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Observation of Orbital Angular Momentum Transfer between Acoustic and Optical Vortices in Optical Fiber

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Acousto-optic interaction in optical fiber is examined from the perspective of copropagating optical and acoustic vortex modes. Calculation of the acousto-optic coupling coefficient between different optical modes leads to independent conservation of spin and orbital angular momentum of the interacting photons and phonons. We show that the orbital angular momentum of the acoustic vortex can be transferred to a circularly polarized fundamental optical mode to form a stable optical vortex in the fiber carrying orbital angular momentum. The technique provides a useful way of generating stable optical vortices in the fiber medium.

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The interaction between acoustic waves and optical waves has been studied in great detail and numerous applications have been developed in either bulk [1] or waveguide [2,3] media based on this interaction. In an acousto-optic interaction, an acoustic wave creates an index grating via the elasto-optic effect, leading to coupling between optical modes. This interaction can also be viewed as a photon-phonon scattering process in which the total energy and linear momentum of the particles are conserved [4]. Angular momentum is another physical quantity that has to be conserved in this interaction. To our knowledge, this issue has not been discussed in photon-phonon interactions because phonons carry no intrinsic spin angular momentum (SAM). However, phonons can carry orbital angular momentum (OAM) by forming vortices in the medium they propagate. Here, we show that this interaction in an optical fiber can lead to transfer of OAM from an acoustic vortex [5–7] to an optical vortex [8–13] and that spin and orbital angular momentum are conserved independently among the interacting photons and phonon. Thus, the acousto-optic interaction can be a useful means to directly generate pure and stable optical vortices in the fiber medium starting from its fundamental mode. Investigation of this interaction is interesting not only from a scientific viewpoint, but also has potential technological implications in optical trapping [14] and quantum communication [15].

In a typical fiber acousto-optic tunable filter (AOTF), the lowest-order acoustic flexural mode is excited and propagates in the fiber. The flexural acoustic wave induces a traveling index perturbation with an antisymmetric cross-sectional profile which can couple the fundamental mode to antisymmetric higher-order modes ($LP_{11}, LP_{12}, LP_{13}, \ldots$) when the phase-matching condition is satisfied [3]. In a single mode fiber stripped of its jacket, these higher-order modes are in fact the cladding modes. We have previously reported an AOTF experiment in which two orthogonally vibrating acoustic waves with the same frequency are launched onto a dispersion compensating fiber [16,17]. These two acoustic waves are even and odd variants of the lowest-order flexural mode and can be conveniently viewed as two transverse waves with orthogonal "polarizations" [18,19]. This dual-grating acousto-optic mode coupling scheme is particularly versatile because of the ability to electronically control the amplitudes and relative phase of these two acoustic gratings. This method can be used to study the interaction between a circularly polarized fundamental mode and a circularly polarized acoustic wave, both of which carry angular momentum.

While the conventional coupled-mode analysis is able to account for the observed experimental results of the coupling between a circularly polarized fundamental mode and acoustic wave, a deeper physical insight can be gained by considering the interaction in terms of optical and acoustic vortices, as it highlights the role of SAM and OAM in the photon-phonon interaction. Optical vortices are optical waves with a helical phase front and carry OAM. One of the characteristics of an optical vortex is that the phase of the beam has an azimuthal dependence which varies by $2\pi l$ under a complete rotation in the azimuthal (ϕ) direction, where l is an integer known as the azimuthal mode number. This azimuthal phase dependence creates a singular point on the beam axis where the phase is undefined. The l is also referred to as the topological charge of the vortex and the associated OAM per photon is $l\hbar$.

Optical vortices can also propagate in waveguides. In circularly symmetric optical fibers, vortices can be formed by combining degenerate even and odd variants of higher-order modes with an azimuthal number $l > 0$ [20]. Pairs of these modes, with a $\pi/2$ phase difference between them, can represent modes that carry both OAM and SAM. The SAM arises due to the orthogonality of the electric fields of these paired modes, while the OAM arises from the azimuthal phase dependence of the beam. Consider the LP_{11} family of modes consisting of TE_{01}, TM_{01}, HE_{21}^{e}, and

HE_{21}^o modes. It can be shown that a combination of HE_{21}^e and HE_{21}^o modes with a $\pm\pi/2$ phase shift between them carries one $\hbar$ of SAM and one $\hbar$ of OAM per photon so that the total angular momentum per photon in these modes is $2\hbar$. A pair of TE_{01} and TM_{01} modes with a $\pm\pi/2$ phase shift between them carries the same magnitude of SAM and OAM as the HE_{21} pair, but with the SAM and OAM having opposing signs, making the total angular momentum equal to zero.

Since the acoustic modes are also guided by the fiber, they have symmetry properties similar to those of optical modes. The flexural waves in an elastic rod (here an optical fiber) are the solutions of the acoustic wave equation with an azimuthal mode number n greater than zero [19]. Similar to optical modes with $l > 0$, each flexural mode can exist in two orthogonal variants: even and odd. Combining the even and odd variants of an nth order mode with a $\pi/2$ phase delay between them creates an acoustic vortex that carries n units of topological charge and an OAM of $n\hbar$ per phonon [5,6].

In order to study the acousto-optic vortex interaction in terms of conservation of angular momentum, it is more natural to express both optical and acoustic modes in terms of bases which are eigenmodes for both SAM and OAM. These vortex modes, namely, circular vortex (CV), unstable vortex (IV), and acoustic vortex (AV) modes are defined in Table I in terms of the conventional HE and EH modes, TM and TE modes, and acoustic flexural modes, respectively. In this representation, the superscripts $\pm$ and subscripts l denote the SAM and OAM, respectively, all in units of $\hbar$ per photon (or phonon) with m as the radial mode number. The AV modes differ from CV modes in that phonons carry zero spin and hence are represented without the superscript. It is worth noting that since the even and odd variants of both HE and EH modes are degenerate modes in circularly symmetrical fibers, the CV modes are also eigenmodes of the propagation constant (β) and therefore of linear momentum. The IV modes, on the other hand, are not eigenmodes of β since the TE and TM modes are nondegenerate eigenmodes of β. Therefore, IV modes are not stable modes. Finally, modes with an azimuthal mode number of zero, ($CV_{0m}^{\pm}$), including the fundamental mode, are not vortex modes and do not carry OAM.

Let us now consider the acousto-optic interaction from the photon-phonon picture. For codirectional propagating optical and acoustic waves, the fundamental mode photon emits a phonon identical to the phonons of the interacting acoustic mode (stimulated emission) and becomes a higher-order mode photon. During this emission process, the photon loses a portion of its linear momentum and energy to the phonon, and therefore, its momentum is reduced to that of a higher-order mode and its frequency is down-shifted by the acoustic frequency. If the initial photon carries a SAM of $\hbar$ and the interacting acoustic mode is a vortex mode with an OAM of $-\hbar$ per phonon, meaning that the emitted phonon has an angular momentum of $-\hbar$, then conservation of angular momentum requires that the generated higher-order mode photon carry a total angular momentum of $2\hbar$. Since spin can account for only one $\hbar$ per photon, the remaining $\hbar$ must be carried by the photon's OAM. Thus, the higher-order mode photon manifests itself as a vortex mode bearing an OAM of $\hbar$ and a SAM of $\hbar$. A closer examination of the acousto-optic coupling coefficient based on vortex modes shows that spin and orbital angular momenta are conserved independently. Let s and l be the SAM and OAM values of the photon modes, respectively. Also, let n be the OAM of the acoustic phonon and the subscripts i and f indicate the initial and final optical modes. Then, the coupling coefficient between two optical modes due to an acoustic vortex can be written as

$$\begin{aligned}\kappa &= \langle CV_{l_i m_i}^{s_i} | \Delta\varepsilon_{np} | CV_{l_f m_f}^{s_f} \rangle = \frac{\omega}{4} \int \mathbf{E}_i(r,\phi)^* \cdot \Delta\varepsilon_{np}(r,\phi)\mathbf{E}_f(r,\phi) r dr d\phi \\ &= \frac{\omega}{4} \int [(\hat{x} - i s_i \hat{y}) e^{-i l_i \phi} F_{l_i,m_i}(r)]^* \cdot e^{-in\phi} T'_{np}(r)(\hat{x} - i s_f \hat{y}) e^{-i l_f \phi} F_{l_f,m_f}(r) r dr d\phi \\ &= \frac{\omega}{4} (\hat{x} - i s_i \hat{y})^* \cdot (\hat{x} - i s_f \hat{y}) \int_0^{2\pi} e^{i(l_i - n - l_f)\phi} d\phi \int_0^{\infty} F_{l_i,m_i}(r)^* T'_{np}(r) F_{l_f,m_f}(r) r dr, \end{aligned} \tag{1}$$

where $\Delta\varepsilon_{np}$ is the perturbation of the permittivity in the waveguide due to the acoustic wave AV_{np} and $*$ denotes complex conjugation. $T'_{np}(r)$ is the product of the elasto-optic coefficient and $T_{np}(r)$. All other terms are defined in Table I. The conservation of spin and orbital angular momenta in the coupling process is evident from this expression. The product of the first two terms containing the two spin values is nonzero if and only if the spins of the two optical modes are identical. This implies the conservation of SAM. Likewise, the azimuthal integral is nonzero if and only if $l_i - n - l_f = 0$. This implies OAM conservation. Previous AOTF experimental results involving interactions between different combinations of circularly polarized fundamental modes and circularly vibrating acoustic waves can now be viewed as stimulated emission of a phonon with an angular momentum of $\pm\hbar$ from a fundamental mode photon with a SAM of $\pm\hbar$ subject to independent spin and orbital angular momentum conservation. Based on Eq. (1), a general expression for transfer of $n\hbar$ units of OAM between acoustic and optical modes can be expressed as

$$CV_{lm}^s \rightleftarrows AV_{np} + CV_{l-n,k}^s \tag{2}$$

such that AV_{np} satisfies the phase-matching condition for

TABLE I. Definition of optical and acoustic vortex modes. In this notation, the superscript denotes the SAM and the subscript denotes the OAM of the modes in units of $\hbar$ per photon. $F_{l,m}(r)$ is the radial dependence of the field profile as defined in Ref. [20] with l and m being the azimuthal and radial mode numbers, respectively. $T_{np}(r)$ is the radial dependence of the acoustic density wave and is calculated by taking the divergence of the displacement field [19]. In the expressions of the field distributions for CV and IV modes, the first factor represents the SAM of the mode, which corresponds to a rotating electric field and shows that these modes are all circularly polarized. The azimuthal phase dependence (the exponential term) is characteristic of a vortex. AV modes also possess such an azimuthal phase dependence which leads to the vortex nature of these modes.

Vortex modes	Vortex modes in terms of conventional fiber or acoustic modes	Field distribution of vortex modes
$CV^{\pm}_{\pm l,m}$	$HE^{e}_{l+1,m} \mp iHE^{o}_{l+1,m}$	$(\hat{x} \mp i\hat{y})e^{\mp il\phi}F_{l,m}(r)$
$CV^{\mp}_{\pm l,m}$	$EH^{e}_{l-1,m} \mp iEH^{o}_{l-1,m}$	$(\hat{x} \pm i\hat{y})e^{\mp il\phi}F_{l,m}(r)$
$IV^{\mp}_{\pm 1,m}$	$TM_{0,m} \mp iTE_{0,m}$	$(\hat{x} \pm i\hat{y})e^{\mp i\phi}F_{1,m}(r)$
$AV_{\pm n,p}$	$A^{e}_{n,p} \mp iA^{o}_{n,p}$	$e^{\mp in\phi}T_{n,p}(r)$

coupling between CV^{s}_{lm} and $CV^{s}_{l-n,k}$. For a contradirectional interaction, the fundamental mode photon absorbs a phonon and converts into a higher-order mode photon whose frequency is up-shifted. Again, spin and orbital angular momenta are independently conserved.

To verify that the generated mode is indeed a vortex mode, it was interfered with a reference plane wave. In this experiment, the appearance of a single-spiral interference pattern is the characteristic signature of a vortex mode with a topological charge of one [8]. The experimental setup is shown in Fig. 1. Two shear-mode piezoelectric transducers (PZTs) are stacked on top of each so that their vibration directions are perpendicular to each other. Two rf signals with the same frequency are applied to the two PZTs. The acoustic waves generated by the PZTs is coupled to the fiber through an aluminum horn. The fiber used for the acousto-optic interaction is a two-mode dispersion compensating fiber (OFS HFDK). One of the characteristics of this fiber is that the individual couplings from the fundamental mode to the higher-order TE_{01}, TM_{01}, and HE_{21} modes are spectrally separated and thus can be resolved [17]. The interaction length of the AOTF is about 30 cm. The laser wavelength is within the C-band (1520–1570 nm) and the acoustic frequency is in the range of 600 to 900 kHz. The fiber at the end of the AOTF is cleaved and the beam is directed to a collimator. A reference beam is delivered to another collimator and the two beams are combined in a beam splitter and projected onto a CCD sensor array. The interference between two fundamental modes is shown in Fig. 2(a). When the AOTF is turned on and the relative amplitude and phase of the two rf signals are adjusted so that an $AV_{\mp 1,0}$ mode is generated in the fiber, the circularly polarized fundamental mode $CV^{\pm}_{0,1}$ can be coupled to the $CV^{\pm}_{\pm 1,1}$ vortex mode. Since the generated vortex modes are frequency down-shifted, we phase-modulated the reference beam at the acoustic frequency to create sidebands having the same frequency as the vortex mode. The interference of the lower sideband with the vortex mode forms a stationary interference pattern and is captured by a CCD camera. Spiral patterns were clearly observed as shown in Figs. 2(b) and 2(c), which show the existence of a vortex mode. This verifies the transfer of OAM from acoustic vortices to optical vortices in the fiber. In Fig. 2(d), the reference beam is slightly tilted. Again, the forklike fringe pattern is characteristic of a vortex. The intensity profile of a vortex mode alone is shown in Fig. 2(e). It should be noted that this type of mode coupling is also possible using two UV-written tilted long period gratings with an appropriate phase shift [21] or by a helical, microbending, long period grating [22].

The acousto-optic interaction provides a novel means to create stable optical vortices with an OAM of ± 1 directly from the fundamental mode of the fiber. Optical vortices have been previously observed in optical fibers. However, they are either generated in free-space before being coupled to the fiber or formed by coupling two orthogonal,

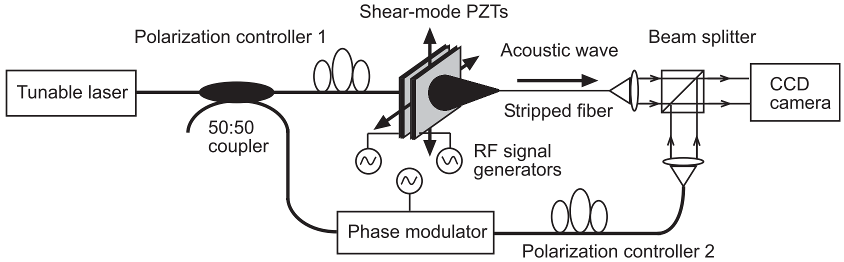

FIG. 1. The experimental setup. Two acoustic waves with orthogonal vibration directions are generated by two PZTs and coupled to the fiber. The input fundamental mode is coupled to higher-order modes by the acoustic grating in the fiber. The output of the AOTF is passed to a collimator and a beam splitter where it is combined with a reference beam and projected onto a CCD sensor array. The reference beam is phase modulated at the acoustic frequency so that it forms a stationary interference pattern with the frequency down-shifted beam at the output of the AOTF.

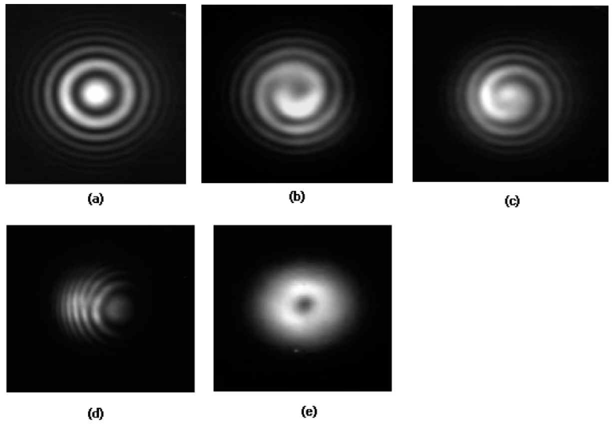

FIG. 2. (a), Interference pattern of the two fundamental modes when the AOTF is turned off. One of the collimators is brought slightly out of focus so that one of the wave fronts is curved and the interference pattern is a group of concentric rings. (b),(c) With the AOTF turned on, the relative phase and amplitudes of the two acoustic waves are adjusted to generate an acoustic vortex on the fiber. The fundamental mode is coupled to the first order vortex mode in the fiber carrying ± 1 unit of OAM. The single-spiral interference pattern verifies the unity topological charge of the mode. (d), The reference beam is slightly tilted. The forklike shape of the fringes shows the singular point in the spiral carrying topological charge of one. (e), Mode intensity pattern of a first order vortex. The reference beam is blocked in this figure. The annular shape with zero intensity in the center is characteristic of these modes.

free-space-generated, Laguerre-Gaussian modes to a stress-induced birefringent fiber [23]. This inevitably excites other nonvortex modes in the fiber. Compared to these methods, a pure vortex mode can be generated in the fiber with 100% mode conversion efficiency via acousto-optic mode coupling. By varying the frequency of the acoustic wave, optical vortices can be generated over a wide range of wavelengths. This method opens the possibility of transferring larger quanta of OAM from fundamental mode to higher-order *CV* modes. For example, by exciting a higher-order acoustic vortex, it is possible to generate higher-order *CV* modes according to Eq. (2). Alternatively, higher-order *CV* modes can be generated by cascading the above-described acousto-optic coupling process in stages so that one unit of OAM is transferred to the optical mode per stage. Since the device is based on mode coupling phenomenon, it is reversible and it can also be used to selectively couple an optical mode with a certain OAM back to the fundamental mode for detection purposes. This detection method provides a much simpler way to detect optical modes with a certain OAM value compared to what has been previously reported [24]. The properties of acoustic and optical vortices in optical fiber and their coupling efficiency are determined by the fiber structure. New ways of controlling these properties may be possible through creative design of microstructured and photonic crystal fibers.

In summary, we have analyzed and verified the generation of an optical vortex carrying orbital angular momentum directly in fiber starting with a fiber fundamental mode for the first time to the best of our knowledge. This is achieved by transferring OAM from an acoustic vortex generated in the fiber. Analysis of the coupling coefficient of this acousto-optic interaction verifies independent conservation of spin and orbital angular momenta.

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