NOTES TOWARD A MATHEMATICAL THEORY OF CULTURE

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Abstract:

Over the last several decades, anthropology has created testable theory comparable to that found in other sciences. This paper summaries one view of that theory, the perspective of rule bound systems theory, or RBS. RBS theory turns out to imply that much of what has been taught and applied as "social statistics" for the least several decades, is based on the wrong combinatorial density function, which in turns helps explain why social forecasting is so often simply wrong. RBS however has made several tested correct forecasts, including in areas that have proven resistant to previous forecasting. RBS theory also has testable implications on broad scale questions of evolutionary biology.

I. INTRODUCTION

Anthropology is a hard science. That is, anthropology can, does and should make testable predictions like any other science. Mathematics is often a useful tool for this work. Like theoretical physics however, math for its own sake is not the purpose of theory. The research papers discussed here under the term "rule bound systems", or RBS, are a tested mathematical theory that connects descriptions of certain cultural structures to predictions of measures which may be found on empirical systems using those structures. They therefore form a testable theory of culture, or at least, of certain important aspects of culture.

The RBS framework is related to but distinct from and complementary to, other areas of social anthropology that have received mathematical treatment: kinship algebras and social networks. The purpose of this note is to summarize developments in rule bound systems theory, not survey developments in all fields. The text describes key RBS papers in Part II; discusses some relationships to kinship and network theory in Part III; summarizes certainly relationships to evolution of cognitive capacities in Part IV; cites the creation of a more complete mathematics for study of culture in Part V; and mentions broader implications in Part VI. Key RBS papers are cited in the style "RBS 1982", with a more complete citation at the end of the note.

Initial development of rule bound systems theory was in description of marriage rules, which led to the ability to draw inferences about certain population measures associated with particular rules. Thus many applications relate to population measures. RBS theory has successfully computed the time path of population measures of the demographic transition of western Europe, given only knowledge of the cultural system changes. It has been successfully applied to cultural systems and populations in India, to various North American cultural systems, and others. More recent RBS papers outline new inferences about cultural structures using the theory of group representations: for example due to Maschke's theorem and Schur's lemma, the "minimal representations" of the RBS theory of marriage structures allow predictions of population characteristics of any sized population. This in turn explains why the demographic statistics of RBS works on all populations, not simply on those near theoretical minimal sizes. This in turn means that those pretty structural diagrams of marriage systems typically drawn by social anthropologists in ethnographies, are actually very powerful analytical tools. Marriage systems also turn out to be structurally programmable automata; which in turn means that cultures, while unique and often literary phenomena, are subject to rigorous analysis. Several papers also develop applications of the theory outside of marriage systems, and show that RBS is a useful tool for many problems of development.

Social anthropology is also not independent of evolutionary theory. A singularity in one of the RBS equations distinguishes cultural systems of humans (and in fact of many mammals, and perhaps birds) from those of social insects. When compared to mechanisms of cultural transmission in diverse species, the singularity also separates systems by mode of transmission of "cultural" information; and therefore separates systems capable of more and less "plastic" development; and thus also, separates species capable of more rapid evolutionary development of cognitive systems from more stable species. Among other consequences, RBS therefore also predicts some conditions for the evolution of self-awareness.

Part of what makes the above results possible, and which distinguishes RBS from much else in mathematical anthropology, parallels the history of physics. One reason quantum theory differs from statistical mechanics is that they rely on different density functions for computing the basic predictions. Something very similar is going on with social anthropology. Many commonly taught (and predictively ineffective) "social statistics" rely on the same density function, the Stirling Number of the First Kind (SNFK), that also underlies much of thermodynamics and statistical biology. SNFK assumes for example that all of the atoms in a room are identical, and all of the locations in the room are different from each other, and then computes a possibility density function (that is, the SNFK) which shows all the possible ways that atoms could be distributed in that room. These traditional statistical models do have proper application for some social phenomena, such as for study of stationarity properties and growth rates associated with age-structured birth and death rates – the specific subject matter of traditional demography.

But the opposite of the SNFK assumptions are required by RBS theory: unique individuals, identical locations - that is, RBS studies unique persons filling a limited set of similar roles. The density function with these properties, and the one required by the algebraic mappings that turn out to underlie social anthropology, is the Stirling Number of the Second

Kind (SNSK). The SNSK underlines all of the successful predictions of the RBS papers. SNSF and SNFK have very different distributional and predictive properties, and relate to different even if inter-related phenomena. If one thinks of "ordinary biology" as requiring SNFK (which it does) and "RBS cognitive science" as requiring SNSK (which it does), and if both statements are true, there should be a nice commutative diagram connecting the measures of both kinds of systems, and there is.

The RBS framework is capable of much richer and more powerful mathematical development, on which some ideas are suggested in this Note. RBS theory also has some very strong philosophical implications due especially to the Stirling Number result mentioned in the above paragraph, which are discussed at the end of the Note.

II. SUMMARY OF BASIC PAPERS OF RULE BOUND SYSTEMS THEORY

Below is a brief summary of the main papers of rule bound systems theory, or "RBS". The papers are listed in logical order of their content. For simplicity papers are cited by the general form "RBS 1982" (for example) for a paper on the topic published in that year. There have been two monographs published on the topic, one in 1975 and the other in 1987; parts of these, such as Chapter 5 of the 1987 monograph, are therefore identified in the form "Chapter 5 RBS 1987". The papers and their contents are:

Chapter 2 RBS 1987: summarizes empirical structures observed by social anthropologists to represent marriage rules and marriage systems. Often, these are compact symmetrical diagrams using descent notation to represent the operation of a rule.

Appendix I RBS 1987: presents a rigorous theory of representation of the empirically described rule structures noted in Chapter 2, and defines concepts of "minimal structure" as the smallest self-reproducing graphical representation of the operation of a rule on a population of discrete generations. Minimal structures are similar to elementary structures in the Levi-Straussian sense. Appendix I was originally completed in 1971, and is related to Chapter 4 of RBS 1975, which expands the notation and derives some properties of a transition matrix representing movements between classes of isomorphic configurations of networks. Related to Appendix I also are papers by Duchamp and Ballonoff (1974 and 1975), which papers include representations of the commutative diagrams connecting population measures and properties of structural classifications, promised at the end of the Introduction above.

Appendix II RBS 1987: reproduces a paper by Schadach which includes summary of theorems on the relationship between algebraic classifications and combinatorial density functions. One major difference of the theory of RBS 1987 Appendix I from traditional social statistics is that the possibility density function underlying the theory of these papers is the Stirling Number of the Second Kind (SNSK), whereas ordinary statistics, based on thermodynamics, uses the Stirling Number of the First Kind (SNFK). Similar results to those in Appendix II are found in the standard combinatorics literature such as in van Lint and Wilson (1992 page 106) or Grimaldi (1989 page 178). This fact explains both why RBS predictions work, and most other social science predictions don't: traditional methods are using the wrong possibility density function. Among other things the SNSK assumes unique individuals, a nice property since biologically individuals are unique.

RBS 1982: refers to two papers published in 1982, described as Ballonoff (1982a and 1982b) in the references below, on "mathematical demography of social systems". These papers derive testable predictions on population measures (material measures on populations), from structural descriptions of RBS theory. They also show, in various appendices and examples, that the predictions work. The papers derive “a demography” (a particular set of population measures) directly from properties of a culture’s minimal structure, then demonstrate that the measurable properties of cultures and cultural changes predicted by this “demography” are in fact found in empirical populations. In other words, these papers help create a testable science for social anthropology, which, also, works.

RBS 1996a: shows that the minimal structures are unique minimal representations of the rules, and therefore, the properties of the minimal structures are properties of any system (of any size) following those rules. RBS 1996a summarizes the points above about density functions, then discusses why traditional information theory is not property defined for cultural systems, but that an information theory is possible for such systems. Such theory must consider that the information maximized by a culture is that related to cultural survival; a still more complete theory would also need to recognize that these results are context-dependent.

The most interesting part of RBS 1996b however is probably the discussion at page 322 and especially as summarized at the bottom of page 324 to top 325, about “mathematical groups in rule bound theory”. These pages show that the rules of a culture have a unique minimal representation. As a result, the minimal structure relates to the rules of any culture of any size - it is not a property of a small size population -- that is, the minimal structure is property of the rule not of the group size. This helps explain why the predictive results of RBS 1982 work. This result is importance for social anthropology generally, since it means that those efficient little diagrams that anthropologists draw to show how marriage systems work are not merely descriptions of "small systems". In fact, the are representations describing rule structures, and operate very much as do structural models of molecules in physics. They make it possible to forecast properties of systems using "bonding rules". These results also say why the 1982 papers on social demography give accurate numerical predictions of measures on so many diverse sizes (and diversely structured) systems.

RBS 1996b: is from the European Meetings on Cybernetics and Systems Research (EMCSR) conference of that year. RBS 1996b shows that the minimal structure diagrams (i.e. those same efficient pictures drawn in so many ethnographies to show how the rules work) are isomorphic to something known in the theory of parallel processing as "butterfly architectures". These are efficient architectures for the organization of parallel processors used for processing classification type information. Since butterfly architectures are efficient processors of classification systems (a special class of information) the analogy says something very important about cultural systems: they are efficient processors of classificatory information. Marriage systems essentially just classify the population and then process based on those classifiers. That result is very important for cultural theory and also cognitive theory: it proves that human cultures are efficient processors of (certain kinds of) information. One always suspected that, but this result proves it!

RBS 1995: shows that many of the "literary" properties of cultures actually make them act as structurally programmable automata; a fact that the work of Mary Douglas helps prove (no doubt much to her consternation, if she knew of the paper).

Chapter 5 RBS 1987: Chapter 5 of the 1987 monograph is a summary of some biochemical and bio-evolutionary evidence related to RBS theory. It discusses that neurological systems of animals may have evolved to provide a secondary, thus initially independent but redundant, means of controlling for effects of errors in more purely biochemical behavioral control systems. Chapter 5 also shows that the equations of RBS 1982 distinguish cultural forms such as found in humans (which seem to rely on cognitive abilities), from those such as found in social insects (which seem to be more completely determined directly by biochemical means), as being within more stable regions of possible forms of systems.

RBS 2000: references a conference paper for the April 2000 EMCSR, entitled "On the Origin of Self-Awareness". This paper discusses a consequence of one of the "demographic" equations of the 1982 papers. Chapter 5 RBS 1987 showed that the equations have consequences on an evolutionary time scale. RBS 2000 shows that a singularity in the basic equation of that theory, coupled with understanding of biomechanics of evolution, separates species which are more and those which are much less, likely to evolve self-awareness. Because evolution is a thermodynamic process, that is, dependent on random molecular processes, then given enough time any species might evolve self-awareness. But species whose "cultural systems" are "above" the singularity, such as mammals and especially humans (and perhaps also birds, though that result is not mentioned in the paper), are much more likely to evolve self-awareness sooner, than species whose measures fall “below” the singularity (such as social insects).

If one puts together all of these results, and with the revision to information theory suggested in RBS 1996a at page 319 (the information maximized by cultural systems is that which maximizes the survival probability of the system), one gets a very interesting overall result: cultural theory is a physically testable, empirically and independently observable theory whose overall structure is based on survivability of structures which process information in a maximally efficient manner, in which the processing is done by unique individuals using common rules. Thus RBS theory has many properties of both biological and physical theory, though it does not directly derive from either. Cultural theory is a separate branch of physical theory.

III. COMPARISON TO KINSHIP AND TO NETWORK THEORY

Much previous work (apart from RBS theory) on mathematical social anthropology focused on kinship algebras and/or on network representations of social systems. The topics are clearly related, since, for example, RBS theory includes notions of kinship algebras (see for example Appendix I of RBS 1987) and results in descriptions of networks in at least some circumstances. And both RBS theory and kinship or network theory relate to group theory representations of marriage systems. But the theories are also distinct.

Some of the topics added by RBS theory can be stated by relating them to a very nice paper summarizing many of the kinship and network theory results, namely White and Jorion (1996). At page 272 White and Jorion observe that with individuals as nodes, the cultural decomposition of the resulting graph of the elementary structure lacks a uniqueness property. This is not quite right, or at least, not if RBS theory is also considered. The objects White and Jorion describe in their figure 14B as elementary structures (in the Levi-Straussian sense) are isomorphic to what RBS theory Appendix I RBS 1987 calls minimal structures: the smallest graph that represents all of the actions and relationships under a given marriage and kinship system which also reproduces itself in the shortest generation time. For all minimal strucures of which this author is aware, one generation is sufficient for self-reproduction of the isomorphic minimal graph. It is isomorphic give or take a few rotations of labels, since these structures are also representable as permutation matricies and thus also as symmetric groups, as is well known.

Page 324 of RBS 1996a expands this analysis. Most previous attention on these structures has been on the fact that they are representable as symmetric or permutation groups. Equally important, is that the marriage matrix of the individuals depicted as nodes in the graph representing the minimal structure has zero trace. (See Duchamp and Ballonoff, 1974 and 1975, and also RBS 1975 Chapter 4). Empirically this is because these structures impose some form of "incest" restriction on marriage within their local groups within the same generation. But this fact is mathematically non trivial, it turns out, in the mathematical theory of group representations, as pointed out in RBS 1996a. Under Maschke's theorem the matrix of any such representation is completely reducible (to a minimal representation with zero trace) and by Schur's lemma, all such representations are equivalent. Since the trace of the marriage matrix of the minimal structure is zero, therefore RBS 1996a infers that the minimal representation (minimal structure of a marriage rule) is unique (the equivalence class of the reduced minimal representation). This is important not only because it shows that the minimal structure representation is unique. More importantly it justifies treating the minimal structure as a property of the rule under which the population is acting. The minimal structure is not a claim that a population is always "small" of size similar to the minimal size; the minimal structure is a tool for deriving mathematical properties of the action of the rule, which properties are relevant to any population using that rule. This is exactly how the inferences of the RBS 1982 papers were made, and RBS 1996a shows this was justified.

Therefore, among other consequences, White and Jorion's statement about the absence of a uniqueness property for systems representing the individuals as nodes is not quite correct. As summarized above from RBS theory, this form of representation can lead to identification of a particular elementary structure (to use the Levi-Straussian term) or minimal structure in the RBS terminology, as the unique minimal structure to represent that cultural system, and thus also to other very strong predictions about properties of systems using particular rules. Kinship theory has concentrated on the cognitive or relational structure of these systems, network theory on the empirical graphs they generate. RBS theory concentrates on the properties of rules and their consequences, especially on population measures, and not explicitly on either the empirical graphs generated nor the cognitive systems required. It is not that one or another perspective is “right” and the other “wrong”. Instead they are complementary perspectives with different purposes and different predictive properties.

For a somewhat similar reason, the statement at page 309 of White and Jorion is not quite correct that at the conclusion of their section 15, regarding the difficulty of “reading back” from models to properties of systems. In fact, one can "read back" from elementary structure type models to predict properties of the systems operating under a given rule. This is what the RBS 1982 papers do, based on the size (group order in fact) of the minimal representation of the elementary structure of the rule, and a few other properties, such as the generational depth of organization required to compute the structure. RBS computes and predicts (and tests empirically) predictions "read back" from the elementary structure, to predict the joint occurrence of certain population measures, namely, "family size", percentage of "married" females, and population growth (or decline) rate; these three values are conjointly predicted by the knowledge of the elementary or minimal structure of the rule. In some cases one can also predict, from the minimal or elementary structure, the bounded range of total empirical population sizes that might use a rule.

RBS theory does not however "read back" to the networks themselves, to predict the possible observations on connectivity in actual networks operating under such rule (except for the case of actual populations which may be as small as the minimal structure representation, in which case the minimal structure predicts the empirical network for those systems which survive and preserve the rule). But this can in principle be done for any size population. It requires more use of combinatorics and of lattices than has been applied so far to these problems. For example, White (JASSS 1999) discusses a notion of "maximal" representations (maximal apparently in a lattice theory sense), while RBS uses a notion of a minimal representation, which is minimal in a population and time count sense; but in terms of the classifications of isomorphic graphical representations, might be the maximal graph in a lattice of possible representations. A further distinction however is that the study of social networks seems to have been focused mainly on analysis of empirical networks, while the interest of RBS theory in networks is in their use to construct theories of what structures are possible, and certain of their properties.

Another relationship between networks and RBS theory is the study of isomorphism classes of graphs of relations. Ballonoff (1976) showed that study of the isomorphism classes of possible graphs of marriage systems forms a category under certain conditions, including with the existence of minimally stable systems. Given this paper and the transition matrix concepts of Ballonoff (1974) pages 190 - 193, it should be possible to construct an algebraic or transition matrix theory of the accessibility of certain graphical configurations from given configurations in prior generations, given a particular marriage and kinship rule. Minimal structure theory (Appendix I RBS 1987) is a limited form of such a theory. The more general theory might be purely algebraic and category-theoretic, or might be constructed on the lines of the work by Lind and Marcus *An Introduction to Symbolic Dynamics and Coding* (1995), which approach seems also related to the social network concepts of White and Jorion.

There are of course relationships between cultural kinship/marriage systems and biological systems of mating. For example, kinship diagrams are often used to describe the biological as well as purely classificatory aspects of descent. But the cultural systems of kinship and marriage are not (quite) biological systems of genetic descent. Cultural systems also allow for adoption and recognize this with the adopted kinship lables, hence mixing up a bit the purely genetic lines of descent. Human systems also can occur with "cheating" -- offspring that are not the biological offspring especially of the male named in the genealogy -- with various forms of tollerance if even detected in the real world. These facts have two consequences:

(1) a regular system of (genetic) mating requires only a small "leakage" in the pattern to become effectively equivalent to a random mating population of equivalent size (see the genetics literature on “effective size” of a mating population, and also Ballonoff 1975b or Chapter 5 of RBS 1987). Those papers used standard genetic measures of effective population size for their analysis; the measure of what is taking place genetically is different from measures of what is taking place culturally. Specifically, the combinatorics of genetic assortment is given by the Stirling Number of the First Kind, which is also why theories of evolution at the genetic level are equivalent to the mathematics of thermodynamics, hence the success of the work of Kimura (1964) and others on that topic. But:

(2) culturally, a different density function is required. The combinatorial density function of classification of a given number of unique individuals into a given number of similar families, where the actual biology of descent is ignored but the families are classified as a limited number of similar roles, is given by the Stirling Number of the Second Kind (see also page 320 of RBS 1996a, on choice of density function). This cultural classification does not depend on the specific genetic tracing of origin, merely the possible assortment of individuals into classes. It is thus RBS’ use of the SNSK density function that makes possible successful predictions of demographic population statistics at the cultural level in the various papers.

IV. RULE BOUND SYSTEMS THEORY AS A THEORY OF COGNITIVE EVOLUTION

A recent book by Terrence W. Deacon, *The Symbolic Species* (1997, W.W. Norton) provides much bio-evolutionary evidence that supports rule bound systems theory. Deacon references cultural systems in a way that implies the necessity for a testable theory of such systems. The RBS papers provide such theory, constructed and empirically tested in much the way required if Deacon’s arguments are correct. This discussion mentions only four citations to Deacon, but many more could be provided: (1) at pages 378 - 379 Deacon refers to the necessity of a symbolic system to be what he calls a “logical” group; (2) at page 383 he notes interaction of the cultural logic or symbolic system with group dynamics; (3) at page 392 he notes the relatively diminished role of pherormones in human and primate communications systems, compared to other species; and (4) at pages 400 - 401 he discusses the consequences of a “Levi-Straussian” view of social structure and of “symbolic culture ... representing a social contract”.

Deacon concludes that the biological evolution of human symbolic capacity was driven in large degree by the increasing relative value of such social capacity. What he does not provide, but which RBS theory does provide, is the social theory which shows that cultural systems have empirically testable properties which yield the conditions required for his biological results to occur, as he predicts.

Regarding Deacon reference (1), it is well known that "Levi-Straussian" representation of cultural systems can be represented as mathematical groups. This result was first published by Weil in the well know appendix to the work of Levi-Strauss, and elaborated as descriptive by for example Harrison White. But the papers RBS 1995 and 1996b showed that these structures are not merely symmetric groups, but this also demonstrates that cultures form efficient processes of classificatory information. RBS theory developed from a rigorous representation of human cultural systems as structures which yield representations as mathematical groups. Chapter 2 of RBS 1987 summarizes typical empirical descriptions made by anthropologists. Appendix I RBS 1987 shows the RBS rigorous description of ethnographic ideas of Chapter 1. But, unlike other work which has largely only described cultures using mathematical groups, RBS theory has also been able to draw very strong predictive conclusions from the mathematical representations. RBS 1996a includes discussion of how RBS representation of cultural rules permits the use of such representation to yield other strong inferences. RBS 1996b shows that such representations are not only groups, but also yield descriptions isomorphic to mathematical objects known as "butterfly architectures". In the theory of parallel processing, butterfly architectures describe the most efficient means of connecting independent parallel processors in a system to process classifications.

Thus, RBS not only supports Deacon's conclusions about the evolution of cultural structure, it also supports that cultural structures provide efficient means of processing the kinds of classificatory information discussed by Deacon. Demonstration that RBS minimal structures are isomorphic to structures known from theory of parallel processing also implies that study of cultures using similar methods, for example, as interacting systems of agents, such as proposed by Fischer and Read (1999) is also likely to produce results - especially if the agent theory includes study of concurrent systems and temporal logics similar to that used in the theory of parallel processing.

Deacon Citation (2) above refers to the necessity for a relationship between cognitive structures, the cultural facts they control, and "group dynamics". The two papers RBS 1982 demonstrate that certain of the properties of the mathematics that represent cultural systems as in RBS 1987 Appendix I, lead directly to testable and tested predictions of relationships between rules and measurable features of group dynamics such as family size, marriage rates, and growth rates. The RBS 1982 papers also discuss tests of this theory on independent data on diverse cultures. One of these demonstrates that the theory correctly predicts the 1000 year history of the "demographic revolution" of Western Europe, based only on knowledge of the cultural changes taking place over that period. There are also other published examples of applications of the theory successfully relating cultural to demographic measures, both in the RBS 1982 papers and elsewhere.

Deacon Citation (3) above refers to the diminished role of pherormones in primate, especially human, communications. Social systems exist using little more than chemical pre-patterning of culture members and their communication, such as in the social insects. But Deacon implies that these would not have the flexibility required for cognitive systems. Chapter 5 RBS 1987 discussed that the "cultural" differences between humans and social insects are related to differences in the flexibility of the means of communication, as derived from divergences in the species' means of implementing communication and control due to biochemical aspects of evolution. It also showed that such differences create social-cultural systems whose properties are predicted by the measures resulting from RBS theory. Using similar logic and other properties of the RBS 1982 paper equations, RBS 2000 shows that a singularity in the RBS 1982 equations imply the conditions for the evolution of self-consciousness, and therefore also why self-consciousness is at the least more likely to be prevalent in creatures with cultural systems (and biologies) similar to those of humans. RBS 2000 shows that the RBS 1982 equations define a threshold above which species are more likely to have cultures using neurologically based "cognitive" communication systems, and below which species are more likely to have more purely chemically (such as pherormonally) controlled behaviors.

Finally, Deacon citation (4) discusses the relationship of "Levi-Straussian" cultural logics to the evolution of social contracts. The structucturalist tradition of course derives from analysis of exchanges which form reciprocal obligations. RBS emphasizes how cultural logics, such as the principles of algebras which may describe them such as "logical" groups, affect operation of large scale cultural rules systems, such as legal systems. Thus, RBS 1994 and an expanded version, RBS unpublished, summarizes how RBS analysis has broader relevance, including on the general analysis of evolution of "social contracts", and on understanding the properties of political-legal systems.

Thus, cultural theory is not merely pleasant discussion of interesting literature. RBS theory can and does provide empirical means of testing claims about human evolution based on analysis of physiological evidence, such as the work of Deacon. Study of culture is as much a part of science, of "hard" testable science, as are the more classical sciences, and is interdependent with them in understanding general and human evolution. Analysis of culture is as much a part of science as are biology, chemistry and physics, and is as necessary.

V. TOWARD A MORE COMPLETE THEORY OF CULTURE

The papers primarily reviewed here, on rule-bound systems theory, are clearly not a full theory of culture. For example, Part III above pointed out some technical differences between the mathematics of RBS and that used for kinship algebras and network perspectives. Not only is the math somewhat different, but the empirical subjects are also somewhat different, though all are obviously necessarily related. Part IV mentions cognition, but clearly also RBS is not a theory of cognition; indeed it makes no attempt to describe cognition; at most is offers some insights on broad features of cognitive evolution related to structural properties of populations.

Obviously a more comprehensive framework is required, which can encompass RBS, kinship algebras, network descriptions, and cognition, as well of course as culture in the most general sense. This author believes that such a theory, or certainly essential pieces of such a theory, does now exist. This can be found in the works of I. V. Ezhkova. Ezhkova is known primarily as a mathematical theorist of artificial intelligence, and especially for creating techniques that make it possible to create context-sensitive AI machines. However, a fully implemented "Ezhkova Machine" is capable of much more than mere traditional AI. An Ezhkova Machine collects experience, from itself and when in networks, from other machines or devices. An Ezhkova Machine can organize this experience into data bases and also knowledge bases, and do so in ways appropriate to particular circumstances, selecting the relevant data and knowledge bases to the particular context. Ezhkova Machines can interact with their environments using symbolic representations of knowledge or of concepts. Based on their common experiences, Ezhkova Machines can create common symbolic languages for mutual communication, as appropriate to their experiences and concepts of experiences. In short, Ezhkova Machines have a ver human like intelligence. It is not entirely inaccurate to say that, mathematically speaking, culture is an Ezhkova Machine. Or perhaps more accurately, cognition - the culture and knowledge of a single individual in a cultural system – can be described mathematically as an Ezhkova Machine.

The work of Ezhkova has been published only in limited form in English. One early paper, in English is Ezhkova (1983) which presents a mathematical method for representing linguistic estimates between concepts with a universal scale measured using fuzzy sets, and then translating these estimates into some given natural language. [This paper is remarkably similar to the more speculative discussions of Lehman (1985 especially pages 39 and 40), on the requirements for a more complete theory of cognition.]. Later publications have elaborated and refined the algebraic framework for this context representation and analysis, such as Ezhkova (1989) and Ezhkova (1990). Ezhkova's notions include sufficient generalization of the underlying algebra to describe the space of possible experiences, and develop an algebra on such objects (using for example ordinary set theory or sentential logic operators).

Since these experiences can represent the knowledge of a single individual Ezhkova Machine, as well as what it has shared with (learned from) other machines or devices, then Ezhkova has created the basic frame work in which do (or represent) all of the mathematics of cognitive representation generally, kinship algebras (which are algebras on the representations of particular facts within the experience of a particular individual), social networks (which describe relationships among individuals within networks), and RBS (which describes rules applied to facts represented in part by kinship or social algebras known to individuals in networks). It is will probably also be found that Ezhkova Machines are a framework for study of cultures as interacting agents, as proposed by Fischer and Read (1999), since agent theory is itself a form of representation of AI which can be encompassed within the theory of Ezhkova Machines.

In a general summary, the Ezhkova Machine framework offers to the theory of culture very much what Hilbert spaces offer to the theory of theoretical physics: a general mathematical space in which to do all of the relevant computations. To fully implement Ezhkova Machines as a framework for all of this, and especially for RBS (which requires a space of possible representations covering many generations of interacting individuals) will probably require some expansion of the existing work. In particular, that the Ezhkova Machine framework of the earlier papers be expanded to show lattices of possible experiences, on which the various algebras (kinship rules, marriage rules, religious structures, etc.) then form filters or operators on these lattices. Much of what is currently done as linguistics and semantics would be representable as classifications and linguistic measures and verbalizations of particular machines in particular contexts. Much of what is now discussed as cultural structures would look like the category theory applicable to the properties of the various possible operators. Even the long term properties of cultural and biological co-evolution (as in Part IV just above) should be treatable as the study of the properties of operators acting on lattices of Ezhkova Machines.

VI. SOME BROADER IMPLICATIONS OF A THEORY OF CULTURE

A truly scientific theory of culture will have broad and important implications, including for practical actions of government, population policy, the philosophy of science, perhaps even for moral theory. This section explores some of those implications from the viewpoint of rule bound systems theory results noted above.

Notions of Social Statistics

Start with the concrete: with what we can count. Some rule bound systems papers do "statistics", but not the usual kind of "social statistics". Current academic notions of "social statistics" involve concepts of what are the right kinds of distributions and density functions to use for essentially everything. The origins of what is often called, in western universities, "the" philosophy of science for social sciences, derives from a trend of thought common in Europe especially in the first half of this century. That trend, under the name "positivism", saw that many aspects of physical, biological and economic sciences have mathematical constructs in common. In particular, for present purposes, positivism saw that statistical mechanics (thermodynamics), statistical biology, and aspects of economics all had the same basic mathematics. So, positivism asserted that all science must share these same mathematical distributions as their foundation. In the United States especially after the Second World War, but also in many European national systems (east and west), this idea took control of most social sciences.

The immediate technical effect in many academic traditions was that anything that did not agree with the positivist assertions on the nature of universal statistical methods was eliminated, as not being "science". Of course, if this were really science, it might not have lasted very long. Science tests its predictions and when they are wrong, changes its assumptions and methods. But "social science" of the positivist sort has been making consistently and spectacularly wrong forecasts for half a century or more. The public at large now discounts "academic forecasts" immediately. And even within the academic world, the idea has arisen that because of these failures it is inherently impossible to do some forms of social science better (or differently) than what they are doing. This is of course a politically convenient notion for those already in control of institutions, but it is not a scientific idea.

Now, technically, what was wrong with the positivist proposition, that all social sciences must share the same mathematical-statistical foundations as certain physical sciences? Consider what thermodynamics actually does. For example, it studies the distribution of atoms of gas in a room, in a physical space. To do this it assumes that all of the atoms are identical, and all of the locations in the room are different from each other. It then computes a possibility density function which shows all the possible ways that atoms could be distributed in that room. This density function is known technically as the Stirling Number of the First Kind (SNFK). Based on this density function it predicts (correctly) that pretty much all the time, the atoms will be evenly distributed in the room. Using the positivist premise, this basic notion then shows up in all manner of discussions. For example, essentially the same statistical logic underlies the idea that "entropy" also governs all of human relationships, since under the most literal application of positivism, the "second law of thermodynamics" applies literally to everything. Much modern political theory is also often based on these same ideas, to the extent they claim modern "social science" as their foundations.

The problem, as demonstrated by the massive failures of so many social science (and political) predictions, is that these methods are not universal to all social phenomena. What is wrong? The positivist assumptions lead to use of the wrong density function for analysis of at least many cultural phenomena. The correct function is derived explicitly in RBS theory. If one looks at the first RBS paper "Theory of Minimal Structures" (originally 1972 but reprinted as Appendix I RBS 1987), one finds a relationship between two generations of a population denominated as a "descent map". The descent map (actually, the inverse descent map, from off-spring to the possible population of parents) is a surjection. That is, while everyone has a parent (indeed, a pair of them) there might be some adults who have no offspring. In the graphical representation of a minimal structure, these are not depicted, but in general it is of course true. This is why the descent map has to be constructed as a surjection. This is a simple idea with a profound consequence.

The combinatorial possibility density function required for distributions of surjections is not the SNFK. Instead, it is the SNSK, or the Stirling Number of the Second Kind. *Notice that the above discussion therefore derived the necessity of use of SNSK from the properties of the descent map in the theory of minimal structures.* While use of SNFK by traditional social science theory and traditional statistical methods is an assumption of positivism, use of SNSK is not an assumption. The various tests of RBS theory in the 1982 papers and elsewhere shows that computations based on the SNSK also yield correct predictions.

RBS and Population Policy

It is useful to begin analysis of the implications of the above by discussing a specific and rather statistically oriented topic. Much "western" population policy has been based on the idea that "family planning" for "stable populations" requires that the ideal family has exactly 2.1 (or so) off-spring; the "extra" 0.1 is supposedly to account for stochastic sex-ratio fluctuations or the like. When this kind of family planning is applied to many cultures, the people in that culture often tend to object, rather strongly. Indeed, even the very "western" Catholic Church tends to object to these kinds of notions of "family planning for stable populations".

The RBS 1982 papers show that many of these objections may be well founded. The equations of those papers show that once we know which rules are used in a culture, then those rules let us compute three concurrent numbers: the average family size, the average proportion of folks having families (the proportion of adults as ascribed parents of offspring classified into recognized marriages or family units), and the growth rate. The 1982 papers show that different cultures have zero growth at different pairs of (family size, percent married) values. So, when traditional "scientific" family planning concentrates only on the family size, ignoring everything else, and tells everyone to have only two children, many cultures quite correctly take this as an attack on their basic cultural values. Of course, most members of most cultures don't know anything about RBS theory, but they know intuitively what makes their own culture work. It is also interesting that "scientific population planning" has totally failed to predict the actual trends of population growth, or its absence. For example, the "population explosion" predicted in the 60's and 70's to wipe out civilization by 1990 did not occur. In fact, world population growth, unpredictably by normal theory, slowed in most places. RBS theory could predict this. One of the RBS 1982 papers computes the path of growth rate changes of the "demographic transition" in western Europe, based on changes in cultural structures. RBS 1998 does a related computation for population growth rates of India, predicting, based on demonstrated changes in cultural structure, that the population growth rates increased when the structure changed earlier in the century, and would be slowing currently. In 1999 measurements of Indian growth rates showed slowing growth rates were indeed the case (for example as reported in the *Washington Post* of Monday August 23 1999 at page 1).

Human Uniqueness And Cultural Relativism

What else do the differences in assumptions of SNFK and SNSK mean for social sciences? One answer has to do with both human and cultural uniqueness. SNFK density functions are equivalent to assume, as noted above, identical particles, and unique locations. SNSK is the equivalent to assume the opposite: unique particles, identical locations. The "particles" in question are humans not atoms, and the "positions" in question are not physical locations but "roles".

But this also matches an intuitive notion of what social theory is, or ought to be, all about.

We can all see that humans are each unique; genetically, we know each is unique. But human cultures fit, or try to fit, most everyone onto a limited number of rather similar roles. Of course, not all roles are the same, and there is more than one role in each culture. But assuming a limited number of roles and unique humans is no worse a variation from reality than the assumptions made to construct a useful theory for physics. In reality in physics, not all atoms are identical: they may have more or less electrons even of the same compound, molecules of the same gas may have different physical configurations (indeed switch them continually), and that gasses in a room may not all be of the same kinds of molecules. None the less thermodynamics does pretty well with those slightly inaccurate assumptions when predicting its proper applications (atoms in a room, etc.). So, the assumptions of SNSK and cultural reality are not a perfect match. Not all humans are totally different (we most all have two arms, etc.). And not all social roles are identical. But this is no worse than how physical theory often differs from physical reality. Thus the most fundamental way that RBS theory differs from traditional social science is that traditionalists assume SNFK (and make wrong predictions), whereas RBS is required by its construction to use SNSK, and turns out to make correct predictions.

RBS theory recognizes that humans are all unique individuals, but that they relate to each other in systems of rule-like relationships, which we call cultures, which try to put those individuals into rather similar roles. However, this does not say that all human cultural systems are identical. Just the opposite, RBS theory requires recognizing that each culture may have its own system of rules and of the roles to which the rules apply, with different consequences for the operation of the system under the rules. The theory enables one to compute the result of applying the particular cultural system of rules to all those unique humans. These results might be rather different for different cultures (that is, for different systems of rules). So, RBS theory also recognizes that human cultures are or may be very different from each other. To understand a culture, one may not simply photo-copy everything known (or believed to be true) about one cultural system, and use those "facts" to predict what will happen in some other system. One needs to understand each culture on its own terms.

At first glance this seems to be just a restatement of "cultural relativism". Indeed, it might be similar to what Benjamin Lee Whorf meant when he first compared the semantics of native North American languages to English in discussing how to describe the risks of fire from a chemical compound. But in modern political applications, certainly in the US, "cultural relativism" has come to mean the assertion that "all cultures are equal" or have “equal value”. This is taken to mean that one assumes them equivalent or identical in their operation, and in how well they work; it leads to never questioning whether they work, or to compare the effects of their operation. A lot of U.S. and indeed, western, foreign policy and economic development theory has been based on this non-comparative version of "cultural relativism". A lot of US and western foreign policy and economic development policy has failed badly. It may be because the traditional form of relativism makes it impossible to clearly see how cultures in fact differ.

It is just the opposite of what RBS theory requires. Recognizing that each culture has its own properties is also not the same as saying that all cultures of all possible shapes are all going to be successful. For example, inherent in the computations of the RBS 1982 papers are two kinds of predictions: certain things that are computed by the theory will occur, and other things therefore will not occur. All those other things that do not occur are also possible cultures. Actually if one examines the detailed construction of the theory it turns out that it computes both success and failure possibilities; or to be more precise, growth and decline risks associated with use of each cultural form. In general, the RBS 1982 papers and others with the various "statistical" numbers in them, usually only present in print the values related to likely successful systems. There is an inherent assumption that over a few million years of human (indeed, mammal, and indeed, general) evolution, the kinds of cultural systems that are likely to fail have mostly been eliminated by a kind of natural selection on cultural forms, and for the most part human cultures will tend to pick the more likely successful options. (See RBS 1987 Chapter 5 or RBS 2000 for a discussion of some effects of RBS selection over evolutionary time scales).

Of course, failed cultures are well known to exist, though usually not for very long. Modern anthropology has often been too delicate to mention this fact, but a more realistic view has entered discussions in recent years. For example Robert Edgerton (in *Sick Societies, Challenging the Myth of Primitive Harmony*, 1992, The Free Press) has usefully opened a more objective analysis of the topic, while Shephard Krech (in *The Ecological Indian: Myth and History*, 1999, W.W. Norton) shows that non-western and pre-historic cultures were not necessarily in perfect harmony with nature.

RBS and Political Systems Analysis

At the political level, it is more difficult to avoid notice of failed systems. In recent times several more oppressive forms of human cultural organization on the large scale have indeed failed. One can predict which kinds of cultural systems, including broad scale political systems, are more or less likely to succeed, by also asking whether they tend to require SNFK or SNSK type assumptions. Political or economic systems that assume all humans are identical will tend to fail. Political or economic systems that assume all humans are potentially very unique, which recognize that humans live within cultures that may have very different properties, and allow for these facts, will tend to succeed. Thus, the political economies known as "Nazi" and "Communist" have both failed rather spectacularly in this century. Both assumed, in somewhat different ways, that all humans are pretty much identical in many ways. The communists especially assumed not only that are all humans pretty much identical in their "needs" but therefore as a result a small group of "scientifically" correct bureaucrats can decide what all those identical people need. This clearly has not worked anyplace where it was tried.

In general the more market oriented approaches have been the more successful ones. Of course it is well recognized that one of the things that makes markets more successful than bureaucracies to manage economies is that they allow for human diversity, which bureaucrats usually do not. But it is also very interesting to notice how the technical theory of market economics ("micro-economics" or sometimes "the theory of the firm") is constructed. Within that theory one finds something called "preference theory" or "utility theory". Look inside preference theory, and one finds that micro-economists assume "the incomparability of individual utilities". That is, they assume each human may have a unique "preference function". Micro-economists make no effort to claim all humans have identical "needs", they merely assume humans will act rationally to achieve whatever each thinks are their own needs. Humans may or may not be fully rational, but the theory based on the assumption has proven quite successful and predictive. This aspect of micro-economics is thus also technically compatible with RBS cultural theory. Micro-economics uses the SNFK to analyze cost and production functions, which are physical processes subject to physical laws, but does not use SNFK methods in the wrong places. To describe human needs and preferences, preference theory uses assumptions equivalent to SNSK, though this author is not aware of a specific reference in that literature doing so explicitly.

Notice that the above points out that when social theories deal with physical processes, such as economic models of cost or production, they need to recognize physical processes that use normal physical science. There is no incompatibility between physical science and social science, provided that the social science is in fact science, not just ideology. Social science does not become science merely by using words and concepts photo-copied from physics. Thus for example an awful lot of silly nonsense has been written about how the "second law of thermodynamics" and/or "entropy" governs all of human society. This was supposedly why the world was going to run out of energy and pretty much all of its other resources, which as for the population explosion, was also supposed to occur by 1990. Which of course it did not. Another version of very much the same false application of purely physical models to social theory are the predictions of "chaos" theory. Chaos theory has often become application of the elaborated thermodynamic models to many problems where it is just the wrong density function. In some very technical areas, chaos theory might predict some social processes. But merely because, for example, a lot of bureaucrats got confused when communists systems collapsed in east Europe, does not mean that "chaos" theory, in the mathematical sense, explains it. Chaos theory in fact did not predict those changes.

On the other hand, a lot of the post-soviet advice given by many western advisors to the former soviet states has also proven to be bad advice. Very often the advisors simply photo-copy western laws and institutions and try to impose that literal structure onto the post-soviet society. Yes, such systems need to develop market economies. But to successfully design a cultural change, one also needs to recognize the cultural properties of the existing systems and design in compatible manner. Even in the west, not all market systems are identical. They differ from each other in many very important details, such as of market rules, of competition laws, of property laws, of political structures and forms of government, of religions and the implications of religion for moral limits of market action, and so forth. What they share is the ability to recognize and allow for human individual uniqueness and for the existence of cultural differences among sub-groups of the national population.

RBS and Long Term Stability of Cultural Forms

The above mentions that specific numerical calculations of RBS relate to prediction of the survival probabilities of, and thus relative times of existence for, specific cultural systems. But the basic structure of the theory may also predict some forms of long term stability. As noted above, many successful human systems - cultures, political structures, economic systems - recognize the existence of human uniqueness within a limited number of cultural roles. Does this apply to the most long-lasting of continuously existing western institutions, the religions? The answer seems to be “yes”. One need not claim that religions are perfect social theorists to reach this conclusion. Take for example the Catholic Bible at Romans 12 - 6 on the duties of Christians: "having then gifts differing according to the grace that is given us, let us prophecy according to the proportion of faith". Now this author can make few claims to knowledge of theology, but it seems this is a statement recognizing the importance of SNSK – unique individuals having common duties (roles to play) to the Church. One can construct a similar argument that the Jewish faith has incorporated SNSK assumptions into its basic theology. (See for example the Notes to Genesis IX 6 or to Deuteronomy XVI 20 in the widely used Soncino Press prayer book *Pentateuch and Haftorahs*, 2nd Edition). The Jewish faith quite explicitly recognizes the notion of a culture organized under law, and that while individuals are unique, different cultures have different laws, resulting in different systems of behavior. Thus, making no claims about theological consequences, but taking western religious institutions as empirical examples of cultural objects, the ones which have been successful for the longest periods of time have properties that RBS theory suggests they need to have to survive.

Now, this is not all there is to say on these subjects. A separate paper tries to develop more general analytical techniques for treating larger-scale political-economic rule systems, using similar principles of analysis in a less mathematical way (RBS 1994 expanded in RBS unpublished, on “Rule Bound Systems as a Tool of Economic Development”). And, the paper RBS 2000 uses properties of the equations of the RBS1982 papers, in connection with some facts of biology, to show that these equations define a threshold in biological evolution separating species that are more likely to evolve self-consciousness from those which are less likely to do so.

Not yet written is that the necessity for successful systems to recognize both human uniqueness and the differing properties of distinct cultures, says something about whether and if so why, in the longer term human cultures might tend more toward "decency". But that is another topic.

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