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# The Local Group: the ultimate deep field

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## ABSTRACT

*Near-field cosmology* – using detailed observations of the Local Group and its environs to study wide-ranging questions in galaxy formation and dark matter physics – has become a mature and rich field over the past decade. There are lingering concerns, however, that the relatively small size of the present-day Local Group ( $\sim 2$  Mpc diameter) imposes insurmountable sample-variance uncertainties, limiting its broader utility. We consider the region spanned by the Local Group’s progenitors at earlier times and show that it reaches  $3 \text{ arcmin} \approx 7$  comoving Mpc in linear size (a volume of  $\approx 350 \text{ Mpc}^3$ ) at  $z = 7$ . This size at early cosmic epochs is large enough to be representative in terms of the matter density and counts of dark matter haloes with  $M_{\text{vir}}(z = 7) \lesssim 2 \times 10^9 M_{\odot}$ . The Local Group’s stellar fossil record traces the cosmic evolution of galaxies with  $10^3 \lesssim M_{\star}(z = 0)/M_{\odot} \lesssim 10^9$  (reaching  $M_{1500} > -9$  at  $z \sim 7$ ) over a region that is comparable to or larger than the *Hubble* Ultra-Deep Field (HUDF) for the entire history of the Universe. In the *JWST* era, resolved stellar populations will probe regions larger than the HUDF and any deep *JWST* fields, further enhancing the value of near-field cosmology.

**Key words:** galaxies: evolution – Local Group – cosmology: observations – cosmology: theory.

## 1 INTRODUCTION

The standard introduction of a paper on near-field cosmology extols the virtues of the Local Group as a cosmic Rosetta Stone that provides archaeological clues left behind by untold generations of stars, clues that may unlock unsolved mysteries in galaxy formation. A frequent concern is that resolved-star studies are inherently limited to a region that is relatively small and likely biased, however, setting fundamental limits on the broader applicability of results based on near-field studies. In this Letter, we show that the volume spanned by the high-redshift progenitors of the Local Group was large enough to have been typical in many respects. The archaeological record imprinted on Local Group galaxies is therefore likely to provide an unbiased view of faint galaxy populations at early times, making near-field observations a powerful complement to direct deep-field studies.

## 2 METHODS

At present, the Local Group consists of two dark matter haloes, each with virial mass of  $\sim 10^{12} M_{\odot}$  (e.g. Klypin, Zhao & Somerville

2002), approaching each other for presumably the first time (Kahn & Woltjer 1959). The comoving Lagrangian volume of the matter contained within the Local Group – i.e. the comoving volume spanned at earlier epochs by the particles in the current-day Local Group – was therefore larger in the past; the same is true for all dark matter haloes. Thus, while the Local Group is a very specific (and non-typical) volume of the Universe today, its properties at earlier times should be closer to an average portion of the Universe. Precisely how much closer is the topic of this Letter.

The spherical collapse model (Gunn & Gott 1972) provides a starting point for understanding the evolution of the Local Group. In this model, the matter within a virialized dark matter halo of radius  $R_{\text{vir}}$  (defined by an average enclosed density of  $\Delta_{\text{vir}}$  times the critical density of the Universe) came from a spherical region with a comoving Lagrangian radius  $r_l$  equal to  $(\Delta_{\text{vir}}/\Omega_m)^{1/3} R_{\text{vir}}$ . For flat cosmologies with  $\Omega_m \sim 0.3$ , this gives  $r_l \approx 7 R_{\text{vir}}$ , indicating that the Local Group must have been significantly larger in comoving size in the early Universe. Cosmological zoom-in simulations, by design, follow the evolution of Lagrangian regions surrounding specific haloes from linear fluctuations to the highly non-linear regime (Katz & White 1993; Oñorbe et al. 2014). The ELVIS suite of  $N$ -body zoom-in simulations (Garrison-Kimmel et al. 2014) provides 12 Local Group analogues simulated from  $z = 125$  to  $z = 0$ , each of which is uncontaminated by lower resolution particles over a

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spherical region having a radius of at least 1.2 Mpc centred on the  $z = 0$  barycenter of the Local Group. In what follows, we use this suite to study the comoving volume probed by the Local Group at higher redshifts.

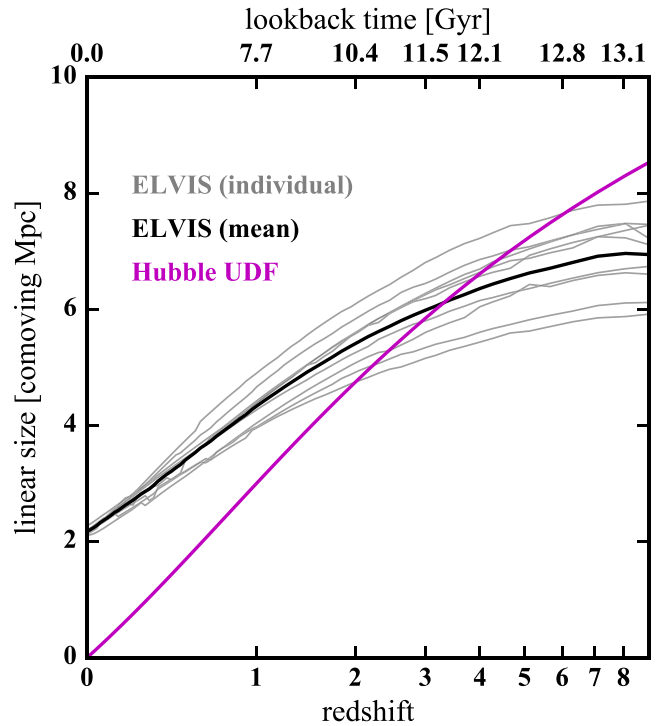
In our analysis, we first eliminate three ELVIS pairs that contain a third large, nearby halo, as these would bias any results. For the remaining nine pairs, we identify all subhaloes within 1.2 Mpc of the Local Group's  $z = 0$  barycenter and track *all* of their progenitors back through time. There are occasionally individual subhaloes that come from regions that are distant from the vast majority of the matter that forms the Local Group; such subhaloes can artificially increase the inferred volume of the Local Group at earlier times. To eliminate these objects, we run a friends-of-friends (Davis et al. 1985) group finder with a large linking length of 400 kpc and retain only the main grouping. In practice, this removes  $\ll 1$  per cent of subhaloes at  $z = 0$ . We then identify the positions spanned by the progenitors of the remaining Local Group subhaloes above the ELVIS completeness limit of  $M_{\text{peak}} = 6 \times 10^7 M_{\odot}$  at each earlier snapshot; this constitutes the 'proto-Local Group' at each epoch. It is important to note that the number of galaxies or haloes in the proto-Local Group is much larger than the number in the Local Group at  $z = 0$  owing to mergers and disruption over time.

If we are only interested in understanding the Local Group itself, this would be sufficient. To place the Local Group in context at higher redshifts, however, we must understand the full environment that the proto-Local Group occupies. We therefore compute, at each snapshot, the minimum cuboid volume defined by the proto-Local Group – i.e. the rectangular cuboid defined by the minimum and maximum comoving coordinate locations of all proto-Local Group progenitors at that time,  $\mathcal{V}_{\text{RC}}$  – and identify all additional haloes in this region (i.e. haloes that appear to be part of the proto-Local Group but that do not end up in the Local Group at  $z = 0$ ). The inclusion of these objects roughly doubles the counts at  $z \sim 7$  within  $\mathcal{V}_{\text{RC}}$ . This doubling is consistent with the extra volume contained in the cuboid  $\mathcal{V}_{\text{RC}}$  region circumscribing a sphere, although the proto-Local Group progenitors are not confined to a spherical region at  $z \sim 7$ . Proto-Local Group haloes dominate the central portion of  $\mathcal{V}_{\text{RC}}$  and the additional haloes populate the outskirts of the volume.

We define the linear size of the proto-Local Group  $l_{\text{LG}}(z)$  as the geometric mean of the three axes defining  $\mathcal{V}_{\text{RC}}(z)$ ; in other words,  $l_{\text{LG}}(z) = \mathcal{V}_{\text{RC}}(z)^{1/3}$ . At  $z = 0$ , the Local Group volume is defined by a sphere of radius 1.2 Mpc, so  $l_{\text{LG}}(z = 0) \approx 2.4$  Mpc (the actual number depends on the distribution of haloes at  $z = 0$  but can never exceed 2.4 Mpc). At higher redshifts,  $\mathcal{V}_{\text{RC}}$  can, in principle, become highly elongated in one or two dimensions. In practice, however, we find that this is not the case: at  $z = 7$ , the median minor-to-major axis ratio is 0.76 and the median intermediate-to-major axis ratio is 0.81, and in only one case is the minor axis smaller than half of the major axis size. The typical  $\mathcal{V}_{\text{RC}}(z = 7)$  is moderately prolate: six of nine simulated proto-Local Groups have a triaxiality parameter  $T$  (see Franx, Illingworth & de Zeeuw 1991) larger than 0.5.

### 3 THE LOCAL GROUP THROUGH TIME

Fig. 1 shows the comoving linear size,  $l_{\text{LG}}(z)$ , of the proto-Local Group going back in time to  $z = 9$ . Thin grey lines show the size of individual Local Group pairs from the ELVIS simulation suite, while the thick black line shows the median value across the ELVIS pairs at each redshift. The linear size of the proto-Local Group increases with increasing redshift, reaching  $\approx 7$  Mpc (comoving) at  $z \sim 7$ . Going back in time, therefore, the Local Group probes a significantly larger (comoving) volume than it does today. To give



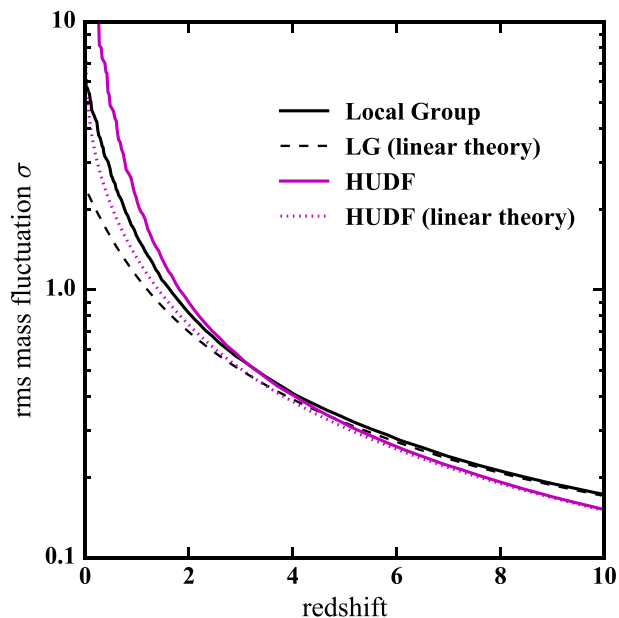
**Figure 1.** The comoving linear extent of the proto-Local Group (black, grey curves) and HUDF (magenta curve) as a function of redshift. For  $z \lesssim 3$ , the proto-Local Group covers an area on the sky that is larger than the HUDF. At earlier times, the HUDF is marginally larger. The typical proto-Local Group reaches a comoving size of 7 Mpc at  $z \sim 7$ , meaning it probes an effective volume of  $\sim 350$  comoving  $\text{Mpc}^3$  in the reionization era.

context to the Local Group's size at earlier epochs, Fig. 1 also shows the comoving linear size of the HUDF (Beckwith et al. 2006, assuming an angular size of  $3.1 \text{ arcmin} \times 3.1 \text{ arcmin}$ ) as a function of redshift (magenta curve). At all epochs later than  $z \approx 3$  (the last 85 per cent of cosmic time), the proto-Local Group covers a larger area on the sky than the HUDF.

It is important to understand how representative such portions of the Universe are at each cosmological epoch. One way to do this is to compute the rms amplitude of density fluctuations  $\sigma$  in regions having volumes equal to  $\mathcal{V}_{\text{RC}}(z)$ . In classical Press–Schechter (1974) theory and its extensions, the typical scale  $M^*$  that is collapsing at a given epoch has  $\sigma_{\text{lin}}(M^*, z) = \delta_c \approx 1.686$  (where subscript 'lin' indicates that the relevant rms amplitude comes from linear theory, extrapolated to the redshift in question). Roughly speaking, scales with  $\sigma(M, z) > 1$  have collapsed while those with  $\sigma(M, z) \ll 1$  are firmly in the linear regime.

In reality, linear theory underestimates  $\sigma(M, z)$ . Cosmological simulations account for effects of non-linear growth; accordingly, we use the *Illustris* suite (Vogelsberger et al. 2014a,b; Nelson et al. 2015) to compute  $\sigma(M, z)$ . At each snapshot, we evaluate the density field for *Illustris*-Dark-1 on a  $256^3$  grid, compute the overdensity  $\delta_i = \rho_i / \langle \rho \rangle - 1$  for each cell  $i$ , then calculate  $\delta(M, z)$  by smoothing the gridded overdensity field with a real-space top-hat filter having a volume equal to  $\mathcal{V}_{\text{RC}}(z)$  (so the mass contained in the volume is  $M_{\text{LG}} = \rho_m(z) \mathcal{V}_{\text{RC}}(z)$ ). The rms amplitude of fluctuations is equivalently characterized by  $\sigma(M_{\text{LG}}, z)$  or  $\sigma(l_{\text{LG}}, z)$ .

The resulting values for  $\sigma(l_{\text{LG}}, z)$  are plotted as a solid black curve in Fig. 2; the linear theory value of  $\sigma(l_{\text{LG}}, z)$  is shown as a dashed black curve. We also compute the same quantities for the HUDF,  $\sigma(l_{\text{HDF}}, z)$ , and plot these with magenta curves. Both the Local

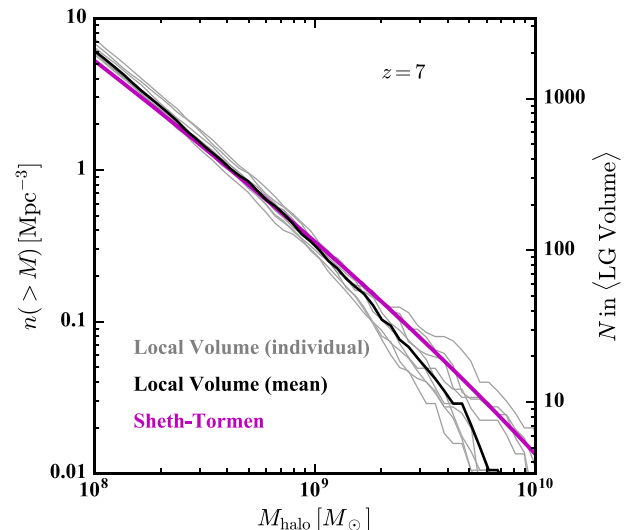


**Figure 2.** The rms mass fluctuation,  $\sigma$ , computed over volumes equal to that covered by the proto-Local Group (black) and HUDF (magenta) as a function of redshift. Also shown are linear theory values for  $\sigma$  over those same volumes. At early times, both regions have  $\sigma \sim 0.2$  and the redshift evolution is well-described by linear theory. The regions become non-linear ( $\sigma \gtrsim 1$ ) at  $z \sim 2$ , by which point linear theory underestimates the true rms amplitude of fluctuations.

Group and the HUDF have  $\sigma < 1$  for  $z \gtrsim 2$ . At high redshift, both probe volumes that are well-described by linear theory. In particular, the volumes probed by the proto-Local Group and by cubic slices of the HUDF ( $\Delta z \approx 0.02$ ) in the reionization era ( $6 \lesssim z \lesssim 10$ ) have  $\sigma(M_{\text{LG}}) \lesssim 0.25$ . For a broad discussion of variance in deep-field galaxy counts, see Robertson (2010).

The results above have established that the proto-Local Group was substantially larger at earlier epochs, large enough to cover a volume that becomes non-linear only after  $z \sim 2$ . A further, and more stringent, test is to compare the mass function in the region defined by the proto-Local Group to the cosmological mass function at earlier times. In Fig. 3, we plot the cumulative comoving number density  $n(>M)$  of haloes within  $\mathcal{V}_{\text{RC}}$  for each ELVIS pair (thin grey lines), as well as the median across the simulation suite (thick black line), at  $z = 7$ . The cosmological expectation, as encapsulated by the Sheth-Tormen (2001) mass function, is plotted as a magenta line. The mass function in  $\mathcal{V}_{\text{RC}}(z = 7)$  matches the cosmological mass function for  $M_{\text{vir}} \lesssim 2 \times 10^9 M_{\odot}$ , with counts in the proto-Local Group region falling below Sheth-Tormen at higher masses (smaller number densities) owing to the size of this region. *The volume covered by the proto-Local Group at  $z = 7$  is therefore a cosmologically representative region for the mass function of haloes with  $M_{\text{vir}}(z = 7) \lesssim 2 \times 10^9 M_{\odot}$ , a remarkable result.*

As indicated by the scale on the right side of the figure, there should be  $\sim 2000$  haloes with  $M_{\text{vir}} > 10^8 M_{\odot}$  (this approximately corresponds to haloes above the atomic cooling threshold) and  $\sim 100$  haloes with  $M_{\text{vir}} > 10^9 M_{\odot}$  in the  $z = 7$  proto-Local Group region. It is this large number of low-mass systems, coupled with the small value of  $\sigma(l_{\text{LG}}, z = 7)$ , that makes mass functions in the proto-Local Group cosmologically representative even in the  $350 \text{ Mpc}^3$  comoving volume at early times (note the small variance in normalizations of the mass functions in Fig. 3). By  $z = 4$ , the proto-Local Group has



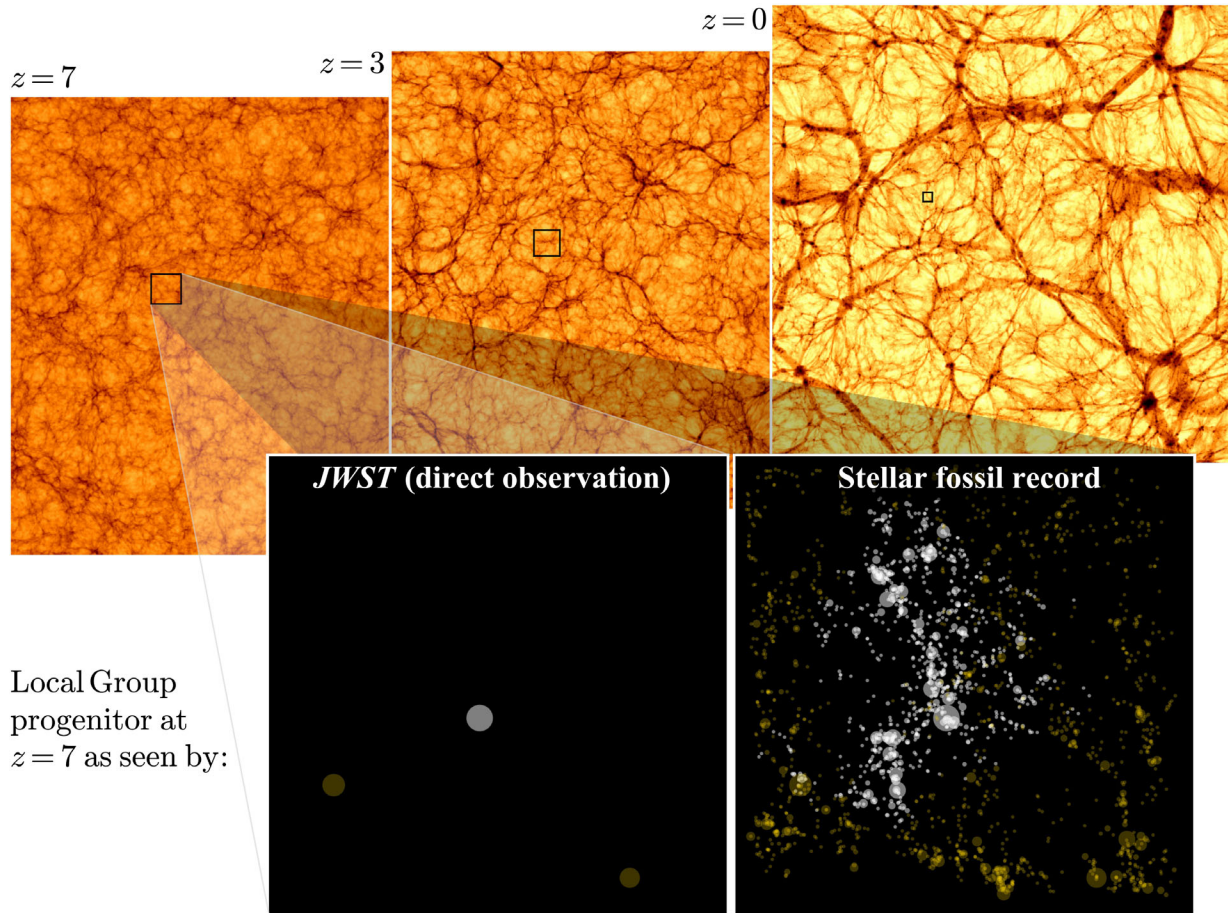
**Figure 3.** Cumulative dark matter halo mass functions for the volume spanned by the proto-Local Group at  $z = 7$  (grey curves: individual ELVIS simulations; black curve: ensemble average). The right y-axis gives the cumulative counts for the average proto-Local Group region in ELVIS. The magenta curve shows the global Sheth-Tormen mass function. At  $z = 7$ , the normalization of the ELVIS proto-Local Group mass function agrees very well with the Sheth-Tormen mass function, indicating the proto-Local Group covers a representative portion of the Universe.

become somewhat more dense, and the equivalent plot of mass functions shows that counts in the proto-Local Group volume exceed Sheth-Tormen expectations by  $\approx 50$  per cent at this epoch. However, the shape of the mass functions still matches that of Sheth-Tormen for cumulative number densities larger than  $10^{-2} \text{ Mpc}^{-3}$  (equivalently,  $M_{\text{vir}} < 5 \times 10^{10} M_{\odot}$ ).

#### 4 DISCUSSION

The previous sections have explored a simple yet important question: given regions that end up as Local Group analogues at  $z = 0$ , how representative is the volume that their progenitors spanned at earlier times? The rough estimate given at the start of Section 2 indicates that a collapsed dark matter halo covered a comoving region that was  $\gtrsim 7$  times larger in the early Universe than at  $z = 0$ , and the results of Section 3 show that the Local Group (which is not a virialized region today) was approximately 3.5 times larger than its present-day size at early times. The proto-Local Group is large enough to be representative at high redshifts both for the matter density it contains and for counts of dark matter haloes with  $M_{\text{vir}} \lesssim 2 \times 10^9 M_{\odot}$ .

These results are only true when starting with Local Group-like regions at  $z = 0$  and considering their properties at earlier times; proto-Local Group-sized regions selected at  $z \sim 7$  are large enough to be representative at that time but will not generally evolve to be Local Groups at  $z = 0$ . The direct, one-to-one connection between galaxies in the  $z = 0$  Local Group and their ancestors in the proto-Local Group is complicated by any mergers and disruption of galaxies in the intervening time; this also means that surviving galaxies at  $z = 0$  represent a lower limit on the number of similar galaxies in the proto-Local Group. While we plan to address this point in future work, we note that the predicted merger histories of low-mass haloes in  $\Lambda$ CDM are such that most present-day dwarfs have not had significant mergers (in terms of stellar mass



Local Group  
progenitor at  
 $z = 7$  as seen by:

**Figure 4.** Thin density slices of the *Illustris* simulation at  $z = 0, 3,$  and  $7$  (upper panels;  $106.5 \times 106.5 \times 1.065$  comoving Mpc), along with the comoving size of the average proto-Local Group at each redshift (top panels). The bottom panels show the galaxies in such a region (which is comparable in linear extent to the HUDF) that are accessible at  $z = 7$  through direct observation with *JWST* in a narrow redshift slice of width  $\Delta z \approx 0.02$  (corresponding to the  $\sim 7$  Mpc comoving size of the proto-Local Group at  $z = 7$ ) and through the stellar fossil record in the Local Group. White symbols indicate objects that end up in the Local Group at  $z = 0$ , while gold symbols represent objects in the proto-Local Group volume at  $z = 7$  but not within 1.2 Mpc of the Local Group barycenter at  $z = 0$ .

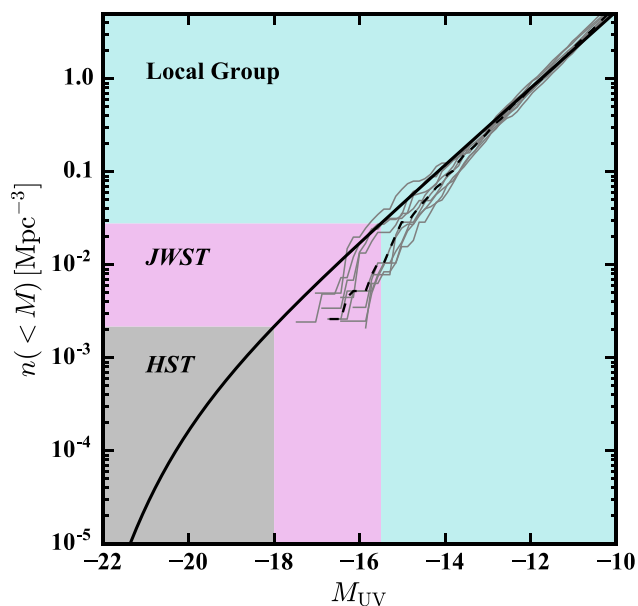
growth) since the reionization era (Deason, Wetzel & Garrison-Kimmel 2014; Boylan-Kolchin et al. 2015); Local Group dwarfs are therefore expected to provide a direct window to low-mass systems in the high-redshift Universe.

The representative nature of the proto-Local Group’s volume, coupled with the similarity between the (1D) size of the proto-Local Group and the HUDF, suggest an interpretation of the Local Group at earlier epochs: *observations of the Local Group can be thought of as providing a (very) narrow slice in time of the HUDF.* Given our ability to measure resolved star formation histories of Local Group galaxies, we can look at the Local Group at a variety of ‘snapshots’ in time. This is the same as looking at a series of thin transverse slices through the HUDF. A complete census of galaxies within 1–2 Mpc of the Local Group with depth to reach the oldest main sequence turn-off would therefore allow a *continuous* look at galaxy formation and evolution – i.e. tracking individual galaxies across cosmic time – in a size equivalent to the HUDF to depths of  $m \sim 38$  ( $M_{UV} \approx -9$ ) at  $z = 7$ .

We emphasize that resolved-star studies of the Local Group provide an almost perfectly complementary view of galaxy formation to deep blank-field (and lensing) observations of the high-redshift Universe with *Hubble*. At  $z \sim 7$ , the faintest galaxies in blank-field *HST* observations are likely to be more massive than the pro-

genitor of the Milky Way at that time (Boylan-Kolchin, Bullock & Garrison-Kimmel 2014). Archaeological studies of Local Group galaxies extend the range of galaxies to at least 8 mag fainter (Weisz, Johnson & Conroy 2014; Boylan-Kolchin et al. 2015; Graus et al. 2016). *HST* is therefore capable of probing galaxy formation over six decades in mass ( $10^3 < M_*/M_\odot < 10^9$ ) and 12–13 Gyr in time over an area comparable to the HUDF via the stellar fossil record.

The power of near-field studies, and their complementarity to direct high- $z$  observations, is emphasized in Figs 4 and 5. For both figures, we assign UV luminosities at  $z = 7$  to ELVIS dark matter haloes via abundance matching based on the global UV luminosity function from Finkelstein et al. (2015) and the Sheth–Tormen mass function. Fig. 4 shows slices through the density distribution of the *Illustris* simulation at  $z = 0, 3,$  and  $7$ , along with boxes indicating the approximate size of the proto-Local Group at each of those redshifts. The insets at  $z = 7$  show galaxies that can be observed either directly in the proto-Local Group with the *James Webb Space Telescope* (*JWST*; left) or through archaeological studies in the Local Group with *HST* (right). *JWST* deep fields will have many more galaxies at a range of redshifts, while the stellar fossil record in the Local Group probes a huge number of galaxies that will be unobservably faint at cosmic dawn.



**Figure 5.** Complementarity of direct observation and stellar fossil record studies of galaxies at high- $z$ . Solid line: cumulative luminosity function at  $z \sim 7$  from deep-field observations using *HST* (from Finkelstein et al. 2015). The best-fitting cumulative luminosity function is extrapolated beyond the HUDF magnitude limit of  $M_{UV} = -18$ . Solid grey curves show the cumulative luminosity functions found in each ELVIS pair based on abundance matching. The agreement in normalization between the luminosity function globally and in the proto-Local Group region is a direct result of the unbiased nature of counts in the proto-Local Group volume at  $z = 7$  (Fig. 3). The approximate blank field limiting magnitude for *JWST* is also shown, along with the corresponding number density. The Local Group probes galaxies that, for the most part, will remain undetected through direct observations in the high- $z$  Universe because of their intrinsic faintness.

Fig. 5 shows the cumulative galaxy luminosity function from Finkelstein et al. (2015) at  $z = 7$  (thick black curve), extrapolated from the *HST* observational limit of  $M_{UV} \sim -18$  to  $M_{UV} = -10$ . Each grey line shows the luminosity function in the proto-Local Group volume from an individual ELVIS simulation, while the dashed black line shows the mean among all ELVIS pairs. Importantly, the agreement in normalization between the global luminosity function and the proto-Local Group regions from ELVIS was not pre-determined but rather is a result of the agreement between the global halo mass function and the halo mass function within the proto-Local Group at  $z = 7$ . The approximate reach of *HST* and *JWST* is shown in the figure, while the proto-Local Group covers the entire region of the figure in which the halo mass functions are non-zero.

The high angular resolution of *JWST* and its exquisite sensitivity to (red) ancient main-sequence turn-off stars make it ideally suited to extend the same census to 3–5 Mpc, meaning it is capable of surveying a region within the Local Volume that is markedly larger than the HUDF (and any *JWST* deep field) at all redshifts. The complementarity with direct observations of galaxies at early cosmic epochs will also be enhanced, as *JWST* will likely detect galaxies as faint as  $M_{UV} \sim -15.5$  in blank fields at  $z \sim 7$  (Windhorst et al. 2006), comparable to progenitors of the Large Magellanic Cloud (Boylan-Kolchin et al. 2015; see Patej & Loeb 2015 for related calculations). The *Wide-Field Infrared Survey Telescope* (*WFIRST*) has potential to be similarly transformative for near-field cosmology. The addition of an optical filter (e.g. *R*-band) would improve

its angular resolution (and temperature sensitivity), allowing it to reach the ancient main sequence turn-offs of galaxies out to  $\sim 5$  Mpc. This modification, combined with *WFIRST*'s extremely wide field of view, would capture full star formation histories over the entire spatial extent of virtually any nearby galaxy in a single pointing, revolutionizing how we study and understand the evolution of low-mass galaxies.

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