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Theoretical Foundations for Centrality Measures[1]

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Three measures of actors' network centrality are derived from an elementary process model of social influence. The measures are closely related to, and cast new light on, widely used measures of actors' centrality; for example, the essential social organization of status that has been assumed by Hubbell, Bonacich, Coleman, and Burt appears as a deducible outcome of this social influence process. Unlike previous measures, which have been viewed as competing alternatives, the present measures are complementary and, in their juxtaposition, provide for a rich description of social structure. The complementarity indicates a degree of theoretical unification in the work on network centrality that was heretofore unsuspected.

INTRODUCTION

J. A. Barnes, an eminent social anthropologist, suggested that the theoretical foundations of research on social networks were rudimentary: "There is no such thing as a theory of social networks; perhaps there never will be. The basic idea behind the metaphorical and the analytic uses of social networks—that the configuration of cross-cutting interpersonal bonds is in some unspecified way causally connected with the actions of these persons and with the social institutions of their society—this remains a basic idea and nothing more. It constitutes what Homans calls an 'orienting statement' rather than a theory with propositions that can be tested" (1972, p. 2).

Other prominent structuralists have echoed Barnes's thoughts (Alba

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1982; Granovetter 1979; Rogers 1987; Wellman 1983). Such criticism, designed to be provocative, somewhat exaggerates the circumstances of the field. In fact, in the development and use of measures of actors' positions in social networks, respectable advances have been made toward a formal theory of social network mechanisms. An important part of this enterprise, and the focus of this article, has been the development of simple numerical indices, so-called measures of network centrality or sociometric status, that describe actors' positions in terms of features of their network environments.[2] These measures have entered into noteworthy empirical studies revolving especially on the distribution of power and influence among individual and collective actors (Marsden and Laumann 1984).

Much of the current thinking about actors' network centrality has been defined by the work of Freeman (1979) and Bonacich (1972*a*).[3] Bonacich's measure of centrality, which is closely related to Hubbell's (1965) measure of sociometric status, Coleman's (1973) measure of power, and Burt's (1982) measure of prestige, has been widely employed in the burgeoning literature on collective actors in interorganizational networks. Freeman's influential analysis and consolidation of the extant literature on centrality have provided a framework for a large number of investigations on power and influence in informal communication networks.

The extant measures of centrality appear to provide competing hypotheses about the relationship between particular structural features of a network and actors' behavior, opinions, or interpersonal influences. Without exception, the proposed hypotheses are not derived from any broader theory but are ad hoc formalizations of plausible ideas. Freeman (1979, p. 217) is properly circumspect about the limitations of such intuitive foundations: "Ideally, measures should grow out of advanced theoretical efforts; they should be defined in the context of explicit process models. Before such models can be developed, however, a certain amount of conceptual specification is necessary; the basic parameters of the problem must be set down. Thus, the introduction of measures in

[2] For evaluation of these efforts see Freeman, Roeder, and Mulholland (1980) and Bolland (1988). Numerous centrality measures have been proposed since the seminal work of Bavelas (1948); among the recent proposals are those of Doreian (1986) and Stephenson and Zelen (1989).

[3] There is another line of work on centrality in exchange networks with restricted flows of resources (Cook et al. 1983; Cook, Gillmore, and Yamagishi 1986; Markovsky, Willer, and Patton 1988; Willer 1986; Marsden 1983). Although Bonacich (1987) suggests that these two lines may be integrated, I do not deal with this possibility in this article.

the present context must be understood simply as a way of clarifying the centrality concept."

Social process foundations are preferred on intellectual grounds. It is satisfying to work with measures that have both clear-cut and elementary theoretical foundations. The appropriate interpretation of measures that lack such foundations is often ambiguous. Social process foundations provide a clarity of meaning but, in doing so, restrict applicability; measures that have been derived from a social process can only be meaningfully applied to situations in which the social process occurs. Hence, measures that have been derived from a particular social process do not necessarily supplant other measures that may have a different theoretical foundation.

The contribution of this article is the derivation of three measures of actors' network centrality from a process model of social influence. The first of these measures—total effects centrality—indicates the total relative effect of an actor on the other actors in the network. The second measure—immediate effects centrality—indicates the rapidity with which an actor's total effects are realized. The third measure—mediative effects centrality—indicates the extent to which particular actors have a role in transmitting the total effects of other actors. Because of their common theoretical origin, these measures are complementary rather than competitive; each addresses a different question that might be posed about the social structure of a group.

FORMAL FOUNDATIONS

The foundations for the centrality measures are laid out in this section of the article. These foundations consist of the process model from which the measures are derived, a structural classification of the social influence networks that may be involved in the process, and relevant properties of the total interpersonal effects that arise from the process.

Opinion Formation Process

The process of opinion formation can rarely be reduced to accepting or rejecting the consensus of others; typically, individuals form their opinions in a complex interpersonal environment in which influential opinions are in disagreement and liable to change. How individual opinions and consensus may form in such complex circumstances is the subject of a formal theory that has been under development by social psychologists and mathematicians since the 1950s (French 1956; Harary 1959; DeGroot 1974; Friedkin 1986; Friedkin and Cook 1990; Friedkin and Johnsen

1990).[4] While the theory is meant to be descriptive of the actual process of opinion formation, it also is consistent with a normative theory of rational decision making (Wagner 1978; Lehrer and Wagner 1981).

The theory, along the lines of a simple recursive definition, stipulates that individuals' settled opinions are developed in a joint process of group-level polarization of opinions (escalation) and individual-level weighting of the opinions of influentials (compromise):

$$\boldsymbol{y}_1 = \mathbf{X}\boldsymbol{b}, \tag{1}$$

and

$$\boldsymbol{y}_t = \alpha\mathbf{W}\boldsymbol{y}_{t-1} + \beta\mathbf{X}\boldsymbol{b}, \tag{2}$$

for $t = 2, 3, \ldots$, where $\boldsymbol{y}_t$ is an $n \times 1$ vector of individuals' opinions at time t, $\mathbf{X}$ is an $n \times k$ matrix of k exogenous variables, $\boldsymbol{b}$ is a $k \times 1$ vector of coefficients for the exogenous contributions, $\mathbf{W}$ is an $n \times n$ stochastic matrix of interpersonal influences ($0 \leq w_{ij} \leq 1$, $\Sigma_{j=1}^{n} w_{ij} = 1$), $0 < \alpha < 1$ is a scalar weight of the interpersonal influences, and $\beta = \delta(1 - \alpha)$ is a scalar containing a coefficient of boundary attenuation $(1 - \alpha)$ and a coefficient of group polarization (δ).

The process (1)–(2) stipulates that all exogenous influences on individuals' opinions are reflected in their initial opinions on an issue, and that, at each subsequent point in time, individuals' opinions are altered by a set of interpersonal influences.[5] It stipulates that interpersonal influences modify the effects of exogenous conditions on opinions; for instance, group pressures toward uniformity diminish the importance of socioeconomic background as an influence on group members' opinions.[6] The

[4] Empirical supports for the theory are reviewed in French (1956) and Friedkin and Johnsen (1990). Friedkin and Cook (1990) are able to reject several models that are inconsistent with the theory. The theory is consistent with the mixed regressive-autoregressive (endogenous feedback) model that has become the standard statistical model for the study of social influence networks (Anselin 1988).

[5] The matrix contains the set of interpersonal influences. French (1956) first proposed that social influence was a finite distributed resource. Following Lewin (1951), he argued that persons' opinions may be tugged in various directions by the influences of their significant others and that individuals deal with these cross pressures by shifting their opinions into positions where the pressures are balanced. French operationalized his theory with the assumption that social influence is distributed evenly among those persons with whom an individual is in direct communication. Subsequently, numerous investigators have held the view that social influence is a finite resource that is distributed among a set of significant others, although they sometimes relax French's assumption of an even distribution of social influence (see Implementation below).

[6] Here the model is consistent with Festinger's (1953, p. 237) speculations about such a redirection of influences: "When a person or a group attempts to influence someone, does that person or group produce a totally new force acting on the person, one

model also makes provision for a "polarizing" or escalation process in which all of the opinions in a group become more extreme as a consequence of interpersonal interactions.[7] In the absence of any such polarization, members' opinions are strictly weighted averages of other members' opinions, and the opinions that are produced by the process must be in the range of the group's initial opinions.

Four useful reduced-form equations can be derived from the process described by equations (1)–(2):[8]

$$\boldsymbol{y}_\infty = \alpha \mathbf{W} \boldsymbol{y}_\infty + \beta \mathbf{X} \boldsymbol{b}, \tag{3}$$

$$\boldsymbol{y}_\infty = (\mathbf{I} - \alpha \mathbf{W})^{-1} \beta \mathbf{X} \boldsymbol{b}, \tag{4}$$

$$\boldsymbol{y}_\infty = (\mathbf{I} - \alpha \mathbf{W})^{-1} \beta \boldsymbol{y}_1, \tag{5}$$

and

$$\lim_{\alpha \to 1} (\mathbf{I} - \alpha \mathbf{W})^{-1} \beta \boldsymbol{y}_1 = \delta \mathbf{W}^\infty \boldsymbol{y}_1. \tag{6}$$

With these equations, the model brings together two previously separate lines of formal work. Equations (3)–(4) establish the formal relationship of the present model with work on mixed regressive-autoregressive models of spatial interaction, including interpersonal influence (Ord 1975; Cliff and Ord 1981; Anselin 1988; Erbring and Young 1979; Doreian 1981; Burt 1982, 1987). Equations (5)–(6) establish the formal relationship of the present model with the work of French (1956), Harary (1959), DeGroot (1974), and Friedkin (1986), all of which are concerned with social structural conditions of reaching consensus.

The process model is consistent with the development of consensus or a pattern of disagreement. Consensus appears as a limiting condition for

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which had not been present prior to the attempted influence? Our answer is No—an attempted influence does not produce any new motivation or force. Rather, what an influence attempt involves is the redirection of psychological forces which already exist."

[7] In the recent social psychological literature, group polarization refers not to the cleavage of a group but to the movement of all group members' opinions toward the same extreme position; see Isenberg (1986) for a review of the experimental literature on this process. Cartwright (1971) has suggested that group polarization is in part an artifact of the social influence process. Within the framework of the model, group polarization that *cannot* be accounted for by the social influence process will be reflected in values of δ greater or less than unity. Such exogenously determined group polarization may drive final opinions outside the range of group members' initial opinions.

[8] The derivations of eqq. (3)–(5) are straightforward. See Friedkin and Johnsen (1990) for a proof of the limit $\lim_{\alpha \to 1}(1 - \alpha)[\mathbf{I} - \alpha \mathbf{W}]^{-1} = \mathbf{W}^\infty$ in eq. (6).

$\alpha \rightarrow 1$ in suitable influence networks (the permitting conditions are defined below). Given a network in which consensus might be attained, it is noteworthy that the model does not inevitably lead to consensus (cf. Abelson 1964); in such networks, depending on α, various patterns of more or less marked disagreement are possible. Horowitz (1962, p. 182) has commented that "any serious theory of agreements and decisions must at the same time be a theory of disagreements and the conditions under which decisions cannot be reached." The present model satisfies Horowitz's criterion.

For the analysis in this article the model is constrained by two simplifying assumptions. I assume that there is no group polarization (i.e., $\delta = 1$). The constraint is in line with Friedkin and Cook's (1990) support of Cartwright's (1971) speculation that group polarization is a by-product of the social influence process; in any event, it does not mislead to omit this parameter. For technical reasons, I also assume that the influence networks are "regular" (the definition of such networks follows).[9] Many structures can be described by regular networks; for example, hierarchies consisting of asymmetric influences can be represented with imbalanced relationships ($1 > w_{ij} > w_{ji} > 0$) in which one weight is close to unity and the other is close to zero.

Classification of Influence Networks

The theory of digraphs (Harary, Norman, and Cartwright 1965) and Markov chains (Kemeny and Snell 1960) provides a classification of influence network structure ($\mathbf{W}$) that is important in this analysis. A network is *disconnected* if its membership can be partitioned into two or more groups between which no influence relations exist; otherwise it is *connected*. A connected network is *strong* (ergodic) if every member has direct or indirect influence on all other members. A connected network

[9] This assumption is useful because it assures the convergence of $\mathbf{W}^\infty$ in eqq. (6) and (11). Periodic $\mathbf{W}$ would not converge. An example would be

$$\mathbf{W} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix},$$

all odd powers of which are

$$\mathbf{W} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

and all even powers of which are

$$\mathbf{W} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

is *unilateral* if, for all pairs of members, at least one member of a pair has direct or indirect influence on the other member. A connected network is *weak* given at least two members neither of whom has direct or indirect influence on the other.

The class of strong networks has two important subclasses: a network is *regular* (aperiodic or acyclic) if some power of its matrix presents entirely positive entries; otherwise, the network is *periodic*.[10] The class of unilateral networks also has two important subclasses: a network is *centered* if it contains a single regular subnetwork ($n \geq 1$) whose members directly or indirectly influence all other network members; otherwise the network is *noncentered*.

Total Interpersonal Effects

The process model describes how initial opinions held by a group's members are transformed by interpersonal influences. Actors' total interpersonal effects are given by an $n \times n$ matrix, $\mathbf{V}$, of reduced-form coefficients that transform initial into final opinions:

$$y_\infty = \mathbf{V} y_1, \tag{7}$$

where

$$\mathbf{V} = (\mathbf{I} - \alpha \mathbf{W})^{-1}(1 - \alpha). \tag{8}$$

I refer to $\mathbf{V}$ as the total interpersonal effects matrix and will base several measures of centrality on it. However, before these centrality measures are introduced, relevant properties of $\mathbf{V}$ need to be described.

1. The matrix $\mathbf{V}$ is row stochastic: its entries are nonnegative ($0 \leq v_{ij} \leq 1$) and each of its rows sum to unity ($\Sigma_{j=1}^{n} v_{ij} = 1$). Hence, an entry in $\mathbf{V}$ gives the relative weight of the initial opinion of actor j in determining the final opinion of actor i.

2. As $\alpha \rightarrow 1$, $\mathbf{V}$ may converge to a matrix ($\mathbf{V}_U$) that transforms a set of heterogeneous initial opinions into consensus; that is,

$$\mathbf{V}_U = \begin{bmatrix} c_1 & c_2 & \cdots & c_n \\ c_1 & c_2 & \cdots & c_n \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ c_1 & c_2 & \cdots & c_n \end{bmatrix}.$$

The distinctive feature of $\mathbf{V}_U$ is the convergence of the total interpersonal effects of each actor j to a constant $v_{ij} = c_j$ ($0 \leq c_j \leq 1$) for all i. The

[10] Given $w_{ii} > 0$ for any i, all strong networks must be regular.

$1 \times n$ vector of these constants, $\boldsymbol{c} = [c_1 \quad c_2 \quad \ldots \quad c_n]$, is a left eigenvector of $\mathbf{W}$,

$$\lambda \boldsymbol{c} = \boldsymbol{c}\mathbf{W}, \tag{9}$$

for $\lambda = 1$ (the largest eigenvalue).

Whether $\mathbf{V}_U$ emerges as $\alpha \rightarrow 1$ depends on the structure of $\mathbf{W}$. In regular and centered influence networks, for $\alpha \rightarrow 1$, the outcome will be consensus regardless of the distribution of initial opinions. In influence networks that are noncentered, weak, or disconnected, for $\alpha \rightarrow 1$, the outcome will be subsets of agreeing actors rather than global consensus. In such a case $\mathbf{V}$ may be rearranged in block diagonal form, for example,

$$\mathbf{V}_B = \begin{bmatrix} c_1 & c_2 & 0 & 0 & 0 & 0 \\ c_1 & c_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & c_3 & c_4 & c_5 & 0 \\ 0 & 0 & c_3 & c_4 & c_5 & 0 \\ 0 & 0 & c_3 & c_4 & c_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & c_6 \end{bmatrix},$$

where each block consists either of a single actor or a larger subset of actors involved in a regular or centered subnetwork.

For $\alpha < 1$, regardless of the connectivity category of the network, particular actors may or may not be in agreement at the end of the social influence process. Global consensus can appear only if there was consensus at the start of the process; hence, the expectation for $\alpha < 1$ is a pattern of disagreement.[11] Total effect matrices for $\alpha < 1$ are referred to as nonuniform ($\mathbf{V}_{\bar{U}}$).

3. The total interpersonal effect of one actor on another is related to the number and length of the various paths and sequences that join them in the network of interpersonal influences.[12] This relationship has the following formal foundation:

$$\begin{aligned} \mathbf{V} &= (\mathbf{I} - \alpha\mathbf{W})^{-1}(1 - \alpha) \\ &= (\mathbf{I} + \alpha\mathbf{W} + \alpha^2\mathbf{W}^2 + \alpha^3\mathbf{W}^3 + \ldots)(1 - \alpha). \end{aligned} \tag{10}$$

Consider an arbitrary term, $\alpha^k\mathbf{W}^k$, in the infinite series ($\alpha\mathbf{W} + \alpha^2\mathbf{W}^2 + \alpha^3\mathbf{W}^3 + \ldots$). If all the nonzero entries in $\mathbf{W}$ were converted to 1's, an entry in $\mathbf{W}^k$ would indicate the number of ways in which interpersonal

[11] Thus $\boldsymbol{y}_\infty = \mathbf{V}\boldsymbol{y}_1$ for $\delta = 1$; since each of the rows in $\mathbf{V}^{-1}$ sum to one, if the outcome is consensus, $\mathbf{V}^{-1}\boldsymbol{y}_\infty = \boldsymbol{y}_\infty = \boldsymbol{y}_1$.

[12] In a path of interpersonal influences ($i \rightarrow j \rightarrow k \rightarrow l$) no actor appears more than once. In a sequence of interpersonal influences the same actor may appear more than once (e.g., $i \rightarrow j \rightarrow k \rightarrow j \rightarrow l$).

influence flows in k-steps from one actor to another in the network; the greater the number of such k-step flows, the larger the expected impact of one actor on the other. The network model qualifies this expectation in two respects. First, the impact of a single k-step flow diminishes with the number of steps involved. Second, the impact of flows that traverse the same number of steps depends on the strengths (αw_{ij}) of constituent links. In short, the total interpersonal effect of one actor on another is a weighted sum of the number of different channels of interpersonal influence that join them in the network, where each channel is weighted according to its length and strength of constituent links.

4. Now consider the sequences of interpersonal influence from actor j to actor i in which actor j appears only once. For $\alpha \rightarrow 1$ in regular influence networks, the average length of these sequences (each sequence weighted according to the strength of its constituent links) is m_{ij}:

$$\mathbf{M} = (\mathbf{I} - \mathbf{Z} + \mathbf{E}\mathbf{Z}_{dg})\mathbf{D}, \tag{11}$$

where $\mathbf{D}$ is the diagonal matrix with elements $d_{ii} = 1/c_i$, c_i is an element of the left eigenvector of $\mathbf{W}$ in equation (9), $\mathbf{E}$ is an $n \times n$ matrix with all entries 1, $\mathbf{Z} = (\mathbf{I} - \mathbf{W} + \mathbf{W}^{\infty})^{-1}$, and $\mathbf{Z}_{dg}$ results from $\mathbf{Z}$ by setting off-diagonal entries to zero (Kemeny and Snell 1960, p. 79). These average lengths completely determine the influence network:

$$\mathbf{W} = \mathbf{I} + (\mathbf{D} - \mathbf{E})(\mathbf{M} - \mathbf{D})^{-1} \tag{12}$$

(Kemeny and Snell 1960, p. 81).

5. The mean length of the sequences of interpersonal influence (i.e., m_{ij}) can be decomposed into the number of such sequences from actor j to actor i via particular other actors:

$$m_{ij} = \sum_{k=1}^{n} t_{(j)ik}, \quad i \neq j \neq k, \tag{13}$$

where $t_{(j)ik}$ is the ikth entry in $\mathbf{T}_{(j)} = (\mathbf{I} - \mathbf{W}_{(j)})^{-1}$ and $\mathbf{W}_{(j)}$ is a matrix obtained by deleting the jth row and column from $\mathbf{W}$ (Kemeny and Snell 1960, pp. 112–13).

CENTRALITY MEASURES

Three centrality measures stem from these formal foundations. This section of the article presents the definitions of the measures along with an analysis of their relationships to measures that have been proposed by Katz (1953), Hubbell (1965), Bonacich (1972*a,* 1987), Freeman (1979), Coleman (1973), and Burt (1982).

Total Effects Centrality

The total effects of actor j on other members of the group are given by the entries in column j of $\mathbf{V}$. Total effect centrality, *TEC*, is defined as the average total effect of an actor:

$$c_{TEC(j)} = \frac{\sum_{i=1}^{n} v_{ij}}{n-1}, \quad i \neq j. \tag{14}$$

For uniform total effects ($\mathbf{V}_U$), there is no variance in the total effects of each actor and, therefore, *TEC* provides an exact description of each actor's relative influence in determining the consensus of opinions in the group.[13] For blocked and heterogeneous total interpersonal effects ($\mathbf{V}_B$ and $\mathbf{V}_{\bar{U}}$, respectively), some caution is warranted. In the case of $\mathbf{V}_B$, an actor may be highly influential in a particular subnetwork but have no influence on the remainder of the network members. In the case of $\mathbf{V}_{\bar{U}}$, the precision of the measure is reduced by the variance of an actor's total effects.

Katz.—The *TEC* measure is closely related to Katz's (1953) index of sociometric status:

$$\begin{aligned}\mathbf{t} &= (\alpha\mathbf{R} + \alpha^2\mathbf{R}^2 + \ldots + \alpha^\infty\mathbf{R}^\infty)'\boldsymbol{e} \\ &= [(\mathbf{I} - \alpha\mathbf{R})^{-1} - \mathbf{I}]'\boldsymbol{e},\end{aligned}$$

where $\mathbf{R}$ is a $n \times n$ matrix in which $r_{ij} = 1$ if actor i is responsive to actor j and $r_{ij} = 0$ otherwise, $0 < \alpha < 1$ is a coefficient of social influence, and $\boldsymbol{e}$ is an $n \times 1$ vector of ones.

Katz's measure is consistent with a viewpoint of centralities as total interpersonal effects that transform individual inputs into outputs; it takes into account all the channels of interpersonal influence that have contributed to the interpersonal effect of one actor on another. However, these total effects are not consistent with the formation of consensus since $(\mathbf{I} - \alpha\mathbf{R})^{-1}$ cannot converge to $\mathbf{V}_U$. Moreover, as Hubbell (1965) has noted, Katz's model has a multiplicative implication; that is, the coefficients in $(\mathbf{I} - \alpha\mathbf{R})^{-1}$ will transform inputs ($\boldsymbol{y}_1$) into outputs ($\boldsymbol{y}_\infty$) such that each actor's output is greater than the actor's input ($\boldsymbol{y}_\infty - \boldsymbol{y}_1 > \mathbf{0}$). Such a multiplicative implication is inconsistent with experimental findings on the interpersonal influence process where it appears that the

[13] In regular and centered $\mathbf{W}$, $\mathbf{V}_U$ emerges for $\alpha \rightarrow 1$. An approximate $\mathbf{V}_U$ is obtained by setting $\alpha \approx 1$ (e.g., .999) and computing $(\mathbf{I} - \alpha\mathbf{W})^{-1}(1 - \alpha)$; see Implementation for a fuller discussion and the Appendix for an illustration.

final opinions of a group's members are virtually always in the range of their initial opinions (Friedkin and Cook 1990).

Hubbell et al.—Five other measures also are consistent with *TEC*. These measures have in common the assumption that the centrality of an actor is a function of the centralities of those actors with whom the actor has interpersonal relations.

Hubbell (1965) has proposed an index of sociometric status that takes account of both the "status of the chooser and the strength at which he chooses" (p. 382):

$$s_i = e_i + r_{i1}s_1 + r_{i2}s_2 + \ldots + r_{in}s_n, \tag{15}$$

where e_i is an exogenous contribution, r_{ij} is the strength at which actor j chooses actor i, and s_j is actor j's status. Hence, an actor will tend to have high status to the extent that other high-status actors have strong ties to the actor. The matrix question for (15) is

$$\begin{aligned} \boldsymbol{s} &= \boldsymbol{e} + \mathbf{R}\boldsymbol{s} \\ &= (\mathbf{I} - \mathbf{R})^{-1}\boldsymbol{e}, \end{aligned}$$

assuming $(\mathbf{I} - \mathbf{R})$ is invertible. Hubbell sets the exogenous contributions, $\boldsymbol{e}$, to a column vector of ones,

$$\begin{aligned} \boldsymbol{s} &= (\mathbf{I} - \mathbf{R})^{-1}\boldsymbol{e} \\ &= (\mathbf{I} + \mathbf{R} + \mathbf{R}^2 + \mathbf{R}^3 + \ldots)\boldsymbol{e}, \end{aligned}$$

so that the measure of an actor's status is computed as a weighted sum of all paths from the members of a network to the actor.

Thus, while Hubbell's index (in its reduced form) appears closely related to the one proposed by Katz (1953), the conceptual foundations are quite different. Hubbell's idea about the social organization of status (15) was to find a more elegant realization in the work of Bonacich (1972*a*, 1987), Coleman (1973), and Burt (1982).

Bonacich (1972*a*) starts with a definition of actors' centrality as a function of the centralities of those actors with whom they are related,

$$\lambda c_i = \sum_{j=1}^{n} r_{ij}c_j, \tag{16}$$

where $\mathbf{R}$ is an $n \times n$ matrix of interpersonal relations ($0 \leq r_{ij} \leq 1$) and λ is introduced as a convenience. It follows that the vector of actors' centralities, $\boldsymbol{c}$, is an eigenvector of $\mathbf{R}$ ($\lambda\boldsymbol{c} = \mathbf{R}\boldsymbol{c}$). To assure that these centralities are nonnegative, $\boldsymbol{c}$ is taken as an eigenvector associated with the largest eigenvalue of $\mathbf{R}$. This eigenvector is then normalized (Bonacich 1972*b;* Mizruchi et al. 1986; Knoke and Burt 1983).

Recently, Bonacich (1987) has proposed a more general form of his eigenvector measure. Actors' centrality again appears as a function of the centralities of those actors with whom they are related,

$$c_i(\beta, \alpha) = \sum_{j=1}^{n} (\beta + \alpha c_j) r_{ij}, \tag{17}$$

where α and β are scalars.

Coleman (1973) has defined the power of an actor along the same lines: $p_j = \Sigma_{k=1}^{n} p_k r_{kj}$, where p_j is the power of actor j and r_{kj} is the dependency of actor k on actor j. Burt's (1982, p. 35) measure of prestige also is along these lines: p_j is the prestige of actor j and r_{kj} is an interpersonal relation of some sort.

The essential social organization of status that is postulated by Hubbell, Bonacich, Coleman, and Burt can be deduced from the process (1)–(2). Since, for $\mathbf{V}_U$, total effect centralities are a left eigenvector of $\mathbf{W}$ associated with eigenvalue 1, that is,

$$\mathbf{c}'_{TEC} = \mathbf{c}'_{TEC}\mathbf{W}, \tag{18}$$

it follows that

$$c_{TEC(j)} = \sum_{i=1}^{n} c_{TEC(i)} w_{ij}. \tag{19}$$

Thus, the process model is consistent with a social organization in which actors are central to the extent that they strongly influence central actors.

Immediate Effects Centrality

Actors with equivalent total effects may vary in the immediacy of their influences. Actors whose effects are transmitted over lengthy sequences of interpersonal influence have less immediate effects than do actors whose effects are transmitted over short sequences of interpersonal influence. Actors with greater immediacy are less dependent on intervening actors.

The immediacy of actor j's influences on other members of the group are given by the entries in column j of $\mathbf{M}$ as shown in (11). Immediate effects centrality, *IEC,* is defined as the reciprocal of the mean length of the sequences of interpersonal influence from actor j to other actors in the network:

$$c_{IEC(j)} = \left(\frac{\sum_{i=1}^{n} m_{ij}}{n - 1} \right)^{-1}, \quad i \neq j. \tag{20}$$

The *IEC* measure takes into account both the lengths and strengths of the sequences of interpersonal influence that connect actors. The larger the *IEC,* the more rapidly the total effects of an actor tend to emerge from the influence process. The computation of this measure is illustrated in the Appendix.

The *IEC* measure is closely related to closeness-based measures of point centrality (Bavelas 1950; Beauchamp 1965; Sabidussi 1966). Freeman (1979, pp. 224–26) suggests that the best of these measures is the one proposed by Beauchamp (1965) in which actors appear central to the extent that the average distance separating them from other actors is small:

$$c_j = \left(\frac{\sum_{i=1}^{n} d_{ij}}{n - 1} \right)^{-1}, \quad i \neq j, \tag{21}$$

where d_{ij} is the length of the shortest path (geodesic) from actor j to actor i in a network.

Closeness-based measures stem from the ideas of independence and efficiency. The independence idea is that central actors do not need to rely on other actors for influence, while peripheral actors must depend on others as intermediaries. The efficiency idea is that influence of central actors spreads more rapidly throughout a network than does the influence of peripheral actors. The indices follow from the plausible inverse relationship between a path's length and contribution to information and influence flows. The shorter the average distance of an actor to other actors, the more direct and efficient is the actor's impact because fewer intermediaries are involved in the transmissions.

Mediative Effects Centrality

The third measure indicates the extent to which an actor transmits the total effects of other actors. From equation (13),

$$\bar{t}_{(k)j} = \frac{\sum_{i=1}^{n} t_{(k)ij}}{(n - 2)t_{(k)jj}}, \quad i \neq j \neq k, \tag{22}$$

is indicative of the contribution of actor j in transmitting the interpersonal effects of actor k (i.e., $\bar{t}_{(k)j}$ is the ratio of actor j's transmissions to nontransmissions of actor k's effects); and

$$c_{MEC(j)} = \frac{\sum_{k=1}^{n} \bar{t}_{(k)j}}{n - 1}, \quad j \neq k, \tag{23}$$

is indicative of the contribution of actor j in transmitting the interpersonal effects of all network members (see the Appendix for an illustration of these computations).

The *MEC* measure is closely related to Freeman's (1979) measure of betweenness centrality. The "betweenness" of an actor is defined as the proportion of all the shortest paths (geodesics) of a network in which the actor is involved as an intervening point:

$$c_j = \sum_{i=1}^{n} \sum_{k=1}^{n} \frac{g_{ik(j)}}{g_{ik}}, \quad i \neq j \neq k, \tag{24}$$

where $g_{ik(j)}$ is the number of geodesics from actor k and actor i that involve actor j as an intervening point and g_{ik} is the number of geodesics from actor k to actor i. The rationale for the measure is that actors involved in many of the paths linking other actors have an opportunity to affect the transmissions that occur through these paths. "It is this potential for control," Freeman argues (1979, p. 221), "that defines the centrality of these points."

Complementary Measures

Clearly *TEC, IEC,* and *MEC* are not alternative measures of the centrality of an actor. Because the measures are complementary it makes no sense to ask which is the best measure of an actor's position in an influence network (cf. Freeman et al. 1980; Bolland 1988; Knoke and Burt 1983). Each measure addresses a different question about the operation of an influence network. The *TEC* measure indicates the total relative effect of an actor on the other actors of the network; *IEC* indicates the immediacy of an actor's total effects; and *MEC* indicates the extent to which an actor mediates the total effects of other actors.

The measures distinguish (*a*) the substantive contribution of an actor to other actors' opinions from (*b*) the structural contribution of an actor as a conduit of other actors' interpersonal effects. The settled opinions of a group need not reflect the initial opinions of actors who are important transmitters of influence or whose immediacy of effects is high. When a group has reached equilibrium, it is an actor's total effect that is the relevant measure of substantive impact. The controlling role of mediating actors is a control over the rapidity with which other actors' total effects are realized. However, given premature termination of the social influence process, it is possible for such processional control to substantively affect the "final" opinions by serving to allocate disproportionate influence to those actors with the highest immediacy.

Table 1 illustrates the three centrality measures in 21 networks that

TABLE 1

CENTRALITY MEASURES FOR ILLUSTRATIVE NETWORKS

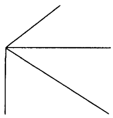
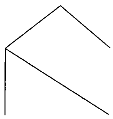
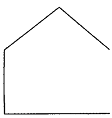
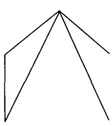
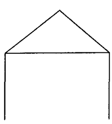
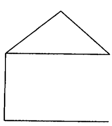
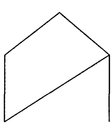
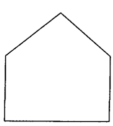
Network	*TEC*	*IEC*	*MEC*
1. 	.154	.080	.375
	.385	.500	1.000
	.154	.080	.375
	.154	.080	.375
	.154	.080	.375
2. 	.231	.133	.667
	.308	.250	.875
	.154	.069	.347
	.154	.069	.347
	.154	.056	.264
3. 	.231	.105	.625
	.231	.167	.750
	.231	.105	.625
	.154	.050	.250
	.154	.050	.250
4. 	.333	.400	.917
	.200	.133	.500
	.200	.133	.500
	.133	.067	.292
	.133	.067	.292
5. 	.200	.143	.533
	.267	.222	.742
	.133	.059	.242
	.133	.059	.242
	.267	.222	.742
6. 	.200	.105	.450
	.267	.222	.792
	.200	.105	.600
	.133	.045	.208
	.200	.105	.450
7. 	.200	.142	.526
	.200	.125	.472
	.200	.142	.526
	.133	.060	.226
	.267	.235	.750
8. 	.200	.133	.500
	.200	.133	.500
	.200	.133	.500
	.200	.133	.500
	.200	.133	.500

Table 1 (*Continued*)

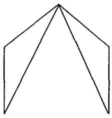
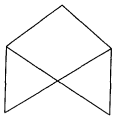
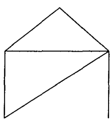
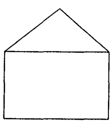
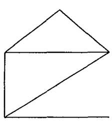
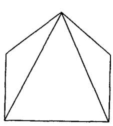
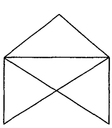
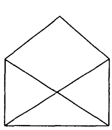
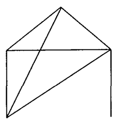
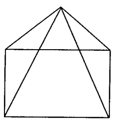
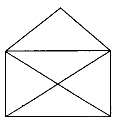
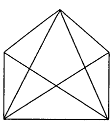
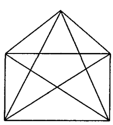
	Network	*TEC*	*IEC*	*MEC*
		.294	.333	.833
		.176	.109	.417
9.		.176	.109	.417
		.176	.109	.417
		.176	.109	.417
		.176	.129	.417
		.235	.214	.625
10.		.176	.129	.417
		.176	.129	.417
		.235	.214	.625
		.176	.125	.431
		.235	.200	.611
11.		.176	.125	.431
		.118	.057	.209
		.294	.333	.817
		.176	.118	.386
		.235	.210	.629
12.		.176	.120	.428
		.176	.120	.428
		.235	.210	.629
		.176	.111	.383
		.235	.199	.624
13.		.235	.200	.683
		.118	.051	.186
		.235	.199	.624
		.263	.286	.714
		.158	.108	.345
14.		.211	.183	.548
		.211	.183	.548
		.158	.108	.345
		.158	.114	.361
		.263	.286	.708
15.		.158	.114	.361
		.158	.114	.361
		.263	.286	.708
		.158	.111	.357
		.211	.187	.548
16.		.211	.185	.523
		.211	.185	.523
		.211	.187	.548

TABLE 1 (*Continued*)

	Network	*TEC*	*IEC*	*MEC*
		.210	.178	.528
		.210	.178	.528
17.		.210	.178	.528
		.105	.050	.167
		.263	.286	.750
		.238	.250	.619
		.190	.167	.470
18.		.190	.167	.470
		.190	.167	.470
		.190	.167	.470
		.143	.100	.302
		.238	.250	.635
19.		.190	.164	.464
		.190	.164	.464
		.238	.250	.635
		.217	.222	.560
		.174	.150	.411
20.		.217	.222	.560
		.217	.222	.560
		.174	.150	.411
		.200	.200	.500
		.200	.200	.500
21.		.200	.200	.500
		.200	.200	.500
		.200	.200	.500

NOTE.—The points of these networks are labeled counterclockwise with first points at 12:00. For each network, the diagonal entries of its adjacency matrix $A = [a_{ij}]$ were set to one and its influence network was computed as $W = [w_{ij}] = a_{ij}/\Sigma_j^n a_{ij}$.

have appeared in studies of network centrality.[14] These networks include all the connected networks from the population of nonisomorphic symmetric networks with five points. The influence networks were computed as $\mathbf{W} = [w_{ij}] = [a_{ij}/\Sigma_j^n a_{ij}]$, where $a_{ii} = 1$ and $a_{ij} = 1$ wherever a line exists between two points (see the diagrams in table 1). This specification of $\mathbf{W}$ follows French (1956). The *TEC, IEC,* and *MEC* measures are

[14] Freeman (1979) employed these networks to illustrate his measures of network centrality. Subsets of the networks appeared in the work of Bavelas (1950) and, more recently, in Stephenson and Zelen (1989). Subsets also appeared in experimental studies of communication networks (Leavitt 1951; Freeman et al. 1980).

restricted to regular networks; such networks do not need to be symmetric, nor do they need to entail the above specification of **W**. I have illustrated the measures in the present fashion because the relationship between network structure and point centrality is easiest to apprehend with such baseline networks.

IMPLEMENTATION

In this final section of the article, I deal with three equations that bear on the use of the centrality measures. How might an influence network (**W**) be described? When is it proper to set the coefficient of social influence (α) to near unity? How might an empirical estimate of α be obtained?

Influence Networks

Operationalization of the centrality measures requires a stance on the likely content of the influence network, **W**. For a suitable matrix (**R**) of interpersonal relations, the entries of **W** may be computed as

$$w_{ij} = \frac{r_{ij}}{\sum_{j=1}^{n} r_{ij}}. \tag{25}$$

The interpersonal relations may be simple adjacencies of communication, kinship, or friendship: $r_{ij} = 1$ if actor i is adjacent (i.e., responsive) to actor j and $r_{ij} = 0$ otherwise. This approach follows Katz (1953) and French (1956); it also coincides with conventional methodological practice in handling spatial autocorrelation in multiple regression models (Anselin 1988; Johnston 1984, p. 308).

Alternatively, **W** may be described with a more refined theory in which r_{ij} appears as a continuous measure of actors' interdependency. Three examples of such measures will be given.

1. Freeman (1980) has proposed a measure of pair-dependency that stems from his work on point centrality:

$$r_{ij} = \sum_{k=1}^{n} \frac{g_{ik(j)}}{g_{ik}}, \quad i \neq j \neq k, \tag{26}$$

where g_{ik} is the number of geodesics (shortest paths) from actor i to actor k and $g_{ik(j)}$ is the number of such geodesics that contain actor j as an intervening point. The measure is an "index of the degree to which a particular point must depend upon a specific other—as a relayer of messages—in communicating with all others in the network" (Freeman 1980, p. 587).

2. Another noteworthy approach is the structural equivalence hypothesis of Burt (1982, 1987). Burt's hypothesis is that two actors are responsive to each other to the extent that they occupy similar positions in a social network:

$$r_{ij} = \left[\sum_{k=1}^{n} [(d_{ik} - d_{jk})^2 + (d_{ki} - d_{kj})^2] \right]^{1/2}, \tag{27}$$

where d_{ij} is an index of the strength of an interpersonal relationship. The index d_{ij} could be a binary measure of adjacency in a communication network, or it could be a more complex measure of tie strength (Burt 1988; Marsden and Campbell 1984; Friedkin 1990).

3. My earlier work (Friedkin 1982, 1983) on information flow and observability of role performance in communication networks indicates support for a simple model of structural accessibility:

$$r_{ij} = 1 - \prod_{k=1}^{2} (1 - \rho^k)^{x_k} \tag{28}$$

where $0 < \rho < 1$ and x_k is the number of k-step communication paths connecting actor i and actor j.[15] In terms of theoretical parsimony, this model lies between the adjacency approach of French (1956) and the structural equivalence approach of Burt (1987). It allows for interdependency in the absence of direct communication (cf. French) and gives considerable weight to the number of actors' mutual communication ties (cf. Burt).

Social Structure of Consensus Production

To ascertain the centralities of actors in the production of consensus (e.g., the collective decisions of a group), actors' total effects are computed as

$$\mathbf{V} = \lim_{\alpha \to 1} (\mathbf{I} - \alpha\mathbf{W})^{-1}(1 - \alpha) = \mathbf{W}^{\infty}, \tag{29}$$

under the condition that $\mathbf{W}$ is a regular network.

For a group that has reached consensus on one or more issues, the centrality scores provide an analysis of the roles of actors in producing

[15] If ρ is the probability that an interpersonal tie will transmit an item of information (e.g., the opinion of an actor), then, if we assume independence, $1 - \rho^k$ is the probability that the information will not be transmitted over a k-step path and $(1 - \rho^k)^{x_k}$ is the probability that not one of x_k independent paths will transmit the information. Hence, the probability that the information will be transmitted by at least one of the one-step or two-step paths connecting actor i and actor j is r_{ij}.

that consensus. For a group that may not have reached consensus, but in which there is a strain toward consensus, the centrality scores describe that particular social structure of status toward which the group is straining.

Estimating the Coefficient of Social Influence

An empirical estimate of α is desirable for an analysis of (*a*) a group with a history of noteworthy, unresolved disagreements or (*b*) a group's handling of a particular issue on which noteworthy disagreements remained unresolved. In general, given noteworthy disagreements in a group, there is little warrant for an assumption that the actors have a *coherent* status; their interpersonal effects are likely to be blocked or heterogeneous. A more refined analysis is called for that would describe the pattern of total effects in the group and explain how *particular* actors or subgroups have come to settle on their opinions.

Given data on a subset ($\mathbf{X}^*$) of the exogenous variables that determine group members' initial opinions on an issue or issues, an empirical estimate for α may be obtained with the model

$$\boldsymbol{y}_\infty = \alpha \mathbf{W} \boldsymbol{y}_\infty + \mathbf{X}^* \boldsymbol{b}^* + \boldsymbol{u}, \tag{30}$$

where $\mathbf{X}^*$ is an $n \times k$ matrix of the exogenous variables, $\boldsymbol{b}^*$ is the $k \times 1$ vector of coefficients for these variables, and $\boldsymbol{u}$ is an $n \times 1$ vector of residuals.

This estimation equation, which is a standard mixed regressive-autoregressive model, can be derived from a reduced-form equation of the process model (3): the scalar β is subsumed into $\boldsymbol{b}$ and the matrix of exogenous variables is partitioned into observed and unobserved parts ($\mathbf{X}\boldsymbol{b} = \mathbf{X}^*\boldsymbol{b}^* + \boldsymbol{u}$), ideally under the condition $(\mathbf{X}^*)'\boldsymbol{u} = \mathbf{0}$. The maximum-likelihood approach for estimating α and $\boldsymbol{b}^*$ is described by Ord (1975); also see Doreian (1981), Cliff and Ord (1981), and Anselin (1988).[16]

CONCLUSION

Three measures of actors' network centrality have been derived from an elementary process model of social influence. The measures are closely related to widely used measures of actors' network centrality. Unlike

[16] Anselin (1989) and Friedkin (1989) provide computer software that will estimate the parameters of mixed regressive-autoregressive models. Friedkin's software also will calculate the centrality measures that are reported in this paper. The software is designed for use with the GAUSS system, version 2.0.

previous measures, which have been viewed as competing alternatives, the present measures are complementary and, in their juxtaposition, provide for a rich description of social structure. The complementarity indicates a degree of theoretical unification in the work on network centrality that was heretofore unsuspected.

New light has been cast on the theoretical foundations of an important family of centrality measures. The sociometric status measure of Hubbell (1965), the centrality measures of Bonacich (1972*a,* 1987), the power measure of Coleman (1973), and the prestige measure of Burt (1982) are based on the tautological definition of status in terms of the status of related others. While the *social organization* of status is precisely formulated in these definitions, the origins of status are left murky.

The present analysis has shown how an actor's status may arise from the flows of interpersonal influence in a network. It has also shown how the essential social organization of status that has been assumed by Hubbell, Bonacich, Coleman, and Burt can be deduced from a micro-level process model of social influence. From the present perspective, the definition of status in terms of other actors' status, while correct, appears less fundamental than the definition of status in terms of an actor's total interpersonal effects.

Coombs reminds us that "a measurement or scaling model is actually a theory about behavior, admittedly on a miniature level, but nevertheless theory" (1964, p. 5). By this criterion, every new proposal of a centrality measure presents new theoretical material. This article offers new theoretical material, but it also may serve to raise the ante in the field by encouraging the construction of somewhat broader theoretical foundations for proposed measures of network centrality.

APPENDIX

This Appendix illustrates the calculation of the proposed centrality measures—*TEC, IEC,* and *MEC*. Let

$$\mathbf{W} = \begin{bmatrix} \frac{3}{5} & \frac{1}{5} & \frac{1}{5} & 0 & 0 \\ \frac{3}{5} & \frac{1}{5} & 0 & \frac{1}{5} & 0 \\ \frac{3}{8} & \frac{3}{8} & \frac{1}{8} & 0 & \frac{1}{8} \\ 0 & \frac{3}{4} & 0 & \frac{1}{4} & 0 \\ 0 & 0 & \frac{3}{5} & \frac{1}{5} & \frac{1}{5} \end{bmatrix}$$

$$\mathbf{V} = [v_{ij}] = \lim_{\alpha \to 1} [\mathbf{I} - \alpha\mathbf{W}]^{-1}(1 - \alpha) = \mathbf{W}^{\infty}$$

$$= \begin{bmatrix} .513 & .260 & .131 & .075 & .021 \\ .513 & .260 & .131 & .075 & .021 \\ .513 & .260 & .131 & .075 & .021 \\ .513 & .260 & .131 & .075 & .021 \\ .513 & .260 & .131 & .075 & .021 \end{bmatrix},$$

and

$$c_{TEC(j)} = \frac{\sum_{i=1}^{n} v_{ij}}{n - 1}, \quad i \neq j.$$

Hence, $TEC = [.513 \quad .260 \quad .131 \quad .075 \quad .021]'$.

$$\mathbf{Z} = (\mathbf{I} - \mathbf{W} + \mathbf{W}^{\infty})^{-1}$$

$$= \begin{bmatrix} 1.148 & -0.112 & 0.106 & 0.132 & -0.009 \\ 0.064 & 1.005 & -0.171 & 0.154 & -0.052 \\ -0.203 & 0.070 & 1.040 & -0.045 & 0.137 \\ -0.620 & 0.659 & -0.347 & 1.388 & -0.080 \\ -0.949 & -0.108 & 0.529 & 0.220 & 1.307 \end{bmatrix},$$

$$\mathbf{Z}_{dg} = \begin{bmatrix} 1.148 & 0 & 0 & 0 & 0 \\ 0 & 1.005 & 0 & 0 & 0 \\ 0 & 0 & 1.040 & 0 & 0 \\ 0 & 0 & 0 & 1.388 & 0 \\ 0 & 0 & 0 & 0 & 1.307 \end{bmatrix},$$

$$\mathbf{D} = [d_{ij}] = \begin{bmatrix} 1.949 & 0 & 0 & 0 & 0 \\ 0 & 3.846 & 0 & 0 & 0 \\ 0 & 0 & 7.611 & 0 & 0 \\ 0 & 0 & 0 & 13.362 & 0 \\ 0 & 0 & 0 & 0 & 48.654 \end{bmatrix},$$

($d_{ii} = 1/v_{ii}$), and $\mathbf{E}$ is an $n \times n$ matrix with all entries equal to one.

$$\mathbf{M} = [m_{ij}] = (\mathbf{I} - \mathbf{Z} + \mathbf{E}\mathbf{Z}_{dg})\mathbf{D}$$

$$= \begin{bmatrix} 1.948 & 4.298 & 7.111 & 20.316 & 64.104 \\ 2.111 & 3.846 & 9.222 & 16.487 & 66.214 \\ 2.631 & 3.596 & 7.611 & 19.146 & 56.993 \\ 3.444 & 1.333 & 10.555 & 13.365 & 67.548 \\ 4.084 & 4.281 & 3.889 & 15.609 & 48.705 \end{bmatrix},$$

$$c_{IEC(j)} = \left(\frac{\sum_{i=1}^{n} m_{ij}}{n-1} \right)^{-1}, \quad i \neq j.$$

Hence, $IEC = [.326 \quad .296 \quad .130 \quad .056 \quad .016]'$.

$$\mathbf{T}_{(k)} = [t_{(k)ij}] = (\mathbf{I} - \mathbf{W}_{(k)})^{-1},$$

where $\mathbf{W}_{(k)}$ is a matrix obtained by deleting the kth row and column from $\mathbf{W}$:

$$\mathbf{T}_{(1)} = \begin{bmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & 1.667 & 0 & .444 & 0 \\ \cdot & .867 & 1.280 & .284 & .200 \\ \cdot & 1.667 & 0 & 1.778 & 0 \\ \cdot & 1.067 & .960 & .658 & 1.400 \end{bmatrix},$$

$$\mathbf{T}_{(2)} = \begin{bmatrix} 3.289 & \cdot & .843 & .035 & .132 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 1.579 & \cdot & 1.684 & .070 & .263 \\ 0 & \cdot & 0 & 1.333 & 0 \\ 1.184 & \cdot & 1.263 & .386 & 1.447 \end{bmatrix},$$

$$\mathbf{T}_{(3)} = \begin{bmatrix} 5 & 1.667 & \cdot & .444 & 0 \\ 5 & 3.333 & \cdot & .889 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 5 & 3.333 & \cdot & 2.222 & 0 \\ 1.250 & .833 & \cdot & .556 & 1.250 \end{bmatrix},$$

$$\mathbf{T}_{(4)} = \begin{bmatrix} 12.195 & 4.512 & 3.122 & \cdot & .488 \\ 9.146 & 4.634 & 2.341 & \cdot & .366 \\ 10.244 & 4.390 & 3.902 & \cdot & .610 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 7.683 & 3.293 & 2.927 & \cdot & 1.707 \end{bmatrix},$$

$$\mathbf{T}_{(5)} = \begin{bmatrix} 35 & 16.667 & 8 & 4.444 & \cdot \\ 35 & 18.333 & 8 & 4.889 & \cdot \\ 30 & 15 & 8 & 4 & \cdot \\ 35 & 18.333 & 8 & 6.222 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix},$$

and

$$\bar{t}_{(k)j} = \frac{\sum_{i=1}^{n} t_{(k)ij}}{(n - 2)t_{(k)jj}}, \quad i \neq j \neq k.$$

For example,

$$\bar{t}_{(2)1} = (1.579 + 1.184)/(3 \cdot 3.289) = .280,$$

$$\bar{t}_{(3)1} = (5 + 5 + 1.250)/(3 \cdot 5) = .750,$$

$$\bar{t}_{(4)1} = (9.146 + 10.244 + 7.683)/(3 \cdot 12.195) = .740,$$

$$\bar{t}_{(5)1} = (35 + 30 + 35)/(3 \cdot 35) = .952,$$

$$\bar{t}_{(1)2} = (.867 + 1.667 + 1.067)/(3 \cdot 1.667) = .720,$$

$$\bar{t}_{(3)2} = (1.667 + 3.333 + .833)/(3 \cdot 3.333) = .583,$$

and so forth.

$$c_{MEC(j)} = \frac{\sum_{k=1}^{n} \bar{t}_{(k)j}}{n - 1}, \quad j \neq k.$$

Hence, $MEC = [.681 \quad .722 \quad .596 \quad .345 \quad .106]'$.

REFERENCES

Abelson, Robert P. 1964. "Mathematical Models of the Distribution of Attitudes under Controversy." Pp. 142–60 in *Contributions to Mathematical Psychology*, edited by N. Frederiksen and H. Gulliksen. New York: Holt, Rinehart & Winston.

Alba, Richard A. 1982. "Taking Stock of Network Analysis: A Decade's Results." *Research in the Sociology of Organizations* 1:39–74.

Anselin, Luc. 1988. *Spatial Econometrics: Methods and Models*. Dordrecht: Kluwer Academic.

———. 1989. "Spatial Regression Analysis on the PC: Spatial Econometrics Using GAUSS." Unpublished manuscript, University of California, Santa Barbara, Department of Geography.

Barnes, J. A. 1972. *Social Networks*. New York: Addison-Wesley Modular Publications 26.

Bavelas, Alex. 1948. "A Mathematical Model for Group Structures." *Human Organization* 7:16–30.

———. 1950. "Communication Patterns in Task Oriented Groups." *Journal of the Acoustical Society of America* 22:271–82.

Beauchamp, Murray A. 1965. "An Improved Index of Centrality." *Behavioral Science* 10:161–63.

Bolland, John M. 1988. "Sorting Out Centrality: An Analysis of the Performance

of Four Centrality Models in Real and Simulated Networks." *Social Networks* 10:233–53.
Bonacich, Phillip. 1972*a*. "Factoring and Weighting Approaches to Status Scores and Clique Identification." *Journal of Mathematical Sociology* 2:113–20.
———. 1972*b*. "Technique for Analyzing Overlapping Memberships." Pp. 176–85 in *Sociological Methodology 1972,* edited by H. Costner. San Francisco: Jossey-Bass.
———. 1987. "Power and Centrality: A Family of Measures." *American Journal of Sociology* 92:1170–82.
Burt, Ronald S. 1982. *Toward a Structural Theory of Action.* New York: Academic Press.
———. 1987. "Social Contagion and Innovation: Cohesion versus Structural Equivalence." *American Journal of Sociology* 92:1287–1335.
———. 1988. "Some Properties of Structural Equivalence Measures Derived from Sociometric Choice Data." *Social Networks* 10:1–28.
Cartwright, Dorwin. 1971. "Risk Taking by Individuals and Groups: An Assessment of Research Employing Choice Dilemmas." *Journal of Personality and Social Psychology* 20:361–78.
Cliff, Andrew, and Keith Ord. 1981. *Spatial Processes, Models and Applications.* London: Pion.
Coleman, James S. 1973. *The Mathematics of Collective Action.* Chicago: Aldine.
Cook, Karen S., Richard M. Emerson, Mary R. Gillmore, and Toshio Yamagishi. 1983. "The Distribution of Power in Exchange Networks: Theory and Experimental Results." *American Journal of Sociology* 89:275–305.
Cook, Karen S., Mary R. Gillmore, and Toshio Yamagishi. 1986. "Point and Line Vulnerability as Bases for Predicting the Distribution of Power in Exchange Networks: Reply to Willer." *American Journal of Sociology* 92:445–48.
Coombs, Clyde H. 1964. *A Theory of Data.* New York: Wiley.
DeGroot, Morris H. 1974. "Reaching a Consensus." *Journal of the American Statistical Association* 69:118–121.
Doreian, Patrick. 1981. "Estimating Linear Models with Spatially Distributed Data." Pp. 359–88 in *Sociological Methodology,* edited by S. Leinhardt. San Francisco: Jossey-Bass.
———. 1986. "Measuring Relative Standing in Small Groups and Bounded Social Networks." *Social Psychology Quarterly* 49:247–59.
Erbring, Lutz, and Alice A. Young. 1979. "Individuals and Social Structure: Contextual Effects as Endogenous Feedback." *Sociological Methods and Research* 7:396–430.
Festinger, Leon. 1953. "An Analysis of Compliant Behavior." Pp. 232–56 in *Group Relations at the Crossroads,* edited by M. Sherif and M. O. Wilson. New York: Harper.
Freeman, Linton C. 1979. "Centrality in Social Networks: Conceptual Clarification." *Social Networks* 1:215–39.
———. 1980. "The Gatekeeper, Pair-Dependency and Structural Centrality." *Quality and Quantity* 14:585–92.
Freeman, Linton C., Douglas Roeder, and Robert R. Mulholland. 1980. "Centrality in Social Networks: II. Experimental Results." *Social Networks* 2:119–41.
French, J. R. P., Jr. 1956. "A Formal Theory of Social Power." *Psychological Review* 63:181–94.
Friedkin, Noah E. 1982. "Information Flow through Strong and Weak Ties in Intraorganizational Social Networks." *Social Networks* 3:273–85.
———. 1983. "Horizons of Observability and Limits of Informal Control in Organizations." *Social Forces* 62:54–77.

———. 1986. "A Formal Theory of Social Power." *Journal of Mathematical Sociology* 12:103–26.

———. 1989. SNAPS (Social Network Analysis Procedures) for GAUSS, Version 1.0. Kent, Wash.: Aptech Systems.

———. 1990. "A Guttman Scale for the Strength of an Interpersonal Tie." *Social Networks* 12:239–52.

Friedkin, Noah E., and Karen S. Cook. 1990. "Peer Group Influence." *Sociological Methods and Research* 19:122–43.

Friedkin, Noah E., and Eugene C. Johnsen. 1990. "Social Influence and Opinions." *Journal of Mathematical Sociology* 15:193–206.

Granovetter, Mark O. 1979. "The Theory-Gap in Social Network Analysis." Pp. 501–18 in *Perspectives on Social Network Research,* edited by P. W. Holland and S. Leinhardt. New York: Academic Press.

Harary, Frank. 1959. "A Criterion for Unanimity in French's Theory of Social Power." Pp. 168–82 in *Studies in Social Power,* edited by D. Cartwright. Ann Arbor, Mich.: Institute for Social Research.

Harary, Frank, Robert Z. Norman, and Dorwin Cartwright. 1965. *Structural Models: An Introduction to the Theory of Directed Graphs.* New York: Wiley.

Horowitz, I. L. 1962. "Consensus, Conflict and Cooperation: A Sociological Inventory." *Social Forces* 41:177–88.

Hubbell, Charles H. 1965. "An Input-Output Approach to Clique Identification." *Sociometry* 28:377–99.

Isenberg, Daniel J. 1986."Group Polarization: A Critical Review and Meta-Analysis." *Journal of Personality and Social Psychology* 50:1141–51.

Johnston, J. 1984. *Econometric Models,* 3d ed. New York: McGraw-Hill.

Katz, Leo. 1953. "A New Status Index Derived from Sociometric Analysis." *Psychometrika* 18:39–43.

Kemeny, John G., and J. Laurie Snell. 1960. *Finite Markov Chains.* Princeton, N.J.: Van Nostrand.

Knoke, David, and Ronald S. Burt. 1983. "Prominence." Pp. 195–222 in *Applied Network Analysis: A Methodological Introduction,* edited by R. S. Burt and M. J. Minor. Beverly Hills, Calif.: Sage.

Leavitt, Harold J. 1951. "Some Effects of Certain Communication Patterns on Group Performance." *Journal of Abnormal and Social Psychology* 46:38–50.

Lehrer, Keith, and Carl Wagner. 1981. *Rational Consensus in Science and Society.* Dordrecht: Reidel.

Lewin, Kurt. 1951. *Field Theory in Social Science.* New York: Harper.

Markovsky, Barry, David Willer, and Travis Patton. 1988. "Power Relations in Exchange Networks." *American Sociological Review* 53:220–36.

Marsden, Peter V. 1983. "Restricted Access in Networks and Models of Power." *American Journal of Sociology* 88:686–717.

Marsden, Peter V., and Karen E. Campbell. 1984. "Measuring Tie Strength." *Social Forces* 63:482–501.

Marsden, Peter V., and Edward O. Laumann. 1984. "Mathematical Ideas in Social Structural Analysis." *Journal of Mathematical Sociology* 10:271–94.

Mizruchi, Mark S., Peter Mariolis, Michael Schwartz, and Beth Mintz. 1986. "Techniques for Disaggregating Centrality Scores in Social Networks." Pp. 26–48 in *Sociological Methodology,* edited by Nancy Tuma. Washington, D.C.: American Sociological Association.

Ord, Keith. 1975. "Estimation Methods for Models of Spatial Interaction." *Journal of the American Statistical Association* 70:120–26.

Rogers, Everett M. 1987. "Progress, Problems, and Prospects for Network Research." *Social Networks* 9:285–310.

Sabidussi, Gert. 1966. "The Centrality Index of a Graph." *Psychometrika* 31:581–603.

Stephenson, Karen, and Marvin Zelen. 1989. "Rethinking Centrality: Methods and Examples." *Social Networks* 11:1–37.

Wagner, Carl. 1978. "Consensus through Respect: A Model of Rational Group Decision-making." *Philosophical Studies* 34:335–49.

Wellman, Barry. 1983. "Network Analysis: Some Basic Principles." Pp. 155–200 in *Sociological Theory 1983,* edited by R. Collins. San Francisco: Jossey-Bass.

Willer, David. 1986. "Vulnerability and the Location of Power Positions: Comment on Cook, Emerson, Gillmore, and Yamagishi." *American Journal of Sociology* 92:441–44.