geometry

234 As shown in Figure 3, the membrane is represented as a set of 100 homogeneous
235 compartments, each $1.63 \mu m$ thick, coupled by Fickian diffusion paths. An interfacial
236 compartment of $1 nm$ thickness is provided at the gas-polymer interface to enable explicit
237 description of adsorption, desorption and transfer of gases to and from the polymer bulk. The
238 other side of the membrane is in contact with a glass substrate and so is not permeable. The area
239 of the membrane is taken to be the footprint of the laser beam used for dye excitation, $2 mm$ by 1
240 mm .

241



242

243Figure 3. Overview of the multi-scale model for a sorption simulation. The membrane is divided
244into 100 compartments, plus one thin interfacial compartment. The area of flux is 2 mm by 1
245mm, which is the area typical of a laser beam.

246

247B. Gas-polymer interactions

248 The experimental diffusion coefficient is used for gas transport throughout the polymer,
249including the surface layers. The maximum concentration of a gas in the membrane is
250determined by its experimental Henry's Law solubility. Swelling was not reported for this system
251and is neglected in the simulations. When present, dynamic volume changes as the membrane
252swells must be included in the model. They will reduce solute concentrations and concentration
253gradients while increasing the diffusion coefficient, and their effect on permeation may vary with
254time.

255 Adsorption and desorption are represented as direct processes involving a gas molecule and a
256surface site, a description used in other studies.¹⁵ Adsorption is broken into two steps: sticking of
257the gas to the membrane surface and diffusion from the surface layer into the bulk. Sticking is
258represented as a reaction between a gas molecule and an empty surface site,



259where the adsorption rate constant, k_{ads} , is the collision frequency of the gas with the surface, k_{coll} ,
260times the sticking probability, μ ,

$$k_{ads} = \mu k_{coll} = \mu (ZA) = \mu \left(\frac{P_{up}}{\sqrt{2\pi m_w k_B T}} \right) A \quad (2)$$

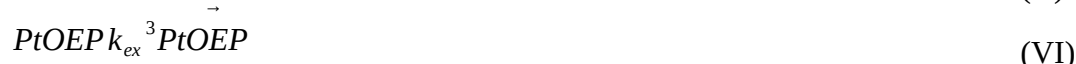
261and Z is the collision frequency per unit area, A is the experimental surface area, p is the
262upstream partial pressure of the gas, m_w is the molecular mass of the gas, k_B is Boltzmann's
263constant, and T is the temperature.³⁴ The sticking coefficient is initially assumed to be 0.1 (which
264is high but not unity since there is no information in the literature for this system); the sensitivity
265of the simulation to this value has been investigated and will be discussed in a later section. We
266use the generic term "site" because we assume that the gas molecules can physisorb to an atom
267type at the surface. The concentration of surface sites is assumed to be that of surface atoms in a
268liquid, 10^{14} atoms/cm².³⁵ Since the surface must be represented in a 3-dimensional compartment
269in Kinetiscope, the sites are distributed through a 1-nm thickness, giving a concentration of 1.66
270mol/L. Once adsorbed, the gas molecule can diffuse from the interface layer to the adjacent bulk

271polymer compartment. Desorption from the membrane is represented as detachment from a
 272surface site, the reverse of Reaction I. Because the polymer and gas molecules are weakly
 273interacting, we base the desorption rate constant on the breaking of a single van der Waals bond
 274in the gas phase, such that $k_{des} = 1.4 \times 10^{11} \text{ s}^{-1}$.³⁶⁻³⁷ The sensitivity of the simulation results to
 275assumptions concerning the surface description is discussed below.

276

277C. Kinetics of dye quenching

278 The dye is assumed to be uniformly distributed throughout the polymer bulk, and its
 279excitation and deexcitation reactions are:



281where PtOEP is in the ground state, ${}^3\text{PtOEP}$ is in the first excited triplet state, and $\text{O}_{2(p)}$ is the
 282oxygen gas within the polymer phase. These steps are the non-radiative decay, radiative decay,
 283triplet-triplet annihilation (TTA), quenching, and excitation steps, respectively. In this section,
 284we describe how the rate constants have been determined for these steps.

285 The photophysics have been determined from studies on PtOEP in polymer environments.^{6-7,}
 286^{33, 38} From the ground state, PtOEP absorbs at 380 nm and 547 nm to enter the first excited state;³⁸
 287greater than 99.9994% of PtOEP molecules excited to the first singlet state undergo intersystem
 288crossing to the first excited triplet state ${}^3\text{PtOEP}$.³³ Therefore, fluorescence from the singlet state is
 289neglected. ${}^3\text{PtOEP}$ emits at 647 nm when well-dispersed in a clear polymer.^{33, 38} For excited-state
 290dye at high concentration or in solution with low viscosity, TTA may occur, in which one ${}^3\text{PtOEP}$
 291molecule quenches another and is itself excited back to the first excited singlet state. Because
 292each excited singlet quickly relaxes to the first excited triplet we do not explicitly track the first
 293excited singlet state in Reaction IV.³³

294 Two types of experiments are examined to construct and validate a kinetic model for PtOEP
 295photoprocesses. One determines the dye's phosphorescence lifetime using pulsed laser

296excitation;^{33, 38} the other determines the emission intensities of oxygen-free and oxygen-saturated
297polymer under continuous laser excitation.⁷

2981. *Dye lifetime by pulsed laser excitation.*

299*Oxygen-free Environment.* The phosphorescence lifetime, τ_0 , phosphorescence quantum yield, ϕ_P ,
300and TTA rate constant are determined experimentally for polystyrene (PS) in Reference³³ and are
301listed in Table 1. The rate constants for radiative decay, k_r , and non-radiative decay, k_{nr} , are
302calculated as

$$k_r = \frac{\phi_P}{\tau_0} \quad (3)$$

303and

$$k_{nr} = \frac{1 - \phi_P}{\tau_0} \quad (4)$$

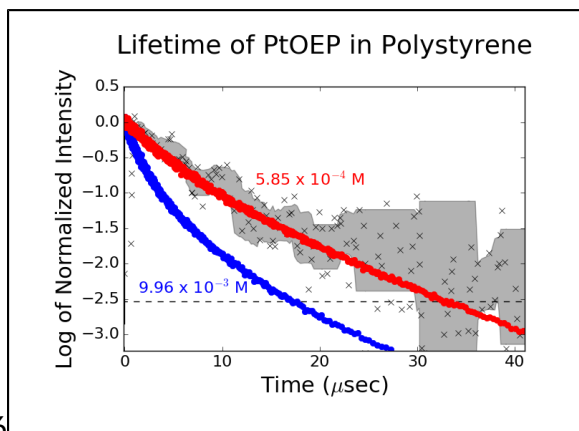
304 Simulations of phosphorescence decay in PS using the reported rate constants were performed in
305 order to establish the validity of the mechanism above (Rxn. II - IV). The intensity of emitted
306 light, I , is calculated from the change in photon production over time. For these simulations, we
307 can omit Reaction VI and start with a population of excited dye molecules. The initial excited-
308 state dye population, $[{}^3\text{PtOEP}]_{init}$, can be calculated using

$$[{}^3\text{PtOEP}]_{init} = \frac{[\text{PtOEP}] \sigma_a \omega_L}{h \nu_L} \quad (5)$$

309where $[\text{PtOEP}]$ is the total concentration of dye molecules, σ_a is the absorption cross section of
310the dye at the exciting wavelength, ω_L is the energy density of the laser beam, h is Planck's
311constant, and ν_L is the exciting frequency. For the experiments of Bansal *et al.* ($\sigma_a = 4.70 \times 10^{-21}$
312m²; $\omega_L = 17.0$ J/m²; $\nu_L = 8.64 \times 10^{14}$ s⁻¹; $[\text{PtOEP}] = 0.0731$ M), this calculation predicts that
313 $[{}^3\text{PtOEP}]_{init}$ is 9.96×10^{-3} M. However, as shown in Figure 4, and explained in the SI, simulations
314using the reported rate constants are in good agreement with experiment only for $[{}^3\text{PtOEP}]_{init}$
315equal to 5.85×10^{-4} M, which is 0.8% of the total dye concentration, $[\text{PtOEP}] = 0.0731$ M. The
316reason for this difference is not clear, but may reflect dye aggregation since clusters of PtOEP
317emit at 790 nm,^{33, 38} which would not be detected in the experiments. Sensitivity studies for
318 $[{}^3\text{PtOEP}]_{init}$ are described in the SI Section 4 and illustrated in Figure S1.

319 For C4PTP, only the lifetime and total dye concentration are reported. It is assumed that the
 320unreported properties are the same as in PS, due to the dye being in a polymer environment in
 321both cases. The rate constants for radiative and non-radiative decay are calculated from the
 322assumed quantum yield and the experimentally measured phosphorescence lifetime. The initial
 323excited-state dye population is assumed to be 0.8% of the total dye concentration, as it is in PS.
 324All values related to the dye lifetime are listed in Table 1.

325



326

327Figure 4. Simulated and experimental phosphorescence decay in air. The experimental data
 328points from Ref. ³³ are marked with black x's, and the gray region indicates the local standard
 329deviation (over 11 data points). The dashed line indicates the lower detection limit of the
 330instrument. The blue markers are for the simulation data using the reported value of $[\text{}^3\text{PtOEP}]_{init} =$
 331 9.96×10^{-3} M. The red markers are for a simulation using a reduced value of $[\text{}^3\text{PtOEP}]_{init} = 5.85 \times$
 332 10^{-4} M.

333

334Oxygen-containing Environment. Data for lifetimes in the presence of air are reported for PS and
 335C4PTP. The introduction of oxygen in air opens a channel for quenching which competes with
 336phosphorescence. The quenching rate constant, k_q , is calculated from the lifetime measured at a
 337partial pressure of oxygen $p_{O_2} = 21$ kPa, assuming that TTA is negligible. Then,

$$k_q = \frac{\frac{1}{\tau} - k_r - k_{nr}}{[O_{2(p)}]} \quad (6)$$

338where $[O_{2(p)}]$ is the concentration of oxygen gas within the polymer, and τ is the lifetime under
 339ambient conditions. Simulations of PS using the reported quenching rate constant (see Table 1)

340 agree with experiment when $[^3\text{PtOEP}]_{\text{init}} = 0.8\%$ of the total dye concentration. Sensitivity of the
 341 lifetime to k_q is discussed in SI Section 5 and illustrated in Figure S2.

342

3432. *Excitation rate constant from continuous excitation of the dye*

344 The emission intensity is a result of the balance between excitation, radiative decay, and
 345 other forms of decay. Since the rate constants for all decay processes in the dye kinetics scheme
 346 are fixed, simulation of the emission intensity can be used to determine k_{ex} . However, the
 347 experimental data are reported as relative intensity values, rather than absolute intensities. The
 348 extrema in intensities are captured in the ratio $B = I_0/I_{\text{eq}} - 1$, where I_0 is the phosphorescence
 349 intensity in the absence of oxygen, and I_{eq} is the intensity for a constant concentration of O_2
 350 consistent with $p_{\text{O}_2} = 21$ kPa. The experimental value is $B = 74 \pm 6$. Simulations of the oxygen-
 351 free and oxygen-saturated steady-state scenarios are performed; then absolute and relative
 352 intensities are calculated. Values for the relative intensity B are plotted as a function of k_{ex} in
 353 Figure S3 in SI Section 6. The value of the ratio is quite insensitive to k_{ex} over a range of 1 to 100
 354 s^{-1} , where B stays between 65 and 67. The median of $k_{\text{ex}} = 50 \text{ s}^{-1}$ is chosen, at which $B = 66$.

355 Time-dependent data.

356 The intensity over time, $I(t)$, was used by Jayarajah *et al.* to determine the solubility
 357 coefficient, S , and the diffusion coefficient, D , for O_2 in C4PTP using a two-parameter fit to the
 358 equation,

$$I(t) = \frac{I_0}{l} \int_0^l \frac{[\text{O}_{2(p)}]_{\text{eq}}}{1 + B[\text{O}_{2(p)}](x,t)} dx \quad (7)$$

359 where

$$B = \frac{I_0}{I_{\text{eq}}} - 1 = 4 \pi N_{\text{Av}} \tau_0 R_{\text{AB}} D S p_{\text{O}_2} \quad (8)$$

360 where l is the membrane thickness, N_{Av} is Avogadro's number, τ_0 is the lifetime of PtOEP in the
 361 polymer environment in the absence of oxygen, $[\text{O}_{2(p)}](x,t)$ is the absolute concentration of O_2
 362 within the membrane at position x at time t , and $[\text{O}_{2(p)}]_{\text{eq}}$ is the equilibrium concentration of O_2
 363 within the membrane, which is equal to $S \times p_{\text{O}_2}$.^{6-7, 39} The equation above assumes that the dye is
 364 both distributed and excited homogeneously throughout the membrane (i.e. the sample is

365optically thin). In the simulations, we also use a homogeneous dye distribution and assume
366homogeneous excitation.

367 The continuous excitation experiments were performed in two modes: (i) sudden replacement
368of N₂ with air, leading to sorption of O₂ and a decay in phosphorescence intensity, and (ii) sudden
369replacement of air by pure N₂, leading to desorption of O₂ and an increase in phosphorescence
370intensity. Because the pressure vs time data are not available to use as input to the simulations,
371the increase or decrease in oxygen partial pressure is assumed to be instantaneous in the
372simulation.

373

374IV. Simulations of the O₂ sensor system

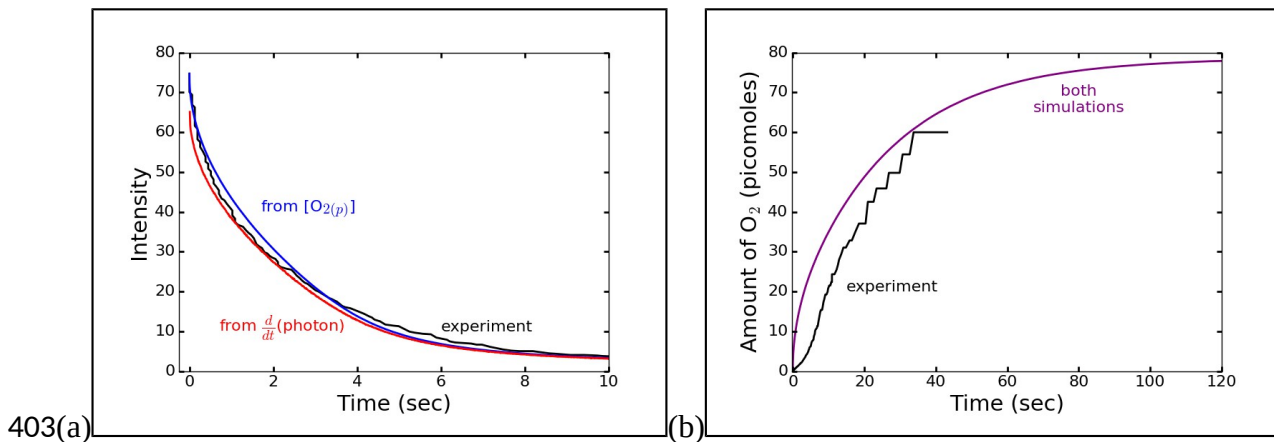
375 Simulations of emitted light intensity as O₂ sorbs into or desorbs from the polymer are used
376to calculate $I(t)$ for comparison to experimental data. The absorption and desorption simulation
377results are compared with the experimental data in two ways: (i) dye kinetics are not simulated
378directly, but the total amount of O₂ predicted to be absorbed in the polymer in the simulation is
379used to calculate the expected $I(t)$ using the reported intensity ratio of $B = 74$ and equations 7-8,
380and (ii) full simulation of the dye kinetics and the photons produced per unit time are directly
381compared to the experimental intensity over time. The use of two methods serves as a check on
382the internal consistency of the experimental data and on the validity of the reaction mechanism.
383The experimental value of B is chosen for comparison in method (i) because the estimation of k_{ex}
384introduces an additional discrepancy between simulation and experimental results.

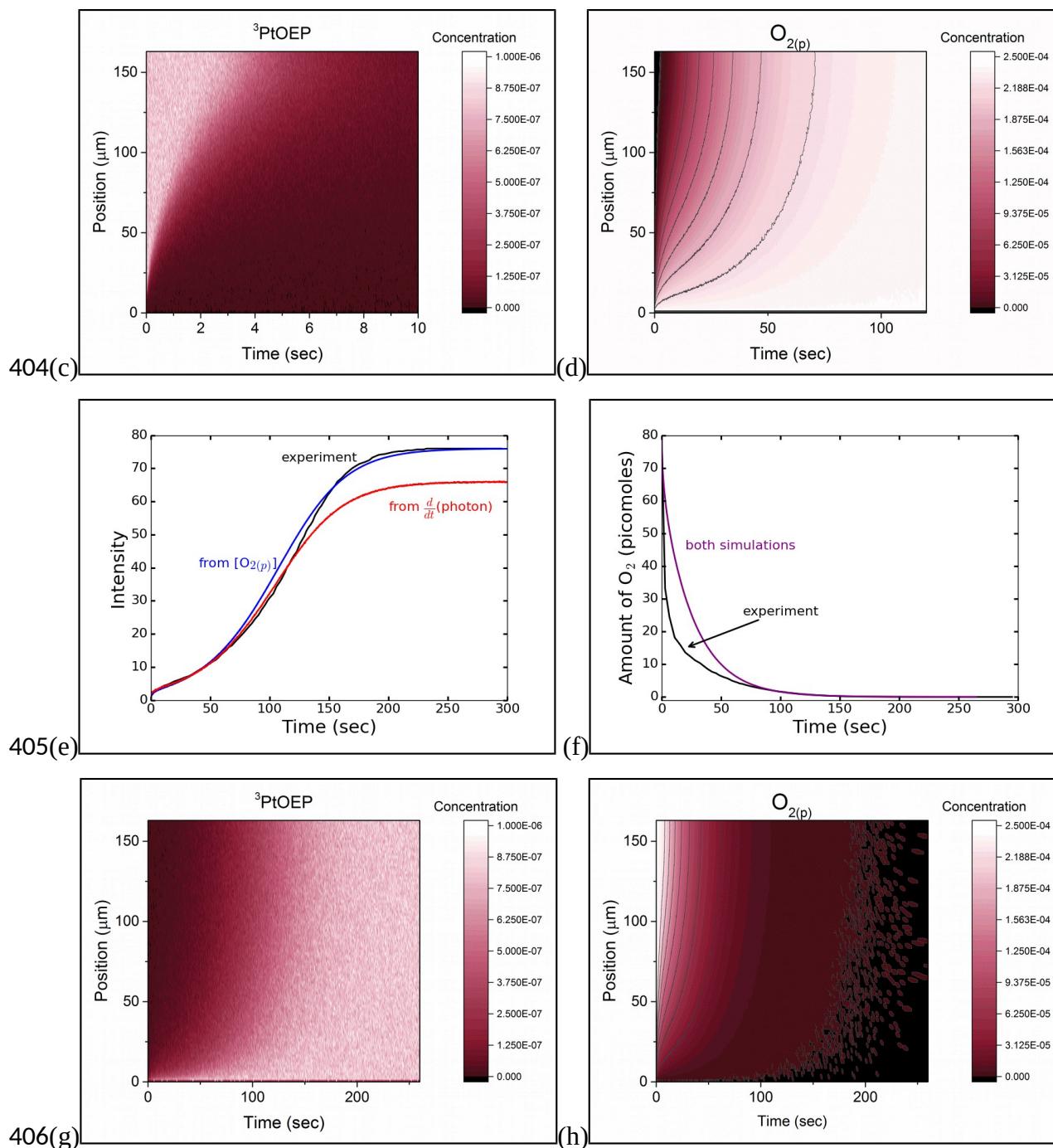
385 The simulations predict the amount and distribution of oxygen gas within the membrane at
386all times. We compare the simulated O₂ amount to experiment, which does not measure O₂ in the
387film directly, by calculating n , the expected amount of O₂ in the membrane, from the
388experimental $I(t)$ curve. We assume $dx = l$ in Equation 7, i.e. that the film is entirely uniform.
389The maximum amount of O₂ in the membrane, n_{max} , is found at equilibrium. $n_{max} = [O_{2(p)}]_{eq} \times V$,
390where $V = A \times l$. Then, rearranging Eq 7-8 gives

$$n = n_{max} \frac{1}{B} \left[\frac{B+1}{I(t)} - 1 \right] \quad (9)$$

391 Simulation results for sorption and desorption are compared to experiment in Figure 5. The
 392 shape of the calculated curves is in good agreement with observations, but there are several
 393 quantitative differences. The initial rise in the simulated amount of O₂ in the polymer shown in
 394 Figure 5b is faster than that calculated from the experimentally measured $I(t)$ using Equation 9.
 395 In the simulation, the amount of O₂ in the membrane compartments near the gas-polymer
 396 interface increases rapidly, and they become saturated with O₂. The rate of O₂ absorption then
 397 slows, limited by O₂ diffusion to the other side of the membrane. The reason for the difference
 398 between simulated and experimental amount of O₂ is unclear. One possibility is that the
 399 assumption of $dx = l$ in the calculation of the amount of O₂ from $I(t)$ (either experimental or
 400 simulated) is not valid, which is supported by the fact that simulated and measured $I(t)$ curves are
 401 in much closer agreement (Figure 5a).

402





407Figure 5. Results of simulation of laser-excited PtOEP and O_2 kinetics in C4PTP. In all the
 408contour plots, darker colors represent lower concentration, and lighter colors represent higher
 409concentration. Position is defined as 0 at the gas interface of the polymer. (a) $I(t)$ when O_2 sorbs
 410into the polymer, calculated from $\text{O}_{2(p)}$ with Eqn. 7 - 8 (blue) and from direct simulation of the
 411dye photophysics (red); (b) The amount of $\text{O}_{2(p)}$ in the polymer as a function of time, comparing
 412that calculated from the experimental data and both simulation types; (c) Contour plot of the
 413concentration of $^3\text{PtOEP}$ through the polymer thickness during 10 s of sorption, showing
 414complete quenching of phosphorescence; (d) Contour plot of the concentration of $\text{O}_{2(p)}$ through

415the polymer thickness, reaching saturation after 120 s of sorption. (e) $I(t)$ when O_2 desorbs from
416the polymer, calculated from $O_{2(p)}$ with Eqn. 7 - 8 (blue) and from direct simulation of the dye
417photophysics (red); (f) The amount of $O_{2(p)}$ in the polymer as a function of time, comparing that
418calculated from the experimental data and both simulation types; (g) The concentration of
419 3PtOEP through the polymer thickness over time during desorption, with recovery of uniform
420phosphorescence by 200 s; (h) The concentration of $O_{2(p)}$ through the polymer thickness over
421time during desorption, uniformly reaching zero between 200 and 250 s.

422

423 Another possibility is the presence of inhomogeneities either in the polymer, dye distribution,
424or emission, that would lead to the same total phosphorescence intensity but no simple
425relationship between light emission and total O_2 in the film. This possibility was tested in two
426ways. First, a simulation in which dye is not present in the top quarter of the membrane gives a
427similar $I(t)$ curve to experiment, showing that emission intensity is insensitive to spatial
428distribution of the dye and therefore to details of the O_2 kinetics. Second, altering the pressure
429profile of O_2 so that it does not instantaneously rise to $p_{O_2} = 21$ kPa, but rather increases over
430several seconds, which is more similar to the experimental procedure, does not change the shape
431of the curve. Finally, the assumption that the sample is optically thin is tested via simulation in
432which only the dye in the top half of the membrane becomes excited; this situation results in an
433intensity profiles that dies off much more quickly than experiment. These considerations suggest
434that Equations 7-8 are qualitatively valid for estimating the full time dependence of O_2
435absorption, but may not be quantitative in all cases.

436 Comparison of Figures 5a and c and Figures 5b and d also shows that while phosphorescence
437decays rapidly, the O_2 concentration in the membrane does not reach saturation until much later.
438In the sorption simulations, O_2 starts to reach the membrane compartments furthest from the
439interface after 2.0 sec. The concentration of 3PtOEP is very small, and is fully quenched 8.1 sec
440after the introduction of O_2 to the polymer. However, the concentration of O_2 continues to rise for
441another 140 sec. Thus, there is not a one-to-one correspondence between the intensity and the O_2
442concentration in the sensor film outside the range of 1 to 50 $\mu\text{mol/L}$ of O_2 , or an external partial
443pressure of 2 to 130 Pa, far below that used in this experiment.

444 The shape of the $I(t)$ desorption curve for C4PTP is also captured well by the simulation
445(Figure 5e). There is an initial small but rapid increase in $I(t)$ because the O_2 in the top membrane
446compartments desorbs in the first few seconds. From then until 50 s, there is slower rise in

447intensity as O₂ diffuses from the lower compartments and desorbs, but the overall O₂
448concentration within the membrane is still high enough to quench most of the dye. A more rapid
449increase in intensity ensues after 50 s, once the O₂ concentration is no longer in excess of that
450needed to quench ³PtOEP. Finally, the maximum intensity is reached around 200 s, once all of
451the O₂ is removed from the polymer. The contour plots shown in Figures 5g and 5h, support this
452interpretation. The simulated O₂ content shown in Figure 5f is higher than that extracted from the
453*I(t)* measurements. This discrepancy may have the same origin as discussed above for the
454sorption case, Figure 5b.

455 Sensitivities of the simulation results to uncertainties have been thoroughly evaluated. Those
456for *D* and *S* are discussed in the SI Section 7 and shown in Figure S4. Those of the assumed
457properties of the gas-polymer interface are presented in Figures S5 and S6 and summarized here.
458Saturation of surface sites by O₂, assuming a sticking coefficient of 0.1, occurs within 10⁻²¹
459seconds at 21 kPa. Reduction of the sticking coefficient to 10⁻¹⁷ has no effect on the O₂ profile,
460nor does increasing the sticking coefficient to 1. The concentration of adsorption or desorption
461sites can also be varied by 1 to 2 orders of magnitude with no effect. Alteration of the surface
462diffusion coefficient does not change the sorption results. Reduction, though not increase, of the
463interfacial diffusion coefficient does alter the desorption results, however. Finally, the desorption
464rate constant can also be reduced or increased by several orders of magnitude without altering
465the oxygen profiles. Because the simulation results are largely insensitive to surface kinetics at
466the rubbery polymer-inert gas interface, this type of simulation is not useful for exploring them,
467therefore we have used molecular dynamics simulations to gain more insight into the processes
468occurring at the surface.

469 V. Permeation of gases through a membrane separator: experiment and 470 simulations

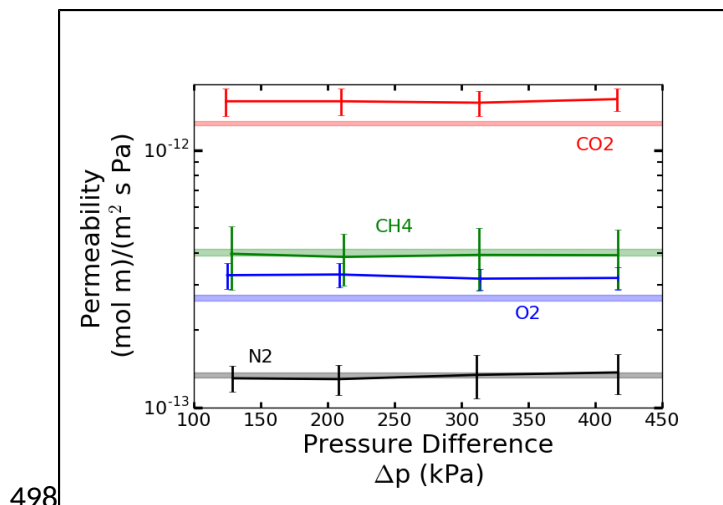
471 Having established a suitable reaction-diffusion framework as described in the preceding
472sections, we use it to examine the fully time-dependent permeation data obtained in this work,
473capturing not only steady-state permeation, but also the downstream pressure rise for steady-state
474and pre-steady-state regimes. In this section, we present the permeation measurements and
475simulations of them. In view of the insensitivity of sorption of gas by rubbery polymer sorption

476to assumptions of gas sticking, we report molecular dynamics calculations to provide a
477description of this process for one system, thus avoiding an arbitrary guess in the model.

478A. Measurements of time-dependent permeation through PDMS

479 In this study, the permeation of the gases N_2 , O_2 , CO_2 , and CH_4 through PDMS is investigated
480at 308 K for various pressure gradients. The set of all experimental data is summarized in Table
481S1. The pressure vs time data are presented in the SI Section 8, and a selection of data is shown
482in Figure S7. The permeability is calculated from the rate of pressure increase downstream from
483the membrane when steady state is reached; differences due sample to sample variations are
484accounted for in the calculation of permeation via Equation 1. The permeability values as a
485function of pressure difference between source and receiver volumes are given in Figure 6. Each
486data point represents an average value of three or more different samples and measurements. The
487leak rate for each measurement is kept under 0.05% of the permeate flux. Propagated
488measurement error ranges from 1 to 4%. The error bars show the sample-to-sample variability,
489which is 10 to 20%. As can be seen in Figure 6, permeability shows no dependence on pressure
490gradient, indicating that the sorption isotherm is also linear. Permeation values for PDMS can
491vary with changes in base to cross-linker ratio, casting method, and annealing temperature. The
492measured values are consistent with previously measured values within experimental error and
493the sample-dependent variability of literature values.⁴⁰⁻⁴²{Merkel, 2000 #27} Higher pressure
494gradients (600 – 850 kPa) were also investigated, but with fewer experiments such that the
495variation between samples was not robustly characterized; so these data are omitted from Figure
4966.

497



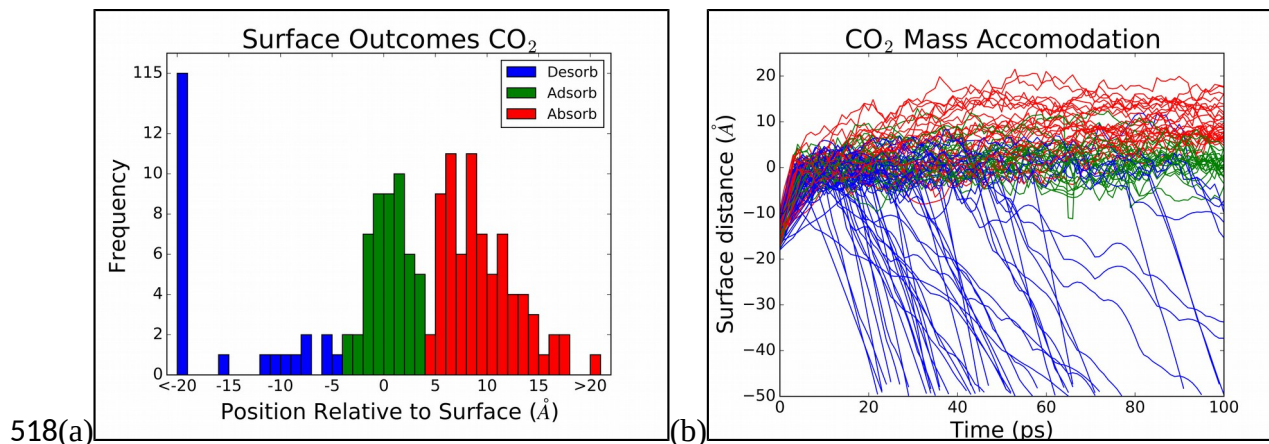
499 Figure 6. Comparison of experimental permeability to literature values. The literature values
500 from Ref. ⁴⁰ are shown with their standard deviation as the shaded regions.

501

502 B. Calculation of sticking probability of CO₂ on PDMS

503 Few data are available on sticking coefficients to of weakly interacting gases to PDMS or
504 other rubbery polymers; therefore, we use molecular dynamics simulations to estimate
505 reasonable values. We found that the sticking process is not kinetically limiting during
506 construction of the permeation model for PDMS, similar to the finding for O₂ in C4PTP, so we
507 have performed calculations for a single gas, CO₂, and assume that its sticking coefficient on
508 PDMS is applicable to the other gases investigated. A series of 250 simulations of CO₂ impacts
509 onto a PDMS surface was performed, and the results are shown in Figure 7 and summarized in
510 Table 2. Some care must be taken in how the classification of type of event is interpreted: the
511 distinction between an adsorbed and absorbed molecule is arbitrary, especially for atoms just
512 below the interface, and the fate of molecules adsorbed on the surface is not clear from the finite
513 simulation time. Thus, sticking in these simulations has a lower bound of 30%, equal to the
514 fraction of absorbed molecules, and an upper bound of 50%, equal to the fraction of absorbed
515 plus adsorbed molecules. The minimum sticking coefficient of 30% is used in the reaction-
516 diffusion simulations for all gas molecules.

517



519 Figure 7. Results of molecular dynamics simulations for CO₂ sticking to PDMS. The surface is
 520 defined as position 0 with positive position indicating the region occupied by polymer and
 521 negative position indicating the empty region. Absorbed molecules are plotted in red, adsorbed
 522 molecules in green, and scattered and desorbed molecules in blue. (a) Histogram showing the
 523 distribution of outcomes from all 250 simulations. Note that the far left blue bar represents
 524 desorption in 115 simulations. (b) Distance from the final Willard surface as a function of time
 525 for 100 randomly selected trajectories.

526

Events	Absorb	Adsorb	Desorb	Total
Number	75	50	125	250
Percent	30	20	50	100

527 Table 2. Results of Molecular Dynamics study of sticking of CO₂ to a PDMS surface.

528

529 The most similar system that has been studied experimentally is the scattering of the O₂ gas
 530 from the surface of the hydrocarbons squalane and dodecane.⁴³ At incident energies of 8 kJ/mol,
 531 twice the average kinetic energy for gas molecules in this study, the oxygen molecules fully
 532 transfer their excess energy to the hydrocarbon surface,⁴³ indicating a sticking probability near
 533 100%. A MD study of carbon dioxide⁴⁴ colliding with hydrocarbon self-assembled monolayers
 534 (SAMs) also shows a large sticking probability of $\approx 70\%$ when the SAMs are terminated with -
 535 -CH₃ or -OH functional groups. The sticking probability falls to $\approx 40\%$ for SAMs terminated with
 536 -CF₃. The reason for such a high sticking probability is explained in a MD study of argon
 537 colliding with hydrocarbon SAMs terminated with -CH₃ and -CF₃. The SAMs terminated with
 538 -CH₃ are able to redistribute the energy of the incoming molecule on the same timescale as the
 539 impact of the atom with the surface by recruiting a large number of low-frequency (inter-chain)
 540 vibrational modes; the SAM's terminated with -CF₃ redistribute the energy more slowly along

541 high-frequency (intra-chain) vibrational modes, resulting in a lower sticking probability.⁴⁵ PDMS
542 contains a large number of low-frequency interactions, and so an energy transfer mechanism
543 similar to -CH₃ terminated SAMs may apply. Thus, we conclude that a sticking probability of 30
544 to 50% is reasonable for a light, inert gas molecule at ambient temperature colliding with a
545 flexible polymer surface. Further study of this type of system, and systems in which there are
546 stronger interactions between the gas and the polymer, would be useful.

547 The absorption mechanism observed in the MD simulations involves CO₂ interacting with a
548 gap between the polymer chains during a gas-surface collision or while transiently physisorbed,
549 and passing directly into the polymer bulk. The simulations did not show that CO₂ has a strongly
550 preferred adsorption site, i.e. atom type, on the PDMS surface. This is expected for gas-polymer
551 combinations with weak interactions and supports our assumption that every surface atom is an
552 available binding site in the reaction-diffusion simulations. If strong hydrogen bonding were
553 possible, the surface area available for adsorption would be reduced.⁴⁶⁻⁴⁷ On the other hand, if
554 roughness were significant the surface sites available would be greater than assumed. The MD
555 simulations show that the ratio of the instantaneous surface area to the nominal surface area is
556 1.1, indicating that the actual, rough, surface area is only 10% greater than the ideally smooth
557 surface assumed in the reaction-diffusion simulations.

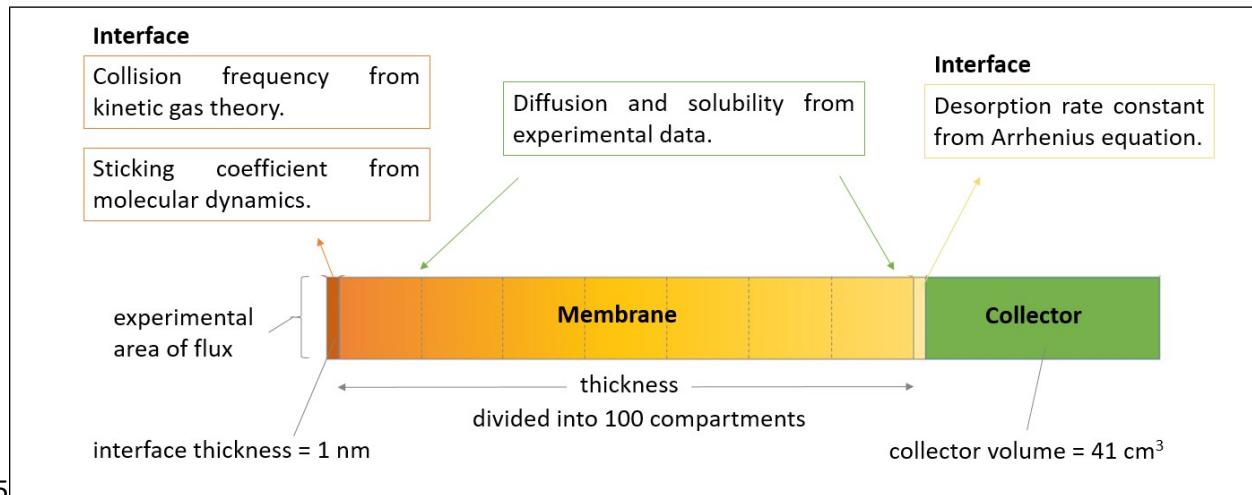
558

559 C. Simulations of time-dependent permeation

560 The model framework constructed for phosphorescence quenching (Figure 3) is modified for
561 PDMS permeation by removing the dye quenching kinetics and replacing the glass support with
562 a gas collector volume. A 1 nm thick interfacial region is provided between the polymer and the
563 gas collector, and the desorption steps occur therein; no desorption occurs at the upstream
564 interface in the permeation simulations. The size of the collection compartment is set equal to the
565 experimental collection volume of 41.3 cm³. The diffusion coefficient of desorbed gases from the
566 interface into the collection chamber is 2.0 m²/s. The surface site concentration of 1.66 mol/L and
567 the desorption rate constant of $1.4 \times 10^{11} \text{ s}^{-1}$ are the same as above. The multi-scale simulation
568 setup is shown in Figure 8. Instrumental characteristics such as volumes and the measured
569 pressure rise in the upstream chamber when gas is initially introduced are included in the model.
570 Because the downstream volume remains at low pressure relative to the surroundings, a small

571 amount of gas can leak into the collection chamber; this effect is included in the simulations for
 572 the sake of completeness. Details on the implementation of this step can be found in the SI
 573 Section 9, and relevant values are listed in Table S2.

574



575

576 Figure 8. Overview of multi-scale model for a permeation simulation. The membrane is divided
 577 into 100 compartments, plus two thin interfacial compartments. An additional compartment
 578 collects gas after its permeation. The thickness and area are based on the particular sample being
 579 simulated. The surface description is the same as for the dye quenching study, except for the
 580 sticking coefficient, which is now known from MD.

581

582 The solubility of each gas in PDMS is taken from the literature.⁴⁰ The permeability measured
 583 under each experimental condition (combination of gas, polymer sample, and upstream pressure)
 584 is divided by the literature solubility to calculate the diffusivity. These experimental conditions
 585 and the values used in simulation are presented in Table 3. We use the literature value for
 586 solubility because solubility varies only over an order of magnitude for a given polymer, whereas
 587 the diffusivity within the polymer may vary over six orders of magnitude for different
 588 permeants.⁴⁸ Only N₂ and CO₂ are presented in the main body, since they represent the lower and
 589 upper limits of permeability in PDMS, respectively; similar data for O₂ and CH₄ are available in
 590 the SI. The adsorption rate constant is calculated for each p_{up} from Eqn. 2 with $\mu = 0.30$.

591

Gas	S (a)	Sample ID	l	A	p_{up}	D
-----	-------	-----------	-----	-----	----------	-----

	mol/(L Pa)		μm	cm^2	kPa	m^2/s
CO ₂	5.68×10^{-7}	L4S2	163	0.20 3	122	2.34×10^{-9}
					206	2.34×10^{-9}
					311	2.39×10^{-9}
N ₂	3.96×10^{-8}	L4S2	163	0.20 3	122	3.27×10^{-9}
					206	3.26×10^{-9}
					416	3.26×10^{-9}
					820	2.80×10^{-9}
		L5S1	165	0.18 7		

592(a) Reference⁴⁰

593Table 3. Input values for multi-scale simulations for comparison to experimental permeation of
594PDMS.

595

596 Experimentally, the upstream permeant gas pressure p_{up} increases over several seconds after
597opening the gas valve. This rise is captured and used as an input to the simulation rather than
598assuming an instantaneous pressure step. The beginning of the rise is used as the zero for the
599simulation time base. The changing internal concentration of the gas in the polymer is set to be
600proportional to the external pressure, p_{up} ,

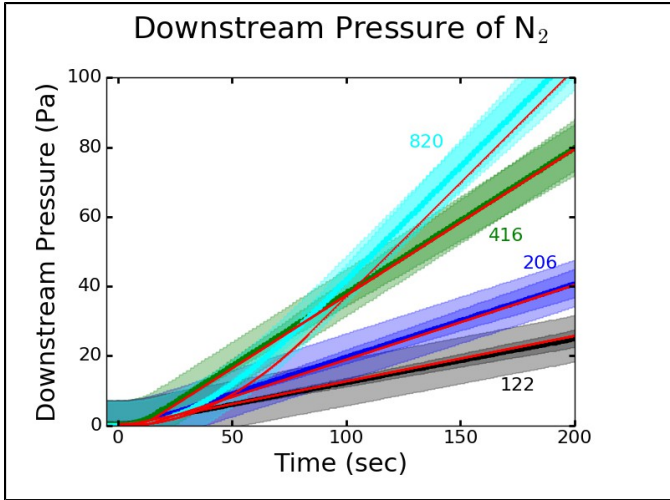
$$[gas_{(p)}] = S p_{up} \quad (10)$$

601where the proportionality constant, S , is the gas's Henry's law solubility over the pressure range
602in this study.⁴⁰ Figures 9a-c and 10 show the permeation curves from experiment and the
603corresponding simulations for N₂ and CO₂, respectively, through PDMS for upstream pressures
604from 122 to 820 kPa. While the experimental standard deviation for samples at higher pressures
605was not thoroughly investigated, the data are sufficient for comparison to simulation. The figures
606for N₂ and CO₂ are highlighted here because they represent the extremes in permeability among
607the gases tested; similar data for O₂ and CH₄ are available in the SI Section 11, Figure S8 and
608Table S3. The agreement for all times, gas types, and pressures is within experimental error,
609confirming that the simulation framework and the model represent the full permeation process

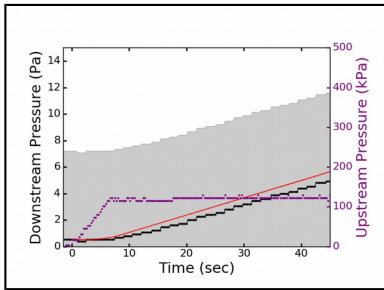
610over a wide range of conditions, Henry's law solubilities, and permeabilities. More details on
611implementation of Eq 10 in the model are presented in the SI Section 10, and relevant values are
612listed in Table S2. The error introduced by ignoring the dynamic solubility increase is illustrated
613in Figure 9d, in which the results of simulations with instantaneous and dynamic increases in
614available binding sites are compared. The simulation with a dynamic increase faithfully
615reproduces the pre-steady-state region of the permeation curve; the simulation with all binding
616sites instantly available only reproduces the steady-state region of the curve. This dynamic
617treatment is especially important for the more permeable gases or where the upstream pressure
618rise takes place over a long time.

619 The simulations add two new insights to dynamical aspects of the system. First, we find that
620inclusion of instantaneous increases in the maximum possible concentration of the permeant in
621the membrane as the permeant pressure increases on the upstream side is required for good
622agreement between the experimental and calculated pressure rise for all gases studied. This
623behavior would need to be incorporated in any new theoretical treatment of solution-diffusion in
624membranes far from steady state. It should be noted that while the computational treatment for
625single permeants is straightforward, it is likely to be more complex when time-varying mixtures
626are involved. Second, we find that the bulk PDMS diffusion coefficient is sufficient to describe
627gas motion at all locations within the polymer. This indicates that tortuosity, polymer chain
628fluctuations, and other detailed, molecular-level characteristics are well-captured by the Fickian
629diffusion description, and do not need to be included explicitly.

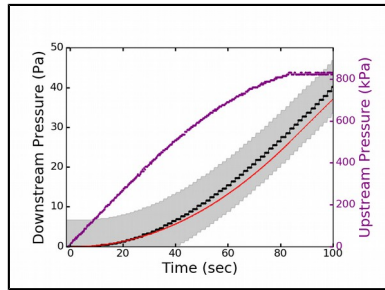
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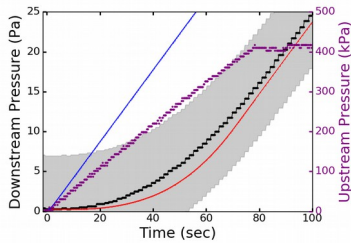
631(a)



632(b)



(c)



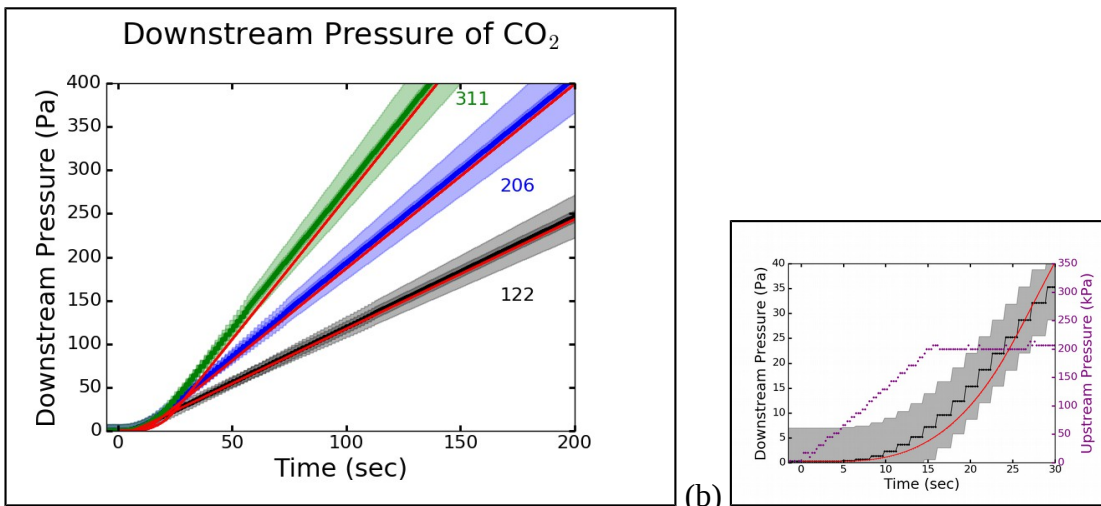
633(d)

634Figure 9. Comparison of simulation and experimental permeation curves for N₂. The simulation 635 results are shown in red. (a) Summary plot for all pressures. The shaded regions represent an 636 experimental sample-to-sample standard deviation of 10%. Each curve is labeled with the 637 upstream pressure in kPa. (b) Pre-steady state detail for $p_{up} = 122$ kPa (c) Pre-steady state detail 638 for $p_{up} = 820$ kPa. The black curves in (b) and (c) are experimental measurements, and the purple 639 curves are the measured upstream pressure rise. The shaded regions represent the accuracy of the 640 downstream pressure transducer. The accuracy of the pressure transducer is the main source of 641 error at low pressures; at higher pressures, the sample-to-sample variation is the main source of 642 error. The lower limit of detection of the downstream pressure detector is 0.1 Pa, resulting in 643 steps in the measured downstream pressure. (d) Comparison of simulation with and without a 644 dynamic increase in maximum absorbate concentration in the polymer. The experimental data for 645 N₂ permeation through sample L5S2 are in black. The upstream pressure rise is in purple. The 646 correct simulation, with a dynamic increase in maximum concentration within the polymer, is 647 shown in red. The simulation with all internal gas absorption sites available at the start of the 648 upstream pressure rise is shown in blue.

61

649

650



651(a)

(b)

652 Figure 10. Comparison of simulation and experiment permeation curves for CO₂. The simulation
 653 results are shown in red. (a) Summary plot for all pressures. The shaded regions represent an
 654 experimental sample-to-sample standard deviation of 10%. Each curve is labeled with the
 655 upstream pressure in kPa. (b) Pre-steady state detail for $p_{up} = 206$ kPa. The black curve is the
 656 experimental measurements, and the purple curve is the measured upstream pressure rise. The
 657 shaded region represents the accuracy of the downstream pressure transducer. The accuracy of
 658 the pressure transducer is the main source of error at low pressures; at higher pressures, the
 659 sample-to-sample variation is the main source of error. The lower limit of detection of the
 660 downstream pressure detector is 0.1 Pa, resulting in steps in the measured downstream pressure.

661

662 **VI. Application of the model: predictive simulations for two rubbery**
 663 **membrane separations systems**

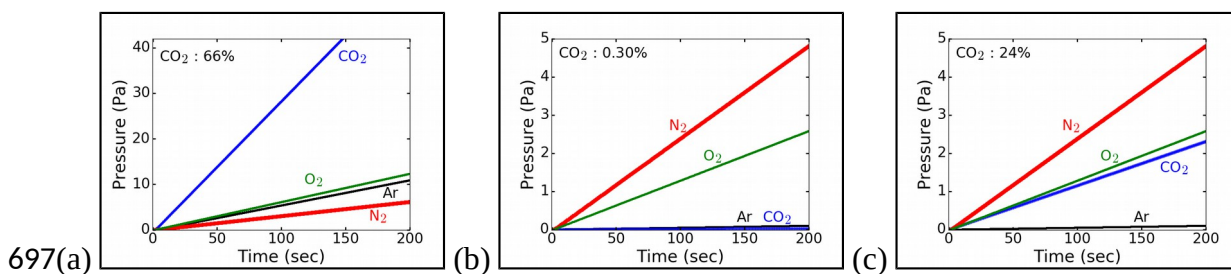
664 Two variations of the reaction-diffusion framework developed in this work are used to
 665 develop models that account for the entire experimentally observed permeation process, both
 666 steady-state and pre-steady-state, for phosphorescence quenching, and gas permeation through a
 667 membrane. The models have no adjustable parameters and are physically realistic, and therefore
 668 predictive. In this section we take one of the models, gas permeation through PDMS, to examine
 669 two other types of systems at an exploratory level: CO₂ separations from air, and solar CO₂
 670 photoelectrochemical reduction. We describe how the system models are set up, then present and
 671 discuss the predictions from the simulations.

672

673A. Simulations of CO₂ separation from air

674 There is currently much interest in use of permeable membranes for CO₂ capture from
 675 ambient air⁴⁹⁻⁵⁰ and we use the PDMS permeation model described above to examine how a
 676 membrane with its properties would function. Experimentally, gases have been shown to
 677 permeate independently in rubbers.⁵¹⁻⁵² Simulation of permeation of a mixture of N₂ and O₂,
 678 using the setup shown in Figure 8, agrees with experiment (see SI Sect. 12 and Fig. S9).⁵² We
 679 apply this model to simulate permeation of mixtures containing several components of dry air:
 680 N₂, O₂, Ar, and CO₂. All relevant input values for the simulations are reported in Table S4. In
 681 Figure 11a, it is shown that the selectivity for CO₂ in a mixture of gases, all at the same partial
 682 pressure, is near to its ideal selectivity ($\approx 10\times$ that of N₂ and $\approx 5\times$ that of O₂). However, the
 683 concentration of CO₂ in air is low, $p(\text{CO}_2) = 0.041 \text{ kPa}$.⁵³ If the concentrations of gases are set to
 684 their concentrations in air (80 kPa for N₂, 21 kPa for O₂, and 0.95 kPa for Ar),⁵⁴ the permeation
 685 of CO₂ nearly ceases, due to its extremely low partial pressure, as shown in Figure 11b. The
 686 simulations enable a target permeability for CO₂ to be estimated for a membrane that would
 687 selectively increase the CO₂ concentration in air. Assuming a CO₂ permeability value 100 times
 688 higher than for PDMS, with that for the other gases held constant, enrichment to 24% is
 689 predicted. This hypothetical situation is not achievable with a single component polymeric
 690 membrane because selectivity and permeability for gases of similar characteristics cannot be
 691 tuned independently.⁵⁵ To identify useful architectures, the model framework presented here can
 692 be extended to evaluate and compare new membrane design concepts prior to experimental
 693 study, such as layered and inorganic-organic composites and supported ionic liquids. Information
 694 on performance characteristics of these systems under intermittent conditions can be obtained
 695 from the simulations.⁵⁰

696



698 Figure 11. Downstream gas composition for upstream mixtures of O₂, N₂, Ar and CO₂ as a
699 function of time. (a) All four gases present at equal upstream partial pressure, resulting in a
700 downstream mixture highly enriched in CO₂. (b) Upstream gas mixture with the same
701 composition as air permeating PDMS; minimal CO₂ permeation is observed due to its low
702 upstream concentration. (c) Upstream gas mixture with the same composition as air permeating a
703 hypothetical polymer with CO₂ permeability 100 times greater than PDMS, resulting in the
704 downstream gas mixture containing 24% CO₂.

705

706 **B. Simulations of solar CO₂ reduction product crossover between electrode chambers** 707 **during a diurnal cycle**

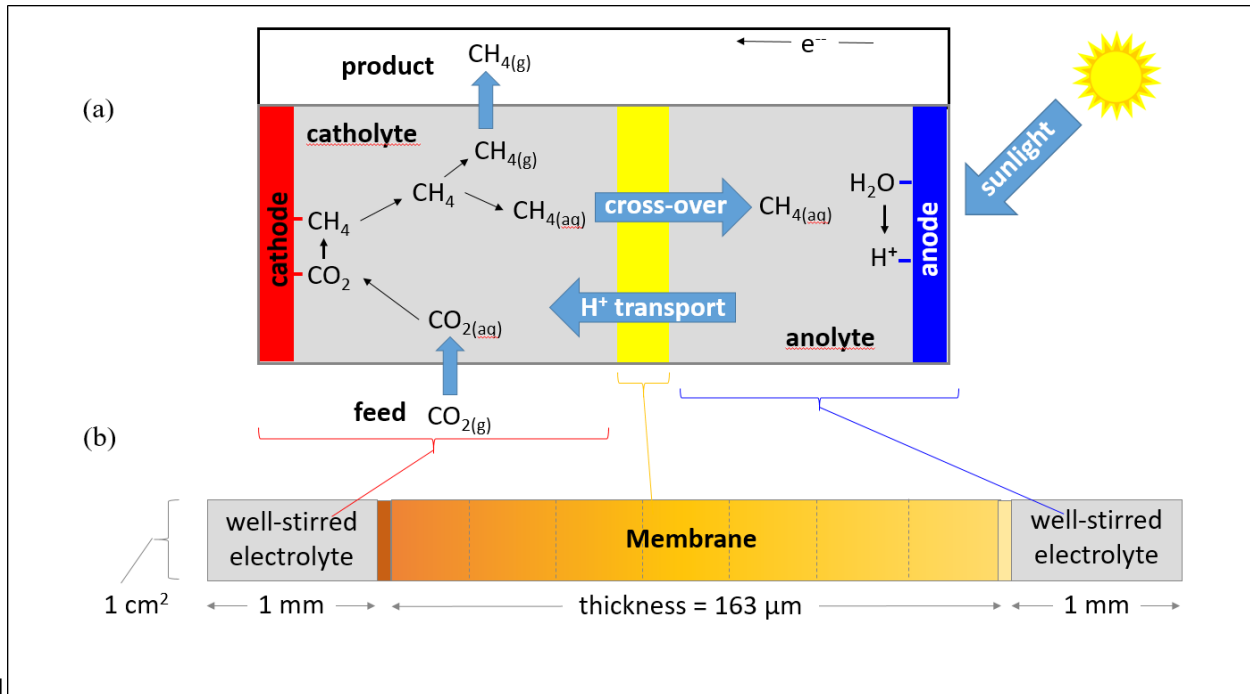
708 Solar fuels generators⁵⁶ utilize membranes to separate the cathode and anode regions, and
709 enable ion transfer between catholyte and anolyte during operation. For an example of the device
710 architecture, see Reference⁵⁷. Reduced products are formed on the cathode side, and oxygen is
711 formed on the anode side. The basic architecture of the commercial membranes used for solar
712 hydrogen generators is a phase-separated polyelectrolyte with glassy regions that are
713 impermeable to H₂ and O₂, typically dissolved in electrolyte or present as gas bubbles, and
714 hydrated channels with good ionic conductivity.⁵⁸⁻⁵⁹ A recent modeling study has shown that the
715 H₂ and O₂ product crossover-blocking property of a membrane for a photoelectrochemical
716 hydrogen generation system is crucial for efficient operation, and can be achieved by reducing
717 conductivity since the crossover path is typically through the hydrated ionic conduction
718 channels.⁶⁰

719 Solar photoelectrochemical devices that convert CO₂ into reduced hydrocarbon and
720 oxygenate products present a much more complicated situation. They usually generate mixtures
721 on the cathode side, where each product may influence the others' transport processes through
722 the membrane, and O₂ on the anode side, which can diffuse in the opposing direction. In a solar
723 CO₂ reduction device (illustrated in Figure 12a), the rate of product formation and the chemical
724 composition of the products are voltage-dependent. The cell voltage is determined by the amount
725 of sunlight incident on the light absorber in the device, varying with time of day, season, and
726 weather.⁶¹ It cannot be assumed that steady-state membrane operation is ever achieved; however
727 multiphysics models generally do so because of the lack of information about their non-steady-
728 state transport.⁶² The glassy matrix blocks significant permeant crossover, but exposure of glassy
729 polymers to CO₂ can cause them to plasticize, resulting in development of a rubbery character
730 over time with increased permeability, and product crossover between the electrodes'
731 chambers.^{55, 60} This will potentially reduce efficiency and selectivity of these systems. Modeling

732 studies examining optimum membrane characteristics for CO₂ reduction systems have not been
733 reported to our knowledge.

734 The model developed in the present work for PDMS permeated by CH₄, a prototypical CO₂
735 reduction product, allows aspects of varying production rate and cross-over to be examined.
736 Assumption of PDMS-like properties is a worst-case scenario, representing a membrane that has
737 been extensively plasticized in use. The primary modification of the kinetic processes in the
738 model is to change the simulated system from a gas-membrane system to an aqueous electrolyte-
739 membrane system. Under such conditions, the interfacial interactions change, but transport
740 within the membrane is unaltered if swelling and water uptake are negligible. The simulation
741 setup for this case is shown in Figure 12b. The surface adsorption rate constant k_{ads} (see Reaction
742 I) is calculated from the Smoluchowski equation for diffusion-controlled reactions, rather than as
743 a scattering process as in the gas-polymer case. Using the diffusion coefficient of methane in
744 water of $2.35 \times 10^{-9} \text{ m}^2/\text{s}$ (at 308 K)⁶³ and an interaction distance of 4 Å, which is twice the van
745 der Waals radius, k_{ads} is equal to $7 \times 10^9 \text{ M}^{-1} \text{ s}^{-1}$. The diffusion coefficient of CH₄ molecules
746 between electrolyte and interface compartments is set to 1 m²/s to represent a rapidly stirred
747 electrolyte. It is assumed that once the permeant has traversed the membrane, it does not back-
748 diffuse. It is also assumed that the catholyte is well mixed but not recirculated, allowing products to
749 accumulate.

750



751

752 Figure 12. Overview of a solar fuels device and its multi-scale model. (a) An example of a solar
 753 fuels device with a face-to-face architecture, in which the cathode, anode, and membrane all
 754 have the same area, and the membrane is separated from the catalyst layers by aqueous
 755 electrolyte. The reduction of CO_2 to CH_4 is shown, including electron transport, proton transport,
 756 carbon dioxide transport, carbon dioxide reduction reaction, and methane desorption, along with
 757 the possibilities for CH_4 to become hydrated and cross over the membrane or to enter gas phase
 758 and be collected in the product stream. Protons for the reduction reaction are generated at the
 759 anode via a water-splitting reaction. Details of the proton transport depend upon the pH, the
 760 buffer system, and the membrane type, and are omitted here. (b) In the multi-scale model, the
 761 catalysts are combined with their electrolyte regions to form a single well-stirred electrolyte
 762 compartment. The catholyte compartment contains a simplified version of all the chemistry
 763 occurring at the cathode surface, representing all the reactive steps and the desorption step by a
 764 single step for methane production. No chemistry occurs in the simulated anode region, which
 765 serves simply as a collection compartment for CH_4 in this work. The geometry is that of a typical
 766 solar fuels device prototype with 1 cm^2 cross-sectional area and 1 mm thick electrolyte
 767 compartments. The interfacial regions between electrolyte and membrane are 1 nm thick.

768

769 If only CH_4 is produced at a typical current density for a solar photoelectrochemical device of
 770 0.010 A/cm^2 , a production rate of $13 \times 10^{-9} \text{ mol}/(\text{cm}^2 \text{ s})$ is expected assuming the catalyst area is
 771 equal to the geometric area of the cathode.⁶⁴ For the combined cathode-catholyte region used in
 772 the simulations, the CH_4 production rate per area converts to a volumetric production rate of χ_{prod}
 773 $= 1.30 \times 10^{-4} \text{ mol}/(\text{L s})$. Because photocurrent scales with time of day, the rate of CH_4 production
 774 would be at its maximum only when insolation is at its maximum.⁶¹ For simplicity, this diurnal

775 variation is described as a simple triangle wave lasting a total of 8 hours, shown in Figure 13a.
 776 An extended period of darkness follows during which no additional CH₄ is produced but that
 777 which remains dissolved in the electrolyte can permeate the membrane. The CH₄ production
 778 reaction step is implemented as a pseudo-0th order reaction



779 where $k'_{prod} = r_{sun} \times \chi_{prod}$, and r_{sun} is the ratio of current sunlight level to the maximum amount of
 780 sunlight. This representation is used because it allows a correct generation rate without
 781 specifying detailed surface kinetics. Once formed, methane may dissolve up to its solubility limit
 782 in water of 0.001 M (for 308 K),⁶⁵ represented as an aqueous solution site, “aq-site,”



783 or bubble out of solution as a gas



784 where the zeroth order rate constants (now equal to the rates) for hydration, $k_{hyd} = 10^{50}$ mol/(L s),
 785 and bubbling out, $k_{bub} = 10^{25}$ mol/(L s), are arbitrarily high to represent nearly instantaneous
 786 processes. The larger rate constant for hydration ensures that the methane molecules remain in
 787 water if an aqueous site is available; if no site is available, the methane molecule will bubble out
 788 of solution instead. It is assumed that the electrolyte is rapidly stirred such that boundary layer
 789 and mass transport limitations are negligible.

790 The transport properties of CH₄ in PDMS and two hypothetical polymers (Hyp1 and Hyp2)
 791 are listed in Table 4. Both hypothetical polymers have a permeability three orders of magnitude
 792 lower than PDMS. For Hyp1, all reduction in permeability comes from a reduction in the
 793 diffusivity; for Hyp2, all reduction in permeability comes from a reduction in the solubility. As a
 794 point of reference, the values of D and S for Hyp1 are similar to those for polystyrene.⁶⁶

795

Polymer	S (mol/L)/(mol/L) (a)	D m ² /s	P_m m ² /s
PDMS (b)	0.467	2.20×10^{-9}	1.03×10^{-9}
Hyp1	0.467	2.20×10^{-12}	1.03×10^{-12}

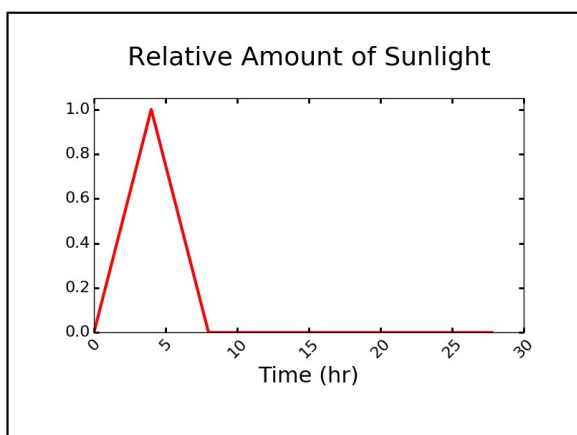
Hyp2	4.67×10^{-4}	2.20×10^{-9}	1.03×10^{-12}
------	-----------------------	-----------------------	------------------------

796(a) The units for solubility are mol/L inside of the polymer per mol/L in the external electrolyte.

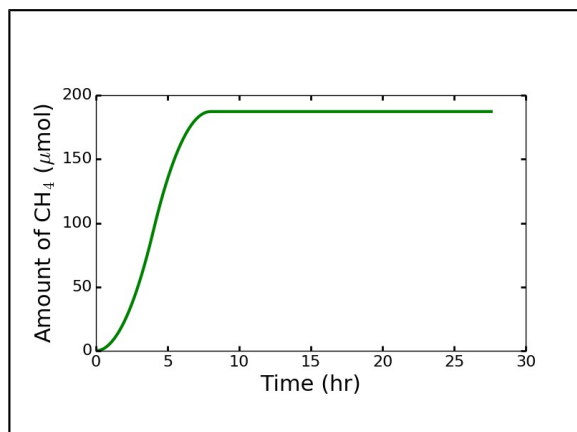
797(b) Reference ⁴⁰

798Table 4. Properties of the membranes used in the diurnal cycle simulations.

799



800(a)



801(b)

802Figure 13. The production of CH₄. (a) The normalized level of sunlight, which is a multiplier for
803the rate constant for CH₄ production. (b) The total amount of CH₄ produced from Reaction VII,
804including both those that will become gaseous and those that will become hydrated.

805

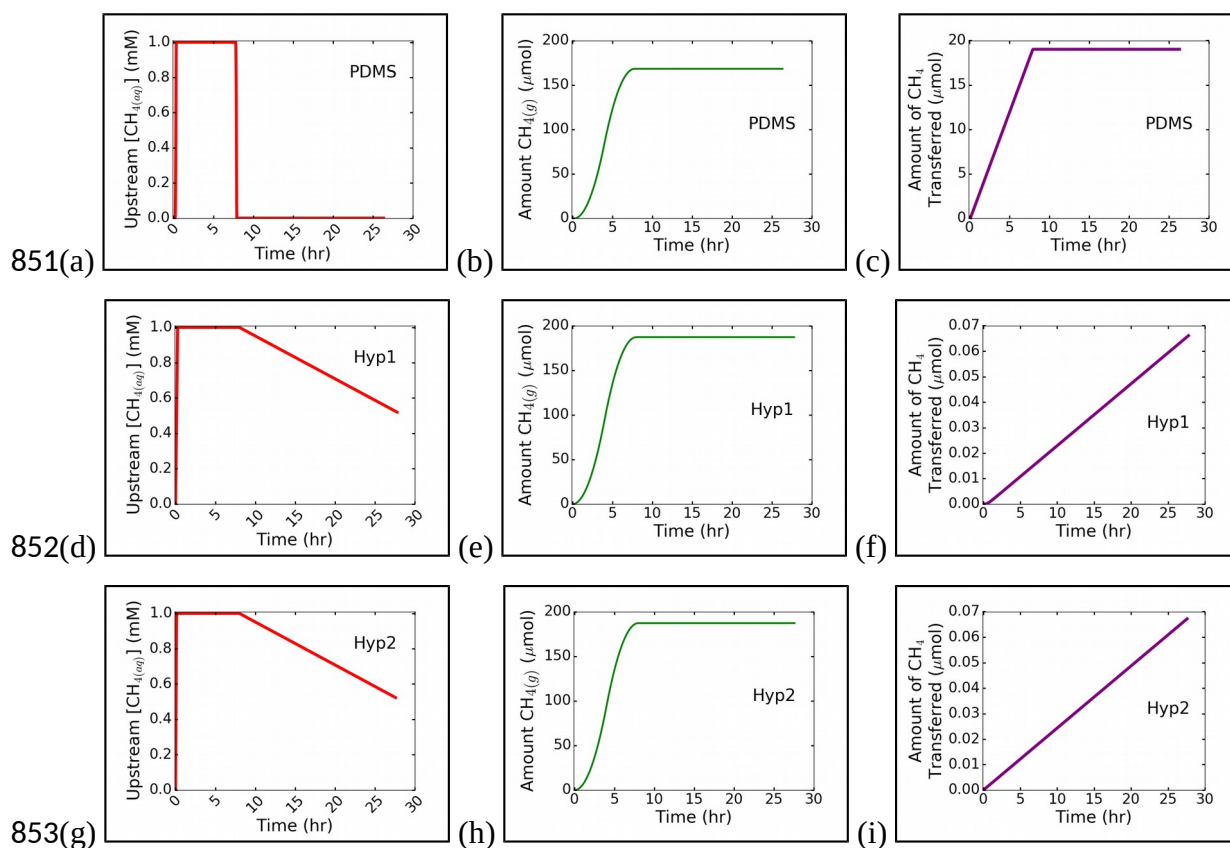
806 The amount of CH₄ produced from Reaction VII over time is shown in Figure 13b and is the
807same for all three simulations. The rate of CH₄ production increases over the first four hours,
808reaches its maximum at the peak of available sunlight, then declines over the next four hours,
809resulting in a total production of 1.87×10^{-4} moles of CH₄. Because the solubility limit for gases

810in water is low, a constant aqueous concentration of CH_4 is rapidly established, as shown for
811PDMS, Hyp1, and Hyp2 in Figure 14a, 14d, and 14g, respectively. Additional CH_4 produced
812bubbles out as a gaseous product, as shown for PDMS, Hyp1, and Hyp2 in Figure 14b, 14e, and
81314h, respectively. For a theoretical, completely blocking membrane, the maximum aqueous
814concentration is reached in 7.8 minutes. Hyp1 and Hyp2 come close to this, with aqueous phase
815saturation occurring within 10 minutes, at which time, sunlight is less than 10% of its maximum
816level. The concentration profiles of CH_4 within all three membranes are available in the SI
817Section 13, Figure S10. The concentration profile of CH_4 within the hypothetical membranes
818also reaches steady state within 10 minutes. In PDMS, these processes take 20 minutes. The
819delay in reaching steady state, compared to the hypothetical membranes, is due to the high
820permeability of PDMS. As CH_4 is entering the aqueous phase near the cathode, it is also being
821removed by transport across the membrane, thus delaying the build-up to its maximum
822concentration. However, this build-up is only a small fraction of the device's operating time.

823 Once CH_4 production ceases, it takes only about 1 minute for the remaining CH_4 to traverse
824the PDMS membrane into the anode chamber. This means that for a very permeable membrane,
825any CH_4 present in the aqueous phase will cross over the membrane. In contrast, the depletion of
826 CH_4 from the catholyte and emptying of the remaining CH_4 within the membrane is estimated to
827take over 40 hours for the less permeable membranes. In this case, the enhanced blocking
828capabilities of these membranes are due not only to lower steady-state permeability but also
829retention of product within the membrane during dark hours, as can be seen in the concentration
830profiles in the SI Section 13, Figure S10. The membrane is a key contributor to the ability to
831recover products from solar fuels devices. The maximum possible amount of product that can be
832recovered is equal to the total amount of gas produced, shown for PDMS, Hyp1, and Hyp2 in
833Figure 14b, 14e, and 14h, respectively. Conversely, the total amount of methane cross-over for
834PDMS, Hyp1, and Hyp2 are shown in Figure 14c, 14f, and 14i, respectively. For PDMS, $\approx 10\%$
835of CH_4 crosses the membrane. Because CH_4 is highly permeable in PDMS, 25% represents an
836approximate upper bound on gaseous product loss, though products with higher solubility in
837water may experience even greater losses. By reducing the permeability by three orders of
838magnitude, recovery can be increased to 99.96%. It does not matter whether this decrease in
839permeability results from changes in diffusivity or solubility. Even with low permeability,
840however, it is unlikely that product cross-over can be prevented completely.

841 This is an interesting situation because the solar-fuels device is operating under continually
 842 changing conditions, so the rate of methane production is changing continually as well. However,
 843 the aqueous concentration of methane is constant over most of the operating time, and the
 844 membrane is essentially experiencing steady-state conditions with respect to this permeant
 845 during daylight hours. Using a more realistic insolation profile or including the fluctuations in
 846 sunlight that are present under real-world operating conditions will have little effect on the
 847 operation of the membrane, at least for the conditions studied herein. Fluctuations in sunlight
 848 levels may play a greater role in other types of systems, for example, when the production rate is
 849 lower, the aqueous solubility is higher, or the permeability is higher.

850



854 Figure 14. Results of membrane crossover simulations during diurnal photoelectrochemical CH₄
 855 production. (a) – (c) PDMS, (d) – (f) Hyp1, (g) – (i) Hyp2. (a), (d), (g) Upstream concentration
 856 of dissolved CH₄ in the cathode chamber stays at the aqueous solubility limit during insolation.
 857 The remaining CH₄ is rapidly depleted by crossing the PDMS membrane but is slowly depleted
 858 for the two hypothetical membranes due to their low permeability. (b), (e), (h) Amount of CH₄
 859 that enters the gas phase and can be collected as a product. (c), (f), (i) Amount of CH₄ that

860 crosses over the membrane and so is not collected as product. The results for the two
861 hypothetical membranes are nearly identical due to their identical permeabilities.

862

86VII. Summary and conclusions

864 In this work we describe a multiscale modeling study of permeation of rubbery polymer
865 membranes by simple inert solutes, with the goal of developing a full description of steady-
866 state and non-steady-state processes. We have chosen to focus on these systems as a simplest
867 case since the polymer transport properties, solubility and diffusivity, are expected to be
868 constant over time. We have confirmed that the permeation process is insensitive to permeant
869 adsorption and absorption kinetics in these systems, consistent with the assumption made in
870 the solution-diffusion model for membrane permeation. Two quantitatively validated models,
871 built on a common reaction-diffusion framework, are reported: quenching of PtOEP
872 phosphorescent dye in C4PTP, an oxygen sensor system, and permeation of O₂, N₂, CH₄ and
873 CO₂ through a PDMS separator. The predictive framework is extended to simulate two
874 additional hypothetical systems, separation of CO₂ from air using a PDMS-like membrane,
875 and crossover of the CO₂ reduction product, CH₄, in a solar-fuels system whose membrane
876 separator is rubbery or glassy, but inert to the permeant.

877 Because the goal is the development of a quantitatively predictive model with no
878 adjustable parameters, the model construction process is dependent on time-dependent
879 permeation data as well as experimental and theoretical information on specific materials
880 properties and processes. Such data are not commonly available in the literature so
881 experimental and MD computational studies have been performed and are reported in this
882 paper. The results show that a simple reaction-diffusion framework simulated using
883 stochastic methods is adequate to capture the details of interfacial processes as well as bulk
884 processes, covering a broad range of length- and timescales. Measured quantities such as
885 solubilities and diffusion coefficients contain enough information about solute-polymer
886 interactions to predict experimental observables such as pressure increases and
887 phosphorescence quenching as investigated here without explicit inclusion of free volume
888 dynamics and tortuosity, although these elements can be built into the model with
889 straightforward modifications.

890 Several new insights to polymer-permeant systems have been obtained through the
891 simulations.

892(i) Time-dependent phosphorescence quenching is related in a complicated way to the
893 amount of O₂ sorbed into a polymer due to the rate mismatch between dye-O₂ interactions
894 and O₂ transport. This suggests that such sensors are best used in conjunction with
895 models such as that presented here to ensure complete data interpretation.

896(ii) The pre-steady state portion of a gas permeation curve contains a great deal of
897 information about the nature of the gas-polymer interaction. In particular, we find it
898 necessary to include a dynamically increasing maximum concentration within the
899 polymer as the upstream permeant pressure increases, in accordance with Henry's law
900 solubility, to reproduce the experimental data successfully. This raises interesting
901 questions about what is occurring inside the polymer during a pressure rise, especially
902 how interfacial processes affect the polymer bulk, and how fast this response is.

903(iii) During intermittent operation such as a diurnal cycle, low-permeability membranes can
904 continue to permit crossover long after product formation has ceased due to slow
905 permeation of residual products in the electrolyte. This effect needs to be taken into
906 account for successful membrane and separation systems design for complex situations
907 such as solar fuels generators and CO₂ capture.

908

909

910

911 **Declaration of interest**

912 Conflicts of interest: none

913

914 **Supplementary Information**

915 Supplementary data related to this article can be found at [http:// link](http://link)

916 Data tables for all figures and additional experimental data can be found in the spreadsheets at
917 <https://doi.org/10.5072/FK2Z31W208>

918

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932

933References

934

9351. Mulder, M., *Basic Principles of Membrane Technology*. Kluwer Academic Publishers: 936Dordrecht, The Netherlands, 1996.
9372. Wijmans, J. G.; Baker, R. W., The Solution-Diffusion Model - a Review. *J Membrane Sci* 938**1995**, *107* (1-2), 1-21.
9393. Crank, J.; Park, G. S., *Diffusion in Polymers*. Academic Press Inc., Ltd.: London and New 940York, 1968.
9414. Shin, D. W.; Guiver, M. D.; Lee, Y. M., Hydrocarbon-Based Polymer Electrolyte 942Membranes: Importance of Morphology on Ion Transport and Membrane Stability. *Chem Rev* 943**2017**, *117* (6), 4759-4805.
9445. Kusoglu, A.; Weber, A. Z., Study of PFSA ionomers using X-ray scattering techniques. 945*Abstr Pap Am Chem S* **2014**, 248.
9466. Jayarajah, C. N. Luminescence Quenching Studies of Oxygen Diffusion in Highly 947Permeable Media. University of Toronto, 1998.
9487. Jayarajah, C. N.; Yekta, A.; Manners, I.; Winnik, M. A., Oxygen diffusion and 949permeability in alkylaminothionylphosphazene films intended for phosphorescence barometry 950applications. *Macromolecules* **2000**, *33* (15), 5693-5701.
9518. Wang, X.-d.; Wolfbeis, O. S., Optical methods for sensing and imaging oxygen: 952materials, spectroscopies and applications. *Chemical Society Reviews* **2014**, *43*, 3666-761.
9539. Bunker, D.; Garrett, B.; Kleindienst, T.; III, G. L., Discrete simulation methods in 954combustion kinetics. *Combustion and Flame* **1974**, *23*, 373-379.
95510. Gillespie, D. T., General Method for Numerically Simulating Stochastic Time Evolution 956of Coupled Chemical-Reactions. *J Comput Phys* **1976**, *22* (4), 403-434.
95711. Hinsberg, W. D.; Houle, F. A., *Kinetiscope*. www.hinsberg.net/kinetiscope. , 2015.
95812. Houle, F. A.; Hinsberg, W. D.; Morrison, M.; Sanchez, M. I.; Wallraff, G.; Larson, C.; 959Hoffnagle, J., Determination of coupled acid catalysis-diffusion processes in a positive-tone 960chemically amplified photoresist. *Journal of Vacuum Science & Technology B: Microelectronics* 961*and Nanometer Structures* **2000**, *18*, 1874.
96213. Houle, F. A.; Hinsberg, W. D.; Sanchez, M. I., Kinetic Model for Positive Tone Resist 963Dissolution and Roughening. *Macromolecules* **2002**, *35*, 8591-8600.
96414. Wiegel, A. A.; Wilson, K. R.; Hinsberg, W. D.; Houle, F. A., Stochastic methods for 965aerosol chemistry: a compact molecular description of functionalization and fragmentation in the 966heterogeneous oxidation of squalane aerosol by OH radicals. *Phys. Chem. Chem. Phys.* **2015**, *17*, 9674398-4411.
96815. Wiegel, A. A.; Liu, M.; Hinsberg, W. D.; Wilson, K. R.; Houle, F. A., Diffusive 969Confinement of Free Radical Intermediates in the OH Radical Oxidation of Semisolid Aerosol. 970*Physical Chemistry Chemical Physics* **2017**, *19*, 6814-6830.
97116. Wallraff, G.; Hutchinson, J.; Hinsberg, W.; Houle, F.; Seidel, P.; Johnson, R.; Oldham, 972W., Thermal and acid-catalyzed deprotection kinetics in candidate deep ultraviolet resist 973materials. *Journal of Vacuum Science & Technology B: Microelectronics and Nanometer* 974*Structures* **1994**, *12* (6), 3857-3857.
97517. Houle, F. A.; Hinsberg, W. D.; Sanchez, M. I.; Hoffnagle, J. A., Influence of resist 976components on image blur in a patterned positive-tone chemically amplified photoresist. *Journal*

- 977of *Vacuum Science & Technology B: Microelectronics and Nanometer Structures* **2002**, 20 (3), 978924-924.
97918. Bowers, K.; Chow, E.; Xu, H.; Dror, R.; Eastwood, M.; Gregersen, B.; Klepeis, J.; 980Kolossvary, I.; Moraes, M.; Sacerdoti, F.; Salmon, J.; Shan, Y.; Shaw, D., Scalable Algorithms 981for Molecular Dynamics Simulations on Commodity Clusters. *ACM/IEEE SC 2006 Conference* 982(SC'06) **2006**, (November), 43-43.
98319. Shivakumar, D.; Williams, J.; Wu, Y.; Damm, W.; Shelley, J.; Sherman, W., Prediction of 984Absolute Solvation Free Energies using Molecular Dynamics Free Energy Perturbation and the 985OPLS Force Field. *Journal of Chemical Theory and Computation* **2010**, 6, 1509-1519.
98620. Guo, Z.; Mohanty, U.; Noehre, J.; Sawyer, T. K.; Sherman, W.; Krilov, G., Probing the α - 987Helical Structural Stability of Stapled p53 Peptides: Molecular Dynamics Simulations and 988Analysis. *Chem. Biol. Drug Des.* **2010**, 75, 348-359.
98921. Banks, J. L.; Beard, H. S.; Cao, Y.; Cho, A. E.; Damm, W.; Farid, R.; Felts, A. K.; 990Halgren, T. A.; Mainz, D. T.; Maple, J. R.; Murphy, R.; Philipp, D. M.; Repasky, M. P.; Zhang, L. 991Y.; Berne, B. J.; Friesner, R. A.; Gallicchio, E.; Levy, R. M., Integrated Modeling Program, 992Applied Chemical Theory (IMPACT). *Journal of Computational Chemistry* **2005**, 26 (16), 1752- 9931780.
99422. McQuarrie, D. A., *Statistical Mechanics*. University Science Books: Mill Valley, CA, 9952000; p 641.
99623. Martyna, G. J.; Tobias, D. J.; Klein, M. L., Constant pressure molecular dynamics 997algorithms. *Journal of Chemical Physics* **1994**, 101 (5), 4177-4177.
99824. Shin, H.; Pascal, T. A.; Goddard, W. A.; Kim, H., Scaled effective solvent method for 999predicting the equilibrium ensemble of structures with analysis of thermodynamic properties of 1000amorphous polyethylene glycol-water mixtures. *Journal of Physical Chemistry B* **2013**, 117 (3), 1001916-927.
100225. *Polymer Data Handbook*. Oxford University Press: 1999; p 1012.
100326. Willard, A. P.; Chandler, D., Instantaneous liquid interfaces. *Journal of Physical* 1004*Chemistry B* **2010**, 114 (5), 1954-1958.
100527. Julin, J.; Shiraiwa, M.; Miles, R. E. H.; Reid, J. P.; Pöschl, U.; Riipinen, I., Mass 1006accommodation of water: Bridging the gap between molecular dynamics simulations and kinetic 1007condensation models. *Journal of Physical Chemistry A* **2013**, 117 (2), 410-420.
100828. Julin, J.; Winkler, P. M.; Donahue, N. M.; Wagner, P. E.; Riipinen, I., Near-Unity Mass 1009Accommodation Coefficient of Organic Molecules of Varying Structure. *Environmental Science* 1010& *Technology* **2014**, 48, 12083-12089.
101129. *Springer Handbook of Materials Measurement Methods*. Springer: Berlin, 2006; p 1207.
101230. "Model Super TJE: Ultra Precision Pressure Transducer." HoneyWell.
- 1013[https://measurementsensors.honeywell.com/Pages/Product.aspx?](https://measurementsensors.honeywell.com/Pages/Product.aspx?category=&cat=Honeywell&pid=stje) 1014[category=&cat=Honeywell&pid=stje](https://measurementsensors.honeywell.com/Pages/Product.aspx?category=&cat=Honeywell&pid=stje) (accessed March 24, 2017).
101531. "Baratron General Purpose Capacitance Manometers: 750C11T." MKS Instruments. 1016<https://www.mksinst.com/product/product.aspx?ProductID=126> (accessed March 24, 2017).
101732. Hodge, K.; Prodpran, T.; Shenogina, N. B.; Nazarenko, S., Diffusion of oxygen and 1018carbon dioxide in thermally crystallized syndiotactic polystyrene. *Journal of Polymer Science* 1019*Part B: Polymer Physics* **2001**, 39, 2519-2538.
102033. Bansal, A. K.; Holzer, W.; Penzkofer, A.; Tsuboi, T., Absorption and emission 1021spectroscopic characterization of platinum-octaethyl-porphyrin (PtOEP). *Chemical Physics* 1022**2006**, 330 (1-2), 118-129.

102334. Engel, T.; Reid, P., *Thermodynamics, Statistical Thermodynamics, & Kinetics*. 2nd ed.; 1024Prentice Hall: 2010; p 602.
102535. Somorjai, G. A.; Li, Y., *Introduction to Surface Chemistry and Catalysis*. Wiley: 2010; p 1026800.
102736. Atkins, P.; de Paula, J., *Physical Chemistry*. 8th ed.; W. H. Freeman and Co.: New York, 1028USA, 2006; p 1053.
102937. Houston, P., *Chemical kinetics and reaction dynamics*. Dover Publications, Inc.: Mineola, 1030New York, 2001; p 518.
103138. Dienel, T.; Proehl, H.; Fritz, T.; Leo, K., Novel near-infrared photoluminescence from 1032platinum(II)-porphyrin (PtOEP) aggregates. *Journal of Luminescence* **2004**, *110*, 253-257.
103339. Yekta, A.; Masoumi, Z.; Winnik, M. A., Luminescence measurements of oxygen 1034permeation and oxygen diffusion in thin polymer films. *Canadian Journal of Chemistry* **1995**, *73* 1035(11), 2021-2029.
103640. Merkel, T. C.; Bondar, V. I.; Nagai, K.; Freeman, B. D.; Pinnau, I., Gas sorption, 1037diffusion, and permeation in poly(dimethylsiloxane). *Journal of Polymer Science, Part B: 1038Polymer Physics* **2000**, *38* (3), 415-434.
103941. Brandão, L.; Madeira, L. M.; Mendes, A. M., Mass transport on composite dense PDMS 1040membranes with palladium nanoclusters. *Journal of Membrane Science* **2007**, *288* (1-2), 112- 1041122.
104242. Berean, K.; Ou, J. Z.; Nour, M.; Latham, K.; McSweeney, C.; Paull, D.; Halim, A.; 1043Kentish, S.; Doherty, C. M.; Hill, A. J.; Kalantar-zadeh, K., The effect of crosslinking 1044temperature on the permeability of PDMS membranes: Evidence of extraordinary CO₂ and CH₄ 1045gas permeation. *Separation and Purification Technology* **2014**, *122*, 96-104.
104643. Lancaster, D. K.; Johnson, A. M.; Burden, D. K.; Wiens, J. P.; Nathanson, G. M., Inert 1047gas scattering from liquid hydrocarbon microjets. *Journal of Physical Chemistry Letters* **2013**, *4* 1048(18), 3045-3049.
104944. Lu, J. W.; Morris, J. R., Gas-Surface scattering dynamics of CO₂, NO₂, and O₃ in 1050collisions with model organic surfaces. *Journal of Physical Chemistry A* **2011**, *115*, 6194-6201.
105145. Alexander, W. a.; Zhang, J.; Murray, V. J.; Nathanson, G. M.; Minton, T. K., Kinematics 1052and dynamics of atomic-beam scattering on liquid and self-assembled monolayer surfaces. 1053*Faraday Discussions* **2012**, *157*, 355-374.
105446. Donaldson, D. J., Adsorption of Atmospheric Gases at the Air-Water Interface. I. NH₃. 1055*Journal of Physical Chemistry A* **1999**, *103* (1), 62-70.
105647. Ogasawara, H.; Horimoto, N.; Kawai, M., Ammonia adsorption by hydrogen bond on ice 1057and its solvation. *Journal of Chemical Physics* **2000**, *112* (19), 8229-8232.
105848. Yampolskii, Y. P.; Pinnau, I.; Freeman, B. D., *Materials Science of Membranes for Gas 1059and Vapor Separation*. John Wiley & Sons Ltd: West Sussex, England, 2006; p 445-445.
106049. House, K. Z.; Baclig, A. C.; Ranjan, M.; van Nierop, E. A.; Wilcox, J.; Herzog, H. J., 1061Economic and energetic analysis of capturing CO₂ from ambient air. *Proceedings of the 1062National Academy of Sciences* **2011**, *108* (51), 20428-20433.
106350. Jones, C. W., CO₂ capture from dilute gases as a component of modern global carbon 1064management. *Annual review of chemical and biomolecular engineering* **2011**, *2*, 31-52.
106551. Dhingra, S. S.; Marand, E., Mixed gas transport study through polymeric membranes. 1066*Journal of Membrane Science* **1998**, *141*, 45-63.
106752. Wu, F.; Li, L.; Xu, Z.; Tan, S.; Zhang, Z., Transport study of pure and mixed gases 1068through PDMS membrane. *Chemical Engineering Journal* **2006**, *117*, 51-59.

106953. "Recent Global CO₂." Earth System Research Laboratory Global Monitoring Division. 1070<https://www.esrl.noaa.gov/gmd/ccgg/trends/global.html> (accessed June 16, 2017).
107154. IUPAC, *Compendium of Chemical Terminology, 2nd ed.* Blackwell Scientific Publications: Oxford, 1997; Vol. 2017.
107355. Sanders, D. F.; Smith, Z. P.; Guo, R.; Robeson, L. M.; McGrath, J. E.; Paul, D. R.; 1074Freeman, B. D., Energy-efficient polymeric gas separation membranes for a sustainable future: A 1075review. *Polymer* **2013**, *54* (18), 4729-4761.
107656. Xiang, C.; Weber, A. Z.; Ardo, S.; Berger, A.; Chen, Y. K.; Coridan, R.; Fountaine, K. T.; 1077Haussener, S.; Hu, S.; Liu, R.; Lewis, N. S.; Modestino, M. A.; Shaner, M. M.; Singh, M. R.; 1078Stevens, J. C.; Sun, K.; Walczak, K., Modeling, Simulation, and Implementation of Solar-Driven 1079Water-Splitting Devices. *Angewandte Chemie - International Edition* **2016**, *55* (42), 12974- 108012988.
108157. Singh, M. R.; Clark, E. L.; Bell, A. T., Effects of electrolyte, catalyst, and membrane 1082composition and operating conditions on the performance of solar-driven electrochemical 1083reduction of carbon dioxide. *Physical Chemistry Chemical Physics* **2015**, *17* (29), 18924-18936.
108458. Allen, F. I.; Comolli, L. R.; Kusoglu, A.; Modestino, M. A.; Minor, A. M.; Weber, A. Z., 1085Morphology of hydrated as-cast Nafion revealed through cryo electron tomography. *ACS Macro 1086Letters* **2015**, *4* (1), 1-5.
108759. Kusoglu, A.; Weber, A. Z., New Insights into Perfluorinated Sulfonic-Acid Ionomers. 1088*Chemical Reviews* **2016**, *117*, 987-1104.
108960. Berger, A.; Segalman, R. A.; Newman, J., Material requirements for membrane separators 1090in a water-splitting photoelectrochemical cell. *Energy & Environmental Science* **2014**, *7* (4), 10911468-1476.
109261. Chen, Y.; Lewis, N. S.; Xiang, C., Modeling and Simulation of the Spatial and Light- 1093Intensity Dependence of Product Distributions in an Integrated Photoelectrochemical CO₂ 1094Reduction System. *ACS Energy Letters* **2016**, *1* (1), 273-280.
109562. Stevens, J. C.; Weber, A. Z., A Computational Study of Optically Concentrating, Solar- 1096Fuels Generators from Annual Thermal Fuel-Production Efficiency Perspectives. *Journal of 1097The Electrochemical Society* **2016**, *163* (7), H475-H484.
109863. Jähne, B.; Heinz, G.; Dietrich, W., Measurement of the diffusion coefficients of sparingly 1099soluble gases in water. *Journal of Geophysical Research: Oceans* **1987**, *92* (C10), 10767-10776.
110064. Clark, E. L., personal communication. 2016.
110165. Clever, H. L.; Young, C. L., "Methane." *IUPAC Solubility Data Series*. Pergamon Press: 1102Oxford, England, 1987; Vol. 27/28.
110366. Rein, D. H.; Baddour, R. F.; Cohen, R. E., Gas solubility and diffusion in a polystyrene- 1104polybutadiene block copolymer. *Journal of Applied Polymer Science* **1992**, *45* (7), 1223-1227.
- 1105