Key Actions in Insight Problems: Further Evidence for the Importance of Non-Dot Turns in the Nine-Dot Problem

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Abstract

Key actions are single actions or behaviors that can be singled out as leading to the solution of a problem. In the nine-dot problem (Maier, 1930), Kershaw and Ohlsson (2004) proposed that non-dot turns are the key action necessary for solution. In two experiments, non-dot turns are further analyzed as the key action necessary for solving the nine-dot problem and its variants. Non-dot turns are found to be predictive of solution, while the classic conception of drawing lines outside the dots does not distinguish between solvers and non-solvers.

Key Actions in Problem Solving

Problem solving in everyday life, as well as the laboratory, can be quite difficult. Often a problem or activity can seem unduly difficult when one does not know the *key action* necessary for completing the problem. A key action can be defined as a single action or behavior that can be singled out as the key to the solution. Examples of key actions abound in everyday life. A proper roux cannot be made without engaging in continual stirring. Algebra problems become routine once one understands how to balance the equation and isolate the variables. Finally, as I have learned one too many times, data will invariably disappear if I have not completed the key action of backing it up!

In laboratory problem solving, many insight problems can be solved through the production of a key action. For example, using the pliers as the pendulum weight is the key action necessary for solving Maier's (1931) two-string problem, and moving objects in three-dimensional space is necessary for the six matches problem (Scheerer, 1963) and the eight-coin problem (Ormerod, MacGregor, & Chronicle, 2002). As a third example, the key action necessary for solving the prisoner and rope problem (Metcalfe & Wiebe, 1987) is to unravel the rope into two strands, then tie the ends of the two strands together to escape the tower.

Sometimes the key action can be realized without much struggle, depending on an individual's prior knowledge. A friend (and fellow insight researcher) worked on a farm growing up, where the splitting of rope to make it longer was a common occurrence. He instantly knew how to solve the prisoner and rope problem; the key action was easy to discover. In other insight problems, however, the key action is not easy to discover. The common use of pliers hinders their use as a pendulum weight in Maier's (1931) two-string problem (cf. Birch & Rabinowitz, 1951). Additionally, other insight problems may have multiple factors of difficulty preventing the discovery of the key action, such as in the nine-dot problem (Maier, 1930).

Finding the Key Action in the Nine-Dot Problem

The nine-dot problem (Maier, 1930; see Figure 1) is quite possibly the most difficult insight problem that has been studied, with a typical solution rate for unaided participants of 0% (MacGregor, Ormerod, & Chronicle, 2001). Problem solvers are required to connect all the dots in a 3 x 3 matrix by using four straight lines, without lifting their pens from the page or retracing any lines.

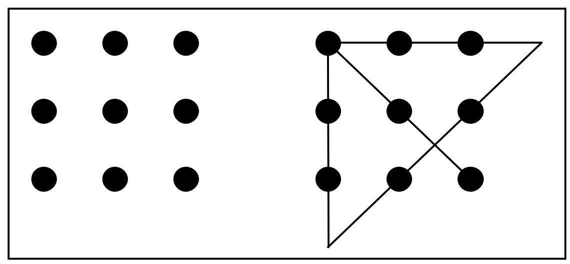

Figure 1: The Nine-Dot Problem and its Solution

The classic conception of the key action necessary for solving the nine-dot problem is that participants should draw lines that extend beyond the dots (Maier, 1930; Maier & Casselman, 1970; Scheerer, 1963). In a related conception, Lung and Dominowski (1985) claimed that the key action was drawing lines that did not begin or end on dots.

Kershaw and Ohlsson (2001) hypothesized that the key action necessary for solving the nine-dot problem was making non-dot turns, or turns that occur in the empty space between dots. The conception of non-dot turns as the key action came about through an inspection of two of MacGregor et al.'s (2001) nine-dot problem variants. The variant with no non-dot turn had a solution rate of 88% after four attempts, while the variant that required one non-dot turn only had a solution rate of 27% after four attempts. Kershaw and Ohlsson (2004) continued in this line of reasoning by explaining that the likelihood of producing a key action is dependent on the cognitive factors that underlie that action. In the nine-dot problem, multiple factors of difficulty are operating that each lower the probability of making a non-dot turn. Kershaw and Ohlsson (2004) distinguished three classes of difficulty: perceptual, knowledge, and process.

Perceptual factors include Gestalt properties of the nine-dot problem such as goodness of figure and figure-ground relationships. Making a non-dot turn requires that one both breaks the good figure of the square and views the white space beyond the dots as part of the problem. Knowledge factors refer to an individual's prior knowledge. Making a non-dot turn is hindered by people's prior experience with dot puzzles, such as connect-the-dot games played by children (cf. Weisberg & Alba, 1981). Process factors include the size of the search space, the specificity of the goal state, and the amount of mental lookahead necessary to find the solution. Making a non-dot turn is difficult because it is not obvious where to draw the first line or what the end state of the problem will be. In addition, people vary in the amount of mental lookahead they possess (cf. MacGregor et al., 2001), which affects the process of making non-dot turns. Kershaw and Ohlsson (2004) showed that perceptual, knowledge, and process factors interact to suppress the probability of producing the key action of non-dot turns in the nine-dot problem.

In the following experiments, non-dot turns are again examined as the key action necessary to solve the nine-dot problem. Experiment I follows up on Kershaw and Ohlsson (2004) but adds an additional possible facilitating factor: giving participants the first line of the solution, which should narrow the search space. Experiment II uses a think aloud methodology to explore what behaviors precede the production of non-dot turns.

Experiment I

Prior research by Kershaw and Ohlsson (2004; Kershaw, Ohlsson, & Coyne, 2003) has shown that increasing the number of non-dot turns leads to greater problem difficulty, such that the more non-dot turns a given problem requires, the harder that problem will be to solve.

Kershaw and Ohlsson (2004; Kershaw et al., 2003) increased solution rates through a training procedure that targeted the multiple factors of difficulty -- perceptual, knowledge, and process -- that hinder the production of non-dot turns. This training procedure was used in Experiment I.

A new facet of the procedure was to give participants the first line of the solution to each target problem. Weisberg and Alba (1981) raised the solution rate of the nine-dot problem to 62% by giving participants the first line in addition to instructing them to go outside of the box set up by the dots. The placement of this first line was chosen based on an analysis by MacGregor et al. (2001). For two of the target problems, the first line extended into the non-dot space.

The addition of the first line influenced the predictions for this experiment. One prediction was that first line would not affect the order nor the magnitude of the solution rates for the five target problems that were reported by Kershaw and Ohlsson (2004), with the 11-dot problem being the easiest and the three-turn problem being the most difficult. A second prediction was that the order of solution rate would remain the same, but that the magnitude would increase for all five problems. A third prediction was that the first line would differentially affect the solution rates for the problems, such that the displaced nine-dot and three-turn problems would show the greatest increase in solution rate due to their first lines cutting into the non-dot space.

Method

Participants and Design One hundred fifty undergraduates from UIC's participant pool participated in the experiment for course credit. No demographic data were collected about the participants.

Participants all received the same training, and one of five target problems.

Materials The first part of the training, the shape training, had a perceptual component in which participants learned to distinguish the shape of the nine-dot problem solution from other shapes (see Kershaw & Ohlsson, 2004; Kershaw et al., 2003). The second part of the training, the dot connecting training, featured problems made of black, filled dots presented on a grid of other unfilled dots as well as problems made of black dots that were alone on the page (see Kershaw & Ohlsson, 2001, 2004; Kershaw et al., 2003). In addition, the training contained a dialogue component in that participants were informed of the purposes of each training task.

The five target problems were taken from Kershaw and Ohlsson (2004; see Figure 2, the nine-dot problem was also used). The problems were modified by adding a diagonal line from the bottom right to the top left of the problem. The placement of the first line was chosen based on an analysis by MacGregor et al. (2001, Experiment 4). Participants were told to treat this line as the first line of the solution that they had to produce.

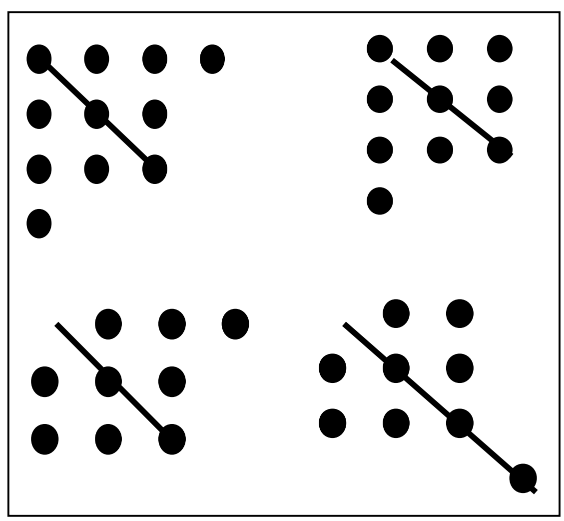

Figure 2: 11-Dot, 10-Dot, Displaced Nine-Dot, and Three-Turn Problems with First Line

Procedure Participants were seen in groups. Participants completed the shape training, and then the dot connecting training. During the shape training, participants were told that the shape they learned was the shape that would be required to solve the target problem. During the dot connecting training, participants were told that it was necessary to draw lines outside the dots and turn in the empty space between dots. Participants were also shown the correct answer for judging a shape or connecting dots for each judgment or problem that was completed. In addition, they were continually reminded that what they were learning in the training would be applicable to the target problem.

After completing the training, participants attempted one of five target problems (the 11-dot, 10-dot, nine-dot, displaced nine-dot, or three-turn). Participants were given four minutes to connect all the dots using four straight lines. They were instructed to view the line in the problem as the first line, and to draw the remaining lines such that all lines could be drawn without lifting their pens from the page or retracing the lines.

Results

Kershaw and Ohlsson (2004) found the following solution rates for the five target problems (in the training condition): 11-dot, 97%; 10-dot, 80%; displaced nine-dot, 50%; traditional nine-dot, 40%; three-turn, 30%. In contrast, the respective solution rates for this experiment were 83%, 60%, 38%, 40%, and 50% (see Figure 3). However, individual chi-square tests between each problem's solution rate for this experiment and Kershaw and Ohlsson's (2004) data were all non-significant (*p*s > .05).

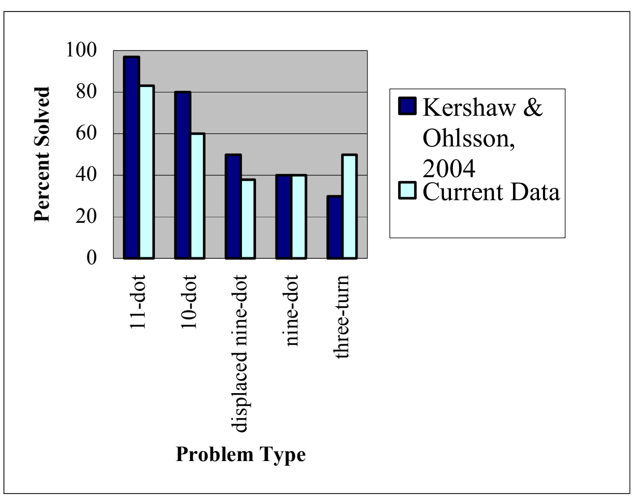

Figure 3: Comparison of Solution Order and Magnitude for the Five Problem Types

Effect of the number of non-dot turns Despite differences in exact solution rates, the new data are similar to those reported by Kershaw and Ohlsson (2004). First, there are overall differences in solution rate between the problems, χ^2 (4, N=150) = 17.02, $p < .05$, $\lambda = .15$. The standardized residuals were examined. The participants who solved the 11-dot problem (25/30 or 83%) caused the greatest standardized residual, 2.2; therefore, this cell made the greatest contribution to the chi-square. When the 11-dot problem was removed from the analysis, the differences between the other problems were not significant, χ^2 (3, N=120) = 4.02, $p > .05$, $\lambda = .09$. Therefore, once a non-dot turn was introduced, all problems became equally difficult.

An alternative way to examine the influence of the number of non-dot turns is to determine the probability of making a non-dot turn (cf. Kershaw & Ohlsson, 2004). The percentage of participants who made any non-dot turns versus correct non-dot turns was calculated for the nine-dot and three-turn problems, both of which require two unassisted (not affected by the first line) non-dot turns. Sixty-five percent (39/60) of the participants made one non-dot turn. Of these participants, 100% (39/39) made two non-dot turns. In contrast, 52% of the participants (31/60) made one correct non-dot turn. Of these participants, 94% (29/31) made two correct non-dot turns.

Effect of drawing lines beyond the dots The nine-dot problem forms a good Gestalt, but the dot groups that make up the other problems do not. The tendency to draw lines that extend beyond the boundary of the dots, the classic explanation of difficulty for the nine-dot problem, was measured across the problem types. The 11-dot problem was excluded from this analysis because drawing lines outside the dots is unnecessary for solution.

Eighty-three percent (99/120) of the participants drew lines outside of the dots. In the 10-dot (24/30), displaced nine-dot (28/30), and three-turn (29/30), participants were equally likely to draw lines outside of the dots, χ^2 (2, N=90) = 5.19, $p > .05$, $\lambda = .07$, despite differences in solution rate. In contrast, participants who attempted the nine-dot problem were less likely to draw lines outside of the dots (18/30). This effect is striking when the nine-dot problem is compared to the three-turn problem, both of which required two unassisted non-dot turns, χ^2 (1, N=60) = 11.88, $p < .05$, $\lambda = .26$. Although the solution rate for these two problems did not differ, the three-turn problem led to a greater rate of drawing lines outside the dots than the nine-dot problem.

Discussion

The results of Experiment I are comparable to Kershaw and Ohlsson (2004) in solution magnitude, the probability of making a non-dot turn, and the prevalence of drawing lines outside the dots. The order of solution rates did differ in that the three-turn problem had the third-highest solution rate in the current data, compared to the lowest solution rate in Kershaw and Ohlsson's (2004) data. However, as in Kershaw and Ohlsson's results, the problems that required non-dot turns did not differ significantly from each other. In addition, individual comparisons between each problem across the two data sets were not significant.

The current data do not support any of the predictions fully. Providing the first line of the solution did not affect the magnitude of solution rate, as predicted, but did affect the order of the solution rate. The solution percentages appear to support the third prediction, that solution magnitude would be affected differentially, but the rate increased for the three-turn problem yet decreased for the displaced nine-dot problem. However, as mentioned above, individual comparisons between the problem types across data sets did not reveal any significant differences.

The current data give further support to the non-dot turn as the key action necessary for solving the nine-dot problem and its variants. As soon as a non-dot turn was introduced, the solution rate dropped by at least 20%. In addition, drawing lines that went outside the dots was not enough to solve the problem. Eighty-three percent of the participants who attempted the 10-dot, displaced nine-dot, nine-dot, or three-turn problems drew lines outside of the dots, but only 47% of the participants correctly solved one of these four problems.

As noted previously, giving participants the first line did not increase the solution rate, as compared to Kershaw and Ohlsson (2004). This finding is interesting compared to similar manipulations used by Weisberg and Alba (1981) and MacGregor et al. (2001). Weisberg and Alba (1981) achieved a solution rate of 62% by giving participants the first line and telling them to go outside the dots. MacGregor et al. (2001, Experiment 4), in contrast, achieved a 6% solution rate after the first 10 attempts, and 47% after 10 additional attempts by giving participants the first line of the nine-dot problem. One explanation, in light of the current data, is that the extensive training used in Experiment I overshadowed any benefit of the first line for the problem variants. Although the solution rate was raised for the three-turn problem, its solution rate was not significantly different than the rate found for the three-turn problem by Kershaw and Ohlsson (2004), nor were there any differences in solution rate across the two experiments for any of the nine-dot problem variants. Untrained participants, in contrast, would most likely benefit from being given the first line, and would thus show differences in comparison to the control group in Kershaw and Ohlsson (2004, Experiment 3).

Experiment II

Experiment I further established non-dot turns as the key action required for solving the nine-dot problem. Experiment I also showed that the classic conception of difficulty for the nine-dot problem, the inability to draw lines beyond the boundary of the dots, did not hold up as a difficulty for the other problem types. However, participants were less likely to draw lines outside the dots for the nine-dot problem, thus supporting the Gestalt factor.

The aim of Experiment II was to examine how participants explore the search space of the nine-dot and 10-dot problems. Both Kershaw and Ohlsson (2004) and Experiment I showed that making non-dot turns is important, but did not show the process that participants go through when making a non-dot turn. Experiment II used a think-aloud methodology to examine the individual thoughts and actions that lead to the making of non-dot turns. Verbal protocols and other trace methods, such as eye movements, have been used effectively to understand the processes involved in achieving insight in problems such as the mutilated checkerboard (Kaplan & Simon, 1990) and in matchstick arithmetic (Knoblich, Ohlsson, & Raney, 2001).

Half of the participants received the training used in Experiment I, and the other half were not trained. The participants received either the nine-dot or 10-dot problem as their target problem. One prediction for Experiment II is that participants who received training will be more likely to solve their target problems, and will show a greater incidence of behaviors that lead to non-dot turns. Based on solution rates found in Experiment I and Kershaw and Ohlsson (2004), no difference in solution rate is expected between the 10-dot and nine-dot problems.

Method

Participants and Design Twenty undergraduates from UIC's participant pool participated in the experiment for course credit. No demographic data were collected about the participants.

The design of Experiment II was a 2 x 2 factorial. The two independent variables were type of training (control and training) and target problem (nine-dot and 10-dot).

Materials The training materials used in Experiment II were the same materials used in Experiment I. The control group did not receive any training. In addition, all participants were given a long division problem as a practice for thinking aloud while solving the target problem. A video camera was used to record each participant's verbalizations and actions.

Procedure Participants were seen individually. As in Experiment I, the participants who received training learned to distinguish the shape of the nine-dot problem solution from other shapes, and learned how to connect dots. They were shown the correct answer for each training exercise and were reminded that the material learned in training would be useful for solving the target problem. Participants in the control group did not receive any training.

Before beginning the target problem, participants practiced thinking out loud by solving a long division problem. Participants were then given four minutes to attempt the target problem. They were told to connect all the dots by using four straight lines, without lifting their pens from the page or retracing any lines. They were instructed to talk out loud while working on the problem. If the participant stopped verbalizing while working on the problem, the experimenter reminded the participant to continue talking. Each participant's verbalizations, as well as his or her actions, were recorded using a video camera.

Protocol transcription Each participant's verbalizations and actions were transcribed into a verbal protocol by either the author or a research assistant. Protocols were constructed so that the participant's words and actions were grouped together. Actions were described in terms of drawing or simulating lines, and were transcribed by using a map that numbered the dots in each problem.

Results

Effects of training and problem type Solution rates for the problems across training types are as follows: 10-dot training, 60% (3/5); 10-dot control, 20% (1/5); nine-dot training, 40% (2/5); nine-dot control, 0% (0/5). A chi-square analysis was conducted to determine the effect of problem type. There was no significant difference between the number of solvers for the 10-dot and nine-dot problems, χ^2 (1, N=20) = .952, $p > .05$, $\lambda = .13$, as predicted.

A second chi-square analysis was conducted to determine the effect of the training. Participants were more likely to solve their target problem when they had received training than when they had not, χ^2 (1, N=20) = 3.81, p = .05, $\lambda = .25$, as predicted.

Analysis of behaviors that lead to non-dot turns Participants' verbal protocols were examined to determine the behaviors that led to making non-dot turns, the key action necessary for solving the 10-dot and nine-dot problems. Based on our previous work (Kershaw & Ohlsson, 2001, 2004; Kershaw et al., 2003), we hypothesized that several actions would show that participants were affected by the training and understood the requirements of the problem: 1) making diagonal lines, 2) making triangle shapes, 3) making the arrow-like shape of the nine-dot problem solution, and 4) making lines that extended beyond the boundary of the dots. For the purposes of this paper, the two actions that will be analyzed are making arrow shapes and making lines that extend beyond the boundary of the dots. In addition, participants' verbalizations may reveal attention to particular areas of the problem, or a rehearsal of strategies.

The participants' verbalizations were surprisingly unhelpful in determining what thoughts preceded making non-dot turns. The majority of participants limited their verbalizations to keeping track of the number of lines they had drawn so far. Only four of the 20 participants verbalized anything about going outside of the dots. Examples of these verbalizations include: "outside the line here" (said while moving a pen from the bottom right dot to the top left dot) and "let's see, I should probably think more about going outside," which was not accompanied by an action.

The use of arrow shapes and lines that extended beyond the dots illustrated the effect of the training in the solution attempts of the participants. Eight participants in the training group made at least one arrow shape, while only three participants in the control group made an arrow shape; this difference was significant, χ^2 (1) = 5.05, $p < .05$, λ = .47. Likewise, all 10 participants in the training group made lines that extended beyond the dots, while only two participants in the control group attempted such dots. This difference was also significant, χ^2 (1) = 13.33, $p < .05$, λ = .78.

In addition, these actions were better indicators of events that precede non-dot turns than participants' verbalizations. Participants who solved their target problems drew arrow shapes and extended lines beyond the dots in the correct places before making non-dot turns. In contrast, some participants who did not solve their target problems also drew arrow shapes, but drew them exclusively inside the dots. Other non-solving participants drew arrow shapes and extended lines, but did not make non-dot turns. As in Experiment I, drawing lines that extended beyond the dots was not enough to solve the target problems. Participants needed to extend their lines in the correct places, and make non-dot turns.

Discussion

The results of Experiment II followed up those of Experiment I and Kershaw and Ohlsson (2004) by showing the importance of non-dot turns in solving the nine-dot (and 10-dot) problem(s). In addition, Experiment II showed, like Kershaw and Ohlsson (2004), the effectiveness of training for raising the solution rate for the nine-dot and 10-dot problems.

Experiment II contributes to this line of research by providing a means to analyze the process of attempting the nine-dot problem (or one of its variants). This initial analysis of the verbal protocols revealed that participants who receive training are more likely to produce actions that are necessary for solving the problem, such as drawing an arrow shape, extending a line beyond the dots, and making a non-dot turn. However, as shown in Experiment I and in Kershaw and Ohlsson (2004), making non-dot turns is a difficult key action to execute. Participants must extend lines beyond the dots in the correct place and form the arrow shape of the solution correctly. Merely extending any line beyond the boundary of the dots will not lead to solution.

General Discussion

Key actions can be identified in many different types of problems and in everyday life, from using pliers as a pendulum weight in Maier's (1930) two-string problem to learning to continually stir a roux. In the nine-dot problem, the key action is making a non-dot turn (Kershaw & Ohlsson, 2001, 2004; Kershaw et al., 2003). While some key actions are easily discovered and produced, making a non-dot turn is hindered by interacting factors of difficulty: perceptual, knowledge, and process.

In Experiments I and II, making non-dot turns was compared to the classic conception of the key action necessary for solving the nine-dot problem, drawing lines that extend beyond the dots (Maier, 1930; Maier & Casselman, 1970; Scheerer, 1963). In both experiments, drawing lines outside the dots was not sufficient to solve a target problem. As a striking example, nearly all the participants in Experiment II readily drew lines outside of the dots in the 10-dot, displaced nine-dot, and three-turn problems. However, less than half of the participants actually solved one of these problems.

Experiments I and II provided further support for Kershaw and Ohlsson's (2004) analysis of the importance of making non-dot turns. Other insight and everyday problems are best solved through different key actions. Further study will allow for the identification of these key actions, and the determination of what cognitive factors underlie the production of such actions.

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