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Algebraic Expression: A Case Study of Mathematical Thinking in the Adult Education Classroom

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in Education

by

Joy Zimmerman

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ABSTRACT OF THE DISSERTATION

Algebraic Expression: A Case Study of Mathematical Thinking in the

Adult Education Classroom

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Professor Noreen M. Webb, Co-Chair

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Knowledge and information are the foundations for economic activity, thus current legislation is designed to strengthen and improve the nation's public workforce system. Despite the heightened visibility of adult mathematics in surveys of adult skills like PIAAC and the implementation of College and Career Readiness Standards for Adult Education, our institutions are failing to meet the needs of adult learners. Recent efforts call for classroom environments where students have opportunities to reason and construct viable arguments. To this end, a descriptive single case study of a collaborative adult education mathematics classroom was

undertaken. This study contributes to our understanding of adult learning in the mathematics classroom. It provides a perspective on how knowledge is constructed through interaction and the opportunities available to students to explain and engage with others' ideas. This study provides an example of how one classroom, centering adults' mathematical thinking, developed over time.

The dissertation of Joy Zimmerman is approved.

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CHAPTER 1: INTRODUCTION

The introduction to this study begins by means of two of my former adult education students, George and Pablo Jesus, and their experience in adult education. This chapter will define and summarize the problem facing adult education programs, teachers, and their students. After describing the challenges students face without a high school credential and the structures that make it challenging to obtain one, this chapter will position adult mathematics as an opportunity, not a barrier to student success.

George, a quiet 35-year-old with cropped hair and heavy shoulders, arrives five minutes early to an adult education mathematics classroom and slips into the seat nearest the door. His recent success in a weight-loss program builds his confidence and prompts his return to adult school. Greg currently collects unemployment and shares with the class that he seeks a recession-resistant career and aspires to be a phlebotomist or lab technician. As a former employee of a company that produced blinds, he is familiar with productivity targets, rate of production, and geometric measurement. However, Greg admits that formal mathematics makes him anxious and describes himself as someone who "cannot do algebra."

Pablo Jesus, a 30-year-old, cheery, round-faced man from Guatemala, enters the same classroom and chooses a seat near the window. Pedro completed sixth grade in his country and departed school to help support his family at the request of his father. As an adolescent, he served as the town hearse by transferring corpses on horseback to the church for burial, an especially challenging task during the winter's rainy season. Pedro immigrated to the United States as a teenager to pursue his U.S. citizenship and a higher standard of living for himself and his family back home. Pedro has a firm grasp on the Pythagorean relationship and measurement due to his roofing and drywall experience, yet algebra and word problems give him pause.

Adult learners' experiences reflect systemic issues that cost the economy billions. George and Pablo Jesus are two of the over 34 million adults in the United States who do not have a high school diploma (U.S. Census Bureau, 2015) and every year they are joined by an additional three million youth who drop out of school (Belfield, Levin, and Rosen, 2012). On average, adults without a high school diploma, like George and Pablo Jesus, earn almost $10,000 less per year than those with a diploma, are more than twice as likely to be unemployed, are three times as likely to live in poverty than adults with some college and are four times more likely to report poor health than adults identified as highly proficient (US Department of Labor Bureau of Labor Statistics, 2015). Moreover, adults without a high school diploma comprise disproportionately higher percentages of the incarcerated and welfare populations and cost the economy approximately $266,000 per individual over a lifetime (Levin and Belfield 2007; Pleis, Ward, and Lucas 2010).

Greg and Pedro Jesus' desire to pursue adult education places them among the one million adults who enroll in adult basic education (ABE) and adult secondary education (ASE) programs annually in community colleges, faith-based organizations, and other not-for-profit organizations (NCES, 2014). The two men aspire to join the 560,000 annual GED recipients and aim to take advantage of the benefits that a high school equivalency affords them, such as increased wages, access to financial aid, higher future earnings and life satisfaction, lower levels of depression and substance abuse, and expanded opportunities in the labor market. (Ou 2008; Song and Hsu 2008; Heckman, Humphries, and Mader, 2010; GED Testing Service, 2014). However, even with the prevalence of adult education programs, there is no assurance that students like George and Pablo Jesus will successfully obtain an alternative high school credential.

The picture in "second chance" programs may not be an encouraging one for George and Pablo Jesus. First, there is a high rate of attrition. Thirty percent of adult learners leave math classes within the first three weeks (National Council of State Directors of Adult Education, 2014). In California, 35 percent of adult learners did not qualify for the National Reporting System (NRS) federal reporting, which restricts data reported to the Department of Education to learners who persisted for more than 12 hours (California Department of Education, 2011). That is, *one-third* of adult learners exited their programs within the first *12 instructional hours*. We also know that high rates of attrition, commonplace in adult education, impacts instruction and student success (Comings, 2008; Comings & Cuban, 2007; Quigley, 1995; Dirkx & Jha, 1994; Beltzer, 1985). Thus, barriers to participation have been extensively researched and generally can be ascribed to one of three categories: situational barriers such as day-to-day personal circumstances, dispositional barriers, which include self-perceptions, and institutional barriers, which encompass an institution's policies and procedures (Cross, 1981; Quigley, 2006). Barriers impeding adults' success in education include prohibitive fees, low confidence, negative past experiences, difficulty navigating enrollment, balancing life roles, caregiving, financial limitations, language barriers, and limited course schedules (see Chao & Yap, 2009; Fike & Fike, 2008, King, 2002; Patterson et al., 2010; Quigley, 2006; Quigley & Uhland, 2000; Reder, 2007; Reder, 1999; Ross-Gordon, 2011; Spellman, 2007).

Second, to the extent that information is available, the picture of instruction within adult education is grim. Adult math education relies on teacher-centered pedagogies, self-paced learning materials, a prescriptive curriculum, few opportunities for students to interact with others, and little or no opportunity for students to bring in or take advantage of the wealth of knowledge, skills, and experiences they bring to the classroom (Gunby, 2012; Misko, 1994). Adults bring lived experiences and a wealth of prior knowledge to the classroom, thus effective instruction must be responsive to these experiences (Knowles, 1980); however, the traditional curriculum does not address or take advantage of the mathematically relevant life experiences and knowledge that adults bring into the classroom. Moreover, instructional practice is constrained by commercial publications and standardized tests. Adult educators are aware of the need to "change the way math is actually taught and learned in an adult literacy community," but little is known about what that looks like (Brover, Deagan, and Farina, 2005).

Third, even for students who manage to complete the required course(s), success rates on proficiency tests are low. For example, of those who met NRS requirements in California, 60 percent completed an Education Functioning Level (EFL), and only 38 percent advanced one or more levels (California Department of Education, 2011). Passing rates for the three High School Equivalency (HSE) vary by exam. Students who opt for Educational Testing Service's HiSet pass at a rate of 59.8%; and those who select the GED and HiSet exams pass at a rate of 75.7 percent and 57.7 percent respectively (McFarland, Rathbun, & Holmes, 2018). Passing rates for the test battery also differ. For example, the GED battery pass rates for 2017 are as follows: Math (79%), Reading Language Arts (85%), Science (91%), and Social Studies (90%). Today, approximately 75 percent of GED completers pass the test, up from 59 percent in 2014 (GED Testing Service, 2017). It is not uncommon for testers to repeat the entire battery or specific subject tests to obtain a satisfactory passing score. Adult educators are well aware that the mathematics subject test presents the greatest barrier to an HSE credential. In 2006, 54 percent of all repeat examinees retested in Mathematics compared to the lowest re-test subjects of Science and Reading at 26 and 28 percent, respectively; and compared with all subject areas, more candidates tested three or more times in Mathematics (Zhang & Patterson, 2010).

A central tenet of adult education and one espoused by the Organization for Economic Cooperation and Development (OECD) is to promote the development of knowledge and competencies to enable adults to actively participate in a globalized economic life (Evans, Wedege, & Yasukawa, 2013). To better understand how education and training systems can nurture these skills and to measure and compare basic skills and competencies among adults in the U.S. and other nations, the Organization of Economic Cooperation and Development developed the Program for the International Assessment of Adult Competencies (PIAAC). The survey assesses adult numeracy, or the ability to "*access, use, interpret, and communicate mathematical information and ideas, to engage in and manage mathematical demands of a range of situations in adult life,*" and locates adults' "skills levels" in the global context (PIAAC Numeracy Expert Group, 2009, pp. 20).

The results exposed the United States for failing to meet the educational needs of its adult citizens (OECD, 2013a). In the first cycle of PIAAC, the United States ranked 21st in numeracy out of 25 countries and below the international average in numeracy. Of note, U.S. adults perform similarly to Israel, a country that invests *half* of the annual K-12 per-pupil expenditure of the United States (National Center for Education Statistics, 2012). The U.S. had the highest percentage of participants (15.8%) scoring *below* the minimum proficiency level the OECD identified as able to succeed with simple problem-solving tasks encountered in daily life and one in three adults demonstrated weak numeracy skills (OECD, 2013). The stark survey results reflect a growing numeracy problem and one that is exacerbated by the demands of a technology-rich work environment and a nation's need to remain economically viable. Although this appears to be an adult education problem, some see it is very much a K-12 issue as well, as the 15-year-olds identified as underperforming by the Program for International Student Assessment (PISA) of today are the adult education students of tomorrow. The emphasis on the mounting need for adult proficiency in mathematics is undeniable; however, the focus on outcome locates the problem of adults' underachievement with the individual is problematic. Media reports of rising unemployment, low test scores, under-preparedness for contemporary jobs and careers, high attrition and drop-out rates, and the slipping U.S. status abroad fuel the fire that adults are deficient in the mathematical skills demanded by the workplace and society at large.

To solve the adult numeracy "problem," the OECD leverages the PIAAC survey results to disseminate practices that influence and legitimate policy intervention in the United States. Surveys like PIAAC are based on a competency model of knowledge, thus advancing a very narrow notion of competence (Tsatsaroni & Evans, 2014). The conceptualization of adult numeracy as competency places the onus on the individual for their own education. Adults are responsible for retraining, *up-skilling*, or acquiring knowledge and bear the burden when the focus of learning shifts from what one *knows* to what one can *do* (Cummings et al., 2019; Goodman et al., 2015; Moore with Jones, 2007). The policymakers and school reformers adopt these deficit framings and aim to repair and prepare broken adults to take their place in the workforce rather than build on the vast knowledge they bring to the classroom. Thus, students like George and Pablo Jesus will likely encounter lessons that convey factual information and recall in classes that emphasize discrete skills instruction (Bedar & Medina, 2001). Moreover, they will likely be exposed to prescribed forms of knowledge with limited opportunities to access mathematical principles that conceptual forms of knowledge can provide.

Researchers have identified numerous reasons that limit participation outside the classroom; however, little is known about what transpires *inside* the classroom and how that may impede or promote adult learning and persistence. A potentially valuable contribution was put forth by Christophel & Gorham (1995) and suggested that the teacher-student relationship in the first three weeks is critical to sustain motivation; thus, “motivation is modifiable” (p. 304). While this aligns with adult learning theory which indicates adult learners enroll with sufficient motivation to succeed (see Knowles, 1998), it also suggests that classrooms can “demotivate” the adult learner (e.g., Bean et al., 1989; Diekhoff & Diekhoff, 1984). Thus, indicating that motivation is not solely the students’ problem and, to some degree, may depend on what transpires in the classroom. While the adult motivation literature is beyond the scope of this dissertation, it does suggest that teachers *can* do something in their classrooms to retain adult learners.

In this dissertation I open the classroom door to investigate what the teacher can do to support learning. Specifically, I investigate the intersection of policy, curriculum, and context in the form of classroom interaction and what it means to actively participate in an adult mathematics classroom. Understanding how adult learners experience classroom processes involves understanding the ways they explain their ideas and engage with others’ ideas and how the teacher supports them to make sense of the mathematics. Examining classroom interaction around students’ mathematical ideas and how it is constructed in the adult education setting may illuminate how adults participate and the learning that transpires in the adult classroom.

CHAPTER 2: REVIEW OF RELEVANT LITERATURE

Introduction

Recent adult education mathematics reform efforts call for classrooms where students, as members of a community of learners, can reason and construct mathematical understandings. This chapter presents the research that forms the foundation of this study of adult learning. An investigation of adult classroom participation is grounded in several intersecting bodies of literature that situate adult teaching and learning of mathematics within sociocultural structures of schooling. I first situate the amorphous field of adult mathematics. Then I discuss adult's mathematical thinking at school and in the workplace. This is followed by an exploration of the foundations of adult learning and a theoretical perspective centering the social nature of learning.

The Field of Adult Mathematics

Adult mathematics education is situated in a field of lifelong learning, vocational education, continuing education, and career and college readiness. It encompasses a range of ideas from equal opportunity to self-actualization and has struggled to define itself in a research field dominated by studies of children's education (Field & Leicester, 2000). The participants are equally diverse. The adult learner is defined not by the level of mathematics being studied but by the diverse student themselves. A retired restauranteur with a love of lifelong learning sits adjacent to a 21-year-old trying to acquire skills for a better paying job and a 35-year-old mother who wants to learn mathematics to support her primary-aged son. 'Adult' refers to people who start, resume or continue their education in formal, informal, or non-formal settings, beyond compulsory schooling age in their respective societies (ICME, 2015). For some, adult education may be their first formal learning experience; for others, there may be a break of years or decades since their last math class. Some return by choice to be a role model for their children or prove to a loved one or themselves that they can succeed, some return with their sights on continuing education and training, and others enroll to fulfill a requirement for a rehabilitation program. The inescapable reality for those working with adults is that mathematics is perceived as difficult to learn, associated with failure, considered a gatekeeper to advanced education, positioned as an obstacle to job advancement, perpetuates inequality in society, and can evoke negative emotions (O'Donoghue, 2000).

Adults require a range of mathematical skills to navigate the demands of an information-rich culture, such as skills to interpret productivity information at work, compare cell phone plans, and follow instructions to assemble a Dyfjord from Ikea. Numeracy is a vital skill and one that highly correlates with economic success (Murnane, Willet, & Levy, 1995). Historically, legislative actions focused on expanding and improving adult basic skills in the United States stress the importance of literacy, with numeracy sporadically mentioned or omitted entirely from public policy (Tout & Schmitt, 2002). When numeracy is subsumed by literacy, the possibility of identifying resources to address inequities between those who are less and those who are more numerate is diminished.

The field of adult mathematics is diverse and requires a broad conception of mathematics beyond classroom mathematics, including specialized mathematics in higher education, school mathematics, vocational mathematics, street mathematics, mathematics for everyday living, and adult numeracy (FitzSimons et al., 2003). The field of adults learning mathematics has been likened to a moorland located at the intersection of social sciences, adult education, and mathematics education (Wedege et al., 1999). Coben (2006) identifies two areas in the expansive and uncultivated domain: mathematics teaching and learning at any level and mathematics in the social context, or numeracy.

The frameworks and theories and conceptions of numeracy are compelling, but of great interest to adult educators is whether they can identify characteristics of effective instruction. The research on instruction and adults' mathematical learning has not kept pace with theory development or conceptualizations in numeracy and thus limited and at its early state of inquiry (Condelli, 2006). Current research on adult learning reflects a "haphazard and unorganized approach…not guided by any theory, approach or school of thought about good pedagogy" (Condelli & Wrigley, 2004, p. 22 in Condelli, 2006).

Adults' Mathematical Thinking

Research on learning mathematics has largely focused on children's learning. Much of the literature may be relevant to adult learners of mathematics; however, findings may require some interpretation. The field of adults learning mathematics joined its oft-researched numeracy cousin as a topic of interest only 30 years ago. In the late eighties, Brookfield's (1986) comprehensive book on adult learning did not include a single entry under mathematics or numeracy. Theoretical insights were few, and the field failed to develop empirical research about how adults learn mathematics. By the mid-nineties, the well-regarded Basic Skills Agency bibliography of basic skills research (BSA, 1994) devoted only 14 out of 300 pages specifically to numeracy. At the same time, the public interest and research tides turned as the Adult Numeracy Practitioners Network (ANPN) held its first annual meeting in the U.S., and the international forum, Adults Learning Mathematics, commenced in the United Kingdom. In 2000, Iddo Gal edited a volume titled Adult Numeracy Development Today: Theory, Research, Practice, a volume identifying a set of frameworks and approaches to developing and researching numeracy-related knowledge, thinking processes, and dispositions (Gal, 2000). Today, there are numerous publications and professional organizations devoted to the field of adult mathematics including the recent emergence of an adult numeracy strand at the quadrennial International Congress on Mathematical Education (ICME).

Tout and Schmitt's (2002) survey of the literature of adult numeracy education revealed a "dramatic absence" of attention paid to developmental mathematics in adult basic education (p. 17). Specifically, they searched the ERIC database for articles published in the United States concerning mathematics education and adult basic education. They found approximately three thousand articles related to each field, yet only nine related to mathematics education of adults in basic education. Of these, only one study addresses how adults think mathematically; the remainder of the articles address non-cognitive topics, such as math anxiety.

The authors of the sole article related to adult learning interviewed and recorded 60 adult students to assess their informal knowledge of percent and its relationship to formal mathematics (Ginsburg, Gal, & Schuh, 2000). The authors coded explanations in response to four percent tasks (explanatory, shopping, visual, and computation) as appropriate or inappropriate; numerical responses as correct, in the ballpark, or incorrect; and solution strategies were categorized. Ginsburg and her colleagues found that nearly all adults in the study, including those with low levels of achievement, demonstrated some conceptual understanding of percent upon entering their programs; however, many showed gaps in their knowledge of percent. Moreover, groups of students with lower achievement performed better on mental and visual tasks than computational tasks. They hypothesize that adults develop verbal associations through repeated encounters with percent within everyday discourse (e.g., 100% pure orange juice, 50% off sale). As a result, they have not fully grasped the underlying mathematical principle in each context and across different contexts.

Adding to this idea, Coben (2000) argues that adult learners operate on a mixture of formal mathematics remembered from the classroom and mathematical understandings derived from adult experiences outside of formal schooling. She likened fragmented adult mathematical knowledge to knowledge of a city acquired by people who travel primarily by underground metro and do not know how the stations on the surface relate to one another. Traditionally compartmentalized instruction combined with equally compartmentalized adult knowledge may be a recipe for disaster. Adult learners who bring specific knowledge of various "stations" to the classroom may not have opportunities to share their expertise with classmates in this framework.

Researchers have found that adult students enrolled in a mathematics course with problems based on a "real world" context had improved mathematical confidence and confidence with everyday tasks (Miller-Reilly, 2000; FitzSimons, 1994; Lehmann, 1987). This suggests that adults may be unaware of the extent of the mathematical knowledge they bring to the math class. Wedege and Evans (2006) constructed three positions adults frequently take around mathematics: "I am not here to learn mathematics; Mathematics – that's what I cannot do; and No, I don't use mathematics at work" (p. 33-34). The learner attitudes highlight the perceived differences between formal and informal mathematics. The latter two positions suggest that once adults have succeeded in applying mathematics, it becomes common sense, not considered 'math,' and thus devalued. Benn (1999) argues that knowledge of mathematics is socially powerful and, unlike common sense, carries power and prestige. Conversely, individuals who rely on common sense mathematics are perceived as socially inferior to those who can explicitly do mathematics (Coben, 1999). Coben (2000) refers to the math adults can do but is not perceived as invisible mathematics (Coben, 2000).

Programs that make the invisible visible have been met with some success. Manly and Ginsburg (2010) have determined that integrating algebraic thinking into adult math instruction and relating algebra to adults' intrinsic, real-world knowledge will enhance understanding in adults and reduce math anxiety. They argue, "algebraic thinking can be a sense-making tool that introduces coherence among mathematical concepts for those who previously have had trouble learning math" (p. 13). It is only recently that the field of adult education has acknowledged the importance of sense-making.

Informal mathematics learning at work

Canario (2008) posits, "the exercise of the work is, in itself a producer of competencies" (p. 29). Learning mathematics in the workplace is informal, often overlooked, and occurs in work-related activities such as training courses or on-the-job skills development. It encompasses social, personal, and cultural knowledge and requires the ongoing need for communication as information, often mathematical, is sought and shared between those in the workplace (Eraut, 2004; FitzSimmons, 2013). Unlike school-based mathematical tasks, a workplace task includes context-specific numbers and units (dollars, pounds, minutes), can have multiple approaches and solutions and requires collaboration (Wedege, 2002b).

This act of recontextualization combines mathematics knowledge with contextual, personal, and sociocultural knowledge in the form of communication (Sfard, 2017). Learning mathematics at work requires working with available tools and employing multimodal communication, including symbols, diagrams, speech, and gestures (Björklund Boistrup & Gustafsson, 2014). As adults adapt to changing technology, tools, and processes, it requires more sophisticated problem-solving skills, interpretation of data, statistical knowledge (FitzSimmons, 2019). Workplace problem-solving is not static; instead, it is often governed by changing monetary, temporal, and legal constraints; addresses a specific need or purpose, relies on two-way communication depending on who holds the information and the authority of the actors, and always has practical consequences (FitzSimons & Boistrup, 2017; Wedege, 2002b). FitzSimmons (2019) argues that the accumulation of mathematical rules is not sufficient for adult learners in the workplace to develop new knowledge. Rather, adults must have access to and build on the integrating structures of the mathematics discipline to solve problems in their local workplace contexts. For example, Madison and Steen (2003) found, based on surveyed employers, that:

> Work-related mathematics is rich in data, interspersed with conjecture, dependent on technology, and tied to useful applications. Work contexts often require multi-step solutions to open-ended problems, a high degree of accuracy, and proper regard for required tolerances. None of these features are found in typical classroom exercises (p. 55).

In the workplace, mathematics is both explicit and transparent. Despite the impact of changing technology in twenty-first-century workplaces, other research shows that mathematics is becoming increasingly invisible (Marr and Hagston, 2007; Straesser, 2015; Thumpston & Coben,1995; Wedege, 2010). Adults feel workplace competence without awareness of the role mathematics plays in their activities (Wedege, 1999; Wedege & Evans, 2006); mathematics encountered in the workplace is viewed as common sense while the math they could not do was mathematics (Coben & Thumpston, 1995; Coben, 1997). Thus, adults fail to recognize that their everyday competencies do not count as mathematics (FitzSimmons, 2002) and are often embedded in professional knowledge and experiences (see Hoyles et al., 2002). Workplace mathematics can be obscured by modern technology, and applying formal, school mathematics to the workplace is not straightforward. For example, Hoyles, Noss, and Pozzi (2001) studied proportional reasoning in nursing. They found that the formulas used by nurses in the workplace differed from the algorithm taught in their training program. Moreira and Pardal (2012) examined masons' use of geometry and arithmetic and found mathematics embedded in their daily practices. Saló I Nevado, Holm, and Pehkonen (2011) examined farmers used various measuring devices to allocate space in a barn to feed calves. They found that the farmers' use of spatial reasoning and number sense allowed them to solve problems in complex situations creatively. Salo i Nevado & Pehkonen (2018) found that cabinetmakers utilized measurement and transformations in their work without labeling it as mathematics. It is evident that mathematics is embedded in diverse workplaces and includes not only arithmetic (Williams & Wake, 2007) and simple algorithms (Hoyles et al., 2001), but proportionality, approximation, basic geometry (Greiffenhagen & Sharrock, 2008).

To many adults, mathematics is perceived to have minimal involvement in their daily practices. For example, estimating the number of bees per hectare and the number of bee visits is made in situ and woven into an individual's numeracy practices (Kane, 2018). Others do not view these practices as 'legitimately' mathematical (Cockcroft, 1982; Coben; 2006). The individual's proximity to their own situation and intimate knowledge with the specific context of their practice accounts for the missing mathematical detail (Noss, Hoyles, & Pozzi, 2000). Adults may draw on embedded mathematic practices only when required. Niss (1994, p. 371) refers to this as the 'relevance paradox' where people fail to see the mathematics they need, yet the "social significance of this mathematics" takes place in "ever increasing scope and density."

Kane proposes "a broad dynamic flexi-model of numeracy practices that bends, twists and responds to each relevant situation" (p. 36).

Gal (2000) elaborates on the issue related to the acquisition of functional (e.g., workplace) versus formal (e.g., school) knowledge. Most school mathematics focuses on simple generative tasks requiring students to manipulate numbers, quantities, or items and generate new ones, and have a correct answer that the teacher can verify. However, numerate situations that arise in the workplace are not the simulated work-related story problems found in classrooms and, as such, cannot be resolved with strategies categorized as right or wrong. Rather, when faced with a numerate situation in context, adults manage them by selecting from multiple courses of action based not only on the type of data or quantitative information available but on resources, consequences, and personal goals and dispositions. Moreover, adults may ask various questions to gather information to help them decide the best course of action. Thus, adults presented with computational problems in classrooms may sacrifice precision for estimation (estimating a tip), overestimate quantities required for a task to build a margin of safety, or delegate responsibility by asking for help (p. 17). Adults may address situations through computationally inefficient means or invented procedures but manage situations based on what is acceptable and reasonable within the context of the situation and ways that make sense to the individual. The author calls attention to a gradual shift in mathematics instruction from teaching procedures and math facts to problem-solving, reasoning, communication, and group processes and advocates for students spending more time "learning what mathematicians do (e.g., conjecture, experiment, check hypotheses, verify results, explain) rather than what mathematicians know (e.g., number facts, computational rules, formulas, proofs)." (p. 13).

Framing Adult Learning

Nearly a century ago, research on adult education focused not on how adult students learn but on whether adults could learn (Thorndike et al., 1928). Approaching adult learning from this behavioral perspective resulted in comparisons of intelligence with children based on timed tests, which led to conclusions that younger meant better. By the 1950s, researchers added problem solving and cognitive development to studies of intelligence, but artificial settings and contexts shared with children made insights questionable. The drive to differentiate adult learning led to the theory-building efforts of Knowles, Tough, and Houle (Merriam, 2001).

Malcolm Knowles' (1980) concept of andragogy refers to the method and practice of adult learning and is often contrasted with pedagogy, the method and practice of children's learning. The six assumptions underlying andragogy distinguish the adult learner from the school-age learner: 1) independent self-concept and capable of self-directed learning, 2) accumulation of life experiences as a resource for learning, 3) readiness to learn orients to changing social roles, 4) immediate application of knowledge resulting in a shift from subject- to problem-centeredness, 5) internal motivation to learn, and 6) adults need to know why they need to know something (Knowles et al., 2005). After much criticism, debate, and discussion, Knowles acknowledged that andragogy is more a conceptual framework than a theory (Knowles, 1989) and defined by the learning situation rather than the learner. Thus, Knowles revised his position of andragogy and pedagogy as situated on a continuum of teacher-directed to student-directed learning rather than placed in opposition to one another with the appropriate instructional approach located somewhere between. Some have criticized his work as atheoretical or challenged the core principles, while others seek to validate it (Grace, 1996; Houde, 2006; Norman, 1999; Merriam, Caffarella, & Baumgartner, 2007; Pratt, 1993). Despite the research that agreed or conflicted with Knowles' criteria, andragogy still serves as the basis for much of the work in adult education and "will continue to be the window through which adult educators take their first look into the world of adult education" (Pratt, 1993, p. 21 in Merriam, 2001).

Knowles (1975), Houle (1961), and Tough (1967, 1971) introduced another model that further served to distinguish adult learners from children – self-directed learning (SDL). Knowles (1975) describes it as follows:

> a process in which individuals take the initiative, with or without the help of others, in diagnosing their learning needs, formulating learning goals, identifying human and material resources for learning, choosing and implementing appropriate learning strategies and evaluating learning outcomes (p. 18).

Tough's (1979) seminal work draws attention to individuals learning with intent and taking the initiative to gain knowledge or initiate a change in themselves. Thus, the goal of adult education is to foster the development of a learner's capacity to be self-directed. Critics of SDL challenge Knowles' notion that all adults are self-directed and suggest that some may be more self-directed than others (Candy, 1991; Merriam & Bierema, 2014). Various SDL models evolved to include not only the learner, but the context and nature of learning, critical reflection, social and political action, and learning processes (Brookfield, 1986, 1993; Collins, 1996; Danis, 1992; Mezirow, 1985). Both SLD and andragogy served as a foundation of adult learning theory and helped shape adult education's identity; however, both have been criticized for "the blinding focus on the individual learner while ignoring the sociohistorical context in which it occurs" (Merriam, 2001). Adult learning theories have emphasized the individual with little regard for the ways individuals shape and are shaped by society. This narrow focus on individual learning rather than its situated nature has shaped the field of adult education to include educators with "diametrically opposed social purposes: from progressive educators seeking social change to technicians seeking to adapt learners in a rigged game of social immobility" (Heaney, 1995, p. 1). Theorists including Knowles (1980), Cross (1981), Kolb (1984), McClusky (1963), and Mezirow (1990) offer important insights into adult learning; however, they did not address a vital component of adult learning – the social and interactive dimensions. The interaction between instructor and student and among students is essential for creating meaning.

Adult Learning as a Social Endeavor

Merriam and Caffarella (1999) contend, "Adult learning does not occur in a vacuum" (p. 22). This differs from behaviorist perspectives, where learning is unchanging and transitive and happens inside the brain, independent from the experience and the context of the learning situation (Brown & Duguid, 1996). Extant research has established the importance of classrooms to shape student participation, learning, and development. In this study, the social setting of the adult classroom is of interest. Classrooms involve multiple participants actively interacting over time to achieve specific goals (Blumenfeld, 1992; Pianta & Allen, 2008; Tseng & Seidman, 2007). Adult education students are unique in that they do not reside on campuses like adults in higher education, nor do they have access to school-sponsored extracurricular activities like their K-12 counterparts. Classrooms may constitute the entirety of an adult student's educational experience, which positions the classroom as a critical context to understand. Thus, what happens inside adult education classrooms matters.

This dissertation adopts the view that learning is not just an individual process; it is a social one. From a sociocultural perspective, learning emerges from social interactions; hence, the theory focuses on the role of participation in culturally arranged activities (Vygotsky, 1978).

While Vygotsky focused on the development of children, sociocultural theory has been extended and applied to adult education.

Lave and Wenger (1991) refuted individualist views of learning in favor of a social and contextual approach. Central to this perspective is that all learning is situated in relation to social practice – the interaction between and among learners, their actions, and their worlds. They examined issues of transfer and developed the issue of situated learning. Learning, according to the authors, is "legitimate peripheral participation in communities of practices." The concept describes learning as it occurs through participation within a community of practice. Participation begins on the edge or periphery as new community members interact with more experienced members and may choose to move from a peripheral level to a more central level and a potential source of learning for others. The community provides the context for the social production of knowledge and sense-making. Moreover, learning occurs through discourse when the community in which the discourse takes place has meaning. To Wegner, meaning is always the product of its negotiation and exists in the process. As Wenger states, "meaning exists neither in us, nor in the world, but in the dynamic relation of living in the world" (p. 54). To situate learning is to place thought and action in a specific place and time and to involve others, the environment, and activities to create meaning. In other words, the learning process takes place in "situated activities." "Learning is a much more complex phenomenon than can ever be limited to a classroom. It is inextricably connected to how we live our lives, and to the excitement, challenge, motivation and support woven through our daily experience" (Argyris, 1994, p. 46).

In Lave's (1988) ethnographic study, she observed adults who were taught "school" calculations to solve problems and discovered that although they could not solve problems in a classroom, they could solve multifaceted problems while grocery shopping. She found the interactions with shoppers and store workers provided the social context, and the coupons and grocery items became the tools. Lave (1996) contends that it is not enough to "add situated contexts to learning experiences," rather "a more promising alternative lies in treating relations among people, tools, activity as they are given in social practice" (p. 7). That is, real-world contexts, like the grocery store, make the best learning environments. For example, Gillies (1991; 1994) studied nurse trainees coping with required mathematical skills and strongly advised hands-on experience before teaching formulas for drug dosage calculations. The hospital's real-world context provided opportunities for nurses to interact around the mathematics in their specific social situations.

This dissertation grounds this research in learning that is social in nature and considers the role that community plays in learning. Learning requires active rather than passive participation and interaction with others where different perspectives are incorporated (Jonassen et al., 1995; Pea, 1993). Thus, learning environments that encourage active participation, interaction, and dialogue provide students the opportunity to construct knowledge and create new meaning from their experiences (Jonassen, Davison, Collins, Campbell, & Bannan Haag, 1995). Dialogue serves as an instrument for thinking as learners explain, clarify, elaborate, and defend their ideas (Brown & Palinscar, 1989; Jonassen et al., 1995; Norman,1993), and through these cognitive processes, we engage in meaning-making (Brown & Palinscar, 1989; Jonassen et al., 1995; Norman,1993).

For Sfard (2017), cognition and communication are fully integrated and conceptualized mathematics as a "well-defined form of communication" (p. 42). She continues, "to think mathematically means communicating – with others or with oneself" in the ways promoted by the mathematical community. Therefore, collaboration serves as a powerful agent of mathematics development.

In adult education, to situate learning means to "create the conditions in which participants will experience the complexity and ambiguity of learning in the real world" (Stein, 1998, p. 1). That is, situated learning is knowledge gained and applied to daily situations. The four premises in the adult teaching and learning process include:

(1) Learning is grounded in the actions of everyday situations;

(2) Knowledge is acquired situationally and transfers only to similar situations;

(3) Learning is the result of social process encompassing ways of thinking, perceiving, problem solving and interacting in addition to declarative and procedural knowledge; and

(4) Learning is not separated from the world of action but exists in robust, complex, social environments made up of actors, actions and situations (Stein, 1998, p. 2).

While many researchers and educators believe learning is a process that exists at the confluence of the individual and their sociocultural environment, only a few studies have addressed adult learning in this way (Forman & McPhail, 1993; Shah, 2017). One significant body of research in K-12 mathematics that has the potential to improve adult education is collaborative learning.

Collaborative learning

Adult educators espouse the need for collaboration and participation; however, there is little empirical evidence for collaborative learning as a productive way to educate adults. Collaboration, to Rochelle and Teasley (1995, in Dillenbourg, 1999), is "a coordinated, synchronous activity that is the result of a continued attempt to construct and maintain a shared conceptualization of a problem" (p. 70). Knowledge is created rather than transmitted from the instructor (Sheridan, 1989), and anyone can participate in shaping and testing ideas (MacGregor 1990; Novotny, Seifert, and Werner 1991).

Collaborative learning represents a significant shift away from the teacher-directed environment and toward one where the teacher facilitates and enters into a process of mutual inquiry. MacGregor (1990) notes that as teachers reframe their traditional role, the process can be challenging as teachers reconcile "one's sense of responsibility about course coverage with one's commitment to enabling students to learn on their own (p. 26). Thus, Imel (1991) recommends facilitators develop methods for "sharing their expertise without usurping the attempts of learners to acquire their own" (p. 4). Adopting these methods is critical for collaborative discourse and can minimize "distortions in communication" due to imbalances in power and influence (Mezirow, 2007, p. 16). Mezirow distinguishes collaborative discourse from everyday discussion and dialogue by its exclusive focus on content and the ensuing assessment of arguments, examination of alternative viewpoints and assumptions, and consideration of evidence (p. 14).

Students working together to learn what a teacher wants them to learn is not collaborative learning, nor is it the extension of the teacher's instruction. It is not having students work together while doing individual assignments, helping peers finish their work, or relying on one or two students to complete the work (Klemm, 1994). The receiving of information by the student fails to foster critical engagement fundamental to meaningful participation. This mode of teaching and learning undermines the learner perspectives and experiences in the construction of knowledge; thus, limiting discursive exchange between the teacher and students (Armstrong & Hyslop-Margison, 2006; Laal & Laal, 2012). What collaborative learning entails is joint intellectual effort by students or by the students and teachers together as they collectively construct understandings or meanings; it is a structure that creates space for student talk; it may involve a pair, small group, or a class; and it is designed to promote effective teaching for the greatest number of students (Dillenbough, 1999; Golub et al., 1988; MacGregor, 1990; Pugach, M. & Johnson, L. J., 1995; Smith & MacGregor, 1992).

Whole-class collaborative inquiry

Students have opportunities to engage in productive conversations and construct knowledge with their peers in whole-class participation structures, namely whole class discussions. Whole-class discussions are used by teachers to promote and extend students' mathematical understandings and develop their proficiency with a range of mathematical practices such as conjecturing, justifying and reconciling (Ball &Bass, 2000).

Despite the interest in collaborative inquiry, the traditional communication pattern in whole-class settings, Initiation-Response-Evaluation (IRE) (Mehan, 1979), is the predominant mode of instruction in adult education (Beder, Medina, & Eberly, 2000). This particular mode of instruction is usually described as offering students limited opportunities for extended talk or extending communication by initiating, clarifying, agreeing, contradicting and arguing (Wedin, 2004 in Wedin & Shaswar, 2019). Despite the limitations of the I-R-E structure, adult education researchers are finding opportunities for learning and student interaction in whole-class participation structures. For example, researchers in Sweden studied interaction patterns within the I-R-E structure and examined interaction patterns that provided rich opportunities for students to use the target language. They claim that language in the whole-class context created linguistic challenges that prompted a negotiation of meaning through extended talk. Moreover, whole-class interaction can be organized to provide opportunities for extended and student-initiated talk (Wedin & Shaswar, 2019). Wedin and Shaswar documented extensive student interaction in the whole-class context, and their work suggests that studying interaction in whole-class adult settings can be a productive endeavor. However, when researching whole-class settings, it is essential to consider and give sufficient attention to the quality of students' engagement rather than the number of exchanges. It is crucial to address the process and quality of learning and avoid confusing student activity for student learning or mistaking group interaction for group participation (Mason, 1992). Thus, it begs the question, what is the nature of productive conversations that foster situations for mathematical learning?

For this, we must turn to our K-12 colleagues for evidence to support more collaborative and student-centered environments. Collaboration, to Rochelle and Teasley (1995, in Dillenbourg, 1999), is "a coordinated, synchronous activity that is the result of a continued attempt to construct and maintain a shared conceptualization of a problem" (p. 70). Whole-class participation structures provide opportunities for students to construct knowledge with their peers. Collaborative environments can support conceptual understanding (Hiebert et al., 1997; Silver & Stein, 1996), increase interest and promote critical thinking (Gokhale, 1995; Totten, 1991), produce more equitable learning environments (Boaler, 1997; Cohen & Lotan, 1997; Silver, Smith & Nelson, 1995), foster positive identities (Boaler & Greeno, 2000; Cobb & Hodge, 2002), and enable adults to draw on their previous experiences by accessing their reservoir of accumulated wisdom and knowledge (Brookfield 1986; Bruffee 1987; Martin 1990; Novotny, Seifert, and Werner 1991). Furthermore, collaborative classrooms require teachers to be responsive to students. To do so requires teachers to create space for and pursue the thinking behind students' contributions. A responsive teacher interprets student ideas and makes in-the-moment decisions on how to best support student understanding. The teacher must have a deep understanding of the content (mathematics in this case), the development of students' mathematical thinking, and the curriculum (Staples, 2007).

Classroom interaction

Few studies focus on how participation impacts adult learning and problem solving, yet K-12 researchers have explored the relationship between student engagement and the development of skills and understanding for decades. Classroom interaction and its effects on learning have been well established (see Forman, 2003; Lampert & Cobb, 2003). Researchers have studied student participation that is productive for learning, specifically explaining one's ideas and engaging with the ideas of others (see Kuhn & Crowell, 2011; Mercer et al., 2019; Waggoner et al., 1995). This dissertation examines these two ways that interaction is enacted and is grounded in the work of Webb and her colleague (2021; 2018; 2014). Explaining one's thinking and engaging with others about their ideas can help students fill in gaps in understanding, clarify material for themselves, acquire new knowledge and perspectives, attend to misconceptions (Barth & Schul, 1980; King, 1992; Peterson et al., 1981; Rogoff, 1991; Saxe et al., 1993; Valsiner, 1987) and construct shared meanings (Roschelle, 1992).

Neil Mercer (1995) investigated the role of language and the development of children's thinking. He developed a taxonomy to differentiate between disputation, cumulative, and exploratory talk, which helps us understand how individuals use language to learn. Exploratory talk or talk used to engage with each other's ideas "represents the more 'visible' pursuit of rational consensus through conversation. More than the other two types, it is like the kind of talk which has been found to be most effective for solving problems through collaborative activity" (p. 105).

Empirical findings from previous studies suggest that active student participation is beneficial to student learning (e.g., Chinn, O'Donnell, & Jinks, 2000; Fuchs et al., 1997; Howe et al., 2007; Veenman et al., 2005; Mercer, Dawes, Wegerif, & Sams, 2004; Brown & Palincsar, 1989; Webb & Palincsar, 1996). For example, researchers have found relationships between giving explanations and learning outcomes (Howe et al., Veenman et al., 2008, 2009); results that suggest students, guided to elaborate their own ideas, demonstrated greater learning outcomes than students without guidance (Gillies, 2004; Howe & Tolmie, 2003; Mercer et al., 2004); and revealed the benefits challenging student ideas, such as testing predictions, rethinking prior knowledge, and revising ideas (Brown, Campione, Webber, and McGilley, 1992).

Empirical research of student interaction in adult education is limited. However, as researchers begin to study classroom interaction, the ways in which adults learn through participation unfolds. For example, Díez-Palomar, Rodríguez, and Wehrle (2006) applied a dialogic framework to study cognitive trajectories in mathematics. They found that adults who understand the abstract meaning of a concept tend to explain it first as a generalization and then through successive concrete examples. Moreover, they suggest that illocutionary approaches encourage dialogue and the joint construction of meaning, while perlocutionary speech acts can both encourage and discourage learning based on the use of positions of power.

Drawing from the idea that learning is connected to reasoning and dialogue, researchers have examined how teacher practices influence student dialogue in the class. Teachers can impact student participation by creating opportunities for argumentation (Foreman et al., 1998; Chinn et al., 2001), asking higher-order questions (Boaler, 1997; Graesser & Person, 1994; Nystrand & Gamoran, 1991), and pressing students to justify their work (Boaler, 1997; Gillies, 2004; Lampert, 1990). In addition, the teacher can promote dialogue in the classroom by establishing communication norms, such as peers probing each other's ideas (Yackel, Cobb, & Wood, 1991), and encourage elaboration by asking students to question each other at a high level

(King, 1992; Mavarech & Kramarski, 2003). Moreover, teachers who press students to explain their ideas rather than provide highly directive help may influence student interaction (Chiu, 2004; Webb et al., 2006: Dekker & Elshout-Mohr; 2004).

Although adults' mathematical thinking differs from children's, much can be gleaned from Cognitively Guided Instruction (CGI) in primary classrooms (Carpenter et al., 2015). Namely, learning mathematics with understanding and the teachers' use of the development of mathematical thinking (i.e., Carpenter et al., 1989; Lampert, 1990). Their conception of learning with understanding includes four themes and may apply to adult education: 1) knowledge is connected, 2) knowledge is generalizable, 3) students describe, explain, and justify their mathematical thinking, and 4) students identify themselves as mathematical thinkers who see that mathematics should make sense and that they have the power to make sense of it (p. 185). Studies show connections between student achievement and teachers' knowledge and the development of children's thinking (Carpenter et al., 1998) and the positive impact of professional development focused on children's thinking (Franke et al., 2001). This dissertation study focused on the ways learning with understanding may transpire in the adult education classroom.

Situating the current study

What transpires inside an adult education classroom is critical. Specifically, what is communicated about the mathematics, how it is communicated, and by whom. The dialogue spoken in class is meaningful. For Freire, dialogue is more than an educational technique; it is at the root of the process of becoming a human being; it is a "moment where humans meet to reflect on their reality as they make and remake it" (Freire & Shore, 1987, p. 98). Examining the talk that occurs inside a classroom can, at a minimum, shed light on the development of adults' mathematical thinking. At most, it can promote a change in consciousness that can liberate one from oppression (Freire & Shore, 1987) and transform the teacher-learner relation in which both teacher and student "become jointly responsible for a process in which all grow" (Cahn, 1997, p. 466). An investigation of classroom interaction between the teacher and students and the students with their peers is imperative for understanding the development of adults' mathematical thinking. A substantial body of literature has explored student participation and teacher practices to support student participation in primary and secondary classrooms; however, the research has not yet explored what this looks like in the adult education mathematics classroom. This dissertation aims to combine research on adult learning, adults' mathematical thinking, and collaboration through a focus on adult student and teacher interaction. Examining classrooms through a participation lens provides a way to examine adults' mathematical thinking and learning and how a teacher can support adult learning. This dissertation investigates classroom interaction in one adult education algebra classroom.

CHAPTER 3: METHODS

An Analysis of Classroom Participation

This chapter presents the research methods and analyses to examine the confluence of adult students' capacity for mathematical ideas and thinking and teacher practices that engender and support that thinking. Specifically, I investigated how the teacher provides opportunities for students to participate and how students take up those opportunities to explain their own and engage with others' mathematical ideas in an adult education algebra class. Over time, interactions during whole-class instruction serve to construct an understanding of classroom participation and the shifts that occur over time. Thus, understanding opportunities for students to participate requires analyses of general participation trends over time and a close examination of specific classroom interactions. In this chapter, the case study design, sample and setting, and procedures for data collection and analyses are described. This dissertation investigates the following questions:

1. In what ways do adult learners participate in an Adult Education mathematics classroom explicitly designed to support student interaction?
    - In what ways are adult students taking up opportunities to participate?
    - In what ways are students explaining and engaging with others' ideas?
    - How does student participation change over time?
2. How does the teacher support students to participate in an Adult Education mathematics classroom explicitly designed to support student interaction?
    - How do particular teacher moves relate to students' opportunities to participate?
    - How does teacher support of student participation change over time? To investigate these questions, this study examines student participation and teacher

support of student participation in one Adult Education algebra classroom where the teacher supported students to problem solve with their peers and participate in whole-class instruction. Various data sources, including fieldnotes and classroom video, were analyzed with respect to types of participation evident in previous research and elaborated below.

A descriptive, single case-study design was used to collect qualitative data from multiple sources related to the adult mathematics classroom. A case study is defined as an investigation and analysis of a single social phenomenon within a bounded system that exists in a specific time and place (Hays, 2004; Merriam, 2009; Yin, 2002). The unit of analysis, or case, is one of Community Adult's mathematics classrooms. This empirical inquiry will investigate a phenomenon, student engagement and teacher support of student engagement, within its real-life context, the classroom. The case was selected because it represents a bounded case where participation can be explored in relation to the sharing of students' mathematical thinking and teacher support of students' mathematical thinking. A single case study was carried out to enable the depth of observation necessary to capture the subtleties of student and teacher interaction and the relational nature of explaining and engaging with others' ideas. Given the research aim, objective, and question, an explanatory approach was adopted. Although the results are not generalizable, the in-depth observations described in this case enable the formulation of a more complete picture of classroom interaction and provide a foundation from which to further an understanding of adult participation, and teacher support of student participation, in an adult education mathematics setting.

Site Selection

The study seeks to understand "what is happening" in the adult mathematics classroom and the "relations linking events," thus, purposive sampling was utilized (Merriam, 1998). I chose a large, urban district serving students of color and an enthusiastic instructor eager to surface students' mathematical thinking through explaining and engaging with others' ideas. To find Ms. Diaz, I first conducted a Google search of adult education programs in the city. I then limited the search to areas within the city with the most diverse populations where students' mathematical strengths may have been systemically underestimated. Then I reached out to several programs via email and heard back from four program administrators. A faculty member at UCLA recommended Community Adult based on my parameters and goals. After a few email exchanges with the administrator and an initial discussion, she introduced me to Ms. Diaz. I met Ms. Diaz in the administrative office. She smiled enthusiastically, looked me in the eye, firmly shook my hand, and invited me to observe her classroom the following week to see if we would be a good fit. We were.

Overview of setting: The Adult Education math class studied over one trimester

Community Adult School serves a diverse community and provides educational resources to support students to attain their goals and empower students to become successful in the workplace, home, and community. The single, focused mission of Community Adult is to prepare students for the future. Community Adult describes itself as a place to prepare adults to become self-directed learners, effective communicators, and creative and resourceful critical thinkers who can analyze, synthesize and solve problems. To meet this end, the school offers free day and evening courses to adults in Adult Basic Education (ABE), Adult Secondary Education (High School Equivalency and High School Equivalency) preparation, English as a Second Language (ESL), Citizenship, and 55+ community education. In addition, the school offers the following short-term career training courses: business management, EKG technician, drafting, nursing assistant (CNA), phlebotomy technician, pharmacy technician, and vocational nursing (LVN). The programs are a first step to access developmental education opportunities, job-related training, and programs that bridge progress towards credit-based certificates and degrees. The school serves adults 18 years of age or older who are not high school graduates nor attending day high school.

Community Adult mirrors the demographic composition of its urban community. The school is nested in an urban school district that serves a large portion of students of color. The racial/ethnic breakdown of the school reported by the state was 94.2% Latino, 5% Black, and .3% White. Nearly all the students in the district qualify for free/reduced-priced meals (90.9%), and 26.7% were classified as English learners. The classroom composition studied here reflects that of the school and district. To serve their diverse student body, the school subscribes to a philosophy rooted in: educational and vocational opportunities for everyone, respect and care, shared responsibility, and students' self-worth. Moreover, Community Adult's motto reflects the school's strong commitment to giving back to the local community, "Enter to Learn, and Depart to Serve."

Community Adult is an ideal research site for four reasons. First, Community Adult has demonstrated a commitment to adult education. Community Adult has been educating adult learners for seven decades and continues to expand adult education programs to learners in the community. Second, the campus is representative of adult schools in California. The adult campus is situated in a large urban center with a large proportion of Latino students and a White minority. Third, the principal and faculty are committed to quality instruction and interested in research that centers adult student thinking. Fourth, the school offers an adult education algebra course.

At the adult education level, there is a movement to better prepare students for post-secondary education and training for jobs in high-demand fields. That is, skilled proficiencies in algebra, like conceptual understanding and problem-solving, can prepare adult students to meet the demands of skilled trades and industries. Moreover, math is ubiquitous in all aspects of life, and the following examples illustrate the need for algebra in adult's lives: personal financial literacy and decision making (e.g., change over time), academic requirements, on-the-job requirements (e.g., applied concepts of proportionality), and entrance-to-employment requirements (Manly & Ginsburg, 2010). Thus, an adult education algebra course is well-suited to observe how a teacher weaves conceptual understanding and reasoning into a traditional curriculum focused on developing basic skills. The tension between skills development and algebraic reasoning found in an adult education algebra classroom is ideal for observing students making sense of mathematics.

The single mathematics classroom studied is one of two mathematics classrooms at the school. Ms. Diaz taught mathematics and science at the school for over two decades, had a preexisting collaborative learning practice, and adopted a student-centered, inquiry-based approach to mathematics instruction. In contrast to her colleague, Ms. Diaz's instruction involved story problems often originating from experiences students disclosed in class, multiple participation settings (whole class and small group), strategy shares at the whiteboard, and a commitment to student interaction. While Ms. Diaz had years of experience supporting students to be actively involved in mathematics, she had less experience in supporting students to explain their ideas and engage with the ideas of others and was selected because of her commitment to improving this area of her practice. Ms. Diaz adhered to the course's established scope and sequence and the administration of the National Reporting System (NRS) mandated assessments.

Participants

Nineteen students enrolled in the Algebra I course and three exited after the first day. Of the remaining 16 students who completed the course, 12 students were present during the three time points in the study. Thus, the study examines classroom interaction between the stable cohort of students (n = 12) and their teacher across the three time points (see Table 3.1).

Table 1. Student Attendance: Time 1, 2, and 3

	Students															
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
Time 1	x	x	x	x	x	x	x	x	x	x	x	x	x	x		
Time 2	x	x	x	x		x	x	x	x	x	x	x	x	x	x	
Time 3		x	x	x	x	x	x	x	x	x	x	x	x	x	x	x

Note. Student 1, 5, 15, and 16 were excluded from the analyses as they were absent from at least one observation at Time 1, 2, or 3.

The class met once a week for twelve weeks on Fridays from 9 am – 3 pm from April 2019 to June 2019. In addition to time in the classroom, I often joined students during their scheduled 30-minute break and special school-organized events. The class did not meet on Good Friday and the week of spring break. I observed 9 out of the 10 classes from 9 am – 3 pm as a participant-observer, however, the data analyzed for this study were collected during the morning portion of the class only (9 am – 11:30 am). Data for this study include observations from Week 3, Week 6, and Week 9. These particular days were selected because they were neither the beginning nor the end of the course and did not conflict with beginning-of-session logistics, content review, special events, or testing. The three days selected reflect typical days in Ms. Diaz's Algebra I classroom.

The Instructor

The Algebra 1 class has a fierce leader at the helm. Like many adult education instructors, Ms. Diaz discovered the field by accident. While working in a lab as a researcher for a petroleum company, she heard stories of the adult education field from her sister who taught, part-time at Community Adult. Seeking more enjoyable and rewarding work, Ms. Diaz exchanged her solo pursuits in the lab for dynamic work in education. She began in the office at Community Adult while working towards her teaching credentials. As a credentialed teacher, Ms. Diaz transitioned to the classroom, has taught math and science at Community Adult, and has never looked back. That was over 20 years ago.

Ms. Diaz approaches teaching like a scientist – systematic and objective. She admits that some of her educational experiments do not always work. She retains, modifies, extends, or abandons lessons based on self-reflection and her perception of student engagement. Ms. Diaz carefully plans daily lessons but arms herself with backup or tangential lessons and tasks when she notices student interest in a particular topic or needs to address a gap in mathematics knowledge. She can adeptly create spontaneous lessons on the fly should the discussion veer in a direction she did not anticipate. Ms. Diaz understands the adult learner and makes a conscious effort to follow their interests to connect mathematics to their lives outside of the classroom. She understands adult learners all have a story, and they come with preconceived ideas of schooling based on their K-12 experiences, responsibilities, the need to feel safe, and they return to school to "get something and move on with their lives." Through her extensive experience in the classroom, Ms. Diaz knows that "you cannot just approach it [math] in one way or you will lose most of the class." She also knows that teachers of adults need to find the delicate balance between student autonomy and handholding and struggles with students who say, "Ms. Diaz, why don't you just tell us?" If she does not navigate the tension successfully, adult students will simply walk away from the non-compulsory adult education setting.

Student participation is essential. While Ms. Diaz values and encourages student participation, she confesses that sometimes she has to stop herself from interjecting. She wants to support students but knows that when she does, she "take[s] away from their learning." For example, Ms. Diaz knew she would lose students if she stood at the board and directly taught the distributive property. Hence, she designed a lesson where the students discovered the property on their own in small groups in a real-world context. She notes that her students experienced frustration and wanted a teacher-led explanation. They asked, "Isn't that what you're here for? To help us? Why don't you just explain it to us. Tell me." Ms. Diaz frequently hears that, but she holds steadfast to her teaching philosophy that adult students learn by collaborating with their peers, which often includes productive struggle. She establishes classroom structures that reflect that philosophy.

The mathematicians

Adult Education students comprise a diverse profile with a wide range of academic histories, life experiences, ages, mathematical knowledge, and racial, cultural, and geographical backgrounds. Many are heavily invested in their familial and workplace responsibilities, and when they enroll in school, they add one more role to their lives, that of a student. Adult education classrooms are dynamic sites of identity, where individual learners seek participation, meaning, and competence. Learners' sense of their own mathematical knowledge and knowledge valued by the mathematics class they belong to form a crucial element of their sense of self. Moreover, identity is a representation that we ascribe to ourselves and others: "Our conception of who we are, our identity, is constituted by the power of all of the discursive practices in which we speak—which in turn 'speak' us" (Chappell et al., 2003, p. 41). Thus, each student is identified as a mathematician. Mathematicians nurture their intuition and recognize the importance of making connections in the building of mathematical meaning. They are reflective about the problem-solving process, a process dependent upon learning through inquiry that involves "making connections, building understanding through knowledge and experience, developing a sense of the possible or even likely" (Burton, 2004).

To meet the learning needs of adult students, it is necessary to understand students' diversity, who they are, and why they decided to re-enroll in school. Unlike traditional K-12 students whose identities revolve around being students, adult learners at Community Adult primarily perceive themselves as parents and employees, and it is through this identity in which they evaluate and prioritize learning. Adult students' identities are rooted in adult responsibility and competing life roles, and it is through this lens, they seek adult education.

Each learner brings their personal struggles and triumphs, mathematical attitudes, racial experiences, and perceptions of self to the classroom every day, which inform how they participate, interact and make sense of the mathematics. Their mathematician identities reveal both shared and divergent experiences. Through those lived experiences, each student has acquired different ways of perceiving, acting, and thinking that enables them to respond to negotiate meaning with peers. Knowledge and meaning are actively constructed within the adult classroom community as adult learners build relationships and learn from each other.

The cohort of twelve students included in the sample comprise a diverse classroom community. The majority of students in the sample were unemployed (67%), native Spanish speakers (58%), and identified as Hispanic or Latino (83%) (see Appendix A). In addition, the students waited an average of ten years before returning to school (see Figure 1) and self-reported average to lower grades in their former high and middle school mathematics classes (see Figure 2). Each student brought with them a story, and often that story included hardship, overcoming obstacles, and ultimately perseverance. Student profiles were included to illustrate the diversity of the individuals who attended class and committed to engaging in problem-solving every day (see Appendix B).

Figure 1. Number of Years Between Compulsory School and Adult Education

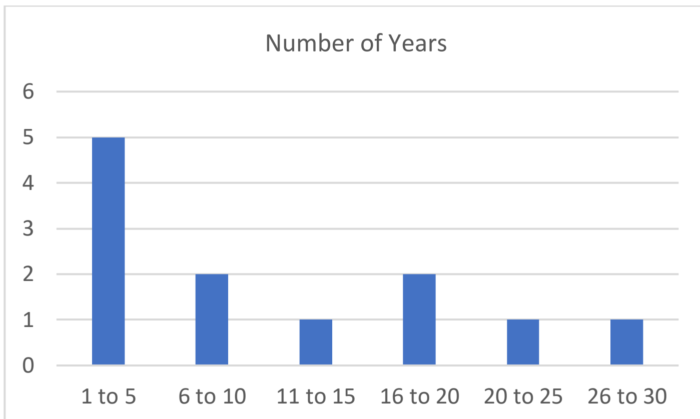
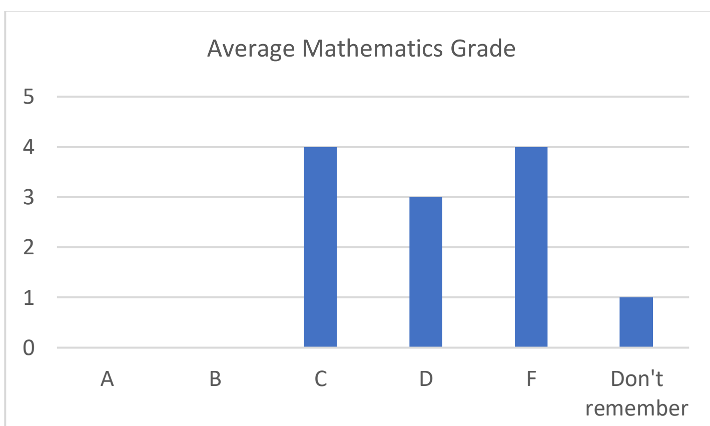


Figure 2. Self-Reported Average Mathematics Grade in Compulsory High or Middle School



The instructor, classroom structure, curriculum, and the students form a dynamic learning community where students' mathematical understanding and engagement are central tenets to learning. Ms. Diaz supports a diverse group of learners in the whole-class context to not only do the work required to complete the course but to do the heavy lifting of mathematical understanding through dialogue, disagreement, and finding one's mathematical voice.

Classroom structure

Adult education students are just that – adults. They have jobs and family responsibilities, doctor's appointments, and car trouble. Ms. Diaz prides herself on high expectations for her students, but she realizes that flexibility and understanding are integral to working with busy adults. A few students trickled into the classroom after dropping children at school or grabbing breakfast after a night shift at a local warehouse. Some students had to leave early for work. Ms. Diaz structured the class in a way that minimized disruption when students entered late or exited early.

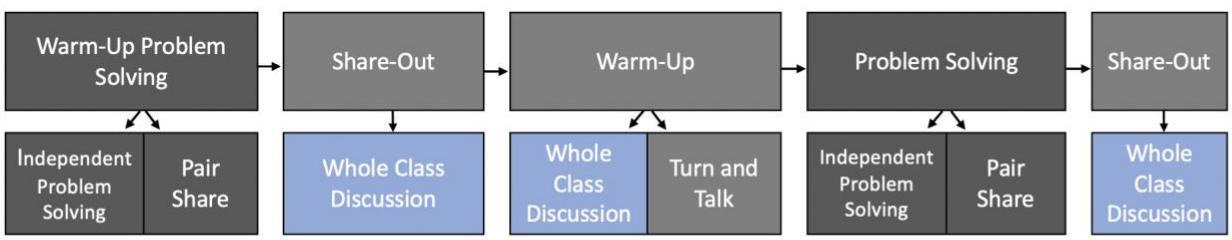
Students entered the classroom, signed the attendance sheet, picked up an entry ticket, found a seat at one of the long tables, and began to work. For those students who arrived early, Ms. Diaz asked them about their work, children, and lives. She attentively listened to their stories to get to know the students and for ways to connect mathematics to their everyday experiences and extend their curiosities. She took note that Lucia arrives, every day, with a bottled Starbucks Frappuccino (the subject of a spontaneous lesson on diabetes and the sugar content of seemingly innocuous drinks) and that Jose and Diego walk 40 minutes to school (the subject of a rate, distance, and time problem). Those who were a few minutes late quickly collected and completed the entry ticket and began the warm-up activity.

Ms. Diaz provided a space where students asked questions, joked with one another, told stories, and shared experiences daily. She structured the class to incorporate multiple transitions and participation settings where students interacted with one another with and without the teacher present. Ms. Diaz built social and cultural capital into the mathematics. For example, an activity based on wages morphed into three students sharing that their managers are skimming from their paychecks. Ms. Diaz reaffirmed their worth and suggested reporting the company to a news agency. This, in turn, led to a discussion of racial injustice, confronting power structures, and protections for undocumented adults afraid to lose their jobs. Amongst the order of operations and algebraic expressions, Ms. Diaz tackled social injustice along with mental and physical health, parenting advice, and career counseling. She is the epitome of the modern hyphenated woman; she is the teacher-counselor-nurse-advisor-aunt-friend-scientist-mathematician.

A typical day in Ms. Diaz's Algebra 1 class had five phases. Class began with warm-up problem solving, followed by a share-out, a warm-up, a second problem-solving phase, and concluded with a whole-class share-out. Each phase can be further distilled into corresponding settings: the warm-up problem-solving phase included independent problem-solving and a pair share; the two share outs consisted of a whole class discussion, the warm-up consisted of a whole-class discussion and brief turn-and-talks; and the whole-class share-out included a whole-class discussion without turn-and-talk. The typical Algebra 1 classroom structure is shown in Figure 3.

The following is a description of the student experience as they proceed through the daily phases and settings. First, students solved entry-ticket problems independently and then shared their strategies with a nearby peer. For example, in Time 1, the entry ticket asked students to find the earnings of a person who works 10 hours making $10, $20, and $30 per hour, respectively, and create a table to summarize their results. Students worked independently for 10 – 15 minutes on their strategies and tables and then shared their solutions and representations with a nearby peer. Second, two or three students volunteered to share their entry ticket strategies in the whole class setting. As the author shared, the teacher commented or asked questions, and the students asked questions or waited to be called on to engage with the author's ideas. Third, students jotted notes and participated in a whole-class discussion that often involved a brief turn and talk. This setting looked like a familiar, direct instruction lesson – the teacher wrote on the board, the students diligently transferred the notes to their notebooks and asked clarifying questions. The warm-up discussion around the daily content was interspersed with brief moments where the teacher instructs the students to turn and talk to their partners about an in-the-moment idea. Fourth, the students work independently to solve problems from a handout or the textbook and, upon completion, check in with their classmates to share their thinking, strategies, and answers. Fifth, two or three students share out their strategies during a teacher-directed whole-class discussion.

Figure 3. Classroom Phases and Settings



For the purpose of this study, I examine the three whole-class settings: warm-up and share-out (identified in blue above). Adult education classrooms are dominated by traditional whole-class interaction. A similar phase and setting diagram for a traditional classroom would contain only one blue whole-class rectangle. Some instructors include brief interruptions for independent work, homework review, and worksheets for fluency practice; however, due to time constraints and a demanding curriculum, most adult educators rely on whole-class instruction and rarely include small group student-student interaction. Thus, I decided to study participation in the whole-class setting, as this is the setting where adult learners may be given the most opportunities to participate in the adult mathematics classroom verbally. Additionally, I collapsed the three whole-class settings to retain a sharper focus on any potential change in student participation and teacher support of student participation over time rather than the subtleties of the setting at this time. Thus, I aim to understand and describe how the whole-class settings as a whole offer space and opportunities for explaining and engaging with each other's ideas over the three time points. The whole-class setting is important for two reasons.

First, the most prominent form of instruction in adult education is espoused by adult learning theories wherein adults take responsibility for learning (Merriam & Caffarella, 1999). This self-directed learning model results in individualized group instruction in learning centers where learners work independently in self-study spaces on computers, workbooks, or packets of individualized, self-paced assignments, and instructors provide help on request (Beder & Medina, 2001; Beder et al., 2006; Mellard et al., 2005; Smith & Hofer, 2003). Some researchers question the efficacy of this model (e.g., Mellard & Scanlon, 2006; Robinson-Geller, 2007). For those programs offering whole-group instruction, only about 25% adopted instruction forms other than the Initiation, Reply, Evaluation (IRE) sequence (Mehan, 1979). Thus, the whole-class setting was ideal for exploring instructional strategies focused on student participation as the self-study model does not offer students opportunities for extended talk.

Second, the whole-class setting has been well studied. Capturing complex interactions in primary and secondary classrooms have been a focus of extensive research driven by the desire to identify practices that best support academic success (Correnti et al., 2015; Kane, Kerr, & Pianta, 2015; Stigler, 2003; Stigler, Gallimore, & Hiebert, 2000). In contrast, there is a paucity of empirical research that focuses on classroom interaction of adult learners (Deil-Amen, 2015). The whole-class setting involves multiple participants interacting over a sustained period of time (Blumenfeld, 1992; Pianta & Allen, 2008; Tseng & Seidman, 2007). Through those repeated interactions, norms become established (Sarason & Klaber, 1985). The whole class setting is well-suited to examine classroom processes that matter for teacher and student interaction.

The mathematics

Ms. Diaz follows a standard adult education Algebra 1 curriculum. The curriculum includes a range of algebraic topics, including an introduction to operations and variables, the use of order of operations to evaluate expressions, identification and use of properties of real numbers, graph and compare numbers, square roots, one- and two-step equations, linear equations, multi-step inequalities, and a cumulative review. Ms. Diaz writes the daily content objectives in the upper left-hand corner of the whiteboard every morning, and the students add the details to their notebooks as an organizer and note-taking strategy. Table 2 contains the objectives for the three time points and includes simplifying expressions and using the order of operations in Time 1, discovering the distributive property in Time 2, and solving equations in Time 3.

Table 2. Ms. Diaz's Algebra 1 Content Objectives: Time 1, Time 2, Time 3

	Time 1	**Time 2**	**Time 3**

Content Objectives	• To simplify expressions including exponents • To use order of operations to evaluate expressions • To classify, graph, and compare numbers • To find square roots • To find the sum, difference, product, and quotient of real numbers	• To use the distributive property to simplify expressions • To solve equations using tables and patterns	• To solve one-step, two-step, and multi-step equations

Typical mathematics problems varied across the three days and included a combination of procedural and conceptual tasks. Each day, students entered the classroom, added their names to the sign-in sheet, and picked up an entry ticket stacked in a neat pile on the same table. For example, students were asked to solve for how much they would earn making $10/hr and working 10, 20, 30, and 40 hours and to summarize their results in a table or to share what they noticed about the expression .8(200 – age) (the expression for finding maximum heart rate). After the entry ticket warm-up, the remainder of the problems on Day 1 were evaluating expressions using the order of operations, such as 52 + 82 – 3(4 – 2)3. On Day 2, students used mental math to find the product of two numbers, for example, 5.2 x 8. Next, students solved problems with both textbook-derived and student-derived context, such as, "Songs are $.99 each on iTunes. Maria purchased 50 songs. How much did she spend in all?" and "Based on the lunch receipt you brought to class and the number of times you go out to lunch per week, how much do you spend on lunch per week?" On Day 3, students solved contextualized and decontextualized equations. For example, students were asked to solve for b when b + 1.1 = -11. They were also asked to write an equation and solve for an unknown within the context of a story, such as, "Karla has 15 flowers. 9 of the flowers are daisies. The rest are roses. How many flowers are roses?" Across time, students have opportunities to explain their mathematical ideas, elaborate on the application of procedures in various ways, and make sense of tasks without, with textbook, and with real-life contexts.

The collaboration

The main focus of Ms. Diaz and my work together was to notice and support students to explain and engage with others' ideas. The skills to engage in a detailed way around the mathematics cannot be developed in isolation. Engaging in mathematically detailed ways requires sophisticated skills, including a minimum level of comfort to interact with classmates, communication skills with partners and in the whole-class context, awareness of sociomathematical norms to support productive interaction around the mathematics, and negotiation work through challenges or disagreements, and listening skills. Before conducting formal observations in Ms. Diaz's class, I observed three of her Algebra classes from the previous trimester and noticed a commitment to building a collaborative environment where she centered many of the skills mentioned above.

During our first meeting, Ms. Diaz shared her background, approach to life, and teaching philosophy that included the following: listen carefully, embrace students' curiosity, be patient and flexible, let go of the notion that every common core standard must be covered, incorporate humor, and model the joy of doing mathematics. I shared my interest in student engagement in the adult education mathematics classroom and some of the research that guided my work. My role as a participant-observer allowed me to observe and more actively participate in the classroom.

This study is not based on a sleek professional development program or a carefully designed intervention to elicit students' mathematical thinking. Rather, this study was born out of curiosity and a desire to understand how adults learn mathematics and how a teacher, like Ms. Diaz, supports them to do so. Ms. Diaz kept her general class structure and curriculum but was open to suggestions for entry ticket problems and the inclusion of turn-and-talks. There were three primary areas of collaboration: in-the-moment noticings, working closely with students, and a daily debrief. Building on Ms. Diaz's foundation of developing effective communication skills and classroom norms, I assumed the role of nudger, noticer, interlocutor, and listener.

In-the-moment noticings occurred during the whole-class context and were similar to a teacher-time-out move enacted during classroom-embedded professional development, like a learning lab. The noticings were brief interruptions that drew attention to something of interest, such as a complete conceptual explanation that demonstrated a creative approach to problem-solving, a task that elicited productive engagement, a change in a student's participation, or a pause and pivot. For example, Marco shared out a strategy reflecting a misconception. Ms. Diaz had been quick to correct students' work until this point, so I paused her and asked what she thought of the students finding the error and supporting Marco to consider an alternative strategy. Ms. Diaz embraced the suggestion and gave students the space to engage with Marco's idea, which allowed Marco to learn from his classmates, take their suggestions under consideration, and revise his thinking.

During the second half of class, when the video cameras were off and occasionally while I was filming, I worked with students during small group problem-solving. While Ms. Diaz circulated the classroom asking students about their strategies, I also joined pairs of students at their table to learn about and support their mathematical thinking. Rather than direct students how to solve problems, I would listen and ask questions, examine student work, encourage students to re-read the story, promote the use of tools and representations, encourage students from neighboring pairs to engage with a student who needed support, and generally support students in any way that I could. For example, on one particular day, Kalifa struggled to make sense of a two-step problem, including measurement division. After re-reading the problem several times, she still had questions, so I suggested drawing a picture to represent the problem. After working out a representation for the problem, Kalifa looked up and said that she initially thought drawing pictures in math class was childish but ultimately found it helpful. As this was Ms. Diaz's class, I aimed to be helpful and respectful of my guest status. Thus, I opted for a quietly supportive, minimally disruptive role.

The daily debrief occurred after the morning session during the students' lunch break. Ms. Diaz and I would discuss several topics, including what went well, what she noticed about students' mathematical thinking, who participated and how, ideas for subsequent lessons, any surprises or challenges, and general thoughts about the day. Some debriefs were brief, and others were more extensive, depending on the particular day. For example, during one debrief, Ms. Diaz shared that the initial turn-and-talk did not go as well as she had hoped. On another occasion, she was surprised by the creative problem solving and the various ways students decomposed numbers. Moreover, on another day, she mentioned that students were uncharacteristically quiet when given instructions to explain to their classmates, which led to a discussion of the tasks that can promote or hinder student engagement.

My Role as a Participant-Researcher

My experiences as a professional developer, member of a state leadership team, program coordinator, and instructor of adult education mathematics catalyzed my interest in the adult learner and classroom interaction. The approach used in this study reflect my commitment to better understand adults' mathematical thinking, explore their contributions to classroom discussions, and center the assets adults bring to the classroom. As a member of the UCLA Mathematics Teaching and Learning Research Group with Megan Franke, Noreen Webb, Marsha Ing, Nicholas Johnson and my graduate student peers, I wondered how I might apply our work in primary classrooms to the adult education setting. I hold a unique position in that I can apply and build on my knowledge of children's mathematical thinking and the ways young people participate to the adult education realm to the adult algebra classroom. Adults are clearly not children. Thus, learning about adults' mathematical thinking and engagement may shed more light on the unique qualities of the adult learner and how to improve and tailor our practice to better serve them. My position as a participant-researcher has permitted me to develop relationships with Community Adult's administration, the Algebra I teacher, and the students. I had the privilege of getting to know the students, their struggles and triumphs, and most importantly, their mathematical ideas. I sat side-by-side with students during problem-solving, walked miles with them during the Walk for Health, and was persuaded to dance (poorly) the Macarena alongside the students at the end-of-the-year banquet. While I identify primarily as a researcher now, I cannot deny the way my identity and background influence the methods I choose, the analyses I conduct, and how I interpret the data. I identify as a mother and a white, middle-class female raised in rural Alaska. I have always been acutely aware of the cultural mismatch in my own adult education classrooms and, as such, did my best to prioritize student experiences and their own thinking, rather than mine, around the mathematics. As an adult educator, I witnessed roofers explain the Pythagorean relationship, drywallers easily estimate area, and cashiers calculate percentages with ease. I understood that adult students have an intuitive understanding of mathematics based on experiences in the workplace and their rich life experiences. I identify with the challenges adult educators face, namely persistence, as adult learners often are forced to choose adult responsibilities over education. During my work in adult classrooms and supporting teacher collaboration, I have believed that teaching involves striving to understand and support the extension of adults' mathematical thinking.

Data collection procedures

Data collected for this study investigated 1) the ways in which adult students participate and 2) how the teacher supports adult students to participate. To reduce researcher bias and ensure the creditability of the findings, I included multiple data sources to answer my research questions (Guba & Lincoln, 1986). The teacher was solicited via email to participate in the study. I met with Ms. Diaz to discuss the details of the study, observed three classes from the winter trimester to assess feasibility and compatibility. When the new trimester began, I recruited students, in-person to participate. Primary data were gathered via video recording and fieldnotes taken during classroom observations.

Video recording

The fieldnotes and classroom video footage presented in this study is drawn from a larger corpus of data collected in the Algebra I classroom. For this study, one classroom was filmed for nine days during the spring trimester (March to June). On each day of data collection, I recorded classroom interaction using one iPad device to record the teacher and six iPod touch devices to record six pairs or triads of students throughout the entire morning lesson (9 am – 11:30 am). The multiple recording devices made it possible to record all the students and the teacher over time. The multiple cameras made it possible to identify individual student contributions at all lesson phases and settings. The small, low-profile iPods sat atop miniature tripods on student desktops and could be moved with ease as the students moved about the classroom.

The purpose of this study is to investigate student participation, teacher support of student participation, and how student engagement and teacher practices may change over time. Thus, three days (one in March, one in April, and one in May) were selected to highlight the range of student explanations and engagement with others' ideas from the beginning, middle, and end of a 12-week course. I anticipated changes in student participation over time, such as increases in the number of complete explanations or the inclusion of different types of teacher questions. The three selected days used a similar lesson structure consisting of two rounds of warm-up activities, problem-solving, and sharing-out of student strategies. While the teacher relied on multiple classroom settings, including individual problem solving, teacher-orchestrated whole-class and share-out, small group, and turn-and-talk structures to elicit student thinking, this study focuses solely on the whole class setting.

The study examines students' opportunities to learn mathematics and the role the teacher plays in supporting students. Specifically, the study includes how the teacher connects students' mathematical thinking to important mathematical ideas and daily learning goals and mediates student contributions to ensure community understanding of others' ideas. Thus, I focus on the multiple whole-class settings of the lesson.

Field notes

Additional data analyzed included daily fieldnotes. I documented closely the nuanced processes of learning as it occurred in the classroom. Throughout this interpretive process, I built upon my prior understanding of student engagement by developing new insight and understanding of the phenomenon. Fieldnotes provided a valuable resource for preserving the in-the-moment experience and offered a means of reflection and understanding of those experiences (Emerson, Fretz, & Shaw, 2011). Producing a record of student engagement as it occurred preserved the essence of the activity without the detrimental effects of retrospective recall. The immediately written fieldnotes added texture to create a world of the algebra classroom on the page. In addition, the detailed description and documented insights were used for developing codes for the final analysis.

I attended to events related to student participation around the mathematics and teacher support of student participation, specifically, explanation and engaging with others' ideas. To that end, fieldnotes were written 1) concurrently with the video recording during the 2.5-hour morning portion of class to capture interesting, significant events that may not have been captured on video, 2) after the video cameras were turned off to document significant events related to student engagement during the remainder of the class, and 3) when there were video-related technical difficulties.

The impressions documented in the fieldnote protocol did, for the most part, remain jottings. More extensive jottings recorded an ongoing, mathematically rich, dialog between students; a complete student explanation; or an in-the-moment lesson based on student interests. The quickly rendered scribbles helped me construct the scene once I left the classroom. While primarily focused on key events or incidents, I also looked for variations from exceptions to emerging patterns. These differences deepened my understanding of interaction in the adult education classroom and encouraged me to examine conditions that may account for variation. During class, I took notes in Microsoft Word by parsing the observation into 20-minute sections, wrote short memorandums after class, and accrued a total of 9 classroom observations.

Analysis of fieldnotes occurred as they were being written and while conducting observations. Preliminary analysis fostered self-reflection, which is crucial for the meaning-making process and revealed emergent themes. Identifying emergent themes in the classroom allowed me to shift my attention in ways that fostered a more developed investigation.

Coding of student participation

Analysis of video-recorded observations began with multiple passes through the video data. The first pass of iPad footage that followed the teacher was streamed in its entirety. A second pass parsed each daily lesson into segments by identifying time stamps of the multiple beginnings and endings of the whole-class setting (independent problem-solving share-out, group warm-up, and group problem solving). During the third pass, timestamps were used to identify whole class segments that were subsequently transcribed in Microsoft Word. The individual student videos were consulted and used to supplement the iPad transcript when student utterances were inaudible or otherwise inadequate.

To capture nuanced features of participation in the adult classroom, the whole class segments were further parsed into episodes. An episode consisted of the related interactions around a particular mathematical strategy or idea (Forman & Ansell, 2002). The episodes afford the researcher a smaller grain size by which to capture specific mathematical ideas that surface in the whole class context and to track how the ideas are taken up by the teacher and students in-the-moment and over time. The episodes were parts of the whole-class interaction that resulted in the negotiation of meaning through students' varied communication with each other and the teacher that offered students space to develop their mathematical thinking. The duration of episodes varied from seconds (a brief share out at the whiteboard without any peer or teacher interaction) to over ten minutes (multiple students making sense of a shared mathematical idea). Despite the variability, each episode offered a glimpse, no matter how brief or extended, into students' mathematical thinking, the participation afforded or constrained by the task and the subtleties of the teacher-student and student-student interactions that follow.

A list of provisional codes (Saldaña, 2013) was developed from existing collaboration research and based on my experiences in Ms. Diaz's classroom. I reviewed the transcripts from the three selected observation days and made notations of significant teacher and student interactions that took place around interesting mathematical ideas. Areas in the transcript were highlighted that corroborated noticings in fieldnotes and documented as potential codes for consideration and review. The transcripts were examined in chronological order to evaluate how the variation in student and teacher participation over time may reveal unique patterns of engagement.

To attend to critical features of student participation based on previous theoretical work, each episode was coded for two prespecified types of participation – student explanations of their own ideas and students engaging with others' ideas (Boaler & Staples, 2008; Brown, Campione, Webber, & McGilly, 1992; Mercer, Hennessy, & Warwick, 2019; Roschelle, 1992; Webb et al., 2018a, 2018b, 2020). Within the two types, aspects of interaction were assigned labels based on the level of detail of the explanation or engagement with others' ideas (see Table 3). Detailed explanations (partial or complete) had to be comprehensible and include details central to students' mathematical idea(s). Engagement with others' ideas required students to explicitly engage with the details of a peer's idea (either referencing or extending the idea). In addition to the prefigured, a priori codes, additional codes emerged during the analysis (Creswell & Poth, 2016). Two emergent codes surfaced when following student ideas in the classroom: sharing ideas without a verbal explanation and sharing of student ideas through teacher explanation. To supplement coding of student interaction, narrative descriptions of each whole class segment were written and synthesized into analytic memos (Saldaña, 2013) to provide a detailed and contextual perspective to complement an individual or series of codes. A comprehensive picture of the details of student participation and what it looks like to do mathematics in the adult education classroom emerged.

Table 3. Operationalizing participation across four types of sharing of student ideas

Type	*Description*	*Example*
Explaining one's own ideas	Students' complete or fully-detailed explanation of their own mathematical ideas	*Solve 2.99 x 3 mentally* It's almost 300, so 300 x 3 is 900 take away the 3 ones to get 897. $8.97.
	Students' partially detailed explanation of their own mathematical ideas	3, 6, 9 – multiply by 3
Engaging with others' ideas	Adding on to or extending another student's mathematical idea	*Solve 22.50 x 13 mentally* He multiplied 20 x 13 first. I added 22.50 three times to get $67.50. Then 22.50 x 10 to get 225 and added them to get $292.50.
	Referencing the details of another student's mathematical idea (without adding on or extending)	Lucia did it the same way. She multiplied by 10 and 3 and added.
Sharing ideas without a verbal explanation	Sharing representations of a mathematical idea without providing a verbal explanation	*Task: Find 10% commission on $380 in sales* *Student approaches the board, writes:* *380 x .10 = 38* *$38.00* *Student walks back to her seat*
Sharing of student ideas through teacher explanation	Explanation of students' mathematical ideas carried out by the teacher	She knew that 10% of $100 is $10, so she added $10, $10, and $10 for each of the three

100 dollars to get 30. Then she knew that 80 is a little less than 100, so it had to be less than $10. She knows that 10 is 10% of 100, so 8 must be 10% of 80. She added 30 + 8 to get $38.

Each participation episode followed a student's mathematical idea from inception to conclusion. Each episode was coded for explanations and engagement with others' ideas, regardless of duration, and attributed to the student author or contributor. Narrative descriptions provided an overview of each episode and noted the sequence of student interaction around a mathematical idea. Students who partially or fully explained in an episode were added to the corresponding cell under each respective category. In addition, students who referenced or added onto the details of others' ideas in an episode were recorded in their individual cells. Students who participated in multiple episodes across the day were coded to capture all their participation. Coding all episodes in which students participate across the lesson provides a comprehensive picture of how students co-construct knowledge through interaction. Each episode was coded in this way and resulted in three tables which include all instances of explaining and engaging with others' ideas (by type) and labeled with each student contributor. Each student contribution was annotated with a narrative note and included the episode, problem, co-contributors, and a coded excerpt from the transcript. The process was repeated for each of the three time points and frequencies were tabulated in Microsoft Excel. For the analyses, students who explained (full or partial) and/or engaged with others' ideas (refer to or extend the details) on one or more occasions were assigned a 1 in each corresponding cell and students who did not take up opportunities to participate (full/partial explain or refer/add on to the details of others') were assigned a 0 in Excel in any relevant cell. For example, a student who fully explained, partially explained, referenced the details of another's idea, and did not extend a peer's idea across the lesson would receive a 1 in each of the first three corresponding cells and a 0 in the extend others' ideas cell, regardless of the number of daily contributions. The proportion of student interaction for each participation type in each time point were calculated.

Coding of teacher support

Teachers' questions can increase the incidence of student explaining (Franke et al., 2009). The ways in which teachers invited students to explain or engage, or how they followed up on students' initial responses across time points, reveals the opportunities presented to students to share their own ideas and collaborate with their peers. Analyzing teacher moves reveals not only the opportunities students have in the whole-class setting, but how the teacher supports the students once they have taken up the opportunity to participate.

To understand how the students and teacher jointly shape the act of doing mathematics, it was necessary to examine the role of the teacher. Only episodes that resulted in a codable level of student engagement were included in the analysis. That is, student interaction coded as fully or partially-detailed explanations or engaging with others' ideas by referring or extending details were included. Episodes that did not rise to a high level of engagement, such as providing the next step to an algorithm or the answer to a calculation, were excluded. To capture detailed features of how the teacher supported students to engage in mathematical conversations, the parsed student episodes were viewed again with attention paid to the moves teachers made to support student interaction. The teacher enacted both initial moves to elicit explanations or engagement with others' ideas and follow-up moves to elicit additional details around students' thinking. Thus, to capture teacher moves identified by the literature and observed in the classroom, each episode was coded for two types of moves: teacher invitation moves and teacher follow-up moves (Franke et al., 2015; Webb et al., 2019). Within each category, both a priori and emergent codes were assigned descriptive labels based on the specific move observed. The details and sequence of teacher moves were added as an additional layer to the coded and annotated student episodes. Qualitative descriptions included patterns of sequenced moves and moves that resonated with developing hunches based on the quantity and quality of student participation over time.

The teacher carried out initial moves to invite students to explain or engage with others' and in-the-moment follow-up moves to elicit additional details and mathematical thinking. While the teacher made a variety of moves, only those moves that led to an explanation or engagement with others' ideas were included in the analysis. Table 4 provides a brief description of the four types of initial invitation moves. Invitation moves to explain or engage can yield high levels of student engagement; however, invitation moves can also result in student engagement around others' ideas or superficial or tangential engagement in the mathematics (Webb et al., 2019). Thus, teachers are often required to follow up on initial invitations.

Table 4. Teacher invitation moves to support student participation

Type		*Description*	*Example*
Eliciting Student Explanations			
EXP Q-G	General question	Asking a general question to elicit explanation	Would you mind sharing your idea?
EXP Q-S	Specific question	Asking a specific question to elicit explanation	Why [did you] minus?
Supporting Student Engagement with Others' Ideas			
EOI Q-G	General question	Asking a general question to explain peer's strategy	When you talked to the person next to you, which one did they do first?

EOI Q-S	Specific question	Asking a specific question to explain peer's strategy	Why do you think she was adding?

To further capture features of the role the teacher plays in supporting student interaction, each episode was coded for teacher follow-up moves. Webb and her colleagues found that teachers followed-up invitation moves with a variety of support moves to probe for details, scaffold to guide or support students, and position students as capable participants. Thus, provisional codes were applied using exploratory coding methods (Saldaña, 2013). The codes drawn from the literature were modified, revised, and deleted during the analysis and resulted in the types of follow-up moves described in Table 5. Episodes were assigned codes characterizing specific aspects of teacher support of student interaction. A table for each of the three time points was created to document the types of support students received. Frequencies were tabulated in Excel and additional tables were created to examine how teacher support may have changed over time.

Table 5. Teacher follow-up moves to support student participation

Type of Follow-Up Move			Example
Eliciting Student Explanations			
T EXP Q-G	General question	Asking general question to elicit additional details	What did you do after that?
T EXP Q-S	Specific question	Asking specific questions to elicit additional details	Where did you get the 40 from?
T EXP PRESS	Press	Pressing for new ideas around or connected to details	What allows me to take away the zero?

T EXP REP	Representation	Asking for representation of the mathematical idea to elicit additional details	Could you write down the steps of what you're saying? And then what? I'm just trying to understand.
T EXP RV	Revoice	Revoicing to elicit further explanation	You are telling me that I have $4 and I'm dividing it among 8 people?
T EXP EX	Student-initiated extension	Taking up student extensions to elicit additional explanations	You're changing the problem on us? What is it?
T EXP POS	Position	Positioning a student who explains as mathematically competent	When you are up there (*sharing at the whiteboard*), you are the teacher.
Supporting Students to Engage with Others' Ideas			
T EOI Q-G	General question	Asking general questions to explain or engage in peer's strategy	Did you ask them why?
T EOI Q-S	Specific question	Asking specific questions to explain or engage in peer's strategy	Where does the .8 come from?
T EOI SPACE	Space	Offering space to engage with others' ideas or reorienting the discussion toward others' ideas	Let's look into Luna's mind. Pay attention.
T EOI YIELD	Yield	Yielding the floor to students to engage with others' ideas	Extended productive student exchanges *without* teacher intervention (>5 student exchanges)
T EOI POS	Position	Positioning students who engage with others' ideas as mathematically competent	(*Directed to a student who added on to a peer's strategy*) That's a good way to see it.
T EOI REF	Refer	Referring students to the details of a peer's idea	Did you hear what she said?

T EOI RR	Recreate representation	Asking students to recreate representation of peer's idea	Don't believe her. Do it yourself.
T EOI CHECK	Check/challenge	Asking students to check or challenge the validity or correctness of a peer's idea	Why don't we check?

The objective of the analysis was to describe and track teacher support and student participation; however, they do not occur in isolation. Student participation and teacher support of student participation is relational in nature. That is, while specific teacher and student utterances reflected in lines of a transcript can be coded as a student's partial idea or a teacher's specific question coded as a follow-up move, meaning is ascribed to the utterances only in connection with each other. The varied ways students participate and how the teacher invites and supports students to participate serve to illustrate what it means to co-construct learning mathematics in an adult education mathematics class. Thus, the transcripts and both teacher and student codes were examined holistically to capture the multidimensionality of interaction in the classroom.

This study provides a comprehensive analysis of student participation and teacher support in the adult classroom setting. Analysis of classroom video footage attended to the ways in which students participated and how teachers provided opportunities for students to explain and engage with others' ideas. These in-depth analyses of classroom interactions detailed what it means to do mathematics in the adult algebra classroom.

CHAPTER 4: FINDINGS

Student Participation in Whole-Class Setting over Time

"[sharing with the class] Makes me feel comfortable because if we need help, we all figure it out step by step as a class which helps us all understand and learn a lot better." – Maria

Analysis of classroom interaction in Algebra 1 across three time points reveals that the quality and quantity of individual student participation and teacher support of student participation during whole-class instruction increases over time. The teacher expanded opportunities for students to explain and engage with others' ideas over time and the students increasingly took up the opportunities over time. In detailing the ways in which the students and teacher engage in the adult mathematics classroom, I begin by providing detailed analyses of the patterns of student participation in Time 1, 2, and 3. The analyses of student participation as evidenced by the increased frequency of student explanations and engagement with others' ideas are followed by an analysis of teacher support of student participation across the three time points. I conclude by summarizing whole-class participation across Time 1, 2, and 3, noting the change in classroom participation patterns, which includes the transition to more detailed student participation and teacher support of student participation.

Overall Student Participation across Time 1, 2, and 3

Ms. Diaz purposely organized her Algebra 1 classroom to provide opportunities for students to explain and engage with each other around their own mathematical ideas. Students participated in a range of ways across time while also advancing Ms. Diaz's goals for their algebraic learning. Students who actively participated may benefit by their own contributions and their peers' engagement with their ideas. Conversely, students who do not participate will have limited opportunities to clarify their ideas, complete partial understandings, or identify misconceptions or gaps in understanding. In an interview, Ms. Diaz spoke about providing opportunities for students to share their mathematical ideas in the whole-class setting:

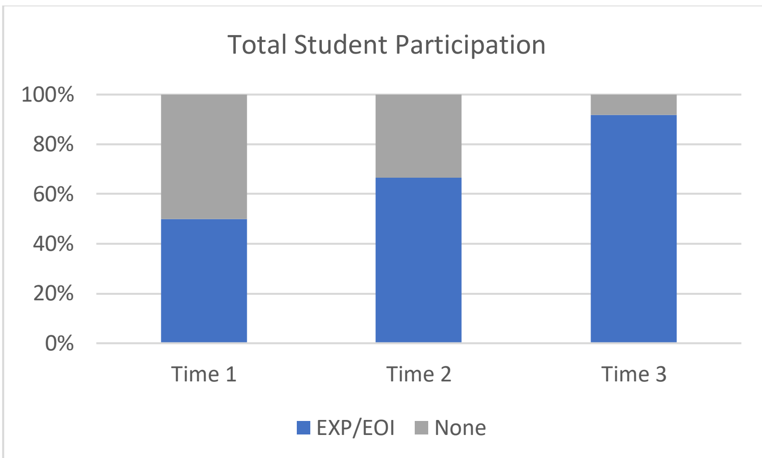
> You know about adult learners, if you give them more responsibility not only in pacing but to teach the class, then I think they really enjoy learning from their fellow classmates because they say if that student up there can do it then I can go to the board, too, and I can show them what I can do even though I'm going to make mistakes I can show them what I'm going to do.

By encouraging students to explain their thinking and engage with their peers' thinking, Ms. Diaz not only signals that she values and strives to understand students' sense-making, but also communicates that students are capable of learning from and teaching their peers. Ms. Diaz creates opportunities for students to share their thinking publicly in whole-class and supports adult students to see themselves as capable doers of mathematics.

Over time, student participation increased over time as did their opportunities to clarify ideas, complete partial understandings, and identify misconceptions. The opportunities arose either by a student-initiated move or following a teacher invitation or follow-up move. In particular, a shift in both general and detailed participation is evident over time. I provide proportional data along with brief descriptors to illustrate the change in student participation over time; specifically, I examine who participates and how.

Figure 4 presents the proportion of student participation based on video analysis of classroom interaction of whole-class conversations. The figure shows the video coding results when partial explanations, full explanations, and refers and extends others' ideas are counted as overall participation. Specifically, the figure answers the question of how many students participate at a codable level at each time point. To accurately capture the percent of student participation in whole-class at each time point, students who explained and engaged with another's idea or those who explained or engaged with others' ideas multiple times are counted only once. Explanations and engaging with others' ideas are coded without regard to accuracy or completeness. Those who do not explain nor engage with others' ideas are not counted as participating in the particular time point.

Figure 4. Percentage of Students Who Explained or Engaged with Others' Ideas at Any Level: Time 1, Time 2, Time 3

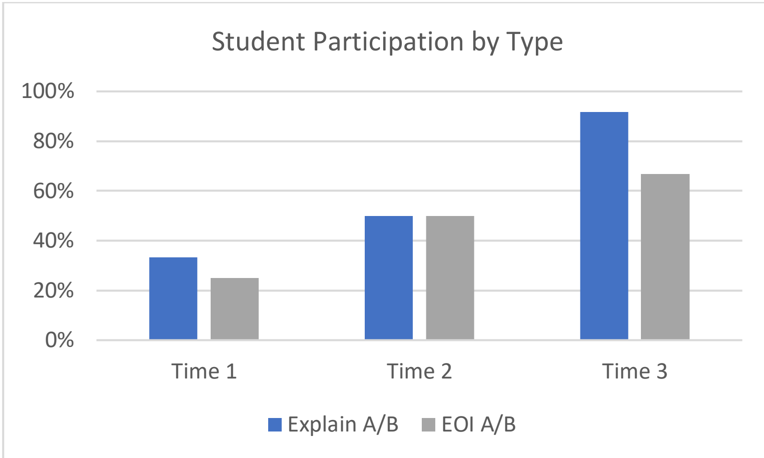


The three time points vary in terms of proportion of students who participated in whole-class instruction. At least 50% of the 12 students participated in either a general or specific way in all three time points. Participation steadily increased over time for both the target and total students. In Time 1, 50.0% of target students explained or engaged with others' ideas; in Time 2, 66.7% of students participated, and in Time 3, 91.7% of students participated at least once. Students participated the least in Time 1 (6 out of the 12 students) and the most in Time 3 (11 out of the 12 students). It is important to remind the reader that students participated in numerous ways throughout the algebra course; however, some participation did not rise to a codable level and is not captured here. Nor does this particular figure capture students who participated in multiple ways throughout the day by, for example, providing more than one explanation.

Student Participation by Type of Engagement

Figure 5 shows the video coding results for the percentage of type of participation in whole-class for each time point. Here, participation is partitioned into explanations (partial and full) and engaging with others' ideas (refers and extends). Students who both explained and engaged with others' ideas are counted once for each category, regardless of the number of contributions. The proportion of students who explained in Time 1, 2, and 3 (33.3%, 50.0%, and 91.7% respectively) and engaged with others' ideas (25.0%, 50.0%, and 66.7%) steadily increased over time. The increase in both the proportion of both levels of explain and engage with others' ideas over time point to the interconnected nature of the two prevalent participation types. As one type of participation increased, the other type increased similarly over time. In Time 1, one-third and one-quarter of students explained and engaged with others' ideas respectively. By Time 2, half of the students explained and engaged with other's ideas and by Time 3, all but one student explained, and two-thirds of students engaged with others' ideas. By Time 3, nearly all students took up opportunities to explain and most students responded to their peer's contributions.

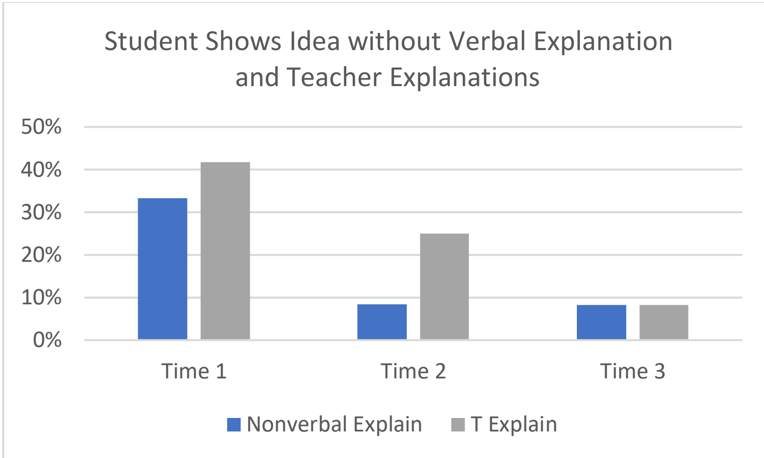
Figure 5. Percentage of Student Participation by Type: Time 1, Time 2, Time 3



Note: Explain A/B combines complete and partial explanations. EOI A/B combines extend and reference others' ideas.

Also presented are proportion of students whose ideas are explained by the teacher and proportion of students who shared their representations with the class but did not verbally explain (Figure 6). In contrast to the increased proportion of student explanations and engagement with others' ideas, the proportion of students whose ideas are explained by the teacher decreased over time (41.7%, 25.0%, and 8.3%) and the proportion of students who shared their ideas without verbally explaining decreased after Time 1 (33.3%) and remain constant in Time 2 and 3 (8.3%). Thus, the proportion of students who provided explanations or engaged in others' ideas may be inversely related to the proportion of students whose ideas are explained by the teacher. In other words, student thinking is on the table in all time points, but there is evidence to suggest that explaining shifts from the teacher to the student-author of the idea over time. Students' learning is connected to reasoning and dialogue. Thus, students who have opportunities for argumentation, justification, and questioning one another at a high level have opportunities to learn.

Figure 6. Percentage of Student Showing Ideas without Verbally Explaining and Students' Ideas Explained by the Teacher: Time 1, Time 2, Time 3

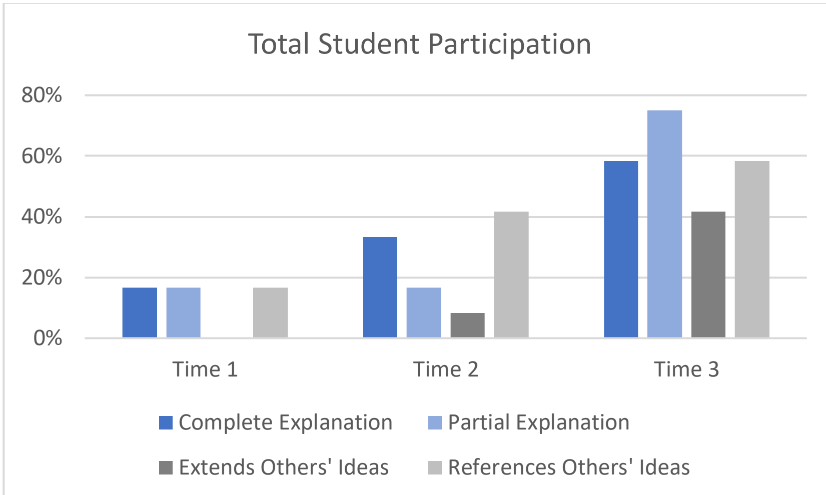


The data suggest that there are differences in the three time points. For example, in Time 1, one-third of students shared their representations on the whiteboard without verbally explaining. The student approached the board, re-represented the strategy written on her paper, the teacher asked the class if they obtained the same (usually "correct") answer, and the student returned to her seat without verbally explaining how she arrived at her answer. Only one student in both Time 2 and 3 shared their thinking in this manner. In other cases, the teacher explained the students' strategies essentially removing the opportunity for the student to explain. The teacher explained five out of the twelve student strategies in Time 1, three in Time 2, and one in Time 3. In both instances, the classroom community did not have opportunities to listen to, ask questions of, or comment on the author's idea. The level of student engagement was limited to nods of agreement or disagreement related to the strategy's correctness. Moreover, engagement with others' ideas in Time 1 was limited and may have been influenced by the number of nonverbal and teacher-voiced explanations.

Student Participation by Level of Engagement

Figure 7 presents the time point profiles for the proportion of students coded by participation type and level of detail: explanation (complete or partial) and engagement in others' ideas (adds on others' ideas and refers others' ideas). The proportion of students who participated in a specific or detailed manner increased across all three time points. Specifically, the proportion of students who provided a complete explanation and extended others' ideas increased 41.6% and 41.7% respectively from Time 1 to Time 3. Of note, there was no evidence of students extending others' ideas in Time 1, yet five out of the 12 students extended a peer's idea in Time 3. This shift may be due to the nature of the task as students completed algorithms with concrete steps to problem-solve, the increased shared authority between the teachers and students, or the increased space the teacher provided for students to engage with others' ideas without teacher interruption. In all time points, students offered complete explanations, however, the higher profile in Time 3 show that more student explanations emerged in the whole-class discussions. Moreover, the proportion of students who provide partial explanations remains constant in Time 1 and 2 (16.7%) and sharply increased in Time 3 (75%). A slightly different participation pattern emerges for students who reference others' ideas. That is, the proportion of students who reference others' ideas steadily increased over time from 16.7% in Time 1 to 41.7% in Time 2 and 58.3% in Time 3. Time 3 has the highest proportion of students participating in a general or partial way with up to three-quarters of students taking up opportunities to participate.

Figure 7. Percentage of Students Participation by Level of Engagement: Time 1, Time 2, Time 3



The profiles extend our understanding of student participation in several ways. First, we see the details of student engagement over time. That is, the profiles capture the shift in participation from teacher talk and non-verbal sharing to more active student engagement as students explain their own and others' mathematical ideas. Second, we see more individual students take up opportunities to participate over time. Specifically, half of students participated in a general or specific manner in Time 1 and by Time 3 all but one student (almost 92%) participated in a general or specific way. Nearly every student had the opportunity to share a mathematical idea with the class or engage with another's idea in whole class by Time 3. Third, students' mathematical thinking is integral to the algebra work in the classroom. The proportion of students who participate in both a general and a specific way increased from Time 1 to Time 3. Student interactions (explain and engagement with others' ideas) are closely aligned with the details (both general and detailed) of adults' mathematical ideas. Specifically, the prevalence of detailed explanations and extension of student thinking in Time 3 suggests that students participate by making the details of their thinking explicit and challenge, question, or add on to their peers' ideas. By Time 3, nearly 60% of students provided fully-detailed correct explanations and almost 80% of students extended a peer's mathematical idea. Fourth, the analyses thus far have addressed explanations and engaging with others' ideas as individual phenomena, with little attention paid to their relational nature.

Typical Episodes of Student Participation at Each Time Point

In Time 1, all of the explanations are procedural in nature as students make sense of and practice simplifying expressions using the order of operations. In Time 2 and 3, students have opportunities to construct their own strategies and peer engagement pivots from procedural inquires and comments to questions and noticings related to the details of students' individual mathematical thinking. As described in detail below, the students explain their thinking and engage in each other's ideas in varying degrees and detail in the public sphere. The analysis is presented chronologically and begins with Time 1.

Time 1: Nonverbal Strategy Share without Peer Engagement

Participation in Time 1 can be characterized as teacher-directed, articulation of procedural explanations, and minimal student engagement in others' ideas. Ms. Diaz led the classroom conversation, asked questions related to procedures and correctness, often explained students' strategies *for* the students, and in most cases, controled the sharing of student ideas via the whiteboard marker or document camera. Similarly, student participation drew on specific procedures or steps and relied on textbook number sets that lack context. Students simplified expressions and shared their ideas in three different ways by providing: a full explanation with choral peer engagement, a partial explanation with minimal engagement, and a nonverbal sharing without peer engagement.

In some cases, particularly in Time 1, students who worked on their ideas in small group or pairs did not describe their ideas at all in whole class. In these cases, a student represented her strategy on the whiteboard, the teacher asked the class if they obtained the same answer, the student returned to her seat, and the next student approached the board to share her strategy. The level of engagement was limited to agreement or disagreement while responding to the teacher's question about correctness. Multiple students did not have the opportunity to share their thinking around the same expression and, while students checked in with the details by following along as they correct their own work, they did not explicitly engage in the details of the author's thinking. The classroom community did not ask questions, note similarities or differences, or comment. Although Maria shared a visual representation of her strategy, she did not verbally explain how she arrived at her answer (Figure 8).

Maria writes $((17 + 7) - (10 \quad 5)^2) + 2^2$ on the whiteboard

1. Maria: Do I write like I have on my paper?
2. T: Do it like you did it.

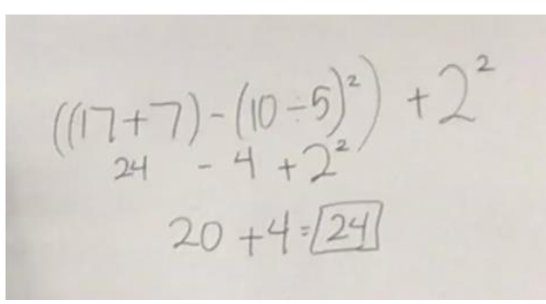
Maria writes 24 – 4 + 2^2

20 + 4 = 24

3. T: Okay class, for #9 she got 24. You all agree?
4. Ss: Yes!

Maria returns to her seat.

Figure 8. Maria's Representation.



Time 2: Students Partially Explain Their Ideas with Some Peer Engagement

Students provide partially and fully detailed explanations in Time 2. The following excerpt is from a mental math exercise to find the product of 2.99 x 3. Ms. Diaz invited students to explain their mental math strategy (Line 1). Aida and Lucia took up the invite (Lines 2, 3) and provided answers – one correct (897) and one incorrect (997). Notice, Ms. Diaz did not call attention to the error. Instead, she asked a follow-up question to elicit additional details (Line 4). In order to multiply 2.99 efficiently, Lucia modified the problem. She eliminated the decimal and applied her knowledge of friendly numbers, "It's [299] almost 300." Lucia multiplied using number facts that can be easily generated, 300 x 3, but made a calculation error to obtain 600 (Line 5). Again, the teacher did not highlight the error. Rather, Ms. Diaz asked a follow-up question to check for understanding (Line 6). In Line 7, Lucia adjusted her initial answer to the correct one (897). Aida, who obtained the correct answer in Line 2, noticed Lucia's calculation error and responded by sharing her skip-count strategy "3, 6, 9" but it interrupted by the teacher (Line 8). By Line 9, Ms. Diaz began to drive the discussion. Later, in Line 13, Aida reiterated her idea, "multiply by 3?" and Lucia followed-up with the idea by adding the next step in Line 14, "and then take away 3."

1. Ms. D: What would you get for 2.99 x 3. How did you solve that one mentally?
2. Aida That would be 897
3. Lucia: 997
4. Ms. D: How did you break it down so you could do it mentally?
5. Lucia: It's almost 300, so 300 x 3 is 600, 600 take away 3
6. Ms. D: You said it was what?
7. Lucia: 897
8. Aida: 3, 6, 9 and then you --

9. Ms. D: 800 okay, what did you say it was? 897. 299 is a difficult number to multiply, isn't it? Right Kalifa? What is an easier number to multiply 3 by? What is the number next to 299?

10. Students: 300

11. Ms. D: 300 is really close to 299, what about if I do 300? What do I need to do to 300 to get it back to 299?

12. Kalifa: Make it more zeros, wait --

13. Aida: Multiply by 3?

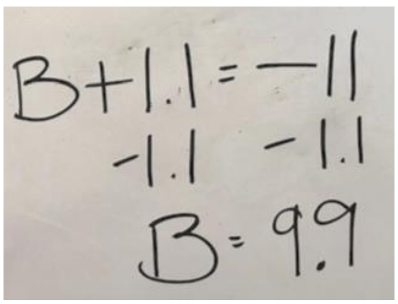
14. Lucia: And then take away 3.

Here, Ms. Diaz and the students were renegotiating norms and developing shared authority. A tension exists between Lines 1-8 and Lines 9 – 14. The first set of dialogue centers student ideas; the second set centers the teacher's ideas. In the beginning of the exchange, students interjected without raising hands and Lucia shared a strategy that was not modeled by the teacher. Aida attended to Lucia's explanation and proposed an alternative counting strategy (that Lucia did not take up). Ms. Diaz focused on the strategy rather than the answer and asked follow-up questions to elicit the details of Lucia's thinking. By the second half of the exchange, Ms. Diaz's authority over the mathematical ideas is evident. Ms. Diaz limited the exchange in four ways: 1) Lucia was not given space to explicitly respond directly to Aida, 2) Ms. Diaz's interruption limited Aida's explanation, 3) Ms. Diaz invited Kalifa (Line 9) but did not follow-up with Kalifa's invalid idea (Line 12), and Ms. Diaz explained Lucia's ideas (Line 9, 11). In this example, the teacher created the representation on the board and any mathematical ideas had to go through her, which was common in Time 2 and may have limited direct student-student interaction. That said, the strategies shared during Time 2 were unique, student-developed strategies.

Time 3: Students Partially Explain and Peers Engage with Their Ideas

I observed students partially explain their ideas to the class community while peers engaged with their ideas. In these instances, students shared a portion of their thinking and their peers showed sustained engagement with the details. In this case, the whole-class community worked together to make sense of Lucia's invalid strategy by offering suggestions that nudged her toward a correct solution. Although the strategy's author, Lucia, took the lead, her peers asked questions, kept tabs on the evolving idea, shared and corroborated feelings of confusion, expressed and clarified misunderstandings, challenged details, and simultaneously worked on and verified calculations. The episode began when Lucia shared an incorrect solution ($B = 9.9$) for the equation $B + 1.1 = -11$ (Figure 9). After much discussion, the students recommended substituting 9.9 for B to check her answer. Confused, Lucia changed her answer to -9.9 without understanding why (still incorrect). The teacher encouraged Lucia to think about the negative integers in the equation as money owed and encouraged her to look at the equation as a story, "You owe me $9.90 and you pay me $1.10. Are you still going to owe me $11?" Lucia and her peers grappled with the application of inverse operations to isolate the variable and attempted to make sense of the memorized "rules" for working with negative. At this point in the whole-class conversation, Lucia had adjusted her answer, B = -9.9, but remained unsure as she and her peers continued to try to make sense of the equation.

Figure 9. Lucia's Initial Strategy.



The following excerpt includes the subsequent conversation and illustrates students engaging with Lucia's representation and partial explanation.

1. Maria: Ms. Diaz, how did she get a -9 there?
2. T: No, she got a 9.
3. Maria: How did she get a 9 there?
4. T: That's why I'm asking her to explain.
5. Irma: We're all lost, man.
6. T: She told Luna she's trying numbers. She said that her answer is –9.9.

Maria engaged with Lucia's idea by asking specific questions to help her make sense of Lucia's idea (Line 1, 3). In Line 4, Ms. Diaz resisted explaining Lucia's idea and instead positioned Lucia as the expert and explainer. Irma used "we" to express the students' general confusion (Line 5). By line 6, Ms. Diaz's noticed Lucia's discomfort and shared information gleaned from small group problem-solving – that Lucia substituted multiple numbers to try to obtain -11. Here, she directed the discussion toward Lucia's strategy and answer, again positioning Lucia as the expert.

7. Kalifa: 11 dollars
8. Lucia: 11
9. T: Well, you had to owe me more because if you pay me $1.10 and you still owe me $11, how much did you --
10. Kalifa: $1.10? Oh, that one is the ten? I thought it was a penny.
11. T: No, it's ten cents.
12. Malena: Then it would be 1.11

In line 7 Kalifa added context to the decontextualized problem and Ms. Diaz took up her idea (Line 9). They suggested thinking about the numbers in terms of money -- negative numbers are monies owed. Kalifa expressed a conceptual misunderstanding related to place value (Line

10), Ms. Diaz corrected her (Line 11), and Malena added on to Kalifa's idea by sharing the total had .10 been one penny (Line 12).

13. T: So how much money did you owe me originally?
14. Maria: Oh! I got it now, I got it!
15. Malena: I think we all got confused with the decimal.
16. Maria: I got it. I got it! So pretty much, you were confusing the crap out of me, pretty much what you had right here, it's like what we were doing at the beginning put 11, 1.1 on both sides, so try to get your *B* by itself.
17. Malena: They are both negative, so –11 and –1.1, instead of subtracting, you're adding because they have the same sign.
18. Maria: Yeah. So instead of adding, yeah, what she just said, so then for this, you get 12.1. That is how much money your originally owed.
19. Lucia: Ahh!

Ms. Diaz referred back to the problem after applying Kalifa's money context (Line 13). After attending to the discussion, Maria had an epiphany (Line 14) and referred to the method Ms. Diaz introduced to balance equations (Line 16). Malena used the "we" pronoun to express the community's confusion around place value (Line 15) and added onto Maria's idea by referencing one of the "rules" for working with negative integers (Line 17). Maria obtained the correct answer and used "owed" to indicate a negative number (Line 18). At this point, Lucia expressed confusion (Line 19).

20. Maria: I think you just –
[T approaches the board. Lucia and the T and speak Spanish to one another as the class listens. Lucia changes her solution on the board to *B* = -12.1]
21. Kalifa: So, she said that you add that 1.1 to both sides and then negative to a positive and you get 12.1?
22. Malena: They have the same sign, they are both negative, add them. You don't take away.
23. Kalifa: They have the same sign, so you add them. Yeah, right.

24. Malena: You don't take away. Then you'll get 12.1

Ms. Diaz interrupted Maria to give Lucia the space she needed to consider her classmates' suggestions and think through her own strategy (Line 20). According to the Spanish speaker who sat to my left, Lucia described and took up her peers' ideas by combining the two negative numbers to obtain -12.1. Kalifa tried to make sense of the strategy (and arbitrary rules) by revoicing a portion of Lucia's strategy (Line 21). Malena added on to Kalifa's thinking and reminded her of one of the "rules" for combining negative numbers to obtain -12.1 (Line 22, 24).

25. T: See how it works, put it in there now. See if –12.1 + 1.1 is going to give you a negative.
26. Maria: A positive.
27. Malena: A negative.
28. Maria: But the answer is positive.
29. T: Which answer is positive?
30. Malena: No, it's a negative 12.1
31. T: Maria, go again, check it.
32. Maria: Yeah yeah yeah.
33. Kalifa: I thought two negatives become a positive.
34. T: Only when you multiply.
35. Malena: When you multiply a negative x a negative, you get a positive. When you add a negative and a negative, you get a negative.
36. Lucia: You are so patient. I would have yelled at me already.

The teacher recommended that the students substitute -12.1 for *B* to check their answer. The remaining lines (26 – 35) sound similar to a "Who's on first" skit and capture adults' attempt to make sense of procedures modeled by the teacher. Students challenged the sign of the answer and the episodes concluded when Malena reiterated the rules for multiplying and adding negative numbers. The episode concluded when Lucia positioned the class community as patient as they gave her space to make sense of the mathematics (Line 36).

I chose this episode in Time 3 because 1) it captures the developing mathematics community and 2) it demonstrates the challenges adult students face when making sense of decontextualized, procedural problems that are so common in adult education classrooms. In this exchange, students did not raise their hands or wait to be called on to participate. Rather, they directly addressed the author of the strategy and spontaneously interjected their mathematical ideas. Lucia held the whiteboard marker – and the floor. There are a number of instances when the students exchanged ideas without the teacher's intervention (Lines 14 – 20, 21 – 24). This free-flowing exchange of ideas between the community and the author reflects students' increased comfort with one another and shared authority. Students used "we" instead of "I" to demonstrate their membership in a mathematical community and Lucia addressed the community as "you" when she highlighted their patience. The teacher positioned students as competent when she deferred to their ideas (Line 4, 6). Students take risks, they make mistakes, and they feel supported doing so.

Decontextualized, procedural tasks can prove challenging for adults. We see that Kalifa, Malena, Maria, and Lucia struggled to remember the seemingly arbitrary rules to working with positive and negative integers. Additionally, both Kalifa and Ms. Diaz added context to support students to make sense of negative integers (Line 7, 9). Despite the limiting nature of procedural, decontextualized tasks, students strove to engage with the author's ideas and their own sense-making. They could have had an opportunity to make sense of the problem had Ms. Diaz encouraged students to approach the problem in ways that made sense to them like she did when assigning conceptual tasks in contexts (see Malena's episode later in this chapter). However, despite the limitations, the students did not abandon the work or their classmates. They persisted by actively participating and supporting Lucia to the end.

Individual Episodes of Student Participation

The analyses presented thus far have described student participation in whole class over time. To understand more about individual students' patterns of increased participation and the openings students with different kinds of mathematical understandings take up across time points, it is necessary to examine an individual student participation profiles. Patterns emerge across the 66 participation episodes – nine from Time 1, 18 from Time 2, and 39 from Time 3 – and will be discussed. Students take up opportunities to explain and engage with others' ideas at each time point; however, no two students participate in the same ways over time. Behind every code and figure illustrating student participation there is an individual student who engages with a mathematical task, communicates with her classmates around problem solving strategies, interacts with the teacher, and explains her mathematical ideas. Students take up different opportunities to participate in detailed ways over time; there is no single "right way" to participate. Among the profiles below are the adult school's upcoming valedictorian, a student who will repeat Algebra I for the third time, two students with learning differences, and a student who exited traditional school directly after primary school. All of the profiles represent students of color, all are identified as "drop-outs" by the adult education system, and many participate at increasingly higher levels across the curriculum.

When communicating with each other in the teacher-directed whole-class setting, students both explained their ideas and engaged with their peers' ideas. In some cases, students explained complete or full ideas; in other cases, students explained their partial ideas; in still other cases, students did not verbally explain, but they represented their ideas, without a verbal explanation, on the whiteboard in front of their peers. At times, peers engaged with the author's ideas and at other times, they did not. In most cases, students explained their own ideas, and in a few cases, the teacher explained the students' ideas. Students explained both teacher-constructed strategies and student-constructed strategies.

Individual student profiles reveal different levels and types of participation over time (see Figure 8). Each profile represents multiple participation episodes for a student. For example, Bianca's profile indicates that she participated once during the duration of the course. Her single episode took place during Time 3 and captures a fully-detailed explanation. Jose's profile captures a total of five participation episodes across the three time points: a fully-detailed explanation in Time 1, two episodes of referring to the details of a peer's idea in Time 2, and one fully-detailed explanation and one partially-detailed explanation in Time 3. Some students participate in every time point, others participate in only one or two time points. Other students participate more frequently, like Malena, whose profile indicates a total of 14 episodes beginning with two partially detailed episodes in Time 1, to one fully-detailed and two partially-detailed episodes in Time 2, and six fully-detailed episodes and three partially-detailed episodes in Time 3.

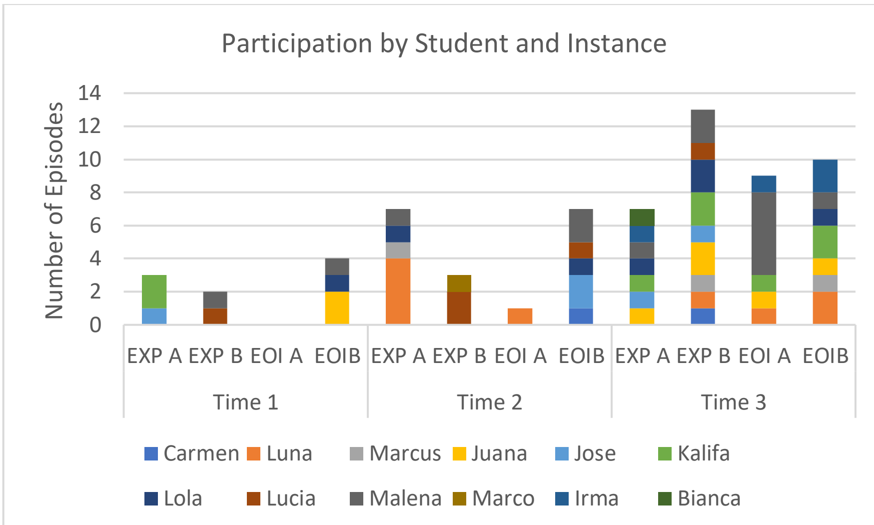
It is clear that individual adult students participate in different ways across the Algebra I course. The twelve individual student participation profiles provide evidence that participation is dynamic, individualized, and ongoing. The twelve profiles capture all participation episodes, not only the highest level, and illustrate the diversity and quantity of individual student participation over time. The nature of participation may vary depending on the mathematical task, the context surrounding the task, teacher support, the students sitting nearby, or comfort with a particular concept. Additionally, on some days the context may be particularly engaging or interesting or the students may feel particularly efficacious or curious. Notice, no two students' participation looks the same.

Figure 10. Individual Student Participation Profiles: Time 1, 2, and 3

Note. Explain A (fully-detailed explanation). Explain B (partially-detailed explanation). EOI A (extends others' ideas). EOI B (references the details of others' ideas).

Figure 9 provides an illustration a composite of the 12 students' participation in whole class across the three time points. The figure shows all of the participation episodes for the 12 students and gives a sense of the total number of instances of each type and level of participation over time. Each student is assigned a color (e.g., Kalifa's participation episodes are green) and the height of the bars are the number of episodes in each category at each time point (e.g., the green and blue bar to the far left indicates three fully-detailed explanations – two from Kalifa and one from Carmen in Time 1).

Figure 11. Total Target Student Participation Episodes by Instance: Time 1, 2, and 3



Note. Explain A (fully-detailed explanation). Explain B (partially-detailed explanation). EOI A (extends others' ideas). EOI B (references the details of others' ideas).

The figure presents the time point profiles for the total number of coded participation episodes for all students in whole-class instruction and includes both general and detailed explanations and engagement with others' ideas, regardless of correctness. Moreover, it is evident that the number of episodes of explain and engage with each other's ideas increase over time with students taking up more opportunities to participate in Time 3. For example, students provided three full and two partial explanations in Time 1; conversely, students provided a total of seven complete and thirteen partial explanations in Time 3. Similarly, there are no instances of extend others' ideas and four instances when students referred to others' ideas in Time 1; in contrast, there are seven instances when students extended others' ideas and 13 instances when students referred to others' ideas in Time 3.

Individual profiles of participation episodes reveal diverse participation profiles, when combined together, indicate a trend toward increased participation over time. Students who take up the fewest opportunities to participate in Time 1, 2, and 3 (Bianca, Carmen, Irma, Marco, and Marcus) do not participate in Time 1. They do; however, participate in Time 2 and 3. Some students consistently take up opportunities to participate over time and explain or engage with others' ideas across the three time points (Malena, Lola, and Lucia). And others participate sporadically with increased participation at one time point (Kalifa, Juana, and Luna). Some students participate in general ways (Marco, Lucia, and Carmen) while others explain and engage with others in more detailed ways (Malena, Luna, Lola, and Kalifa). That is to say that no one profile fits all students. While there were instances when students spontaneously explained and engaged with their classmates, the Ms. Diaz's support served to help students probe more deeply into their own and their peers' mathematical ideas.

To support students' continued understanding, Ms. Diaz provided multiple opportunities for students to make sense of the mathematics and offered support that built on the knowledge that adults brought with them to the classroom. The next section explores the details of teacher support of students' mathematical thinking and engaging with others' ideas.

Summary of student participation

Student participation is dynamic and changes over time. The proportion of Algebra 1 students who participated at any level increased over time from 50% to 91.7%. Similarly, the proportion of students explaining (Time 1: 33.3%; Time 3: 91.7%) and engaging with others' ideas (Time 1: 25%; Time 2: 66.7%) at any level increased over time. In contrast, the proportion of students sharing their ideas without verbal explanations (Time 1: 33.3%; Time 3: 8.3%) and listening to the teacher explain their own ideas (Time 1: 41.7% Time 3: 8.3%) decreased over time. Over time, students increased their participation in both general and specific ways. Moreover, individual students participated in different and often increased ways over time.

Teacher Support of Student Participation

This section highlights the teacher's role in supporting student participation by examining the teacher moves to support student explanation and engaging with others' ideas in the whole-class context. First, I provide an overview of the broadening range of specific ways in which Ms. Diaz supports student to explain their own ideas and engage with the ideas of others across time. Second, I identify the proportion of students who receive support from the teacher to explain their own ideas or engage in the ideas of others. Third, illustrate the trend toward more detailed teacher support and a broadened range of moves by spotlighting Malena's participation across time. Fourth, I summarize the evolving nature of Ms. Diaz's support of student participation over time.

Teacher talk in Algebra 1 constitutes approximately two-thirds of total class talk time. Ms. Diaz's talk time includes community building, facilitating social emotional activities, sharing announcements, discussing current events, providing instructions, unpacking problems, and most importantly, supporting productive mathematical conversations. How the teacher does or does not support students' mathematical thinking can influence the extent of details that are shared and the opportunities students have to engage with their classmates' ideas. While the major focus is on differences between the three time points for all students in the classroom, I also illustrate the details of teacher support and the ways she elicits students' thinking over time.

Teacher Support of Student Explanation

I observed the teacher enact a range of moves to support students to explain their strategies. To elicit student thinking, the teacher asked a variety of questions across the three time points. Asking questions can promote mathematical discourse and increase participation by eliciting student thinking and engaging with others' ideas as well as modeling the types of questions that students can ask each other. The teacher can elicit additional student thinking by asking follow-up questions, scaffolding, adding context or real-world scenarios to support meaningful connections, pressing for additional student thinking, and encouraging peers to engage in the author's ideas (see Webb et al., 2019). Importantly, eliciting additional details supports both the author to make public the details of their mathematical thinking and the classroom community to make sense of the author's ideas.

As will be described below, the range of teacher moves to support student participation can increase the occurrence and extent of student explaining by encouraging students to share their ideas in the public whole-class space and pressing students to clarify and justify their mathematical thinking. In contrast, the absence of teacher support can both limit students to explain their strategies and, at times, provide space for students to play a larger role in engaging with their classmates' strategies. To elicit additional details from a student author of a strategy in whole-class, Ms. Diaz probes the student's thinking by asking both general and specific questions related to the ideas shared in the public space. In many cases, the probing of students' explanations uncovers details or further elaboration about problem-solving strategies. However, in some cases, Ms. Diaz's probing of students' explanations serves to highlight the correct or incorrect execution of procedures rather than foster a deep understanding of concepts or students' problem-solving strategies.

Teacher Support of Students' Engaging with Others' Ideas

Consistent with other research (Webb et al., 2019, Franke et al., 2015), I found that the teacher both initiated invitation and in-the-moment, follow-up moves to encourage support engagement with others' ideas. A description of the two types is described below.

Teacher Invitation Moves

I observed the teacher carry out a variety of initial moves to invite student engagement, such as invite students to explain another student's strategy ("What did your partner do?"), attend to another's strategy in a general way ("Did you see what she did?"), attend to another's strategy in a specific way ("Why do they multiply by .8?"), make connections to another student's strategy ("Is that like Luna's strategy?"), and investigate and correct to identify and help the author address an error (Can you help Marcus and find his mistake? Can you help him?). At times, student engagement ensues from the teacher's initial move. For example, when students sought to find Marcus's error, they closely attend to the details of his strategy, provide an alternate explanation (in this case, substituting a variety of numbers in the inequality to prove Marcus wrong), and offer a correct solution.

In the following example, the teacher's initial move invited students to share a portion of their partner's strategy after a brief turn-and-talk (Line 1). The teacher invited the entire class community to reflect on their partner's thinking and asked volunteers to share their partner's idea. Every student is given the opportunity to make sense of their partner's idea and make public an idea other than their own. Here, Juana takes up the invitation and explains a portion of Jose's strategy.

1. T: So, when you talked to the person next to you, which one did they do first? Did they tell you what they did first?
2. Juana: They (Jose) did division first, they broke it down – they divided. 6 divided by 2 is three –

Teacher invitation moves to explain and engage in others' ideas in Time 1 were limited to general questions (see Table 6). Contrastingly, Ms. Diaz enacted both general and specific moves in Time 2 and 3 to invite students to explain or engage with the ideas of others. As mentioned previously, the procedural nature of Time 1 may limit student participation and teacher support of student participation. Classroom conversations around procedures result in follow-up moves around those same procedures. The teacher's moves were limited to correct solutions and correct strategies. Additionally, a number of problems were solved chorally around agreed-upon steps set forth by the teacher. Thus, individual students did not receive specific support related to their individual ideas. Moreover, due to the procedural nature of the problems in Time 1, there was little room to elicit further details such as, comparing strategies or predicting, from students or their classmates.

Table 6. Teacher invitation moves to support student participation: Time 1, 2, and 3

Category of Invitation Move	Teacher Move	Time 1	Time 2	Time 3
Explain				
General Question	Would you mind sharing your idea?	•	•	•
Specific Question	Why [did you] minus?		•	•
Engage in Others' Ideas				
General Question	When you talked to the person next to you, which one did they do first?	•	•	•
Specific Question	Why do you think she was adding?		•	•

Note. The "•" indicates that the teacher enacts the move at least once during the time point.

At other times, the teacher's request to engage with others' ideas did not result in productive conversation around the mathematics; thus, the teacher carries out follow-up moves to elicit additional engagement around students' mathematical ideas.

Teacher Follow-Up Moves

I observed the teacher make a number of impromptu follow-up moves, which are moves based on the details of student thinking in the whole-class context. It should be noted that the specific questions asked were not a bulleted list of pre-planned questions, a hierarchical list of question stems, nor a heuristic process designed to streamline the messy business of teaching. Rather, follow-up moves take place in-the-moment and rely on the teacher's extensive knowledge of mathematics and adult students' mathematical thinking. They are enacted in the moment and build on students' mathematical ideas.

In the whole-class context, the teacher's follow-up moves supported students to make sense of and engage with their peers' mathematical ideas, to help students notice patterns and broader mathematical concepts, and help students question each other and expert's (e.g., the textbook, a supervisor, the news) ideas to become critical thinkers and consumers of mathematics outside the classroom. Consistent with Webb et al. (2019), I observed the teacher ask specific and general questions, restate and challenge another's ideas, add on and refer to another's ideas, provide space for students to explain, and position students as competent (see Table 7).

At times, the teacher enacted moves unique to the adult education classroom. Ms. Diaz supported students to explain or engage with other's ideas in five different ways that are unique to the adult education classroom: develop a representation of their thinking for the classroom community (Representation), extend or create a new context for an existing problem (Student Extension), yields the floor to support students to engage without teacher intervention (Yield), challenges students to engage with a student's representation by recreating it (Recreate Representation), and encouraging students to check the correctness of a student's strategy by developing their own to compare solutions (Check). Contrary to engagement in primary classrooms, the teacher in the adult algebra class rarely enacted moves to explicitly support students to use the move. For example, the teacher asked, “Do you agree with her?” rather than coached students to share their thinking with a conversation stems, such as, “Say, ‘I’m disagreeing because…’” It may be adults’ life experiences at home and in the workplace that prepare adult students to challenge, negotiate, and offer suggestions in the classroom or it may be the adult educator’s need to support students in ways that are sensitive to their position as experienced adults rather than children in a classroom.

Table 7. Teacher follow-up moves to support student participation: Time 1, 2, and 3

Category of Follow-Up Move	Teacher Move	Time 1	Time 2	Time 3
Eliciting Student Explanations				
General Question	What did you do after that?	•	•	•
Specific Question	Where did you get the 40 from?		•	•
Press	What allows me to take away the zero?		•	•
Representation	Could you write down the steps of what you’re saying? And then what? I’m just trying to understand.		•	•
Revoice	You are telling me that I have $4 and I’m dividing it among 8 people?		•	•
Student Extension	You’re changing the problem on us? What is it?		•	•
Supporting Students to Engage with Others’ Ideas				
General Question	Did you ask them why?	•	•	•

Specific Question	Where does the .8 come from?		•	•
Space (Pauses interaction and reorients toward Ss idea)	Let’s look into Luna’s mind. Pay attention.		•	•
Yield (gives Ss the floor to EOI)	Extended productive student exchanges *without* teacher intervention (>5 student exchanges)			•
Position	That’s a good way to see it.	•	•	•
Refer	Did you hear what she said?		•	•
Recreate Representation	Don’t believe her. Do it yourself.			•
Check	Why don’t we check?			•

Note: The “•” indicates that the teacher enacted the move at least once during the time point.

One notable move unique to the adult algebra classroom is the Student Extension move. The adult education teacher took up student generated problems tangential to the teacher-generated problems. Adult students posed, “What if” questions related to the current subject. For example, Aida extended the teacher’s original problem to find four groups of $7.25 and, as a student making sense of decimals asked, “Say it was just 4. 4 times 7.25? So, it would be 28.100?” On another occasion, a student spontaneously wanted to develop an equation for the number of seconds her 8-year-old son has been alive. And on another occasion, students discussed the rate needed to walk in order to arrive at the school’s Health Fair in 40 minutes. Students asked a series of questions:

What if you go and stop?

What about if you have small feet? Short legs?

What about the distance of your legs?

Have a short person walk the same distance as a tall person and see how long the tall person and the short person take and then average it out to see how much each of them took.

What about if, even if you have long legs, but you are lazy?

Ms. Diaz followed the students' natural curiosity and built in spontaneous, in-the-moment lessons to maximize the wonderings and stories students tell related mathematics.

The teacher carried out various in-the-moment follow-up moves in whole-class. In Time 1, the teacher enacted general probing questions, revoicing, and referencing students' ideas while in Time 3, she exhibited all of the moves in the aforementioned table. That is, the teacher increased the variety of support moves over time. The teacher demonstrated that she is able to adapt her support moves to better respond to classroom community's developing ideas. Although follow-up moves to engage students with others' thinking were not as prevalent as moves to elicit student explanation, the teacher carried out a number of effective support moves for students to engage with each other's ideas.

Here I provide an example of the teacher's support moves to engage students with others' ideas within the context of explanation in the whole-class setting.

In this example, students work to solve a math story from the textbook to find the total number of aces served in 70 games when the player ace average is .3. The math problem requires a lengthy discussion around tennis, matches, and the definition of "ace." With time, the students make sense of the story and, after the teacher asks for a volunteer, Lola accepts the Expo marker and approaches the board to share her thinking. Lola adds a written representation to the whiteboard. Then, Ms. Diaz elicits additional details from Lola and supports students to engage with her idea. The teacher support moves help Lola further detail her thinking and help the students clarify their understanding of Lola's strategy.

1.	T:	Who would like to share? [Lola volunteers and begins to document her strategy on the whiteboard] So I see that she [Lola] is writing the information that they gave us in the paragraph.
2.	Lola:	The total aces that he served during the 7 games is for the whole 7 games, not each game is 21. It's 70 [games]. The average is 0.3, that means is about around 1/3 of the whole one, right? So, 21 is about 1/3 of the 70.
3.	T:	Question, teacher! How did you get that that is about 1/3? How is .3 about 1/3
4.	Lola:	Because I thinking about centimeters that has 10 millimeters. .3 is 3 + 3 + 3 is 9 is just one more milli-- millimeter in Spanish. I don't know how to say in English. To make the whole one, so it's about 1/3 of a whole. And then, the aces 21 multiply 21 x 3 will be 63 is close to 1/3 of 70. That makes me think that the answer is reasonable.
5.	Irma:	She rounded, huh?
6.	T:	Did you hear what she said?
7.	Maria:	Yeah, she did .3 is the same as 1/3. Three 21's is 63 is close to 70.
8.	T:	What makes you think that her answer is reasonable?
9.	Irma:	She double-checked it.
10.	T:	She says that 21 is about 1/3. I still have a question. How did she get that it is about 1/3? She said it, she said something about it.
11.	Malena:	She knows 1/3 is the same as .3. 3 + 3 + 3 + 1 is like 21 + 21 + 21. 21 + 21 + 21 is close to 70. [continues interchange]

The teacher's initial move (Line 1) invites Lola to share her thinking and written representation. After Lola's explanation (Line 2), the teacher positions Lola as a "teacher" and someone who has mathematical authority with ideas worth sharing with the class (Line 3). Also, in Line 3, the teacher asks a specific follow-up question to elicit additional details about Lola's use of 1/3. Lola further details her strategy (Line 4) and Irma spontaneously engages with her own thinking around Lola's idea (Line 5). The teacher references Lola's strategy in a general way (Line 6), which prompts Maria to make sense of and reference the details of the strategy. The teacher asks a general follow-up question (Line 8) to which Irma explains with little detail (Line 9). Noticing that the students still have not made sense of Lola's thinking around 1/3, the teacher continues to probe for students' thinking. She references Lola's details again and carries out a follow-up move that asks a specific question about Lola's strategy (Line 10). After the teacher's final probe around this idea, Malena explains and adds onto Lola's thinking. The exchanges transpire in the whole-class public space and thus support the classroom community to engage in Lola's strategy.

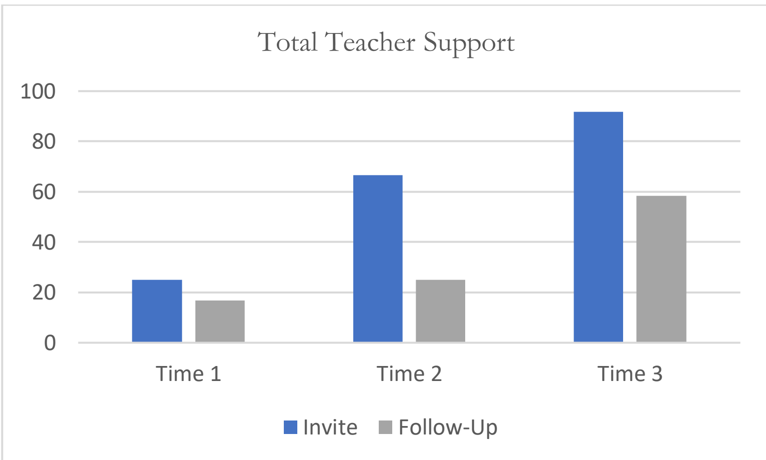
This example illustrates the interplay and relational nature of explanation and engaging with others' ideas. Teacher moves do not occur in isolation, rather the moves work together in concert to elicit student's mathematical thinking and engagement with others' ideas. In this excerpt, the teacher support moves position Lola as a competent mathematician, reference and highlight aspects of Lola's thinking, and through repeated questioning, elicit the details of students' thinking around Lola's idea. Ms. Diaz's support moves focused students' attention on their understanding of 1/3 or .3 and make visible Lola's personalized sensemaking of three tenths.

The teacher elicited student thinking in different ways across the three time points. In Time 1, Ms. Diaz's support paralleled the procedural nature of the task as she drives the students' work forward. In Time 2, teacher support of student explanations reflected the conceptual nature of the task and supports students to develop and make sense of student-constructed strategies. In Time 3, teacher support varied by task, context, and cognitive demand of the task and included space for students to support others' thinking without teacher involvement. The results reflect the nature and limitations of teacher support of student participation at each time point.

Incidence of Teacher Support Over Time

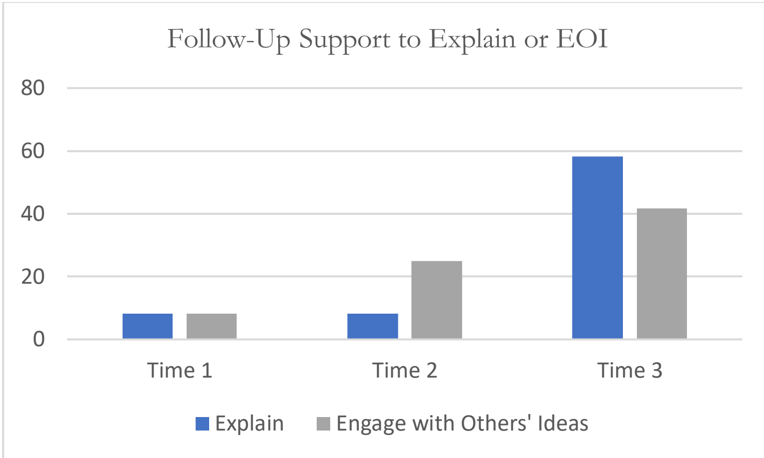
As documented in the previous section, the proportion of students who explained and engaged with others' ideas increased over time. Similarly, the incidence of teacher support of explanation and student engagement that were directed to specific students increased over time. Specifically, the proportion of individual students who receive initial invitations to participate increased over time from 25.0% in Time 1, to 66.7% in Time 2, and 91.7% in Time 3 (Figure 4.10). That is, nearly all of the students received an invitation to explain in Time 3 compared to one quarter of students in Time 1. The proportion of teacher's follow-up moves follow a similar trend. Teacher support for students generally increased from Time 1 to Time 3 (16.7%, 25.0%, and 58.3). That is, seven out of the 12 students in Time 3 received an invitation to explain their idea or engaged in others' ideas compared to two students in Time 1.

Figure 12. Percentage of Individual Students who Received an Invitation and/or Follow-Up Teacher Support to Explain or Engage with Others' Ideas: Time 1, 2, and 3



An examination of teacher follow-up support reveals a similar trend. That is, the incidence of teacher follow-up support of explanation and student engagement that were directed to specific students increased over time. The proportion of individual students who received follow-up support to explain remained steady or increased over time from 8.3% in Time 1, to 8.3% in Time 2, and 58.3% in Time 3. Similarly, the proportion of individuals students who received follow-up support to engage in others' ideas increased from Time 1 to Time 2, and Time 3 (8.3%, 25.0%, and 41.7% respectively). The teacher supported six additional students to add details to their explanation and four more students to further engage with each other's ideas from Time 1 to Time 3. Of interest is the higher proportion of follow-up support moves to engage with other's ideas as compared to support to explain in Time 2. This is the single time point when the teacher supported a higher proportion of students to engage with others' ideas. As a reminder, this is Luna's Day when she dazzles her classmates with her mental math and ability to decompose numbers in a variety of ways. The teacher and students' genuine curiosity may have driven the engagement to understand the range of strategies on this day. The teacher supported students to engage with Dora and her peers' mental math ideas as a way to make sense of the ideas herself and support students to pursue their own understanding of Dora's ideas.

Figure 13. Percent of Individual Students who Received Follow-Up Teacher Support to Explain or Engage with Others' Ideas: Time 1, 2, and 3



There are other marked differences between time points in the ways the teacher probed for details and supported students to engage with others' ideas in different ways. For example, teacher support in Time 1 primarily focused on answers, correctness, and procedural fluency. Teacher questions in Time 1 include, "Why can't ___ be right?" "What did you do after that? Then you got what?" and "Is that correct?" When a student described a procedure for simplifying an expression using the order of operations, teacher support was limited to correctness of procedures and solutions. Errors were treated as procedural missteps and students often choral recite step after step until a solution is obtained. In contrast, the teacher in Time 2 asks students to use mental math and their own receipts to calculate the total amount they spend on lunch in one week. The task required students to problem solve. The students decomposed and combined numbers in different ways that make sense to them. In short, there was a lot to talk about beyond correctness. The teacher asked questions to elicit student thinking, probe specific details of students' ideas, and encourage engagement with others' ideas: "How did you break it down so you could do it mentally?" "She gives us a fraction, what does a fraction mean?" and "Do you see what she is saying? She said 24 is really close to 25, right?" The teacher expressed genuine curiosity on this particular day and was surprised by the creativity of some of the students' strategies. Ms. Diaz asked questions that helped her, and the classroom community, make sense of students' strategies.

In Time 3, the teacher centered both procedural fluency and conceptual understanding. She asked more follow-up questions to probe for understanding and she used a wider range of follow-up moves. Notable in Time 3 is how Ms. Diaz addressed invalid ideas or incorrect solutions. Rather than immediately correcting errors or highlighting missteps, Ms. Diaz revoiced and challenged students' thinking in Time 3, "So, you are telling me, let's see, let's rationalize this. I have $4 and I'm dividing it among 8 people?" Moreover, Ms. Diaz supported students to develop and engage in others' representations. She supported students to re-create representations of their strategies on the board rather than place completed work on the document camera and in some cases, applied their own strategies to check the accuracy of their classmate's work. To further support students to engage with others' ideas, Ms. Diaz yielded the floor to the students on numerous occasions in Time 3. Rather than interrupt or drive the conversation based on her own ideas or learning goals, Ms. Diaz follows the students' ideas.

A close analysis of student participation necessitates paying attention to specific ways teachers support students to engage in the mathematics in context at each time point. Ms. Diaz's actions occurred in-the-moment as she drew from an expanding range of moves. Over time, she was more adept at enacting, sequencing, and adapting moves to elicit and respond to the unfolding of students' mathematical thinking. The teacher increasingly focused on eliciting student thinking beyond the initial invite and an increased her support to understand of the details underlying students' mathematical contributions.

The analyses presented thus far have described teacher support of student participation in whole class over time. To understand more about individual students' patterns of increased participation and the openings students with different kinds of mathematical understandings take up across time points, it is necessary to examine an individual student participation profile. To demonstrate the range of teacher support of student participation in whole-class, I will illustrate a single student's participation across the three time points. I chose to highlight Malena not only because she demonstrated increased participation over time, but because she expressed and overcame her tentativeness to participate over time. I will describe the ways Malena explains and engages with others' ideas and highlight the evolving nature of Ms. Diaz's support of her participation across time.

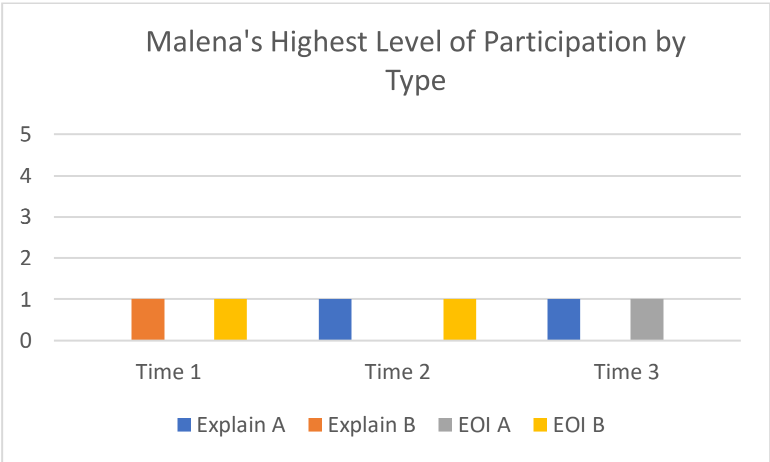
Individual Student Participation Profile: Malena

When Malena stepped into the Algebra I classroom in her 30's, she felt like a "scrawny 16, 15-year-old again." She used to sit outside of her mathematics class in high school because stepping *inside* the room was almost unbearable. In an interview, Malena shared that she is "very uncomfortable with math" and finds it difficult to explain her mathematical ideas to the teacher and her peers. She knows her math facts up to six and prefers the calculator for everything else. Although Malena identifies as someone who is "not good at math," her burgeoning participation in the adult education mathematics classroom suggests otherwise. Malena eagerly sits in the front row and by Time 3 is the first to support a student struggling at the whiteboard by suggesting next steps or correcting an error. Malena's 5'8" frame, openness to share her opinion and provide advice, eagerness to support her classmates, and low, commanding voice help establish her position as a classroom matriarch and one who evolves into someone who likely has the solution.

Malena's participation is not unlike her classmate's. There is hesitancy in Time 1 and a deep-seated association between mathematics and desperate error-avoidance. In fact, she describes gripping fear and her hot, red face after realizing she made an error at the whiteboard in front of the entire class. However, by Time 3, she realized that incorrect or invalid solutions are an integral part of doing mathematics and has grown to feel comfortable sharing at the white board because, "nobody gets upset and you don't feel judged."

A closer look at Malena's participation profile reveals that she participated in more detailed ways over time (see Figure 4.12). In Time 1, Malena explained and engaged with her peers in a less detailed way by partially explaining an idea and referencing the details of others' ideas. By Time 2, Malena began to participate in a more specific way. That is, she not only referenced the details others' ideas, but she provided a fully-detailed explanation. By Time 3, Malena explained and engaged in others' ideas in a detailed way by providing a complete, fully-detailed explanation and extending the details of her peers' ideas.

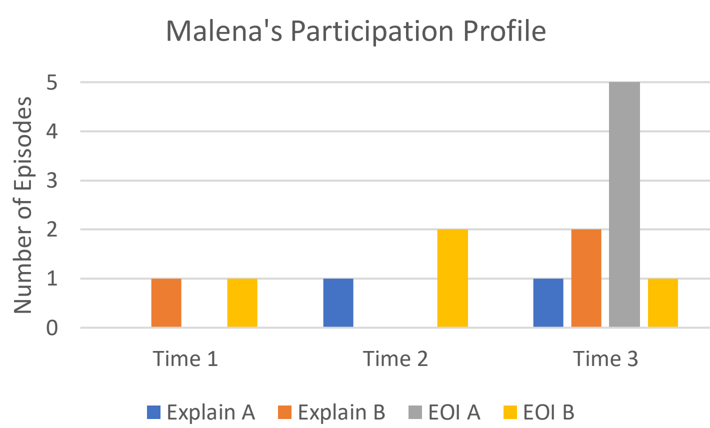
Figure 14. Malena's Participation Profile by Highest Level: Time 1, Time 2, Time 3



Note. This table captures whether Malena did (1) or did not (0) participate by explaining in a fully-detailed (Explain A) or partially-detailed (Explain B) way and/or engaging with others' ideas by extending (EOI A) or referencing (EOI B) their ideas.

Not only did Malena's mathematical contributions grow more detailed over time, but the frequency of contributions increased over time. Figure 4.13 illustrates Malena's participation at Time 1, 2, and 3. At Time 1, she participated at a general level by explaining a partial idea and referencing the details of her peer's idea. In Time 2, she explained at a fully-detailed level once and referenced the details of her peers' ideas twice. By Time 3, Malena participated in nine instances across whole class instruction. That is, she engaged in both a general and detailed way by providing partial (1 instance) and fully-detailed explanations (2 instances) as well as referencing (1 instance) and extending (5 instances) the details of others' thinking. Of note are the five instances of extending others' ideas in Time 3. On this day, Malena extended her peers' mathematical ideas in a range of productive ways, which include adding additional mathematical details to her peers' strategies and challenging her classmates' ideas.

Figure 15. Malena's Participation Profile by Type and Episode: Time 1, Time 2, Time 3



Note. This table captures Malena's total number of episodes, including explaining in a fully-detailed (Explain A) or partially-detailed (Explain B) way and/or engaging with others' ideas by extending (EOI A) or referencing (EOI B) their ideas.

In this study, Ms. Diaz used a range of invitation moves (the initial request for students to explain or engage with others' ideas) to support Malena to explain and engage with others' mathematical ideas. These moves to support Malena occurred across time points and mathematical content. Ms. Diaz's invitations for Malena to explain or engage with others' ideas became more detailed over time. There were no initial teacher moves in Time 1; however, in Time 2, the teacher supported Malena to explain and engage with her peers' ideas by asking general questions. By Time 3, Ms. Diaz invited Malena to explain and engage with her peers by asking specific questions.

Table 8. Type of Invitation Move Enacted by the Teacher to Support Malena's Participation: Time 1, 2, and 3

Category of Invitation Move	Teacher Move	Time 1	Time 2	Time 3
Explain				
General Question	Would you mind sharing your idea?		•	
Specific Question	Why [did you] minus?			•
Engage in Others' Ideas				
General Question	When you talked to the person next to you, which one did they do first?		•	
Specific Question	Why do you think she was adding?			•

Note: The "•" indicates that the teacher enacts the move at least once during the time point.

In some cases, initial invitation moves did not result in Malena explaining or engaging with others' ideas in a detailed way. Ms. Diaz used a range of support moves (the request following the invitation) to help Malena engage with others' mathematical ideas. She pursued Malena's engagement in increasingly detailed and broad ways over time. Specifically, Ms. Diaz's moves served to nudge Malena's participation, support her to add details to her own or others' ideas, or elicit mathematical ideas (e.g., reasoning around place value) embedded in Malena's ideas that were not addressed. To pursue these ends, Ms. Diaz drew on an increased variety of support moves over time. Ms. Diaz did not enact follow-up moves to support Malena's participation in Time 1. It should be noted; however, that Ms. Diaz supported students to participate in ways that did not rise to a codable level including asking yes-no questions, fill-in-the blank type questions, or answer-only questions. Additionally, she provided social-emotional support, financial advising, college advising, and time management support, which is not coded here but was integral to Ms. Diaz's community building and workforce development goals. In Time 2, Ms. Diaz supported Malena after an invitation by enacting single moves and a sequence of a combination of moves. Specifically, the teacher supported Malena to explain by asking both general and specific questions, pressing for details and supporting Malena to make her think explicit by creating a representation of her idea on the board. Moreover, she supported Malena to engage in others' ideas by referencing her idea in the context of making connections between student ideas. By Time 3, Ms. Diaz enacts additional moves to support Malena's participation, which include revoicing her ideas, yielding the floor to provide opportunities for Malena to engage with her classmates without teacher intervention, positioning Malena as a competent mathematician and author, engaging with the details of others' ideas by recreating representations and checking or challenging the validity of students' strategies.

Table 9. Teacher Follow-Up Moves Enacted by the Teacher to Support Malena's Participation: Time 1, 2, and 3

Category of Follow-Up Move	Teacher Move	Time 1	Time 2	Time 3
Explain				
General Question	What did you do after that?		•	•
Specific Question	Where did you get the 40 from?		•	•
Press	What allows me to take away the zero?		•	•
Representation	Could you write down the steps of what you're saying? And then		•	•

	what? I'm just trying to understand.			
Revoice	You are telling me that I have $4 and I'm dividing it among 8 people?			•
Student Extension	You're changing the problem on us? What is it?			
Engage with Others' Ideas				
General Question	Did you ask them why?			
Specific Question	Where does the .8 come from?			
Space (Pauses interaction and reorients toward Ss idea)	Let's look into Luna's mind. Pay attention.			
Yield (gives Ss the floor to EOI)	Extended productive student exchanges *without* teacher intervention (>5 student exchanges)			•
Position	That's a good way to see it.			•
Refer	Did you hear what she said?		•	
Recreate Representation	Don't believe her. Do it yourself.			•
Check	Why don't we check?			•

Note: The "•" indicates that the teacher enacts the move at least once during the time point.

Ms. Diaz invited Malena to participate and enacted moves to further support Malena to explain and engage with other's ideas in Time 2 and 3. In Time 1, Malena participated in class, but was not supported to explain or engage with others' ideas. Ms. Diaz's invitation moves became more specific over time and her follow-up moves broadened over time to include additional explanation and engagement moves. To further illustrate the nuances of teacher support of student ideas and the relational nature of explanation and engagement with others' ideas, I provide detailed descriptions of typical participation episodes for Malena at each time point. As the details of Malena's thinking emerge over time, the details of teacher support also expand.

Malena's Detailed Participation Profile by Time Point

Time 1: General Participation around Procedural Tasks

Participation in Time 1 can be characterized as teacher-directed, articulation of procedural explanations, and minimal student engagement in others' ideas. Ms. Diaz directed the classroom conversation, asked questions related to procedures and correctness, often explained students' strategies *for* the students, and in most cases, controlled the sharing of student ideas via the whiteboard marker or document camera. Similarly, student participation drew on specific procedures or steps and relied on textbook number sets that lack context. Students simplified expressions and shared their ideas in three ways and provided: a full explanation with choral peer engagement, a partial explanation with minimal engagement, and sharing ideas without a verbal explanation or peer engagement.

In some cases, students provided complete procedural explanations while the class solved the problem in parallel with the speaker. Choral peer engagement is possible due to the procedural nature of the task. A problem that requires a prescribed, step-by-step solution has only one strategy and does not require students to generate their own mathematical ideas, thus, students are able to anticipate the next step. As a result, individual explanations in Time 1 are often shared explanations. The teacher may select the lead, or an individual may assert herself during the choral solution to claim authorship.

In the following example, Ms. Diaz asked students to solve the expression 52 + 8^2 – 3(4-2)^3 following a traditional order of operations lesson delivered via direct instruction. After students shared five different answers, the teacher posed a common question to the classroom community, “How did you solve it?”. In this example, Ms. Diaz positioned herself at the white board with marker in hand, revoiced student ideas, commented and posed questions, and documented the students’ shared procedural strategy. The teacher and students participated together and worked on the details of the mathematics. She leveraged the different answers as a learning opportunity while also positioning the student community as capable of doing the mathematical work. On this day, Ms. Diaz did not select a specific student to share out. Rather, she took up a choral idea that emerged from the community (solve what is inside the parentheses first) and primarily relied on the community’s choral responses for the remainder of the solution. Students shared the first step of the strategy and Ms. Diaz recorded and restated the idea on the whiteboard and asked if everyone agrees (Line 1). Then Ms. Diaz invited the community into the conversation; however, the procedural nature of the task limits the conversation to calculations and adherence to prescribed steps (Line 3, 5, 9).

1. T: So, let’s write it down. Parentheses first. Everyone agree?
2. Ss: Yes.
3. T: What is 4 take away 2?
4. Ss: 2
5. T: Parentheses are still there. [writes (2) underneath the (4 – 2)] Like that? What’s next?
6. Ss: Exponents!
7. T: Exponents?
8. Ss: Yeah.
9. T: Alright. Which one?
10. Malena: 8 x 8 is 64.
11. T: 8 x 8
12. Malena: And then it’s 2 x 2 x 2. 8.
13. T: 2 x 2 x 2 [writes and says] which is 8.
14. Ss: 8

15. Lucia: And then bring down the 52.
16. T: Okay. Bringing it down. [writes]
17. Ss: And then you multiply 3 x 8
18. Ss: And then you multiply by -3
19. T: And then we multiply.
20. Ss: -24
21. T: So, minus 24.
22. Ss & Malena: And then you add 52 + 64. Then you get 116.
23. Ss: Add the 52 and 64.
24. T: Do I add first?
25. Ss: Yes.
26. T: So, I had 52 + 64 and that's going to be?
27. Ss: 116
28. T: 116, right?
29. S: Yeah.
30. T: And then you're going to subtract the 24?
31. Ss: Yeah.
32. T: So last is subtraction.
33. Ss & Malena: And you get 92

Malena assumed the lead (Line 10) of the choral strategy. She followed the teacher's prescribed procedure and detailed her calculations (Line 10, 12, and 20). Her classmates followed along, step-by-step and verbalized the anticipated solution (Lines 14, 15, 18, 21, and 24). Malena solved the expression with ease as the teacher documented the steps, often repeating Malena and her peers' steps (Lines 11, 13, 16, 19, 21, 26, 28, 30, 32). In Lines 22 and 23, while Malena and the students combined the addends, Melena increased her volume in an attempt to maintain her authorship of the strategy. Here (Line 24), the teacher assumed control of the explanation and asked for the next step when one had already been provided. In line 26, she again asked a question related to a calculation that had already been completed. The teacher then completed the explanation in the form of a question, “And then you’re going to subtract the 24?” before Malena had the opportunity to explain (Line 30). Malena did not verbalize the final step and immediately volunteered the answer simultaneously with her peers (Line 33). She may have intentionally skipped the subtraction step in an effort to verbalize the answer before the teacher. In Time 1, tension between the teacher and students surfaced during interchanges like this one. The teacher struggled to relinquish control; that is, she moved the class forward at her own pace and took up the opportunities students might have had to share their own thinking. Instruction, in Time 1, relied on teacher-directed strategies rather than student’s mathematical thinking and student-directed strategies. Moreover, teacher support of student participation consisted of inviting students to share the next steps, asking general questions (“what’s next?”), and completing procedural steps for them.

Time 2: Detailed participation around a new mathematical approach

Participation in Time 2 reflects the increased rigor of the task. Unlike Time 1, students in Time 2 were asked to solve problems using mental math situated around authentic contexts. Due to the nature of the task and an emphasis on students’ mathematical thinking, multiple students were able to provide different solutions to the same problem. The classroom community strived to understand the authors’ unique mathematical ideas and thus, students primarily referred to the details of the author’s mathematics rather than extended the author’s mathematical ideas. The example that follows highlights a typical participation sequence in Time 2: multiple complete explanations around a single problem. Malena offers one of the explanations.

The procedural task in Time 1 limited the level of student participation, specifically, students explained teacher-directed solutions focused on the correct procedure to solve each problem, deviations from the procedure were met with identification and correction of the error, and whole-class conversations featured limited opportunities to engage with others' ideas. In contrast, multiple students explained their strategies for a single problem and students took up opportunities to engage with other's ideas at different levels in Time 2. Although, at times, there was little overt peer engagement with the details of the author's strategy, the class is exposed to numerous strategies related to students' mathematical thinking. In Time 2, I observed three students explaining their unique strategies for the same problem. Students in the class engaged with the authors' ideas in general ways and attended to the details of their classmate's strategies. In this example, the teacher elicited the details of students' mathematical thinking in productive ways, which included supporting students to share representations of their thinking, asking specific and general questions, revoicing student ideas, and referring to students' strategies.

In Time 2, the teacher asked students to share receipts collected from their take-out lunches or grocery store purchases and calculate the amount of money each student spends per week on lunch. Some students shared receipts from fast food restaurants and calculated their total based on the three days they attend classes at the adult school. Others packed lunches and calculated the price of an orange and a homemade sandwich and found the total based on the five days they attend classes per week. Luna surprised her classmates and the teacher by decomposing numbers in creative ways to solve the cost-of-lunch problems. Ms. Diaz encouraged students to give Luna's approach a try as they solve the problems "in their heads." It was Malena's turn to share her receipt and math story. Malena's lunch included a bottle of water pulled from a 24-pack she recently purchased at the grocery store. As she calculated the cost per bottle, Ms. Diaz asked if she accounted for the $ .05 CRV (California Refund Value) built into price of every bottle. Malena was not. This led to a discussion about CRV and the inclusion of an in-the-moment task for the class to use mental math to find the CRV for Malena's 24-pack of water bottles.

The following excerpt illustrates Malena and Luna giving detailed explanations of their mental math strategies to solve Malena's CRV problem. Initially, Marcus offered to share his thinking, approached the board, and solved 24 x .05 using the standard algorithm. The teacher expressed surprise that Marcus was able to apply the standard algorithm in his head. Ms. Diaz acknowledged Marcus and Luna's contributions and positions both Luna and Marcus as competent (Line 1). In response to the teacher's request to apply Luna's strategy, Marcus suggested decomposing the 24 by subtracting the 4. The teacher wrote "20 – "on the board, paused and responded to the students' confusion (Line 4) by asking for a volunteer to extend Marcus's idea and create a representation on the whiteboard (Line 5). She asked Malena to approach the board to explain. Malena accepted the offer to participate, approached the board, constructed her representation, and turned to face the class. Ms. Diaz moved the discussion forward by encouraging Malena to explain her thinking (Line 7). Malena verbalized her strategy and added to her representation; she multiplied the 2 (from the 24) and the 4 (from the 24) by 5 (Line 8). She wrote the two products, 10 and the 20, next to one another and combined them by erasing the 10's zero and replacing it with a decimal. Ms. Diaz asked a general question to elicit additional details from Malena (Line 9). Malena responded by repeating her strategy in Line 10 with more detail. When the teacher pressed Malena to explain her rationale for adding the decimal between the 1 and the 20 (Line 11), Malena responded, it's "in my head" (Line 12). Ms. Diaz pressed again to ensure Malena's thinking is made explicit for herself and her peers. Malena responded by sharing that the solution is only one [rational] number (1.20) not two separate numbers (10 and 20).

Ms. Diaz invited Luna into the classroom conversation by asking her to show her thinking on the whiteboard (Line 18). Luna briefly referenced the 24 in Malena's strategy by pointing to her representation on the board, opted to use the friendlier 25, and multiplied 25 x 5 to obtain 125 (Line 19). She then subtracted 5 (obtained from the five 1's she added to create the 25's) from the 125 to reach 120 and, with the teacher's prompting, added the decimal in Line 21 to obtain $1.20 in CRV. In Line 23, Jose made the connection between this strategy and the strategy Luna shared earlier (Line 23) which prompted Luna to partially detail her strategy again. Ms. Diaz encouraged Jose to attend to Luna's explanation by referencing Luna's explanation, revoiced Luna's strategy, and asked a general question to elicit additional details (Line 26). Ms. Diaz revoiced Luna's response and re-represented her strategy on the whiteboard (Line 28). Luna continued with her explanation (Line 30) only to be interrupted by the teacher who explained the remainder of Luna's strategy (Line 31). The teacher pressed students to make comparisons and connections across multiple strategies, including Malena's (Line 33). The students made the connection between the authors' mental math strategies and the distributive property (Line 34, 36). Marcus, Malena, and Luna publicly shared their correct and valid strategies to find the CRV with their peers (Figure 4.8).

1. T: How can I do this by breaking it down like Luna did? Marcus's is good because it's fast. But how can I apply Luna's technique?
2. Marcus: Instead of 24 you can take away the 4.
3. T: Instead of 24 I can take away the 4? [T writes 20 -]
4. Ss: I don't get it.
5. T: Who wants to come and show me? Come on Malena, show us.

Malena grumbles.

6. Jose: I believe in you, Marlene!

Malena approaches the board, writes 2 x 5, 4 x 5., and turns to face the class.

7. T: I can't see inside your head. It's hard for me to translate.

8. Malena: It's 2 x 5 and 4 x 5. [writes 10 under 2 x 5 and 20 under the 4 x 5. Erases the 0 in the 10 and writes 1.20]

9. Ms. D: Can you do that again? I can't see inside your head. It's hard for me to translate.

10. Malena: Separated the 2 and the 4 instead of using the 24. 2 x 5 and then 4 x 5, that's 10, that's 20. I take off the zero [erases the zero from the 10 so she's left with 120. Adds a decimal between the 1.20]

11. Ms. D: What allows me to take away the zero?

12. Malena: In my head.

13. Ms. D: Why does that make sense in your head to take away the zero?

14. Malena: Because it's not a whole number. It's only one number. Instead of 2, it's only a 1.

15. Ms. D: Instead of a whole number, it's a *rational* number?

16. Malena: Hmmmm.

17. Luna: Ms. Diaz, I made the same thing that I made before. I multiplied 25 x 5 = 125 minus 5 is 120.

18. Ms. D: Come show us. I can't see inside your head. Come show us over here. [taps on the board]

Luna approaches board.

19. Luna: 25 in place of the 24 [points to Malena's strategy to the right side of the board]. 25 x 5 is going to be 125 minus 5 is 120.

20. Ms. D: So, she pays $120 in CRV?

21. Luna: No. [adds the decimal to make $1.25]

22. Ms. D: Oh. A *dollar* twenty-five.

23. Jose: You made this like the other one, too?

24. Luna: I made the same thing, I just subtract because it's not 25, it's 24. So, I subtract five.

25. Jose: Yeah, I understand.

26. Ms. D: [to Jose] Do you see what she is saying? She said 24 is really close to 25, right? She said instead of writing 24, she wrote 24 + 1, that gives her 25. And then what did you do?

27. Luna: I multiplied by 5.

28. Ms. D: So, she multiplied by 5, 5 cents for the CRV, right? [writes 24 + 1 = (25)5 = 125]. 5 quarters equals a dollar twenty-five.

29. Jose: Ooohhhh.

30. Luna: So, I subtract 5 cents because --

31. Ms. D: Because you have to take away the 1 x 5, correct? Because you have to take away that 1. One times the five is? 5. And that's how she gets 1.20 with the decimal. [Writes 1(5) = 5, 125 – 5 = 120]

32. Luna: Yeah. That's in my head.

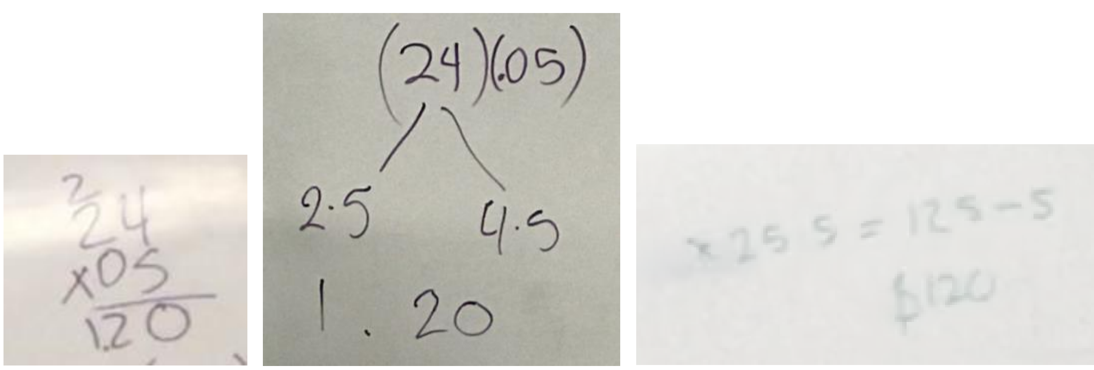
33. Ms. D: Do you notice a pattern from all that she showed us, even the one that Malena showed us?

34. Students: Every time they use the distributive property.

35. Ms. D: Every time they are repeating something, over here they use the 3, over here they use the 5.

36. Malena: We are using the distributive property.

Figure 16. Marco, Malena, and Luna's Strategies



Malena's participation in Time 2 reveals a shift in opportunities for students to participate and teacher support of their participation. Examining the ways that Malena participated reveals that her mathematical ideas surfaced in different ways in Time 2. She participated by providing a complete and correct explanation (Line 8, 10, and 14). Malena was held accountable for ensuring her strategy made sense to not only the teacher, but to her peers (Line 7, 9, 11, and 13) and she was positioned as mathematically competent by a classmate (Line 6). Malena had mathematical authority and was recognized as someone with ideas worth sharing in the public whole class setting (Line 5). Rather than explaining Malena's strategy, Ms. Diaz provided space for her to explain and acknowledged Malena's authorship (Line 7). Moreover, the teacher and Malena's peers were all participants in a larger mathematical community who want to make sense of the work together rather than assign correctness. Malena overcame her initial hesitation to share a correct and complete strategy and successfully applied Luna's approach to solving mental math problems by decomposing numbers in ways that make sense to her. The exchanges transpired in the whole-class public space and thus supported Malena and the classroom community to engage in and build understanding and support the development of Kalifa's strategy.

Time 3: Student thinking takes center stage

Over time, the responsibility of explaining, eliciting students' thinking, and engaging with each other's ideas shift from the teacher to the students as students assume roles that are more traditionally the teacher's. The depth and breadth of support moves enacted by the students increased over time from correcting mathematical errors in Time 1 to a variety of student support moves in Time 3. First, students publicly supported their peers by referencing and attending to the details of the author's strategy by solving calculations ("8 x 8 is 64"), asking specific and general questions ("How did you get that?" and "Why did you multiply by 15 instead of 14?"), noticing similarities and differences ("I separated the big number instead of multiplying the whole thing"), and making real-life connections ("Slope is like the ramp on my treadmill").

Second, students added on or extended the details of the author's ideas by challenging misinterpretations ("That's a negative number, it's like money you *owe*"), problem posing (after solving for the total spent on lunch for one week, "What if we figured out the total for two weeks? Or a year?"), offering suggestions ("You could use a table"), and jointly constructing strategies in the public sphere while the classroom community acts together as a sole peer partner. Third, students demonstrated that they attend to the author's work or their peers' participation by pointing, nodding, applauding, or commenting ("Wait. I don't get it!") and expressing feelings of solidarity ("I had a hard time with this part, too"). And fourth, students positioned each other as competent ("You are so smart, you should be a teacher!").

The following excerpt from Time 3 illustrates the details of one student's explanation as it unfolds over time with the support of Malena and the classroom community while solving the problem, "A ride at the fair costs $2. How much money does it cost for 8 people to go on the ride?" The variable, *z,* is used to represent the total cost. Kalifa shared her answer with the class as she wrote 4/8 = 2 on the board (Line 1). Ms. Diaz challenged Kalifa's invalid strategy by revoicing her idea (Line 2). Kalifa did not respond with additional details, so Ms. Diaz asks a specific question that incorporates Kalifa's ideas (Line 4). Kalifa held fast to her idea (Line 5), resulting in another specific question designed to challenge the validity of her idea. Here, Ms. Diaz created space for Kalifa's peers to offer suggestions (Lines 8, 9, 10). Malena took up the opportunity and suggested that Kalifa multiply 8 x 8. Kalifa replied with an incorrect calculation and the class, aware of the details of Kalifa's thinking, responded, "no no no" in disagreement (Lines 11, 12, 13). Irma offered another idea (Line 14) that Kalifa did not have time to take up before the teacher intervened. Sensing Kalifa's confusion, Ms. Diaz enacted a move to offer Kalifa space to process her peers' suggestions and rethink her own strategy. Kalifa believed the solution was $4 (Line 16) which prompted Ms. Diaz to repeat a portion of the math story and ask a specific question (Line 17). Kalifa expressed confusion (Line 18) and Ms. Diaz responded with a more direct statement around the validity of her solution (Line 19). Kalifa acknowledged her misconception (Line 20) and Irma immediately offered a different approach involving substitution (Line 21). Irma's last suggestion prompted Kalifa to detail her thinking (Line 22). Malena and Marcus challenged Kalifa's solution and interpretation of the story (Line 23, 24), which prompted Kalifa to restate a portion of the story (Line 25). After two missteps (Line 11, 23), Malena offered a valid suggestion that builds on Marcus's idea (Line 26). Maria began to validate Kalifa's misconception (Line 27) and was interrupted by Kalifa taking up Malena's idea (Line 28). Malena added additional details and guided Kalifa toward a valid strategy (Line 29). Irma positioned Malena as competent (Line 30, 41) and Malena and Lola pressed Kalifa to finish the strategy by asking general questions (Line 31, 32). Kalifa completed a valid representation of her strategy on the board by writing and counting the 8 2's to obtain 16 (Line 33). Ms. Diaz positioned Kalifa's idea as valid, "There you go!" and asked her to reference her own idea on the board. Ms. Diaz summed up Kalifa's idea and asked for the final calculation to confirm the validity of her idea (Line 36) to which Kalifa confirmed. Malena positioned Kalifa competently (Line 37) and in the final exchange, Kalifa returned to her initial, invalid idea and asked Malena to corroborate her idea as an incorrect, but reasonable error to make. Kalifa's classmates worked together to support Kalifa to apply a valid and complete solution to the story (Figure 4.9).

1. Kalifa: Alright, so, I got 4. I got 4. [writes 4/8 = 2]
2. Ms. D: So, you are telling me, let's see, let's rationalize this. I have $4 and I'm dividing among 8 people.
3. Kalifa: Yeah.
4. Ms. D: So, if I only have 4 dollars and I'm going to divide it among 8 people, would each person get 2?

5. Kalifa: 2!

6. Ms. D: Each person gets 2?

7. Kalifa: No. Well, I don't know. So -- [erases the numerator, /8 remains]

8. Luna: You need to multiply the 8 times 2.

9. Malena: Put a z on top of the 8, and then you multiply to find z.
Kalifa writes z/8 = 2

10. Irma: Put the z, put the *z* because we are going to find the number that's the z.

11. Malena: Multiply 8 by 8

12. Kalifa: 36

13. Students: No no no.

14. Irma: Just put the multiplication.

15. Ms. D: Hold on, hold on, class. So, Kalifa, you are on the right track. So that number 4 doesn't work. So how much money do you need to have so each person has $2 if you have 8 people?

16. Kalifa: Alright. I'm gonna need $4.

17. Ms. D: So, $4, and there's 8 people. And each one is going to get $2?

18. Kalifa: You said I got $8??!!

19. Ms. D: No, you said that you have $4 and 8 people. It doesn't work, does it?

20. Kalifa: No.

21. Irma: Plug it in.

22. Kalifa: I just went like this [writes 8 tallies on the board] and then I circled 2, 2, 2, and 2 [draws circles around 4 groups of 2 tallies] and that's how I got 4.

23. Malena: I got 14.

24. Marcus: You need to give it to 8 people though. Eight is the people, right? 8 people need to get $2.

25. Kalifa: 8 people need to get $2.

26. Malena: You have to put like 8 people. Make 8 circles. [Kalifa erases her 4 groups of 2 tally marks]

27. Maria: I know where she got confused because if we look at the problem --

28. Kalifa: Make 8 circles? I know how to make the circles.

29. Malena: 1, 2, 3, 4, 5, 6, 7, 8 [counts as Kalifa draws one row of 8 circles] Now put 2 dollars in each. [Kalifa writes 2 inside each of the circles]

30. Irma: I like Malena's.

31. Malena: How many do you have?

32. Lola: How many dollars would you need to give out to the people?

33. Kalifa: 2, 4, 6, 8, 10, 12, 14, 16. 16.

34. Irma: Yeah!

35. Ms. D: There it is. Now look at it. If I have 16 dollars and 8 people, what is 16 divided by 8?

36. Kalifa: 8. Yes, it's 16 divided by that, oh, it's 2. Duh. [writes 16/8 = 2]

37. Malena: Good job, Kalifa!

38. Students: Yeah! [applause]

39. Kalifa: Nah!

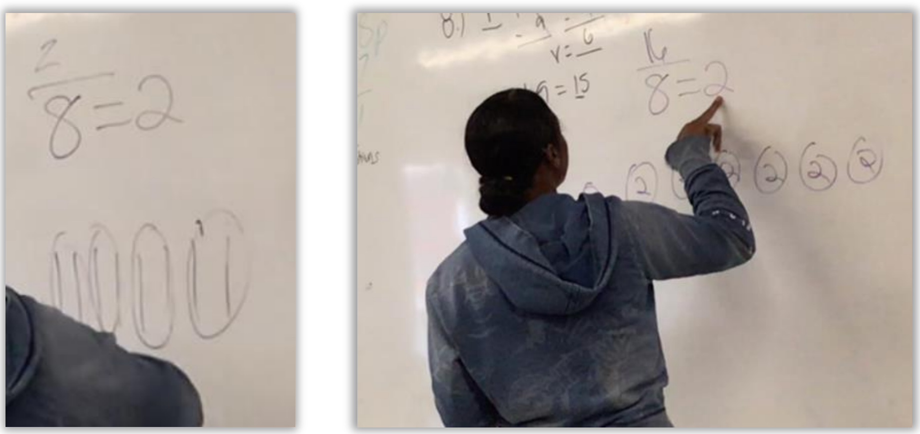
40. Ms. D: That is a very good way to see it, Malena. Very good.

41. Irma: She could be a teacher, man.

42. Ms. D: Go for it, Malena.

43. Kalifa: Thank you, Malena! But you see how I got 4?!

Figure 17. Kalifa Explains her Strategy.



Note: She first draws 4 groups of 2 (left) and then, with Malena's support, produces a valid representation of the story using eight circles with 2 in each (right).

In Time 3, intellectual authority is shared between Ms. Diaz and the students in supporting student thinking. The teacher more readily promoted student participation and valued the contributions made in the whole-class context and, in turn, students like Kalifa and Malena took risks to engage in classroom conversations and the sharing of ideas. The moves employed by the teacher created opportunities for Malena and her peers to contribute mathematically, which legitimizes multiple ways of participating. Ms. Diaz refrained from comments solely based on correctness and instead focused on sense-making which opens more spaces for students to explain and engage with others' ideas. Malena was able to critique the reasoning of her classmate and add onto the author's ideas (Line 23 and 26). In addition, both the teacher and students positioned the author and her contributors (including Malena) as competent and celebrated their successes together (Lines 30, 37, 40, 41, and 43). Moreover, there were numerous episodes where students engaged with others' ideas without teacher intervention (Lines 7-14; Lines 21-34). Ms. Diaz yielded the floor to her students which provided opportunities for students to make sense of and engage with Kalifa's ideas.

Summary of teacher support of student participation

Analyses of teacher support in the whole-class context show that all time points offer opportunities for teacher support of high-level student participation. In the classes observed over time, a substantial portion of teacher support to invite students to participate (91.7%) and follow-up support (58.3%) of students occurred in Time 3 with a smaller proportion of students receiving the same support in Time 1 (invitation 25%, follow-up support 16.7%). The teacher enacted follow-up support to explain (58.3%) and engage with others' ideas (41.7%) to a higher proportion of students in Time 3 as compared with Time 1 (explain 8.3%, engage with others' ideas 8.3%). Moreover, the teacher expanded her range of support moves over time to include engaging with others' representations by re-creating them or developing different strategies to check for validity and correctness and yielding the floor to create space for students to engage with others' mathematical thinking on their own.

CHAPTER 5: DISCUSSION

Introduction

This study contributes to our understanding of student participation and teacher's support of student participation in the adult education algebra classroom. Students' participation and teacher's support of students' explanations and engagement with other's ideas broaden over time in the whole-class context. Analysis of classroom interactions reveals the dynamic nature of student participation; student explanations and engagement with others' ideas surface and develop in relation to the temporal, the complexity of the task, the teacher's in-the-moment support moves, and the details of students' mathematical thinking.

Ms. Diaz created space and opportunities for adult students to make sense of and engage in the mathematics in different ways over time. Participation over the three time points illustrates the variety of contributions made by students and reflects the evolution of how the teacher leverages students' explanations and engagement with others' ideas to advance the instructional goals of Algebra 1.

Emerging explanation and engagement with others' ideas over time

Ms. Diaz's classroom captured the emerging possibilities of instruction that increased productive student participation. Analyses of teacher and student interactions revealed the particular ways students explained their own ideas and engaged in others' ideas and the ways the teacher supported them in-the-moment and over time. What counts as productive participation was identified in the context of whole-class discussions as ideas shifted from the periphery towards the center of classroom discourse over time. Over time, Ms. Diaz increased space and support to solve problems in ways that made sense to students. She also presented students with more opportunities for students to engage mathematical ideas that originated from their peers rather than herself. Furthermore, adult students were introduced to student-directed mathematical ideas and representations, thus having opportunities to engage with alternative perspectives and reasoning in relation to their own. Over time, class time devoted to the sharing of ideas was gradually supplemented and, in some cases, replaced by the sharing of student ideas. The teacher as an author and curator of ideas transformed to some degree to a facilitator eliciting ideas other than her own.

Participation through engagement with others' ideas and explanations will not necessarily produce productive discourse around mathematical sense-making. The relational nature of participation illustrated in the three time points suggests that the variety of ways students participate can be *limited or promoted* by the teacher's individualized follow-up moves, the nature of the task, and *inconsistent or consistent* centering of student ideas. That is, the extent to which students explain and engage with others' ideas were successful in revealing students' developing mathematical thinking was dependent upon the teacher creating space for students to share their ideas, expressing and valuing student understanding, designing robust problem-solving tasks, and creating a community where students can safely take risks.

The range of ways Ms. Diaz supported student participation across the three time points, along with an increased focus on students' mathematical ideas, created opportunities for different types of participation beyond I-R-E patterns typically found in the whole-class context. She began to listen more and tell less and found more ways to encourage students to elaborate their thinking. Her support of student thinking shifted from asking general invitation and follow-up questions in Time 1 (e.g., Can you share your strategy? Can you say more?) to specific invitation and follow-up questions that relied on knowledge of students' specific mathematical ideas (e.g., What does the 40 represent? What did you do after you multiplied by 6?). In other words, instead of asking, “How did you solve that?” to every student, Ms. Diaz tailored her support by using what was shared of the authors’ ideas to probe for student thinking around a concept or elicit additional details. In Time 1, Ms. Diaz expected students to participate similarly – approach the board, re-represent their work, and turn to the class for agreement. By Time 3, Ms. Diaz enacted multiple moves suggesting that she did not expect every student to participate the same. In some instances, the teacher asked for students to explain and re-represent their strategies at the whiteboard, in other instances, she supported students to ask questions, pose their own extension problems, to sit quietly and make sense of the mathematics on their own. Yet, in other instances, Ms. Diaz remained quiet and gave space for students to explain and engage with others’ ideas on their own, without intervention.

There is evidence of a shift in power from Time 1 to Time 3, which may have created opportunities for increased student engagement and the emergence of what some identify as egalitarian dialogue (Flecha, 2000). In Time 1, Ms. Diaz explains the mathematical ideas of 41.7% of students; by Time 3, she explains the ideas of 8.5% of the students. Similarly, in Time 1, 33.3% of students share their ideas without verbally explaining, and by Time 3, only 8.3% of students share their ideas without a corresponding verbal explanation. The findings suggest that the dialogue may have become more egalitarian by Time 3. That is, the teacher and all of the students had the same opportunities to share their knowledge based on their own understandings rather than classroom power relations positioning the teacher-as-expert.

There was a sharing of authority and acceptance of responsibility in the classroom as students worked toward a common goal of mathematical understanding. Ms. Diaz valued student contributions and abilities, thus gradually relinquishing some of her power over time, giving students space to explain their own ideas and verbally share their mathematical thinking. She often referred to a student author as “the teacher” and yielded the conversational floor and the whiteboard marker to them. The shift in participation like this does not only represent a change in activity and behavior, but it may also involve a transformation of roles and developing new identities – identities linked to new knowledge and skill (Wenger, 1998). Lave (1996) states, “... crafting identities is a social process, and becoming more knowledgeably skilled is an aspect of participation in social practice ... who you are becoming shapes crucially and fundamentally what you ‘know’” (p. 157). A consideration of redistribution of classroom authority dynamics and identity development is important in adult education contexts where all actors in the classroom are adults with myriad life and mathematical experiences.

Context matters

There is some evidence to support that context matters in the learning process. Luna provides some evidence of this claim. She volunteers at a local church thrift shop that operates without a cash register. Luna’s co-workers admire her ability to quickly calculate totals “in her head” and make change. During Time 2, students shared their individual lunch receipts and used mental math to calculate totals spent on lunch per week. Each problem was unique to an individual students’ context (individual totals; some purchased lunch two days a week, others give days a week). Luna used her experience with mental math at work to creatively decompose numbers in the classroom. In contrast, during Time 1, she struggled with procedures and did not explain or engage with others’ ideas in any way. The problems in Time 1 lacked the context necessary for students to apply everyday experiences to make sense of the problem. Conversely, Luna provided four complete explanations and extended the details of a peers’ idea in Time 2. In Time 3, the tasks included both context-free computational problems and story problems with context – some related and some unrelated to adults’ lives (e.g., flowers, tickets, aces in a tennis match, math problems remaining on a test). On this day, Luna provided one partial explanation, extended another student's idea, and referenced the details of two student's ideas. Although she interacted four times, only two of those interactions were highly detailed interactions. Luna's experience reinforces the importance of context in adult education (Lave, 1988; Schliemann & Acioly, 1989). Consistent with research (Díez-Palomar et al., 2006), adult in Ms. Diaz's class found meaningful ways to connect with formal mathematical ideas and understand new mathematical concepts or processes.

When faced with problems free of context, Ms. Diaz employed a lecture-type format to demonstrate how to solve an example problem. Independent practice followed. Ms. Diaz produced procedural explanations. When asked to explain their solutions, the students explained similarly. The subsequent discussion focused on student agreement and the correct application of the procedure. Stigler et al. (1999) referred to these two phases as the internal acquisition and subsequent application phases. Consistent with their findings, Ms. Diaz, like teachers in the United States, viewed problem solving as the end goal. However, when Ms. Diaz presented tasks situated in the context of students' lives and requiring students to develop strategies that made sense to them (the lunch receipt task in Time 2 and the story problems task Time 3), the role of problem-solving shifted to understanding the mathematics. This was evident in the types of questions asked by the teacher, "Class, do you agree?" "Is she correct?" to "I can't see inside your head. Can you show us how you did that?" "Can you try it Luna's way? [decomposing]." In the case of the receipt task, student problem solving and explaining came first and was followed by an explicit discussion around the underlying distributive property concept. Consistent with the research, not all tasks in Ms. Diaz's class are created equal, and the different tasks require different kinds of student thinking (Stein et al., 2000), and the level and kind of thinking determine what students learn (Hiebert et al., 1997).

The invisible becomes visible

Adults have been described as having “spiky [mathematical] profiles” on performances assessments like PIAAC (Gillespie, 2004, p. 4-6). That is, some adults may find items on the “data and chance” domain more difficult than the “dimension and shape” domain, while the opposite is true for others. Like the classroom, some adults find measurement division problems more difficult than finding the area of a two-dimensional shape. This can be attributed to distinctive life experiences. So, when policymakers propose a “minimum level of numeracy to cope with the demands of adult life,” the generalizing claims classify adults with diverse life and mathematical histories as one group and assumes their numeracy demands are the same. Assessments like PIAAC cannot possibly assess the full range of adults’ mathematical knowledge in all of the contexts in which adults participate. Adults’ mathematical understandings have been compared to a road peppered with potholes. This implies a deficit perspective and one where teachers must “fix” or mend the road with tar and gravel. Adults’ mathematical assets may not be visible on national and international assessments, in the workplace, or in even the classroom; however, adults know a great deal about mathematics from their daily lives. Or, to pursue the metaphor, the foundation on which the road is built is solid and could be used to leverage and extend the foundation from below. Leveraging and extending the foundation from below occurs in a public space where adults’ mathematical thinking is made explicit.

Studies like this that focus on participation and learning could move the field of adult education in a direction that promotes and supports what students *can do* rather than what they *cannot do* or what prevents them *from doing*. This begins in the classroom by making mathematical thinking explicit and visible. As members of the classroom community, there were instances where students built upon their peers' patchwork of mathematical knowledge and made their thinking visible in Ms. Diaz's class. For example, the teacher supported Malena to make her thinking visible in the public domain:

1. T: I can't see inside your head. It's hard for me to translate.
2. Malena: It's 2 x 5 and 4 x 5. [writes 10 under 2 x 5 and 20 under the 4 x 5. Erases the 0 in the 10 and writes 1.20]
3. Ms. D: Can you do that again? I can't see inside your head. It's hard for me to translate.
4. Malena: Separated the 2 and the 4 instead of using the 24. 2 x 5 and then 4 x 5, that's 10, that's 20. I take off the zero [erases the zero from the 10 so she's left with 120. Adds a decimal between the 1.20]
5. Ms. D: What allows me to take away the zero?
6. Malena: In my head.
7. Ms. D: Why does that make sense in your head to take away the zero?
8. Malena: Because it's not a whole number. It's only one number. Instead of 2 [separate numbers], it's only a 1.

In line 1, Ms. Diaz supports Malena to make her mathematical ideas explicit and public; thus, the classroom discussion becomes a social event in which students are able to participate. In order for Malena's peers to engage with her idea by questioning, challenging, or adding onto her idea, her thinking must first be made visible. In line 2, a portion of Malena's strategy is brought into the public sphere, but only part of her strategy is verbalized. Ms. Diaz asks Malena to "do it again" and repeats, "I can't see inside your head" to reinforce classroom norms around explaining one's own ideas. Malena adds additional details to her explanation in (line 4), but Ms. Diaz wants to know how she obtained 1.20 from the ten written next to the 20 on the board.

Malena responds with, “In my head” (line 6), and Ms. Diaz follows up with a specific question around her understanding of place value (line 7), which prompts Malena to further elaborate on her thinking (line 8).

What the reader does not know is that Malena works as a cashier in a local grocery store. Malena’s classmates position her as an expert and mathematically competent due to workplace experience and her at-homeness with numbers. She developed a family budget at Ms. Diaz’s request (in an attempt to curb her expensive Starbucks habit) and can estimate the price of individual items purchased in bulk at Costco as she comparison shops. When Ms. Diaz presses for Malena to explain her thinking during whole-class discussion she not only presses her to explain beyond her initial explanation, but she supports Malena to transfer the invisible math in her head to the whiteboard and through her verbal explanation. In turn, Malena elaborates on her idea and others have opportunities to engage with her ideas in the public sphere. The transfer of Malena’s knowledge of workplace mathematics to the classroom occurs *in situ* and through interaction with the teacher and her classmates. Her knowledge is generated and constructed in the public sphere. Thus, students’ mathematical thinking can be generated and made visible in the adult education classroom.

Promise of the Study

This small case study in Ms. Diaz’s class offers promising results. The study reinforces the importance of a teacher’s role to support collaborative inquiry in her classroom and offers a vision of what is possible in adult education. Adult students demonstrated increased participation over time. Moreover, analyses that focused on teacher support of student participation also indicated increased support over time. Although I cannot claim that increased attention to students’ mathematical thinking was solely responsible for observed differences in participation, or that the same study can be successful in all adult education environments, it is reasonable to assert that our collaboration had an impact on student participation and reflected what a classroom focused on students' mathematical thinking can be.

The data reported in Chapter 4 reveals changes in student explanations and engagement with others' ideas and the broadening of teacher support moves. These changes may be, in some sense, the natural growth over the course of a trimester and a feature of developing membership in a classroom community. However, the changes also suggest increased learning opportunities as students participate in instruction evolved to include more collaborative inquiry. Malena's aforementioned episodes offer three snapshots of typical engagement in Ms. Diaz's algebra class and provide the reader with a perspective from which to gain a sense of how classroom engagement evolved over time. The episodes captured the nature of student participation in the adult algebra classroom while at the same time, afford a detailed view of characteristics of the classroom and the participants' specific actions. The changes in participation may indicate a significant shift for Ms. Diaz as she began to see the full scope of the role of student engagement in teaching and learning mathematics. I argue that the collaboration, individualized support, renegotiation of norms, engagement opportunities across structures, and multidimensional tasks afforded to students played a role in facilitating teacher and student growth around classroom participation.

Teacher-Researcher Collaboration

There may have been contextual factors that supported students to participate at higher levels and the teacher to broaden and individualize her practice. First, Ms. Diaz volunteered for the project and was not compensated for her time. Thus, she had some innate interest or prior level of commitment for engaging in this work to center students' mathematical ideas. Second, the collaboration between the researcher and the teacher likely facilitated some aspects of participation. I provided additional classroom support by working with individuals and pairs of students during independent and small-group classroom structures. In addition, I called attention to students' contributions in the whole-class context and our daily debrief sessions, provided suggestions for students to engage with others' ideas, designed and offered suggestions for entry ticket problems, and expressed my enthusiasm when a particular task produced productive mathematical talk. My contributions may have kept students' mathematical thinking at the forefront of Ms. Diaz's mind as she reflected, planned, and supported her students.

Individualized Support

I claim that the nature of Ms. Diaz's support, specifically invitation and follow-up moves, affected the development of students' explanations and engagement with others' ideas over time. Conversely, students' increased participation may have affected Ms. Diaz's moves to support student thinking and engagement. Ms. Diaz played an important role in whole-class discussion from the beginning; however, her role had evolved to accommodate more student thinking over time. Analysis of Ms. Diaz's moves to support students' mathematical thinking in Chapter 4 indicated that her range and specificity of questioning students had expanded. In general, in Time 1, whole-class discussions focused on solutions to mathematics problems or performance evaluations rather than shared understandings of students' strategies. Ms. Diaz's evaluation of students' work was central to instruction and positioned her as the 'primary knower' (Nassaji and Wells, 2000). Classroom conversations were directed toward the teacher with very little student-student interaction. In addition, Ms. Diaz took up opportunities to explain or correct students' strategies which may have limited opportunities for students to do the work themselves.

By Time 3, Ms. Diaz was less likely to control the conversation and provided space for student-student interactions and sense-making. She enacted a broad range of invitation and follow-up moves to elicit student explanations and engagement with the details of others' thinking tailored, in-the-moment to each student. She listened more and asked more questions that encouraged student reasoning around a specific idea or detail. As Ms. Diaz's questions shifted, students' responses also shifted. Students were more apt to initiate conversations with their classmates and contributed in increasingly detailed ways. Moreover, after Ms. Diaz enacted the "space" move, students had opportunities to follow up on or elaborate on their initial queries with their peers.

Re-Negotiated Social and Sociomathematical Norms Across Participation Structures

The changing learning environment may have affected the positive shift in student participation. That is, the rules or expectations – the social norms -- Ms. Diaz and the students developed within the classroom were renegotiated over time to center students' mathematical thinking and engagement with students' mathematical ideas. One way to transform a traditional classroom to a more inquiry-based one is to develop or negotiate social and sociomathematial norms (Cobb, Wood, & Yackel, 1991). The authors (1991) describe social norms as one's beliefs about his/her "own role, others' roles, and the general nature of mathematical activity in school and sociomathematical norms as "mathematical beliefs and values" (p. 177).

Ms. Diaz and the students renegotiated the classroom's norms over time. For example, in Time 1, norms existed that defined Ms. Diaz as the authority. At times, students were expected to listen, memorize strategies, and regurgitate their skill development on assignments. At other times, they were expected to participate by sharing their thinking at the whiteboard and checking their answers with the student author at the board – with Ms. Diaz as the central figure in discussions. All roads led to Ms. Diaz. By Time 3, a slight shift had occurred. While Ms. Diaz remained an authority figure, she was not *the* sole authority in the room.

The shift did not originate from the teacher alone. Rather, collective classroom activity is "jointly established by the teacher and student as members of the classroom community" (p. 178). Participation evolved to include practices beyond the raising of hands and moving chairs to work in parallel with a peer. Sociomathematical norms, or normative discussions specific to the mathematics (p. 461), emerged over time in the classrooms' learning community. For example, sharing a written solution at the whiteboard and returning to one's seat was no longer acceptable by Time 3. Ms. Diaz turned students around midway to their seats to reinforce a renegotiated norm that sharing at the board includes a verbal explanation with embedded opportunities for students to engage beyond the strategy's correctness. The teachers and students negotiated sociomathematical norms to include the use of creative approaches and methods to problem-solving, acknowledgment of the value of both fully and partially developed ideas, and justifications. The renegotiated norms may have contributed to changes in the ways students think (Gonzales & DeJarnette, 2015; McClain & Cobb, 2001). Moreover, the changing discussions in the algebra classroom and the sharing of authority may have helped students become the primary knowers.

Engagement Across Participation Structures

Although this case study focused on the whole-class participation structure, it is likely that engagement in one classroom structure may have influenced engagement in the other structures. The small group problem-solving structure may be particularly salient here. Throughout the trimester, students were grouped in pairs or triads to work on various problem-solving tasks. The problem-solving context shifted from one of hesitancy, discomfort, and silence to a boisterous, energetic, take-advantage-of-all-the-space-in-the-room affair.

During problem-solving in Time 1, students were asked to solve problems on their own and check in with a partner, “Try it out and discuss it with someone sitting near you.” Students in the front row were instructed to choose a partner sitting behind them in the second row. One could hear the click-click-click of the analog clock in the front of the room. Students froze and looked around the room as if to consult each other regarding the legitimacy of her request. Ms. Diaz followed up by asking students to physically pick up their chairs, turn them around to face the back row, and work with a partner now assigned by the teacher. After slowly and tentatively turning their chairs, students complied by working on their own problems and briefly checked in with their partners to compare solutions. After many minutes, the silence was broken by low whispers of a few voices asking, “I got 21. What did you get?” This task did not lend itself to collaborative investigation. As such, students worked in parallel rather than collaboratively and, in some cases, did not attempt the problem.

Over time, Ms. Diaz adapted her expectations to focus on the process of collaboration rather than the outcome. By Time 3, students were expected to work together to collectively make sense of and devise strategies to solve problems. They were encouraged to ask questions of others’ strategies and support partners who needed help. Ms. Diaz had adjusted her expectations to include understanding rather than comparing answers and, in turn, the students adjusted their own expectations. During the transition from small group problem-solving to share-out, Ms. Diaz stated the following:

> I went around and people were telling me, Ms. Torres, I think I’m going to multiply 70 x .3 and I ask why? Why did you do that? I looked at the equations there and somebody

> was very honest and said, well, I just copied from that person over there. [student laughter] But the point is that, the point is you have to ask that person why? Okay? Not that that person is incorrect and their math is wrong. It's, "But why am I doing this?" 'Cause remember, the whole point is that you *understand* what you are doing, not that you want to get the answer right. That's cool and all to pass the class, but are you understanding it? So think about it like that.

Over time, students exhibited less tentativeness and, in many cases, expressed an eagerness to collaborate. By Time 3, students quickly selected partners for small group problem-solving and initiated collaborative work without prompting from Ms. Diaz. The room lit up with conversation and movement. Those who finished early or needed extra support buzzed around the room to consult with pairs of classmates like bees collecting nectar. Each pair's workspace expanded from the physical area surrounding the two classmates to the area of the entire classroom and I believe this physical and intellectual expansion may have influenced the whole-class context. Ms. Diaz changed her expectations and redefined what it means to participate. She did more than just encourage participation; she described more specifically what students should contribute. Ms. Diaz encouraged students to take risks, share partially detailed ideas, and rely on and support their community of mathematicians. She encouraged students to ask questions and interject during whole-class discussions, challenge one another, and take the initiative – all in the service of students' mathematical understanding.

Realized Potential of a Mathematical Task

The shifting discussion around the tasks, or the realization that each task can potentially promote or hinder student engagement, may also have contributed to increased student participation. The tasks that Ms. Diaz selected and developed over the trimester varied greatly in their demands and opportunities for student engagement and higher-order mathematical thinking. As such, it may be difficult to parse the demands and opportunities afforded by the task from the changes in students' engagement. However, a close examination of the engagement around specific algebra tasks provides some insight into the nature of student engagement and the level of discussion related to the task. For example, a small group task from early in the trimester required students to complete a mathematics-related crossword puzzle that did not require any mathematical thinking. Students completed the puzzle together, approached a nearby pair, compared their puzzles, and gazed uncomfortably around the room until Ms. Diaz began the share-out. The brief share-out involved students standing amongst the pairs, agreeing in the public space that they obtained the same "answers" as the pair group with whom they consulted. This task structure is very different from other tasks that provide opportunities for students to explain their thinking, for example, where students use real-world contexts (the lunch receipts) to generalize a method to solve for the total amount an individual spent per week. The lunch receipt type task requires substantively different mathematical work from the crossword puzzle.

Ms. Diaz's tasks varied greatly; however, each day included some cognitively demanding mathematical tasks (Stein, Grover, & Henningsen, 1996), some procedural tasks, and some tasks that may have been selected for purposes other than developing students' mathematical proficiencies, such as community building. Each day students worked on tasks with a range of contexts: arbitrary, textbook-derived contexts; teacher-derived contexts; and decontextualized problems. And each day, students took up opportunities to explain their thinking. I argue that the change in participation over time lies in the discussion around the tasks, specifically, who had the opportunity to explain their thinking and what type of task supported it. That is, students had more opportunities to explain their conceptual thinking over time, despite the inclusion of similar tasks. For example, Ms. Diaz led the discussion *and the explanations* of the conceptual task in Time 1. Students completed the problem-solving exercise; however, Ms. Diaz explained their thinking to the class, thus limiting students' opportunities to explain their own ideas. A subsequent task on that day required students to simplify a series of expressions located in the textbook. Students were encouraged to share their procedural solutions with the class, which were limited by the nature of the task's complexity or lack thereof. By Time 3, students explained their own thinking, and students engaged with their peers around the conceptual tasks while Ms. Diaz listened. The teacher shifted *who* does the conceptual explaining from herself to the students. I hypothesize that this shift occurred because she observed the capabilities of each student and entrusted them with more of the conceptual heavy-lifting. With a heightened awareness of student engagement, she may have noticed the procedural tasks yielded very little beyond demonstration of a new skill. Ms. Diaz may have also noticed the lively and often messy discussions around algebraic thinking that yield some frustration, some laughter, and a lot of shared understanding. Not only did Ms. Diaz diversify the tasks, but she also learned to diversify opportunities for classroom participation. Focusing on opportunities to participate and the multidimensionality of tasks may have springboarded Ms. Diaz's growth around effective facilitation of mathematical discussions.

Implications

The results of this study have important implications for practice and future research. Despite the compelling evidence of inquiry-oriented, collaborative learning environments to promote student learning, traditional modes of instruction still dominate the educational landscape (Hiebert & Stigler, 1999; Jacobs et al., 2006). Ms. Diaz's class is no exception. Much of Ms. Diaz's class resembled traditional modes of instruction. It can be challenging to support student participation in interactive practices that center students' mathematical thinking. In the adult education classroom, there are a number of factors that may contribute to the well-established use of typical modes of instruction: pressure to cover content in a limited amount of time, hesitancy to relinquish or yield control, deficit perspectives, difficulty accommodating the needs of both the group and the individual; the reluctance of learners to accept their peers as legitimate sources of knowledge; and lack of meaningful collaborative learning tasks (Bruffee 1987; MacGregor 1990; McKinley 1983; Novotny, Seifert, and Werner 1991; Sheridan 1989). Moreover, limited use of collaborative practices can be attributed to a developing understanding of students' mathematical practices and the activity required to enact these practices successfully (Boaler, 2003).

Although there are barriers to designing and implementing collaborative learning environments, the knowledge gained may outweigh the challenges. By examining student participation and teacher support of student participation in the adult algebra classroom, I have illustrated how Ms. Diaz promoted problem-solving and the active engagement in the learning process. The interactions between the teacher and students, among students, and within the classroom context generate a collective understanding and deeper learning can occur. All of Ms. Diaz's students have the opportunity to participate and create meaning through explanations and engaging with others' ideas. The information within this community of learners has the potential to be shared outside the classroom with family, colleagues, and other communities of practice. The learning that occurs in the classroom impacts each actor and reinforces their commitment to their common interest -- schooling and mathematics.

Paying attention to and centering students' mathematical thinking and the interactive practices that support student understanding provides insight into the ways students make sense of the mathematics. An increased understanding of the ways students interact in the classroom and develop their mathematical understanding lends itself to a more innovative way of designing classroom environments that support student learning. Examining who has opportunities to participate, who takes up those opportunities, and how they take up those opportunities is important to study as it may perpetuate inequitable opportunities for participation and learning and adversely impact the students adult education is most likely to serve. There is a need for teacher professional development on ways to create space for more negotiation of meaning. This may be particularly salient for classrooms relying on individualized group instruction based on theories espousing the importance for directing their own learning experiences (Merriam & Caffarella, 1999).

Limitations

A few limitations of this study are to be mentioned. First, a prominent critique of a single case study is the issue of external validity or generalizability. It is an unavoidable and valid criticism of this and other single case studies. As is common in case studies, the sample size is small and idiosyncratic. There are no claims that this study is typical. I am unable to know empirically the extent Ms. Diaz's classroom is similar or different from other adult algebra classrooms and I certainly cannot establish probability that the data collected in her classroom is representative of some larger population. Thus, the conclusions drawn from this study may not be transferrable to other participants or contexts. Nevertheless, the data showed potential in developing classrooms centering students' mathematical thinking. This case study generated new thinking around participation in the adult mathematics classroom and can be judged in other contexts and settings and compared to other studies of adult participation. Moreover, this case study establishes a provisional understanding of adult participation even though it was based on

12 adult learners and can add to the dearth of knowledge on interaction in the adult mathematics classroom.

Although case studies are often criticized, there is an important qualifier to the argument. The intention of this study is particularization rather than generalizability and, as such, the criticism can be "mitigated by the fact that its capacity to do so [is] never claimed by its exponents; in fact, it is often explicitly repudiated" (Eckstein, 1975, p. 134). Research subjectivity is also a valid limitation and may reflect the less formalized and researcher-independent features of the methodology (Verschuren, 2003). Despite the issues, researchers suggest that a case study contain no greater bias toward establishing the truth or validity than other methods of inquiry (Flyvbjerg, 2006). The exploratory qualitative approach may be subjective; however, in the case of analyzing adult participation using video data, the context-specific, holistic, exploratory nature of this study and its contribution to theory-building is particularly well-suited to case study methodology.

Second, this case study has not examined the ways in which employment histories and current status shape and are shaped by classroom interactions. Nor has this study explicitly explored issues of age, race, class, gender, or primary language. Adult researchers often study race, class, and gender as a way to identify students most at risk or most likely to exit programs. This study did not explore how Ms. D used asset-based practices or culturally relevant pedagogy to better meet the needs of her students. This study does; however, illustrate the possibilities of how the adult mathematics classroom can be modified to better support the visibility of adults' mathematical thinking and what participation looks like when authority is shared among participants.

Learning is an ongoing, dynamic process, so time was a limitation in this work in that only 12 weeks were dedicated to data collection efforts. It would have been interesting to extend the study to include other communities of practice, such as the geometry or pre-algebra classrooms. Not only to have a longer period of time to make sense of adults' mathematical thinking, but to explore the interplay of multiple communities of practice and the learning that occurs within and between them. Moreover, it would have been beneficial to explore students' mathematics learning in the workplace. Specifically, exploring the connections adults make between the mathematics in the workplace and the formal mathematics of school.

Future Research

To my knowledge, this study is the first to apply video analysis to explore participation in the adult education algebra classroom; however, it is imperative for additional research to be conducted in the area of adult education mathematics and student participation.

The evidence of productive student participation in the whole class setting in the adult algebra classroom generates a number of issues for further investigation. First, the nature and frequency of student participation and the teacher practices that support it may not be the same in the different phases of the whole-class setting. For example, how students explain their own ideas or engage with others' ideas may vary across the warm-up and share-out phases of the whole-class setting. Similarly, the ways in which teachers support productive student participation may differ across the phases of the whole-class setting. For example, teacher practices to support student participation may vary based on the goals and tasks of the warm-up and share-out. Supporting teachers to effectively elicit student thinking and build on students' mathematical understanding necessitates attention to the nuances of the whole-class phases.

Second, student participation does not begin and end in the whole-class setting. Thus far, in the research described here, I have opted to focus on student participation and teacher support of student participation in one classroom setting. However, ongoing student participation and teacher support is not bounded by classroom setting. Student participation may differ over the course of a lesson and across multiple classroom settings. For example, the ways in which students explain their ideas or engage with others' ideas may differ during turn-and-talk or small group problem-solving settings. Effective teacher practices that support productive student engagement may also vary across settings. Similarly, the norms and tasks that teachers pose may differ by setting and reflect the goals and nature of each setting. When the objective is to help teachers effectively engage students, it is not enough to understand participation in whole-class settings in isolation. Adult educators require a more complete understanding of the progression of student ideas across settings and the unique affordances of participation in each setting.

Third, much could be gained by examining the potential relationship between student achievement and persistence, student participation, and teacher support of student participation in the adult education classroom. That is, a fuller understanding of adult education mathematics classrooms will be gleaned from investigating the nature of the relationship between explaining one's ideas and engaging with others' ideas, teacher instructional moves, and outcomes valued by adult educators. As I reflect on the complex business of educating adults, several questions arise: Is there a relationship between student achievement and teacher support of student participation? Is there a relationship between persistence and teacher support of student participation? And if so, is the relationship an indirect one through a mediating variable, such as student participation, as evidenced by research in the elementary classroom (Ing et al., 2015)?

Fourth, the need to attend to the development of students' mathematical thinking has implications for classroom practice. For example, by offering students various and multiple opportunities to interact, the risk of individuals dominating interactive space may become less prominent. However, the risk of individuals who are hesitant to participate in public, whole-class participation structures may increase. Thus, more research needs to be conducted on the social impact of classroom interaction, the effects of different interactive patterns, and the experiences and learning of those students who choose to participate by actively listening.

Concluding Remarks

The aim of this case study was, in part, to observe and describe the development of adults' mathematical thinking and the role that classroom participation plays in the development of their thinking. This study was an analysis of a dynamic classroom community where the opportunities for social interaction between students and the in-the-moment decisions made by the instructor impact the lives of every student seated in the adult algebra classroom this year and the next. In many ways, this study represents the possibilities and potential for student learning when the teacher provides space for student dialogue and problem-solving. The algebra classroom in this study represents the first iteration of this work and it is my hope that the findings from this research provide a trajectory for others interested in student interaction and ways that teachers can support student interaction in the adult education context.

APPENDIX

Appendix A. Student Demographics

Students reported their employment status, race, and native language on the end-of-session survey. Sixty-seven percent of students were unemployed while enrolled in Community Adult (see Figure A1). The cohort of students in the study closely paralleled the demographics of the school district: 83% Hispanic or Latino, 8% Black, and 8% Native American (see Figure A2). Instruction was provided in English in this particular math course, thus the 42% of native Spanish speakers were bilingual and the remaining 58% were native English speakers (see Figure A3).

Figure A1. Employment Status

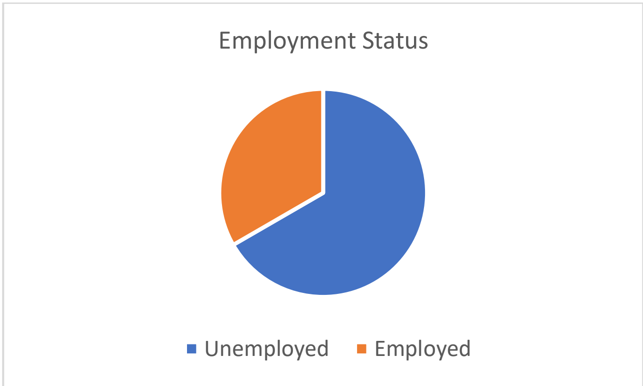


Figure A2. Race

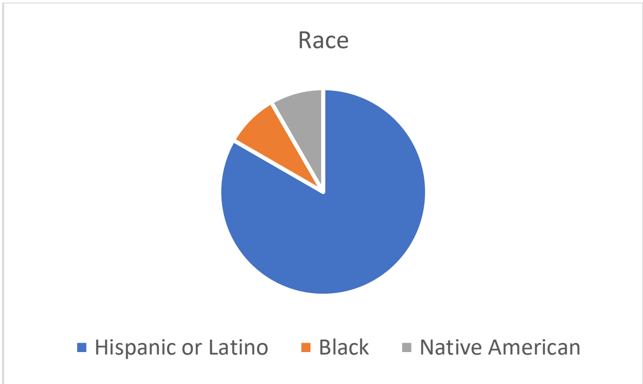


Figure A3. Native Language

Appendix B. Student Profiles

Carmen

Carmen, a Mexican-American with a soft voice and a shy smile, sat in the back row, prepared for the day with an open notebook and a sharpened pencil. She exited high school to care for her newborn and has had completing school on her bucket list ever since. It was the same child, now 13, who encouraged her to return to school. In Algebra 1, she most valued working with classmates and described the ideal partner as someone who makes her laugh and "does not take over the work". She found that partnership with classmate, Jose. During small-group problem-solving the pair could be observed sitting close together with their work in between them, eyes and pencils darting from one page to the other, with frequent laughter emanating from the pair. Cristal shared her mathematical ideas as she worked closely with her partners; however, she was reluctant to volunteer her strategies publicly in the whole-class. She plans to enroll in a phlebotomy technician program and, as money and time permit, take additional health-related classes at the local community college.

Luna

Luna emigrated from Guatemala to the United States 28 years ago. Luna did not have the opportunity to study beyond sixth grade in her country due to her parents' limited financial resources and their belief that girls need not be educated beyond primary school. Young women in Luna's village went to work after primary school, and Luna was no exception. She learned that she has a natural penchant for calculating totals and making change while selling bread in her village. Luna did not have a calculator, so she relied on mental math to quickly combine numbers and soon others were asking her to calculate for them. When she moved to the United

States, she continued to practice mental math at the market, so she did not lose her ability, "you need to learn the prices. If you buy five pounds of this and this is the price, you need to know how much to charge." Luna now works for her church on Sundays and is well-known among her parishioners for her mental math skills. The church does not own "machines" for calculating, so the other employees rely on Luna for quick calculations when it is busy. Luna is the mathematics head-of-household in her family; she teachers her husband to use mathematics in his daily life and she helps her daughter with mathematics homework. It was not until pre-algebra in the adult school that Luna questioned her abilities, "I'm a little confused with what to do with it, when you use letters and math. It makes it a little scary when they start putting letters in math." That said, in Algebra 1, Luna demonstrates her mathematical prowess by sharing her mental math strategies for decomposing numbers during problem-solving. Her classmates refer to the approach to problem solving as, "Luna's way." The shy, cheery, 5'2" woman, who arrives with a homemade lunch every day, quietly selects a seat in the back row. Luna does not waste words; she will raise her hand or interject only when she feels there is something important, mathematically, to add to the discussion.

Marcus

Marcus's five-inch Mohawk hairstyle, fanning atop his head like an iguana's spikes, feels antithetical to his jovial and humble personality. It would be easy to assume that this young, six-foot-tall, Latinx man in his early 20's with the big smile and infectious laugh, steady employment with a shipping company, and an aptitude for solving equations. Math came easy to Marcus until high school when a teacher placed him in accelerated math. Marcus struggled with the accelerated pace of the class and his assigned grade plummeted. Marcus left high school during the second half of his freshman year due to homelessness. He embodies the “never give up approach” and epitomizes persistence in adult education. When describing Marcus, teachers cited Paul of Tarsus, “Obstacles produce perseverance; perseverance, character; and character, hope.” Marcus has overcome numerous barriers and has emerged as a natural leader who is the first to volunteer for a campus event or support his classmates’ understanding of mathematics. At Community Adult, Marcus feels motivated to learn and comfortable asking questions. He credits his supportive teachers for making topics understandable and connecting subjects to his real life. Marcus plans to study software engineering at a local community college in the fall of this year.

Juana

Juana’s cheery attitude brightened the classroom, but that positivity was tempered by a healthy dose of realism manifested through life experience. Despite regularly using math at the grocery store, when purchasing clothing, and at work, Juana “was not so sure” she was “good at math.” After three months of learning mathematics in the classroom, she feels “very confident” about mathematics and “very good” about herself now. Juana exited high school with one month remaining due to a family issue that necessitated relocation to a neighboring state. She returned, four years later, to finish. In class, Juana feels “comfortable” sharing her mathematical ideas and if she or her classmates need help, “we all figure it out step by step as a team which helps us understand and learn a lot better.” Juana, a bilingual speaker with a soft, confident voice, asked carefully considered questions of her classmates’ strategies and tried to put herself “in their shoes so [she] can understand them from their point of view.” Although one of the quieter students in class, when she did speak, she shared thoughtful insights that demonstrated an understanding of her peers' work and was always open to others' opinions and ideas when it was her time to share.

Jose

The students described Jose as "hilarious" and "funny" -- not an insecure-distraction-type humorous, but a genuinely good-hearted, laugh-out-loud funny. At not quite 20, Jose brings a joyous, wide-eyed youthfulness to the class. Every day, he entered into the classroom with enthusiasm and a smile after walking 40 minutes through his urban neighborhood to get there. Jose used the money he would spend on roundtrip bus fare to help support his mother. He was never absent or late. Jose's willingness to attempt any mathematical problem with an infectiously positive attitude made him a class favorite, especially during share-out. When sharing a strategy at the whiteboard, he spoke in a way that invited the classroom to join him on his mathematical journey; everyone was invested in his success that sometimes required repeated attempts, additional peer support, and some cheering. Jose requires extra time to solve problems, but that extra time serves him well as he decomposes numbers in creative ways and solves problems in ways that make sense to him. He is eager to share his mathematical thinking and his explanations have a story-telling aspect to them and often take a circuitous route to completion.

Kalifa

Kalifa, a petite Black woman in her 20's, was the first student to make a real-world connection to slope as the ramp angle of the treadmill at the gym. When she presses the up and down arrows on the treadmill console the slope either increases or decreases. Kalifa preferred visual representations to standard algorithms and abandoned the "rules" for combining positive and negative integers when she realized that owing money is a negative number. Kalifa did not hesitate to interject during whole-class instruction to express her confusion over a problem, concept, or strategy and would ask questions in rapid succession until she understood. When Kalifa solved a problem and obtained the correct answer, she would express disbelief aloud, “I can’t believe I got it!” Her spunky attitude and unapologetic questioning made her a central member of the class. Kalifa often nibbled on a McDonalds hamburger after break, shared stories of the single day she spent at her high school the year she exited and welcomed her classmates as they pulled up chairs to support her sense-making.

Lola

Lola arrived early to class and marched to a seat in the center of the front row. She studied ESL at Community Adult and curiosity, not a high school credential, led her to her first mathematics class. Her curiosity paid off. Lola found the English language challenging, but numbers made sense. Lola accepted a challenge from Ms. Diaz and explained to the class how to derive the formula for the area of a circle on Pi Day. She closely followed her classmates’ board work and was one of the first to find an error or question the validity of a strategy. Lola’s self-assured demeanor and desire to learn positioned her as an expert amongst her peers. After completing middle school in Mexico, Lola worked and attended technical education courses in nursing and secretarial studies and would like to continue her nursing studies in the fall. Her knowledge of mathematics is rooted in the real world. For example, Lola explained that she works part-time at a rate of $60 per day. She can quickly calculate how many days it will take to obtain enough money to pay for her cell phone and electric bills. When she goes out to eat with her four children, she has $25 to spend. Knowing that her family has to “find something that we can eat equally," children forego the costly milkshake and spend the money that "corresponds" to them by looking closely at the menu and ordering, "something less expensive." In class, when standing at the board sharing a strategy, Lola occasionally felt nervous, but "when you're sure [about a solution] …you stand there and feel proud of yourself. It's exciting at the same time."

Lucia

Lucia uses math daily when cooking and helping her children with homework and shared what she has learned in class with her son and daughter. However, Lucia is "not sure" if she is good at math. She equates "good" with calculating two step equations in her head without the use of paper and pencil to solve them, "[the first number] is erased very fast in my mind, so when I want to add another amount, often the last result has already disappeared." What she did not realize is that most, if not all, of her classmates required paper and pencil, too. That said, when she solved a problem on her own without teacher support, she felt "like a genius." She had to calm her nerves when sharing in whole-class due to her struggles with English pronunciation, but she forged ahead and viewed it as an opportunity to learn from her classmates. Lucia, a woman in her early 30's with long brown hair and soft features, expressed genuine curiosity for mathematics and would often ask questions that extended an existing problem or created an entirely new one based on a mathematical curiosity.

Malena

When Malena first stepped into a Community Adult classroom for the second time eight months ago, she felt like a "scrawny, little 15, 16-year-old again." All of the discomfort and nervousness she experienced as a teenager came rushing back. Over a decade ago, Malena left high school because a pregnancy forced her out. After giving birth, Malena attempted continuation school but found learning difficult with a newborn. She tried again when her two children were three and four years old, this time at a local adult education program where "misbehaved teenagers distracted adults from their work." On the first day of class, the teacher handed Malena a photocopied packet of work and said, "Here's your work. If you're able to do it, good, and if you can't, then too bad." After two months, she "couldn't stand it anymore and I just got out of there, too." She enrolled in Community Adult for two trimesters and had planned to return in the fall when Malena's new manager would not accommodate her school schedule. She had to make the difficult decision to prioritize work and leave school again. Months later, to her surprise, Malena received a phone call from the Community Adult principal who asked if she had earned her diploma. It was then that she decided to return to school for the fourth and final time. In class, Malena sits in the front row and is the first to support a student struggling at the whiteboard by suggesting next steps or correcting an error. Malena's 5'8" frame and low voice help establish her position as a classroom matriarch and one who likely has the answers.

Marco

Marco consistently chose a seat closest to the door where he could almost touch the doorjamb with an outstretched arm. He speaks slowly and deliberately and walks with a gaze angled slightly toward the floor, as if deep in thought. His unassuming classroom presence belied his strong 6'2" physical presence. Marco preferred applying the standard algorithm to problems and earned his moniker, "the human calculator" by quickly calculating the answer to addition, subtraction, multiplication, and division problems for his classmates and the teacher during whole-class. He volunteered calculations from his seat and worked diligently with his partners;

however, explaining his mathematical ideas with the class required some gentle encouragement from Ms. Diaz. Marco is a young man with a past and his commitment to Algebra 1 reflects his determination to forge a life of his own choosing.

Irma

Irma's attempt to sneak into class after 9 a.m. was foiled by Ms. Diaz and met with an "at-least-I-made-it" grimace that captured her ability to juggle school and a full-time job. While reluctant to volunteer on her own, Irma would lumber to the whiteboard when called upon and deliver a detailed explanation that would often surprise and impress her classmates. Mathematics, for Irma, made sense when the teacher or problem provided context. She tried to set up equations based on key words in the story and when that failed her, she re-read the problem, made sense of the context, and successfully solved the problem in her own way which may or may not have included a variable. Irma held back in the mathematics she shared with the class, but she contributed in other meaningful ways. She often spoke about injustices like discrimination or racism. For example, during a lesson on wages, she shared how managers skimmed off of her and other Mexican-American employees' paychecks. Irma frequently shared stories or connections to help her, and her classmates, make sense of the mathematics.

Bianca

Bianca, a soft-spoken Latinx with long, wavy auburn hair in her early 30's, returned to school to "make enough money" for herself and her two children. Working closely with, and learning from, an attorney during a recent divorce piqued Bianca's interest in the legal system. She has already applied for financial aid to attend community college in the fall. She knows that studying law will be challenging, but she tells me without hesitation, “I can do it.” She enjoys Ms. Diaz’s class because, “she has much patience with us asking questions and if we didn’t get it the first time and we keep asking the same thing all over again, she wouldn’t get frustrated with us.” Bianca uses math every day to quickly calculate the total of weighed items purchased at the meat counter in her local grocery store to ensure she’s within her budget or determines the cost of a sweatshirt after the discount using mental math.

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