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A Novel Framework for Integration of Abstracted Inspection Data and Structural Health Monitoring for Damage Prognosis of Miter Gates

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Abstract

Operational condition assessments, using a discrete rating system, are frequently used by field engineers to assess inland navigation assets and components. Challenges such as the occasional inability to perform inspections (such as the case with locks watered in an operational state) and protocol requirements requiring ratings even when they aren't inspected lead to highly abstracted inspection data, which are also very prone to human error and misinterpretations due to inspections protocol. On the other hand, some navigational locks are equipped with structural health monitoring (SHM) systems to continuously perform assessments from data obtained *in situ*. This paper aims to develop a novel **hybrid** damage prognosis framework for miter gate component of navigational locks, by mitigating effects of human errors on the condition assessment and integrating the highly abstracted inspection data with the SHM. It overcomes two main challenges, namely (1) there is no physical or empirical model available to model the loss-of-contact degradation in the gate, and (2) the mismatches between the inspection data and the SHM system due to data abstraction. A practical case of monitoring loss-of-contact quoin block demonstrates the efficacy of the proposed framework.

Keywords: Miter Gates; Transition Matrix; Human Error; Gap Growth Model; Damage Estimation; Uncertainty

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Nomenclature

$a_t, a(t)$	=	gap length at time t
a_e	=	gap failure threshold
$a(i, j+k), a_{i, j+k}$	=	i -th realization of the gap length at the $(j+k)$ -th time step
$\mathbf{a}_s(\boldsymbol{\theta}, \mathbf{e})$	=	Samples/realizations obtained of the gap length degradation model parametrized by $\boldsymbol{\theta}$ and \mathbf{e}
\mathbf{e}	=	vector of estimated parameters, e_i , of mapping function, $h_s(a(t))$
$f_e(\mathbf{e} \boldsymbol{\theta})$	=	joint PDF of e_i given $\boldsymbol{\theta}$
$g(t, \boldsymbol{\theta})$	=	degradation model of the miter gate damage gap at time t given $\boldsymbol{\theta}$
$\hat{g}(a_{k+1}, \mathbf{x}_{k+1})$	=	FE model or surrogate model as a function of a_{k+1} and \mathbf{x}_{k+1}
$g_{opt}(\boldsymbol{\theta}; \boldsymbol{\beta}, \mathbf{R})$	=	cost/error function to tune degradation model given $\boldsymbol{\theta}, \boldsymbol{\beta}$, and \mathbf{R}
$h_{OCA}(a_t, \boldsymbol{\beta})$	=	protocol mapping function given $\boldsymbol{\beta}$ to map gap length at time t to OCA ratings
$h_s(a(t))$	=	estimated mapping function to map gap length at time t to OCA ratings
$I_{j,t+1}$	=	inspected state I_j (e.g. A, B, C, D, F or CF) at time $t+1$
I_{t+1}^u	=	underlying true OCA rating at time $t+1$
I_{t+1}^{obs}	=	reported OCA rating from field engineers at time $t+1$
$I_{j,t+1}^s$	=	inspected state I_j (e.g. A, B, C, D, F or CF) at time $t+1$ obtained from degradation model
n_{MCS}	=	number of samples of stochastic degradation model at each time step
N_d	=	distinct degradation stages
N_{PF}	=	number of samples used in the state estimation
N_s	=	number of strain sensors providing data

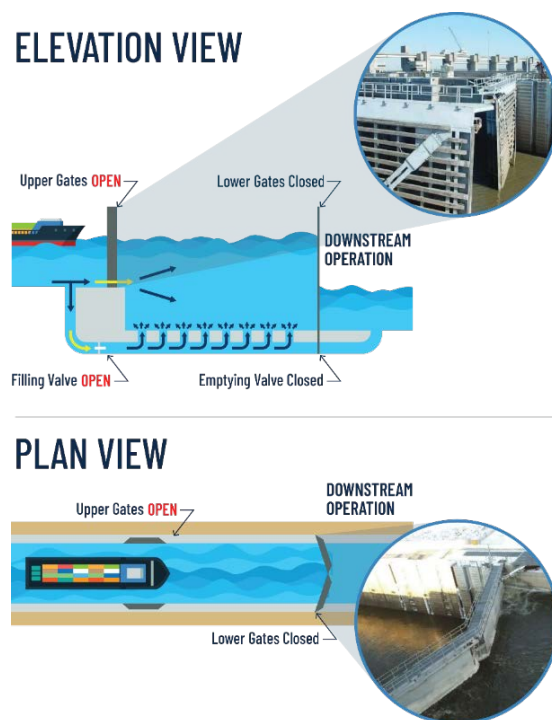
N_t	=	total number of simulation time steps for stochastic degradation model
\mathbf{P}	=	rating transition matrix
$\mathbf{P}_{\text{human}}$	=	human observation error matrix
\mathbf{P}_{OCA}	=	true OCA transition matrix
$\mathbf{P}_{\text{Report}}$	=	reported OCA transition matrix
$\hat{\mathbf{P}}(\boldsymbol{\theta})$	=	simulated transition probabilities of the OCA ratings from the degradation model simulation for given $\boldsymbol{\theta}$
P_{ik}^h	=	probability that the reported OCA rating is k given that the true OCA rating is i
P_{ij}^{OCA}	=	probability of transitioning from true OCA rating i at time t to true OCA rating j at $t+1$
P_{kq}^R	=	probability of transitioning from reported OCA rating k at time t to reported OCA rating q at $t+1$
$\Pr\{\cdot\}$	=	probability operator
q_v	=	time-invariant parameter that controls the rate of cooling
Q	=	degradation model parameter to be estimated
Q_i	=	degradation model parameter at degradation stage i to be estimated
R	=	OCA rating obtained from continuous monitoring
\mathbf{s}_i	=	set of strain measurement data at time step t_i
$\mathbf{s}_{1:k}$	=	set of strain measurement data collected up to t_k
s_{iN_S}	=	strain measurement data at time step t_i at the N_S location
t_l, t_u	=	lower and upper bounds of the time duration of interest (e.g. 1 year)
t_m	=	time when damage threshold is reached
$T_{q_v}(t)$	=	artificial temperature (a time-varying global parameter)
T_{RUL}	=	remaining useful life
$U(t)$	=	stationary standard Gaussian process

w	=	degradation model parameter to be estimated
w_i	=	degradation model parameter at degradation stage i to be estimated
\mathbf{x}_{k+1}	=	other FE model inputs such water levels and temperature in miter gates
$X(t)$	=	stationary lognormal stochastic process
β	=	vector of parameters of protocol mapping function
$\Delta\theta(t)$	=	a trial jump distance of the variable $\theta(t)$
ε	=	measurement noise
ζ_x	=	parameter that controls the correlation of $X(t)$ over time
θ	=	vector of model parameters of degradation
$\theta_j, \theta(t)$	=	vector of model parameters of degradation stage j (or at time t)
$\Lambda(E)$	=	indicator function such $\Lambda(E) = 1$ if event E is true and $\Lambda(E) = 0$ if event E is false
μ_i	=	mean of Gaussian random variable, e_i
σ_e	=	standard deviation of e_i uncorrelated measurement noise, ε
σ_i	=	standard deviation variable of degradation stage i
σ_{obs}	=	standard deviation of
σ_x	=	standard deviation of $X(t)$
$\phi(\cdot)$	=	PDF of the standard normal distribution

31 1 Introduction

32 Miter gates are common hydraulic steel structures that facilitate passage of boats and
33 watercraft through inland navigation systems as shown in Figure 1. In the United States, the
34 U.S. Army Corps of Engineers (USACE) maintains and operates 236 locks at 191 sites [1]. A
35 closure of a lock due to maintenance or repairs can cost up to \$3 million per day to the US
36 economy [2]. This is underscored by the fact that more than half of these structural assets,
37 including miter gates, have surpassed their 50-year economic design life [3]. To help prioritize

38 maintenance and repairs, operational condition assessment (OCA) ratings are performed by
39 USACE inspectors via visual inspections [4]. However, the OCA ratings are highly abstracted
40 and are assigned at a varying frequency, which varies from every year to occurring to a
41 maximum of every 5 years [5]. Recently, several miter gates were equipped with SHM systems
42 that collect strain measurement data in real time [6]. These continuous monitoring systems aim
43 to provide insight regarding deteriorating gates. However, a framework that integrates visual
44 inspections and SHM for damage diagnosis and prognosis has not been developed yet.



45
46 **Figure 1:** Navigation along miter gates

47 This paper first gives an overview of the type of damage present in some components of
48 miter gates and how these components are condition-rated based on the field OCA ratings.
49 Section 3 briefly reviews current approaches for failure prognostics of miter gates through the
50 integration of OCA transition matrix with continuous structural health monitoring and proposes
51 a new approach for damage diagnosis and prognosis via a new degradation model derived by
52 mapping the abstracted inspection data into a multistage discrete-time degradation model. The
53 damage diagnosis and prognosis consider the human errors of field engineers in the inspection

54 data. The integration of the derived degradation model with physics-based finite element (FE)
55 model updating will also be studied to perform online damage diagnostics and estimation of
56 the miter gate's remaining useful life. Finally, Section 4 summarizes the important findings of
57 this work and suggest further steps to be taken.

58 Even though this paper considers a specific application in miter gate damage assessment
59 and prognosis, the developed framework is quite generic; it is easily adaptable to other
60 structural monitoring applications that involve abstracted condition rating data (e.g., like the
61 OCA) and online health monitoring system, such as other miter gate failure modes (e.g.,
62 corrosion or pre-tension loss) or other structures including bridges [7–9], pavements [10,11],
63 offshore structures [12], and others [13].

64 The contributions of this paper are summarized as: (1) it addresses bias in the OCA ratings
65 in the state-transition matrix caused by human observation errors; (2) it maps the abstracted
66 rating state-transition matrix to a failure evolution model; (3) it demonstrates a failure
67 diagnostics and prognostics procedure using structural health monitoring systems based on the
68 failure evolution model; and (4) it demonstrates the developed framework on the very practical
69 case of monitoring loss-of-contact quoin block damage (resulting in “gaps” between the gate
70 and support wall).

71 In summary, this paper proposes a novel hybrid approach for condition-based maintenance
72 where abstracted OCA ratings subjected to human reporting errors are used to derive a
73 degradation model. Simultaneously, a SHM system is used for damage diagnostics and
74 prognostics based on the derived degradation model. The proposed approach overcomes the
75 challenges that there is no viable degradation model available and there is substantial
76 heterogeneity (i.e., physics-based simulation data, OCA rating data, errors in the OCA rating
77 data, and strain measurement data) in the sources used to inform damage prognostics of miter
78 gate components. Note that, the role of prognosis includes predictions of the future state that

79 inform reliability estimates of the system [14–16]. Predictive capabilities allow informed life
80 cycle management, which target to optimize a certain system performance criterion [17] (e.g.
81 cost, availability, reliability, etc.). Moreover, prognosis capabilities enable engineers to turn
82 available data into information that enhance the current knowledge of the system and also
83 provides a policy to maintain the system optimally.

84

85 **2 Problem Statement**

86 As mentioned above, there are significant economic implications caused by navigation lock
87 closure, and how to prioritize repairs or other maintenance actions for miter gate components
88 is paramount to minimizing the consequence costs. To understand the prioritization process,
89 there is a need to estimate the extent of damage (i.e., damage diagnosis), and to predict the
90 evolution of damage into the future (i.e., damage prognosis). Any prognosis action
91 fundamentally requires a degradation model of some kind. Ideally, this model would be built
92 from existing time series data or by data generated using a physics-based knowledge of the
93 degradation/failure process. However, in many real-world applications such as with this miter
94 gate case, the lack of existing time series data correlated to deteriorating components and the
95 lack of understanding of the physics behind the damage mechanism evolution impose
96 additional challenges to performing damage prognosis.

97 As mentioned, OCA ratings are a primary tool used to inform the structural condition state.
98 An OCA rating is a categorical rating given by an inspector, who bases the evaluation on a
99 rating system developed by the USACE Asset Management team, which involves engineering
100 knowledge and information of pre-existing inspections. This rating system classifies structural
101 and non-structural components as A (Excellent), B (Good), C (Fair), D (Poor), F (Failing) and
102 CF (Completely Failed). More detailed definitions can be found in [3]. These ratings are given
103 at the component level of the structural asset (e.g., the miter gate quoin blocks in this paper).

104 These discrete ratings are highly abstracted, assigned at varying time intervals, and are very
105 prone to human error and to misinterpretations due to inspections protocol [16]. However, these
106 ratings can provide information regarding transitions between different damage rating
107 categories, which may be used to build a degradation model parametrized according to the
108 deterioration of the OCA inspection ratings. In this application, the deterioration of a quoin
109 block component in a miter gate (“damage”) is manifested as a “gap” that results in loss of
110 contact beyond the “regular gap” tolerance ($\sim 1/32$ in.) between the quoin block attached to the
111 gate and the quoin block attached to the wall that supports the gate laterally. The “regular gap”
112 tolerance allows a miter gate to operate and closes when the gate is subjected to hydrostatic
113 loading. The formation of an undesirable “damage gap” beyond the tolerance controls the
114 lateral boundary condition of a miter gate, and significant changes can lead to higher
115 strain/stress in critical components (e.g., the pintle) of the gate. The “gap” or “damage gap” in
116 the subsequent sections of this paper is thus the target damage mechanism considered in this
117 work. More details regarding the different miter gates components mentioned (e.g. quoin
118 blocks, pintle, etc.) can be found here [2].

119 From historical inspections, a database of the OCA ratings for quoin blocks and other
120 components is available for the past several years, which provides information of the gap
121 transition over the year at the abstracted OCA rating level. Even though the OCA ratings are
122 very prone to human errors, they are the only available data source that contains some form of
123 degradation information of the gate at present. The problem that needs to be solved is how to
124 utilize the abstracted information to effectively perform failure prognostics. In this paper, these
125 reported ratings would be used to build a transition matrix. This reported transition matrix
126 would be combined with a human error matrix to improve the prognosis capabilities of the
127 damage mechanism. This human error matrix will quantify the ability of the inspector to
128 perform correct assessments and false positives/negatives assessments. Diagnosis and

129 prognosis using data-driven models built from solely inspection data (i.e. OCA ratings),
130 however, may lead to large uncertainty in the failure prognosis as shown in previous studies
131 [16,18] and in the case study section.

132 Beyond these condition ratings, however, structural health monitoring (SHM) systems have
133 been developed for the miter gates to measure their distributed point strain response during
134 operation, providing continuous data streams which may be mined for damage-related
135 information. The SHM measurement systems are coupled with validated high-fidelity physics-
136 based finite element (FE) models [16,19–22], allowing for inference/estimation of the damage
137 gap using the strain measurements. This approach provides more confident estimates of the
138 damage gap state over time. While it is true that the SHM system increases gap inference
139 capabilities, it cannot be used directly to predict the gap degradation over time, since the
140 physics of the gap degradation is complex and not fully understood; SHM alone is not enough
141 to inform decisions regarding prioritizing preventive maintenance.

142 As described above, however, the historical OCA ratings nevertheless do contain
143 information that may be used to understand the gap degradation over time, even though it is
144 highly abstracted and may be contaminated by human observation errors or bias. Synthesizing,
145 rather than separating, OCA rating transition information and SHM system information has the
146 potential to improve an integrated state awareness (damage state) and state prediction (future
147 damage state).

148 The two lines of enquiry that are addressed in this paper, therefore, may be summarized as
149 follows:

- 150 (1) How should the highly abstracted OCA rating transition information be connected with
151 a high-fidelity FE model for useful integrated damage diagnosis and prognosis?
- 152 (2) How should the effects of errors in the OCA rating transition information be mitigated
153 for the damage diagnosis and prognosis?

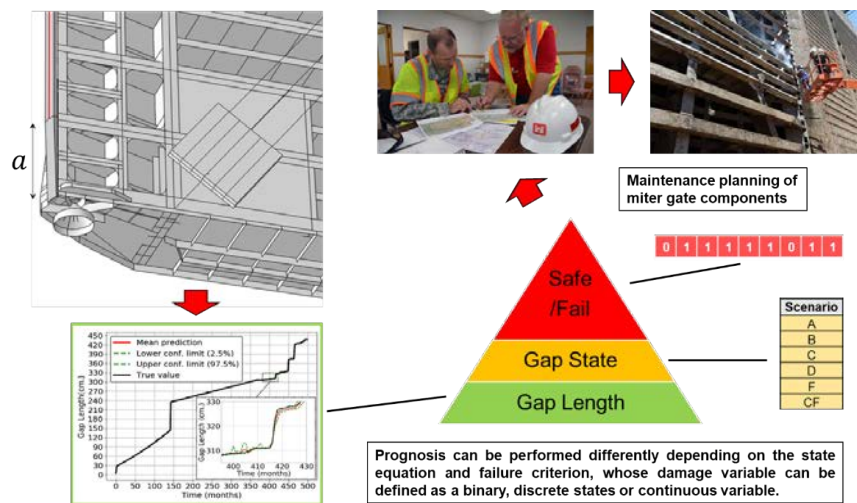
154

155 3 Proposed Method

156 In this section, a brief review of current methods for failure prognosis of miter gates is
157 summarized. After that, the proposed method is explained in detail.

158 3.1 Overview

159 Figure 2 shows the state (damage) variable hierarchy for bearing gaps in a quoin block.
160 This figure shows a hierarchy pyramid that contain three different ways that the gap can be
161 described. The most basic one would use a binary system that would define the state as
162 damaged or undamaged, as time evolves. The next one would be based on discrete state-
163 transition system such as the OCA ratings. For the two ways mentioned, the determination of
164 these deterioration or damage labels would be based on an asset management protocol.



165

166 **Figure 2:** State (damage) variable hierarchy for bearing gap in quoin block

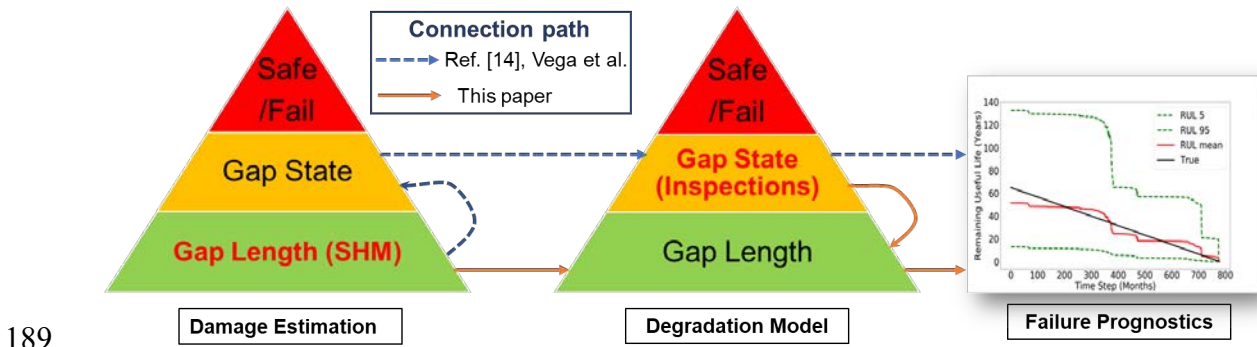
167 Based on a large historical OCA database, the number of times that a component
168 transitioned from one rating category to another (as determined by engineering expert
169 elicitation) over a given inspection time step can be determined to generate the rating transition
170 matrix [23]. The transition matrix \mathbf{P} (see Eq. (1)) is defined as a square matrix with nonnegative
171 values that represents how some process “transitions” from one state to the next. In this

172 application, an inspected state at time t , $I_{i,t}$, (with $i = 1 \dots 6$, corresponding to the 6 letter ratings
 173 specified above), will transition to inspected state at time $t + 1$, $I_{j,t+1}$, $j = 1 \dots 6$, according to

$$174 \quad \mathbf{P} = P(I_{j,t+1} | I_{i,t}) = \begin{bmatrix} P(I_{1,t+1} = A | I_{1,t} = A) & \cdots & P(I_{6,t+1} = CF | I_{1,t} = A) \\ \vdots & \ddots & \vdots \\ P(I_{1,t+1} = A | I_{6,t} = CF) & \cdots & P(I_{6,t+1} = CF | I_{6,t} = CF) \end{bmatrix}. \quad (1)$$

175 In Eq. (1), only the upper triangular components were considered to simulate component
 176 deterioration; the lower triangular components would represent improvements or repairs
 177 (transitions from a worse condition to a better condition), and for the purposes of this analysis,
 178 they were ignored. Further details on this transition matrix can be found in [16,24,25].

179 Furthermore, the bearing gaps may also be modelled at the continuous level (i.e. gap-length
 180 level at the bottom of the pyramid) based on continuous structural health monitoring (SHM)
 181 systems. In order to address the above-mentioned *first line of enquiry*, which is to connect the
 182 highly abstracted OCA rating transition information with a high-fidelity FE model for useful
 183 integrated damage diagnosis and prognosis, Vega et al. [16] developed a hybrid prognostic
 184 approach by converting the continuous level into gap-state level as illustrated in Fig. 3. Even
 185 though the approach developed in [16] allows for the integration of SHM with Markov analysis
 186 for integrated damage diagnosis and prognosis, the component degradation modeling at the
 187 discrete state-transition level could lead to wide uncertainty in the prognostics even when using
 188 recursive model updating.



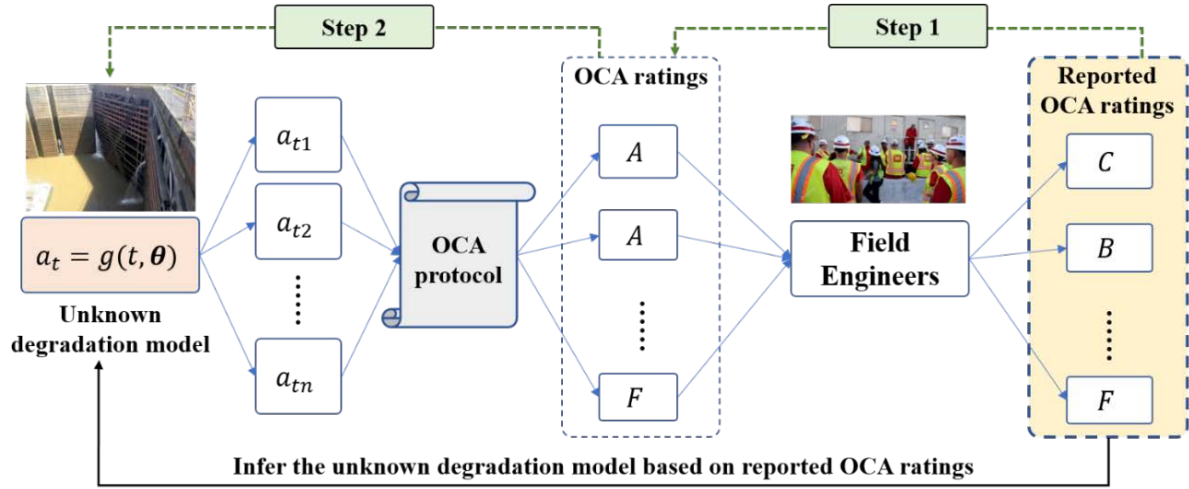
190 **Figure 3:** Comparison of the connection paths between damage estimation and degradation
191 model for the methods presented in Vega et al. [16] and this paper

192 In this paper, as illustrated in Fig. 3, instead of converting the damage estimation at gap-
193 length level into abstracted gap-state level for prognostics, the degradation model is built at the
194 continuous gap-length level by tuning the degradation model parameters to agree with the
195 Markov transition matrix built from the OCA ratings (gap-state level). After that, failure
196 prognostics at the gap-length level is performed. The goal is to meaningfully increase the
197 confidence in the miter gate failure prognostics beyond on what is was proposed in [16] to
198 achieve an effective and useful decision-making capability. In addition to the tuning of
199 degradation model parameters using data at gap-state level, a new approach will also be
200 developed to address the errors in the OCA transition matrix due to human observation
201 variability, thereby addressing the *second line of enquiry* mentioned above).

202 Let $a_t = g(t, \theta)$ be the underlying degradation model of the miter gate damage gap, where
203 a_t is the gap length at time t , and θ is a vector of model parameters. Fig. 4 shows the
204 relationship among the degradation model, OCA ratings, and the reported OCA ratings by the
205 field engineers. As shown in Fig. 4, the OCA protocol maps the gap length, a_t , (i.e., the output
206 of the unknown degradation model) into OCA ratings as if the protocol were strictly and
207 accurately followed by the field engineers. Due to human observation error and variability,
208 however, the OCA ratings reported by the field engineers as indicated in Fig. 4 may not be the
209 same as the “true” rating that better represents the condition; this is proven true for inspectors
210 in many application domains [26].

211 One of the objectives of the proposed method is to infer the unknown degradation model,
212 $a_t = g(t, \theta)$, using *the reported OCA ratings*, which include the human variability or errors in
213 the rating reporting process. The inferred degradation model will then be used for *integrated*

214 damage diagnostics and prognostics of the miter gate. As shown in Fig. 4, the inference of the
 215 unknown degradation model in the proposed framework is accomplished through two steps:



216
 217 **Figure 4:** Relationship among the gap degradation, OCA ratings, and the reported OCA
 218 ratings

- 219 • **Step 1:** Mapping of the reported OCA ratings to the underlying condition for a given
 220 OCA protocol, by considering the human observation errors of field engineers in
 221 reporting.
- 222 • **Step 2:** Estimation of the degradation model parameters (θ) based on the obtained true
 223 OCA ratings (i.e. true OCA transition matrix).

224 In the next section, these two steps will be explained in detail.

225 3.2 Mapping of the reported OCA rating transition matrix to the true transition matrix

226 In order to map the reported OCA rating transition matrix to the underlying “true” OCA
 227 transition matrix, the underlying true OCA rating is defined at time t as I_t^{tr} and that at $t+1$ as
 228 I_{t+1}^{tr} , the reported OCA rating from field engineers at time t as I_t^{obs} and that at time $t+1$ as I_{t+1}^{obs} .
 229 Based on these definitions, the true OCA transition matrix \mathbf{P}_{OCA} (i.e. OCA “ideal” protocol is
 230 strictly followed) is denoted as

231
$$\mathbf{P}_{\text{OCA}} = \begin{bmatrix} P_{11}^{OCA} & P_{12}^{OCA} & \dots & P_{16}^{OCA} \\ 0 & P_{22}^{OCA} & \dots & P_{26}^{OCA} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & P_{66}^{OCA} \end{bmatrix}, \quad (2)$$

232 where $P_{ij}^{OCA} = \Pr\{I_{t+1}^{tr} = j \mid I_t^{tr} = i\} \triangleq P(I_{j,t+1}^{tr} \mid I_{i,t}^{tr})$, $\forall i = 1, 2, \dots, 6; j = i, \dots, 6$ represents the
 233 probability of transitioning from **true** OCA rating i at time t to **true** OCA rating j at $t+1$.

234 Similarly, the reported transition matrix, built from the OCA ratings reported by field
 235 engineers, is denoted as

236
$$\mathbf{P}_{\text{Report}} = \begin{bmatrix} P_{11}^R & P_{12}^R & \dots & P_{16}^R \\ 0 & P_{22}^R & \dots & P_{26}^R \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & P_{66}^R \end{bmatrix}, \quad (3)$$

237 where $P_{kq}^R = \Pr\{I_{t+1}^{obs} = q \mid I_t^{obs} = k\}$, $\forall k = 1, 2, \dots, 6; q = k, \dots, 6$ is the probability of transitioning
 238 from **reported** OCA rating k at time t to **reported** OCA rating q at $t+1$, based on the reported
 239 OCA ratings. In addition, from the reported OCA ratings the state probabilities
 240 $\Pr\{I_t^{obs} = k\}$, $k = 1, 2, \dots, 6$ and $\Pr\{I_{t+1}^{obs} = q\}$, $q = 1, 2, \dots, 6$ may also be obtained.

241 The goal of Step 1 of the proposed method (see Fig. 4) is to map $\mathbf{P}_{\text{Report}}$ to \mathbf{P}_{OCA} . To achieve
 242 this goal, the human observation error matrix is defined as

243
$$\mathbf{P}_{\text{human}} = \begin{bmatrix} P_{11}^h & P_{12}^h & \dots & P_{16}^h \\ 0 & P_{22}^h & \dots & P_{26}^h \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & P_{66}^h \end{bmatrix}, \quad (4)$$

244 in which $P_{ik}^h = \Pr\{I_t^{obs} = k \mid I_t^{tr} = i\}$ is the probability that the reported OCA rating is k given
 245 that the true OCA rating is i .

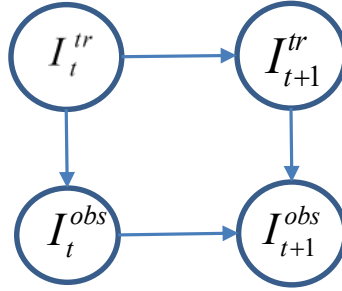
246 Based on the above definitions of \mathbf{P}_{OCA} , $\mathbf{P}_{\text{Report}}$, and $\mathbf{P}_{\text{human}}$, the reported and true OCA
 247 ratings are connected using a Bayesian network as shown in Fig. 5.

248 From the above Bayesian network, the following conditional probability tables (CPTs) are
 249 obtained:

$$\begin{aligned}
 250 \quad & \Pr\{I_t^{obs} = k \mid I_t^{tr} = i\} = P_{ik}^h, \forall i = 1, 2, \dots, 6; k = 1, 2, \dots, 6; \\
 & \Pr\{I_{t+1}^{obs} = q \mid I_{t+1}^{tr} = j\} = P_{jq}^h, \forall j = 1, 2, \dots, 6; q = 1, 2, \dots, 6;
 \end{aligned} \tag{5}$$

251 and

$$\begin{aligned}
 252 \quad & \Pr\{I_{t+1}^{obs} = q \mid (I_{t+1}^{tr} = j, I_t^{obs} = k)\} \\
 & = \frac{\Pr\{I_{t+1}^{obs} = q, I_{t+1}^{tr} = j, I_t^{obs} = k\}}{\Pr\{I_{t+1}^{tr} = j, I_t^{obs} = k\}}, \\
 & = \frac{\Pr\{I_t^{obs} = k \mid I_{t+1}^{obs} = q, I_{t+1}^{tr} = j\} \Pr\{I_{t+1}^{obs} = q \mid I_{t+1}^{tr} = j\} \Pr\{I_{t+1}^{tr} = j\}}{\Pr\{I_{t+1}^{tr} = j, I_t^{obs} = k\}}.
 \end{aligned} \tag{6}$$



253
 254 **Figure 5:** A Bayesian network connecting the observed and the true OCA ratings

255 Since the lower triangular components of $\mathbf{P}_{\text{Report}}$ are all zero, the following marginal
 256 probability is written

$$257 \quad \Pr\{I_{t+1}^{tr} = j, I_t^{obs} = k\} = \sum_{w=k}^6 \Pr\{I_{t+1}^{obs} = w, I_{t+1}^{tr} = j, I_t^{obs} = k\}. \tag{7}$$

258 With the above CPTs, the task is to obtain the true OCA transition matrix by solving
 259 $\Pr\{I_{t+1}^{tr} = j \mid I_t^{tr} = i\}, \forall i = 1, 2, \dots, 6; j = 1, \dots, 6$ in the Bayesian network shown in Fig. 5. Using
 260 $\Pr\{I_{t+1}^{obs} = q\}, q = 1, 2, \dots, 6$, the following marginal probability is written

$$\begin{aligned}
261 \quad \Pr\{I_{t+1}^{obs} = q\} &= \sum_{j=1}^6 \Pr\{I_{t+1}^{obs} = q, I_{t+1}^{tr} = j\}, \forall q = 1, 2, \dots, 6; \\
&= \sum_{j=1}^6 \Pr\{I_{t+1}^{obs} = q | I_{t+1}^{tr} = j\} \Pr\{I_{t+1}^{tr} = j\}, \forall q = 1, 2, \dots, 6,
\end{aligned} \tag{8}$$

262 which may be elucidated more clearly in matrix form as

$$263 \quad \begin{bmatrix} \Pr\{I_{t+1}^{obs} = 1\} \\ \Pr\{I_{t+1}^{obs} = 2\} \\ \vdots \\ \Pr\{I_{t+1}^{obs} = 6\} \end{bmatrix} = \begin{bmatrix} P_{11}^h & P_{12}^h & \dots & P_{16}^h \\ P_{21}^h & P_{22}^h & \dots & P_{26}^h \\ \vdots & \vdots & \ddots & \vdots \\ P_{61}^h & P_{62}^h & \dots & P_{66}^h \end{bmatrix} \begin{bmatrix} \Pr\{I_{t+1}^{tr} = 1\} \\ \Pr\{I_{t+1}^{tr} = 2\} \\ \vdots \\ \Pr\{I_{t+1}^{tr} = 6\} \end{bmatrix}. \tag{9}$$

264 Based on Eq. (9), $\Pr\{I_{t+1}^{tr} = j\}, \forall j = 1, 2, \dots, 6$ may be solved using $\mathbf{P}_{\text{human}}$ and

265 $\Pr\{I_{t+1}^{obs} = q\}, q = 1, 2, \dots, 6$. In this paper, a constrained least-squares method is used to solve

266 Eq. (9) to ensure that the obtained probability estimates are in the range of $[0, 1]$. In order to

267 estimate $\Pr\{I_{t+1}^{tr} = j | I_t^{tr} = i\}, \forall i = 1, 2, \dots, 6; j = i, \dots, 6$, a derivation of the term

268 $\Pr\{I_t^{obs} = k, I_{t+1}^{obs} = q\} = P_{kq}^R \Pr\{I_t^{obs} = k\}$ is performed (see **Appendix A** for derivations) as

269 follows:

$$\begin{aligned}
&P_{kq}^R \Pr\{I_t^{obs} = k\} \\
270 \quad &= \sum_{i=1}^6 \sum_{j=i}^6 \left(\frac{\Pr\{I_t^{obs} = k | I_{t+1}^{obs} = q, I_{t+1}^{tr} = j\} P_{jq}^h \Pr\{I_{t+1}^{tr} = j\}}{\sum_{w=k}^6 \Pr\{I_t^{obs} = k | I_{t+1}^{obs} = w, I_{t+1}^{tr} = j\} P_{jw}^h \Pr\{I_{t+1}^{tr} = j\}} P_{ik}^h \right) \Pr\{I_{t+1}^{tr} = j, I_t^{tr} = i\}. \tag{10}
\end{aligned}$$

271 In order to make $\Pr\{I_{t+1}^{tr} = j | I_t^{tr} = i\}, \forall i = 1, 2, \dots, 6; j = i, \dots, 6$ solvable given the current

272 available information ($\mathbf{P}_{\text{Report}}$ and $\mathbf{P}_{\text{human}}$), a conditional independence is assumed, given by

273 $\Pr\{I_t^{obs} = k | I_{t+1}^{obs} = q, I_{t+1}^{tr} = j\} = \Pr\{I_t^{obs} = k | I_{t+1}^{obs} = q\}$. This is a reasonable assumption for the

274 Bayesian network structure given in Fig. 5, since the resulting joint probability mass function

275 $\Pr\{I_{t+1}^{obs} = q, I_{t+1}^{tr} = j, I_t^{obs} = k\}$ satisfies the constraints of all the current given information in

276 $\mathbf{P}_{\text{Report}}$ and $\mathbf{P}_{\text{human}}$. Based on this assumption, the conditional probability and Bayes rule are

277 exploited

$$\begin{aligned}
 & \Pr\{I_{t+1}^{obs} = q, I_{t+1}^{tr} = j, I_t^{obs} = k\} \\
 278 & = \Pr\{I_t^{obs} = k \mid I_{t+1}^{obs} = q\} P_{jq}^h \Pr\{I_{t+1}^{tr} = j\} = \frac{P_{kq}^R \Pr\{I_t^{obs} = k\} P_{jq}^h \Pr\{I_{t+1}^{tr} = j\}}{\Pr\{I_{t+1}^{obs} = q\}}, \forall q \geq k. \quad (11)
 \end{aligned}$$

279 Substituting Eq. (11) into Eq. (10) as follows

$$\begin{aligned}
 & P_{kq}^R \Pr\{I_t^{obs} = k\} \\
 280 & = \sum_{i=1}^6 \sum_{j=i}^6 \left(\frac{P_{kq}^R \Pr\{I_t^{obs} = k\} P_{jq}^h \Pr\{I_{t+1}^{tr} = j\}}{\Pr\{I_{t+1}^{obs} = q\}} \frac{P_{ik}^h}{\sum_{w=k}^6 \left(\frac{P_{kw}^R \Pr\{I_t^{obs} = k\} P_{jw}^h \Pr\{I_{t+1}^{tr} = j\}}{\Pr\{I_{t+1}^{obs} = w\}} \right)} \right) \Pr\{I_{t+1}^{tr} = j, I_t^{tr} = i\}. \quad (12)
 \end{aligned}$$

$$\begin{aligned}
 & \frac{P_{kq}^R \Pr\{I_t^{obs} = k\} P_{jq}^h \Pr\{I_{t+1}^{tr} = j\}}{\Pr\{I_{t+1}^{obs} = q\}} P_{ik}^h, \text{ it follows that} \\
 281 & \text{Defining } P_{ijkq} \triangleq \frac{P_{kq}^R \Pr\{I_t^{obs} = k\} P_{jq}^h \Pr\{I_{t+1}^{tr} = j\}}{\sum_{w=k}^6 \left(\frac{P_{kw}^R \Pr\{I_t^{obs} = k\} P_{jw}^h \Pr\{I_{t+1}^{tr} = j\}}{\Pr\{I_{t+1}^{obs} = w\}} \right)} P_{ik}^h,
 \end{aligned}$$

$$282 \quad P_{kq}^R \Pr\{I_t^{obs} = k\} = \sum_{i,j=1}^6 P_{ijkq} \Pr\{I_{t+1}^{tr} = j, I_t^{tr} = i\} \quad (13)$$

283 which again elucidated in matrix form is

$$\begin{aligned}
 & \begin{bmatrix} P_{J,1} \\ P_{J,2} \\ \vdots \\ P_{J,20} \\ P_{J,21} \end{bmatrix}_{21 \times 1} = \begin{bmatrix} P_{J,1,1}^h & P_{J,1,2}^h & \cdots & P_{J,1,20}^h & P_{J,1,21}^h \\ P_{J,2,1}^h & P_{J,2,2}^h & \cdots & P_{J,2,20}^h & P_{J,2,21}^h \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ P_{J,20,1}^h & P_{J,20,2}^h & \cdots & P_{J,20,20}^h & P_{J,20,21}^h \\ P_{J,21,1}^h & P_{J,21,2}^h & \cdots & P_{J,21,20}^h & P_{J,21,21}^h \end{bmatrix}_{21 \times 21} \begin{bmatrix} P_{J,1}^{OCA} \\ P_{J,2}^{OCA} \\ \vdots \\ P_{J,20}^{OCA} \\ P_{J,21}^{OCA} \end{bmatrix}_{21 \times 1}, \quad (14)
 \end{aligned}$$

285 where $P_{J,x}^R = P_{kq}^R \Pr\{I_t^{obs} = k\}$, $P_{J,y}^{OCA} = \Pr\{I_{t+1}^{tr} = j, I_t^{tr} = i\}$, $P_{J,x,y}^h = P_{ijkq}^h$, and the indices are

286 related to each other by

$$x = \begin{cases} q, & \text{if } k = 1 \\ (q - k + 1) + \sum_{s=1}^{k-1} (6 - s + 1), & \text{otherwise} \end{cases}, \forall q \geq k, \quad (15)$$

288 and

$$y = \begin{cases} j, & \text{if } i = 1 \\ (j - i + 1) + \sum_{s=1}^{i-1} (6 - s + 1), & \text{otherwise} \end{cases}, \forall j \geq i. \quad (16)$$

290 Using Eq. (14), $P_{j,y}^{OCA} = \Pr\{I_{t+1}^{tr} = j, I_t^{tr} = i\}, \forall i = 1, 2, \dots, 6; j = i, \dots, 6$ may be solved
 291 similarly as in Eq. (9) using the constrained least-squares method. Using the above equations
 292 (Eq. (5) through (16)), the reported OCA rating transition matrix $\mathbf{P}_{\text{Report}}$ is mapped into the
 293 underlying true OCA rating transition matrix \mathbf{P}_{OCA} considering the human observation errors
 294 $\mathbf{P}_{\text{human}}$.

295 As shown above, the estimation of the \mathbf{P}_{OCA} matrix depends on the $\mathbf{P}_{\text{human}}$ matrix, which is
 296 assumed to be known in this work. However, when it is unknown, there are two approaches to
 297 estimate the $\mathbf{P}_{\text{human}}$ matrix. One way is to do a benchmark study using a statistically significant
 298 set of data focused on visual OCA ratings, similar to [26]. This consists on bringing inspectors
 299 to asses miter gate component with previously known damage condition to estimate
 300 $P_{ik}^h = \Pr\{I_t^{obs} = k | I_t^{tr} = i\}$. The other approach is to make the best possible estimation of $\mathbf{P}_{\text{human}}$
 301 , using previously collected data to inform a prior distribution for the parameters of the
 302 degradation model (described in the next section, which can be later updated using the
 303 continous SHM data). This second approach, when used in conjunction with Bayesian
 304 methods, is more desirable since it enables the continous updating of the degradation model
 305 for a specific case/structure using SHM data. Further work that is beyond the scope of this
 306 paper would be required to fully address any of these mentioned approaches. The next section

307 will discuss how to estimate the degradation model parameters θ of $a_t = g(t, \theta)$ using the
 308 transition matrix \mathbf{P}_{OCA} .

309 3.3 Estimation of the degradation model parameters

310 As noted in Step 2 in Fig. 4, in order to establish a connection between the degradation
 311 model $a_t = g(t, \theta)$ and the OCA transition matrix \mathbf{P}_{OCA} , a mapping function is defined for the
 312 OCA protocol as below

$$313 \quad R = h_{\text{OCA}}(a_t, \beta) = \begin{cases} I_{1,t} = A, a \in [0, \beta_1] \\ I_{2,t} = B, a_t \in [\beta_1, \beta_2] \\ I_{3,t} = C, a_t \in [\beta_2, \beta_3] \\ I_{4,t} = D, a_t \in [\beta_3, \beta_4] \\ I_{5,t} = F, a_t \in [\beta_4, \beta_5] \\ I_{6,t} = CF, a_t \in [\beta_5, \infty) \end{cases}, \quad (17)$$

314 where R is the OCA rating, a_t is the gap length, and $\beta = [\beta_1, \beta_2, \beta_3, \beta_4, \beta_5]$ is a vector of
 315 parameters of the mapping function related to the OCA protocol.

316 In the proposed method, the unknown parameters θ are estimated for given set of
 317 parameters β that define the mapping function (i.e. Eq. (17)), given the degradation model
 318 $a_t = g(t, \theta)$ and the true OCA transition matrix, \mathbf{P}_{OCA} , shown in Sec. 3.2. After that, diagnostics
 319 and prognostics are performed based on the estimated θ .

320 The task of estimating θ relies on solving the following optimization problem

$$321 \quad \begin{aligned} \theta^* &= \arg \min_{\theta} \{g_{\text{opt}}(\theta; \beta, \mathbf{P}_{\text{OCA}})\}, \\ &s.t. \theta \in \Omega, \end{aligned} \quad (18)$$

322 where $g_{\text{opt}}(\theta; \beta, \mathbf{P}_{\text{OCA}})$ is a cost function of the optimization model, and Ω is the domain of
 323 θ . In the above optimization model, the cost function $g_{\text{opt}}(\theta; \beta, \mathbf{P}_{\text{OCA}})$ is defined as

$$\begin{aligned}
324 \quad \mathcal{G}_{opt}(\boldsymbol{\theta}; \boldsymbol{\beta}, \mathbf{P}_{OCA}) &= \left\| \hat{\mathbf{P}}(\boldsymbol{\theta}) - \mathbf{P}_{OCA} \right\|_2, \\
&= \sum_{i=1}^6 \sum_{j=1}^6 (\hat{P}(I_{j,t+1}^s | I_{i,t}^s; \boldsymbol{\theta}) - P(I_{j,t+1}^r | I_{i,t}^r))^2,
\end{aligned} \tag{19}$$

325 in which $I_{i,t}^s$ and $I_{j,t+1}^s$ are the inspected state (e.g. A, B, C, D, F or CF) at time t and $t+1$
326 respectively obtained from the degradation simulation and mapping function, $h_{OCA}(a_t, \boldsymbol{\beta})$. For
327 $P(I_{j,t+1}^r | I_{i,t}^r) \triangleq \Pr\{I_{t+1}^r = j | I_t^r = i\}$, the reader can refer to the definitions of Eq. (2),
328 $\hat{\mathbf{P}}(\boldsymbol{\theta}) \triangleq \{\hat{P}(I_{j,t+1}^s | I_{i,t}^s; \boldsymbol{\theta}), i = 1, 2, \dots, 6; j = i, \dots, 6\}$ is the simulated transition probabilities of the
329 OCA ratings from the degradation model simulation for given $\boldsymbol{\theta}$, and \mathbf{P}_{OCA} is the true OCA
330 transition matrix (i.e. Eq. (2)) obtained from Sec. 3.2 based on the reported OCA transition
331 matrix and human observation error matrix.

332 It should be noted that, theoretically speaking, the optimization model Eq. (19) may also
333 be formulated directly from the reported OCA transition matrix \mathbf{P}_{Report} perspective by coupling
334 the approach developed in this section with the forward uncertainty propagation of the OCA
335 ratings based on the human error observation matrices. That kind of formulation may be
336 considered as an alternative approach to the proposed method and will be compared in future
337 work. The benefit of using \mathbf{P}_{OCA} in Eq. (19) is two-fold: first, the identification of \mathbf{P}_{OCA} in Sec.
338 3.2 allows to perform failure prognostics with \mathbf{P}_{OCA} instead of \mathbf{P}_{Report} using the approach
339 developed in [16]. Using \mathbf{P}_{OCA} to replace \mathbf{P}_{Report} in transition matrix-based prognostics will
340 improve the accuracy of failure prognostics since \mathbf{P}_{OCA} mitigates the effects of human
341 observation errors. Second, the formulation given in Eq. (19) eliminates process of uncertainty
342 propagation step from \mathbf{P}_{OCA} to \mathbf{P}_{Report} in estimating $\boldsymbol{\theta}$, which reduces the complexity of the
343 optimization process.

344 As shown in Eq. (19), the estimation of $\hat{\mathbf{P}}(\boldsymbol{\theta})$ for a given $\boldsymbol{\theta}$ is the key for the optimization-
 345 based method to minimize the L2 error norm between the underlying true OCA transition
 346 matrix, \mathbf{P}_{OCA} , and the estimated transition matrix $\hat{\mathbf{P}}(\boldsymbol{\theta})$ obtained from the estimated multi-stage
 347 continuous degradation model. The next section will discuss in detail on how to estimate $\hat{\mathbf{P}}(\boldsymbol{\theta})$
 348 for a given $\boldsymbol{\theta}$. After that, an explanation will be given of how to solve Eq. (19) based on the
 349 estimation of multi-stage continuous degradation model.

350 3.3.1 Prediction of OCA rating transition matrix $\hat{\mathbf{P}}(\boldsymbol{\theta})$ for given $\boldsymbol{\theta}$

351 (a) Selection of degradation model

352 As mentioned earlier, there is a need for a degradation model whose OCA transition matrix
 353 prediction, $\hat{\mathbf{P}}(\boldsymbol{\theta})$, resembles the true OCA transition matrix, \mathbf{P}_{OCA} . A variation of the stochastic
 354 model proposed by Yang and Manning [27], which is referred as the Yang and Manning model
 355 and reviewed in Appendix B, is used. This model allows flexibility when considering the
 356 abstracted OCA data and the lack of the understanding of the physics of the damage evolution
 357 of bearing gaps.

358 To account for the effect of degradation stages over continuous time, the Yang and
 359 Manning model (see Appendix B for details) is generalized as below

$$360 \quad \frac{da(t)}{dt} = \exp(\sigma(t)U(t))Q(t)(a(t))^{w(t)}, \quad (20)$$

361 where $U(t)$ is a stationary standard Gaussian process with auto-correlation function given by
 362 Eq. (51) in Appendix B, $\sigma(t)$, $Q(t)$, and $w(t)$ are parameters determined through gap length
 363 $a(t)$ as follows

$$364 \quad \begin{cases} \sigma(t) = \sigma_j \\ Q(t) = Q_j, \text{ where } j = h_s(a(t)), \forall j = 1, \dots, N_d, \\ w(t) = w_j \end{cases} \quad (21)$$

365 in which N_d is the number of degradation stages, $j = h_s(a(t))$ is a function that discretely
 366 maps gap length $a(t)$ into degradation stages as below

$$367 \quad j = h_s(a(t)) = \begin{cases} 1, & \text{if } a(t) \in [0, e_1], \\ 2, & \text{if } a(t) \in [e_1, e_2], \\ \vdots & \\ N_d, & \text{if } a(t) \in [e_{N_d-1}, \infty), \end{cases} \quad (22)$$

368 where $e_i < e_{i+1}$, $\forall i = 1, 2, \dots, N_d - 2$ are the threshold gap lengths that determine the transition
 369 of degradation stages. Note that the mapping function $j = h_s(a(t))$ for the gap growth model
 370 is different from the mapping function (i.e. $R = h_{OCA}(a_t, \boldsymbol{\beta})$) defined by the OCA protocol. The
 371 mapping function $j = h_s(a(t))$ is governed by the underlying degradation physics, while
 372 $R = h_{OCA}(a_t, \boldsymbol{\beta})$ is defined by the engineers using OCA protocols.

373 Moreover, in order to account for the randomness of the threshold gap lengths that govern
 374 the transition of degradation stages, e_i , $\forall i = 1, 2, \dots, N_d - 1$ are described as Gaussian random
 375 variables as follows

$$376 \quad e_i \sim N(\mu_i, \sigma_e^2), \forall i = 1, 2, \dots, N_d - 1, \quad (23)$$

377 with mean μ_i and standard deviation σ_e .

378 In the discrete time domain, the above degradation model is rewritten as

$$379 \quad a(t_{k+1}) = a(t_k) + \exp(\sigma(t_{k+1})U(t_{k+1}))Q(t_{k+1})(a(t_k))^{w(t_{k+1})}, \forall k = 1, 2, \dots, N_t, \quad (24)$$

$$380 \quad \begin{cases} \sigma(t_{k+1}) = \sigma_j \\ Q(t_{k+1}) = Q_j, \text{ where } j = h_s(a(t_k)), \forall j = 1, \dots, N_d, \\ w(t_{k+1}) = w_j \end{cases} \quad (25)$$

381 where N_t is the number of analysis time steps in the time duration of interest.

382 To summarize, in the selected degradation model, the parameters $\boldsymbol{\theta}$ of the degradation
 383 model include the following parameters

$$384 \quad \boldsymbol{\theta} \triangleq \left\{ \boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \dots, \boldsymbol{\theta}_{N_d}, \zeta, \mu_1, \mu_2, \dots, \mu_{N_d-1}, \boldsymbol{\sigma}_e \right\}, \quad (26)$$

385 where $\boldsymbol{\theta}_j \triangleq \{\sigma_j, Q_j, w_j, j = 1, 2, \dots, N_d\}$.

386 The next section will discuss the prediction of $\hat{\mathbf{P}}(\boldsymbol{\theta})$ for a given $\boldsymbol{\theta}$.

387 (b) Prediction of $\hat{\mathbf{P}}(\boldsymbol{\theta})$ using the degradation model

388 Based on the above degradation model, for given $\boldsymbol{\theta}$ and \mathbf{e} , according to the derivations
 389 given in **Appendix C**, $\hat{P}(I_{j,t+1}^s | I_{i,t}^s; \boldsymbol{\theta}, \mathbf{e})$, $\forall i = 1, 2, \dots, 6; j = i, \dots, 6$, are estimated based on
 390 the degradation simulation as follows

$$391 \quad \begin{aligned} & \hat{P}(I_{j,t+1}^s | I_{i,t}^s; \boldsymbol{\theta}, \mathbf{e}) \\ & \approx \frac{1}{(N_t - 12)n_{MCS}} \sum_{k=1}^{N_t-12} \frac{\sum_{q=1}^{n_{MCS}} \Lambda((\beta_{i-1} \leq a_{q,k} < \beta_i) \cap (\beta_{j-1} \leq a_{q,k+12} < \beta_j))}{\sum_{q=1}^{n_{MCS}} \Lambda(\beta_{i-1} \leq a_{q,k} < \beta_i)}, \end{aligned} \quad (27)$$

392 where $\Lambda(\cdot)$ is an indicator function defined in Eq. (58) in Appendix C and $a_{q,k}$ is the simulated
 393 q -th realization of gap length at time step t_k (see Appendix C for details).

394 The above probability estimate is conditioned on $\boldsymbol{\theta}$ and \mathbf{e} . After considering the
 395 uncertainty in threshold gap lengths, $\mathbf{e} = [e_1, e_2, \dots, e_{N_d-1}]$ that determine the transition of
 396 degradation stages, the marginalization of $\hat{P}(I_{j,t+1}^s | I_{i,t}^s; \boldsymbol{\theta})$ may be written as

$$397 \quad \begin{aligned} \hat{P}(I_{j,t+1}^s | I_{i,t}^s; \boldsymbol{\theta}) &= \int \hat{P}(I_{j,t+1}^s | I_{i,t}^s; \boldsymbol{\theta}, \mathbf{e}) f_e(\mathbf{e} | \boldsymbol{\theta}) d\mathbf{e}, \\ &= \int \int \dots \int \hat{P}(I_{j,t+1}^s | I_{i,t}^s; \boldsymbol{\theta}, \mathbf{e}) \prod_{k=1}^{N_d-1} \phi\left(\frac{e_k - \mu_k}{\sigma_e}\right) de_1 de_2 \dots de_{N_d-1}, \end{aligned} \quad (28)$$

398 where $f_e(\mathbf{e} | \boldsymbol{\theta})$ is the joint PDF of e_i , and $e_i < e_{i+1}, \forall i = 1, 2, \dots, N_d - 2$, and $\phi(\cdot)$ is the PDF of
 399 the standard normal distribution.

400 In this paper, a sampling-based approach is employed to estimate Eq. (28). Using the above
 401 equations and derivations in Appendix C, $\hat{\mathbf{P}}(\boldsymbol{\theta}) \triangleq \{\hat{P}(I_{j,t+1}^s | I_{i,t}^s; \boldsymbol{\theta}), i = 1, 2, \dots, 6; j = i, \dots, 6\}$
 402 may be estimated for given $\boldsymbol{\theta}$. The estimated $\hat{\mathbf{P}}(\boldsymbol{\theta})$ may then be used in Eq. (19) to obtain the
 403 parameters $\boldsymbol{\theta}$ of the degradation model. Table 1 provides a pseudocode for this process.

404 **Table 1:** Estimation of $\hat{\mathbf{P}}(\boldsymbol{\theta})$ for given $\boldsymbol{\theta} \triangleq \{\boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \dots, \boldsymbol{\theta}_{N_d}, \zeta, \mu_1, \mu_2, \dots, \mu_{N_d-1}, \sigma_e\}$

Step	Description
1	Initialization: Generate samples of $U(t_1), \dots, U(t_{N_t})$ for a given correlation length ζ , samples of $e_i < e_{i+1}, \forall i = 1, 2, \dots, N_d - 2$ based on $\mu_1, \mu_2, \dots, \mu_{N_d-1}, \sigma_e$, and initial samples of $a(t_0)$
2	Sort the samples of $e_i < e_{i+1}, \forall i = 1, 2, \dots, N_d - 1$
3	For $k = 1, 2, \dots, N_t$:
4	Map gap length $a(t_{k-1})$ into degradation stage using Eq. (22)
5	Obtain samples of $a(t_k)$ using Eqs. (24) and (25)
End	
6	Obtain samples of $a(t_k), k = 1, 2, \dots, N_t$
7	Reshape the data and obtain samples of $a(t_k)$ and $a(t_k + 12)$
8	Compute $\hat{\mathbf{P}}(\boldsymbol{\theta})$ using Eqs. (27) and (28) for a given $\boldsymbol{\beta}$ defined in Eq. (17)

405 The next section discusses how to estimate $\boldsymbol{\theta}$ by solving the optimization model given in
 406 Eq. (19).

407 3.3.2 Estimation of degradation model parameters $\boldsymbol{\theta}$

408 In this paper, the Generalized Simulated Annealing (GSA) method is used to solve the
 409 optimization problem. This method is a stochastic approach for approximating the global
 410 optimum of the cost function shown in Eq. (19). The GSA method is mainly used when
 411 processing complicated non-linear objective functions with a large number of local minima.

412 The Cauchy-Lorentz visiting distribution is used to generate a trial jump distance $\Delta\theta(t)$ of the
 413 variable $\theta(t)$,

$$414 \quad \Delta\theta(t) \propto \frac{[T_{q_v}(t)]^{-\frac{D}{3-q_v}}}{[1 + (q_v - 1) \frac{p^2}{2}]^{\frac{1}{q_v-1} + \frac{D-1}{2}}}, p \sim U(0,1), T_{q_v}(t) = T_{q_v}(1) \frac{2^{q_v-1} - 1}{(1+t)^{q_v-1} - 1}, \quad (29)$$

415 where D is the dimension of the variable space, $T_{q_v}(t)$ is the artificial temperature (a time-
 416 varying global parameter), and q_v is a time-invariant parameter that controls the rate of
 417 cooling. To avoid local minima, the trial jump uses an acceptance probability using a
 418 Metropolis algorithm. In other words, the proposed trial jump is always accepted if it is
 419 downhill and it is accepted with a probability if the jump is uphill, which allows to explore the
 420 space outside the local minima. For more details on this method, the reader is referred to
 421 [28,29].

422 After the parameters θ are estimated, the degradation model can be used for damage
 423 diagnostics and prognostics, which is briefly discussed in the next section.

424 3.4 Diagnostics and prognostics of using the degradation model

425 Let $\mathbf{s}_t = [s_{t1}, s_{t2}, \dots, s_{tN_s}]$ be the strain measurement data at time step t , where N_s is the
 426 number of strain sensors providing data. The degradation model $a_t = g(t, \theta)$ obtained in Sec.
 427 3.3 can then be used for failure diagnostics and prognostics using the approach presented in
 428 Vega et al. [16], using the following state and measurement equations,

$$429 \quad \begin{aligned} \text{State equation: } a_{k+1} &= a_k + \exp(\sigma_{k+1} U_{k+1}) Q_{k+1} (a_k)^{w_{k+1}}, \\ \text{Measurement equation: } \mathbf{s}_{k+1} &= \hat{g}(a_{k+1}, \mathbf{x}_{k+1}) + \varepsilon, \end{aligned} \quad (30)$$

430 where a_{k+1} , a_k , σ_{k+1} , U_{k+1} , Q_{k+1} , and w_{k+1} are, respectively, $a(t_{k+1})$, $a(t_k)$, $\sigma(t_{k+1})$, $U(t_{k+1})$,
431 $Q(t_{k+1})$, and $w(t_{k+1})$ given in Eq. (24). The term $\hat{g}(a_{k+1}, \mathbf{x}_{k+1})$ is a model (e.g., the FE model)
432 for the prediction of strain response for given gap state a_{k+1} and other input variables \mathbf{x}_{k+1} such
433 as water levels and temperature. The measurement noise ε is assumed to be normal,
434 $\varepsilon \sim N(\mathbf{0}, \sigma_{obs}^2 \mathbf{I})$, with uncorrelated structure characterized by the standard deviation σ_{obs} .

435 Since the original FE model $\hat{g}(a_{k+1}, \mathbf{x}_{k+1})$ is usually expensive, a trained and verified
436 surrogate model, $\hat{g}(a_{k+1}, \mathbf{x}_{k+1})$, is usually used to replace the original FE model. In this paper,
437 a Kriging surrogate modelling method is employed as it can effectively quantify the uncertainty
438 in the prediction, which is advantageous over pointwise-estimate surrogate modelling methods,
439 such as Neural Networks, Support Vector Machine, etc.

440 The equations above can then be solved recursively in a timely manner as been discussed
441 in Vega et al. [16]. Based on the failure diagnostics and prognostics of the gap growth, the
442 remaining useful life of a miter gate can be estimated at every time step t_k as

$$443 \quad \Pr\{T_{RUL} \leq t_m | \mathbf{s}_{1:k}\} = \frac{1}{N_{PF}} \sum_{i=1}^{N_{PF}} \Lambda\{a(i, j+k) > a_e, \exists j = 1, 2, \dots, m\}, \quad (31)$$

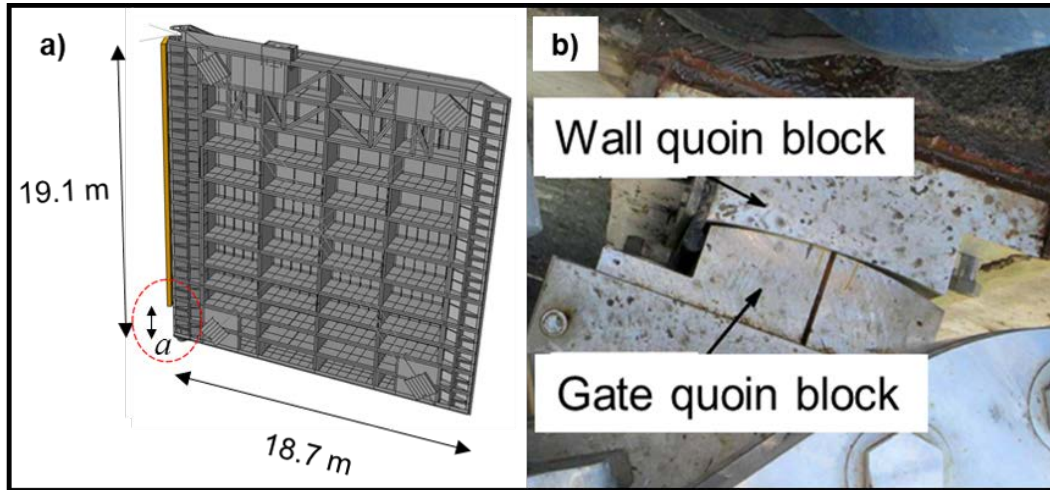
444 in which T_{RUL} stands for the remaining useful life, N_{PF} is the number of samples used in the
445 state estimation using Eq. (30), a_e is the gap failure threshold, and $a(i, j+k)$ is the i -th
446 realization of the gap length at the $(j+k)$ -th time step. In the next section, a miter gate case
447 study is used to demonstrate the effectiveness of the proposed framework.

448

449 **4 A Case Study**

450 One of the primary concerns of USACE engineers for inspection, maintenance, and repair
451 are the condition of the quoin blocks [3]. Commonly, the deterioration of the quoin blocks is

452 broadly manifested as a small bearing “gap”. The formation of this gap is due to the contact
 453 degradation between the quoin block attached to the gate and the quoin block attached to the
 454 wall that supports the gate laterally. The formation of the bearing gap can be detected using
 455 sensor data or from features derived from this data [2,19,30–32]. Figure 6 idealizes the loss of
 456 contact in the physical-based FE model and shows the top view of the quoin blocks.



457
 458 **Figure 6:** a) Gap formation at the bottom of the quoin blocks and b) the top view of the
 459 contact between the quoin blocks [33]

460 The term $\hat{P}(I_{j,t+1} | I_{i,t}, \theta)$ is the derived transition matrix obtained from the stochastic
 461 degradation model. To calculate this matrix, it is necessary to map the gap length value from
 462 its continuous form to the discrete OCA ratings using β defined in Eq. (17). β is also needed
 463 in the evaluation of gap length using OCA ratings by the field engineers. Table 2 shows the
 464 mapping between gap length, $a(t)$, to its corresponding OCA rating. For the values on this
 465 table, the mapping is assumed to be known and would be treated as the inspection policy.

466 **Table 2:** Mapping from gap length, $a(t)$, to discrete OCA ratings.

Gap length (cm)	OCA rating
$0 \leq a < 76.2$	A
$76.2 \leq a < 152.4$	B
$152.4 \leq a < 228.6$	C
$228.6 \leq a < 304.8$	D
$304.8 \leq a < 381$	F
$a > 381$	CF

467 For the OCA ratings given in the above table, an example of the report OCA transition
 468 matrix $\mathbf{P}_{\text{Report}}$ is given as

$$469 \quad \mathbf{P}_{\text{Report}} = \begin{bmatrix} 7.76e-1 & 2.13e-1 & 5.25e-3 & 2.16e-3 & 1.85e-3 & 2.47e-3 \\ 0 & 9.28e-1 & 4.40e-2 & 1.74e-2 & 7.94e-3 & 2.60e-3 \\ 0 & 0 & 8.70e-1 & 1.19e-3 & 6.64e-3 & 4.78e-3 \\ 0 & 0 & 0 & 9.40e-1 & 5.03e-2 & 9.39e-3 \\ 0 & 0 & 0 & 0 & 8.65e-1 & 1.35e-1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \quad (32)$$

470 As discussed in Sec. 3, the reported OCA transition matrix may have errors due to the
 471 human observation errors of the field engineers. Next, a demonstration is presented of how to
 472 obtain the underlying true transition matrix based on the human error matrix using the proposed
 473 method. After that, a discussion is presented on how to obtain a gap degradation model and
 474 how to use it to perform diagnostics and prognostics.

475

476 **4.1 Mapping the reported OCA transition matrix to the true OCA transition matrix for**
 477 **different human error scenarios**

478 As indicated by [26], this human error/performance may be evaluated to quantify the
 479 reliability or accuracy of these inspections. For demonstration purposes, four different cases as
 480 shown in Eqs. (33) to (36) will be evaluated to see the effect of human error on the OCA
 481 transition matrix and the degradation model. Case 1 assumes that the inspection is performed
 482 without any human observation errors, in other words, $\mathbf{P}_{\text{human}}$ would be the identity matrix.
 483 Case 2 represents the behavior of an inspector that regularly tends to assess a structural
 484 component to be in a better condition than reality. For example, as shown in Eq. (34), there is
 485 a 4% probability that an inspector reports a rating A to a structural component when in reality
 486 the true state of the component belongs to rating B. Contrarily, Case 3 represents an inspector

487 that tends to be very conservative. For example, as shown in Eq. (35), there is a 5% probability
 488 that an inspector reports a rating F to a structural component when in reality the true state of
 489 the component belongs to rating D. Case 4 represents a case in between Case 2 and Case 3.

490
$$\mathbf{P}_{\text{human}}^{\text{case1}} = \mathbf{I}_{6 \times 6}, \quad (33)$$

491
$$\mathbf{P}_{\text{human}}^{\text{case2}} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0.04 & 0.96 & 0 & 0 & 0 & 0 \\ 0 & 0.40 & 0.60 & 0 & 0 & 0 \\ 0 & 0.03 & 0.17 & 0.80 & 0 & 0 \\ 0 & 0 & 0 & 0.03 & 0.97 & 0 \\ 0 & 0 & 0 & 0 & 0.03 & 0.97 \end{bmatrix}, \quad (34)$$

492
$$\mathbf{P}_{\text{human}}^{\text{case3}} = \begin{bmatrix} 0.60 & 0.40 & 0 & 0 & 0 & 0 \\ 0 & 0.90 & 0.08 & 0.02 & 0 & 0 \\ 0 & 0 & 0.90 & 0.10 & 0 & 0 \\ 0 & 0 & 0 & 0.95 & 0.05 & 0 \\ 0 & 0 & 0 & 0 & 0.98 & 0.02 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad (35)$$

493
$$\mathbf{P}_{\text{human}}^{\text{case4}} = \begin{bmatrix} 0.9 & 0.1 & 0 & 0 & 0 & 0 \\ 0.05 & 0.9 & 0.03 & 0.02 & 0 & 0 \\ 0.04 & 0.06 & 0.8 & 0.05 & 0.035 & 0.015 \\ 0.015 & 0.035 & 0.05 & 0.8 & 0.6 & 0.04 \\ 0 & 0.015 & 0.035 & 0.05 & 0.8 & 0.1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad (36)$$

494 As shown in Eq. (10), the true OCA transition matrix (\mathbf{P}_{OCA}) may be obtained after knowing
 495 the reported OCA transition matrix ($\mathbf{P}_{\text{Report}}$, Eq. (32)) and the human observation error ($\mathbf{P}_{\text{human}}$,
 496 Eqs. (33) through (36)). Using the different cases for human observation errors mentioned
 497 earlier, the true OCA transition matrix for each case is shown in Eqs. (37) to (40) respectively.

$$498 \quad \mathbf{P}_{OCA}^{\text{case1}} = \begin{bmatrix} 7.76e-1 & 2.13e-1 & 5.25e-3 & 2.16e-3 & 1.85e-3 & 2.47e-3 \\ 0 & 9.28e-1 & 4.40e-2 & 1.74e-2 & 7.94e-3 & 2.60e-3 \\ 0 & 0 & 8.70e-1 & 1.19e-3 & 6.64e-3 & 4.78e-3 \\ 0 & 0 & 0 & 9.40e-1 & 5.03e-2 & 9.39e-3 \\ 0 & 0 & 0 & 0 & 8.65e-1 & 1.35e-1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad (37)$$

$$499 \quad \mathbf{P}_{OCA}^{\text{case2}} = \begin{bmatrix} 7.02e-1 & 2.89e-1 & 7.01e-3 & 0 & 0 & 2.48e-3 \\ 0 & 9.08e-1 & 7.03e-2 & 1.06e-2 & 8.26e-3 & 2.49e-3 \\ 0 & 0 & 8.42e-1 & 1.47e-1 & 6.04e-3 & 4.73e-3 \\ 0 & 0 & 0 & 9.48e-1 & 4.55e-2 & 6.71e-3 \\ 0 & 0 & 0 & 0 & 8.60e-1 & 1.40e-1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad (38)$$

$$500 \quad \mathbf{P}_{OCA}^{\text{case3}} = \begin{bmatrix} 7.89e-1 & 2.02e-1 & 3.02e-3 & 1.42e-3 & 1.87e-3 & 2.35e-3 \\ 0 & 9.50e-1 & 2.72e-2 & 1.19e-2 & 8.10e-3 & 2.48e-3 \\ 0 & 0 & 8.40e-1 & 1.48e-1 & 4.27e-3 & 7.46e-3 \\ 0 & 0 & 0 & 8.66e-1 & 1.17e-1 & 1.74e-2 \\ 0 & 0 & 0 & 0 & 8.69e-1 & 1.31e-1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad (39)$$

501 and

$$502 \quad \mathbf{P}_{OCA}^{\text{case4}} = \begin{bmatrix} 5.63e-1 & 4.34e-1 & 3.17e-3 & 0 & 0 & 0 \\ 0 & 9.37e-1 & 4.11e-2 & 1.27e-2 & 7.80e-3 & 1.15e-3 \\ 0 & 0 & 8.93e-1 & 9.66e-2 & 8.35e-3 & 1.59e-3 \\ 0 & 0 & 0 & 9.29e-1 & 7.13e-2 & 0 \\ 0 & 0 & 0 & 0 & 9.14e-1 & 8.61e-2 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad (40)$$

503 The human observation error has a significant effect on the true OCA transition matrix.

504 For Case 1, the true OCA transition matrix ($\mathbf{P}_{OCA}^{\text{case1}}$, Eq. (37)) is equal to the reported OCA

505 transition matrix ($\mathbf{P}_{\text{Report}}$, Eq. (32)) and consistent when human observation error is not present.

506 For Case 2, the true OCA transition matrix ($\mathbf{P}_{OCA}^{\text{case2}}$, Eq. (38)) shows a decrease on the majority

507 of the transition probabilities located in the diagonal when Cases 1 and 2 are compared. In

508 other words, the degradation model should tend to deteriorate faster at the beginning.

509 Contrarily, the true OCA transition matrix (\mathbf{P}_{OCA}^{case3} , Eq. (39)) for Case 3 shows that the majority
510 of the transition probabilities located in the diagonal shows an increase when Cases 1 and 3 are
511 compared. Note that not all the diagonal elements show a decrease due to the *error*
512 *cancellations* in first and second assessments of the OCA ratings. But in general, the
513 degradation model of Case 3 degrades slower than that of Case 1 (as shown in the results in
514 Sec. 4.2). As expected, Case 4 (i.e. Eq. (40)) shows some of the diagonal entries increase while
515 the other diagonals entries decrease when Cases 1 and 4 are compared. Even though effects of
516 the human observation errors on the transition matrix is very complicated due to the “error
517 cancellation” in the OCA ratings, the proposed approach can account for the complicated
518 effects by mapping the reported OCA transition matrix to the true OCA transition matrix.

519 In the next subsection, the underlying degradation models will be identified based on the
520 obtained OCA transition matrices of different level of human observations errors.

521

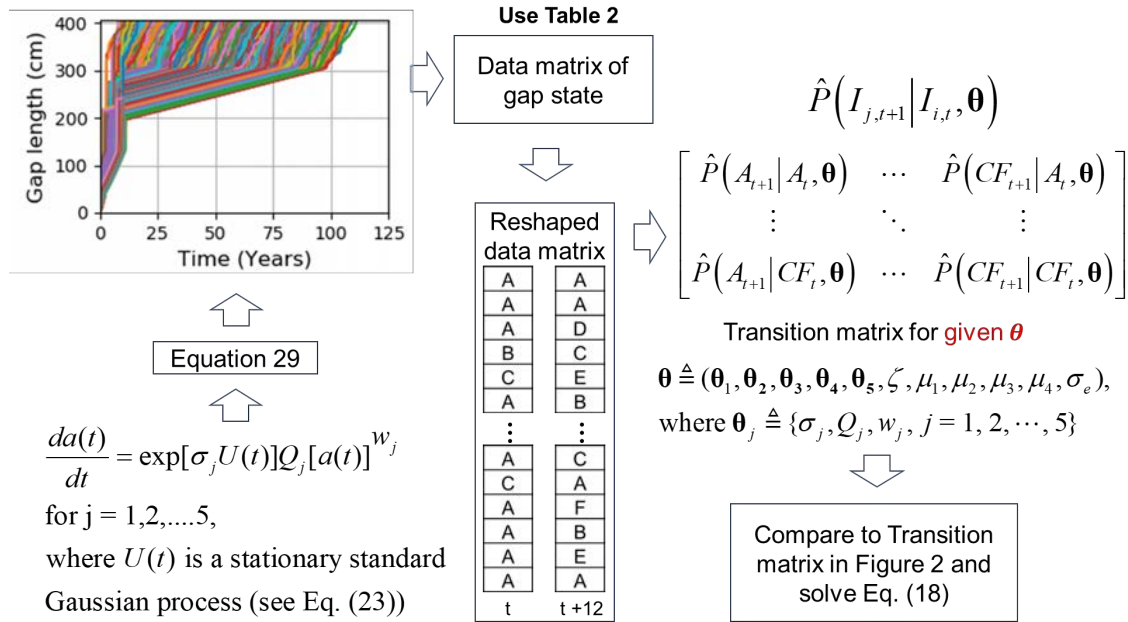
522 4.2 Gap growth modeling based on OCA transition matrix

523 Figure 7 shows a flowchart of how to obtain the transition matrix from the stochastic
524 degradation model, which is used to estimate the gap growth model parameters based on the
525 OCA transition matrices obtained above.

526 Figure 8 shows the cumulative minimum error after each iteration of the stochastic
527 degradation model after tuning 21 parameters for four different cases (i.e. Eq. (33) through
528 (36)). The GSA optimization algorithm successfully achieves a very small error for each case.

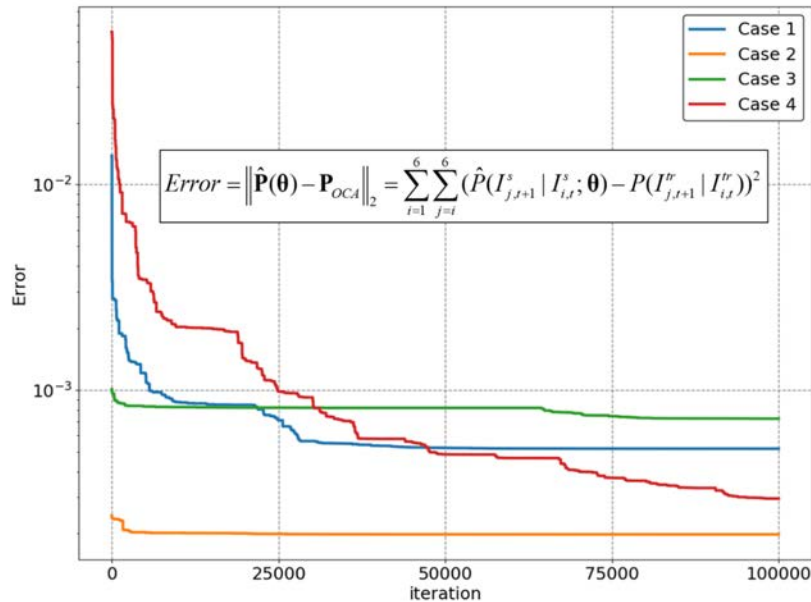
529 Figure 9 presents the simulated gap growth curves corresponding to the four scenarios after
530 identifying the optimal parameters of the gap growth model using GSA. Comparing the gap
531 growth curves of Case 2 to 4 with Case 1, similar conclusions can be obtained as that from
532 comparing the OCA transition matrices (i.e. Eq. (37)-(40)). For Case 2, the degradation model

533 should tend to deteriorate faster at the beginning as shown in Fig. 9, which can also be seen in
 534 Fig. 10 when comparing Case 1 and 2. Contrarily, for Case 3, the degradation model should
 535 tend to deteriorate slower as shown in Fig. 9, when Cases 1 and 3 are compared.



536
537

538 **Figure 7:** Flowchart to obtain simulated transition matrix from a gap degradation model

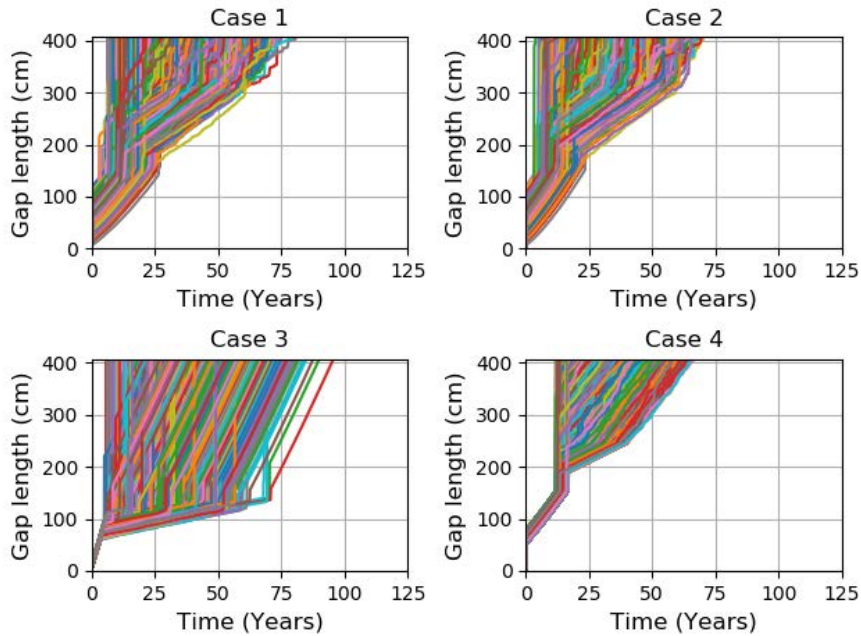


539
540

Figure 8: Cumulative minimum error after each iteration

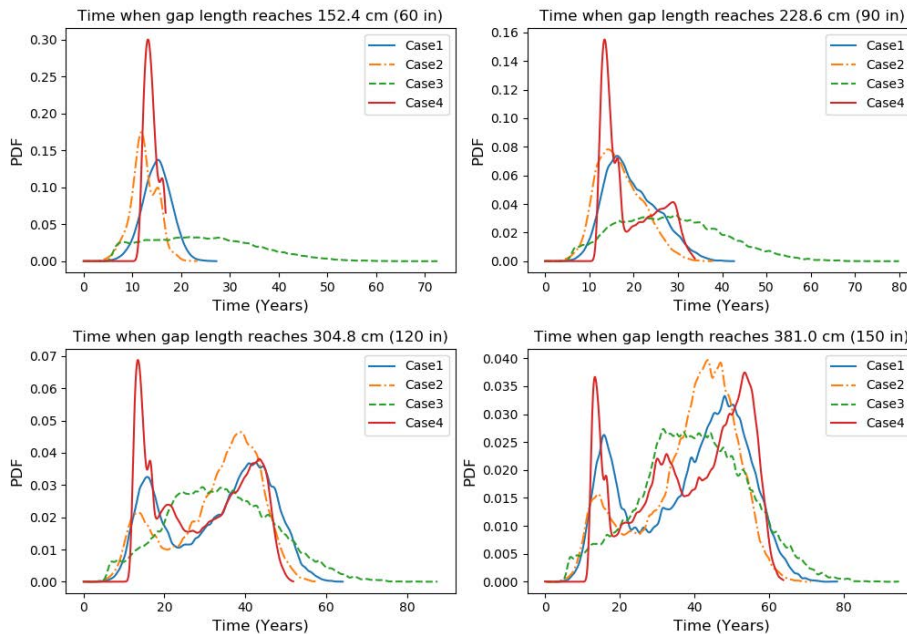
541 Fig. 10 shows the time distribution when the curves shown in Fig. 9 exceed four different
 542 thresholds. As expected, the time distribution for Case 2 shifts to earlier time region (i.e. left)

543 compared to its counterpart of Case 1. Conversely, the time distribution for Case 3 shifts
 544 towards later time region (i.e. right) if compared to Case 1. Consistently, the result for Case 4
 545 in general shows time distributions between that of Case 2 and 3.



546
 547

Figure 9: Gap growth model comparison for different human error cases



548
 549
 550

Figure 10: Time distribution when gap length, a , exceeds different damage thresholds for different human error cases

551 The above results show that the proposed method is able to effectively investigate the
 552 effects of human errors on the OCA transition matrix and the gap growth of the gate over time.

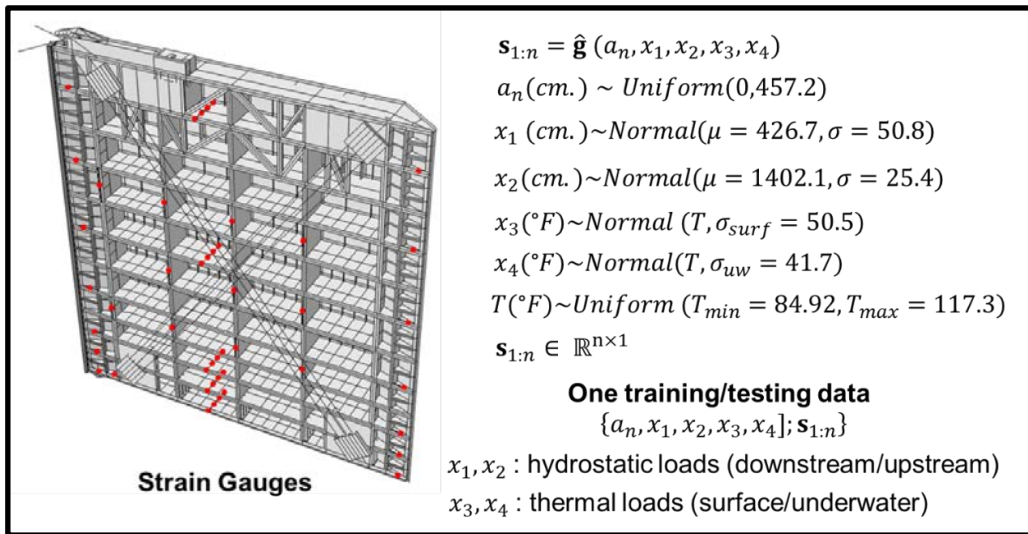
553

554 4.3 Bearing gap diagnosis and prognosis using SHM and gap growth modeling

555 Fig. 11 shows the locations where the strain gauges are installed based on the SHM strain
556 network installed at the Greenup miter gate (Kentucky, USA). Data is extracted from a FE
557 model of this gate to train a Kriging surrogate model.

558 Two different surrogate models are built, one that would be used to generate the synthetic
559 data (representing the true physics) and the other to be calibrated during the estimation process.

560 In other words, one surrogate model is built to mimic the reality and the other one to mimic the
561 FE model in the estimation process. Both surrogate models are built from the input and outputs
562 of the FE model after space filling its parameter space. Figure 12(a) shows the updated
563 predictions of the gap length against the true damage using the proposed gap growth model in
564 the estimation process.

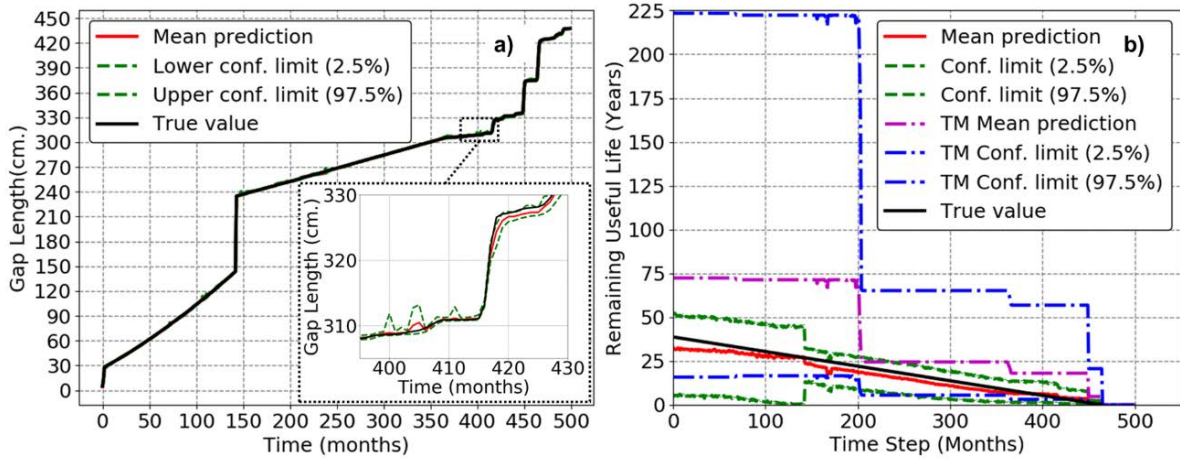


565
566

Figure 11: Sensor locations, and data generated to train surrogate model

567 As shown in Figure 12(b), the proposed method can accurately capture remaining useful
568 life (RUL) while effectively performing damage detection (i.e. Fig. 12 (a)). In addition, the
569 results show that the uncertainty in the RUL estimate can be reduced significantly by mapping
570 the OCA transition matrix into a higher-precision gap growth model, compared to that of the
571 transition matrix-based method as reviewed in Sec. 3. The jumps in Figure 12(b) are attributed

572 to the discrete nature of the OCA ratings, which are more pronounced in the predictions using the
 573 TM based approach. More details of the TM approach can be found in [16]. Results of this case
 574 study demonstrate the efficacy of the proposed method.



575 **Figure 12:** (a) Damage detection over time, and (b) RUL using the proposed method (where
 576 “TM” stands for the transition matrix-based approach as reviewed in [16])
 577

578

579 **5 Discussion**

580 Failure prognostics plays a vital role in proactively scheduling maintenance activities to
 581 avoid catastrophic failures, which improves reliability of civil infrastructure and reduce overall
 582 life-cycle costs [34–37]. In recent years, data-driven approaches have been developed using
 583 neural networks [24,38,39], deep learning [40], and other machine learning-based approaches
 584 [41–44] to correlate sensor monitoring data with system degradation and in order to predict
 585 system failures. For structures like miter gates, however, historical continuous monitoring data
 586 is not available, which makes the state-of-the-art neural network-based approaches
 587 inapplicable for failure prognostics of a miter gate. Instead, highly abstracted rating data are
 588 available, which contain some kind of degradation information. Along with the highly
 589 abstracted data, a high-fidelity physics-based finite element model has been developed to
 590 provide some physical understanding of the gate strain response under different conditions. To
 591 fully leverage the information of the abstracted ratings and the high-fidelity physics-based

592 simulation model, a new prognostic approach is required. To this end, this paper develops a
593 novel hybrid failure prognostic approach by integrating the highly abstracted OCA ratings with
594 structural health monitoring data.

595 The developed approach tackles the issue that no viable degradation model available exists
596 for failure prognostics by mapping the corrected OCA transition matrix into a continuous-space
597 degradation model using an optimization-based method. As an optimization-based approach,
598 it is possible that there may be non-unique solutions. To address this issue, the authors plan to
599 develop a fully Bayesian approach to quantify the uncertainty in various model parameters and
600 continuously update the model parameters during the monitoring process methods such as
601 dynamic Bayesian networks. Moreover, more constraints to the optimization model and the
602 OCA transition matrix need to be added in the future to address the potential non-uniqueness
603 issues in the estimation process.

604 In this paper, a Yang-Manning degradation model is assumed as a potential degradation
605 model. Even though this flexible model allows capturing various gap-growth behavior classes
606 without requiring detailed understanding of the underlying physics, it may not accurately
607 represent the gap degradation pattern in reality. The assumed model may conflict with the
608 subsequent measurement data obtained through an SHM procedure and then affect the
609 inference of the damage states of the system. This is related to the potential model form
610 uncertainty of the assumed degradation model. To address this challenge, the following two
611 research topics are worth investigating in the future: (1) Bayesian model selection and updating
612 using monitoring data to select the best degradation model from multiple candidate models and
613 dynamically updating the model parameters; and (2) dynamic model uncertainty quantification
614 to automatically correct the assumed degradation model during the monitoring process [45].

615 As mentioned earlier, the framework presented in this work can be applied to other
616 structures with SHM systems installed where very little information about the deterioration

617 rate of a component or system exists, but abstracted inspection data based on ratings are
618 available. For example, this methodology can be used for other structural components of miter
619 gates with different failure modes (e.g., corrosion or pre-tension loss) or even other structures
620 including bridges, pavements and offshore structures due to the availability of inspection
621 ratings performed by several transportation and private agencies.

622

623 **6 Conclusions**

624 This paper presents a novel **hybrid** framework for failure diagnostics and prognostics for
625 bearing damaged gaps in the quoin block components of a miter gate. **This framework is based**
626 **on integrating abstracted inspection data and structural health monitoring data, with the**
627 **following information as inputs:**

- 628 • **Historical visual inspection data given in rating/discrete form;**
- 629 • **Previous knowledge of the human observation errors (i.e., P_{human});**
- 630 • **A validated physics-based simulation model of the system;**
- 631 • **A known damage threshold to predict the failure;**
- 632 • **Structural health monitoring data (e.g., strain in the present case) at different locations.**

633 This work is especially useful when the evolution of the damage mechanism is not well
634 known or understood either due to the lack of enough data that relates damage to sensor
635 information or the lack of a physics-based model that describes the evolution of the damage. It
636 is assumed that the only available data that describes the damage evolution are based on
637 abstracted rating assessments such as the OCA ratings. An approach is first proposed to map
638 the reported OCA transition matrix into the underlying true OCA transition matrix. Based on
639 that, the proposed framework successfully integrates a stochastic degradation model built from

640 the OCA Markov transition matrix and shows how this model is suitable for integration with
641 continuous monitoring.

642 The damage diagnosis via physics-based FE model updating using the degradation model
643 proposed provides satisfactory results. Also, to demonstrate the improvement on the gap length
644 prognosis, the updated over time RUL was compared against its true value. Results of a case
645 study show that (1) the proposed framework can effectively address the issue of human
646 reporting errors in the OCA ratings in the prognostics of miter gate, and (2) the uncertainty in
647 the RUL estimate can be reduced significantly using the proposed framework.

648 Note that, this approach can be applicable to different components in miter gates, which may
649 have different transition matrices values. However, further work needs to be done to extend this
650 methodology from miter gate components to the miter gate system level (e.g. including all critical
651 miter gate components); that work would need to focus on how failure mode probabilities from
652 multiple causes/sources are correlated and propagate towards a more global limit state failure
653 definition. In this paper, optimization-based methods are employed to identify the underlying
654 true OCA transition matrices as well as the gap growth model parameters. These procedures
655 can be integrated together in a full-Bayesian framework. The development of the full-Bayesian
656 framework and the investigation of other alternative approaches will be studied in the future.

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661 **Appendix A: Derivation of $\Pr\{I_t^{obs} = k, I_{t+1}^{obs} = q\}$**

662 The marginalization of $\Pr\{I_{t+1}^{obs} = q, I_t^{obs} = k\} = \Pr\{I_{t+1}^{obs} = q | I_t^{obs} = k\} \Pr\{I_t^{obs} = k\}$ is shown
663 as follows

$$\begin{aligned}
664 \quad \Pr\{I_{t+1}^{obs} = q, I_t^{obs} = k\} &= \sum_{i=1}^6 \sum_{j=i}^6 \Pr\{I_{t+1}^{obs} = q, I_t^{obs} = k, I_{t+1}^{tr} = j, I_t^{tr} = i\}, \\
&= \sum_{i=1}^6 \sum_{j=i}^6 \Pr\{(I_{t+1}^{obs} = q, I_t^{obs} = k) | (I_{t+1}^{tr} = j, I_t^{tr} = i)\} \Pr\{I_{t+1}^{tr} = j, I_t^{tr} = i\}.
\end{aligned} \tag{41}$$

665 According to the Bayesian network given in Fig. 5, it follows that

$$\begin{aligned}
666 \quad &\Pr\{(I_{t+1}^{obs} = q, I_t^{obs} = k) | (I_{t+1}^{tr} = j, I_t^{tr} = i)\} \\
&= \Pr\{I_{t+1}^{obs} = q | I_{t+1}^{tr} = j, I_t^{obs} = k\} \Pr\{I_t^{obs} = k | I_t^{tr} = i\}, \\
&= \frac{\Pr\{I_{t+1}^{obs} = q, I_{t+1}^{tr} = j, I_t^{obs} = k\}}{\sum_{w=k}^6 \Pr\{I_{t+1}^{obs} = w, I_{t+1}^{tr} = j, I_t^{obs} = k\}} P_{ik}^h.
\end{aligned} \tag{42}$$

667 Substituting Eq. (42) into Eq. (41) yields

$$\begin{aligned}
668 \quad &\Pr\{I_{t+1}^{obs} = q, I_t^{obs} = k\} \\
&= \sum_{i=1}^6 \sum_{j=i}^6 \left(\frac{\Pr\{I_{t+1}^{obs} = q, I_{t+1}^{tr} = j, I_t^{obs} = k\}}{\sum_{w=k}^6 \Pr\{I_{t+1}^{obs} = w, I_{t+1}^{tr} = j, I_t^{obs} = k\}} P_{ik}^h \right) \Pr\{I_{t+1}^{tr} = j, I_t^{tr} = i\}.
\end{aligned} \tag{43}$$

669 The following is obtained from the numerator of Eq. (6)

$$670 \quad \Pr\{I_{t+1}^{obs} = q, I_{t+1}^{tr} = j, I_t^{obs} = k\} = \Pr\{I_t^{obs} = k | I_{t+1}^{obs} = q, I_{t+1}^{tr} = j\} P_{jq}^h \Pr\{I_{t+1}^{tr} = j\}, \tag{44}$$

671 where $\Pr\{I_{t+1}^{tr} = j\}$ is solved in Eq. (9). Then, combining Eqs. (43) and (44) yields

$$\begin{aligned}
672 \quad &P_{kq}^R \Pr\{I_t^{obs} = k\} \\
&= \sum_{i=1}^6 \sum_{j=i}^6 \left(\frac{\Pr\{I_t^{obs} = k | I_{t+1}^{obs} = q, I_{t+1}^{tr} = j\} P_{jq}^h \Pr\{I_{t+1}^{tr} = j\}}{\sum_{w=k}^6 \Pr\{I_t^{obs} = k | I_{t+1}^{obs} = w, I_{t+1}^{tr} = j\} P_{jw}^h \Pr\{I_{t+1}^{tr} = j\}} P_{ik}^h \right) \Pr\{I_{t+1}^{tr} = j, I_t^{tr} = i\}.
\end{aligned} \tag{45}$$

673

674 **Appendix B: A stochastic crack growth model by Yang and Manning [27]**

675 A simple second order approximation for a stochastic crack growth model was proposed

676 by Yang and Manning [27], given by

677
$$\frac{da(t)}{dt} = X(t)Q(a(t))^w, \quad (46)$$

678 where Q and w are parameters that need to be estimated, and $X(t)$ is modelled as a stationary
 679 lognormal stochastic process with a unit mean and an auto-covariance function [27]

680
$$\text{cov}(X(t_1), X(t_2)) = \sigma_x^2 \exp(-\zeta_x |t_2 - t_1|), \quad (47)$$

681 in which σ_x is the standard deviation of $X(t)$, and ζ_x controls the correlation of $X(t)$ over
 682 time. If ζ_x^{-1} approaches to zero, $X(t)$ is a stationary lognormal white noise random process,
 683 and the degradation model achieves its most non-conservative stochastic performance. On the
 684 other hand, if ζ_x^{-1} approaches infinity, $X(t)$ is a lognormal random variable, and the model
 685 becomes the most conservative.

686 In this paper, a model that is similar to the Yang and Manning model is selected since it
 687 does not require a good understanding of the physics and maintains appropriate growth-law
 688 features at the same time. The model is given by

689
$$\frac{da(t)}{dt} = \exp(\sigma_i U(t))Q(a(t))^w, \quad (48)$$

690 in which $\sigma_i > 0$ is a degradation stage-dependent variable and $U(t)$ is a stationary standard
 691 Gaussian process with auto-correlation function given by

692
$$\text{cov}(U(t_1), U(t_2)) = \exp(-\xi |t_2 - t_1|), \quad (49)$$

693 where ξ is a correlation related parameter similar to Eq. (47). In addition, it is assumed that
 694 the degradation model $a_i = g(t, \theta)$ consists of N_d distinct degradation stages ($N_d = 5$ in the
 695 studied case). Thus, the multi-stage gap growth model is defined as

696
$$\frac{da(t)}{dt} = \exp(\sigma_i U(t))Q_i(a(t))^w, i = 1, 2, \dots, N_d, \quad (50)$$

697 where $a(t)$ is the gap length at time t , σ_i is a standard deviation variable of degradation stage
 698 i , and Q_i and w_i are degradation stage-dependent constants.

699

700 **Appendix C: Estimation of $\hat{P}(I_{j,t+1}^s | I_{i,t}^s; \theta, \mathbf{e})$ based on the simulation of gap growth**

701 As mentioned previously, $\hat{\mathbf{P}}(\theta) \triangleq \{\hat{P}(I_{j,t+1}^s | I_{i,t}^s; \theta), i = 1, 2, \dots, 6; j = i, \dots, 6\}$, for a given

702 $\mathbf{e} \triangleq \{e_1, e_2, \dots, e_{N_d-1}\}$, $\hat{P}(I_{j,t+1}^s | I_{i,t}^s; \theta, \mathbf{e})$ is given by

$$703 \quad \hat{P}(I_{j,t+1}^s | I_{i,t}^s; \theta, \mathbf{e}) = \frac{P(I_{j,t+1}^s \cap I_{i,t}^s; \theta, \mathbf{e})}{P(I_{i,t}^s; \theta, \mathbf{e})}, \quad (51)$$

704 where

$$705 \quad P(I_{i,t}^s; \theta, \mathbf{e}) = \begin{cases} \Pr\{0 \leq a(t) < \beta_i\}, & \text{if } i = 1, \\ \Pr\{\beta_{i-1} \leq a(t) < \beta_i\}, & \text{if } 1 < i < 6, \forall i = 1, 2, \dots, 6 \\ \Pr\{\beta_{i-1} \leq a(t) < \infty\}, & \text{if } i = 6, \end{cases} \quad (52)$$

$$706 \quad P(I_{j,t+1}^s \cap I_{i,t}^s; \theta, \mathbf{e}) = \Pr\{\beta_{i-1} \leq a(t) < \beta_i \cap \beta_{j-1} \leq a(t+12) < \beta_j\}, \quad (53)$$

$\forall i = 1, 2, \dots, 6; j = i, \dots, 6,$

707 in which $\beta_0 = 0$, $a(t)$, and $a(t+12)$ are obtained through the degradation model given in Sec.
 708 3.3.1, conditioned on given θ and \mathbf{e} , and $\beta_i = \infty$ or $\beta_j = \infty$ if $i=6$ or $j=6$. The two time steps
 709 used in Eq. (53) are t and $t+12$ since the inspection interval in the forthcoming case study is
 710 one year, and the unit of the time step of the discrete time degradation model (i.e., Eqs. (24)
 711 and (25)) is one month.

712 Since the inspection time t can be any time in the lifetime of the gate, Eqs. (51) through
 713 (53) are rewritten as follows

$$\begin{aligned}
& \hat{P}(I_{j,t+1}^s | I_{i,t}^s; \boldsymbol{\theta}, \mathbf{e}) \\
714 \quad & = \int_{t_l}^{t_u} \hat{P}(I_{j,t+1}^s | I_{i,t}^s; \boldsymbol{\theta}, \mathbf{e}, t) f(t) dt, \quad (54) \\
& = \int_{t_l}^{t_u} \frac{\Pr\{\beta_{i-1} \leq a(t) < \beta_i \cap \beta_{j-1} \leq a(t+12) < \beta_j\}}{\Pr\{\beta_{i-1} \leq a(t) < \beta_i\}} \frac{1}{t_u - t_l} dt,
\end{aligned}$$

715 where $f(t)$ represents the distribution of the time duration of interest. This distribution is
716 assumed as a uniform distribution bounded by t_l and t_u , which are respectively the lower and
717 upper bounds of the time duration of interest.

718 In general, Eqs. (54) is analytically intractable due to the complicated transition between
719 stages, even though several analytical expressions have been developed for the degradation
720 model with only one stage based on assumptions and simplifications [27]. In this paper, a
721 simulation-based method is employed. For a given $\boldsymbol{\theta}$ and \mathbf{e} , the degradation of the gap is first
722 simulated using the discrete-time model given in Eqs. (24) and (25). From the simulation, the
723 samples obtained of the gap length are denoted as
724 $\mathbf{a}_s(\boldsymbol{\theta}, \mathbf{e}) \triangleq \{a_{i,j}, i = 1, 2, \dots, n_{MCS}; j = 1, 2, \dots, N_t\}$, where $a_{i,j}$ is the i -th realization of the gap
725 growth curve at time step t_j , n_{MCS} is the number of samples at each time step, and N_t is the
726 total number of simulation time steps. Based on the simulated samples of the gap growth, Eq.
727 (54) is approximated as

$$728 \quad \hat{P}(I_{j,t+1}^s | I_{i,t}^s; \boldsymbol{\theta}, \mathbf{e}) \approx \frac{1}{N_t - 12} \sum_{k=1}^{N_t-12} \frac{\Pr\{\beta_{i-1} \leq a(t_k) < \beta_i \cap \beta_{j-1} \leq a(t_k + 12) < \beta_j\}}{\Pr\{\beta_{i-1} \leq a(t_k) < \beta_i\}}. \quad (55)$$

729 In the above equation, $\frac{\Pr\{\beta_{i-1} \leq a(t_k) < \beta_i \cap \beta_{j-1} \leq a(t_k + 12) < \beta_j\}}{\Pr\{\beta_{i-1} \leq a(t_k) < \beta_i\}}$ is estimated using \mathbf{a}_s as

$$\begin{aligned}
& \frac{\Pr\{\beta_{i-1} \leq a(t_k) < \beta_i \cap \beta_{j-1} \leq a(t_k + 12) < \beta_j\}}{\Pr\{\beta_{i-1} \leq a(t_k) < \beta_i\}} \\
730 \quad & \approx \frac{1 \sum_{q=1}^{n_{MCS}} \Lambda((\beta_{i-1} \leq a_{q,k} < \beta_i) \cap (\beta_{j-1} \leq a_{q,k+12} < \beta_j))}{n_{MCS} \sum_{q=1}^{n_{MCS}} \Lambda(\beta_{i-1} \leq a_{q,k} < \beta_i)}, \quad (56)
\end{aligned}$$

731 where $\Lambda(E)$ is an indicator function such $\Lambda(E) = 1$ if event E is true and $\Lambda(E) = 0$ if event

732 E is false. In the above equation, event E represents $(\beta_{i-1} \leq a_{q,k} < \beta_i) \cap (\beta_{j-1} \leq a_{q,k+12} < \beta_j)$

733 and $\beta_i \leq a_{q,k} < \beta_{i+1}$.

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