UNIVERSITY OF CALIFORNIA, SAN DIEGO

Talking Math, Blogging Math

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by

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The thesis of Linda Marie Mathews is approved and it is acceptable in quality and form for publication on microfilm and electronically:

Chair

University of California, San Diego

2009

DEDICATION

To Bob and Rosie. Thank you.

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ABSTRACT OF THE THESIS

Talking Math, Blogging Math

by

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Chris Halter, Chair

Talking Math, Blogging Math is a curriculum designed to aid middle school Pre-Algebra students' mathematical problem-solving through the use of academic language instruction, explanatory proofs, and online technology (blogging). *Talking Math, Blogging Math* was implemented over a period of ten weeks during the 2008 – 2009 school year. The school where the curriculum was implemented is a non-traditional classroom-based charter school. The 7th, 8th and 9th grade students attended class twice a week.

The goals of the *Talking Math, Blogging Math* curriculum were to increase use and understanding of mathematical terminology, to facilitate students' use of problem-solving by teaching them metacognitive strategies, and to use blog technology to extend the Pre-Algebra community of practice beyond the classroom. The students received direct instruction in academic language related to the mathematical content being studied. Students learned metacognitive strategies in the context of simple two-column and paragraph explanatory proofs. Students communicated with each other on the blog about mathematical concepts covered in class.

Methods for evaluating *Talking Math, Blogging Math* included examination of students' work, blog entries, and interviews. Final analysis of *Talking Math, Blogging Math* revealed that students used varied metacognitive processes when creating mathematical proofs. These metacognitive processes transferred to their general problem-solving. The findings suggest that the students increased their use and understanding of mathematical vocabulary. In addition, the students created a community of discourse based on the explanatory proofs.

I. INTRODUCTION

Recently I was blogging with my Pre-Algebra (7th, 8th, and 9th grade) students on our class wiki. A wiki is a webpage that anyone can edit to add content or to update information (Richardson, 2009). I had posted a question about equivalent equations to our class blog on the wiki that I wanted my students to discuss. As I read our current thread, I noticed that one of my students was having problems understanding the concept because she was having difficulties understanding the mathematical terminology. After several posts back and forth, she finally understood that adding the same values to both sides of the equation did not change the solution. Her last post ended with “Math is cool!” She had found success in her own mathematical understanding through the use of technology mediated academic language.

If you had difficulties following some of the “tech speak” in the previous paragraph, you are not alone. However, this is a lexicon that is very familiar to our middle school and high school students. *The Horizon Report* published by New Media Consortium (2007) identified user-created content and social networking as areas that would “have significant impact on college and university campuses within the next five years” (p. 6). The PEW Internet & American Life Project (Lenhart, Madden, & Hitlin, 2005) found that in 2004 over 87% of teens used the Internet and over half of teens went online daily. An informal survey that I took in my Pre-Algebra class indicates that my middle-school students are regular users of many Internet applications including user created content and social networking. I decided to create a curriculum that tapped into the students interest in technology.

But first let me describe North County Charter Academy, the school where I teach. My school is a non-traditional, non-classroom, based public charter school in the northern part of San Diego County. The middle and high school students attend content area classes at our site twice a week. My Pre-Algebra class is composed of 7th, 8th, and 9th grade students. For some of these students, this is the second time they are taking Pre-Algebra. Our Pre-Algebra class meets for a total of 3 hours each week. The students are all from a local community that is mostly white and upper middle class. They are all native speakers of English.

When my students are not in class , which is 60% of the time, they are expected to complete work at home. If students have questions about the homework, they can email me or post questions to the class wiki. In addition, I post supplementary materials to the wiki. My class and I have started to communicate with each other using a web log or blog that is available on the wiki. Even though students can email me or post questions to the wiki, there are many times that the students do not attempt the homework because they do not know how to approach a problem. My students often have problems describing their problem-solving methods and expressing themselves mathematically. In addition, many students have difficulty understanding the mathematical terminology that they hear in the classroom or read in their textbooks. As a result, these students struggle in Pre-Algebra and continue to struggle in Algebra 1. Their access to the content has been limited by their understanding of the academic language used in the mathematics classroom.

The California Mathematics Framework (2005) cautioned that students need to have a strong grounding in the fundamentals of Algebra in order to be successful in more advanced math courses. Thus, it is imperative that Pre-Algebra students develop a good understanding of basic algebraic problem-solving and reasoning in order to have a good foundation for Algebra I and higher math classes. I developed the *Talking Math, Blogging Math* curriculum to help my students gain better understanding of mathematical terminology and their problem-solving processes.

II. ASSESSMENT OF NEED

"Participation for all clearly does not translate automatically into success for all" (Rosin, Barondess, & Leichty, 2009, p.1).

In June 2008, the California State Board of Education mandated that the Algebra I California Standards Test be the test used for federal accountability for students in the 8th grade. Unfortunately, many California math students are not prepared to take Algebra I in 8th grade (Rosin, Barondess, & Liechty, 2009). As a result, many students take the required math courses a year behind the California State Board of Education guidelines. Figure 1 shows the 2008 participation levels for the CST Algebra I Test and the CST General Mathematics Test. Just over half (51%) of California's 8th grade students attempted the CST Algebra I in 2008. The majority of California's 9th grade students (53%) took the CST Algebra I in 2008, a year behind the schedule mandated by the *California Mathematics Framework*.

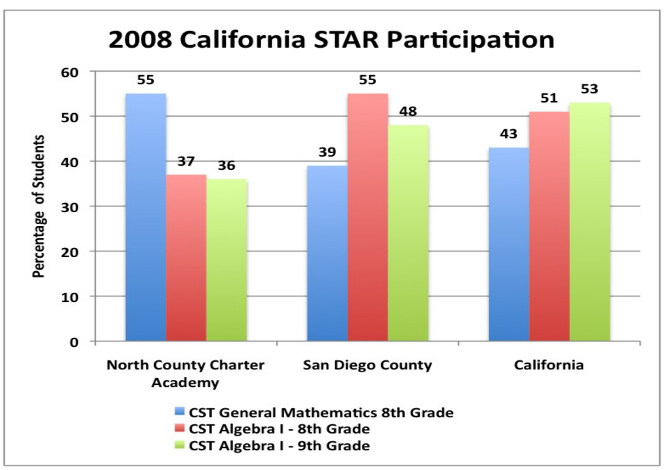


Figure 1: Percentage of students taking CST Algebra I and CST General Mathematics (California Department of Education, 2008).

The data for San Diego County is a similar to the data for the state of California.

A little more than half of 8th grade students (55 %) and about half of the 9th grade students in San Diego County (48%) took the CST Algebra I in 2008. At North County Charter Academy, only one third of the 8th grade and 9th grade students took the CST Algebra I in 2008. Almost 55% of the 8th grade students at North County Charter Academy took the CST General Mathematics instead of the CST Algebra I.

Of California's 8th grade students that took the Algebra I CST in 2008, approximately one third (31%) scored below basic or far below basic (see Figure 2). Moreover, a majority (53%) of California's 9th grade students who took the CST Algebra I test in 2008 scored below basic or far below basic. The statistics for San Diego County and North County Charter Academy are analogous to those of the entire state.

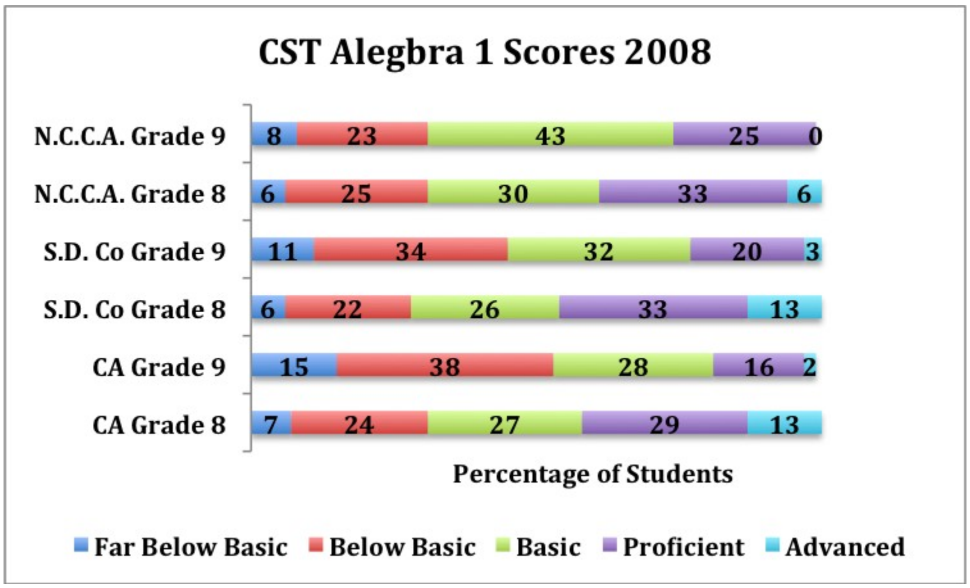


Figure 2: CST Algebra 1 8th and 9th Grade Scores for California (CA) 2008, San Diego County (S.D. Co), and North County Charter Academy (N.C.C.A) (California Department of Education, 2008).

These statistics for the 2008 STAR testing show that students who took Algebra I in 8th grade, the schedule called for by the California Mathematics Framework, were more likely to score proficient or higher on the CST Algebra I than students that took Algebra I in 9th grade, a year behind the recommended timeframe. In addition, over a third of the students who took the 2008 Algebra CST in 9th grade had taken the test in 2007 (Rosin, Barondess, & Liechty, 2009). While there are several reasons why the students were retaking Algebra I, these numbers imply that many students were not successful.

Although the grade 7 CST Mathematics test was not designed to be an assessment for Algebra I readiness, research has shown that standardized tests and grade 7 exit grades are predictors of a student's success in Algebra I (Belli & Gatewood, 1987; Flexer, 1984). One could assume that the 30% of California students, 24% of San Diego County students, and 22% of North County Charter Academy students who scored below basic or far below basic on the 7th grade CST Mathematics do not have adequate math skills to succeed in Algebra I in 8th grade (Figure 3). From my own classroom experience, those students who scored basic and below on the grade 7 CST Mathematics struggled when they were placed in Algebra I in 8th grade.

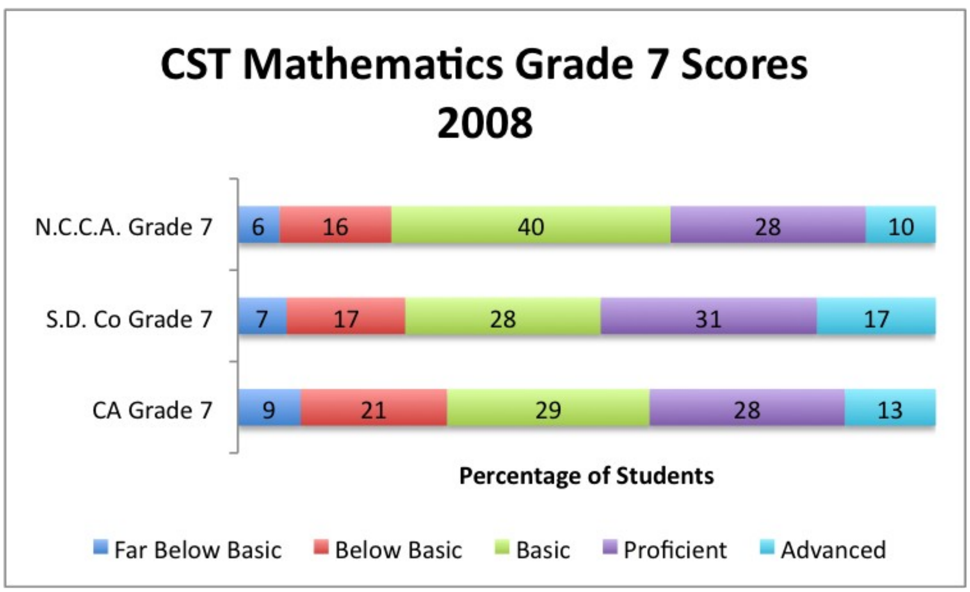


Figure 3: CST Mathematics Grade 7 Scores 2008 (California Department of Education, 2008).

Although *The Nation's Report Card: Mathematics 2007* (Lee, Grigg, & Dion, 2007) documents that overall math scores have steadily increased from 1990 – 2007, the National Assessment of Educational Progress (NAEP) math assessment test scores for California's 8th grade students still lag behind the national average. The (NAEP) math assessment for 8th grade tests knowledge of concepts in algebra, geometry, number properties and operations, and data analysis and probability (Lee, Grigg, & Dion, 2007). As shown in Figure 4, in 2007, 31% of the nation's 8th grade students scored at or above proficient on the NAEP math assessment. In contrast, only 24% of California's students scored at the proficient or advanced level (Lee, Grigg, & Dion, 2007). The ultimate goal is to have all students score at proficient or above on the NAEP assessment.

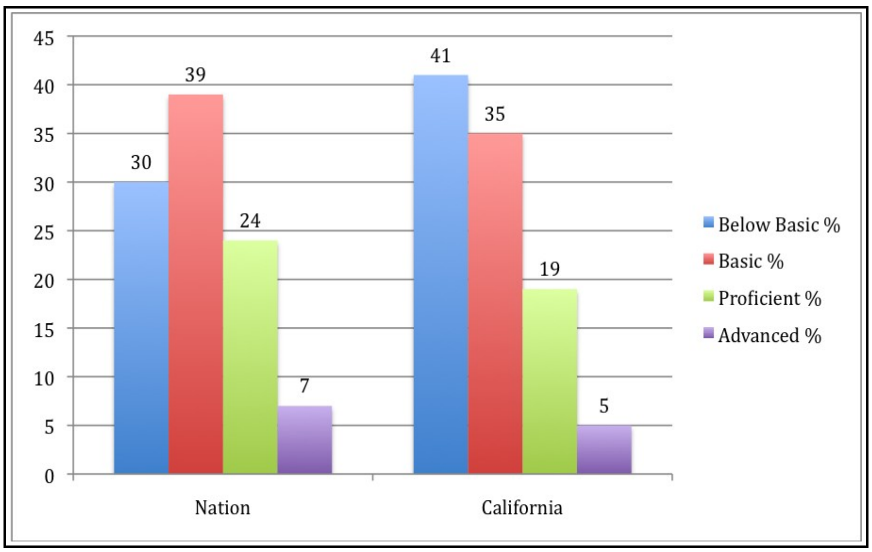


Figure 4: NAEP test results 2007.

It is clear from state and national test data that many of our students are not reaching mathematical proficiency in California's classrooms. Research has shown that students have difficulty moving from arithmetic reasoning to the more abstract algebraic reasoning (Booth, 1981; Herscovics & Linchhevski, 1994). One possible reason is that students have difficulty using and understanding the language of mathematics (Monroe & Panchyshyn, 1995; Thompson & Chappell, 2007; Zevenbergen,2000). In addition, many students are not aware of the problem-solving processes that they use and therefore make limited use of different problem-solving approaches (Pape & Smith, 2002; Schoenfeld, 1987).

The National Council of Teachers of Mathematics (NCTM) standards calls for a change from the teacher-driven classroom to a student-centered classroom in which students use the language of mathematics to describe their mathematical problem-solving (National Council of Teachers of Mathematics, 2000). In the student-centered classroom, the teachers help their students learn through "a process of mathematical communication in social contexts" (Forman, 2003). The idea that learning is social is the basis of the sociocultural theory (Vygotsky, 1986; Wink & Putney, 2002) that has been driving the mathematics instruction reform movement. The next chapters discuss sociocultural reform and what it looks like in a mathematics classroom.

III. LITERATURE REVIEW

As stated in the last chapter, current research into mathematics reform has suggested the adoption of a sociocultural model of learning in the mathematics classroom based on the ideas of Vygotsky and his contemporaries. Forman (2003) notes that these sociocultural theories have influenced research, including mathematical research, since the 1970s. The acceptances of sociocultural theory and research continued to grow and to develop in the 1980s and 1990s due to increased access to the works of Vygotsky and other members of the Soviet sociohistorical school (Minick, Stone, & Forman, 1996). The increased awareness of the works of Vygotsky and others led to research based on the application of the Soviet sociohistorical concepts to the study of cognition. These studies led to increased interest in how social interaction is organized within institutional contexts, including the development of interpersonal relationships in specific learning communities of practice. Mathematical communities of practice are learning communities that share norms for doing mathematics including a common language for describing mathematical concepts and problem-solving (Forman, 1993; Lave and Wenger, 1991; Zevenbergen,2000)

Vygotsky proposed that learning is a process that is meadiated by language and occurs through collaboration with others (Vygotsky, 1986; Wink & Putney, 2002). The role of the teacher in the learning process is to act as a mediator and mentor in order to guide students through their personal zones of proximal development. The zone of proximal development (ZPD) describes the difference of a learner's development level when they are attempting independent problem-solving versus doing expert assisted problem-solving (Bruner, 1985; Vygotsky, 1986). Tharpe and Gallimore (1988) propose that instructional conversation assists the learner in moving through his or her zone of proximal development. The instructional conversation adds new meanings to learners' existing mental structures through learner-teacher interaction.

Lave and Wenger (1991) postulate that a learner becomes a member and participates in a community of practice through a process called "legitimate peripheral participation" (p. 29). Legitimate peripheral participation allows the novice to learn from more capable peers or experts in a community of practice by engaging in the actual practice of the more capable peer or expert. Zevenbergen (2000) states that students achieve legitimate peripheral participation in a mathematics community of practice through their familiarity with and ability to use and understand the mathematics register. Students' levels of linguistic competence determine whether they become a full participant in the mathematical community of practice (Zevenbergen, 2000). The teacher's role in the classroom community of practice is to aid students in realizing which mathematical practices are acceptable and which are not according to the teacher's and students' prior negotiation of sociomathematical norms (Cobb, Wood, & Yackel, 1996; Yackel and Cobb, 1996).

In review, sociocultural theory states that learning is social in nature. The interaction of a learner with a more capable peer or expert can help learners move through their zone of proximal development through the use of instructional conversations in communities of practice. Learners interact with these communities of practice with different levels of legitimate peripheral participation. A learner belongs to many communities of practice in school, at home, in their neighborhood, and in the culture at large. Learners can belong to many online communities of practice as well.

Mathematical Discourse

Sfard, Forman, and Kieran (2001) stated that learners interacting in small groups and taking part in whole class discussions have replaced the traditional mathematics classroom that featured silent passive learning. Many of these changes are due to the NCTM standards that call for students to use communication to organize their mathematical thinking and to communicate this thinking to others, including teachers and peers. This new focus of research resulted in a shift away from experimental methods using laboratory problem-solving to descriptive studies of everyday activities inside and outside the classroom. These new non-interventional studies produced dramatic results showing that people who fail on school mathematical tasks are able to perform mathematical cognitive functions in a non-school setting (Sfard, Forman, & Kieran, 2001). Additional research (Forman, 2003) showed that sociocultural theory contributes to teaching and learning when teachers take in account students' prior-knowledge and in-school and out-of-school experiences.

The new sociocultural based research required the use of different sources of unit analysis. The most widely known units of analysis for sociocultural research include discourse, activity, and practice (Sfard, 2001; Sfard, Forman, & Kieran, 2001; Van Oers, 2001). Van Oers (2001) defined mathematical activity as the actions developed by human beings to deal with different relationships in their cultural and physical environments. Mathematical practice encompasses the cultural values, rules, and tools brought to an activity. A mathematical community of discourse includes learners metacognitively understanding mathematical activities. The concepts of mathematical activity, practice, and discourse were used to define "real mathematical activity" as legitimate participation in a mathematical practice. This participation encompasses discursive mathematical actions and legitimate mathematical activities based on the participant's cultural history. Van Oers (2001) stated that the social process of mathematical learning could be viewed through the lens of apprenticeship learning or a process of legitimate peripheral participation in which the learner gradually increases participation. The teacher has an important and active role in defining the classroom activity and discourse and in helping students take part in mathematical practice.

Sfard (2001) proposed a communication approach to cognition that describes thinking as a form of interpersonal communication. She described the learning of mathematics as an initiation to mathematical discourse. This mathematical communication is shaped and regulated by mediating tools and meta-discursive rules. The mediating tools shape the object-level aspects of the mathematical discourse. Learners use the mediating tools to communicate with themselves and each other. The meta-discursive rules are the behind the scene rules that allow and determine the general path of communicational activities. The meta-discursive rules are the sociocultural norms of what it means to do math. Cobb, Wood, and Yackel (1996) described how the teacher and students negotiated meaning in the classroom through two levels of discourse. One level of discourse dealt with the negotiation of mathematical meaning and the other level of discourse dealt with the negotiation of how to talk about math. The meta-discursive rules encompass the cultural norms, values, and beliefs about communication that teachers and students bring into the classroom (Sfard, 2001).

While communication in mathematics classroom can help students move through their zone of proximal development, many students may not know how to “do” mathematical discourse. Thompson and Chapell (2007) note that mathematical communication includes the use of speaking, reading, writing, and listening. When teachers ask the students to explain their reasoning through speech or writing, both the teacher and the students gain insight into the students' levels of understanding (Thompson and Chapell, 2007). However, students often have difficulty describing their reasoning because they do not know how (Schoenfeld, 1987) or because they do not know the mathematical language needed to describe their reasoning (Thompson and Chappell, 2007; Zevenbergen, 2000). In addition, students' ability to use and understand mathematical discourse depends on their exposure to the language of mathematics outside of the classroom (Sfard, 2001; Zevenbergen, 2000). Students who have had prior exposer to mathematical language outside of school are better positioned for success than those students who have had limited exposure (Zevenbergen, 2000). In order for students to have an active role in the negotiation of mathematical meaning in the classroom, the sociomathematical norms and the communication skills needed for mathematical discourse and reasoning need to be explicitly taught (Cobb, Wood, & Yackel, 1996; Lampert, 1990; Sfard, 2001; Sfard & Kieran, 2001; Van Oers, 2001; Zevenbergen,2000).

In response to the NCTM's call for students to have a more active role in their learning, Lampert (1990) explored the creation of a mathematics community of discourse in the her 5th grade classroom. Lampert wanted to create a social situation in her classroom that used different rules of engagement than those normally seen in a classroom. She wanted to change her students' ideas of what it means to know and do mathematics. Lampert collected observational data during a specially designed learning unit centered around exponents. The questions that Lampert asked during this unit required students to reflect on mathematical assumptions and what it meant to "know math." Lampert, as the teacher, guided the student's learning by modeling the student's roles, guiding the solutions, and solving problems with the class. Lampert's observational data over the course of the unit showed that the students did engage in mathematical discourse. She noted, however, that her students had exhibited types of discourse (silence, pressuring peers, face-saving behavior) that might not be conducive to mathematical learning.

In another study that looked at mathematical discourse, Goos (2004) described the results of an action research project she performed in an 11th and 12th grade classroom in Australia over a two year time period. Data collected for this study included classroom observation, interviews with teachers and students, and recall interviews where students and teachers viewed and interpreted videotape excerpts of the instruction. The research results focused on the teaching and learning practices that lead to the creation of a community of practice in a secondary mathematics classroom. Goos noted that the teacher and students negotiated practices that led to participation in this community of practice. In the study classroom, the teacher invited the students to actively discuss and investigate the concept being taught. He guided the discussion to help the students build their knowledge of the concept. The teacher and students used scaffolding and peer collaboration to move through their zones of proximal development. In addition, the students made connections between prior knowledge and mathematical concepts to internalize their learning.

Goos described the teacher's attitudes and expectations regarding mathematical instruction and learning that are framed in sociocultural theory. Some of these attitudes and expectations included that teacher scaffolding of mathematical inquiry develops mathematical thinking and that students can create and test mathematical thinking through the use of instructional conversation in a community of practice.

Cobb, Wood, and Yackel (1996) defined two levels of discourse that they saw in a mathematics classroom as talking about and performing mathematics and talking about talking about mathematics. When talking about and performing mathematics, the teacher implicitly guided the students in the negotiation of mathematical meaning and the renegotiation of the social norms of the classroom. The teacher's discourse was more explicit when talking about talking about math. Cobb, Wood, and Yackel stated that the relationship between these two levels of discourse was in the form of a dialogue that influenced mathematical discussions and problem-solving that followed. They concluded that “students’ mathematical learning is influenced by both mathematical practices and the social norms negotiated and institutionalized by the classroom community” (Cobb, Wood, & Yackel, 1996, p. 114).

In their research, Sfard, Forman, and Kieran (2001) examined how ineffective communication can scuttle interactive mathematical learning. They had started to implement a research project that was to examine how a teaching unit could stimulate and support algebraic thinking. However, during the collection of data, Sfard, Forman, and Kieran noticed that many of the students, including their case study pairs, could not easily communicate with each other about mathematics. Sfard, Forman, and Kieran (2001) examined the object-level and meta-level discourse patterns of the case study pairs. Sfard, Forman, and Kieran’s analysis of the students’ discourse led them to modify some of their beliefs about learning and doing mathematics. They noted that just because students are communicating does not mean that they are working together in a synergistic manner. They concluded that students need to have strong motivation and communication skills in order for learning-by-talking to work.

While teachers and students use mathematical language to communicate about math, to build understanding about mathematical concepts, and to assess and self-reflect on mathematical learning , the language of mathematics has its own specialized vocabulary, syntax, and lexical density (Thompson & Rubenstein, 2000; Zevenbergen, 2000). Zevenbergen (2000) notes that teachers often assume that all students can decode this mathematical register and do not take in consideration the different language experiences the students have had at home and in school. Unfortunately, students who cannot master this mathematical register find it difficult to become legitimate participants in the mathematics classroom community (Lave & Wenger, 1991; Zevenbergen, 2000). Students need to master the specialized vocabulary, semantic structure, and syntactic structure of mathematical discourse in order to become successful learners of mathematics (Thompson & Chappell, 2007; Zevenbergen, 2000).

Mathematical Vocabulary Instruction

Monroe and Panchyshyn (1996) identify four categories of mathematical vocabulary that are used to describe mathematical concepts. Technical vocabulary describes specific mathematical concepts. Examples of mathematical technical vocabulary include words such as “quotient” and “integer”. The definitions of many of these technical terms contain other technical vocabulary. Students often have problems learning and remembering these terms because they occur in such a restricted context.

Subtechnical vocabulary has parallels in everyday vernacular but carries a different meaning or a more precise meaning in mathematics than in everyday English (words such as “origin” and “slope”). Students may understand the meaning of the words in everyday language or in other content areas, but may not be able to conceptualize the meaning in mathematics. For example, the word “degree” has different meanings in mathematics and science. Monroe and Panchyshyn (1996) further note that many terms can have multiple specialized meanings in mathematics (such as “degree” and “vertex”).

General vocabulary or everyday vocabulary can present problems for students as well. Although much of the vocabulary in mathematical textbooks is general vocabulary, many of these words do not occur in the students’ other reading texts (Monroe & Panchyshyn, 1996). In addition, many mathematical terms are homophones with every day words (“pi” and “pie”). Related mathematical vocabulary can cause a confusion of meaning (“expression” and “equation”) (Thompson & Rubenstein, 2000).

Monroe and Panchyshyn (1996) observe that the highly abstract symbolic vocabulary of mathematics can be very difficult for students to master. The symbolic vocabulary includes the many abbreviations used in math. For example, students need to learn that foot or feet can be represented by the abbreviation ft. or the symbol ‘ (as in 2’ 4”). These pitfalls make understanding and using mathematical vocabulary difficult for many students.

Thompson and Rubenstein (2000) offer several strategies for facilitating vocabulary development. These strategies include introducing a concept before labeling the concept, discussing problems in groups, and writing about mathematics. Thompson and Rubenstein (2000) suggest the use of word origins to help students make connections between everyday and specialized mathematical vocabulary. Gay (2008) suggests the use of graphic organizers and analogies to help students understand the differences between examples and non-examples for a particular term or concept.

Jackson and Phillips (1983) investigated how vocabulary instruction in ratio and proportion would affect the understanding of seventh grade students. Jackson and Phillips found that students who had received direct mathematical vocabulary instruction showed higher achievement on verbal and computational tasks than the students in the control group.

While the language of mathematics can sometimes be difficult for students to understand, direct instruction in mathematical vocabulary can help students gain understanding. Some strategies teachers can use to help students are to use graphic organizers, to employ examples and non-examples, to introduce word origins and related words, and to write about math using the vocabulary. Explanatory proofs provide one way to write about mathematical problem-solving.

Self-Regulated Learning and Metacognition

Zimmerman (2002) describes self-regulation as a “self-directive process by which learners transform their mental abilities into academic skills” (p. 65). Self-regulated learners are proactive learners that use metacognition or knowledge and awareness of their thinking to achieve academic goals. Zimmerman (2002) posits that self-regulated learning traverses through three cyclical phases that occur before, during, and after the learning effort takes place. Each phase is comprised of different self-regulatory processes.

During the first phase, which Zimmerman (2002) identifies as the forethought phase, self-regulated learners use task analysis and self-motivation processes. The task analysis processes involve goal setting and planning of strategies. The self-motivation processes involve processes such as self-efficacy and outcome expectations. Self-efficacy involved learners’ beliefs about their learning capabilities. Learners’ self-efficacy has a direct effect on their personal learning outcome expectations.

Zimmerman (2002) calls the second self-regulatory phase the performance phase. During the performance phase, learners use metacognitive processes such as self-control and self-observation to implement the goals and strategies that they identified in the forethought phase.

During the third phase, self-reflection, learners reflect on and evaluate their learning efforts. Learners’ compare their learning performance against a benchmark they created or that was created for them. Evaluation of their learning processes may lead learners to increase or decrease their self-satisfaction about learning. These self-reflections influence students’ self-efficacy in subsequent learning processes. Much of the research on self-regulated learning in mathematics has focused on metacognition and self-efficacy (Garofalo & Lester, 1985; Schoenfeld, 1987; Pape & Smith, 2002).

Pape and Smith (2002) examine the ways that teachers can help students become self-regulated learners in the mathematics classroom. They explain that the mathematics reform content and process standards (NCTM, 2000) require students to become self-regulated learners of mathematics. In the process area of problem-solving, Pape and Smith note that nearly three quarters of the students in their investigation tended to directly translate word problems without making a mental model of the problem. Successful problem solvers use these mental models and focus on structural rather than surface features. Pape and Smith emphasize the need for teachers to create a classroom environment in which students use metacognitive thinking. Garofalo and Lester (1985) define metacognition as explaining and justifying mathematical reasoning, setting goals, making plans of action, monitoring their progress, and reflecting on the outcomes in order to create new goals and action plans.

Garofalo and Lester (1985) identify a framework that includes four categories of activities used to perform mathematical problem-solving. These categories are orientation, organization, execution, and verification. Each phase includes a set of distinct metacognitive behaviors. During the orientation phase, learners use behaviors that help them assess and understand a math problem. These behaviors include the use of comprehension strategies, analysis of information, and assessment of familiarity, task difficulty, and chances of success. During the organization phase, learners identify problem-solving goals, plan the overall approach to solving the problem, and plan the steps needed to implement the overall plan. During the execution phase, learners monitor the progress of the plans made in the organization phase and make decisions between different solution paths. Finally, during the verification phase, learners evaluate their problem-solving plans and execution and determine if the solution is consistent with the problem conditions.

Schoenfeld (1987) notes that, overall, students do not have a good understanding of their thought processes. Novice mathematicians tend to read a problem and then go immediately to solving the problem. These novice math learners tend to stick to a particular way of solving a problem even if their problem-solving technique is not getting anywhere. In contrast, expert mathematicians spend more time analyzing the problem and planning how best to solve the problem. When expert mathematicians reach a dead-end in their problem-solving, they step back and reanalyze the problem and change their plans accordingly. Expert mathematicians spend more time analyzing the problem and planning how best to solve the problem than novice math learners. Schoenfeld (1987) suggests teacher modeling of metacognitive behavior, whole class discussions about problem-solving, and problem-solving in small groups as ways to help students become more aware of the processes of problem-solving.

Explanatory Proofs

Traditionally proofs have performed many different functions in mathematics (Hanna, 2000). Some of these functions have included persuasion, verification, systemization, communication, exploration, and explanation (Hanna, 2000; Knuth, 2002). In grades K-12, mathematical instruction emphasized logical reasoning and proof in higher-level topics such as geometry and calculus while mathematics instruction in general arithmetic and algebra stressed proficiency and skill development (Yackel & Hanna, 2003). However, as part of the mathematics reform movement, the NCTM (2000) standards call for the study and use of proofs at all levels of mathematics.

Hanna (1990, 2000) and Hersh (1993) identify three different perceived roles of proofs in mathematics education. The first role, the formal proof, is a syntactic structure that is based on axiomatic sentences and inferences made about those sentences. The goal of a formal proof is to prove the truth of the initial statement. Hanna (1990, 2000) notes that the use of formal proofs was adopted by universities and in secondary education as part of the new math curricula in the space race era of the 1960s.

The second role of proof that Hanna describes, acceptable proof, relies more on the mathematical meaning, or semantics, that a mathematician draws from the proof. Mathematicians' judgment is used to determine the validity of an argument rather than the formal structure of an argument. Mathematicians use this form of proof to communicate with other mathematicians.

The third role of proof that Hanna describes is the teaching or explanatory proof. An explanatory proof describes why a particular concept is true rather than proving truth through deductive reasoning. Hersh (1993), Hanna (1990, 2000), and Knuth (2002) note that explanatory proofs can aid student insight and understanding of mathematical properties and theorems. Yackel and Hanna (2003) notes that the use of explanatory proofs in elementary and secondary education can enhance understanding and communication. However, mathematical argumentation and mathematical reasoning are skills that must be taught if students are going to learn how to think mathematically (Yackel & Hanna, 2003). Students often have problems with the conceptual understanding, mathematical language, and structure associated with formal proofs, especially when the students are doing proofs for the first time in high school (Yackel & Hanna, 2003).

Communication, Mathematics, and Technology

In addition to the more rigorous math standards called for at the state and local levels, students are facing new technology requirements (NCTM, 2000; California Department of Education, 2005). The NCTM, as part of its chapter on Principles for School Mathematics, states that “technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning” (p. 11). The Mathematics Framework for California Public schools calls for all students to be familiar with basic computer skills, different computer applications, and the Internet. According to MacBride and Luehmann (2008) teachers are now faced with the challenge of incorporating new technology into their curricula and classroom. On the plus side, this new technology capitalizes on students’ familiarity with different forms of online communication. Groth (2008) notes that technology such as discussion boards and blogs allow for teachers to monitor multiple conversations about math at one time and to use the blog discourse as a formative assessment tool to guide what is taught in the classroom.

Richardson (2009) notes that over ninety percent of our students use social web technologies such as MySpace and instant messaging in their personal lives. The PEW Internet & American Life Project (Lenhart, Madden, & Hitlin, 2005) research data showed similar results. The PEW Internet & American Life Project (Lenhart, Madden, & Hitlin, 2005) surveyed a sample of 1,100 children between ages 12 to 17 and their parents or guardians. The survey found that in 2004 over 87% of teens used the Internet and over half of teens went online daily. In addition, teens preferred communicating through instant messaging (IM) rather than email. The PEW Internet & American Life Project (Lenhart, Madden, & Hitlin, 2005) found that most students become avid Internet users in the 7th grade. But, as Richardson (2009) observes, educators have been very slow to use what he calls the Read/Write web because most educators lag behind their students in the use of technology in their everyday lives.

The Read/Write web allows users to easily create content through the use of applications such as wikis and blogs. Weblogs were one of the first widely adopted tools of the Read/Write web (Richardson, 2009). Richardson (2009) defines weblogs as “an easily created, easily updateable website that allows an author (or authors) to publish instantly to the Internet from any Internet connection” (p. 17). Weblogs or blogs are now being used in conjunction with video and audio technology on sites such as YouTube and MySpace.

Richardson (2009) identifies several ways in which blogs can be used to improve student learning. First, blogs allow teachers and students to construct content that becomes part of a larger body of knowledge on the Internet. Thus, students’ work has a potential audience beyond the immediate classroom. Blogs can also help students who are uncomfortable speaking up in the classroom environment to become active participant in classroom discussions online.

Blogs allow teachers to connect with their students’ outside of the classroom. Teachers and students can collaborate with each other and other students, teachers, and professionals working on the same content. Students can use the information in the classroom blogs and other blogs on the web to build expertise on a specific topic or in a particular content area. Blogs provide a way for this collaborative work to be archived for later reflection.

Richardson (2009) observes that students need the opportunity to use web technology as part of the development of their technological literacy. He explains that classroom blogs can be used as classroom portals, online archives, collaborative space, and e-portfolios for students and teachers alike. The research of MacBride and

Luehmann (2008) and Pyon (2008) demonstrate some of the different ways that blogs can be used in the classroom.

In their case study, MacBride and Luehmann (2008) examined the use of web logs (blogs) in one high school mathematics classroom. MacBride and Luehmann wanted to explore how blogs could be used in secondary math and science classrooms by examining the intent, use, and perceived value of an existing high school classroom blog. The case study data collected by MacBride and Luehmann included teacher interviews, student interviews, and one year's worth of blog data.

MacBride and Luehmann identified several themes that defined the teacher's motivation to blog. These themes encompassed a wish to increase collaborative learning, to nurture a community of learners, to create a student centered learning environment, to provide the students a place for reflection, and to provide class enrichment. In addition, they identified several perceived benefits of blogging. MacBride and Luehmann found that blogging allowed students to have a real audience, motivated the student to do improve their work, provided an opportunity for the students to learn from each other, invited reflection and discussion about their work, and provided a wider audience for their work.

In a similar study, Pyon (2008) studied the impact of blogs on mathematical discussions in a third grade classroom. Pyon wanted to see if the blog would help her third graders improve their metacognitive thinking skills In addition, Pyon wanted to see of her students could construct knowledge when using the blog as an online classroom. Pyon's students used the blog to discuss ideas and solutions for multistep mathematical problems. She posted multistep story problems every other week for a period of two months.

Over the course of the research, Pyon noted that her students became adept at blogging and that an online community of practice had formed. She also noted that her students interacted more positively with each other online than they did in the classroom. Pyon concluded that her action research showed that the blogs helped her students regulate their own thinking and learning and led to increased metacognition. However, both Pyon (2008) and MacBride and Luehmann (2008) stressed the importance of having practices in place to scaffold mathematical discourse.

In conclusion, many middle school students are already avid users of Read/Write technology on the Internet (Lenhart, Madden, & Hitlin, 2005). Teachers feel that this technology can increase collaborative learning, nurture a community of learners, create a student centered learning environment, give the students a place for reflection, and provide class enrichment. Students can benefit from this technology in that they have a real audience, feel motivated to improve their work, can learn from each other, and can reflect on and discuss their work with a wider audience. However, Richardson (2009), Pyon (2008), and MacBride and Luehmann (2008) all note that the use of blogs need to be structured and scaffolded for the students for the blogs to reach their full potential.

Summary

Mathematical reform driven by the NCTM (2000) standards stresses the importance of communication as part of learning in a mathematics classroom. Researchers have described what this reform will look like through the lens of sociocultural theory. Many researchers (Cobb, Wood, & Yackel, 1996; Lampert, 1990; MacBride & Luehmann, 2008; Pyon, 2008; Sfard, Forman, & Kieran, 2001; Van Oers, 2001) worked at creating communities of practice in their classrooms that encouraged mathematical discourse. The students were assisted through their zones of proximal development through interaction with their teacher and other students. Their studies showed that the mathematical discourse would need to be scaffolded so that students would be successful at creating meaning and learning from the discourse. It is important that students practice the use of mathematical terminology in context in both spoken and written form in order to become full participants in the mathematics community of discourse. Teachers can use explanatory proofs and mathematics terminology instructions to scaffold the development of students' metacognitive behaviors and to practice using the mathematics register.

IV. CURRICULUM REVIEW

Traditional mathematics curriculum has viewed learning as a "treatment" to be given to the students in order to increase their mathematical knowledge (Lampert, 2003). Often this "treatment" is given in the form of direct instruction with the goal of the students taking away new knowledge. The NCTM (2000) standards call for a more active role for students in their learning. But how does the NCTM-driven reform look in the classroom?

Calls for Curriculum Reform

Both the National Council of Teachers of Mathematics' *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics, 2000) and the *Mathematics Framework for California Public Schools* (California Department of Education, 2005) call for changes in the mathematics curriculum used in our schools. The NCTM standards envision a mathematics curriculum that has interconnected topic strands that are organized so students can see how the ideas build on and connect to one another. The standards call for teachers to plan their instruction around mathematically important ideas that include foundational ideas (place value, equivalence, functions, etc.) and mathematical reasoning skills (making conjectures, deductive arguments, etc). In addition, mathematics curriculum should build on ideas taught in previous grades so that the student is accumulating mathematical knowledge.

In contrast, the *Mathematics Framework for California Public Schools* (California Department of Education, 2005) calls for a mathematics curriculum that balances proficiency in basic computational and procedural skills, development of conceptual understanding, and proficiency in problem-solving. According to the *Mathematics*

Framework for California Public Schools (California Department of Education, 2005), mathematics instruction should be balanced around three components of proficiency: basic computational and procedural skills, development of conceptual understanding, and mastery at problem-solving. The California framework and standards were a reaction to the standards put forth by NCTM. The controversy surrounding the adoption of the California state standards is documented in several places, including Schoenfeld's (2004) article *The Math Wars*. Because California, Texas, and New York standards drive textbook adoption in most states, the back to basics approach called for in the *Mathematics Framework for California Public Schools* (California Department of Education, 2005) is reflected in many of the nation's textbooks (Schoenfeld, 2004).

There have been many new curriculums created based on the NCTM (2000) reform standards. One example of these new curricula is the QUASAR (Quantitative Understanding: Amplifying Student Achievement and Reasoning project) (Silver & Stein, 1996). This project was implemented over a period of five years in middle schools in six urban school districts. The goal of the QUASAR project was "to provide instruction that will encourage student development of conceptual understanding and thinking, reasoning, and problem-solving in mathematics" (Silver & Stein, 1996, p. 481). Features of the QUASAR curriculum include the promotion of mathematical communication, thinking, reasoning, understanding, and problem-solving. Silver and Stein (1996) state that the majority of the tasks presented to the students involved sophisticated mathematical reasoning, multiple solution strategies, and explanation or justification of their solution. The students frequently participated in mathematical discourse and collaborated on different mathematical activities. Silver and Stein (1996) conclude that the QUASAR project students outperformed their counterparts at similar schools on the NAEP Grade 8 math assessment. In addition, evidence from their studies suggest that the QUASAR instruction provides students access to algebra in 9th grade.

Another curriculum based on NCTM is The Connected Mathematics curriculum, which was developed out of the Connected Mathematics Project (CMP) funded by the National Science Foundation (Ridgway, Zawojewski, Hoover, & Lambdin, 2003). This curriculum, designed for 6th, 7th, and 8th grade students, stresses attention to the connections that students can make between the mathematical ideas they are learning and their interests and experiences outside of the classroom. The goal of the curriculum is to provide students exposure to mathematical thinking and problem-solving and to prepare students for higher mathematics at the high school and college level. Grade 6 students work on number theory, probability and statistics, two-dimensional measurement and geometry, rational numbers, and spatial visualization and reasoning. The curriculum introduces 7th grade students to variables and representations of relationships, similarity, proportional reasoning, integers and the number line, linear relationships, three dimensional geometry and measurement, probability, and number sense. In the 8th grade, students explore representing relationships, the Pythagorean Theorem, exponential relationships, quadratic relationships, algebraic reasoning, symmetry and transformational geometry, statistics and probability, and combinations. The Connected Math curriculum calls for teachers to help their students make sense of mathematics by clarifying the context of a problem, assisting the students in their mathematical inquiry, and then summarizing the students' strategies and ideas. The teacher's role is to guide the students to make connections in their learning.

The Wisconsin Center for Educational Research and the Freudenthal Institute at the University of Utrecht in the Netherlands collaborated with middle school teachers to develop the *Mathematics in Context (MiC)* curriculum based on the Dutch Realistic Mathematics Education approach (Romberg & Shafer, 2003). The creators of the MiC curriculum wanted to create a curriculum program that would meet the NCTMs vision of learning a mathematics in a supportive learning community (Romberg & Shafer, 2003). Encyclopedia Britannica publishes a commercial version of *Mathematics in Context*.

Unlike most middle school curriculum, the MiC curriculum targets math education in grades 5 – 8. In each grade level, the students cover content related to numbers and number theory, basic algebra, measurement and geometry,and probability and statistics over the course of 10 units. The goal over the course of the curriculum is to move from informal to more formal mathematical reasoning in each strand. The MiC curriculum provides detailed teacher's guide for each unit at each grade level.

The *Middle Grades MATH Thematics* is a grade 6-8 curriculum created by the *Six through Eight Mathematics (STEM) Project* at the University of Montana (Billstein & Williamson, 2003). The goal of *MATH Thematics* is to help students become independent learners that can communicate connections between mathematical concepts and apply mathematics to other content areas and real life activities. The MATH Thematics curriculum is organized by grade level strands. These strands cover number, measurement and geometry, statistics, probability, algebra, and discrete mathematics. In addition, the curriculum stresses the concepts of proportional reasoning, multiple representations, patterns and generalizations, and mathematical modeling. There are eight content modules for each grade. McDougal Littel, a Houghton Mifflin Company, publishes *MATH Thematics (Billstein & Williamson, 2003)*.

Chappell (2003) reviewed the preliminary research studies for Connected Mathematics (Ridgway, Zawojewski, Hoover, & Lambdin, 2003), Mathematics in Context (Romberg & Shafer, 2003), and Middle Grades MATH Thematics (Billstein & Williamson, 2003). Chappell notes that the evaluation of these studies shows that students' conceptual and procedural mathematics understanding may have increased due to these new curriculums. However, Chappell states that long term longitudinal research needs to be conducted to determine the long term benefits of the programs.

Mathematical Pedagogy Practice

There have been many different views as to what counts as real math instruction in the classroom. The subject matter of mathematics has been viewed as the mastering of arithmetic operations, the study of abstract structures, and problem-solving of real life situations using symbolic tools. The traditional sender-receiver model stressed by the traditional view of mathematics learning has been replaced by the sociocultural approach which stresses that math is a cultural activity and learning emerges out of communities of practice (Van Oers, 2001).

Hiebert (2003) described the traditional curricula and pedagogy styles used in the United States. In traditional pedagogy practice the teacher answers previously assigned homework, demonstrates the new material, and leads the students through some practice problems that the students will complete later on their own. In this model, the teacher transmits the mathematical knowledge to the students through direct instruction (Van Oers, 2001). The teacher student interaction uses Initiation-Reply-Evaluation (IRE) sequences (Mehan, 1998). This sequence seeks known information from students that the teacher either confirms or corrects. This traditional model does not encourage discussion or interaction between the teacher and the students or students and other students.

Researchers interested in mathematical reform have called for a change in classroom discourse practices so that students have a more active role (Forman, 2003). The traditional classroom IRE discourse patterns do not encourage discussion in mathematical communities of practice. Herbel-Eisenmann (2007) noted that teachers and textbooks often act as the major authority in the classroom. The textbook's authority comes from its use as an answer source. The textbooks also provide feedback about the correctness of the answer.

The important lesson from the research is that curriculum design can influence how and what the teacher teaches and the students learn. In order for students to have an active role in the mathematics classroom, these traditional discourse and pedagogical patterns need to be changed. Zemelman, Daniels, and Hyde (2005) address some of these pedagogical practices in their discussion about best practices for the mathematics classroom.

Best Practices for Mathematics

Zemelman, Daniels, and Hyde (2005) compiled these best practices from "summary reports, meta-analyses of instructional research, accounts from exemplary classrooms, and landmark professional recommendations" (p. 5). Zemelman, Daniels, and Hyde (2005) call for curriculum that is student-centered, develops higher-order thinking, and emphasizes collaborative learning that allows students to benefit from the power of social interactions. Schooling that is student-centered builds on the students' interests and curiosity. As such, the learning should be experiential, holistic, and authentic. Students should be aware of the big picture – even if that big picture is complex. Finally, students also need to learn how to be metacognitive about their learning.

Zemelman, Daniels, and Hyde (2005) propose that teachers need to help all students to recognize that "mathematics is the science of patterns" (p. 112). The goal of mathematical instruction should be to help all students create a deep conceptual and procedural understanding of mathematics. Teachers need to help students make connections, create representations, use reasoning, develop proofs, communicate ideas, and do problem-solving. Zemelman, Daniels, and Hyde (2005) stress that if students are to become fluent in arithmetic computation, they must master counting, number relations, place value, the meaning of operations, and fact strategies.

Zemelman, Daniels, and Hyde (2005), in conjunction with the NCTM standards, call for students to have opportunities to build their algebraic knowledge through all the grades (K-12), to learn geometric and measurement concepts through hands-on activities and to tie statistical concepts to real-world applications. Teachers should use learning assessments to determine what students know and to influence planning of future teaching and learning activities.

Educational Blogging

Henry Farrell (in Downes, 2004) listed several ways that blogs are currently used in education. Teachers have often replaced their webpages with blogs because blogs are easier to update than webpages especially for information rich in text. Teachers also use blogs for keeping lists of web links that they want their students to access. These blogs can act as a type of annotated bibliography of website links if the teacher adds website descriptions.

Richardson (2009) identifies several ways in which blogs can be used to improve student learning. First, blogs allow teachers and students to construct content that becomes part of a larger body of knowledge on the Internet. Thus, students' work has a potential audience beyond the immediate classroom. An interesting effect of using blogs for classroom discussions is the creation of equity in the discussion (Downes, 2004; Pyon, 2008). Students who might not feel comfortable contributing to discussions in the classroom setting often are willing participants in the classroom blog.

Teachers often use blogs as a common place for a class to discuss class readings. In this case a group of individuals are asynchronously contributing to the blog. This is an easy way for a class to share information with other members of the class and the larger community. Teachers and students can collaborate with each other and other students, teachers, and professionals working on the same content. Students can use the information in the classroom blogs and other blogs on the web to build expertise on a specific topic or in a particular content area. Blogs provide a way for this collaborative work to be archived for later reflection.

MacBride and Luehmann (2008) and Pyon (2008) investigated the use of blogs in the mathematics classroom. They noted that there were several perceived benefits to blogging including helping students regulate their own thinking and learning, providing an opportunity for the students to learn from each other, and creating an on-line mathematics community of practice. Richardson (2009) observes that students need the opportunity to use web technology as part of the development of their technological literacy.

Mathematical Curriculum Used At North County Charter School

Glencoe Pre-Algebra

In my classroom we use the California edition of Glencoe Pre-Algebra (Malloy, Price, Willard, & Sloan, 2005). The textbook, chosen by the school curriculum committee, is aligned to the *Mathematics Content Standards for California Public Schools: Kindergarten Through Grade Twelve (1997)*. The textbook is designed for traditional mathematics pedagogy where the teacher teaches a concept and the students do problems in order to practice the new concept or skill. The textbook comes with a wide range of support materials that include work sheets, assessments, and open-ended assessments. Students can also take quizzes online at the Glencoe Pre-Algebra website. The text is available online to registered users. I do not always follow the order of lessons recommended in the textbook. I bring in handouts from different mathematical sources to supplement the text. Because my school relies on the students to study at home on the days they are not at the academy, my students often have to read the description and instructions on how to do a type of problem without my help.

Compass Learning

Compass Learning Odyssey Math is a problem-based online instruction and assessment tool that allows students to work on areas of weakness and to extend their learning based on their current level of mathematical understanding. The students' curriculum level is determined by their scores on the Measure of Academic Progress (MAP) test that they take at the start of the school year and at the end of the school year. The Northwest Evaluation Association (NWEA) provides the MAP test suite. As students work through different lessons and problem sets, they take quizzes that allow them to assess their learning. The teacher can assign specific content exercises for the students. The lessons align to NCTM standards and the California Mathematics Framework. The lessons focus on algebraic concepts, problem-solving, and reasoning skills. Pre-Algebra students at North County Charter School are expected to complete 30 minutes of Compass Learning exercises each week.

Summary

The NCTM *Principles and Standards for School Mathematics* (NCTM, 2000), *Best Practice: Today's Standard for Teaching & Learning in America's Schools* (Zemelman, Daniels, & Hyde, 2005), and the *Mathematics Framework for California Public Schools* (California Department of Education, 2005) call for reform of the mathematics curriculum in K-12 schools. The NCTM standards (NCTM, 2000) and Zemelman, Daniels, and Hyde (2005) call for curricula that consists of interconnected topic strands that emphasize mathematically important ideas and mathematical reasoning skills. They call for curricula to be student centered rather than the the traditional mathematics model where the teacher transmits the mathematical knowledge through direct instruction.

The California framework (California Department of Education, 2005) calls for curricula that stresses proficiency in basic computational and procedural skills, developing conceptual understanding, and developing proficiency in problem-solving. One goal of California framework is to prepare all students to study algebra by the 8th grade. The California framework recommends that mathematics instruction follow and curricula support a three phase model. In the first phase, the teacher introduces the new concept, asks questions and checks for understanding. The students should be actively involved by asking questions, having discussions, or listening to and thinking about the teachers presentation. During the second phase the teacher provides scaffolding so that the learners can make the transition from teacher-supported to self-regulated. During the third phase students work independently.

Research on curricula based on the NCTM standards suggest that students' conceptual understanding may have increased due to the new student centered curricula. In addition, research on NCTM reform based curricula that includes the use of technology suggests that technology, specifically blogging, is a useful tool in the mathematics classroom.

V. TALKING MATH, BLOGGING MATH

"If school lessons are to involve learners doing mathematical work, classrooms will not be silent places where each learner is privately engaged with ideas"

Lampert and Cobb (2003).

Constructs and Goals

In the NCTM reform based mathematics classroom, the teacher develops a community of practice in which the students begin to learn the language of mathematics (the mathematics register). In this community of practice, the teacher and students use a common language to discuss math and to explain their mathematical reasoning (Forman, 2003; Lampert & Cobb, 2003; Zevenbergen, 2000).

In the *Talking Math, Blogging Math* curriculum, students gain understanding of the mathematics register through spoken and written discourse with the teacher and with other students. Students use mathematical discourse to describe their understanding of their problem-solving processes (metacognition). The teacher and students work together in a community of practice to create understanding of the mathematical concepts.

The overall goal of the *Talking Math, Blogging Math* curriculum is to increase students' understanding of how they do math. The combination of instruction in mathematical vocabulary, mathematical discourse, and explanatory proofs in a sociocultural classroom setting helps students to determine, to apply, and to describe what they know about mathematics. This larger goal can be further divided into three distinct goals that involve the constructs of mathematical discourse, metacognition, and community of practice.

Goal 1: Increase the use and understanding of mathematics register in mathematical discourse in the classroom.

Mathematical discourse can present problems for students because of its specialized syntax, lexical density, and content specific vocabulary (Zevenbergen, 2000). Students’ difficulties using mathematical language become problematic when teachers use student’s mastery of written and spoken mathematical language as a tool to assess mathematical learning and understanding (Thompson & Rubenstein, 2000). Students' lack of mastery of the mathematics register can lead to the student and teacher having very different understandings during mathematical discussions. Therefore students need to receive explicit instruction in mathematical language so that they become comfortable with writing math and talking math (Lampert, 1990; Sfard, 2001; Zevenbergen, 2000).

One way to help students learn and use mathematics register is to provide instruction on key mathematical terminology and give the students the opportunity to use the vocabulary in context in the classroom (Forman, 2003; Thompson & Rubenstein, 2000; Zevenbergen, 2000). Students need to master the technical, subtechnical, general vocabulary, and symbolic vocabulary of the mathematics register (Monroe & Panchyshyn, 1996). Once students use and understand the mathematics register, they can become legitimate participants in the mathematics classroom community of practice (Lave & Wenger, 1991; Zevenbergen, 2000).

Goal 2: Increase student’s mathematical metacognition.

One goal of the mathematics reform movement is to make students more aware of the connections that help build mathematical understanding. That being said, many students of mathematics never learn to look beyond the surface structures of a problem when performing mathematics problem-solving (Pape & Smith, 2002). They directly translate the words of a problem into a mathematical sentence without trying to create a mental representation of the problem (Pape & Smith, 2002). Because most students are unaware of their thought processes when they solve mathematical problems, students may try to use a more complex mathematical concept to solve the problem when a simpler concept would be more efficient (Schoenfeld, 1987).

Novice learners of mathematics need to receive instruction on how to do problem-solving. In addition, novice learners of mathematics need to learn how to understand their thinking process during problem-solving (Pape & Smith, 2002; Schoenfeld, 1987). Direct instruction in mathematical terminology can help students express their understanding. The use of explanatory proofs, proofs that help explain mathematical constructs, can help students envision and explain what they are doing during problem-solving (Hanna, 2000; Hersh, 1993). While proofs are more commonly performed in high school and college math, simple proofs can be used in the pre-algebra classroom to aid in understanding. Two-column and paragraph proofs can be used to scaffold the explicit learning of metacognitive behaviors used during problem-solving. Examples of two-column and paragraph proofs are described in the activities section of this chapter.

Goal 3: Extend a classroom community of practice so that the teacher and students are still "talking math" together when students are working at home.

The sociocultural aspects of mathematics reform stress the creation of communities of discourse or communities of practice in the mathematics classroom. Teachers and students create a community of practice in their classroom through their shared negotiation of what it means to learn. Instructional practices in these reform classrooms need to focus on the emphasis of higher-order thinking and problem-solving (Forman, 2003). The univocal discourse patterns often used by teachers, where the teacher lectures and the students listen, need to be augmented with dialogical discourse patterns that allow students to be an active part of their mathematical learning. Students can negotiate mathematical meaning as part of a classroom community. Schoenfeld (1987) notes that small group work can help students learn metacognitive skills needed to gain deeper understanding of mathematical problem-solving.

A classroom community of practice does not need to be confined to the physical classroom. Today's communication technology allows students and teachers to continue their mathematical discourse outside of the classroom. Research has shown that teachers and students can create mathematical communities of practice through the use of technology (MacBride & Luehmann, 2008; Pyon, 2008). Blogs (weblogs) allow students and teachers to continue creating mathematical understanding asynchronously outside of the classroom. By participating in a mathematics course blog, students can continue to practice problem-solving, metacognitive strategies, and mathematical discourse on their computer at home.

Activities Supporting *Talking Math, Blogging Math*

Figure 5 shows the interconnection of instructional activities for the *Talking Math, Blogging Math* curriculum. The curriculum is designed to aid students in the mastery of using mathematical terminology through the use of direct instruction in math vocabulary, two-column proofs, paragraph proofs, class discussions, and discussions on the course blog. The mathematical terminology learned through direct instruction is used in discussions in class, on the blog, and for preparing simple proofs. Students' mathematical metacognition increases through the use of simple proofs, classroom discussions, and written discussions on the blog. Students' time working with their classmates is extended by blogging together about mathematical concepts.

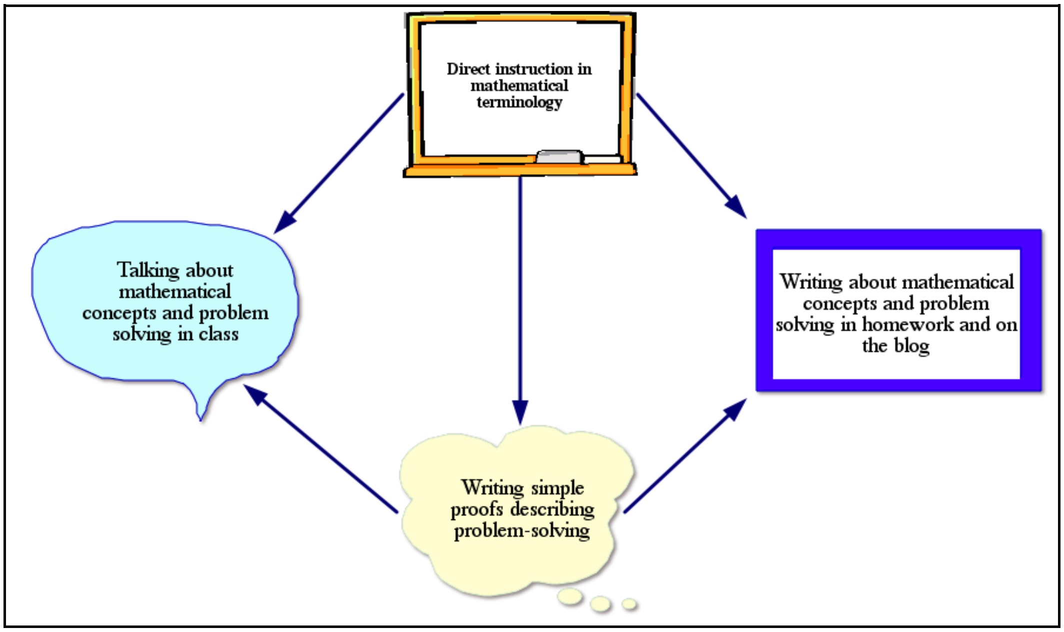


Figure 5: Interconnections between curriculum items.

Vocabulary Instruction

Vocabulary terms are introduced before the concept that uses the vocabulary is taught. The students explore the definition, word roots, related words, examples, and non-examples for each term. This information is recorded on the graphic organizer shown in Figure 6. There are six sections in the graphic organizer. In the Terms and Definitions section, students write down the mathematical term and its definition. In the Word Roots section, students write down any prefixes and word roots that can help them better understand the mathematics term and to identify related mathematical terminology. In the Related Words section, the students list any homonyms, synonyms, antonyms, non-math meanings for the term, and other words that share the same word root. In the Example and Non-Examples fields students should add a drawing, a description, an equation, or anything else that helps them understand what the term represents and does not represent. In the Additional Notes section, the students enter any additional information that helps them better understand the term.

Term and Definition	Word Roots	Related Words
1. The vocabulary term, *2. The definition of the term,* *3. Page number in the text where the word can be found.*	*The Greek, Latin, or Other language roots of the vocabulary term.*	*In this section, list any:* *1. Homonyms,* *2. Synonyms,* *3. Antonyms,* *4. Non-math meanings for the word,* *5. Words with same root.*
Example	**Non-Example**	**Additional Notes**
Add an example here that illustrates the term. This can be a drawing, a description, an equation, or anything else that helps the student understand the term.	*Add an example here that illustrates what the term is not. This can be a drawing, a description, an equation, or anything else that helps the student better define the term. The non-example can be an illustration of a term or concept that is often confused with the vocabulary term.*	*In this section, note any abbreviations for the term, any related concepts, or any additional information that adds to understanding.*

Figure 6: Algebra Vocabulary Graphic Organizer.

Writing Explanatory Proofs

The students use two-column proofs and paragraph proofs to describe their thought process while they are solving an equation or problem. The teacher models how to solve problems using the vocabulary terms taught for the concept being covered. The students practice the writing of the proofs in class, on the blog, and in their homework. The students work together to create the proofs in class and on the blog. The explanatory proofs provide a common structure for students to think about mathematics, discuss mathematics, and do mathematics.

Blogging math in the Pre-Algebra Classroom

This curriculum can be implemented using proprietary or non-proprietary blogging software applications. However, there are additional security and safety considerations if the students are using blogging software that does not allow the instructor to limit who has access to the blog. Each student needs to have access to a computer either at home or at school. It is important to allow enough time for students to use computers at school or elsewhere to participate in the blog if they do not have a computer at home. Finally, it is important to create guidelines for communication on the blog.

Every two weeks I post math questions related to the Pre-Algebra unit that the students and I are working on in the classroom. These questions are taken from the textbook, teacher support materials for the textbook, or *Algebra to Go* (Kaplan, 2000). The students post responses to the questions that detail the mathematical reasoning they used to solve the problem. In addition, the students post comments and questions related to solutions posted by other members of the class. The teacher may post hints to help students that are struggling or post clarifications of the problem if needed. The teacher guides rather than controls the discussion.

Assessment

The teacher examines the students' homework, blogs, and classwork to check for understanding. If the homework, blogs, and classwork show that students are having problems with certain concepts or vocabulary, the teacher will review and reteach the concepts and vocabulary. The students also complete quizzes during the unit and a unit test at the end of each unit. The questions on the quizzes and tests ask the students to explain the reasoning they use to solve each problem. The students are also quizzed and tested on the mathematical vocabulary learned for the unit.

VI. IMPLEMENTATION OF TALKING MATH, BLOGGING MATH

This chapter describes the implementation of *Talking Math, Blogging Math* in a non-traditional, home study based middle school classroom. The goals of the *Talking Math, Blogging Math* curriculum are to increase students' use and understanding of the mathematics register and to extend the classroom community of practice outside of the physical classroom through the use of a class blog. The implementation of *Talking Math, Blogging Math* took place at North County Charter Academy over a period of ten weeks in spring 2009.

On the days that the Pre-Algebra class meets on site at North County Charter Academy, classroom instruction is a mixture of direct instruction and group learning. I introduce or review a topic, take questions from the students, and model problems that I would like them to work on in groups. While the students are working in groups, I move throughout the room in order to monitor their conversations. I listen to make sure that everyone in the groups is participating, that everyone is on task, and that the students are working together. In addition, I check for understanding. If I notice students struggling, I will give them one-on-one attention or ask a peer to give them one-on-one attention. After the students have completed their work, we discuss the problems as a class.

When the students are not on site, they can ask questions about the homework via email or posting questions to the class wiki. To help with homework, I post supplementary materials to the wiki. My class and I have communicated intermittently throughout the school year using a blog available on the class wiki. The blog communication has consisted of asking questions about homework problems and working on math problems as a group. *Talking Math, Blogging Math* grew out of these earlier attempts to extend our classroom community of practice outside of the classroom walls.

The blog that the students and I use for our math discussions is located on our classroom wiki. The wiki and blog discussion pages are invitation only. In the case of my classroom, the students, the principal, and I are the only members. All of my students have access to the wiki, however two of my students need to use the computers at school to complete their on-line assignments. The students and I created guidelines for communication on the blog. We talked about what rules my students thought were important when they talk to other people online. They mentioned that they wanted to avoid flame wars (posting text that is designed to engender a strong reaction from others). My students also noted that they didn't want to start to receive all sorts of messages sent to the group that had nothing to do with math or school such as chain letters and sales pitches. This would be considered spamming. One of my students proposed that the blog could be used for all of them to talk to each other (not necessarily about math). This idea was not included in the math blog, noting that they already had other ways to socialize with each other outside of school. We discussed what type of language should be used on the blog. We came to an agreement that offensive language should be avoided. We also agreed that the abbreviations used when texting (text speak) should not be used because not everyone could understand text speak. My students and I created the following list of basic ground rules for blogging:

1) No flaming.

2) No foul language.

3) No internal spamming.

4) For math conversations only.

5) Use proper grammar and spelling - no text speak!

While the students usually do a good job of self-monitoring, I keep an active presence on the blog. I would monitor the discussions and add comments if I felt the students needed a hint or to keep students on task. We used the blog for mathematical discussions four times before the implementation of *Talking Math, Blogging Math*. The blog posts created during those blogging sessions will be used as a baseline for comparison with the blog posts created during this implementation.

The design of *Talking Math, Blogging Math* allows for the curriculum to be used with the existing classroom Pre-Algebra textbook. In my classroom we use Glencoe's *Pre-Algebra* (Malloy, Price, Willard, & Sloan, 2005). At the time of implementation, we were just starting to work on solving simple algebraic equations. I chose to stagger the implementation of the contextual vocabulary instruction, use of proofs, and blogging because I wanted the students to be comfortable using one part of the curriculum before I introduced another part. The implementation of *Talking Math, Blogging Math* took place over the course of several instructional units. In this section, I am going to focus on how *Talking Math, Blogging Math* was implemented for the equations/inequalities unit and the function/linear equation unit.

Equations and Inequalities Unit

During the first week of the equations/inequalities unit, I introduced paragraph proofs to my students. My students had been introduced to two-column proofs earlier in the year. I started the lesson by introducing how to solve a simple algebraic equation. I demonstrated how to solve an equation, thinking aloud as I solved it. I then asked the students to help me while I solved another problem on the board. An example of the paragraph proof is found in Figure 7.

Paragraph proof for 3(g - 3) = 6. First I use the distributive property to simplify the equation. The equation now reads 3g – 9 = 6. I use the property of equality addition to add 9 to both sides of the equation. The equation is now 3g = 15. I use the property of equality division to divide both sides of the equation by 3. The solution of the equation is g = 5.

Figure 7: Example of paragraph proof.

I then invited the students to break into three small groups to work together on solving additional problems on a handout I provided them. I monitored the three groups as they worked and provided assistance as needed. After the students had practiced solving equations for about 20 minutes, we came back together as a class.

I modeled for the students how to write a two-column proof using one of the problems that they had solved in their groups. An example of a two-column proof that we created is shown in Figure 8.

Statement	Reason
3(g - 3) = 6	**Given**
3(g) – 3(3) = 6	**Distributive property**
3g – 9 = 6	**Substitution**
3g – 9 + 9 = 6 + 9	**Property of equality addition**
3g = 15	**Substitution**
$\frac{3g}{3} = \frac{15}{3}$	**Property of equality division**
g = 5	**Substitution**

Figure 8: Example of two-column proof.

I asked the students why they thought they were learning two column proofs. The most common answer was “so you know if we got the problem right.” I asked my students if they thought doing proofs could be useful for them. After some silence, I revised my question to ask my students if they thought proofs could help them learn algebra. A couple of the students answered that the proofs could help them describe what they were doing when they solved equations. We were running short of time, so I assigned the students nine equations to solve for homework. They were to describe the solution of the equations using two-column proofs. I told them that we would be working on the proofs together the next time we met as a class. Figure 9 shows a proof created by one of the students in the first homework assignment.

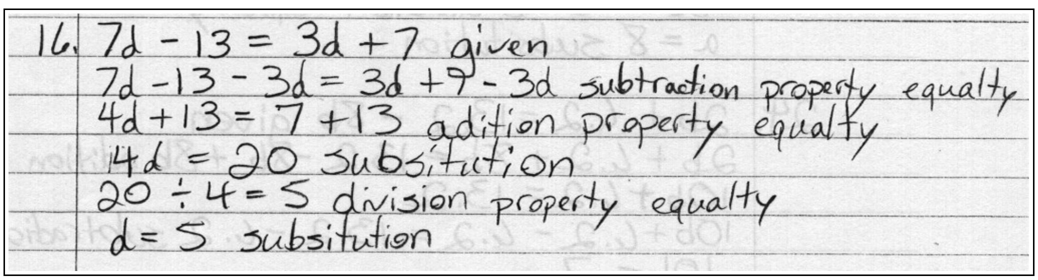
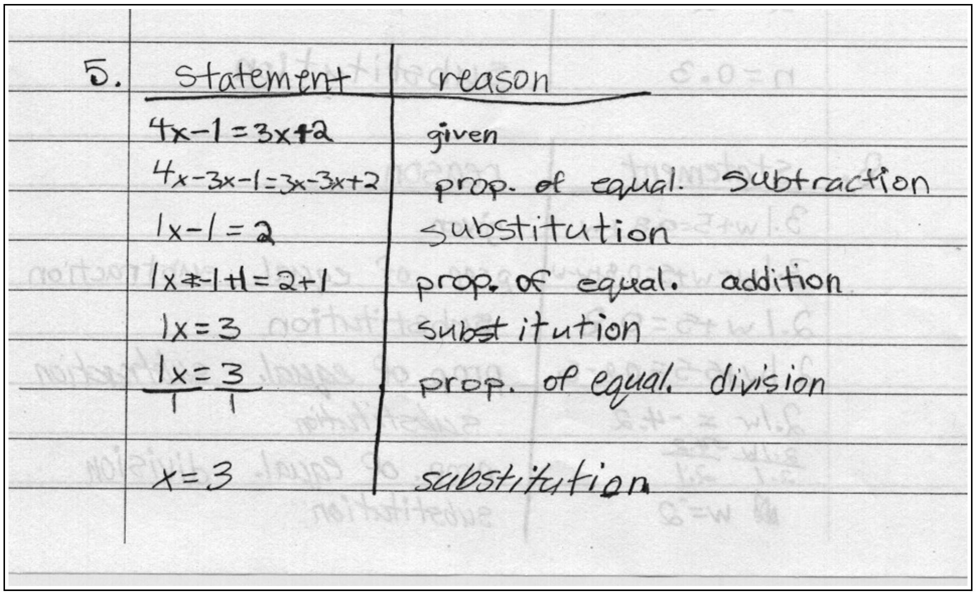

16. 7d - 13 = 3d + 7 given
7d - 13 - 3d = 3d + 7 - 3d subtraction property equalty
4d + 13 = 7 + 13 adition property equalty
4d = 20 subsitution
20 ÷ 4 = 5 division property equalty
d = 5 subsitution

Figure 9: Elizabeth's homework proof.

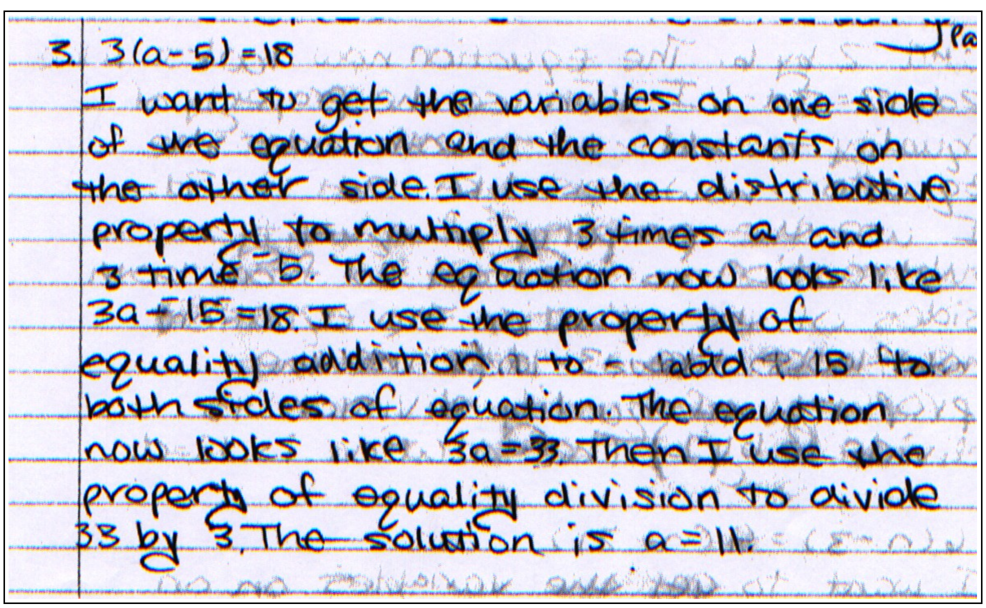
Our next class meeting, we went over the proofs together. Most of the students had made an attempt to do the proofs. After we had discussed the homework, I had the students work together on writing more two-column proofs in small groups. I again modeled how to write a paragraph proof. I had the students work together writing paragraph proofs for some sample equations. For homework, the students wrote two-column proofs for five problems and paragraph proofs for two additional problems. The two-column proofs created by the students showed a better understanding of the proof format and better use of mathematics vocabulary. An examples of a student's proof written for this homework assignment can be seen in Figure 10.



5. Statement	reason
4x−1=3x+2	given
4x−3x−1=3x−3x+2	prop. of equal. subtraction
1x−1=2	substitution
1x−1+1=2+1	prop. of equal. addition
1x=3	substitution
1x/1=3/1	prop. of equal. division
x=3	substitution

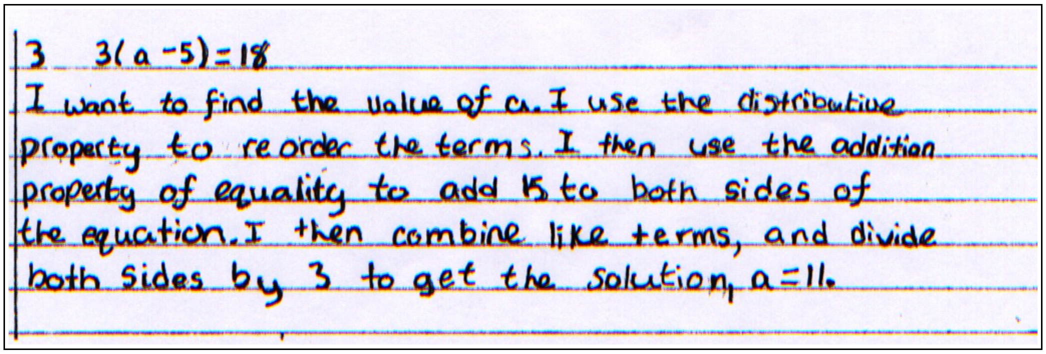
Figure 10: Danielle's two-column proof.

Figure 11 contains an example of a paragraph proof created for a homework assignment. Acacia's paragraph proof is one of the more detailed paragraph proofs written for this solution. In contrast, Jasmine's paragraph proof shown in Figure 12 is not as detailed, although she shows the same solution as Acacia.



3. 3(a−5)=18
I want to get the variables on one side of the equation and the constants on the other side. I use the distributive property to multiply 3 times a and 3 time 5. The equation now looks like 3a−15=18. I use the property of equality addition to add 15 to both sides of equation. The equation now looks like 3a=33. Then I use the property of equality division to divide 33 by 3. The solution is a=11.

Figure 11: Acacia's paragraph proof.



3 3(a − 5) = 18
I want to find the value of a. I use the distributive property to reorder the terms. I then use the addition property of equality to add 15 to both sides of the equation. I then combine like terms, and divide both sides by 3 to get the solution, a = 11.

Figure 12: Jasmine's paragraph proof.

During the second week of implementation, I introduced the use of the vocabulary graphic organizers. I chose to wait to introduce the formal definitions because I wanted the students to be exposed to the concepts first. I felt that the definitions would be more meaningful if the students had a context with which to associate the definitions. The first two terms I introduced were “expression” and “equation”. I consulted two resources for vocabulary information in addition to the Pre-Algebra textbook for this implementation. I used *Algebra to Go* (Kaplan, 2000) for many of the definitions and Schwartzman’s (1994) *The Words of Mathematics: An Etymological Dictionary of Mathematical Terms Used* for information concerning word origins.

I started the lesson by asking the students to tell me what they thought was the definition of the word “expression”. They suggested that expression could mean an example, a solution, and a problem. I then asked the students to look up the definition in their textbook. After I gave the students the information about the Latin roots for the word, and we discussed different meanings for the word expression. Their definitions included a look on your face, a specific phrase, and expressing yourself.

At this point, the students and I listed several examples and non-examples for the mathematical meaning of expression. I now introduced the relationship between expressions and equations. Under the examples for expression, we listed several examples and noted that expressions do not have an equal sign while equations do. Next, we defined the term “equation”. Figure 13 gives an example of the vocabulary graphic organizers for the term expression. An example of the vocabulary graphic organizer for the term equation can be seen in Figure 14.

Term and Definition	Word Roots	Related Words
Expression = a mathematical phrase that is a combination of one or more variables, constants, and operation symbols. An expression does <u>not</u> contain an equal sign.	Latin prefix and root: Ex – out Press, prim – squeeze	The word expression has other meanings that aren't used in math. It can mean: 1. A look on your face 2. Speaking out 3. Word phrases with special meanings

Example	Non-Example	Additional Notes
2π 2πr 8-b Lw 5 An expression does NOT contain an equal sign.	C = 2πr 8-b = 5 5 = 5 A = lw An equation has an equal sign	Expressions are part of an equation.

Figure 13: Expression vocabulary graphic organizer.

Term and Definition	Word Roots	Related Words
Equation = shows the equal relationship between two expressions.	Latin root: Equa, equi – equal, even	1. Equal 2. Equals 3. Equal sign 4. Equate 5. Equality 6. Equator
Example	**Non-Example**	**Additional Notes**
$X + 3 = 10$ $7y – 5 = 4y + 7$ $15 + 25 = 40$ The expressions on either side of the equal sign represent the same quantity.	$X + 3$ 2^5 $7n < 3x^2 + 2$ $15 + 25$ These are expressions but not equations.	Expressions are part of an equation. Analogies: phrase:sentence expression:equation

Figure 14: Equation vocabulary graphic organizer.

During the third week of the implementation, I gave my students a short quiz on the terms expression and equation. I wrote the questions shown in Figure 15 on the board. I gave the students approximately 10 minutes to answer the questions.

Vocabulary Quiz 1

1. What is an expression?
2. What is an equation?
3. How are expressions and equations related?

Figure 15: Vocabulary quiz 1 - expression and equation.

Most of the students were able to define an expression and an equation. However, there still seemed to be some confusion about the relationship between an expression and an equation. Figure 16 shows responses that three of the students wrote in response to the questions. The samples are representative of the other students in the class. Jeremy, Danielle, and Acacia knew that expressions do not contain equal signs while equations do. However, they all had problems clearly describing the relationship between expressions and equations. After the quiz, we reviewed the graphic organizers that we had created for expression and equation. We discussed that equations are made up of one or more expressions.

1. What is an expression?

   Jeremy: An expression is a combination of numbers and can include addition, subtraction, multiplication, and division. An expression does not have an equal sign. 2 + 2

   Danielle: 2π, ½, 2πr, 8 – b, 20%, lw, 5, 0.5

   Acacia: an expression is a math problem with no equal sign. 2^7

2. What is an equation?

   Jeremy: An equation consists of expressions. An equation must have an expression and an equation has an equal sign. 4n + 2 = 24

   Danielle: x + 3 = 10, 7y – 5 = 4y + 7, 5 = 5, 15 + 15 = 30

   Acacia: An equation is a math problem with an equal sign. 7 – 3 = 4

3. How are expressions and equations related?

   Jeremy: They are related because an equation always has and [sic] expression in it. Without the expression it's not an equation.

   Danielle: Expressions are part of an equation. Without the equal sign you only have a [sic] expression not an equation.

   Acacia: Expressions are equations with no equal sign.

Figure 16: Vocabulary quiz 1 responses.

The first blogging assignment came during week four of the implementation of *Talking Math, Blogging Math* I chose to post a question about equations. Figure 17 shows the actual blog post.

Blog for 2/26 - 3/5
Dear students,

Please solve the three equations below. Write your solutions in the form of a paragraph proof. Once you have completed your proofs, read through the proofs of two other students. Post a comment about their solution. Did they solve the proof the same way you did? Can you help your classmate correct a mistake? Does their proof describe all of the problem-solving steps?

If you try a proof and get stuck, it is okay to ask for help from your classmates on the blog. That's what we are here for! Please keep the blogging ground rules in mind when you are posting.

1) 5n + 12 = 3n

2) 7x + 4 = 3x + 16

3) 4(y - 7) = 2(2y + 41)

Figure 17: Blog assignment 2/26 – 3/5.

The students were given a week and a half to respond to the math questions and to respond to the posts of other members of the class. This period of time gave students who do not have access to a computer at home time to use the school computers or to make other arrangements. Students were given participation points for posting to the blog. Figure 18 and Figure 19 show students' blog entries in response to the prompt shown in Figure 17.

Posted Feb 26, 2009 3:59 pm

Jeremy re: Blog for 2/26 - 3/5

1) 5n + 12 = 3n

First I combined like-terms by subtracting 5n to each side of the equal sign. Next, I used the property of equality division to divide -2 to each side of the equal sign. Finally, I ended with the substitution of -6=n.

5n-5n + 12 = 3n-5n

12 = -2n

12/-2 = -2n/-2

-6 = n

2) 7x + 4 = 3x + 16

I started by combining like terms and subtracted 3x to both sides of the equal signs. Next I used property of equality subtraction by subtracting 4 to each side of the equal sign. Next I divided 12 to each side of the equal sign by using the property of equality division. I ended with 3=x.

7x-3x + 4 = 3x-3x + 16

4x + 4 = 16

4x + 4-4 + 16-4

4x = 12

4x/4 = 12/4

x = 3

3) 4(y-7) = 2(2y+41)

First I used the distributive property. Next I used the property of equality subtraction. This cancels out the variable ending in NUL SOLUTION.

4y-28 = 4y+82

(4y-4y)-28 = (4y-4y)+82

NUL SOLUTION

Figure 18: Jeremy's blog posts.

Posted Feb 27, 2009 1:16 pm
Bailey
Bailey re: Blog for 2/26 - 3/5
1. 5n+12=3n
First I use the property of equality subtraction, subtracting 5n by each side of the equal sign. Substitution of 12n=-2n. Then I divide-2 from each side of the equal sign using property of equality division. Last I end with the substitution of -6=n.
2. 7x+4=3x+16
I start by subtracting 3x from each side of the equal sign, using the property of equality subtracting. Substitution of 4x+4=16. Using property of equality subtracting, i subtract 4 from each side of the equal sign. 4x=12 is the substitution. Next I divide 4 from each side of the equal sign using the property of equality division. In the end I get x=3 with substitution.
3. 4(y-7)=2(2y+41)
As I start I use the distributive property, substituting 4y-28=4y+. Next I subtract 4y from each side of the equal sign, which ends because it cancels out the variable, so the problem is called a nul solution.

Figure 19: Bailey's blog posts

As I monitored the blog, I noticed some common themes that I wanted to address with the students. As shown in Figure 20, I noticed that some of the students were forgetting to write the negative sign when writing the difference of 3n and 5n. In addition, some of the students were having problems using the distributive property. Several of the students noted that the solution to problem 3 was a null solution, but several had indicated that the solution achieved "when the variables cancel out" would always result in a null solution. I wanted to remind them that another possible solution could be all numbers depending on the solution. Finally, I asked students to consider whether the properties used to solve the equation had a set order of application. We discussed these questions when we were together in class as well.

Posted Mar 4, 2009 4:43 pm
lmathews re: Blog for 2/26 - 3/5
Great job guys! Some things that I noticed:
remember that 3n - 5n = -2n
a null solution occurs when the equality statement is not true as in problem 3, not if the variables cancel each other out. What would a solution of 3 = 3 be called?
When the distributive property is applied to the last problem
4(y-7)=2(2y+41) you can rewrite the problem as:
4(y) - 4(7)= 2(2y) + 2(41)
4y -28 = 4y + 82
For our blog purposes, please use parentheses or an asterisk to indicate multiplication.

You might want to check out this link to help answer the question I posted earlier:
http://www.purplemath.com/modules/solvelin3.htm

Another question:
Is the way you solved the equations the only way to go about it? Does it matter what order you apply the properties of equality? Try solving equation 2 using a different sequence of steps.

Figure 20: Ms. Mathews' follow-up post.

After we started the unit about how to solve and graph inequalities in week three, I introduced the term “inequality” in week four. I asked the students to write down what they thought the term meant on a separate sheet of paper. The class shared their ideas for the definition with each other. The definitions that the students created included “doesn’t have an equal sign”, “form of expression”, and “numbers aren’t equal.” We then worked together to complete the vocabulary graphic organizer for the term inequality. Figure 21 shows the term definition, word roots, related words, examples, non-examples, and additional information we recorded for the term inequality. I had the students write examples of number line graphs for inequalities and a coordinate plane graph for an inequality on the back of the graphic organizer for later reference. After the students had worked with solving two-step inequalities for two class sessions, I had them add the properties of inequalities (addition, subtraction, multiplication, and division) to their inequality definition graphic organizer. We also noted the cases where multiplication or division of both sides of an inequality by a negative number causes the direction of the inequality to be reversed. The students then practiced creating proofs for inequalities that required multistep solutions.

Term and Definition	Word Roots	Related Words
Inequality - a mathematical sentence that compares two unequal expressions using one of these symbols: <, >, ≤, ≥, ≠	Latin roots in = not equa, equi = equal	less than < greater than > less than or equal to ≤ greater than or equal to ≥ not equal to ≠
Example	**Non-Example**	**Additional Notes**
4n + 10 ≤ 50 x > 5 t < 2 10 ≥ m ≥ 2 y ≠ 7	x = 4 5 3x + 3 = 9	Inequalities can be graphed on a number line or on a coordinate plane. An expression with < or > has an open dot or parentheses at the end. An expression with ≤ or ≥ has a closed dot or bracket at the end.

Figure 21: Inequality vocabulary graphic organizer.

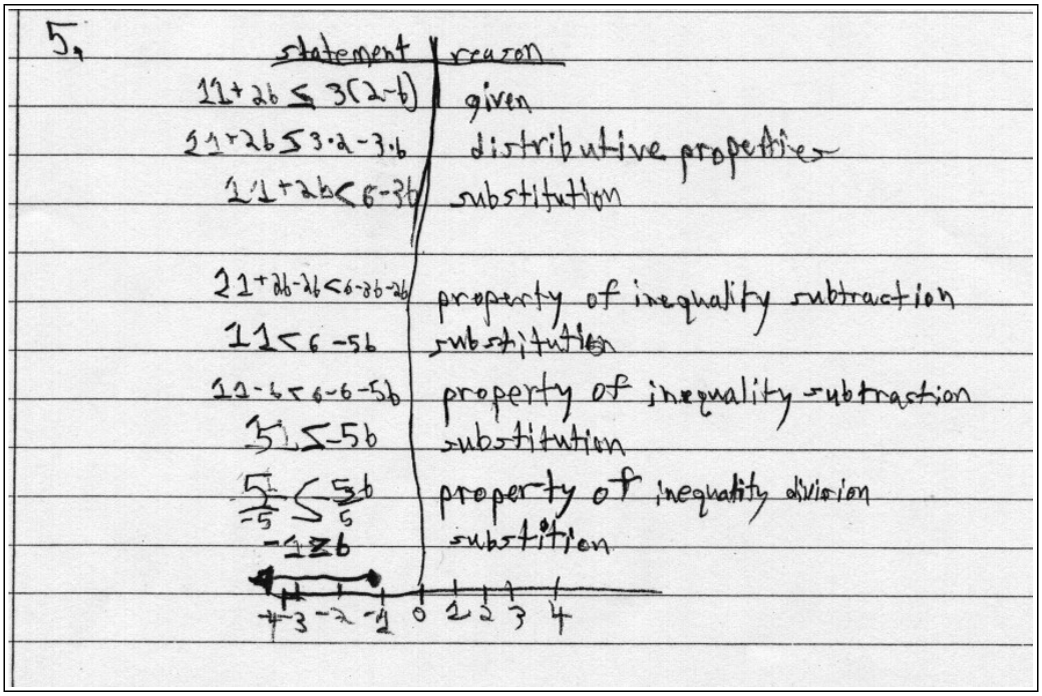


Figure 22: Jeremy's inequality proof.

After the vocabulary for inequality had been introduced, we worked on creating proofs for inequalities for several class periods. The students were able to create proofs for multi-step inequalities as seen in Figure 22. Jeremy was able to describe the steps he used to solve the equation. However, the inequality represented in the proof incorrectly changes from less than or equal to (<=) to less than (<) and then back to less than or equal to (<=) during the course of the proof. Albert does correctly indicate that the inequality changes due to both sides of the inequality being divided by negative five (-5).

We continued to use both two-column and paragraph proofs throughout the semester. We used the proofs as a starting point for the introduction of new concepts such as functions and linear equations. The proofs allowed us to review important vocabulary and properties that would aid in the understanding of the new concepts. The proofs also helped reinforce the use of previously learned vocabulary.

Until week five of the implementation, we had blogged as a class. Not all of the students were participating in the blog even though they were receiving participation points. When asked later, the students told me that they did not like the asynchronous nature of the blog. I noticed that the students tended to post their responses and then make minimal comments on the work of their classmates. There was really never a discussion of math. I decided to change the blog format so that students were going to be working in small groups. Since we were blogging over spring break, the blog groups were given the option of emailing or blogging since many of the students were not going to be at home. This time I asked the students to work together in small groups (two to three students) on questions about equations and inequalities. The students were to determine if a statement was always, sometimes, or never true and then justify their response. As before, there were several students that chose not to participate in the blog discussions. The students that did participate in the blog discussions seemed to engage in more discussion when working in small groups then they did when blogging as a class. An excerpt from an exchange between two students can be seen in Figure 23.

Bob:
> for #1 I got always true
> the example I got 5x +3=2x +6 which equils [*sic*] to 5x+5x+3=5x+2x+6 and that ended up at
> 10x + 3 = 7x + 6
> the description I got is because if you add the same # to both sides of the = sign, the > > solution will be the same.
>What did you guys get for the anser [*sic*] to #1 ?

Jeremy:

I got the same, always true for #1. It is true because of the property of equality addition. One example would be (-2x) + 8 = 5x - 2. This is true because if you add 2x to both sides, the answer doesn't change. This is because of the property of equality addition.

(-2x)+2x + 8 = 5x+2x - 2

Another example would be 5x - 4 = (-1x) + 4. Again this is true because of the property of equality addition

5x+1x + 8 = (-1x)+1x - 4

For #2, I got always true. How 'bout you? This is because of the property of equality subtraction.

An example would be 2x + 5 = 4x - 8 This is true because if you used property of equality subtraction and subtracted 2x to both sides, the answer would not change.

2x-2x + 5 = 4x-2x - 8.

Another example would be 2x +24 = 8 + x. This is the same as the first example because we subtract x to both sides, which is the property of equality subtraction.

2x-x + 5 = 8 + x-x

Figure 23: Bob and Jeremy's blog discussion about equations and inequalities.

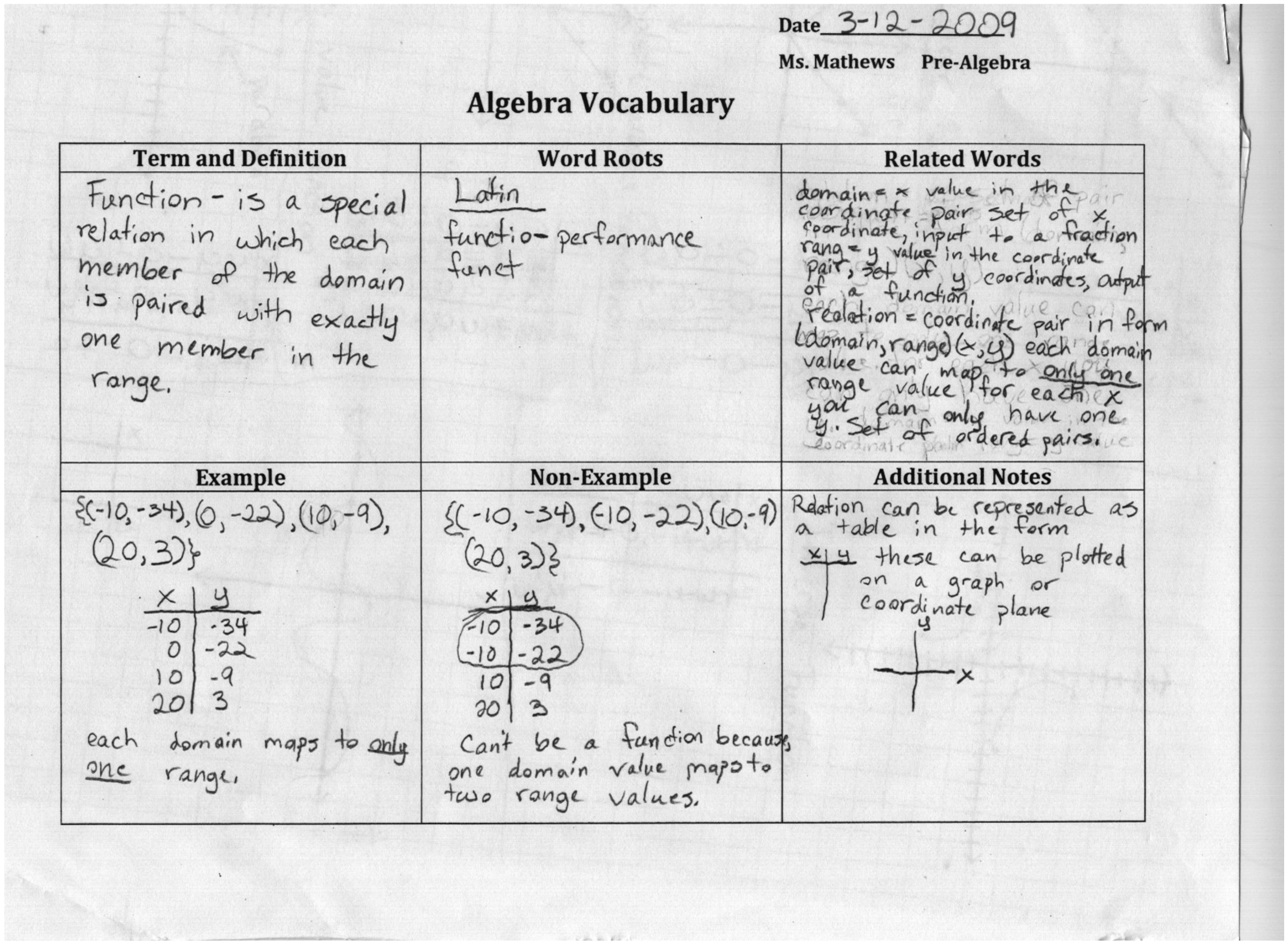
Before we discussed their solutions to the blog questions as a class, I asked the students how I could make the blog more useful to them. I told them that I noted that several of the students were just not participating in the blog work. I suggested that we brainstorm ideas about how to improve the blogging. The students came up with several suggestions. They noted that they would rather be using instant messaging or texting because they didn’t always have access to a computer but they did have access to their phone. They also told me that they liked the option of using email in addition to blogging. The students mentioned that they felt limited by the blog because they thought it would be easier to post pictures of their work than having to write explanations. I pointed out that I wanted them to write explanations but that we could explore video explanations in the future. The students all noted that they liked working in small groups better than as a whole class as they felt more comfortable. I told them that we would continue to work in small groups and that I would make sure that those students who did not have access to the blog because of computer problems at home could continue to email each other rather than use the blog. My only condition for emailing was that the students needed to send me a copy of their email.

Functions and Linear Equations Unit

During week seven of the implementation we started a unit on functions and linear equations. After we had worked together identifying functions, the students came up with several definitions for the term. Their definitions included “an equation”, “the ending of an equation”, “something with input and output”, “what it does”, and “a process.” The students looked up the definition of a function in their textbook and entered this definition in their graphic organizers. Figure 24 shows the definition that we used for function. After they wrote down the definition, I asked the students if they could tell me the meaning of the terms “relation”, “domain”, and “range”. These terms were used as part of the definition copied from the textbook. They looked up the terms relation, domain, and range and added them to the Related Words section of the vocabulary graphic organizer as shown in Figure 24. The students used examples and non-examples from the textbook to complete the Example and Non-Example sections in the graphic organizer. An example of a student’s graphic organizer for the term "function" is shown in Figure 25. In the additional notes field of the graphic organizer, the students noted different ways that functions can be represented.

<table>
<tr><th>Term and Definition</th><th>Word Roots</th><th>Related Words</th></tr>
<tr><td>function – a special relation in which each member of the domain is paired with exactly one member in the range.</td><td>Latin roots
functio-, funct- performance</td><td>domain: x-value in a coordinate pair; set of x coordinates; input to a function
range: y value in a coordinate pair; set of y coordinates; output of a function
relation: coordinate pair (x,y) or (domain,range); set of coordinate pair; each domain value can map to only one range value</td></tr>
<tr><th>Example</th><th>Non-Example</th><th>Additional Notes</th></tr>
<tr><td>{(-10, -34), (0,-22), (10, -9), (20,3)}
<table>
<tr><th>x</th><th>y</th></tr>
<tr><td>-10</td><td>-34</td></tr>
<tr><td>0</td><td>-22</td></tr>
<tr><td>10</td><td>-9</td></tr>
<tr><td>20</td><td>3</td></tr>
</table>
Each domain (x) maps to only one range (y)</td><td>{(-10, -34), (-10,-22), (10, -9), (20,3)}
<table>
<tr><th>x</th><th>y</th></tr>
<tr><td>-10</td><td>-34</td></tr>
<tr><td>-10</td><td>-22</td></tr>
<tr><td>10</td><td>-9</td></tr>
<tr><td>20</td><td>3</td></tr>
</table>
Not a function – the domain (x) value -10 maps to two range (y) values</td><td>A relation can be represented as a table in the form.
<table>
<tr><th>domain</th><th>range</th></tr>
<tr><td>(x)</td><td>(y)</td></tr>
</table>
These coordinate pairs can be plotted on a graph (coordinate plane).</td></tr>
</table>

Figure 24: Function vocabulary graphic organizer.



Date 3-12-2009

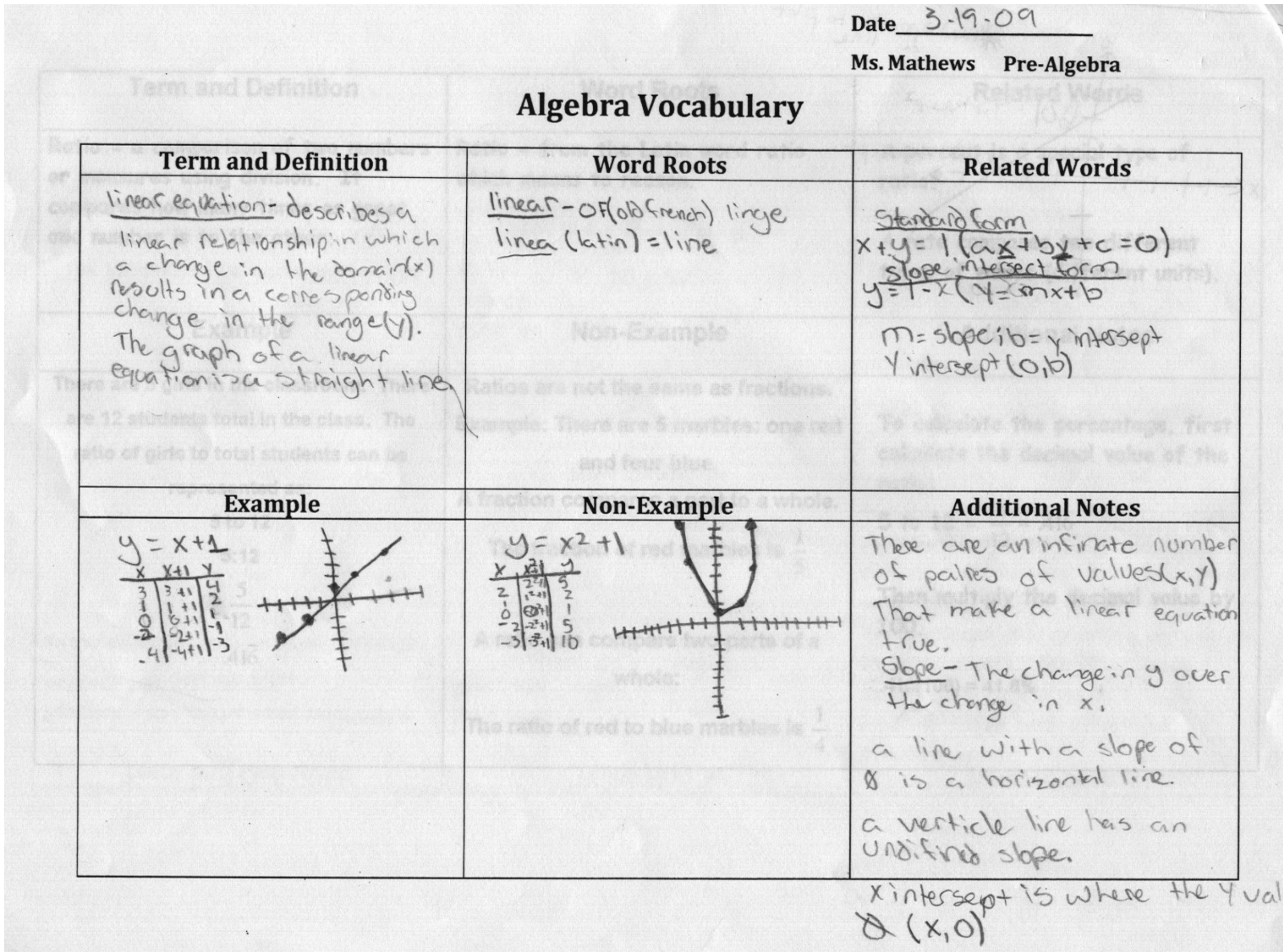
Ms. Mathews Pre-Algebra

Algebra Vocabulary

Term and Definition	Word Roots	Related Words
Function - is a special relation in which each member of the domain is paired with exactly one member in the range.	Latin functio- performance funct	domain = x value in the coordinate pair set of x coordinate, input to a fraction range = y value in the coordinate pair, set of y coordinates, output of a function. relation = coordinate pair in form (domain, range) (x, y) each domain value can map to only one range value for each x you can only have one y. Set of ordered pairs.
Example	**Non-Example**	**Additional Notes**
{(-10, -34), (0, -22), (10, -9), (20, 3)} x | y: -10 | -34; 0 | -22; 10 | -9; 20 | 3 each domain maps to only one range.	{(-10, -34), (-10, -22), (10, -9), (20, 3)} x | y: -10 | -34; -10 | -22; 10 | -9; 20 | 3 Can't be a function because one domain value maps to two range values.	Relation can be represented as a table in the form x | y these can be plotted on a graph or coordinate plane y, x

Figure 25: Elizabeth's vocabulary graphic organizer for function.

After the introduction of the vocabulary for functions during week seven, the students and I worked together on a several problems that explored properties of functions. We discussed how linear functions allowed the solution set to be represented by a line on a graph. The next week, week eight, I introduced the vocabulary for linear functions. The students looked up the definition for linear equation in their textbooks. I explained that the term linear comes from the Latin word linea which means line. We then discussed what change in y and change in x meant on a coordinate plane. The students noted that a line's slope describes its steepness. A line's slope is defined as the change in y over the change in x. They wrote down the two formats that a linear equation has (slope intercept and standard). We also noted that the solution set for a linear equation is the coordinate pairs for the infinite number of points on the line described by the linear function and that a horizontal line has a slope of 0 and the slope for a vertical line is undefined. Figure 26 shows the graphic organizer completed by one of my students. During week ten of the implementation the students had a vocabulary quiz on the mathematical terms associated with linear equations.



Date 3-19-09

Ms. Mathews Pre-Algebra

Algebra Vocabulary

Term and Definition	Word Roots	Related Words
linear equation- describes a linear relationship in which a change in the domain(x) results in a corresponding change in the range(y). The graph of a linear equation is a straight line	linear - OF(old french) linge linea (latin) = line	Standard form x+y=1 (ax+by+c=0) slope intersect form y=1-x (y=mx+b) m= slope; b= y intersept Y intersept (0,b)
Example	**Non-Example**	**Additional Notes**
y = x+1	y = x²+1	There are an infinate number of pairs of values(x,y) That make a linear equation true. Slope- The change in y over the change in x. a line with a slope of 0 is a horizontal line. a verticle line has an undifind slope. x intersept is where the y val 0 (x,0)

Figure 26: Bob's graphic organizer for linear equations.

During weeks eight through ten, the students worked in small groups to practice creating function tables for linear equations and graphing the equations. They also practiced using the x intercept and the y intercept to graph the line for a linear equation. The blog assignment for this unit given in week nine asked the students to explore how changing the slope or the y intercept value in a linear equation affects the graph of the line. The students divided up into groups of 2 to 4 people. Some students emailed each other rather than blogging because they could not get access to the blog due to technical problems with their computers at home. During week ten of the implementation, we discussed the blog assignment as a class after the given time to blog was over. The students and I continued to use vocabulary instruction, proofs, and blogging for units throughout the semester.

VII.EVALUATION OF TALKING MATH, BLOGGING MATH

The goals of the *Talking Math, Blogging Math* curriculum were to increase students' use and understanding of mathematical terminology, to improve students' mathematical metacognition, and to extend the math classroom community of practice outside of the physical classroom through the use of class discussion spaces or blogs. In order to evaluate the success of these goals, I collected students' written work, and students' posts to the blog during each unit. I also conducted individual student problem-solving interviews at the end of the implementation. During these interviews I would ask students to describe how they would go about solving different types of problems. To check for understanding of mathematical vocabulary, I gathered students' mathematics vocabulary quizzes, blog posts, and student interviews for examination. In addition, I conducted individual student interviews at the end of the implementation period to get their opinions about the effectiveness of the class blog, the proofs, and the vocabulary instruction. I had planned to use recordings of the students' classroom discourse collected during the implementation period, but the students were shy and self-conscious when the recording device was placed near them. They would stop talking or talk about subjects not related to math. As a result, I could not use this data. The curriculum goals, constructs, and data collection techniques are presented in Figure 27.

In order to get the most data possible for my analyses, I chose to focus on the students who attended class regularly and were consistent about turning in their homework. Students who did not attend class regularly missed much of the instruction about vocabulary and paragraph proofs. These students also missed the opportunity to work with their classmates on a regular basis.

Out of the class of nine students, six students, four girls and two boys, met the criteria of attendance and homework completion. Of these students, there were four 7th grade students, one 8th grade student, and one 9th grade student. The 8th grade student and the 9th grade student were taking Pre-Algebra for the second time.

Goal	Construct	Data Collected
To increase the use of mathematical terminology.	Mathematical discourse	1. Student interviews. 2. Students' vocabulary brainstorming and quiz data. 3. Students' blog posts. 4. Students' work showing two-column and paragraph proofs
To increase mathematical metacognition.	Mathematical discourse, metacognition	1. Students' blog posts. 2. Students' work showing two-column and paragraph proofs 3. Students' interview data.
To extend the classroom community of practice.	Community of practice	1. Students' interview data. 2. Comparison of mathematical discourse in blog and interviews.

Figure 27: Goals and evaluation strategies.

Increase Use and Understanding of Mathematical Terminology

My first goal for the *Talking Math, Blogging Math* curriculum was to increase students' use and understanding of mathematical terminology in their written work, blog posts, and class discussions. To evaluate whether the students showed increased use of mathematical terminology, I examined students' work, interviews, and blog posts. The students' work and blog posts were collected over a ten-week period. The student interviews took place at the end of the implementation of *Talking Math, Blogging Math*. To evaluate whether students showed increased understanding of the mathematics terminology, I examined students' vocabulary brainstorming and quizzes collected over the span of the implementation.

In order to see if there was a change in the students vocabulary use during the implementation, I analyzed students' blog entries collected pre-implementation, a second set of blog entries completed a month into the implementation, students' paragraph proofs, and students' interview data collected at the end of the implementation. Due to many student absences, only two students had completed all of these assignments. Therefore, I chose to focus on work collected from one 7th grade boy, Jeremy, and one 7th grade girl, Acacia.

The assignments chosen were the pre-implementation blog (completed November, 2008), paragraph proofs 7-1 #31 and 7-1 #33 (completed week 2), paragraph proofs 7-2 #3 and 7-2 #6 (completed week 3), blog about equations (week 4), and the post implementation interview questions # 5 and #6 (completed mid-May, 2009). I coded the students' work looking for the use of the mathematical academic vocabulary that we discussed in class and the use of related mathematical academic vocabulary. Examples of related mathematical academic terminology included "parentheses" and "combine like terms". All of the coded assignments involved paragraph proofs. I coded but chose not to include vocabulary related to operations and relations because the students tended to use the symbol or write out the word interchangeably.

Jeremy

Figure 28 shows Jeremy's vocabulary usage over the different assignments. In the pre-implementation blog, Jeremy used a few vocabulary terms introduced in the textbook. He identified that he needed to "combine like terms and constants." Jeremy noted that he needed to do operations to both sides of the equation, but he did not identify any of the underlying properties used. In the first homework assignment following the start of mathematical academic vocabulary instruction (7-1 # 31), Jeremy's use of the mathematical academic terminology increased from 4 words used to 11 words used in the correct context. However, in the next four homework assignments (7-1 #33, 7-2 #2, 7-2 #6, 2/26 blog # 2), Jeremy's use of the vocabulary highlighted as part of classroom instruction leveled off to an average of 6 terms. Even so, his mathematical vocabulary usage was still higher than that seen in the pre-implementation blog. During the final interviews, Jeremy's usage of mathematical academic terminology increased to 12 terms for interview question #6 and 9 words for interview question #5. In these interview questions, Jeremy employed terms associated with proofs (given, substitution) and identified of the properties that he had applied to solve the equation.

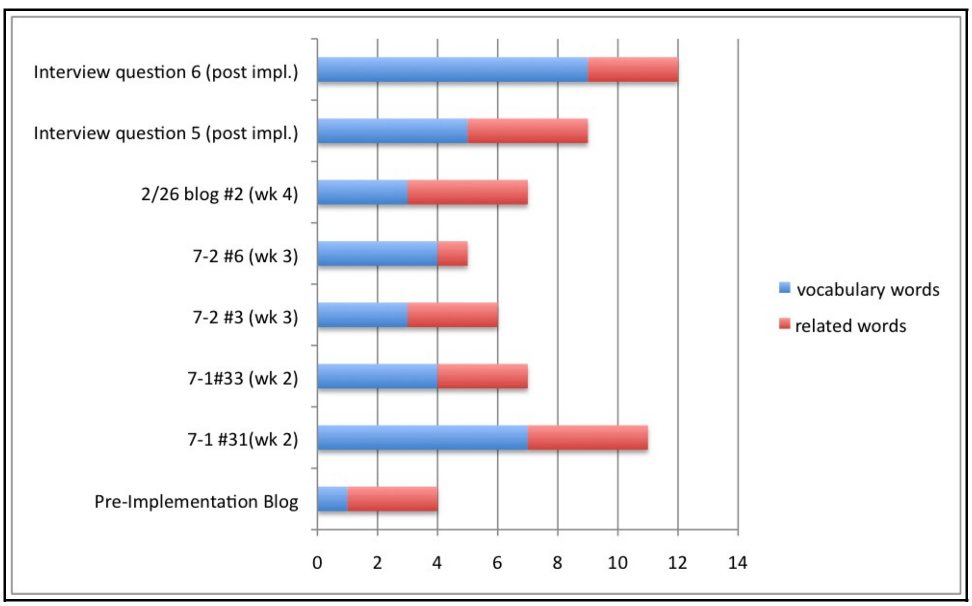


Figure 28: Jeremy's usage of mathematical academic language.

Acacia

Analysis of Acacia's work (Figure 29) showed that she made use of much of the mathematical academic language presented in class. In her pre-implementation blog, Acacia employed terms such as equation and equivalent equation. She did not describe any properties that she used to solve the equation. In the homework assignments following the implementation of vocabulary instruction, the number of mathematical vocabulary terms Acacia used increased an average of three words with each assignment. In these homework assignments, Acacia described the properties that she used to solve the equations in addition to using correct terminology for the different parts of the equation. Acacia's usage of mathematical academic language decreased to 11 or 12 terms for the last three assignments. The most of the terms she used in the last three assignments described properties and actions used to solve the equations.

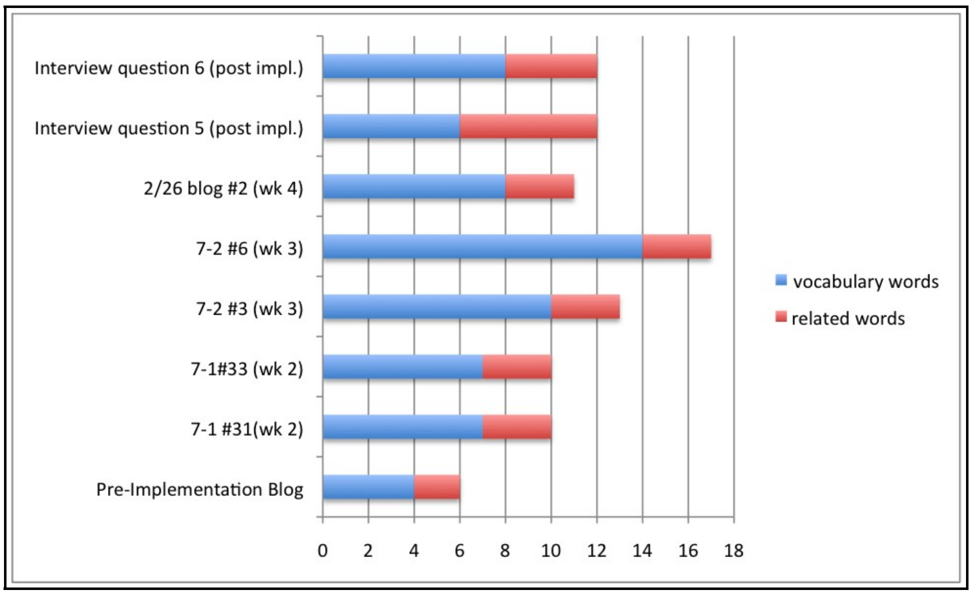


Figure 29: Acacia's usage of mathematical academic language.

Both Jeremy and Acacia showed a slight increase in their use of mathematical terminology in context. In my classroom observations, I also noted that the students used more mathematical terminology when they were working together on problem-solving, especially associated with the explanatory proofs. Perhaps one reason that both Jeremy's and Acacia's usage of mathematical terminology varied was that students need to hear and use vocabulary many times before the vocabulary becomes internalized (Thompson & Chappell, 2007). Because of the low frequency of our class meetings (twice a week), the internalization of the mathematics vocabulary would have been a slower process than if the class had met more often (Harmon, Hendrick, & Wood, 2005; Zevenbergen, 2000).

In order to see if there was a change in the students' understanding of the mathematical register, I examined students' vocabulary quizzes. The students' took the vocabulary quizzes a week or two after the content vocabulary and concepts had been introduced in class. The quizzes covered concepts such as equations, expressions, solution sets, linear equations, and slope. I introduced these concepts on the vocabulary graphic organizers and in class discussions The quizzes required the students to write short answers rather than match a term to a definition. The vocabulary quizzes I examined spanned the range of the implementation. Four students completed all of the vocabulary quizzes. Of these four students, three were in 7th grade and one student was in 8th grade. There were three girls and one boy.

I assessed the vocabulary quizzes using the rubric shown in Figure 30. Categories on the rubric checked for understanding of the mathematical concepts associated with the vocabulary, the clarity and completeness of the response, and the correct use of mathematical terminology in the explanation. Each vocabulary question received a score for each of the rubric categories. All of the scores assigned were in category three and two. The evaluation criteria used for each question are described in the following paragraphs.

Category	4	3	2	1
Explanation	The explanation addresses the question. The explanation given is detailed and easy to understand.	The explanation addresses the question. The explanation given is easy to understand and some detail is provided.	The explanation addresses the question only superficially the question. The explanation provides few details.	No explanation is provided or the explanation does not address the question.
Mathematical Concepts	The explanation provided shows complete understanding of the mathematical concepts.	The explanation provided shows good understanding of the mathematical concepts	The explanation provided shows some understanding of the mathematical concepts.	The explanation provided shows little or no understanding of the mathematical concepts.
Mathematical Terminology and Notation	Correct mathematical terminology is used. All vocabulary words are used correctly in the explanation.	Correct mathematical terminology is usually used. Most vocabulary words are used correctly in the explanation.	Correct mathematical terminology is sometimes used. Vocabulary words are sometimes used correctly in the explanation.	Correct mathematical terminology is not used. Vocabulary words are not used correctly in the explanation.

Figure 30: Rubric used to assess student understanding of vocabulary.

The questions for vocabulary quiz 1 are shown in Figure 31. In order to receive a score of 4 on the rubric, the students needed to explain that an expression is a mathematical phrase that is a combination of one or more variables, constants, and operation symbols, that the expression is part of an equation, and that an equation does not contain an equal sign. For question 2, the students needed to respond that an equation indicated an equal relationship between two expressions and that the expressions on either side of the equals sign represent the same quantity. For question number 3, the students needed to indicate that an equation is made up of multiple expressions.

1. What is an expression? Give an example.
2. What is an equation? Give an example.
3. How are expressions and equations related?

Figure 31: Vocabulary quiz 1 given week 2.

When we initially brainstormed for a definition for expression as a class, most of the students were unable to give more that a one-word definition. They described an expression as a problem, an example, or a solution. Many of the students simply responded that they didn't know. The responses the students gave showed more understanding, but still echoed some of the students' initial definitions (Figure 32). For example, Acacia continued to define an expression as a math problem. She did not indicate that expressions could be combinations of numbers, variables, and operations. Jeremy, on the other hand, provided a more detailed description of an expression, but did not mention that expressions could contain variables. All of the responses indicated that an expression did not contain an equals sign. But there did not seem to be understanding of why not having an equal sign was relevant or that the expression could be evaluated.

Acacia: An expression is a math problem with no equal sign. Example: 2^7

Jeremy: An expression is a combination of numbers and can include addition, subtraction multiplication and division. An expression does not have an equals sign. Example: 2 + 2

Sara: An expression is part of an equation, but with no equals sign. Example: 3x + 2

Michaela: An expression contains addition, subtraction, multiplication, and division. It does not have an equal sign, unlike an equation. Example: b + c

Figure 32: Answers to "What is an expression?"

The definitions that the students provided for "what is an equation?" (Figure 33) all indicated that an equation has an equals sign. Their definitions, however, did not describe why the equals sign was important. Not one of the students indicated that the expressions on either side of the equals sign are equivalent. Jeremy, Sara, and Michaela all indicated that equations contain expressions. Sara repeated the analogy that an equation is a mathematical sentence.

Acacia: An equation is a math problem with an equals sign. Example: 7 – 3 = 4.

Jeremy: An equation consists of expressions. An equation must have an expression and an equation has an equal sign. Example: 4n + 2 = 24

Sara: An equation is a mathematical sentence with an equals sign. Example: 5y + 2 = 17

Michaela: An equation unlike an expression has an equal sign. It contains expressions. Example: x + y = 8

Figure 33: Response to "What is an equation?"

When asked how mathematical expressions and equations are related, Jeremy, Sara, and Michaels focused on the idea that equations contain expressions. Acacia's definition indicated that she felt the only difference between the two was that an equation contained an equals sign and an expression did not. Acacia and Sara still focused on the absence of the equal sign. While Sara noted that both equations and expressions are mathematical sentences, she described the main difference as being the lack of an equals sign.

Acacia: Expressions are equations with no equals sign.

Jeremy: They are related because an equation always has an expression in it. Without the expression, it's not an equation.

Sara: Expressions and equations are related because they are both mathematical sentences and because an expression is part of an equation, but without the equals sign.

Michaela: Expressions and equations are related because equations are made up of expressions.

Figure 34: Answers to "How are expressions and equations related?"

While the students' understanding did seem to improve, their understanding of the mathematical concepts associated with the academic language was incomplete. Much of the lack of understanding centered around the meaning of “equals” and what an equals sign indicates. Kieran (1981) notes that the idea of the equals sign as a “do something” operator rather than an equivalence relation persists into adolescence for many students. My students seem to still see “equals” as indicating an operation rather than a relation.

The questions for Vocabulary quiz 2 (Figure 35) inquired about linear equations and slope. In order to score a 4 on the rubric, the students needed to answer that a linear equation represented a function in which a change in the domain value (the x value) resulted in a corresponding change in the range (y value). Most importantly, they needed to indicate that a linear equation represented a line. For question 2, the students needed to describe the slope as a ratio between the change in the range (x) value to the change in the domain (y) value. They also could have indicated that the slope was “rise over run”, but they needed to further explain what rise and run defined.

1. What is a linear equation?
2. Why does a line’s slope indicate?

Figure 35: Vocabulary quiz 2 given week 4.

In their descriptions about the nature of a linear equation, only one of the students mentioned that a linear equation represented a line. This had been mentioned in the graphic organizer created for this term. Acacia and Michaela showed understanding that a linear equation was a function that could be used to identify coordinate pairs. Jeremy and Sara labeled the different values in a slope intercept equation, but they had mixed success identifying what actually would be graphed. Sara indicated that the slope and y-intercept could be graphed while Jeremy focused on the intercepts. These responses and the students’ homework showed me that the students were still having difficulties understanding the relationships between linear functions and the lines the functions represented.

Acacia: A linear equation helps you find the y and x coordinates. An example of a linear equation is y = 3x + 2. You can plug in any number for x, solve, and gives you the y and x coordinate.

Jeremy: A linear equation is an equation that is y = mx + b. The equation contains a slope (m), the y-intercept (b), and x=intercept (x). A linear equation is used to find the x and y intercept of a line.

Sara: a linear equation is an equation where the slope and y-intercept can be graphed. it normally uses the equation y – mx + b, where m = the slope and b= the y intercept.

Michaela: A linear equation is an equation that you solve and graph. Also, in solving the equation you find out the x and y coordinates.

Figure 36: Answers to "What is a linear equation?"

The definition of slope was difficult for the students to master as well. The students could identify that the slope was rise over run and that the slope corresponded to the change in y over the change in x. But their homework showed that they were still not always sure how to calculate the slope. The students were also able to identify that the slope indicated how steep a line was and that the slope could be positive or negative. Jeremy's response indicated that he does not completely understand the definition of a line when he stated that the slope shows how narrow a line is.

Acacia: A line's slope indicates how steep it is, if it's positive or negative, and if they are parallel. It represents the change in y over the change in x or the rise over run.

Jeremy: The slope indicates how the line moves. It is also what x is multiplied by. The slope is the change of y over the change in x (rise and run). The slope indicates if the line is positive or negative, or how narrow it is.

Sara: A line's slope indicates when the line crosses the x-axis, how steep the line is, if it is a positive or negative slope, and if the lines are parallel.

Michaela: A slope's line indicates the change in y over the change in x. Also called the rise and run. The slope indicates the steepness, if it's positive of negative, and if it's parallel or not.

Figure 37: Answers to "What does a line's slope indicate?"

In vocabulary quiz 3, the students addressed questions concerning system of equations. For question 1, the students needed to indicate that the solution set is a point that falls on both lines or a coordinate pair in order to score a 4 on the rubric. For question number 2, the students could solve the equation using function tables or substitution. They needed to provide an explanation of their problem-solving process. For question 3, the students could indicate that the lines were parallel by noting that the lines had the same slope, or by creating function tables and graphing the lines. For this evaluation, I am going to focus on questions 1 and 3 as they are more closely related to the vocabulary instruction.

1. What is the solution set when you solve for a system of equations?
2. Explain how you would solve this system of equations:

   y = -3x – 2 and y = x – 2
3. Why does the system of equations y = 2x + 3 and y = 2x – 5 not have a solution. Hint: graph the lines.

Figure 38: Vocabulary quiz 3 given week 8.

The students' responses to questions about system of equations (Figure 39) showed that they knew that the solution should be a point or coordinate pair where the two lines intersect. Jeremy gave a more detailed response indicating how the solution could be reached. We had spent several class periods investigating how to solve a system of equations. The students found using function tables to graph the two lines easier than using substitution to determine the solution.

Acacia: The solution set is a point on a graph where the line intersects. The solution set is a coordinate pair.

Jeremy: The solution set for when you solve a system of equations is where two lines intersect. You find two points on the two lines given, graph them, and see where they intercept. You will see the x and y coordinate pair.

Sara: The value of x or y, a coordinate point where two lines cross.

Michaela: The solution set when you solve for a system of equations is one point on the graph where the two lines intersect.

Figure 39: Answers to "How would you solve for this system of equations?"

In the final question of the quiz (Figure 40), most of the students were able to identify that the lines were parallel, but only after graphing the lines. The students were not able to tell that the lines were parallel from looking at the equations alone.

Acacia: The system of equations does not have a solution because the lines are parallel and do not intersect.

Jeremy: There is no solution for lines $y = 2x + 3$ and $y = 2x - 2$ because the two lines are parallel. This means they will never cross.

Sara: Because they do not contain the second part of the equation which gives you the x or y value, because they are parallel.

Michaela: It does not have a solution because the lines never cross each other. They are parallel.

Figure 40: Answers to "Why does this system of equations not have a solution?"

Increase Use and Understanding of Mathematical Terminology: Summary

Zevenbergen (2000) noted that students must crack the code of the mathematics register in order to have legitimate participation in the mathematics classroom. While the short 10 week duration of the implementation and the resulting small data set create difficulties for making generalizations about the data, the findings suggest that my students were gaining a better understanding of the mathematics register. Both Jeremy and Acacia showed a slight increase in their usage of mathematical understanding. Jeremy's use of the mathematical terminology increased from 4 words in the pre-implementation blog to an average of 9 words over the last three assignments included in this study. Acacia's use of the mathematical terminology increased from 6 words in the pre-implementation blog to an average of 12 words over the last three assignments included in this study In addition, based on my observations in class, the students used more mathematical terminology when they were working together on problem-solving. The students included in the data exhibited a slightly better understanding of the mathematical terminology as seen through their responses in the vocabulary quizzes given during the implementation of *Talking Math, Blogging Math*. Prior to implementation, many of these students had not been able to provide a definition for the terms when we brainstormed about them in class.

Research in content area vocabulary learning notes that students need to hear and use vocabulary many times before the vocabulary becomes internalized (Thompson & Chappell, 2007). The low frequency of our class meetings (twice a week) and the low participation in the blog meant that the internalization of the mathematics vocabulary would have been a slower process, especially since many of the terms used in mathematics are low frequency words or have different meanings in general vocabulary (Harmon, Hendrick, & Wood, 2005; Zevenbergen, 2000).

In their post-implementation interviews, all of the students indicated that they found the vocabulary instruction useful. They also indicated that they referred to the graphic organizers for word meanings and concepts associated with the terminology. The use of two-column and paragraph proofs standardized the vocabulary associated with problem-solving. This helped the students and me to have common terminology to use in our discourse about mathematics content and procedures in the classroom, in the homework, and online.

Goal 2: Increase Student's Mathematical Metacognition.

My second goal for the *Talking Math, Blogging Math* curriculum was to increase my students' mathematical metacognition through the use of explanatory proofs. I proposed that explanatory proofs could be used as a scaffolding tool to help students learn the different metacognitive behaviors associated with problem–solving (Garofalo and Lester, 1985; Pugalee, 2001). In order to check for these different metacognitive behaviors, I examined five students' blog posts, written work, and final interview for evidence of the metacognitive behaviors. The students included in the coding had completed at least three of the five problem sets included in the coding. I coded the students' work for evidence of the use of metacgonitive strategies as described by Pugalee (2001) during each of the four problem-solving phases (orientation, organization, execution, and verification) described by Garofalo and Lester (1985). Pugalee (2001) had created lists of activities that problem-solvers use during each of Garofalo and Lester's phases. During the orientation phase, problem-solvers use strategies such as reading and rereading the problem to make an initial analysis of a problem. During the organization phase problem-solvers identify problem-solving goals and subgoals, organize the data, and start to implement an overall plan. During the execution phase, problem-solvers perform the calculations, monitor the progress of the subgoals and goals, and redirect their efforts of needed. During verification, the problem-solvers evaluate their decisions and check computations. I used Pugalee's (2001) descriptions of to create rubrics for each phase. I used these rubrics to determine the attributes and characteristics of the problem-solving phases in the students' work. I found that there was not always a clear distinction between one category and another.

For the orientation phase, I looked for actions that showed the students' initial plans for solving the problems. These behaviors included re-reading the problem aloud, rewriting the problem, commenting on the problem, identifying how a proof would be set up, and commenting on their perceived ability to work the problem. Figure 41 shows examples drawn from the students' written work and their interviews. I analyzed data from the initial blog, two subsequent blogs, paragraph proof homework, two-column proof homework, and the interviews conducted with the students at the end of the implementation.

Problem-solving Phase	Associated metacognitive behaviors	Evidence in students' work:
Orientation (Garofalo and Lester, 1985; Pugalee, 2001)	• reading/rereading • initial/subsequent representations • analysis of information and conditions • assessment of problem difficulty (Pugalee, 2001, p. 245)	"Seven more than twice a number. That's greater than, I think. Yeah, that's greater than. Twice a number, 2x, oh I see now" – Jeremy "(rereading problem) so seven more than twice a number equals 17" –Acacia "So the problem is 5x minus 2 equals 13" – Danielle "(reads problem out loud) Well the solution is 17" – Jasmine "I want to find out the value of x" – Jasmine "First I would [create] the column" – Elizabeth "equation: 7x + 4 = 3x + 16" – Danielle "3(g-3) = 6, given" = Jeremy

Figure 41: Scoring of students' work: Metacognitive behaviors during orientation phase.

During the organization phase of problem-solving, the students identified the steps they were going to use to solve the equations. In two-column proofs, evidence for the organization phase could be found in the reason column. Ordering phrases such as “first I use”, “next, I will”, and “then I would”, and “finally” were counted as being part of the organization phase. Figure 42 shows examples of what I coded as evidence for the organization phase from the students proofs, blog posts, and interviews. There was more coded evidence for this phase than any of the other problem-solving phases.This is not surprising as Schoenfeld (1987) noted that novice problem-solvers often tend to read the problem and do the calculations. These activities are part of the organization and execution phases. In addition, the novice problem-solvers tend to continue with a solution technique even if they are getting nowhere.

Some of the data that I coded as being part of the organization phase could also have been coded to be part of the verification phase. The distinction I made was whether or not the reason for completing the problem was given before or after the execution phase. If it was given before, I counted the reason as being part of the organization phase. For example, one of the students wrote “so I subtract 4s from both sides. 3s minus 4s plus 66 equals 4s minus 4s plus 48.” I would code the “so I subtract 4s from both sides” as part of the organization phrase and “3s minus 4s plus 66 equals 4s minus 4s plus 48” as part of the execution phase. In contrast, if the reason for the step was given after the execution phase, I counted the step as part of the verification phrase. For example, in the excerpt “so it’s -2c < 6. Substitution”, I coded “so it’s 2c<6” as part of the execution phase and “substitution” as part of the verification phrase. The reason statements in the two-column proofs were counted as part of the organization phase.

Problem-solving Phase	Associated metacognitive behaviors	Evidence in students' work:
Organization (Garofalo and Lester, 1985; Pugalee, 2001)	• identifying goals and subgoals • making a global plan • implementing a global plan • drawing diagrams and organizing data into other formats (Pugalee, 2001, p. 245)	"First I would use the distributive property to get rid of the parentheses" – Jeremy " So to get the variables to one side and the constants on the other" – Elizabeth "I use the property of equality division to divide 33 by 3" – Acacia "Then I would see how many times 160 goes into 720, so…" – Danielle "I combine like terms- Jasmine

Figure 42: Scoring of students' work: Metacognitive behaviors during organization phase.

During the execution phase, students performed the calculations needed to solve the problem. As the students performed the calculations, they monitored their own progress to see if their plan of action needed to be adjusted. In two-column proofs, the steps performed during the execution phase were found in the statements column. Figure 43 shows what behaviors were scored as being part of the execution phase. These statements indicated the intermediate steps taken to solve the equation. The execution phase statements usually followed organization phase statements or preceded verification statements.

Problem-solving Phase	Associated metacognitive behaviors	Evidence in students' work:
Execution (Garofalo and Lester, 1985; Pugalee, 2001)	• performance of local goals • monitoring progress of local and global goals • performing calculations • redirecting efforts (Pugalee, 2001, p. 245)	"5x equals 15" – Jeremy "Which would then give you 3s plus 66 minus 4s is equal to 4s plus 48 minus 4s" – Elizabeth The equation now looks like 6n – 4n + 26" – Acacia "So I get w equals negative 24" – Danielle "72 times 100 is 7200" - Jasmine

Figure 43: Scoring of students' work: Metacognitive behaviors during execution phase.

Students reassessed the problem-solving decisions and verified their computations during the verification phase of problem-solving. The actions that were coded as being part of the verification phase included correcting the name of properties used, changing the overall problem-solving plan, correcting errors made during computation, and giving the rationale behind the steps taken. Figure 44 gives examples of statements coded as being part of the verification phase of problem-solving. The first blog entries and paragraph proof entries showed very little use of metacognitive behaviors associated with verification of problem-solving. In contrast, the students used self-checking and self-correction quite often during their final interviews.

Problem-solving Phase	Associated metacognitive behaviors	Evidence in students' work:
Verification (Garofalo and Lester, 1985; Pugalee, 2001)	• evaluating decisions • correcting computations (Pugalee, 2001, p. 245)	"It can't be five, it has to be four" – Jeremy "Wait, do I want to subtract 66 from both sides?" – Elizabeth "or you could add, er, wait, you could add 1 to both sides" – Acacia "So it would be 7x, no, 7 plus 2x equals 17" – Danielle "Oh wait, that is 3c actually, so c - 1 – 3c" - Jasmine

Figure 44: Scoring of students' work: Metacognitive behaviors during verification phase.

After I coded the students' work using Pugalee's (2001) list of behaviors, I examined the data to see if there was a change in students' metacognitive behaviors over time. I chose this approach due to the small number of students in my class. In the first blog homework, the students used organization and execution behaviors, but very little orientation and no verification behaviors. The only orientation behavior seen was the rewriting the initial problem. The blog entry in Figure 45 is representative of the entries of all of the students. The phrases "first you would take…", Next you would add…", "now to solve", and "after that, you subtract…" indicate metacognitive behavior associated with the organization phase of problem-solving. The rewriting of the individual computational steps shows behaviors associated with the execution phase. A few of the students rewrote the equation before they started to solve it (orientation phase), but none of the students checked their computations or evaluated their decisions (verification phase). The students' beginning paragraph proofs showed the same use of behaviors associated with the organization and execution phases.

> First you would take 10x+4 and put it in parentheses
> (10x+18)+4=48
> Next, you would add 4 to 48
> (10x+18)+4=48+4
>
> Solving the problem:
> Now to solve you would combine like terms and constants
> (10x+18)+4+48+4
> 10x+22=52
>
> After that, you subtract 22 from each side
> 10x+22=52
> -22-22
> 10x=30
>
> Finally you divide 10 from each side
> (10/10)x=30/10
> 1x=3
> So x=3

Figure 45: Entry from first blog.

Later blog entries showed a more varied use of metacognitive behaviors associated with the four problem-solving phases. Jasmine's paragraph proof shown in Figure 46 is representative of the students' responses for this the assignment. Jasmine used several different problem-solving behaviors. Jasmine started the proof by restating the problem and determining her overall problem-solving goal (orientation phase). Jasmine then defined some short-term goals (organization phase). She determined that she was going to apply the distributive property to reorder the terms, apply the property of equality to the equation, combine like terms, and divide both sides by three. Jasmine also described intermediate solution steps during the problem-solving process (execution phase).

3. 3(a - 5) = 18

   I want to find the value of a. I use the distributive property to reorder the terms. I then use the addition property of equality to add 15 to both sides of the equation. I then combine like terms and divide both sides by 3 to get the solution a = 11.

Figure 46: Paragraph and two-column proof homework assignment, Jasmine.

At the end of the implementation, the students' problem-solving interview responses showed that they all used all four of the problem-solving phases. Figure 47 shows Acacia's problem-solving process for the problem "72 is what percent of 160?" Acacia oriented the problem by describing how she set the problem up (orientation phase). Acacia then stopped to verify her equation when she added, "I think that's it" (verification phase). She then cross-multiplied ("72 times 100 equals 160 times x") (execution phase). Acacia realized that her solution of 72,000 was incorrect and restated the solution as 7,200 (verification phase). Acacia continued to work the problem until she reached the solution of 45 percent.

4) And then, 72 over 160 times x over 100. I think that's it. so 72 times 100 equals 160 times x. That would be 72,000 I think. Oh. so 7,200 equals 160x. Divide 160 from both sides. [much calculating, erasing, recalculating] so its 45 percent.

Figure 47: Acacia's problem-solving for the problem "72 is what percent of 160?"

Danielle's response to the same problem, shown in Figure 48, demonstrated the use of all of the four problem-solving phases as well. She started by restating the problem (orientation phase). She then described her plan for solving the problem (organization phase). Danielle verified her execution of the problem-solving steps several times while she worked. Finally, Danielle stated her solution.

72 is what percent of 160. Well you can use a calculator [but without a calculator] I would do… I did 160 divided by 72 er 72 divided by 160 and I would want to put a decimal point and add a zero to make it 720. Then I would see how many times 160 goes into 720, so... Okay then I decided to use the first number that popped into my head which is 4. I did 160 times 4 which is 640. And that's as far as I can go so far so 160 goes into 720 4 times. So I would subtract 640 from 720 and I would get 80 and then I would want to do another zero and bring it down. Then that's 800 160 times 4 is 640 then adding 160 would bring me to 800. 160 goes into 800 five times. So 72 is 45 percent of 160. I took the number out of 100.

Figure 48: Danielle's problem-solving for the problem "72 is what percent of 160?"

Increase Student's Mathematical Metacognition: Summary

In order to determine a change in the use of metacognitive strategies due to the use of explanatory proofs, I coded the students work using rubrics based on the metacognitive behaviors Pugalee (2001) identified for each of Garofalo and Lester's (1985) metacognitive phases. The students' range of problem-solving strategies increased over the course of the curriculum implementation. The students' first blog posts showed use of organization and execution strategies, but no use of orientation or verification strategies. In contrast, the students' problem-solving interviews conducted at the end of the implementation of *Talking Math, Blogging Math* exhibited the use of all of the problem-solving behaviors defined by Garofalo and Lester (1985) and Pugalee (2001).

The students' responses in the final interview indicated they felt that proofs provided support for them while they were working on a problem. Their responses demonstrated that they all viewed the proofs as a tool to be used to help them during the problem-solving process. One common thread in their responses was that the proofs provided a step-by-step method for problem-solving.

The students readily used the paragraph and two-column proofs as a common tool when discussing mathematics in the classroom and (to a limited amount) on line. The proofs provided a common discourse that the students could use when faced with the goal of understanding their individual problem-solving processes. Classroom discussions about problem-solving centered around the proofs. During the post-implementation interviews, several of the students set up the solutions to the given problems as two-column proofs without being asked to do so. In addition, the students indicated in their interviews that they found the proofs useful because the proofs gave them a step by step process to follow when solving-problems. The scaffolding provided by the proofs allowed the students to be legitimate participants in the classroom community. Because they had a common language they could use to discuss problem-solving, the students could help each other reach their zones of proximal development. Overall, the proofs were successful in helping students describe and understand their problem-solving processes.

Goal 3: Extend Classroom Community of Practice: Strategy and Findings

My third goal for *Talking Math, Blogging Math* was to extend my classroom community of practice through the use of the class blog. Because of the limited amount of time that I have with my students in the classroom, I wanted to use the classroom blog to continue our Pre-Algebra classroom math discussions. In order to collect my students' perceptions of their mathematical understanding, I interviewed the students at the end of the implementation of *Talking Math, Blogging Math*. I wanted to know whether the students felt that they were learning and gaining new understanding about their problem solving (metacognition) when they participated in a blog with other students in the class.

The responses collected from the interview indicate that students like to work in groups, but not necessarily on the blog. Although my students liked to use technology and used it readily at home, my students did not like the blog because they found it difficult to connect with each other. In general, students were frustrated by the asynchronous nature of blogging and their lack of access to computers. In their *Teens and Social Media* report, Lenhart, Madden, Macgill, and Smith (2007) note that students prefer to use synchronous forms of communication, such as telephones, cell phones, instant messaging, or talking in person, to email or blogging. Lenhart, Madden, Macgill, and Smith (2007) also found that adolescents who regularly used the Internet were more likely to blog on social networking sites or their personal blogs than teens that had less regular access to the Internet. Many of my students did not have regular access to the Internet and therefore probably did not regularly post to blogs online, contrary to my initial assumption that the students were "digital natives" that use the Internet daily.

Most of the students expressed a desire to use email or instant messaging instead of the blog. This is not too surprising considering that the PEW Internet & American Life Project (Lenhart, Madden, & Hitlin, 2005) identified the instant messaging as the communication method of choice for middle and high school students. Other technologies that the students suggested include web cams, email, and video blogs (video audio streaming).

Overall, the students indicated that they liked to work in groups in class but not on the blog. Because of the small number of students in my class, the groups were made up of 2 to 4 students. My students saw many advantages to working in groups in class including the opportunity to learn from each other, to have another person review their work for possible mistakes, and to use each other as sounding boards for ideas. However, the students noted in their post-implementation interviews and in class discussions that they did not like the asynchronous nature of the blog. The students did not like to wait for a response from the other students. They indicated that they would rather use a synchronous form of communication such as instant messaging or the telephone.

But the desire to work with another student was not clearly evidenced outside the classroom. When asked about to whom they go for help when they are having difficulties understanding or solving a math problem at home, only two of the students responded that they would email another student. Those students mentioned emailing another student only after listing other people that they would ask first. Students were more likely to ask for help from a family member (sister, father, grandfather), to look at the book, or search the Internet for help. However, when asked whom they would ask for help from when at school, the students all mentioned that they would ask a classmate. The data indicates that students are more willing to ask for help from someone that can give them immediate feedback rather than having to wait for a response by email or on a blog.

Extend Classroom Community of Practice: Summary

Although Pyon (2008) and MacBride and Luehmann (2008) both saw communities of practice coalesce through the use of blogs, I did not see the same results in my classroom. Perhaps one reason my students were not able to create this community of practice was the infrequency of our blogging due to technology constraints and the short duration of the blogging implementation. Many of the students had only intermittent access to the web at home or were competing with siblings for use of the home computer. Therefore, each blog was spread out over a week or two so that all students could get to a computer and participate. As a result, there were fewer blogging opportunities than I had anticipated. I had hoped that we could blog once or twice a week.

The students found waiting to receive replies to their posts frustrating. They indicated in our classroom discussions and their post-implementation interviews that they preferred more immediate forms of communication such as instant messaging. While 28% of online teens have created and used blogs, instant messaging, the telephone or cell-phone, and face-to-face conversations remain the communication methods of choice among teens (Lenhart, Madden, Macgill, & Smith, 2007). Another form of communication that we used during implementation, email, has fallen out of favor with teens. Just 14% of teens email each other every day compared to the number who talk to their friends every day using instant messaging (28%) or the cell phone (35%) (Lenhart, Madden, Macgill, & Smith, 2007).

Perhaps another reason that the students had trouble using the blog was that the students were not experienced users of blogs. MacBride and Luehmann (2008) stated that the students' blogging needed to be scaffolded. I made the assumption that the students were familiar with blogging and the type of writing that is associated with blogging. Research shows that students who do not have regular access to the Internet are less likely to access and to use blogs. This meant that my assumption that the students were experienced users of blogs was probably not true for my students.

Blogging tends to be a reflective form of writing that asks the user to reflect on and discuss the content of the blog (Richardson, 2009). Perhaps my students were not ready cognitively to participate in this type of reflective activity without different levels of scaffolding that met the different phases of cognitive development of my students. During the middle school years (ages 12 – 14) students are making the transition from the concrete operational phase of cognitive development to the formal operational phase of cognitive development (Piaget, 1972/2008). In addition, middle school age students tend to like to work alone or with a single partner (Wood, 2007). Participation on the blog increased when I let the students work together in small groups rather than as a class. However, there were still several students that chose not to participate.

Talking Math, Blogging Math **Discussion**

The *Talking Math, Blogging Math* curriculum showed success in increasing students' use and understanding of mathematical discourse and metacognition, but had mixed success in extending our classroom community of practice beyond the classroom. Analysis of student work suggested an increase in the use of mathematical terms over time. In addition, the students indicated in their post interviews that they found the vocabulary instruction helpful. In fact, many of the students used the vocabulary graphic organizers as an instructional aid. There were indications that the vocabulary instruction helped the students create a common discourse that they could use to communicate with each other about mathematics. The proof vocabulary provided the students and me a common discourse to use when discussing problem-solving. An additional benefit of the the vocabulary instruction was that it helped me identify where there were misunderstandings between my discourse and the students.

The students showed a wider and more varied use of mathematical problem-solving behaviors at the end of the implementation of *Talking Math, Blogging Math* than at the beginning. At the start of the implementation, the students mainly used behaviors associated with the execution phase of problem-solving (Pugalee, 2001). During the implementation, the students started to use more orientation, organization, and verification behaviors. The proofs provided the students with a common frame of reference to utilize when problem-solving individually and as a group. It is important that mathematics teachers realize that the students often need instruction in the language of mathematical problem-solving and need scaffolding on how to describe their problem-solving. In order to become metacognitive learners of mathematics.

Although other research studies have indicated that a blogging community of practice can be created in a classroom (Luehmann & McBride, 2008; Pyon, 2008), ours never coalesced around the blog. I think that several factors led to our lack of success. First, blogging was not the technology of choice for my students. This may have been due to the fact that they were not regular users of blogs or that they were not cognitively ready for the reflective thinking used while blogging. My students did not like the asynchronous nature of the blog. Because of technological constraints that my students faced, we were only able to blog once every week or two. In addition, my students were much more comfortable working in smaller groups than as a class as a whole. Participation increased after I allowed my students to work with just one or two other people, but only slightly. While the students were becoming more comfortable with using mathematical terminology in the classroom, they were not comfortable talking about problem-solving online.

Interestingly, while the students were not able to extend the community of practice based on the blog activity, the students did create a discourse community in the classroom based on the explanatory proofs. The proofs provided a common language and structure that the students could use to “do math” and to describe their problem-solving. Before the implementation, there was very little mathematical conversation during group work, and many of the students preferred to work individually. After the implementation, the students communicated more about their problem-solving during group work. In addition, the students were able to identify what they did not understand. The proofs proved to be a successful scaffolding tool for the gaining access to and gaining understanding of the language of mathematics. The students had greater insight into their problem-solving because they now had the language to describe their problem-solving processes.

VIII. CONCLUSION

Curricular Goals Revisited

This thesis describes the implementation and evaluation of the *Talking Math, Blogging Math* curriculum. The goals of the curriculum were to increase students' use and understanding of mathematical terminology, improve students' metacognition through the use of explanatory proofs, and to extend the classroom community of practice through the use of a class blog. While the amount of student data examined was small, I believe that the analysis of the data shows promise for the use of mathematical vocabulary instruction and the use of explanatory proofs. The explicit teaching and use of the mathematical register helped students have better understanding of what it meant to “do math”.

While I did not see a dramatic change in the amount of mathematical terminology used, I did see progress in my students understanding and use of mathematical terminology. In our classroom discussions, the students seemed more comfortable “talking math”. In addition, the students had a common discourse to use when discussing math with each other. The vocabulary and the proof structure provided scaffolding for problem-solving. The brainstorming we did at the beginning of the vocabulary lessons showed me that many of the students were having problems understanding the terminology used in their textbooks and in class.

During the student interviews, I asked the students whether they found the vocabulary instruction useful. All of the students gave positive responses. The students indicated that they referred to the graphic organizers when they had questions about word meanings. Other students used the examples and non-examples shown on the vocabulary graphic organizer to guide their problem-solving. I think that over time the graphic organizers can become a tool that the students use regularly to gain contextual understanding of mathematical terminology. Several of the students gave feedback on how to improve the graphic organizer. These suggestions included giving more space to write, adding references to page numbers in the book, and more specific additional notes.

I had postulated that writing about math would help my students determine what they understood and help them identify where their understanding stopped. I decided to use explanatory two-column and paragraph proofs to help my students express their problem-solving in writing. I hoped that this more formal type of mathematical writing could provide scaffolding for my students' problem-solving. The students' responses indicated that proofs provided scaffolding for them while they were working on a problem. However, it was not always clear whether the students really understood the problem-solving or whether they had just memorized a series of steps.

Unexpectedly, I found proofs to be a good formative assessment tool. When we discussed problem-solving in class or on the blog, it was easy for me to look at the students' work and see where they were struggling or where they had made mistakes. One of my students touched upon this when asked if she had a better understanding of what she did when she solved a problem. Acacia noted "It is also easier for you to help us because you can see where we got it wrong."

The use of proofs helped students describe and understand the different phases of problem-solving. The students exhibited the use of more varied problem-solving behaviors, especially verification behaviors. While the use of the blog did not lead to the creation a community of practice outside of the classroom, the students did create a learning community that used the proofs as a common tool for understanding the problem-solving process.

The last goal of the *Talking Math, Blogging Math* curriculum was to extend the classroom community of practice using the blog. I thought that adolescents would find group work beneficial and would enjoy working together online. Informal observations that I made in my Pre-Algebra classroom seemed to support this assumption. Prior research on blogging seemed to support my assumptions as well (MacBride & Luehmann, 2008; Pyon (2008).

While my students did like to work together in small groups, these groups were usually, but not always, divided by gender. My students noted that they liked working in groups because they learned from each other, but that there were sometimes disagreements that they needed to work through before they work on the math. These disagreements are not uncommon for middle school students (Wood, 2007).

My assumption that my students would willingly use the blog to communicate with each other outside of class did not factor in the communication preferences, the different developmental levels, and the different levels ofInternet usage of my students. During a class discussion, my students indicated that they did not like the asynchronous nature of the blog. They wanted to be able to work with their group synchronously like they do in class. When I asked the students what technologies that they would rather use, a majority mentioned instant messaging (IM). My students noted that they frequently used instant messaging to talk with each other. However, in an informal poll at the beginning of the year, many of my students indicated that they use social networking sites such as FaceBook. The communication on these sites tends to be asynchronous in nature. So there may be a stigma attached to using this technology for school rather than for social networking.

In addition, blogging is a reflective form of communication. Nardi, Schiano, Gumbrecht, and Swartz (2004) note that bloggers blog to record events in their life, as an outlet for feelings and thoughts, as a reflective exercise (writing and thinking), and as a forum for communities to share information. The fact that the blog required reflective thinking and writing may have discouraged some of the students from participating. Middle school students may not have practiced reflective thinking in the lower grades. In addition, many of these students may be at different stages of emotional and cognitive development (Wood, 2007). In retrospect, I should have provided more scaffolding for the students before and during our blogging. However, I still feel that blogging can become a useful and powerful tool in continuing the mathematical community of discourse.

Evaluation of the *Talking Math, Blogging Math* curriculum shows that explicit instruction in mathematical terminology and scaffolding of the use of the terminology can help students access the mathematical register. The students became more confident in describing their problem-solving because they had a common language that they could use to describe their thought processes. The use of the explanatory proofs helped students learn to understand and provided a common way to describe their problem-solving to others. The *Talking Math, Blogging Math* curriculum helped my students talk and think like mathematicians.

Implications for Practice and Research

The implementation and evaluation of *Talking Math, Blogging Math have* made me better aware of the difficulties that students face in the Pre-Algebra classroom. I had assumed that many of the terms that my students struggled with were words that were familiar to the students and therefore did not need further review. This, however, was not the case. My students were not understanding the math language or applying different meaning to the mathematics we were discussing in class. As a result, the students were not comfortable participating in mathematical discourse (spoken and written). In the future, I plan to make vocabulary instruction a central part of my classroom instruction from the beginning of the school year. Once students become comfortable using the graphic organizer, I would like them to use the graphic organizer on their own as an individualized tool. I also plan to scan the students' completed graphic organizers so that they cab become part of a student created mathematical dictionary on our class wiki. Students need to have a good understanding of the mathematical register before they tackle Algebra and higher math.

The use of explanatory proofs as a metacognitive and scaffolding device for problem-solving shows promise. I was surprised at how the students created a community of discourse around these tools. I plan to continue to use proofs in my classroom. I am interested to see if the students can eventually move beyond procedural understanding to deeper understanding of their problem-solving. I would like to add other forms of mathematical writing to my classroom as well. Perhaps math journals would be a tool that could be started and scaffolded in class and eventually moved online.

I still think that technology has an important role to play in my classroom. I realize now that I cannot make the assumption that my students have mastered all forms of technology. The *Talking Math, Blogging Math* curriculum asked my students to use technology as a reflective tool, something that they may not have been ready to do. Thus, I need to demonstrate and scaffold the type of usage I want to see. The research on blogging that I reviewed all indicated that the online community was created over the course of a school year. Perhaps we will see more success with a longer implementation period.

My students also wanted to have a more immediate connection with the other students while they were working online. There are many Read/Write web applications that are available and that are being developed. Further research is required to determine which applications will meet the criteria of immediacy and ease of use my students desire while supporting their metacognitive development.

APPENDIX I

Talking Math, Blogging Math

Combining the use of vocabulary instruction, explanatory proofs, and technology to improve mathematical understanding.

Created by

Linda Mathews

2009

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Curriculum Description

The *Talking Math, Blogging Math* curriculum is designed to aid students in the mastery of mathematical terminology through the use of direct instruction in math vocabulary, two-column proofs, paragraph proofs, class discussions, and discussions on the classroom blog. The curriculum grew out of my observations that my Pre-Algebra math students had difficulty understanding and were uncomfortable using mathematical terminology.

The *Talking Math, Blogging Math* curriculum is composed of three curricular units. The vocabulary curricular unit involves the use of a graphic organizer for direct instruction in mathematics terminology. The proof curricular unit uses explanatory two-column and paragraph proofs to help students understand and express their mathematical reasoning using the mathematical terminology. The technology curricular unit uses current online communication technology such as email and weblogs to help students continue to practice the use of mathematics terminology outside of the classroom. Each of these units can be used standalone or in conjunction with the two other units depending on students' prior experience and instructional needs. The technology module assumes that students have access to computers either at home or at school.

How the different units of *Talking Math, Blogging Math* work together to help students learn and use mathematical terminology is shown in Figure 1. The mathematical terminology learned through direct instruction is used in discussions in class, on the blog, and for writing simple proofs. The students use this terminology to help describe their problem-solving processes. In addition, the students' understanding of their problem-solving processes increases through the use of the simple proofs, classroom discussions, and written discussions on the blog. Students' time working with their classmates is extended by blogging together about mathematical concepts and different problem-solving approaches. The teacher can use the proofs and online communication to determine whether students show understanding of a concept or whether the students need additional instruction in a concept.

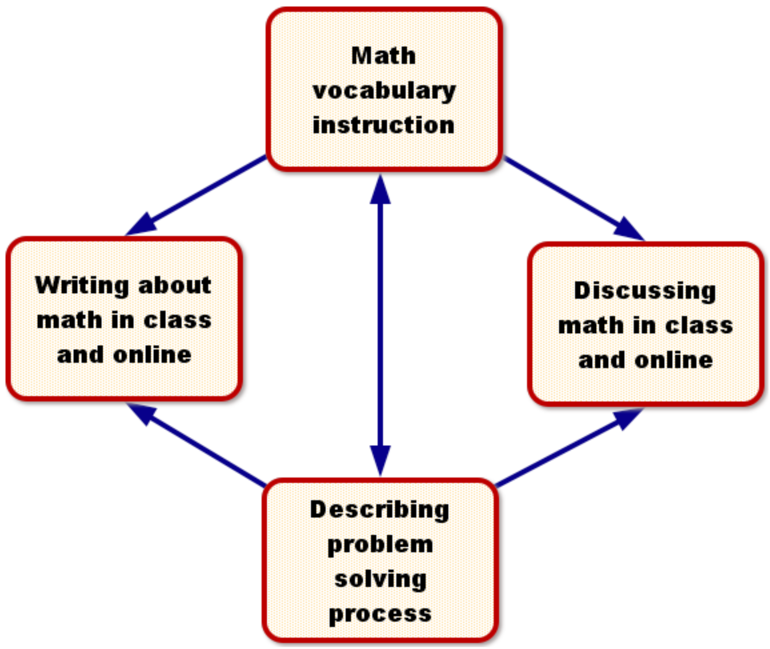

Figure 1: Interconnections between curriculum activities.

Goals

The overall goal of the *Talking Math, Blogging Math* curriculum is to increase students' understanding of how they do math. The combination of instruction in mathematical vocabulary, mathematical discourse, and explanatory proofs helps students to determine, to apply, and to describe what they know about mathematics. This larger goal can be further divided into three distinct goals that are concerned with the use of mathematical discussions between students and the teacher and students and each other (mathematical discourse), helping students understand what they do when they solve a problem (metacognition), and the creation of a classroom environment where mathematical inquiry and mathematical discourse take place (a mathematics community of practice).

Goal 1: Increase the use and understanding of mathematical terminology in mathematical discourse in the classroom.

Students often have trouble learning and using mathematical terminology. One way to help students learn and use mathematical terminology is to provide instruction on important terminology and give the students the opportunity to use the vocabulary in the classroom. The mathematics teacher needs to model correct use of mathematical vocabulary in the classroom. In addition, the mathematics teacher needs to model how to use mathematical terminology when describing problem-solving in writing through the use of explanatory proofs. The study of mathematical terminology provides students a tool that they can use to describe their problem-solving and to ask questions about the mathematical concepts covered in class.

Goal 2: Increase student's mathematical metacognition.

Schoenfeld (1987) states that most students are unaware of their thought processes when they solve mathematical problems. As a result, students often try to use a more complex mathematical concept when a simpler concept would be more efficient. Novice mathematics learners also attempt to directly translate a problem-solving technique to a new situation without thinking about whether the problem-solving construct is appropriate to the new situation. Students need to have explicit instruction in metacognitive strategies that help them understand their problem-solving process.

Once students have a better understanding of their thought process while solving problems, they can develop a deeper understanding of the mathematical concepts.

Goal 3: Extend a classroom community of practice so that the teacher and students are still "talking math" together when students are working at home.

Teachers and students create a community of practice in their classroom through their shared negotiation of what it means to learn. The mathematics reform movement calls for the mathematics classroom communities of practice to be places where mathematical inquiry and mathematical discourse take place. Instructional practices in these reform classrooms need to focus on the emphasis of higher-order thinking and problem-solving (Forman, 2003). Students need to be an active participant in their mathematical learning.

A classroom community of practice does not need to be confined to the physical classroom. Today's communication technology allows students and teachers to continue their mathematical discourse outside of the classroom. By participating in a classroom blog, students can continue to practice problem-solving, metacognitive strategies, and mathematical discourse on their computer at home. Students can continue to "think math" while discussing the homework assignments with other students or the teacher.

The *Talking Math, Blogging Math* curriculum is aligned to California and National Council of Teachers of Mathematics (NCTM) standards for mathematical reasoning and use of mathematical language. In addition, the vocabulary instruction unit meets California English-Language Arts standards for vocabulary development grades 6 through 12. The vocabulary instruction using the vocabulary graphic organizer can be beneficial to English Language Learners as well. The following section introduces the unit plan for *Talking Math, Blogging Math*.

Unit Plan

The *Talking Math, Blogging Math* unit plan below details the three curricular modules. The format for the unit plan was adapted from the unit plan template in *Understanding By Design* (Wiggins & McTighe, 2006). The desired results section describes the standards covered by the curriculum and the desired learning outcomes. The assessment evidence section describes what evidence can be used for assessing the desired outcomes. The assessment evidence listed in this section can be used to check for understanding as well as to determine what the students know.

Title: Talking Math, Blogging Math	Subject: Pre-Algebra and above
Topic: Mathematical Vocabulary	Grades: 6 - 12

Desired Results
Established Goals: *NCTM Standards (NCTM, 2000):* Instructional programs from prekindergarten through grade 12 should enable all students to: • monitor and reflect on the process of mathematical problem-solving. • organize and consolidate their mathematical thinking through communication; • communicate their mathematical thinking coherently and clearly to peers, teachers, and others; • analyze and evaluate the mathematical thinking and strategies of others; • use the language of mathematics to express mathematical ideas precisely. *CA Mathematics Content Standards Grade 7(California State Board of Education, 1997):* Mathematical Reasoning 2.5 Use a variety of methods, such as words, numbers, symbols, charts, graphs, tables, diagrams, and models, to explain mathematical reasoning.

Desired Results
2.6 Express the solution clearly and logically by using the appropriate mathematical notation and terms and clear language; support solutions with evidence in both verbal and symbolic work. *CA English Language Arts Content Standards Grade 6(California State Board of Education, 1997):* 1.0 Word Analysis, Fluency, and Systematic Vocabulary Development 1.2 Identify and interpret figurative language and words with multiple meanings. 1.3 Recognize the origins and meanings of frequently used foreign words in English and use these words accurately in speaking and writing. *CA English Language Arts Content Standards Grade 7(California State Board of Education, 1997):* 1.0 Word Analysis, Fluency, and Systematic Vocabulary Development 1.2 Use knowledge of Greek, Latin, and Anglo-Saxon roots and affixes to understand content-area vocabulary. 1.3 Clarify word meanings through the use of definition, example, restatement, or contrast. *CA English Language Arts Content Standards Grade 8 (California State Board of Education, 1997):* 1.0 Word Analysis, Fluency, and Systematic Vocabulary Development 1.3 Use word meanings within the appropriate context and show ability to verify those meanings by definition, restatement, example, comparison, or contrast. *CA English Language Arts Content Standards Grades 9 and 10 (California State Board of Education, 1997):*

Desired Results

1.0 Word Analysis, Fluency, and Systematic Vocabulary Development

1.1 Identify and use the literal and figurative meanings of words and understand word derivations.

1.2. Distinguish between the denotative and connotative meanings of words and interpret the connotative power of words.

CA English Language Arts Content Standards Grades 11 and 10 (California State Board of Education, 1997):

1.0 Word Analysis, Fluency, and Systematic Vocabulary Development

1.2 Apply knowledge of Greek, Latin, and Anglo-Saxon roots and affixes to draw inferences concerning the meaning of scientific and mathematical terminology.

Understandings:	Essential Questions:
Students will understand: • The importance of accurately using mathematical terminology to explain problem-solving. • The importance of using metacognitive processes during problem-solving.	• Do I understand the mathematical terminology? • Can I define the math terminology used for this unit? • Can I explain my problem-solving process to my teacher, my peers, or myself? • Are there other ways to solve the problem? • Can I express how I came to a solution for a problem in writing?

Desired Results	
Students will know… • How to use correct mathematical terms when talking about math. • How to do two-column algebraic proofs. • How to do paragraph proofs. • How to reflect on their mathematical problem-solving • The definitions of mathematical vocabulary	Students will be able to… • Explain their problem-solving process using correct mathematical terminology through verbal communication and writing. • Use their knowledge of word roots to determine the meaning of a term. • Correctly define mathematical terminology used in class.

Assessment Evidence

Assessment Evidence
Performance Tasks: Students will complete a graphic organizer for new terms introduced during the unit. They will provide a definition for the term, information about the origin of the term, give examples of related terms, write or draw examples of the term, provide non-examples for the term, and note any additional information that will clarify the meaning of the math term. Students will use the math terms that they defined when they are talking about math and writing about math. The students will also explain problem-solving for problems using two-column proofs and paragraph proofs. The proofs will be used for homework problems, in class work, and on the class blog.
Key Criteria: • The correct use of mathematical vocabulary • The use of mathematical vocabulary when writing about math. • The use of mathematical vocabulary in the context of class discussions.
Other Evidence: • Students' written work. • Vocabulary Graphic Organizers. • Transcripts of the discussions on the blog. • Transcripts of class discussions. • Vocabulary quizzes. • Chapter tests that contain two-column or paragraph proofs.

Learning Plan – Vocabulary Instruction

Learning Plan – Vocabulary Instruction
Learning Activities: Note: The activities described below are not tied to a specific math concept, but rather provide a structure for teaching the math concepts. I designed these activities to be used with math concepts covered in Pre-Algebra. However, they can be used with any math topic.
a) Introduction • Preview the math concepts that are to be learned in the day's lesson using the mathematical terminology for the concept. • Introduce new vocabulary that is needed for that day's lesson. If none is needed, review already introduced mathematical terminology needed for the concept.
b) Initial scaffolding/Modeling Help the students complete a vocabulary graphic organizer for each new mathematical term. • Brainstorm about what the possible meanings are for the term. Then read the definition from the glossary of the textbook. Have the students record the term and the definition in the Term and Definition field of the graphic organizer. • Provide the students with the word roots for the word. Have the students write this information in the Word Roots section of the graphic organizer. • Brainstorm with the students about words that could be related to the term being defined. If they are not sure, have them look in an etymological dictionary. Record this information in the Related Words field of the graphic organizer. • Ask the students if there are any non-math meanings for the word. If there are, record them in the Related Words field of the graphic organizer as well. • Ask the students to provide examples for the term. Tell them they can use their textbook. Write their suggestions on the board. Discuss with the students which examples best illustrate the definition of the term. The examples can be made of words, drawings, equations, expressions, or anything else that helps define the term. • Ask the students to provide non-examples for the term. There may already be some on the board! Write their suggestions on the board. Discuss with the students why these are non-examples. • Have the students record abbreviations, related concepts, or any other information

Learning Plan – Vocabulary Instruction
that helps clarify the definition of the term in the Additional Notes section. • Model how the term is used when discussing a math concept or a solution. • Model how the term is used when discussing a math concept or a solution.
c) Independent work • Have the students work together in small groups on problems that are related to the concepts being covered. Monitor the students' progress by observing the groups as they work.
d) Debriefing/Reflection Ask students to describe their problem-solving process to each other and the class.
e) Assessment Monitor the group work to see if and when students are using the math terminology correctly. Monitor the homework to see if and when students are using the math terminology correctly. Vocabulary quizzes

Learning Plan – Explanatory Proofs

Learning Plan – Explanatory Proofs
Learning Activities: Note: The activities described below are not tied to a specific math concept, but rather provide a structure for teaching the math concepts. I designed these activities to be used with math concepts covered in Pre-Algebra. However, they can be used with any math topic..
a) Introduction • Preview the math concepts that are to be learned in the day's lesson using the mathematical terminology for the concept. Ask the students to describe their problem-solving steps for a simple equation. List all of the steps on the board.
b) Initial scaffolding/Modeling • Model writing a paragraph using the students' problem-solving steps. • Next, model a two-column proof for the same problem. Make sure to identify the properties used for solving the equation. • Model paragraph proofs and two-column proofs for several more related problems. The number of problems demonstrated depends on students' understanding.
c) Independent work • Have the students work together in small groups on problems that are related to the concepts being covered. They should work together to write paragraph or two-column proofs for each problem. • Monitor the students' progress by observing the groups as they work. • For homework, have the students continue to write proofs for each solution.
d) Debriefing/Reflection • Have students share their two-column and paragraph proofs with each other. Discuss how the proofs can differ and why. • Ask students to describe their problem-solving process to each other and the class.

Learning Plan – Explanatory Proofs
e) Assessment • Monitor the group work to see if and when students are having problems. • Monitor the homework to see if and when students are having problems. • Quizzes including use of proofs. • Tests including the use of proofs.

Learning Plan - Technology

Learning Plan – Technology
Learning Activities: Note: The activities described below are not tied to a specific math concept, but rather provide a structure for teaching the math concepts. I designed these activities to be used with math concepts covered in Pre-Algebra. However, they can be used with any math topic. Although I used blogging for my curriculum, this module can be adapted to other applications as well. It is assumed that students have access to computers at school and at home.
a) Introduction • Preview the math concepts that are to be learned in the day's lesson using the mathematical terminology for the concept. • Introduce new vocabulary that is needed for that day's lesson. If none is needed, review already introduced mathematical terminology needed for the concept. • Review paragraph and two-column proofs.
b) Initial scaffolding/Modeling • Have the students practice logging into their blog accounts. • Have the students write a brief message to other students in class. • Have the students read each others messages. • Brainstorm and create a set of "etiquette" rules for the students to follow on the class blog. • Decide as a class how the class will use the blog. • Blog as a whole class or in smaller groups • Use the blog for homework help • Use the blog to share solutions • Use the blog for math discussions

Learning Plan – Technology
• Create a paragraph proof for a simple equation. • Practice posting the paragraph proof to the blog. • Practice commenting on the paragraph proofs as a class.
c) Independent work • On the class blog, post a problem similar to the problems being discussed in class. • Have the students post their work and solutions to the blog. Their work should be in the form of a paragraph proof. (Make sure to monitor the blog posts and have a blog etiquette agreement in place.) The frequency of posting problems on the blog is dependent on the students' access to computers outside of school.
d) Debriefing/Reflection • Discuss the blog work in class. • Have students respond to the posts of other students.
e) Assessment. • Monitor the blog to see if and when students are having problems. • Monitor the blog for student participation.

Vocabulary Instruction

The vocabulary terms can be introduced before the concept that uses the vocabulary is taught or after inquiry into the concept has taken place. The students should explore the definition, word roots, related words, examples, and non-examples for each term. This information is recorded in the six sections of the graphic organizer shown in Figure 2.

In the Terms and Definitions section, students should write down the mathematical term and its definition. The source for the mathematical definition can be the students' textbook or another source preferred by the teacher. Before the students look up or are provided the definition, the teacher should ask the students what they think the term means. This process allows the teacher to determine prior knowledge and possible areas of misunderstanding. The students' definition ideas can noted in the additional notes or related word sections to help clarify the meaning of the term

Next, the students should write down any prefixes and word roots in the Word Roots section that can help them better understand the mathematics term and identify related mathematical terminology. A great source for information about word roots can be found in Schwartzman's *The Words of Mathematics: An Etymological Dictionary of Mathematical Terms Used in English*. Many of the larger collegiate level dictionaries contain etymological information as part of their word definitions as well.

In the Related Words section, the student can list any homonyms, synonyms, antonyms, non-math meanings for the term, and other words that share the same word root. This is a great place to list multiple meanings for words that the students mentioned at the beginning of the vocabulary exercise. Words that are related to the concept being defined should be placed here as well. For example, the term slope could be listed in this section as part of related words for the mathematical term linear equation.

In the Example and Non-Examples fields students should add drawings, descriptions, equations, or anything else that helps them to understand what the term represents and does not represent. The students can use their textbooks and other resources to look up possible examples and non-examples. Brainstorming for examples and non-examples as a class helps students better understand what is an example or a non-example. It is wise to review the examples and non-examples a few days after these are initially introduced to help students clarify the concept they are learning.

In the Additional Notes section, the students can add any additional information that helps them better understand the term. This can include related formulas, definitions of related terms, reminders, or clarifications. Students can use this section to make connections between the term being defined and other terms they know. Figure 3 shows a completed graphic organizer for the term function. Figure 4 shows the blackline master for the vocabulary graphic organizer.

Term and Definition	Word Roots	Related Words
1. The vocabulary term, 2. The definition of the term, 3. Page number in the text where the word can be found.	***The Greek, Latin, or Other language roots of the vocabulary term.***	***In this section, list any: 1. Homonyms, 2. Synonyms, 3. Antonyms, 4. Words that the term might have be confused with, 5. Related words.***
Example	**Non-Example**	**Additional Notes**
Add an example here that illustrates the term. This can be a drawing, a description, an equation, or anything else that helps the student understand the term.	***Add an example here that illustrates what the term is not. This can be a drawing, a description, an equation, or anything else that helps the student better define the term. The non-example can be an illustration of a term or concept that is often confused with the vocabulary term.***	***In this section, note any abbreviations for the term, any related concepts, or any additional information that adds to understanding.***

Figure 2: Description of vocabulary graphic organizer fields.

Term and Definition

function – a special relation in which each member of the domain is paired with exactly one member in the range. Each domain value can map to only one range value. The input and output values for a function can be plotted on a coordinate plane as (domain, range).

Word Roots

Latin:

functio, funct = performance

Related Words

coordinate pair: an ordered pair of numbers (x,y) that give the location of a point on a plane.

domain: x value in a coordinate pair; set of x coordinates; input to a function.

range: y value in a coordinate pair; set of y coordinates; output of a function.

Example

{(-10,-3),(0,-22),(10,-9),(20,3)}

x	y
-10	-3
0	-22
10	-9
20	3

Each domain (x) maps to only one range (y).

Non-Example

{(-10,-3),(10,-22),(10,-9),(20,3)}

x	y
-10	-3
10	-22
10	-9
20	3

Can't be a function because <u>one</u> domain value (x) maps to two range values (y)A relation can be represented in table form.

Additional Notes

The coordinate pairs can be plotted on a graph or coordinate plane.

The graph of a function can indicate the type of underlying function.

Figure 3: Example vocabulary graphic organizer – function.

Term and Definition	Word Roots	Related Words
Example	**Non-Example**	**Additional Notes**

Figure 4: Vocabulary graphic organizer

The NCTM standards and California state standards identify several mathematical terms as part of the concepts that Pre-Algebra students should know. A list of these terms can be found in Figure 5. Students should know and be able to use these terms as part of their spoken and written discourse in the mathematics classroom.

mean	**circumference**	**histogram**	**rectangle**
measures of central tendency	**coefficient**	**inequality**	**rhombus**
median	**common denominator**	**integers**	**samples**
mode	**commutative property**	**intercept**	**scatter plot**
nonlinear function	**congruence**	**interquartile range**	**similar**
number line	**congruent**	**inverse**	**simplify**
number theory	**constant**	**least common multiple**	**slope**
numerator	**coordinate pair**	**length**	**solve**
order of operations	**coordinate plane**	**linear function**	**square**
paragraph proof	**coordinates**	**prime numbers**	**square roots**
parallel	**denominator**	**probability**	**squares**
parallelogram	**diameter**	**product**	**sum**
percent of change	**difference**	**properties of equality**	**three dimensional object**
perimeter	**distributive property**	**proportion**	**transformation**
place value	**divisibility (rules)**	**Pythagorean theorem**	**triangle**
polygon	**equation**	**quotient**	**two dimensional object**
population	**equivalence**	**radius**	**two-column proof**
absolute value	**estimation**	**rate**	**unit**
angle	**expression**	**ratio**	**variable**
area	**function**	**rational numbers**	**volume**
box plot	**graph**		
	greatest common factor		

Figure 5: Terms that are used in the NCTM standards (National Council of Teachers of Mathematics, 2000) and the California state standards (California Department of Education, 1997).

Vocabulary Instruction Resources

Vocabulary Instruction Resources
California State Board of Education. (1997). *English–language arts content standards for California public schools, kindergarten through grade twelve*. Sacramento, CA: California Department of Education. *These standards describe what English - Language Arts content California students should be learning and mastering at each grade level.*
California State Board of Education. (1997). *Mathematics content standards for California public schools: Kindergarten through grade twelve*. Sacramento, CA: California Department of Education. *These standards describe what mathematics content California students should be learning and mastering at each grade level.*
Davis, Jon D. (2008). Connecting students' informal language to more formal definitions. *Mathematics Teacher*, 101(6), 446 – 450. *Davis describes how teachers can help students learn the definitions of formal mathematical terms by first creating and using their own informal mathematical terminology.*
Gay, A. S. (2008). Helping teachers connect vocabulary and conceptual understanding. *Mathematics Teacher*, 102(3), 218 – 223. *Gay notes that in order to understand and communicate mathematics, students need to know the meaning of mathematics vocabulary words. Gay describes vocabulary strategies and activities that can be used in middle and high school mathematics classrooms.*

Vocabulary Instruction Resources
Jackson, M. B., & Phillips, E. R. (1983). Vocabulary instruction in ratio and proportion for seventh graders. *Journal for Research in Mathematics Education*, 14(5), 337-343. *Jackson and Phillips researched whether direct mathematics vocabulary instruction helped 7th grade students' achievement during a unit covering ratios and proportions. Jackson and Phillips found that the vocabulary instruction did improve the students' comprehension of mathematical terminology.*
Kaplan, A. (Ed.). (2000). Algebra to go: A mathematics handbook. Wilmington, MA: Great Source Education Group. *Algebra to Go is a great student and teacher reference book for Algebra concepts and terminology. This book is not a text book, but rather a good secondary source for students seeking clarification or an alternate explanation of a concept.*
National Council of Teachers of Mathematics (2000). *Principles and standards for school mathematics*. Reston, VA: Author. *This publication documents the National Council of Teachers of Mathematics' recommendations for the teaching and learning of mathematics for grades K-12. These recommendations are based on the six principles of equity, curriculum, teaching, learning, assessment, and technology.*

Vocabulary Instruction Resources
Rubenstein, R. N. (2000). Word origins: Building communication connections. *Mathematics Teaching in the Middle School*, 5(8), 493. *Rubenstein describes how learning the origins mathematics vocabulary can facilitate the reinforcement of the concepts of mathematics. Rubenstein gives examples of roots and related words for several common terms used in middle school mathematics.*
Rubenstein, R. N. (2007). Focused strategies for middle-grades mathematics vocabulary development. *Mathematics Teaching in the Middle School*, 13(4), 200-207. *Rubenstein describes different ways that mathematical terminology can be a source of confusion to students. She provides a summary of different vocabulary challenges for students. Rubenstein then describes several different focused language strategies that teachers can use to help students understand mathematics terminology.*
Rubenstein, R.N., & Schwartz, R.K. (2000). Word histories: Melding mathematics and meanings. *Mathematics Teacher*, 93(8), 664 – 669. *Rubenstein and Schwartz describe how learning the origins mathematics vocabulary can facilitate the reinforcement of the concepts of mathematics. Rubenstein and Schwartz give examples of roots and related words for several common terms used in Algebra, Geometry, and general mathematics.*

Vocabulary Instruction Resources
Schwartzman, S. (1994). *The Words of mathematics: An etymological dictionary of mathematical terms used in English*. Washington, D.C.: Mathematical Association of America. *Schwartzman's work describes the origins of over 1500 mathematical terms used in English.*
Thompson, D. R., & Rubenstein, R. N. (2000). Learning mathematics vocabulary: Potential pitfalls and instructional strategies. *Mathematics Teacher*, 93(7), 568-574. *Thompson and Rubenstein describe general, oral, written, and kinesthetic strategies to help students access the meaning of mathematical vocabulary.*

Explanatory Proofs

As part of the mathematics reform movement, the NCTM (2000) standards call for the study and use of proofs at all levels of mathematics. Traditionally proofs have performed many different functions in mathematics (Hanna, 2000). Some of these functions have included persuasion, verification, systemization, communication, exploration, and explanation (Hanna, 2000; Knuth, 2002).

Hanna (1990, 2000) and Hersh (1993) identify three different perceived roles of proofs in mathematics education. The first role, the formal proof, is a syntactic structure that is based on axiomatic sentences and inferences made about those sentences. The goal of a formal proof is to prove the truth of the initial statement. Hanna (1990, 2000) notes that the use of formal proofs was adopted by universities and in secondary education as part of the new math curricula in the space race era of the 1960s.

The second role of proof that Hanna describes, acceptable proof, relies more on the mathematical meaning, or semantics, that a mathematician draws from the proof. Mathematicians' judgment is used to determine the validity of an argument rather than the formal structure of an argument. Mathematicians use this form of proof to communicate with other mathematicians.

The third role of proof that Hanna describes is the teaching or explanatory proof. An explanatory proof describes why a particular concept is true rather than proving truth through deductive reasoning. Hersh (1993), Hanna (1990,2000), and Knuth (2002) note that explanatory proofs can aid student insight and understanding of mathematical properties and theorems.

Students should be instructed in how to use two-column proofs and paragraph proofs to describe their thought process while they are solving an equation or problem.

The teacher should model how to solve problems using the vocabulary terms taught for the concept being covered. The students will have the opportunity to practice writing these proofs in class, on the blog, and in their homework. The students should be given the opportunity to work together to create the proofs in class and on the blog.

Paragraph Proof

A paragraph proof allows students to describe their problem-solving process in narrative form. The students should include a sentence for each step of the process. They should identify algebraic and number properties that describe their reasoning. The final statement should show the solution(s) for the problem. Figure 6 shows an example of a simple paragraph proof used to show the problem-solving steps for the equation 3(g – 3) = 6.

Paragraph proof for 3(g - 3) = 6.
First I use the distributive property to simplify the equation. The equation now reads 3g – 9 = 6. I use the property of equality addition to add 9 to both sides of the equation. The equation is now 3g = 15. I use the property of equality division to divide both sides of the equation by 3. The solution of the equation is g = 5.

Figure 6: Paragraph proof example.

Two-Column Proof

A two-column proof is a more formal method used to describe problem-solving reasoning. The proof is set up as a T-chart with the headings shown below. Each line of the proof should show a step in the problem-solving process (the statement) and the mathematical reasoning used to create that step (the reasoning). The final statement should show the solution(s) for the problem. Figure 7 shows an example of a two-column proof used to show the problem-solving steps for the equation 3(g – 3) = 6.

Statement	Reason
3(g - 3) = 6	**Given**
3(g) – 3(3) = 6	**Distributive property**
3g – 9 = 6	**Substitution**
3g – 9 + 9 = 6 + 9	**Property of equality addition**
3g = 15	**Substitution**
$\frac{3g}{3} = \frac{15}{3}$	**Property of equality division**
g = 5	**Substitution**

Figure 7: Two - column proof example.

Explanatory Proof Resources

Explanatory Proof Resources
California State Board of Education. (1997). *Mathematics content standards for California public schools: Kindergarten through grade twelve*. Sacramento, CA: California Department of Education. *These standards describe what mathematics content California students should be learning and mastering at each grade level.*
Hanna, G. (1990). Some pedagogical aspects of proof. *Interchange,* 21(1), 6-13. *Hanna discusses three different ways that proofs can be used in mathematics education. In addition to the formal proof and the acceptable proof, explanatory proofs can help students understand key mathematical concepts.*
Hanna, G. (2000). Proof, explanation and exploration: An overview. *Educational Studies in Mathematics,* 44(1/2), 5-23. *Hanna explains the different roles that proofs play in mathematics. Mathematicians use proofs for verification, explanation, communication, discovery, and exploration. Hanna proposes that students and teachers can use explanatory proofs as part of their mathematics curriculum.*
Hersh, R. (1993). Proving is convincing and explaining. *Educational Studies in Mathematics,* 24(4), 389-399. *Hersh distinguishes between two purposes of proof in mathematics. He notes that in research the mathematician uses proof to convince others. However, in the classroom, the purpose of proof is to explain and clarify important concepts.*

Explanatory Proof Resources
Knuth, E. J. (2002). Secondary school mathematics teachers' conceptions of proof. *Journal for Research in Mathematics Education*, 33(5), 379-405. *Knuth notes that mathematics teaching reform calls for the teaching and use of proofs in the secondary classroom. Knuth examines how a teacher's perception of what a proof is will determine what type of proof is used in the mathematics classroom.*
Schoenfeld, A. H. (1994). What do we know about mathematics curricula? *Journal of Mathematical Behavior,* 13(1), 55-80. *Schoenfeld discusses how questions about content, tracking, problem-based curricula, and the role of proof have influenced the teaching of mathematics.*

Blogging About Math

In addition to the more rigorous math standards called for at the state and local levels, students are facing new technology requirements (NCTM, 2000; California Department of Education, 2005). The NCTM, as part of its chapter on Principles for School Mathematics, states that “technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning” (p. 11). The Mathematics Framework for California Public schools calls for all students to be familiar with basic computer skills, different computer applications, and the Internet. According to MacBride and Luehmann (2008) teachers are now faced with the challenge of incorporating new technology into their curricula and classroom. On the plus side, this new technology capitalizes on students’ familiarity with different forms of online communication. Groth (2008) notes that technology such as discussion boards and blogs allow for teachers to monitor multiple conversations about math at one time and to use the blog discourse as a formative assessment tool to guide what is taught in the classroom.

Richardson (2009) notes that over ninety percent of our students use social web technologies such as MySpace and instant messaging in their personal lives. The PEW Internet & American Life Project (Lenhart, Madden, & Hitlin, 2005) supports this. The PEW Internet & American Life Project (Lenhart, Madden, & Hitlin, 2005) surveyed a sample of 1,100 children between ages 12 to 17 and their parents or guardians. The survey found that in 2004 over 87% of teens used the Internet and over half of teens go online daily. In addition, teens prefer communicating through instant messaging (IM) rather than email. The PEW Internet & American Life Project (Lenhart, Madden, & Hitlin, 2005) found that most students become avid Internet users in the 7th grade. But, as

Richardson (2009) observes, educators have been very slow to use what he calls the Read/Write web because most educators lag behind their students in the use of technology in their everyday lives.

The Read/Write web allows users to easily create content through the use of applications such as wikis and blogs. Weblogs were one of the first widely adopted tools of the Read/Write web (Richardson, 2009). Richardson (2009) defines weblogs as “an easily created, easily updateable Website that allows an author (or authors) to publish instantly to the Internet from any Internet connection” (p. 17). Weblogs or blogs are now being used in conjunction with video and audio technology on sites such as YouTube and MySpace.

Richardson (2009) identifies several ways in which blogs can be used to improve student learning. First, blogs allow teachers and students to construct content that becomes part of a larger body of knowledge on the Internet. Thus, students’ work has a potential audience beyond the immediate classroom. Blogs can also help students who are uncomfortable speaking up in the classroom environment to become active participant in classroom discussions online.

Next, blogs allow teachers to connect with their students’ outside of the classroom. Teachers and students can collaborate with each other and other students, teachers, and professionals working on the same content. Students’ can use the information in the classroom blogs and other blogs on the web to build expertise on a specific topic or in a particular content area. Blogs provide a way for this collaborative work to be archived for later reflection.

Finally, Richardson (2009) observes that students need the opportunity to use web technology as part of the development of their technological literacy. He explains that classroom blogs can be used as classroom portals, online archives, collaborative space, and e-portfolios for students and teachers alike. The studies detailed in the next paragraphs demonstrate some of the different ways that blogs can be used in the classroom.

In the *Talking Math, Blogging Math* curriculum, blogs are used to create a collaborative space that students can use to discuss math outside of the classroom. The blog should be a place where students can help each other with math and where the teacher can provide clarification and explanations as needed. The blog discussions can be done at the whole classroom level or in small groups. The teacher should actively monitor the discussion, but should try to not monopolize the instruction. Richardson's (2009) *Blogs, Wikis, Podcasts, and Other Powerful Web Tools for Classrooms* gives a good overview of weblogs, guidance for creating a classroom blog, and links to examples of successful classroom blogs.

I have included some examples of questions that my students and I have addressed on our classroom blog in Figure 8. I start each blogging assignment by discussing the assignment in class and then I have the students work together on part of the assignment in class. The students then work together on the blog to complete the assignment. I allowed the students a week to blog together because some of my students do not have access to the wiki at varying times during the week. I monitor the blog as the students work, but I only intervene if necessary. Once the students have discussed the questions on the blog, we discuss the questions together as a class in the classroom. I use the students' blog transcripts and our discussions in class as formative assessments to determine what the students have mastered and what needs to be revisited.

Questions about Equations and Inequalities
Tell whether the statement is always true, sometimes true, or never true. Explain why – give examples or counterexamples. Please write your answers on a separate piece of paper. 1. The same variable term can be added to both sides of an equation without changing the solution of the equation. 2. The same variable term can be subtracted from both sides of an equation without changing the solution of the equation. 3. An equation in the form *a*x + *b* = c cannot be solved if *a* is negative number. 4. The solution of the equation $\frac{x}{3}$ = 0 is 0. 5. The solution of the equation $\frac{x}{0}$ = 3 is 0. 6. In solving an equation of the form *a*x + *b* = *c*x + *d,* the goal is to rewrite the equation in the form *variable = constant*. 7. In solving an equation of the form *a*x + *b* = *c*x + *d,* subtracting *c*x from each side of the equation results in an equation with only one variable term in it. 8. The same variable term can be added to both sides of an inequality without changing the solution set of the inequality. 9. The same variable term can be subtracted to both sides of an inequality without changing the solution set of the inequality. 10. Both sides of an inequality can be multiplied by the same number without changing the solution set of the inequality. 11. Both sides of an inequality can be divided by the same number without changing the solution set of the inequality. 12. Suppose *a* > 0 and *b* < 0. Then *ab* > 0.

Figure 8: Sample questions about inequalities.

Blogging Resources

Blogging Resources
Brescia, W. F. J., & Miller, M. T. (2006). What's it worth? The perceived benefits of instructional blogging. *Electronic Journal for the Integration of Technology in Education, 5*, 44-52. *Brescia and Miller conducted a study to determine if there were any instructional advantages of blogging in a college classroom setting. Their findings suggested that the most valuable aspects of blogging were the reinforcement of course engagement and the repetition of exposure to coursework.*
Downes, S. (2004). Educational Blogging. *EDUCAUSE Review*, 39(5), 14-26. *Downes describes how blogs have been adapted for use in the classroom environment. He gives a brief definition of blogs and blogging applications. Downes also explains how blogging is currently being used in various classrooms.*
Goos, M. (2004). Learning mathematics in a classroom community of inquiry. *Journal for Research in Mathematics Education*, 35(4), 258-291. *Goos examined the steps teachers took to create a community of practice using mathematical inquiry in their 11th grade classrooms. Goos described methods that teachers can use to help students become members in a community of mathematical inquiry. These methods included scaffolding of concepts, use of peer collaboration, and the combining of real world and theory to explain mathematical concepts.*

Blogging Resources
Groth, R. (2008). Analyzing online discourse to assess students' thinking. *Mathematics Teacher,* 10(4), 422 – 427. *Groth found advantages and disadvantages to using on line discussion boards. He described strategies to use to make on line discussion boards (including blogs) an effective tool for assessing students understanding and learning.*
MacBride, R., & Luehmann, A. L. (2008). Capitalizing on emerging technologies: A case study of classroom blogging. *School Science and Mathematics*, 108(5), 173-183. *MacBride and Luehmann examined how a high school teacher integrated blogging into his math curriculum. They found that the teacher had multiple intentions for creating the blog and that the teacher utilized the blogs in multiple ways. MacBride and Luehmann concluded that the students and teacher both perceived the blogs as a worthwhile investment of time and resources.*
New Media Consortium. *The horizon report: 2007 edition*. NMC and Educause. Retrieved September 30, 2008, from http://www.nmc.org/horizon/2007/report. *The Horizon Report: 2007 Edition identified different technologies that could have an impact on teaching and learning in both the near future and in the coming decade. The Horizon Report: 2007 Edition named blogging as a technology that would have great impact on teaching and learning in the next five years.*

Blogging Resources
Pyon, S. (2008). Why math blogs? *Teaching Children Mathematics*, 14(6), 331 – 335. *Pyon implemented blogging in her third grade classroom to determine if blogging could help her students' metacognitive skills. Pyon found that blogging was an effective tool for all of the students in her class including the English Language Learners. The blogs helped her students gain important critical thinking and metacognitive skills.*
Richardson, W. (2008). *Blogs, wikis, podcasts, and other powerful web tools for classrooms (2nd ed.).* Thousand Oaks, CA: Corwin Press. *Richardson's book guides teachers through the implementation and use of read/write technologies in the K-12 classroom. Richardson includes chapters on blogs, wikis, podcasts, and other emerging web technologies.*

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