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Publication Date

1969-08-18

To be presented at 7th International
Conference on High Energy Accelerators,
Yerevan, Armenia, USSR, August 27—
September 2, 1969.

UCRL-19213
Preprint
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AEC Contract No. W-7405-eng-48

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ABSTRACT

In the development of a resonant extraction system for the Bevatron it has been necessary to consider the effects of the strong nonlinearities in the synchrotron guide field. The variation of particle oscillation frequency with amplitude and distortions from sinusoidal form have been analyzed. Effects on the resonant growth process have been calculated and observed. Guided by the analysis, it has been possible to achieve satisfactory behavior with only minor alterations of the guide field. Use of a small spill-control perturbation is discussed and performance of the extraction system is reported.

Introduction

The plans for the Bevatron resonant extraction system were described in a previous paper.[1] As tests of the system have proceeded, effects of the variation of focusing with radius became evident, interfering with the study of the resonant process and limiting flexibility for future use. Although it is difficult to remove nonlinearities, it has been possible to compensate the troublesome effects and obtain a satisfactory extraction behavior.

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*Work supported by the U. S. Atomic Energy Commission.

We shall remind the reader that the Bevatron resonant extraction operates at an initial betatron frequency of $2/3 + \Delta\nu$, with $\Delta\nu$ about -0.02. It is controlled by a single magnet which has three independent coils: (a) the perturbation exciting the resonant growth, (b) the spiller controlling the rate of extraction, and (c) the first septum. This magnet is plunged close to the accelerated beam. Figure 1 shows the perturbation and spiller in relation to the closed orbit and to the 0.13-inch septum. Because one is not restricted to excitation by the lowest-order multipole (in this case, sextupole), the particle trajectories may be adjusted by shaping the perturbation. This flexibility could also be utilized to offset nonlinearities in the synchrotron guide field, but we desired a more nearly linear behavior in the developmental stage and some understanding of the nonlinear effects.

Effects of Nonlinear Guide Field

The radial dependence of betatron frequency with closed orbit position in the Bevatron at 5.7 GeV is shown in Fig. 2. The departure from constant gradient is considerable and produces a variation in average frequency, $\bar{\nu}$, with oscillation amplitude. Because the motion of particles near resonance is critically dependent upon $\bar{\nu}$, a variation with amplitude can modify or arrest the growth of amplitude before particles reach the extraction septum.

To estimate the deviation of $\bar{\nu}$ from the value ν_o at small amplitude, we assume the particle to be oscillating in a restoring force of the form

$$F = -\nu_o^2 X + C_2 X^2 + C_3 X^3,$$

for which the anharmonic oscillation frequency is readily calculable.[2]

This form does not match the Bevatron guide fields, but we feel justified in terminating the expansion because the oscillation frequency is strongly influenced by the restoring force near maximum amplitude. Hence, for each amplitude, a, the constants C_2 and C_3 were evaluated to make the forces at ±a each equal to those in the Bevatron. The equation to determine C_2 and C_3 becomes

$$F(\pm a) = \mp \nu_o^2 a + C_2 a^2 \pm C_3 a^3 = \int_{R_o}^{R_o \pm a} \nu^2 \, dR.$$

Using these constants, the frequency at each amplitude was calculated. This estimate indicated substantial variations of $\bar{\nu}$ within the 6-inch amplitude needed to extract.

For more detailed calculation of particle motion we later developed a computer program that used the measured small-amplitude ν values for the Bevatron. (See Appendix A.) From this program we may obtain values of $\bar{\nu}$ vs. amplitude, and example results for two closed-orbit radii are shown in Fig. 3, curves A and A'. Case A shows little frequency deviation out to the required 6 inches, and extraction at this radius was possible. For extraction with Case A', the frequency could be shifted to give the proper value of $\Delta\nu$ at small amplitudes, but the deviation of 0.01 at 6 inches would arrest amplitude growth before particles reach the septum.

A second effect of the nonlinearities arises from the departure of the motion from sinusoidal form. The effect of a perturbation on amplitude and phase must be calculated with details of the relation between these quantities in the nonlinear oscillation being taken into consideration. In this it is a simplification to have an azimuthally local perturbation, since the motion in any small portion of an oscillation may be considered to be sinusoidal with frequency appropriate to the local focusing force.

This approximation was used to calculate the deflection which, when applied once each M turns, would be needed to produce a fixed point of order M. Let the average betatron frequency be

$$\bar{\nu} = \frac{N}{M} + \Delta\nu,$$

with $\Delta\nu$ small and N and M integers. Divide the M turns into two portions: $\frac{N}{\bar{\nu}}$ turns to complete N oscillations, and a small fraction $M \frac{\Delta\nu}{\bar{\nu}}$ of a turn in which the excursion y is sinusoidal with a frequency ν_p appropriate to the guide–field restoring–force near the fixed point. A deflection $\Delta \frac{dy}{d\theta}$ at the center of the latter portion produces a fixed point. The motion of the particle near the fixed point is described by

$$y = y_o \cos \nu_p \left(\theta - M\pi \frac{\Delta\nu}{\bar{\nu}}\right)$$

and

$$y_o = y_p / \cos \nu_p M\pi \frac{\Delta\nu}{\nu},$$

where y_p is the displacement of the fixed point from a reference closed orbit. The local deflection needed, obtained from the slope at $\theta = 0$, is

$$\Delta \frac{dy}{d\theta} = 2y_p \, \bar{\nu} \left(\frac{\nu_p}{\bar{\nu}}\right) \tan M\pi \left(\frac{\nu_p}{\bar{\nu}}\right) \Delta\nu ,$$

and differs from that for linear motion to the extent $\nu_p/\bar{\nu}$ differs from unity. The value of ν_p may be obtained from the ν–vs–R relation using

$$\nu_p^2 = \frac{1}{y_p} \int_{y=0}^{y_p} \nu^2 \, dR.$$

A typical magnitude of $\frac{\nu_p}{\bar{\nu}}$ for $y_p = 6$ in. in the Bevatron is ≈ 1.1.

In the experimental program, the use of currents in the Bevatron pole-face windings provided a limited control of guide-field nonlinearities. Because of limitations on current, we could not simply remove the gradient variations; but it was possible to compensate particular effects. As noted before, a similar control of any particular set of these nonlinear effects is offered by the adjustment of the perturbation shape, but for our tests the pole-face windings were more suitable.

We were concerned at first with the variation of $\overline{\nu}$ with amplitude, but we learned that we could correct ν by means of pole-face winding currents; this correction did aid extraction in some cases, but did not reduce abnormal behavior in all situations. When we recognized and allowed for the effect of local focusing force in altering the action of perturbation, we found agreement between predicted and observed results of gradient changes.

Particle motions from computer calculations for three cases of ν-vs-R are shown in the figures. As noted before, Case A in Fig. 2 is the unaltered guide field. The phase diagram for that case is given in Fig. 4a; a stable fixed point appears at -6.5 inches, but particles still proceed past the septum satisfactorily. Case B has a linear change in ν-vs-R of -0.02 inch and is of interest because it has little change of ν with amplitude (see Fig. 3). However, in this case, observed experimentally, growth per turn is very small at the septum, with larger growth appearing at smaller amplitudes. Also, not all beam spilled out with reasonable perturbation strength. The phase diagram of Fig. 4b supports this behavior, our interpretation being that high restoring force in the vicinity of the perturbation has produced the stable third-order fixed point on the beam side of the septum. In Case C, the gradient was altered to reduce the restoring force near the perturbation. A small secondary effect is seen in the average frequency in Fig. 3, but experimental behavior was improved, and fluctuation of parameters had less influence on good growth per turn at the septum. This case is also shown in the phase diagrams of Fig. 4a. Pole-face windings needed for this compensated case are less than one half what is needed for less satisfactory extraction through attempts to produce a simpler-appearing ν-vs-R relation.

Beam Spill Control

A shaped field of variable amplitude has been used to control spillout of the beam while other parameters of the accelerator are held constant. The shaped spiller coil is added to the perturbation magnet. It provides a flexible control for either slow or fast spillout, and should have a simple, fast response useful in design of feedback systems.

The shape of the spiller field has been chosen to cancel most of the sweeping of angle at the septum as particles of progressively smaller initial betatron amplitudes are extracted. Figure 1b shows successive sequences of particle positions as the spiller amplitude increases and moves the unstable fixed point toward the closed orbit during spillout. A fair degree of coincidence of angle and growth per three turns at the septum has been achieved. Considerable variations in the spiller design is possible as experience adds new specifications.

Performance

The scale of the Bevatron beam size is suggested by noting that about 70% of the particles are within a total radial amplitude of ± 1.5 inches; the phase oscillations account for about ± 1/4 inch.

The growth of amplitude per resonance-period (three turns) has been found, in good agreement with calculations, to be about 0.8 inch at the septum of the first extraction magnet. The septum is located at a radius 5.5 inches smaller than the closed orbit. Figure 5 shows a radioautograph of a gold foil irradiated at this location over a 4-hour period.

Experimental observations have shown that all internal beam participates in the resonant process and reaches the first septum magnet. Therefore the beam measured at this first extraction magnet has become the reference for extraction efficiency.

With the new 0.13-inch-thick septum and the growth per resonance period of 0.85 inch, the theoretical extraction efficiency is 85%. Measurements of the relative amount of ^{149}Tb produced by irradiation of gold foils show a survival of $\approx$ 87% of the beam at the entrance to the second magnet. An aperture restriction that the beam encounters at the transition into the external beam channel intercepts $\approx$ 20%, as indicated by measurement of 65% survival outside the synchrotron. The emittance of the external beam, full area in the radial plane, has been estimated to be about 1 cm mrad.

The extracted beam current shows little or no rf modulation after the rf system has been turned off for a few milliseconds. However, a slow vertical growth takes place, which is presently under investigation.

With a narrow target at the same azimuth as the first extraction magnet the amplitude growth can be traced back several resonance periods. Whenever the shadow of the target falls on the septum a minimum of beam current is lost. This "orderly" behavior of the resonant excited beam encourages us to plan an electrostatic septum upstream to extract the beam which ordinarily would be intercepted by the septum of the first magnet.

Acknowledgments

We are indebted for support of this work to the Bevatron staff headed by Dr. E. J. Lofgren. For their close cooperation we wish to thank R. J. Force and his collaborators; as well as J. R. Guggemos and M. M. Tekawa. J. B. McCaslin, A. R. Smith, and L. D. Stephens from the Health Physics Department made our gold–activation runs possible.

APPENDIX A

The calculation by computer of the motion of a particle through extraction in the Bevatron including guide field nonlinearities was based on the following approach.

Assume the Bevatron is composed of four quadrants separated by field-free drift sections. At each radius, the gradient in a quadrant is $\frac{dB}{dR} = - n \frac{B}{R}$, a value calculable from the measured data on ν-vs-R for small oscillations (Fig. 2). The field is then constructed from

$$B = B_o + \int_{R_o}^{R} \frac{dB}{dR} dR,$$

where B_o is the reference value at the stable closed orbit radius R_o.

The equations of motion in the radial plane in polar coordinate are

$$P_R = P_\theta \frac{dx}{d\theta} \frac{1}{R} ,$$

$$\frac{dP_R}{d\theta} = P_\theta - e\ BR,$$

$$P_\theta^2 = P_o^2 - P_R^2 ,$$

where P_R, P_θ are the transverse and longitudinal momenta and x the particle excursion from the closed orbit. If we substitute

$$\frac{dx}{d\theta} \frac{1}{R} = \frac{du}{d\theta} ,$$

these equations can be combined to give

$$\frac{d^2u}{d\theta^2} = 1 - \frac{BR}{B_oR_o} = - ku,$$

where

$$k = \frac{BR}{B_oR_o} (1-n).$$

The value of k is calculated from the constructed field B, and we assume it to be constant in a short azimuthal interval $\Delta\theta = \theta_2-\theta_1$, so that the motion is locally a portion of a sinusoid. The transformation from θ_1 to θ_2 can then be written as

$$\begin{pmatrix} u \\ \frac{du}{d\theta} \end{pmatrix}_2 = T \begin{pmatrix} u \\ \frac{du}{d\theta} \end{pmatrix}_1 ,$$

where

$$T = \begin{pmatrix} \cos\sqrt{k}\,\Delta\theta & \frac{1}{\sqrt{k}}\,\sin\sqrt{k}\,\Delta\theta \\ -\sqrt{k}\,\sin\sqrt{k}\,\Delta\theta & \cos\sqrt{k}\,\Delta\theta \end{pmatrix} .$$

A computation program traces the motion through successive azimuthal intervals by joining segments of sinusoids having the appropriate parameters.

The particle coordinates in radial phase space are obtained from the equations

$$R_2 = R_1\, e^{(u_2-u_1)}, \quad x_2 = R_2-R_o, \quad \text{and} \quad \left.\frac{dx}{ds}\right|_2 = \left.\frac{du}{d\theta}\right|_2 .$$

A local perturbation is readily added as an angular deflection dependent upon radius as desired.

References

1. Proceedings of the Sixth International Conference on High Energy Accelerators, Cambridge, CEAL 2000, p. 343 (1967).
2. L. D. Landau and E. M. Lifschitz, Mechanics, p. 84 (Pergamon Press, 1960).

Figure Legends

Fig. 1a	Perturbation and spiller shape.
Fig. 1b	Phase diagram for Case C during extraction by spiller.
Fig. 2	Radial betatron frequency as a function of radius.
Fig. 3	Radial betatron frequency as a function of amplitude.
Fig. 4a	Full phase diagram for Case C with part of Case A shown.
Fig. 4b	Full phase diagram for Case B.
Fig. 5	Beam-spot size at first septum magnet.

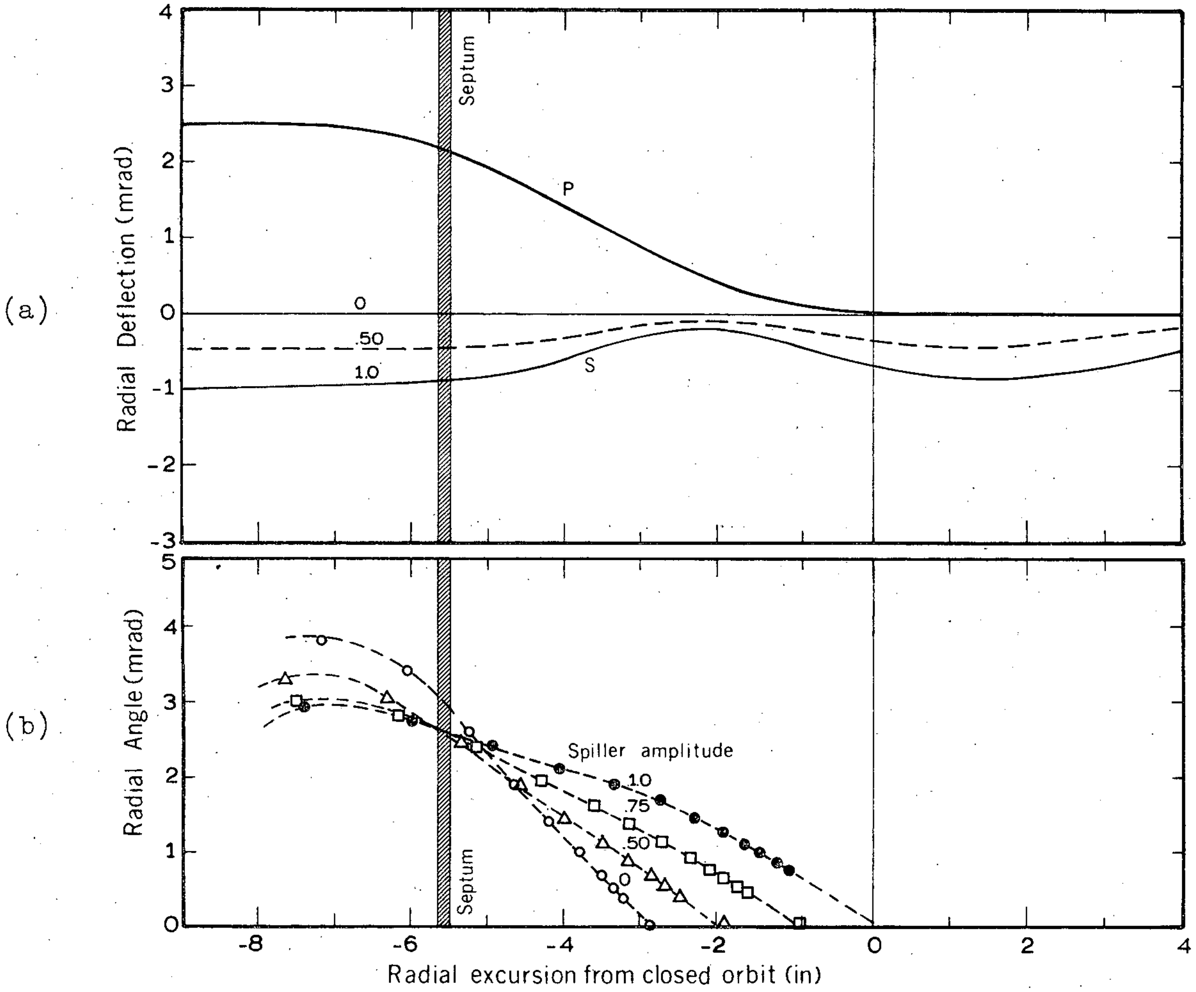
(a)
Radial Deflection (mrad)
4
3
2
1
0
-1
-2
-3
Septum
P
0
.50
1.0
S
(b)
Radial Angle (mrad)
5
4
3
2
1
0
Septum
Spiller amplitude
1.0
.75
.50
0
-8
-6
-4
-2
0
2
4
Radial excursion from closed orbit (in)

Fig. 1a, Fig. 1b

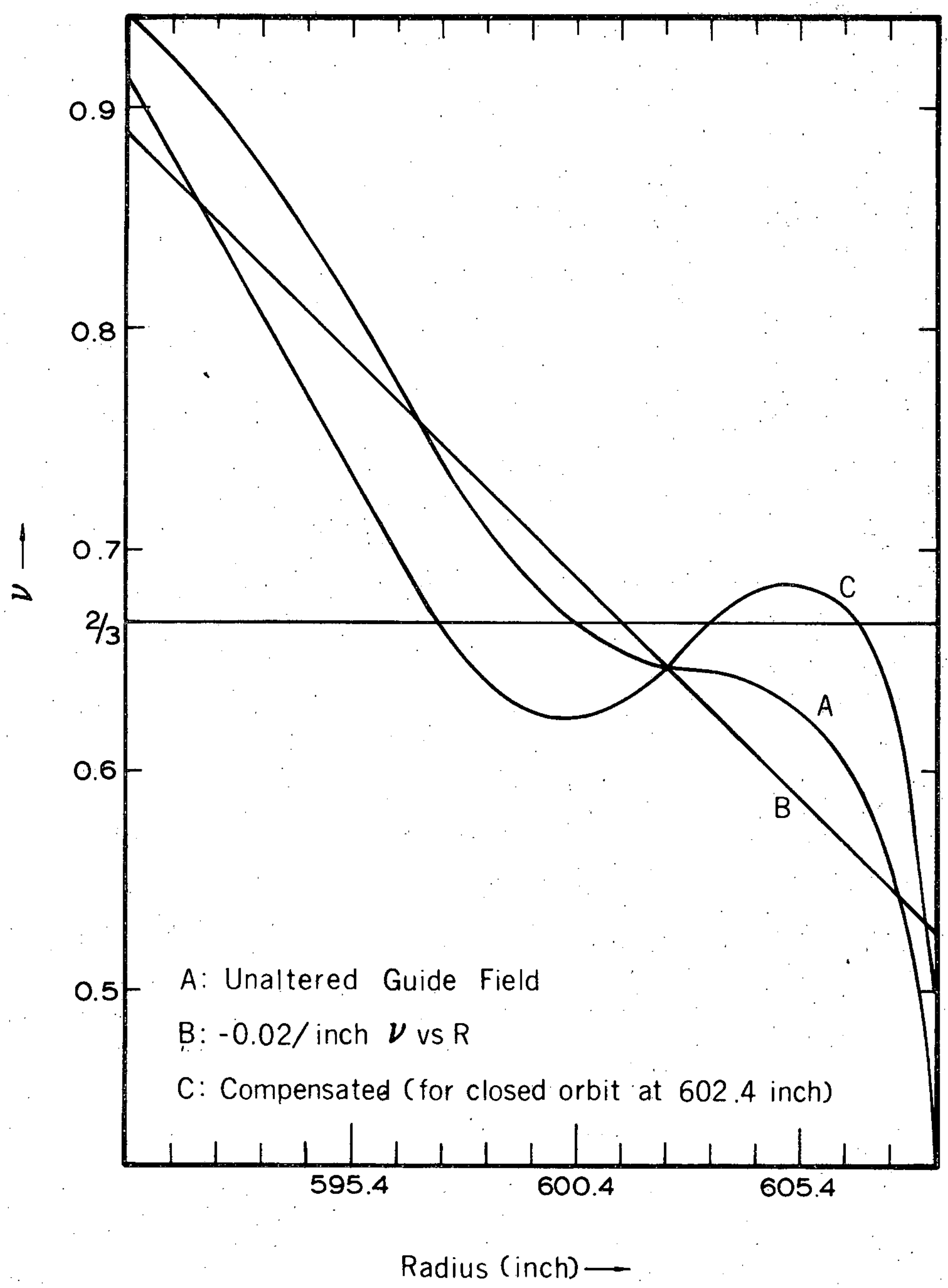
0.9
0.8
0.7
2/3
0.6
0.5
ν
C
A
B
A: Unaltered Guide Field
B: -0.02/inch ν vs R
C: Compensated (for closed orbit at 602.4 inch)
595.4
600.4
605.4
Radius (inch) ⟶

Fig. 2

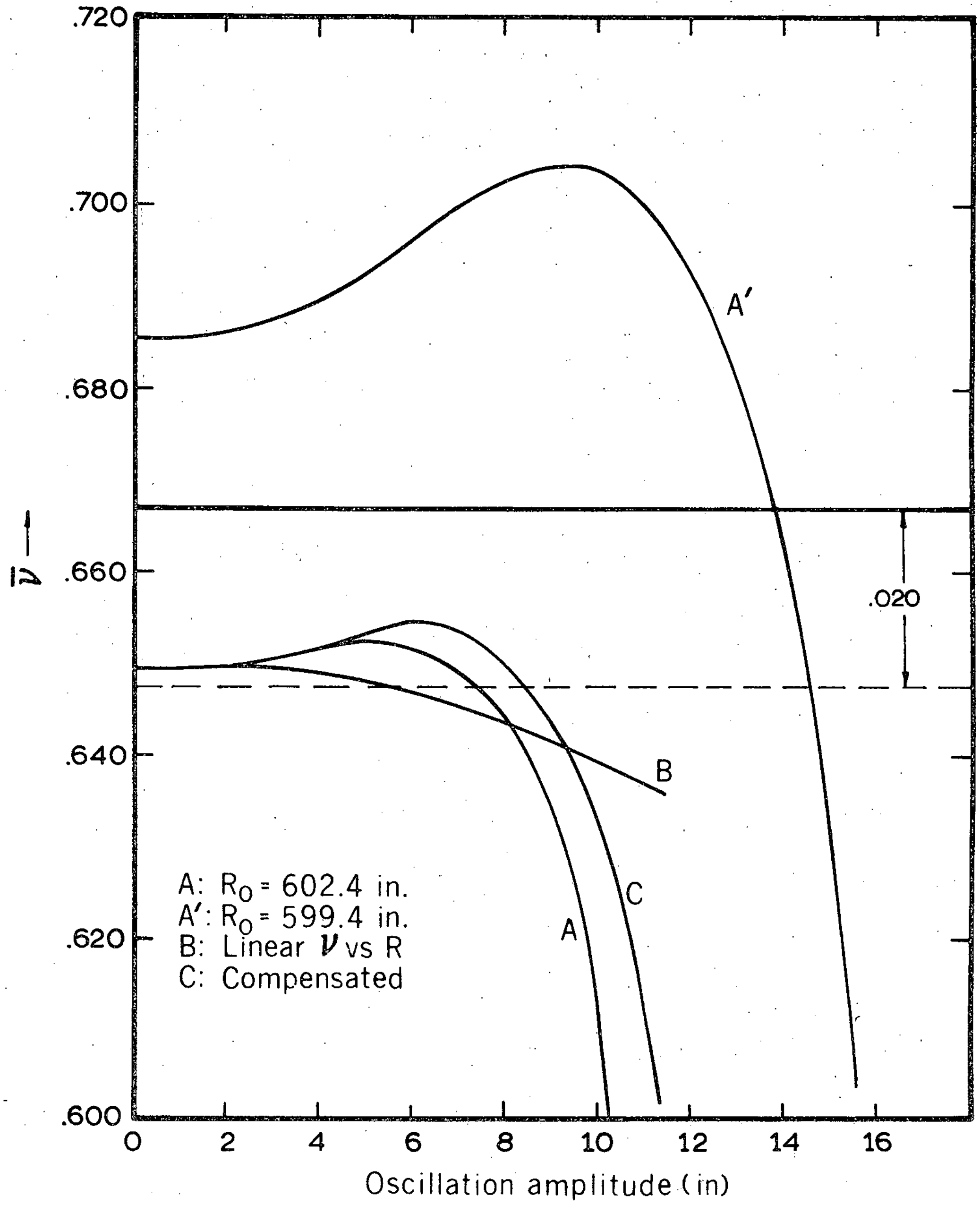
.720
.700
.680
.660
.640
.620
.600
$\bar{\nu}$
A′
.020
B
C
A
A: $R_0 = 602.4$ in.
A′: $R_0 = 599.4$ in.
B: Linear ν vs R
C: Compensated
0
2
4
6
8
10
12
14
16
Oscillation amplitude (in)

Fig. 3

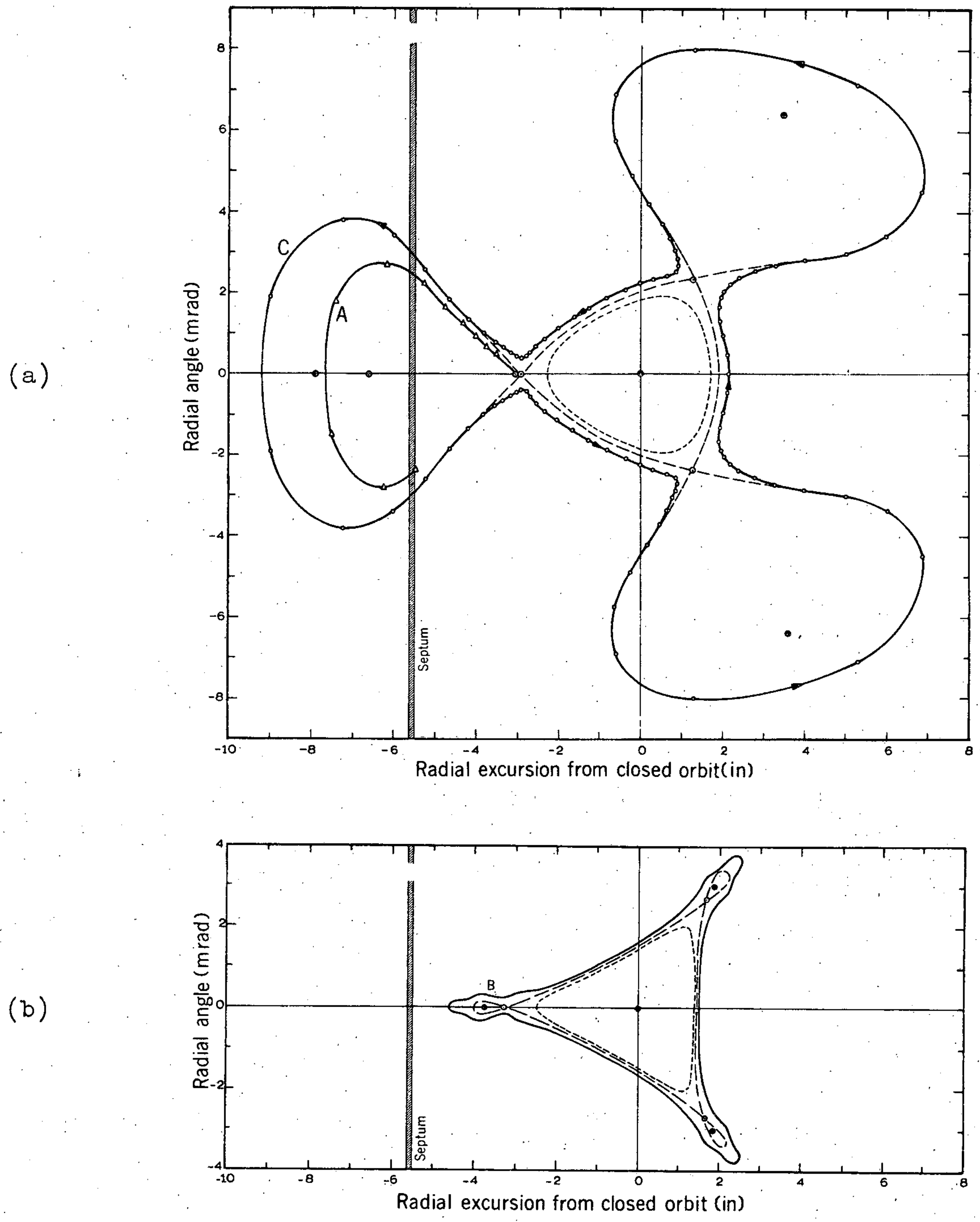
(a)
C
A
Septum
Radial angle (mrad)
8
6
4
2
0
-2
-4
-6
-8
-10
-8
-6
-4
-2
0
2
4
6
8
Radial excursion from closed orbit(in)
(b)
B
Septum
Radial angle (mrad)
4
2
0
-2
-4
-10
-8
-6
-4
-2
0
2
4
6
8
Radial excursion from closed orbit (in)

Fig. 4a, Fig. 4b

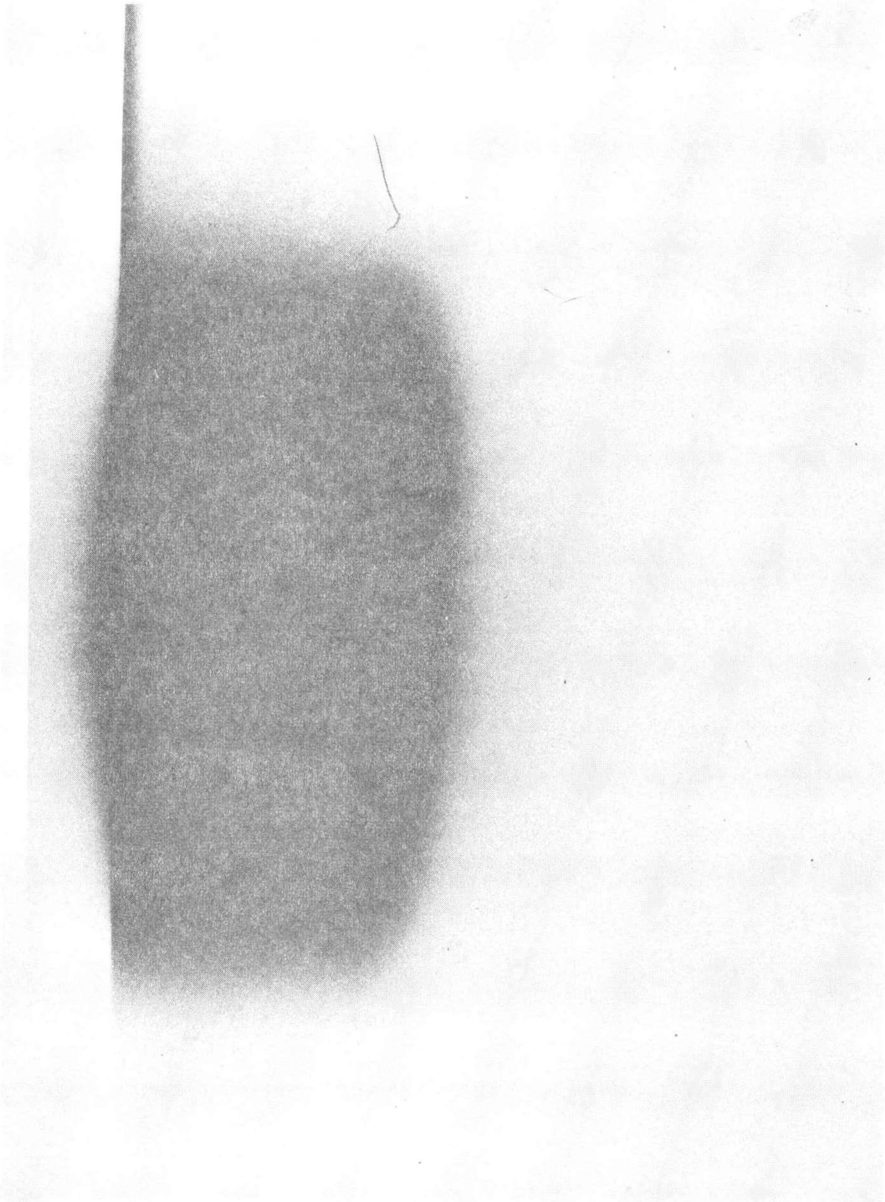

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Fig. 5

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