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11 Equation Section 1 Unsupervised Machine Learning for Detecting Soil 2 Layer Boundaries from Cone Penetration Test Data

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10 **Abstract**

11 Cone penetration test (CPT) data contains detailed stratigraphic information that is useful in a wide variety of applications.
12 Separating a CPT profile into discrete layers is an important part of many analyses such as critical layer selection in
13 liquefaction triggering analysis, effective stress seismic ground response analysis, analysis of pile shaft and tip resistance,
14 and soil-pile interaction analysis. The discretization of the profile into layers is often done manually, relying on the
15 judgment of the analyst. This manual approach is cumbersome for datasets that include large numbers of CPT profiles
16 [such as the Next Generation Liquefaction (NGL) database and the New Zealand Geotechnical Database] and it may not
17 be consistent or repeatable because different analysts may discretize a given CPT log in different ways. To overcome these
18 difficulties, we present an approach to automatically divide a CPT profile into discrete layers. Automated layer detection is
19 performed using an unsupervised machine learning technique called agglomerative clustering in combination with two
20 cost functions to identify an optimal number of layers. The algorithm is illustrated using CPT profiles from the NGL
21 database, where the approach is being used in the development of liquefaction triggering and manifestation models.
22 Although the algorithm shows promise for replicating our judgment regarding layering, we recommend visual review of
23 the layering produced by the algorithm to check for reasonableness given the site geology and intended use of the CPT
24 data.

25 **Introduction**

26 Cone penetration test (CPT) data is one of the most valuable resources for subsurface characterization by
27 geotechnical engineers. CPT data is used in a large variety of applications from identifying soil types to estimating
28 static and dynamic shear strength of soil. By typically sampling at 1 cm intervals, an individual CPT test may
29 contain thousands of data points, which provide essentially continuous profiles of tip resistance (q_c), sleeve friction
30 (f_s), and sometimes pore pressure (u_2) over the length of the CPT profile. Most geotechnical engineering applications
31 require grouping the CPT data within the site's stratigraphic profile into a discrete number of layers of consistent
32 soil type and behavior. Examples include liquefaction triggering evaluation, including identification of a critical
33 layer, ground response analysis to evaluate earthquake site response, evaluation of the axial and lateral capacity of
34 deep foundations, and many others.

35 Selection of layers is often based on the judgment of an engineer or geologist with the goal being to select layers
36 that have similar geologic origin and soil properties but are distinct from the materials above and below them. The
37 number and thickness of layers selected to represent the profile depends on the intended application. This process is
38 subjective and hence unrepeatably when based entirely on analyst judgment because different analysts (or the same
39 analyst at a different time) may choose different layer boundaries. Additionally, manual layer selection becomes
40 inefficient when large numbers of profiles require interpretation. Therefore, the engineering community needs an
41 algorithm that can efficiently assign layers to CPT profiles with repeatable, objective results, thereby removing bias
42 that can be introduced by a sole analyst or small group of analysts. The aim of this paper is to describe and propose
43 such an algorithm based on an unsupervised machine learning procedure that, along with a small number of existing
44 alternate approaches (described in the remainder of this section), enables robust analysis of CPT profiles.

45 ***Existing Layering Algorithms***

46 A number of techniques have been developed to automate identification of simplified profiles from CPT data. For
47 example, Wang et al.^{1,2,3} and Cao et al.⁴ developed a Bayesian approach to assign layer boundaries and assign a
48 probability that soil within a particular layer falls within a soil behavior type category. Ching et al.⁵ developed a
49 procedure that utilizes the wavelet transform method to distinguish sudden changes in CPT tip resistance from
50 smaller amplitude changes due to within-layer soil variability. These methods are rather complicated, require a
51 significant number of calculations, and only consider one parameter (soil behavior type or tip resistance). Cao et al.⁴
52 proposed a Bayesian identification method based on the soil behavior type index, I_c .⁶ Ntritsos and Cubrinovski⁷
53 developed an algorithm that minimizes the within-layer coefficient of variation of q_{cINcs} and I_c for the purpose of
54 developing finite element meshes for one-dimensional ground response analysis. Their method is conceptually and
55 computationally simpler than many previous methods and was shown to produce similar results to analyzing the full
56 profile with respect to liquefaction potential. Ntritsos and Cubrinovski⁷ caution that the algorithm may result in
57 fictitious layers at layer boundaries and indicate that their algorithm is not intended to replace engineering judgment.
58 Molina-Gómez et al.⁸ more recently utilized a multivariate hierarchical clustering approach to identify stratigraphic
59 layers at a site in the Tagus River Valley where gel push sampling was performed in combination with CPT testing
60 to confirm soil types. Layers need not be vertically continuous in their algorithm (e.g., a layer may have another
61 layer within it). They suggest that their algorithm is well-suited to identifying layers at other experimental sites. In
62 addition to identifying layers based on a vertical profile, some of the methods (e.g., Wang et al.^{2,3}) assess the lateral
63 spatial variation of stratigraphy within a site where multiple CPT soundings and/or boring logs are available. We
64 recognize that automated lithology detection of rock strata based on geophysical data has been studied by statistical,
65 wavelet, and, more recently, machine learning procedures in the petroleum exploration industry but will not be
66 discussed here as it is a significantly different application compared to our method.

67 *Motivation for Automated Layer Identification Algorithm*

68 A motivation for the work described in this paper was the need to create discretized representations of individual
69 CPT profiles at sites in the Next Generation Liquefaction (NGL) database^{9,10}. Such profiles are required for the
70 development of new liquefaction triggering and manifestation models. Our algorithm was developed independently
71 from, and concurrently with, the methods by Ntritsos and Cubrinovski⁷ and Molina-Gómez et al.⁸ and bears some
72 similarities to both methods, as well as having some advantages. It is similar to the method by Ntritsos and
73 Cubrinovski⁷ in that it seeks a set of layers that reduces the within-layer variance. It differs from their method in that
74 it uses unsupervised machine learning rather than prescribed rules for assigning layers, which is advantageous since
75 the algorithms are widely available in Python packages. Our method is similar to that of Molina-Gómez et al.⁸ in
76 that it utilizes an unsupervised machine learning technique, hierarchical clustering, to identify layers. However, it
77 differs from their method in two important respects. First, our algorithm requires that layers be vertically contiguous,
78 whereas theirs allows for non-contiguous layers. Second, our algorithm selects the optimal number of layers based
79 on a cost function that is unbiased with respect to the maximum penetration depth, whereas theirs utilizes an
80 automated algorithm to select the number of layers. We show herein that the automated method they adopted results
81 in bias wherein thicker layers are identified for deeper profiles, and thinner layers for shallower profiles, whereas our
82 algorithm is unbiased with respect to the total depth of the CPT sounding. Our proposed method only considers
83 vertical layering and does not consider horizontal spatial variability because it is intended to be used at a single CPT
84 location. A reduced dataset that contains only locations with multiple CPT soundings in proximity would be
85 required to extend the method for horizontal interpretation, which is beyond the scope of this paper.

86 We consider the existence of multiple automated algorithms to provide a beneficial measure of epistemic
87 uncertainty, which is important for quantification of overall uncertainty in engineering analyses. No single algorithm
88 will best suit the needs of all users and all applications; therefore, it is useful for different algorithms to utilize a
89 range of different approaches to quantify uncertainties related to layer identification decisions. The following
90 sections present some details about CPT measurements and data analyses and introduce, describe, and provide
91 examples on how to apply our proposed layer detection algorithm.

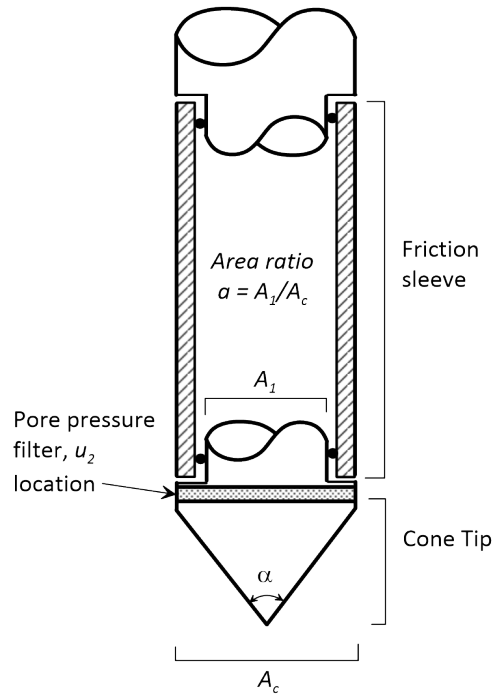
92 **Cone Penetration Test**

93 The CPT probe measures tip resistance, sleeve friction, and sometimes pore pressure (Fig. 1) (e.g., Robertson⁶,
 94 Lunne et al.¹¹). A hydraulic press pushes the cone into the ground generally at a rate of 2 cm/s. The cone tip
 95 resistance, q_c , is equal to the measured force on the cone tip divided by the cone area, A_c , and the sleeve friction, f_s , is
 96 the force acting on the friction sleeve divided by the surface area of the sleeve. Commonly A_c is 10 to 15 cm², and
 97 the cone tip angle is $\alpha = 60^\circ$. The most common location for pore pressure measurement is between the cone tip and
 98 the friction sleeve, which is deemed the u_2 location. When pore pressure is measured, the corrected tip resistance is
 99 computed as,

100
$$q_t = q_c + u_2(1-a) \tag{1}$$

101 where a is the net area ratio of tip, usually between 0.6 and 0.8 depending on cone design. Eq. 1 accounts for the
 102 influence of water pressure acting downward behind the cone tip on the measured tip resistance. Measurements are
 103 generally recorded at 1 cm intervals.

104



105

106 **Figure 1.** Cross-section schematic of cone penetration test probe.

107

108 Various quantities are often computed from CPT measurements, and utilized to identify soil characteristics. Cone tip
 109 resistance and sleeve friction increase with depth in uniform soil due to increasing effective stress with tip resistance
 110 generally being high relative to sleeve friction in coarse-grained soils and vice versa in fine-grained soils. To assess
 111 fundamental soil properties, the normalized cone resistance, Q_m , defined by Eq. 2 is typically used, where σ_{vo} is in-
 112 situ vertical total stress, σ_{vo}' is in-situ vertical effective stress, p_a is atmospheric pressure (101.325 kPa), and n is an
 113 exponent that defines the soil-type-dependent relationship between σ_{vo}' and q_t . Furthermore, normalized sleeve
 114 friction, F_r , is defined by Eq. 3. These dimensionless quantities are combined to define the soil behavior type index,
 115 I_c^6 , defined by Eq. 4. The exponent n depends on I_c as defined by Eq. 5, and Eqs. 3, 4, and 5 therefore form an
 116 implicit system of equations that is solved by iteration.

117

$$Q_m = \left(\frac{q_t - \sigma_{vo}}{p_a} \right) \left(\frac{p_a}{\sigma_{vo}'} \right)^n \tag{2}$$

118

$$F_r = \frac{f_s}{q_t - \sigma_{vo}} \times 100\% \tag{3}$$

119

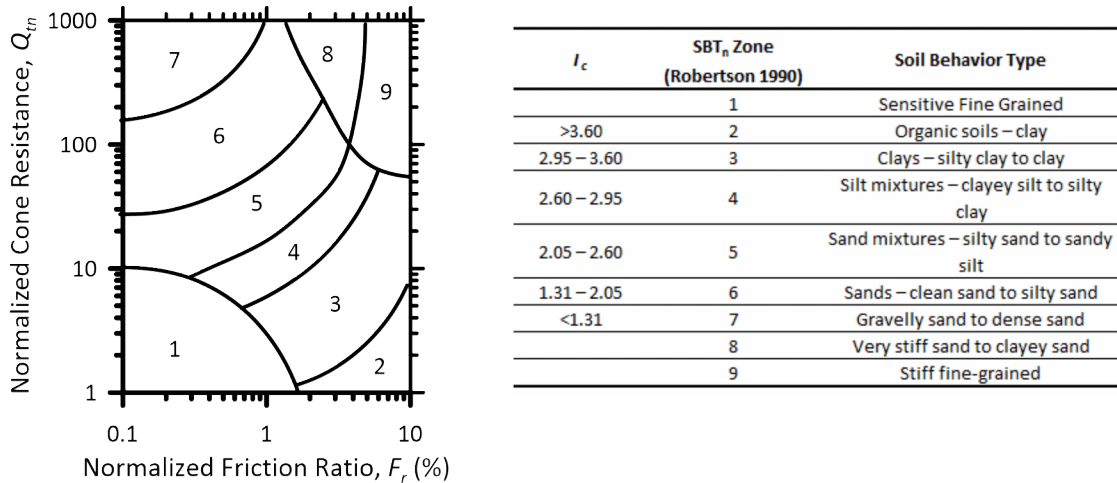
$$I_c = \sqrt{(3.47 - \log Q_m)^2 + (\log F_r + 1.22)^2} \tag{4}$$

120

$$n = 0.381 I_c + 0.05 \left(\frac{\sigma_{vo}'}{p_a} \right) - 0.15 \tag{5}$$

121 **Soil Behavior Type**

122 Robertson⁶ (1990) found that soil behavior type can be classified based on contours of Q_m vs. F_r shown in Fig. 2.
 123 Soils that cluster within SBT_n zones 2 through 7 are separated by contours that approximately follow the range of I_c
 124 values in Fig. 2, and exhibit soil behavior type that increases in coarseness from $SBT_n=2$ (Organic soils and clay) to
 125 $SBT_n = 7$ (gravelly sand to dense sand). Sensitive fine-grained soils, very stiff sand to clayey sand, and stiff fine-
 126 grained soils do not have a unique I_c range associated with their behavior type.



127

128

Figure 2. Soil behavior type based on CPT measurements.

129 **Thin Layer and Transition Zone Effects**

130 Due to its physical dimensions, the CPT probe averages out soil properties within a zone of influence near the cone
 131 tip. As a result, the cone may render measurements near layer interfaces that imply incorrect soil behavior type. For
 132 example, as a cone transitions out of a stiff sand layer with $SBT_n = 6$ into a soft underlying clay layer with $SBT_n = 3$,
 133 there will likely be a transition zone in which $SBT_n = 4$ and 5 will be measured even though silty soil does not exist
 134 in these zones. Furthermore, when a cone is advanced through a thin sand layer sandwiched between two softer clay
 135 layers, the tip resistance measured at the center of the sand may be lower than the resistance that would be measured
 136 in a uniform profile of the same sand.

137 A number of algorithms have been developed to identify transition zones, where an interface between different types
 138 of soil result in CPT measurements that may not accurately reflect the soil at that depth. For example, CPeT-IT¹²
 139 provides an algorithm for identifying interface zones, along with the SectionMaker software for assigning layers
 140 within a cross-section based on CPT measurements. Boulanger and DeJong¹³ developed an inverse-filtering
 141 algorithm to recover the “true” CPT soil properties from the measured properties by accounting for the influence of
 142 the layered profile on the CPT measurements. Their algorithm tends to increase the tip resistance in stiff layers near
 143 the boundaries with softer layers, and to a lesser degree it also decreases the tip resistance in soft layers near the
 144 boundaries with stiff layers. Other authors have pointed out the limitations of the Boulanger and DeJong¹³
 145 algorithm¹⁴ and have begun introducing refined algorithms¹⁵.

146 *CPT Corrections*

147 Although the layer identification algorithm presented here is general and could be used for many different CPT
 148 applications, our specific focus is the evaluation of liquefaction. CPT is a preferred tool for characterizing site
 149 conditions for liquefaction analysis due to its repeatability and the nearly continuous profile that it provides. Both
 150 corrected cone tip resistance and liquefaction resistance depend fundamentally on soil density and fines content.
 151 However, the dependencies are different, which requires adjustments to the measured cone tip resistance to render a
 152 quantity that relates more directly to liquefaction resistance. Namely, corrections are applied to account for the
 153 influence of σ_{vo}' and fines content, FC . The overburden- and fines-corrected cone tip resistance, q_{cINcs} , is defined in
 154 Eq. 6, where the overburden correction factor, C_N , is defined by Eq. 7¹⁶. The fines correction in Eq. 6 is intended to
 155 account both for the reduced stiffness and strength of sandy soils containing fines (which affect tip resistances) and
 156 the effects of fines on the cyclic resistance of the soil to liquefaction triggering. Liquefaction triggering relationships
 157 typically utilize q_{cINcs} to define cyclic liquefaction resistance of sand-like soils (e.g., Moss et al.¹⁷, Boulanger and
 158 Idriss¹⁶).

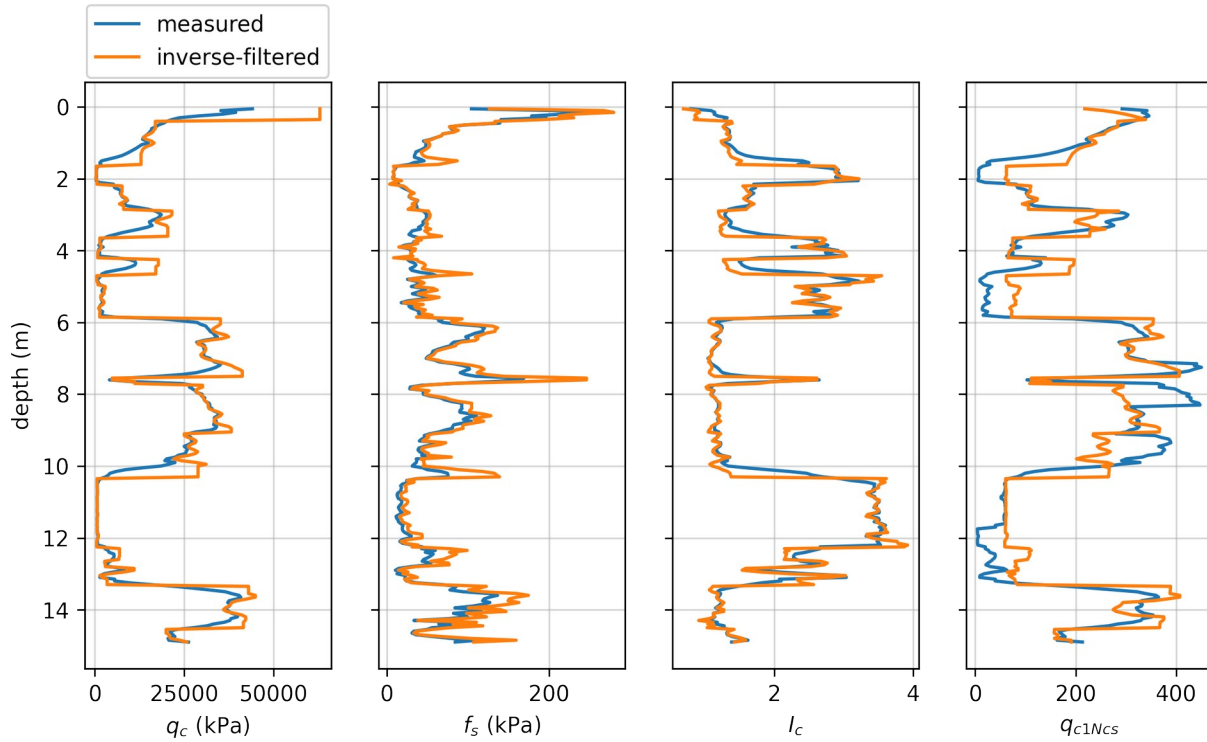
$$159 \quad q_{cINcs} = C_N \frac{q_t}{p_a} + \left(11.9 + \frac{C_N}{p_a} \frac{q_t}{14.6} \right) \exp \left[1.63 - \frac{9.7}{FC + 2} - \left(\frac{15.7}{FC + 2} \right)^2 \right] \quad (6)$$

$$160 \quad C_N = \left(\frac{p_a}{\sigma_{vo}'} \right)^{n_f} \leq 1.7 \quad (7)$$

$$161 \quad n_f = 1.338 - 0.249 \left(q_{cINcs} \right)^{0.264} \quad (8)$$

162 *Example CPT Profile*

163 An example CPT profile, UC-4, obtained at Moss Landing (California) near Sandholdt Road is shown in Fig. 3. This
 164 site exhibited severe manifestations of liquefaction due to the 1989 M6.9 Loma Prieta earthquake^{18,19}. The CPT
 165 profile shows that this site consists of alternating layers of fine-grained and coarse-grained materials. Note that
 166 coarser-grained materials with lower I_c tend to have higher q_c and q_{cINcs} . Furthermore, the averaging of cone
 167 penetration tip resistance near layer boundaries is evident, for example at a depth near 6m. The inverse-filtered CPT
 168 data¹³ have sharper edges due to being corrected for layer transition effects. We consider the inverse-filtered profiles
 169 to provide a more accurate representation of the true soil properties, and utilize the inverse-filtered profiles for the
 170 remainder of this paper.



171

172

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Figure 3. Cone penetration test data for UC-4 at the Moss Landing site near Sandholdt Road, which exhibited liquefaction manifestations due to the 1989 M6.9 Loma Prieta earthquake^{18,19}.

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175 Layer Identification Algorithm

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In this section, we first summarize main features of the theoretical framework behind the tools used to produce our layer identification algorithm. We then provide a description of how it was implemented and details on how it should be used. Clustering or cluster analysis is an unsupervised machine learning approach that categorizes data based on common attributes²⁰. K-means clustering categorizes data based on the aggregate distance between the data point and the centroid of each cluster, where distance is measured in the parameter space of the variables included in the clustering algorithm^{21,22}. Gaussian mixture models assign probabilities that each data point belongs within each cluster based on the cluster statistics, and may be thought of as an extension of K-means clustering that also considers covariance among variables. The number of clusters is provided as an input parameter, and the algorithm assigns data to clusters in a manner that minimizes the sum of within-cluster variance. We perform K-means and Gaussian mixture model clustering using standardized values for cone tip resistance and soil behavior type index defined by Eqs. 9 and 10, where μ_q , σ_q , μ_{I_c} , and σ_{I_c} are the mean and standard deviation of q_{c1Ncs} and I_c for the entire profile, respectively.

188

$$\hat{q}_{c1Ncs} = \frac{q_{c1Ncs} - \mu_q}{\sigma_q} \tag{9}$$

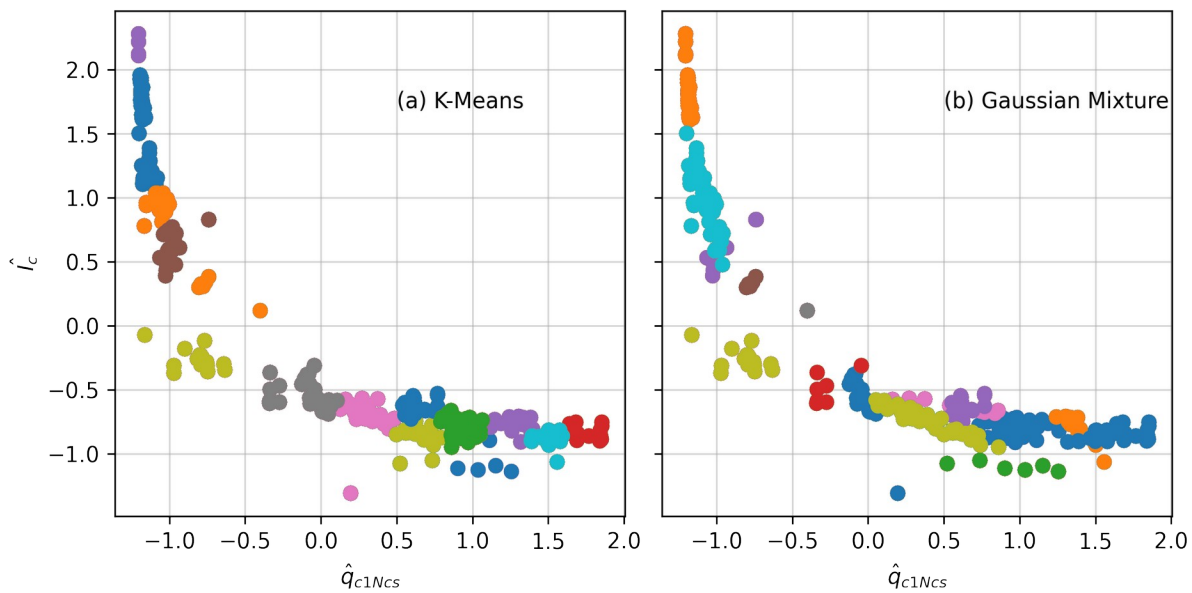
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$$\hat{I}_c = \frac{I_c - \mu_{I_c}}{\sigma_{I_c}} \tag{10}$$

190 Standardizing the data prior to clustering is important, particularly when the parameter space contains variables of
 191 different units and significantly different ranges. Without standardization, variables with higher numerical values
 192 may be inadvertently weighted more heavily than variables with smaller numerical values in the distance
 193 calculation. For example, q_{c1Ncs} for liquefaction applications generally varies from about 50 to 300, while I_c varies
 194 only from about 1.0 to 3.5.

195 ***K-Means and Gaussian Mixture Model Results***

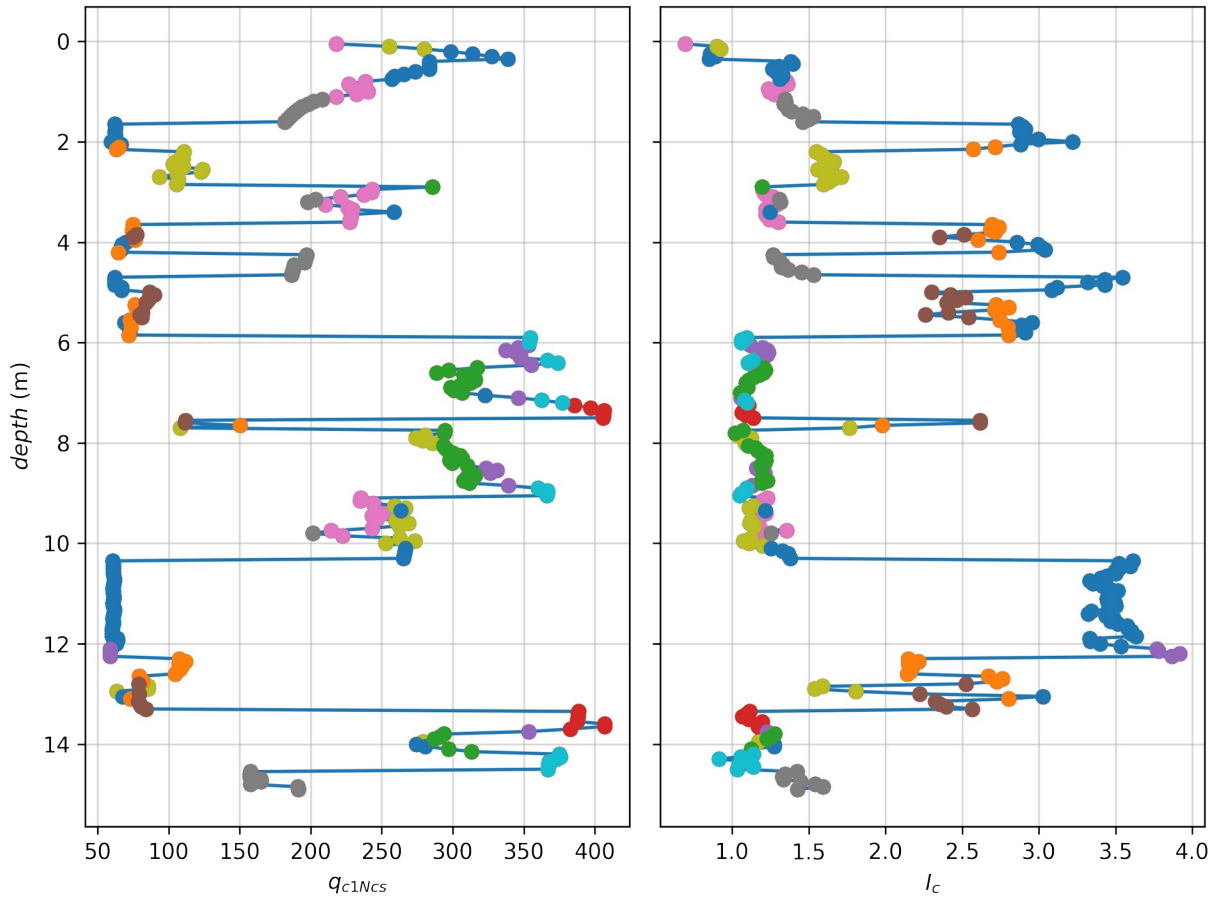
196 Fig. 4 shows results for K-means and Gaussian mixture model clustering each with 16 clusters. Calculations were
 197 performed using the Python package Scikit-learn²³ with default input parameters. Both algorithms group data into
 198 clusters that are close to each other in $\hat{q}_{c1Ncs} - \hat{I}_c$ space, thereby showing promise for grouping data based on
 199 similarities in soil composition. The algorithms exhibit subtle differences in their clustering of the data, with the
 200 Gaussian mixture model resulting in differently shaped clusters than K-means in some cases (Fig. 4). These
 201 approaches to clustering data are similar in concept to the soil behavior type assignments by Robertson⁶ in that soils
 202 in different regions in $\hat{q}_{c1Ncs} - \hat{I}_c$ space are expected to exhibit different soil behavior type. However, the SBT_n
 203 regions defined by Robertson⁶ are fixed in $Q_m - F_r$ space, whereas the clusters are determined simply by proximity to
 204 other data points. We selected $\hat{q}_{c1Ncs} - \hat{I}_c$ as the clustering parameters rather than Q_m and F_r because the former is
 205 more relevant for liquefaction assessments.



206
 207 **Figure 4.** Clustering algorithm results for the UC-4 CPT profile using (a) K-means and (b) Gaussian mixture
 208 modeling.

209
 210 As shown in the profiles in Fig. 5 for the K-means clustering algorithm, these algorithms do not cluster the data into
 211 spatially contiguous layers (e.g., the green colored cluster occurs over the depth intervals 6.5-7.5 m, 7.9-9.0 m,
 212 and 13.8-14.2 m). The reason is that these algorithms cluster data based only on their similarities in $\hat{q}_{c1Ncs} - \hat{I}_c$ space,
 213 and do not consider the fact that the data are hierarchically ordered based on depth. The Gaussian mixture model,
 214 which is not shown in Fig. 5 for brevity, produces similar results in that the clusters are not vertically contiguous.

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Figure 5. CPT profiles for UC-4 based on K-means clustering. Common coloration indicates that depths are associated with the same cluster, e.g. pink intervals at 1, 3, and 9.7 m depths.

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220 *Agglomerative Clustering*

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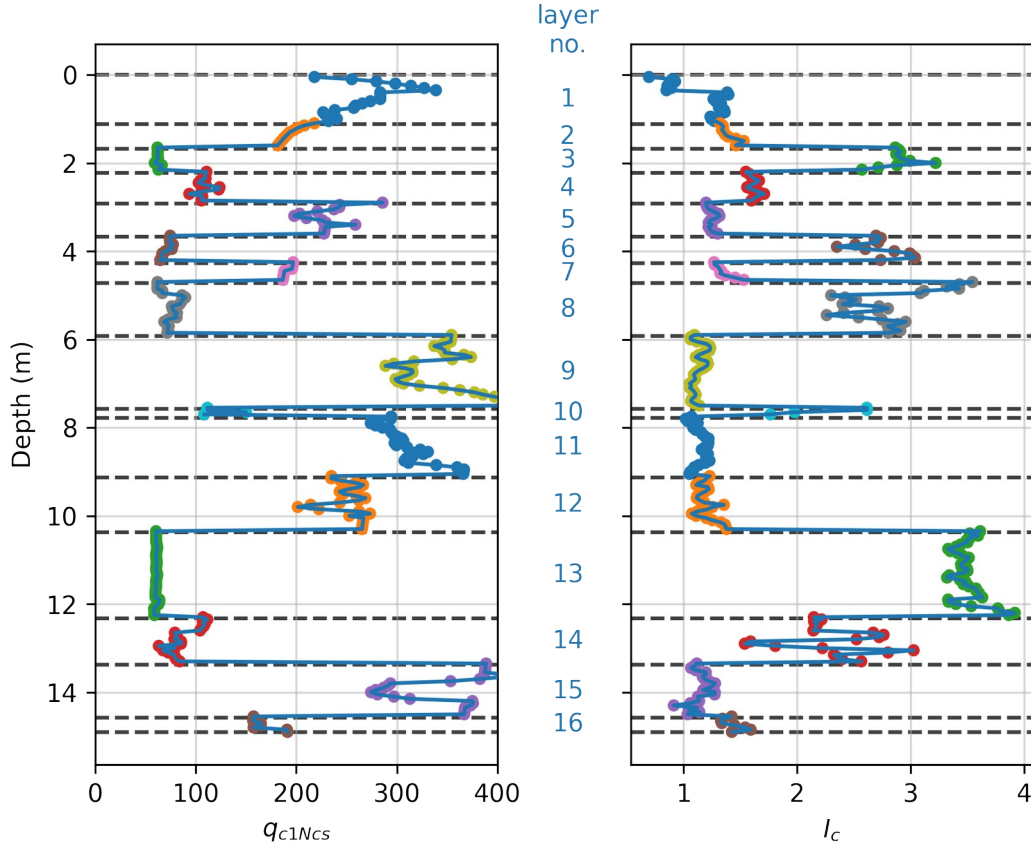
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We turn to agglomerative clustering, which is a form of hierarchical clustering that groups data based on a cascading “tree” of clusters computed using distances between points²⁰, to produce clusters that form vertically contiguous layers. A nearest-neighbor matrix is provided to the clustering algorithm to specify which points are permitted to be considered when assigning clusters. For sequentially ordered data such as CPT data, the nearest neighbor matrix is tri-diagonal with ones on the diagonal and the two adjacent diagonals, and zeros elsewhere. This differs from the approach of Molina-Gómez et al.⁸ that used a more fully populated nearest neighbor matrix. In our approach, a particular data point is constrained to belong to the same cluster as the point above and the point below (or both) or to constitute its own cluster, but it cannot belong to the same cluster as a distant neighbor unless all of the points in between are part of the same cluster. The algorithm then clusters data by minimizing the collective within-cluster variance for the total number of clusters specified. The resulting data are plotted in Figure 6 for the UC-4 CPT profile using a total of 16 clusters. In this case, the clusters are organized into vertically contiguous layers in a manner that reflects their depositional sequence and is similar to how layers might be assigned using human judgment. In this respect, our approach differs from that of Molina-Gómez et al.⁸ which permits clusters to be vertically non-contiguous.



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Figure 6. CPT profiles for UC-4 using agglomerative clustering with tri-diagonal nearest neighbor matrix.

237 Number of Layers

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A crucial consideration in the clustering algorithm is selection of an appropriate number of clusters (i.e., layers). In the preceding examples we have manually set the number of clusters as 16. Here we seek an algorithm capable of selecting the optimal number of clusters, which is expected to vary depending on profile depth and complexity. The goal is to separate the CPT data into contiguous layers with similar soil properties using the fewest clusters possible. The optimal number of clusters is therefore subjective, and different analysts would likely select different numbers of layers for a given CPT profile. Our goal is therefore to identify a method for automatically assigning the number of layers in a manner that captures the stratigraphic details important for liquefaction evaluations.

245 *Distortion Score*

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In agglomerative clustering, a distortion score, J_D , is often utilized to identify the optimal number of clusters, and is defined for the two-standardized-variable case considered here in Eq. 11,

$$J_D = \frac{\sum_{i=1}^N \left[\left(\hat{q}_{c1Ncs_i} - \mu_{\hat{q}} \right)^2 + \left(\hat{I}_{c_i} - \mu_{\hat{I}_c} \right)^2 \right]}{\sum_{i=1}^N \left[\hat{q}_{c1Ncs_i}^2 + \hat{I}_{c_i}^2 \right]}$$

248

(11)

249 where μ_{q_i} and μ_{ic_i} are the mean values of \hat{q}_{c1Ncs} and \hat{I}_c , respectively, for the i^{th} cluster (i.e., subscript i is the
 250 index for clusters and identifies values of these parameters for each individual cluster), and N is the total number of
 251 data points in the profile. Note that J_D decreases as the number of clusters, K , increases, and by definition is equal to
 252 zero when $K=N$ because every point would constitute its own cluster and the numerator would be zero. The optimal
 253 number of clusters therefore cannot be computed by minimizing the distortion score, but rather is a compromise
 254 between reducing the distortion score while retaining the smallest possible number of clusters that adequately
 255 categorizes the data.

256 ***Thickness-Dependent Cost Function and Combined Cost Function***

257 We define a cost function, J_T , that penalizes the average layer thickness within a profile using Eq. 12.

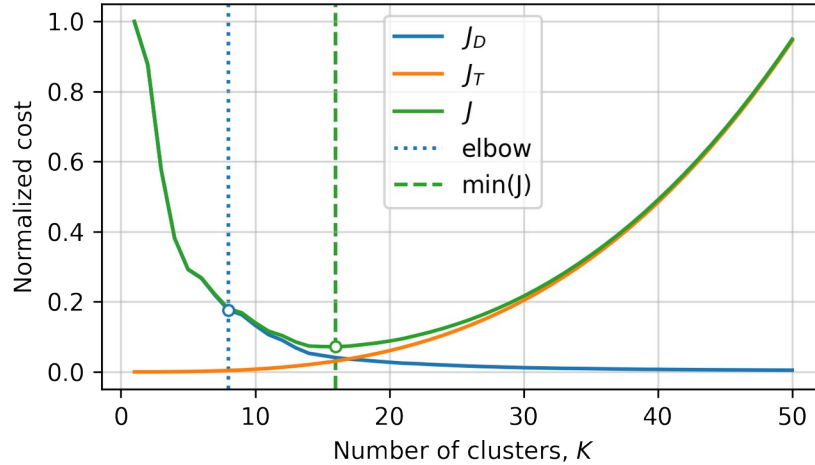
$$J_T = 0.2 \left(\frac{0.5m}{t_{avg}} \right)^3 \quad (12)$$

258
 259 The average thickness is defined as $t_{avg} = z_{max}/K$, where z_{max} is generally the total depth of the CPT profile. Note that
 260 predrilling is sometimes necessary for CPT profiles, in which case the first depth at which data is recorded is
 261 nonzero. In those cases, z_{max} is the difference between the deepest and shallowest CPT measurement. The purpose of
 262 Eq. 12 is to penalize selection of a high value of K if it results in average layer thicknesses that are too small to be
 263 considered geotechnically significant. Based on inspections and analyses of hundreds of CPT profiles in the NGL
 264 database, we believe that 0.5 m is a fairly thin stratum, and we set the coefficients in Eq. 12 such that $J_T = 0.2$ for this
 265 condition. The cubic form of Eq. 12 was adjusted until the achieved average layer thickness accorded well with our
 266 judgment. A combined cost function is then defined in Eq. 13, where w_D and w_T are weights assigned to the
 267 components of the cost function. We herein utilize $w_D = w_T = 1.0$, but these weights can be adjusted based on user
 268 judgment in a site- or region-specific manner.,

$$J = w_D J_D + w_T J_T \quad (13)$$

270 ***Elbow and min(J) Methods***

271 We consider two methods for utilizing the distortion score and the combined cost function to select the optimal
 272 number of layers. First, the “elbow” method graphically interprets a plot of J_D vs. K , which has a negative curvature
 273 over the full range of K , but flattens as K increases (Figure 7). The optimum value of K (9 in the case of Figure 7) is
 274 identified on the basis of curvature having decreased to a sufficiently low level, which is subjective. As such, the
 275 elbow method is based only on J_D and not on J_T . We utilize the Yellowbrick²⁴ Python package to implement the
 276 elbow method which identifies the point of maximum curvature of the J_D vs. K curve and assigns that as the
 277 optimum number of layers. The silhouette method²⁴ is also often utilized to identify the optimal number of clusters.
 278 This method is based on a so-called “silhouette” value that measures the similarity of data points within a cluster
 279 compared to other clusters. We found it to produce similar results to the elbow method. Thus, results from this
 280 method are not reported in Figure 7. Molina-Gómez et al.⁸ utilize the silhouette method to define the number of
 281 clusters in their algorithm.

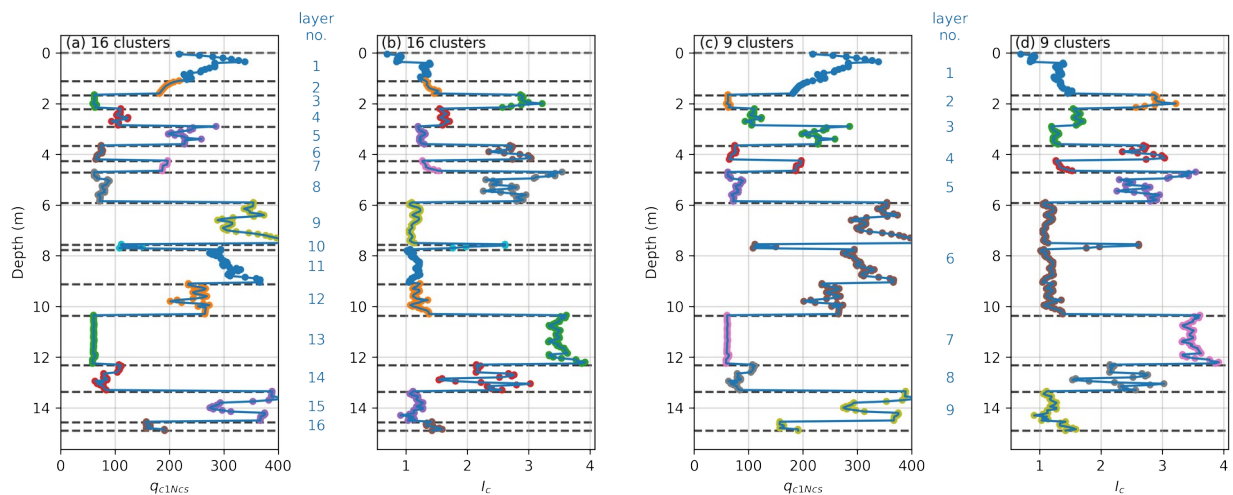


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Figure 7. Cost functions and layer selection for CPT profile UC-4.

284 We also apply an alternative method in which K is selected as the point where J (from Eq. 13) is minimized. For this
285 reason, we call this the $\min(J)$ method. The combined cost function is minimized for $K = 16$ clusters for the example
286 of CPT UC-4 in Figure 7.

287 Profiles of 16 and 9 layers are shown in Fig. 8, where (a) and (b) have 16 layers by using the $\min(J)$ method,
288 whereas (c) and (d) have 9 layers by using the elbow method. The primary differences between these two profiles
289 are in layers number 3, 4, and 6 for the 9-layer profile. These layers clearly contain within-layer regions that are
290 vertically contiguous with different q_{c1Ncs} and I_c values (e.g., the layer for the 2.2-3.8 m depth range), yet they are
291 clustered together in the 9-layer profile. By contrast, they are separated into different layers in the 16-layer profile.
292 The 16-layer profile accords better with our judgment, and similar observations observed across diverse profiles with
293 a wide range of depths (as described in the next section) causes us to prefer use of the $\min(J)$ approach over the
294 elbow method when selecting the number of layers. We recognize that a different curvature threshold in the
295 application of the elbow method would have produced a different number of layers and, possibly, a solution that
296 accords better with our judgment. However, the superiority of the $\min(J)$ method is related to the fact that it is based
297 on layer thickness, which is a physically meaningful quantity, whereas the gradient of J_T vs. K used in the elbow and
298 silhouette methods does not have a clear physical meaning.



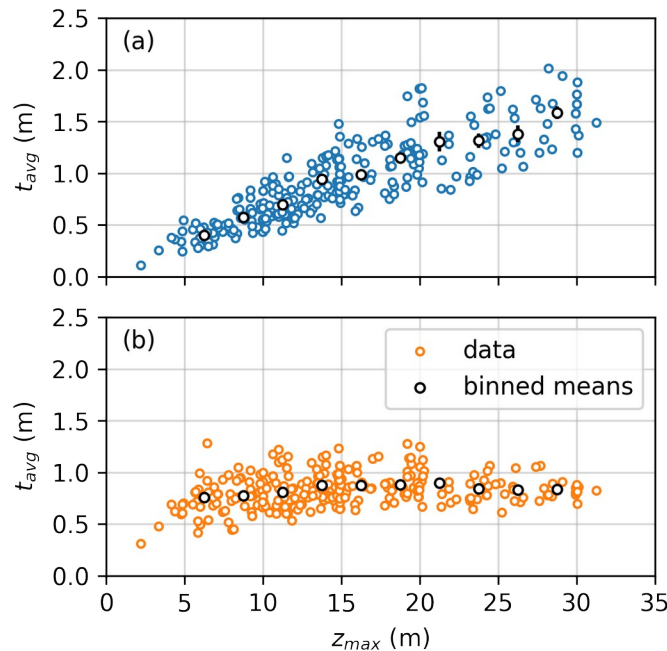
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Figure 8. Profiles of q_{c1Ncs} and I_c with 16 layers by using the $\min(J)$ method (a and b) and 9 layers by using the elbow method (c and d).

302 **Calculations for Many CPT Profiles**

303 Calculations of the optimal numbers of layers were performed for a total of 272 CPT profiles contained in the NGL
 304 database^{9,10}. Both the elbow method and the min(*J*) method were utilized to select the optimal number of layers. We
 305 expect that t_{avg} should be independent of z_{max} because t_{avg} depends upon vertical heterogeneity of the soil profile,
 306 which is controlled by the geological processes that formed the soil deposit, whereas z_{max} arises from a decision
 307 controlled by the objectives of the site investigation. For example, z_{max} may be higher for a site investigation for a
 308 pile-supported tall building with a corresponding deep zone of influence than for a single-story building supported
 309 by spread footings with a corresponding shallow zone of influence.

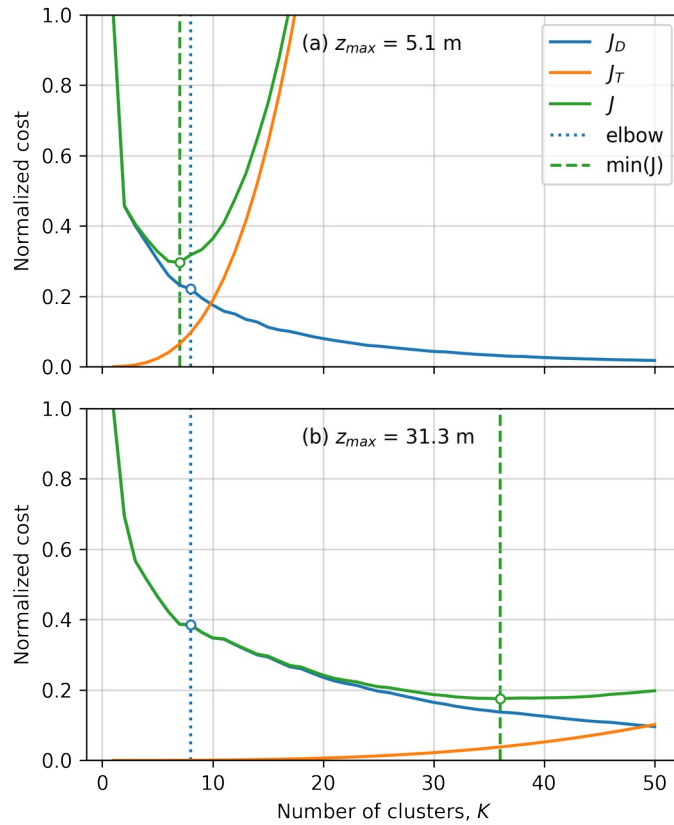
310 Values of t_{avg} vs. z_{max} are plotted in Fig. 9. The elbow method exhibits a strong positive correlation in which t_{avg}
 311 increases essentially linearly with z_{max} . This is an undesirable outcome since we anticipate t_{avg} to be independent of
 312 z_{max} . By contrast, values of t_{avg} are essentially independent of z_{max} using the min(*J*) method, particularly for values of
 313 $z_{max} > 12$ m. For liquefaction triggering evaluation, profiles shorter than about 15m may miss layers that could
 314 potentially liquefy and produce surface manifestation. In this regard, the slight bias in the min(*J*) method for shallow
 315 profiles has little practical impact.



316
 317 **Figure 9.** Average layer thickness, t_{avg} , versus total CPT profile length, z_{max} for (a) elbow method and (b) min(*J*)
 318 method

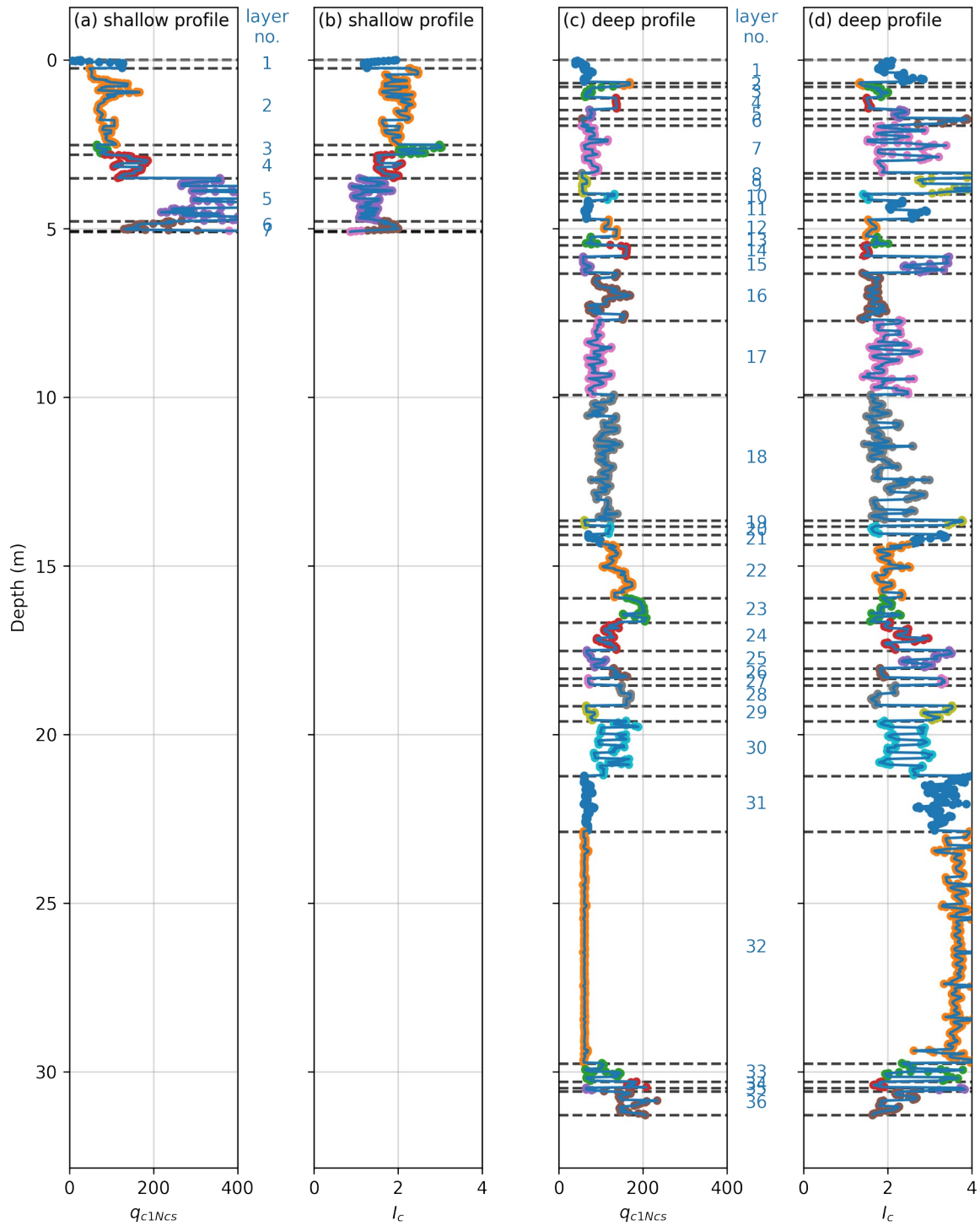
319 The influence of maximum depth on average layer thickness is further explored in Fig. 10, which illustrates
 320 normalized cost versus number of clusters for (a) a shallow profile with $z_{max} = 5.1$ m from CPT_8933 at Site 76 in
 321 Edgecumbe, New Zealand, and (b) a deep profile with $z_{max} = 31.3$ m from CPT001 at the Inage site in Urayasu City,
 322 Japan (CPT names are those reported in the NGL database). Note that the J_T functions are significantly different for
 323 these two profiles because the same average thickness in Eq. 12 produces fewer layers for the shallow profile than
 324 for the deep profile. For the shallow profile, the elbow method indicates that 8 sublayers is ideal ($t_{avg} = 0.64$ m), while
 325 the min(*J*) approach provides 7 layers ($t_{avg} = 0.73$ m). These results are very similar. By contrast, for the deep profile,
 326 the elbow method indicates that 8 layers is ideal ($t_{avg} = 3.9$ m), while min(*J*) provides 36 sublayers ($t_{avg} = 0.87$ m).
 327 These results are significantly different, and the average layer thickness using the elbow method is too large to
 328 capture potential critical layers of sand-like soil with low q_{cINcs} .

329 Note that when $K=8$, J_D is near 0.2 for the shallow profile and near 0.4 for the deep profile. A fundamental limitation
 330 of the elbow method is that it considers only the curvature of the cost function, and not the value of the cost function
 331 itself.



332
 333 **Figure 10.** Normalized cost versus number of clusters for (a) a shallow profile with $z_{max}=5.1$ m corresponding to
 334 CPT_8933 at Site 76 in Edgecumbe, New Zealand, and (b) a deep profile with $z_{max}=31.3$ m corresponding to CPT001
 335 at the Inage site in Urayasu City, Japan.

336 The two profiles are illustrated in Fig. 11 with a common depth axis to illustrate the clear differences in the
 337 maximum penetration depth. The average layer thicknesses determined using the min(J) method are similar for these
 338 two profiles despite the different total depths. Furthermore, it is clear that reducing the number of layers for the
 339 deeper site from 36 (using the min(J) method) to only 8 (using the elbow method) would result in significantly
 340 higher average layer thickness, and would miss much of the stratigraphic detail within that profile.



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 343

Figure 11. Profiles of q_{c1Ncs} and I_c for (a) and (b) a shallow profile corresponding to CPT_8933 at Site 76 in Edgecumbe, New Zealand, and (c) and (d) a deep profile corresponding to CPT001 at the Inage site in Urayasu City, Japan.

344 Conclusions

345 This study developed an unsupervised machine learning approach for identifying layers from cone penetration test
346 data and selecting the optimal number of layers (or clusters). The clustering parameter space consisted of \hat{q}_{c1Ncs} and
347 \hat{I}_c , which are standardized values of the overburden-corrected clean sand equivalent cone tip resistance, q_{c1Ncs} , and
348 the soil behavior type index, I_c . The clustering algorithm utilizes the Scikit learn Python package, which is widely
349 available and easy to implement. We utilize agglomerative clustering with a tridiagonal nearest neighbor matrix to
350 identify vertically contiguous soil layers.

351 A crucial aspect of the proposed algorithm is selecting the optimal number of clusters. The elbow method, a
352 traditional approach commonly utilized in machine learning, did not perform well for our application because the
353 resulting average thickness of the soil layers was strongly dependent on the maximum depth explored by the CPT.
354 We posit that soil stratigraphy is independent of the maximum depth to which the CPT probe is advanced. To
355 overcome this limitation, we introduced a supplemental cost function that penalizes small average layer thicknesses.
356 The optimal number of clusters is selected at the minimum point of this cost function added to the normalized
357 distortion score. This approach produced an average layer thickness that is essentially independent of maximum
358 depth, which is a desired outcome. Compared with manual assignment of layer boundaries, our method is automated
359 and rapid, and shifts human judgment from a case-by-case basis (which is not repeatable) to selection of input
360 parameters in the clustering algorithm (which is repeatable).

361 All calculations presented herein were performed on inverse-filtered CPT data rather than on raw recorded CPT
362 data. We believe this is more appropriate because CPT measurements are influenced by soil layering, and the
363 inverse filtering attempts to recover the “true” CPT profile. Although not shown in this paper, we found that the
364 proposed algorithm often grouped transition layers into a single cluster. In this manner, the algorithm may be useful
365 for application to raw measurements as well, provided that the analyst properly accounts for these transition zones
366 for liquefaction evaluation or other applications.

367 The proposed algorithm provides a convenient means for rapidly developing a tentative layering profile for further
368 engineering evaluation. Modeling parameters were adjusted to accord with our judgment regarding layer
369 assignments. However, the algorithm may do a poor job identifying layers in some situations, and we urge users to
370 review the layering that arises from the algorithm and to exercise their own judgment and available geological
371 knowledge in assigning layers for their particular application before proceeding with calculations. For instance,
372 users could calibrate the J_T function by adjusting the t_{avg} or the weights in Eq. 13 based on their dataset and intended
373 application.

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