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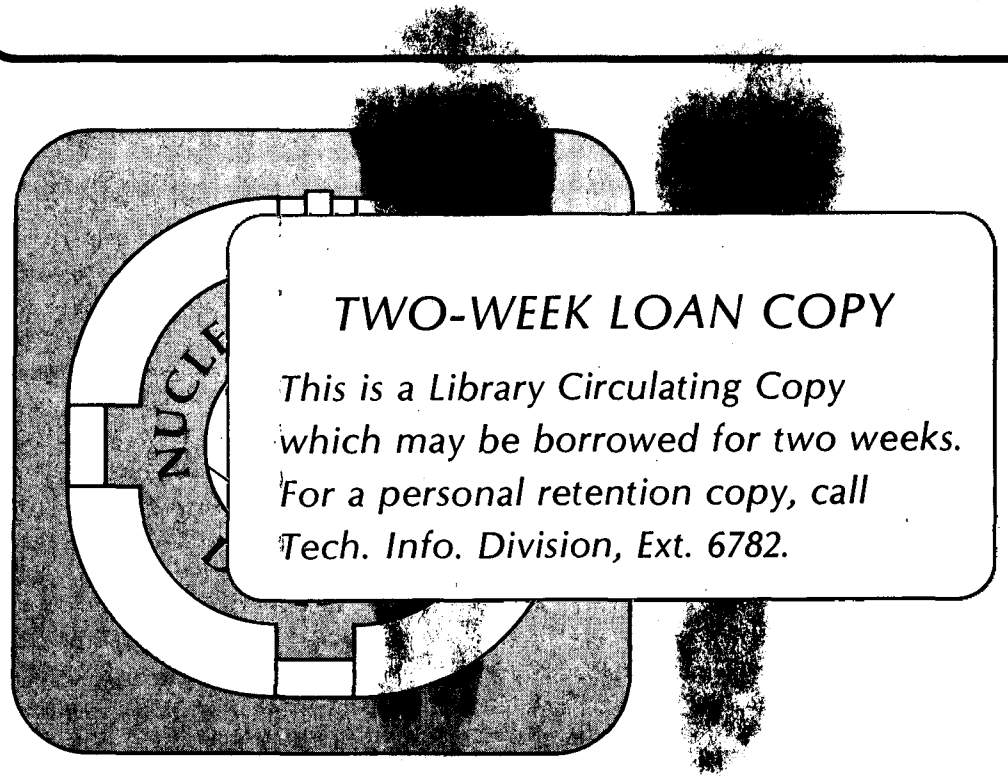

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Pion Production as a Probe of the Nuclear Matter Equation of State

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Abstract:

It is shown that thermal and chemical equilibrium are approached during the high density stage in central nucleus-nucleus collisions and that the yield of produced pions is determined at that time. A chemical model with Rankine-Hugoniot compression is used to extract a nuclear matter equation of state from the observed pion yield assuming a partition of the internal energy per nucleon into thermal and compressional energy fractions.

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The primary objective in studying relativistic nucleus-nucleus collisions is to determine the response of nuclear matter under conditions of high temperature and compression.[1-3] In order to relate observables to state variables, it is necessary to consider the rapid variation in space and time of the collision process. It has been suggested[4] that the time dependence of a reaction may be determined by studying the properties of particles emitted preferentially at various stages in the collision process. In our earlier work[5] the total pion multiplicity was proposed as an observable that is sensitive to the high density stage of central collisions where densities several times that of normal nuclear matter are expected to be reached. Since this stage occurs at the turning point between initial compression and later expansion when nuclear conditions and variables are relatively constant, it may survive long enough for equilibrium to be established. If so, the total pion multiplicity should be a sensitive measure[6,7] of the nuclear matter equation of state.

In this Letter the conditions for establishing chemical and thermal equilibrium in various stages of the collision are examined. A chemical model is used to study pion production and extract the nuclear compressional energy as a function of density. The results are in close agreement with our previous analysis which depended upon the validity of intranuclear cascade calculations, and lend credence to the extracted equation of state. The present method offers possibilities for extension to higher energies where cascade calculations face increasing difficulties.

In nucleus-nucleus collisions below $E_{lab} \sim 2$ GeV/u the system can be considered as a mixture of nucleons, deltas, and pions. The approach to chemical equilibrium (in which the abundances of the constituents are in equilibrium) is determined by the inelastic cross sections for $NN \leftrightarrow N\Delta \leftrightarrow NN\pi$. The approximate rate constant for approach to chemical equilibrium of the Δ's is given by[8]

$$\lambda = \langle \sigma v \rangle \rho_N^2 / 2\rho_\Delta^{eq} \qquad (1)$$

where ρ_N and ρ_Δ^{eq} are the nucleon and Δ densities in equilibrium. The rate constant λ is proportional to the thermal energy average $\langle \sigma v \rangle$ of the cross section for $NN \leftrightarrow N\Delta$ times the relative pair velocity v. To estimate λ for the high density stage it is assumed that $T = 100$ MeV, $\rho_\Delta^{eq} = 0.2\ \rho_N \simeq 0.6\ \rho_o$ with $\rho_N = 3\ \rho_o$, and $\langle \sigma v \rangle \approx 1.5 \times 10^{23} fm^3 s^{-1}$. The resulting equilibration time constant is $\tau \equiv \lambda^{-1} \approx 6 \times 10^{-24}$ s. Intranuclear cascade calculations have been used[5,10] to estimate the duration of the high density stage which is found to be 2τ and 3.5τ for central collisions of Ar + KCl and La + La, respectively, long enough for chemical equilibrium to be reached. To make a similar analysis for thermal equilibrium, i.e., equipartition of energy among the various kinetic degrees of freedom, the total cross sections must be used. The resulting time constant is even shorter, so that both thermal and chemical equilibrium exist at the end of the high density stage.

To discuss the expansion phase of the interaction it is again necessary to establish a time scale. If the equilibration time constants are short compared with this time scale, chemical and thermal equilibrium can be

considered to exist at each time during the expansion, the composition and temperature of the system adjusting continuously. In cascade calculations the central density falls a factor of two every 10^{-23}s during expansion. The equilibration time constants at a later stage in the expansion will be larger. For example at $\rho = 0.7\ \rho_0$, assuming an expansion somewhere between isoergic and isentropic,[11] the chemical equilibration time constant from Eq. 1 lies between the values $3 \times 10^{-23}\text{s} < \tau < 2 \times 10^{-22}\text{s}$, much longer than the characteristic time of the expansion 10^{-23}s. Thus chemical equilibrium cannot be maintained, contrary to the assumption made in the fireball[12] and hydrodynamical model[13] studies of pion production, and the pion yield will freeze out near the high density stage rather than late in the expansion. This is the effect noted in the cascade calculations which formed the basis of our previous analysis of the pion yield.[5] The thermal equilibration time is determined by the large πN cross section and is much shorter, so that thermal equilibrium is maintained until later in the expansion. The pion yield is established by the $\pi + \Delta$ abundance at chemical freeze-out, and is unaltered by the pion-number-conserving interchange $\pi N \leftrightarrow \Delta$. Furthermore, during expansion the rapidly diminishing number of Δ's (due to $\Delta \rightarrow \pi N$ decay) make it progessively less likely for "π absorption" to occur via $\Delta N \rightarrow NN$.

The measured pion multiplicity can now be used to determine the thermal energy content of the system at the end of the high density stage. To accomplish this, the idealization of complete equilibrium at the end of the high density stage and its freeze-out with the onset of expansion will be assumed. The total c.m. excitation energy per baryon in a system at temperature T and density ρ is given by[8,12,14]

$$E - m_N = \frac{3}{2} T + \langle n_\pi \rangle \langle E_\pi \rangle / B + \langle n_\Delta \rangle \langle m^* \rangle / B + W(\rho, T = 0) + E_{flow} \quad (2)$$

where the left-hand side is the kinetic energy per baryon in the c.m. frame. The first three terms on the right-hand side describe the thermal excitation energy contained in baryons, pions, and in resonance mass excess m^*. The fourth term is the ground state compressional energy and the last term is the flow kinetic energy, both at density ρ. At the time of maximum density when the final pion multiplicity is established, the compressional energy is a maximum and there is no kinetic flow energy[15] which will appear only as the system expands. Thus, at the time of maximum density Eq. 2 can be written in a simplified form

$$\epsilon = E_T(\rho, n_\pi, n_\Delta) + E_C(\rho, T = 0) \quad (3)$$

where ϵ is the incident c.m. kinetic energy, E_T the thermal excitation, and E_C the compressional energy each taken per participant. Here medium effects[16] on the thermal baryon spectrum are ignored, the medium being assumed to exert its influence through the compressional energy only.

In our previous approach[5] the observed pion multiplicity was used to calculate E_T by means of the cascade model. The difference between the bombarding energy ϵ and E_T then gave the compressional energy E_C. This quantity was plotted as a function of the density ρ, also extracted from cascade model calculations, to derive the nuclear equation of state. A similar relationship will now be derived without reference to the cascade model.

In the new approach a relativistic chemical model[12] which assumes an equilibrium mixture of N, Δ and π was used to predict the total pion multiplicity. This model requires the density and total c.m. energy as input. The density is also necessary to derive a nuclear equation of state. It was obtained using the relativistic Rankine-Hugoniot relation[6]

$$\rho/\rho_o = \gamma_{c.m.}[1 - \rho_o E_{lab}/2P]^{-1} \tag{4}$$

where ρ/ρ_o describes the shock compression in a medium at rest in the center of mass, and $\gamma_{c.m.}$ is the beam Lorentz factor with respect to the c.m. frame for an incident lab kinetic energy per nucleon E_{lab}. The quantity P is the pressure of the medium, related to the equation of state $W(\rho)$ and temperature T by the relation

$$P = \rho^2 \partial W(\rho)/\partial\rho + \rho T \quad . \tag{5}$$

Equations (4) and (5) were solved within the framework of the chemical model, by the following iterative technique:

1. Assume a trial density dependence $\rho(\epsilon)$.
2. Using this density, calculate the total $\pi + \Delta$ multiplicity at energy ϵ using the chemical model.
3. By comparing the experimental and calculated values of the $\pi + \Delta$ multiplicity as a function of ϵ, extract the compressional energy $E_C(\epsilon)$.
4. With the trial density and the extracted compressional energy, the function $W(\rho)$ is obtained.

5. Using this $W(\rho)$ and the temperature T obtained from the chemical model in step 2, obtain the pressure P from equation (5).
6. Using this value of the pressure, obtain a new value $\rho(\varepsilon)$ from equation (4). Repeat the procedure using $\rho(\varepsilon)$ in step 1 until no further change in ρ is found.

Use of the trial value $\rho/\rho_0 = 2\gamma^2_{c.m.}$ led to rapid convergence of the procedure. The results are shown in Fig. 1, together with the results previously obtained from cascade calculations and the experimental data. The experimental points are slightly different from those presented in Ref. 5, which were for impact parameters up to $b = 2.4$ fm, defined by a central trigger. The present data were obtained from measurements in the minimum bias trigger mode. At each bombarding energy the ratio of the pion multiplicity to the observed proton participant number was found to be a constant as shown in Fig. 2. The yields at $Z_1 + Z_2$, where the proton participant number equals the total nuclear charge, were taken to correspond to $b = 0$. They are plotted in Fig. 1 where[17] $\langle n_{\pi+\Delta} \rangle = 3\langle n_{\pi^-} \rangle$. The cascade and chemical results for the pion yields are seen to be in close agreement. The compressional energies indicated by the horizontal arrows are therefore also similar. Fig. 3 shows the compressional energies for both models, plotted against the density. The results are again rather similar. Also shown in Fig. 3 is an equation of state calculated by Boguta and Stöcker,[1] using a relativistic field theory approach for nuclear matter with an incompressibility constant $K = 240$ MeV and effective mass $m_{eff} = 0.75\ m_N$.

In conclusion, it is reasonable to use chemical and thermal equilibrium calculations for the high density stage of nucleus-nucleus collisions, when considering nucleons, pions and deltas. A distinction must be made between

chemical freeze-out which for $\pi + \Delta$ occurs at the end of the high density phase and thermal freeze-out which occurs later. It will be of interest to investigate further the expansion stage to determine how much the temperature changes and whether there really is a well-defined thermal freeze-out. In the present analysis of the total pion yield, the density and compressional energy were obtained using the chemical model and the Rankine-Hugoniot relationship. The results are similar to those previously obtained using the cascade model and lend confidence to the equation of state derived from the experiments.

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Figure Captions

1. The $\pi + \Delta$ yield per participant nucleon is plotted as a function of c.m. energy (lower axis) and laboratory energy (upper axis) for Ar + KCl. The solid circles are the experimental results for zero impact parameter. The dashed and dotted curves correspond to the Cugnon cascade and the chemical model predictions, respectively, and the triangles are predictions of the Yariv-Fraenkel[18] cascade. The horizontal arrows represent the values of the compressional energy per nucleon E_c determined at each experimental point.
2. The mean π^--multiplicity observed in Ar + KCl reactions at 1.0, 1.2, 1.4, 1.6 and 1.8 GeV/u plotted as a function of the observed number of proton participants Q. Only the interpolating lines are shown except for 1.0 and 1.8 GeV/u.
3. Values of the compressional energy, minus 10 MeV binding energy, plotted as a function of the density in units of normal nuclear density ρ_o derived from the data using the cascade and chemical model, respectively. Only statistical errors are given. The curve is a relativistic field theoretical prediction (Ref. 1) with K = 240 MeV and $m_{eff} = 0.75\ m_N$.

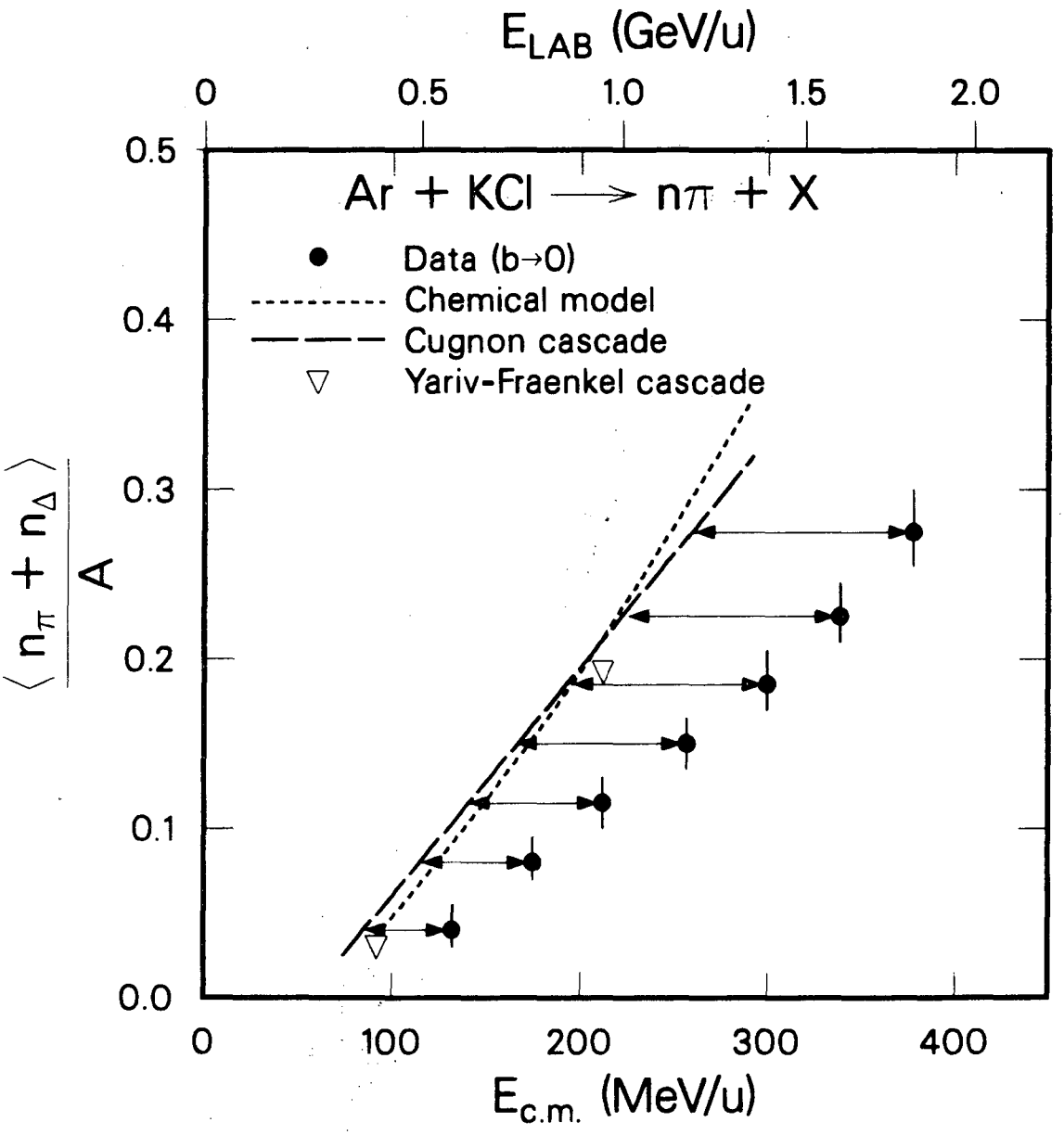

Fig. 1

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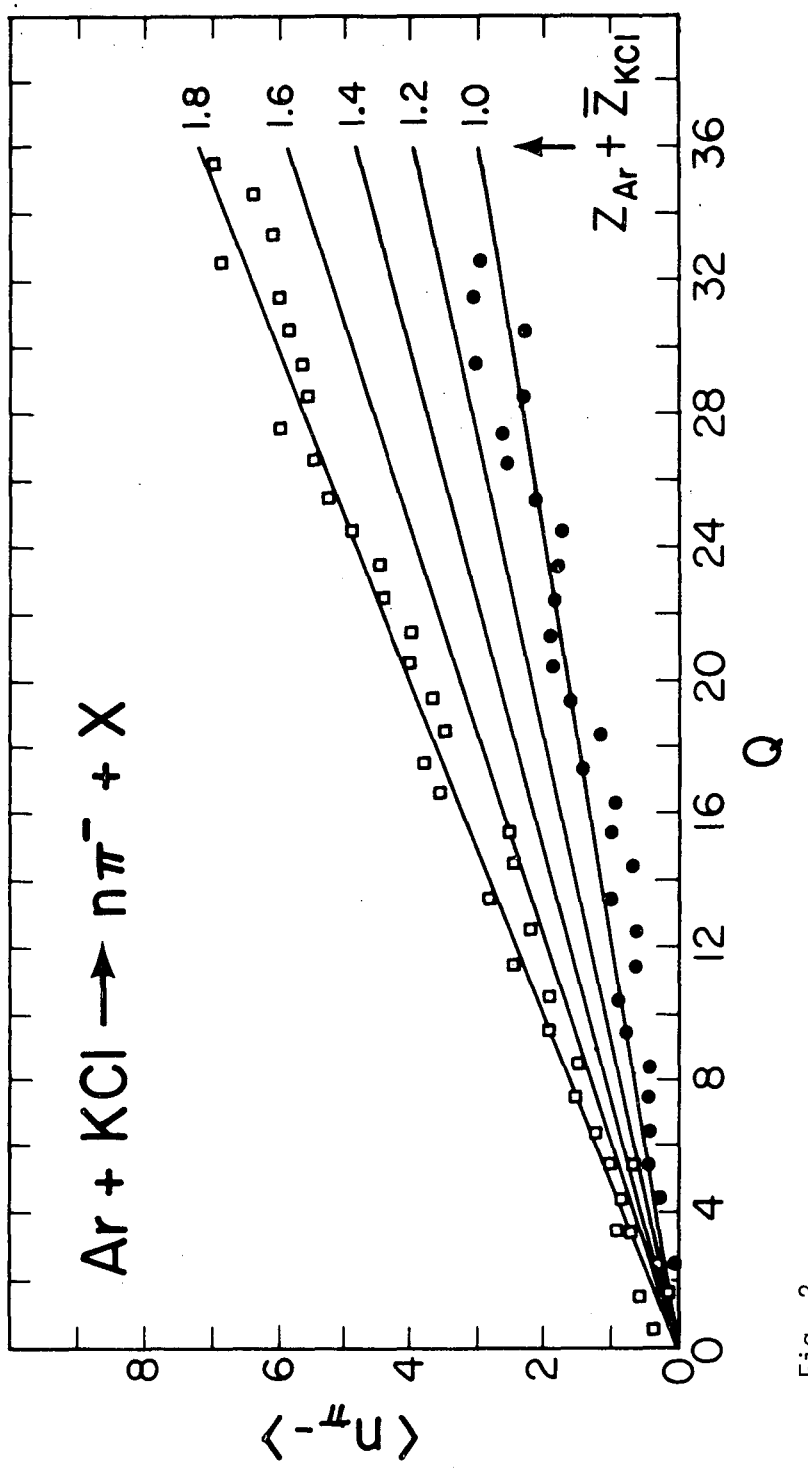

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Fig. 2

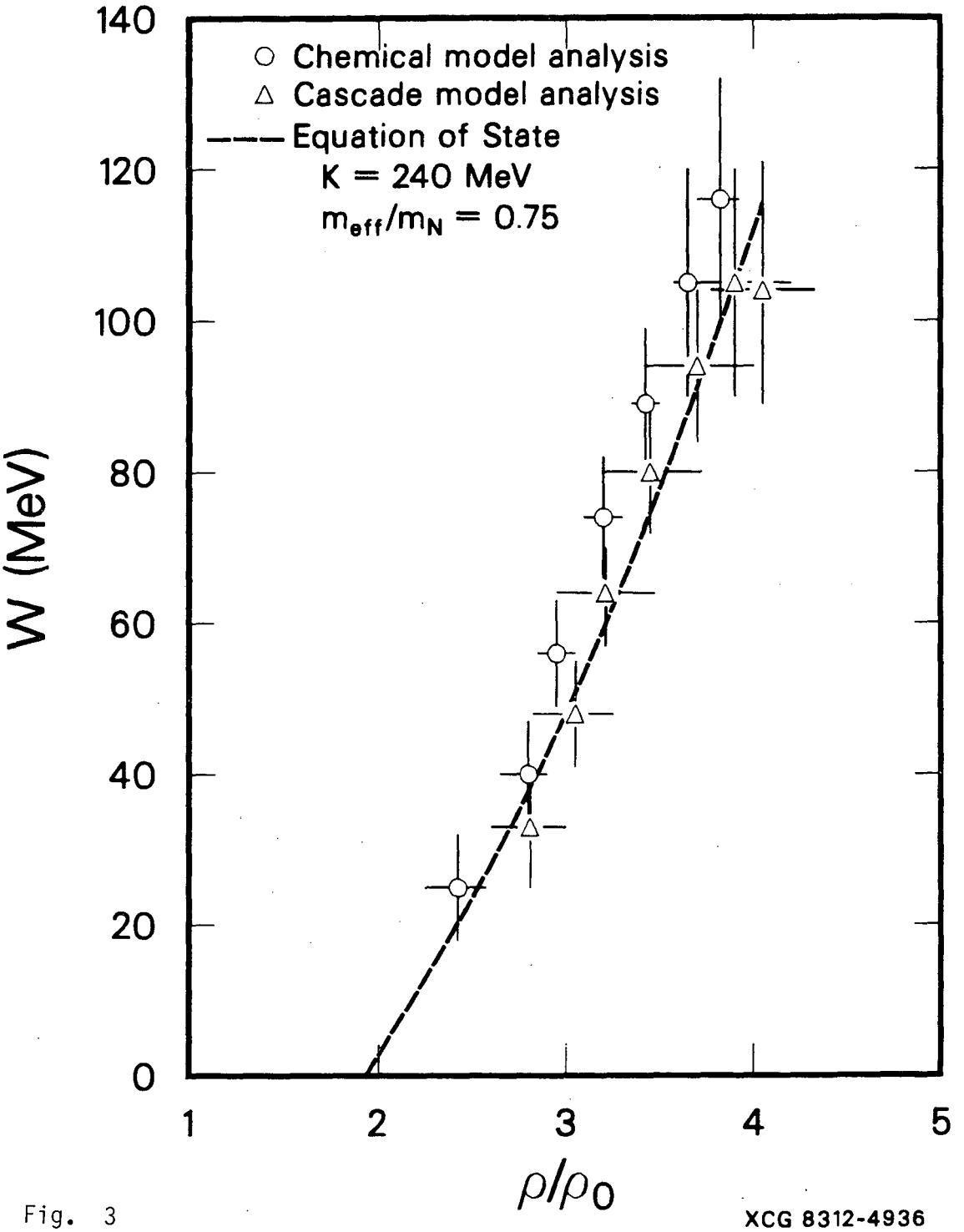
140
120
100
80
60
40
20
0
W (MeV)
1
2
3
4
5
ρ/ρ0
○ Chemical model analysis
△ Cascade model analysis
—— Equation of State
K = 240 MeV
meff/mN = 0.75
XCG 8312-4936

Fig. 3

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