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Oversimplification: a reply to White

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Why favour simpler hypotheses? Roger White has argued 'that the assumption which supports favouring simplicity is far more modest than it first seems'. (White 2005: 205) I will critically examine these arguments.

White imagines a 'black box' machine with a dial and a pointer, each with a large but finite number of positions. When the dial is turned, the pointer changes its position. White supposes that we know only that there is a mechanism in the box. He doesn't tell us about the parts inside: how many there are, and what their properties might be.

What function is calculated by the mechanism inside the box? Presumably, checking each possible dial position would take too long, so we check a subset of them. For any such subset of dial settings and pointer readings there will be a large number of functions compatible with those readings. A principle of favouring simpler functions would facilitate a choice among functions compatible with a data set.

White considers the following simplicity favouring principle (**SFP**):

> **SFP**: The greater the complexity of a function *f*, the less probable it is that the machine computes *f*.

Without this principle it isn't obvious why we should prefer one possible function to any other.

White distinguishes two senses of complexity[1]: the complexity of the mechanism in the box and the complexity of the function the mechanism produces (hereafter 'machine complexity' and 'functional complexity', respectively).

> The complexity of the mechanism in the box is roughly a matter of how many different kinds of parts it contains, all of which are intricately linked so that a change in any part would make a major difference to the workings of the mechanism. (2005: 206)

He next characterizes *functional* complexity:

> The complexity of a function is harder to characterize, but I will assume that we recognize it well enough when we see it. For instance a linear function is intuitively characterized as simple, and higher order polynomials as more complex. (2005: 206)

I will criticize both of these characterizations later, but let us continue with White's argument for the moment. White tells us that these two senses of complexity, mechanical and functional, bear an important relation to one another. Let us call this important relation (**IR**):

> (**IR**): *The more complex the function, the more complex the mechanism required to compute it* (White 2005: 206)

In other words, the more complex the function, the more 'number of kinds of parts' we need. Thus White claims we can derive a measure for functional complexity—a function *c* that maps each possible function onto the least degree of machine-complexity required to compute it. (2005: 206) So I think we can say:

> $c(f)$ = minimum 'number of kinds of parts' needed to compute f

[1] For purposes of my discussion here, simplicity and complexity are two directions on a linear scale. I use the terms interchangeably to describe what White is trying to quantify.

Notice some features of this setup: first, the more complex the machine, the more possible functions it can compute; there are more possible ways to put more parts together than fewer. (2005: 207) Second, there is a maximum number of kinds of parts, or number of parts[2], due to physical limitations of the system (only so many can fit inside). (2005: 206) Third, the machine-complexity levels are discreet—there is one complexity level for each 'number of kinds of parts', and nothing in between. (2005: 205-206)

Now for White's derivation of the SFP (2005: 206-207):

Let F = the machine computes function f.
Let P(F) = the probability that the machine computes function f.
Let M_i = the mechanism in the black box is of complexity i

White then restates the simplicity favouring principle as:

SFP: P(F) decreases as $c(f)$ increases.

The probability that the machine computes the function f can be given as:

$$P(F) = P(F|M_1)P(M_1) + P(F|M_2)P(M_2) + \ldots + P(F|M_n)P(M_n)$$

where n is the maximum degree of complexity for a machine that could fit in the box. Also, a machine cannot compute a function whose complexity measure is greater than the complexity of the machine, i.e. $P(F|M_i)=0$ if $c(f) > i$. If that is the case, then

$$P(F) = \sum_{i=c(f)}^{n} P(F \mid M_i)P(M_i)$$

One set of crucial values in this formula are the conditional probabilities, $P(F|M_i)$. In order to evaluate these, White takes note of an important fact: *the number of functions that a machine can compute increases rapidly with the complexity of the machine.*

This means that as the complexity of the machine increases, there will be more functions at each complexity level that will share the probability space *given* that we know that machine is of that complexity level. Because of this, White claims that as M_i increases, the conditional probabilities $P(F|M_i)$ decrease rapidly.

White doesn't mention it, but all this shows is that the *average* conditional probability goes down as machine complexity level goes up. *If* we assume a roughly even distribution over the possibilities *given* that complexity level, invoking a principle of indifference of sorts, the conditional probability $P(F|M_5)$ will be higher than, for example, $P(F|M_{10})$. If we didn't distribute the credences roughly evenly given a complexity level, it would be perfectly consistent for there to be some functions for which $P(F|M_5)$ is lower than $P(F|M_{10})$.

But White proceeds. He then introduces the following 'modest' assumption (**A**) that he thinks supports the SFP.

(A): *the machine is not considerably more likely to be complex than simple*

White claims that the only way to avoid adopting his SFP is if we strongly bias our credences towards high machine complexity levels, that is, deny (A). He doesn't say why such a bias does this, but I think the rough idea is that since he claims that each $P(F|M_i)$ term in the summation decreases rapidly as i increases, we can offset this decrease by increasing each corresponding prior probability of $P(F_i)$. Any other distribution of prior probabilities will fail to offset the dramatic drop off in $P(F|M_i)$, hence White's claim that the only assumption we need is (A).

However, it isn't clear why offsetting the decrease in $P(F|M_i)$ avoids the SFP. He doesn't explain, but here is what I think he might have in mind: suppose for some function f_{10}, $c(f_{10}) = 10$. This function will receive a share of the probability space from machine complexity level 10, level 11, level 12, and so on to

[2] White switches between 'number of parts' or 'number of kinds of parts'—it's not clear which he has in mind.

the maximum complexity level (it will receive nothing from complexity levels below 10 as on White's account those machine complexity levels are insufficiently complex to generate the function). A simpler function f_8, where $c(f_8) = 8$, will receive the same shares of the probability space as f_{10}, but those shares of machine complexity level 9 and 8 as well. Thus absent bias towards more complex functions we should prefer f_8. But even if we bias our credences towards M_1 - M_n strongly towards more complex machines as White suggests, the simpler function will *still* be more probable as f_8 will still receive all the shares of the probability space as f_{10} does, plus those at level 9 and level 8. The simpler function will be more probable no matter what our prior probabilities in M_1-M_n may be, assuming that these priors and the terms $P(F|M_i)$ are the same in calculating the probability of each function. So if this interpretation is right, then White's argument looks suspiciously strong—he wouldn't need (A) at all in order to support his claims since it doesn't matter what the values of the priors are. Thus I confess puzzlement as to how White thinks (A) fits into his arguments.

Let us now consider White's argument for adopting (A) itself. White says that the only way to avoid the SFP is by *denying* (A) and strongly biasing our credences towards complex machines. He appeals to our sense that a strong bias towards complex machines seems arbitrary and unreasonable. Presumably we want to avoid making arbitrary and unreasonable assumptions (the essential motivation behind his paper), so we're left with adopting (A) the SFP he thinks follows.

But this is a questionable move. A proposition being arbitrary gives no positive support to its negation—at best it supports withholding judgment. And if a proposition is arbitrary, then so is its negation. So any consideration of being arbitrary can't differentiate between the two. To take a simple example, suppose that I have flipped a coin and have yet to reveal which side it landed on. Simply assuming that the coin landed heads would be unreasonably arbitrary, but it doesn't follow from this that we should think that the coin landed tails. White's argument is fallacious.

There are other serious objections to White's arguments.

One is that the assumption that functional complexity correlates with the 'number of kinds of parts' required is unwarranted—**(IR)** is false.

Suppose that the machine has two parts: each is the same irregular, not-remotely-elliptical gear. We can generate an enormously complex function on White's measure with just these two parts. We could increase the complexity of the function by increasing the number of bumps and curves on the gear we've designed, or decrease it by using smoother shapes. If we measure complexity by counting number, or number of kinds, of parts, no matter how we count there will be more complicated machines that can compute only much more simple functions than machines that are less complex.[3]

Now White could say that the irregular gears in fact make for a high level of machine complexity. But how then is machine complexity being measured? If not, as White said originally, by the number and/or kinds of parts, but instead by the 'complexity of the parts' individually, we will need some way to measure part complexity. But there is no way to do this that won't fall back on some measure of functional complexity as used to describe the shapes of parts.

Another objection is that White's characterizations of complexity are suspect. With regards to *functional* complexity, the example of the linear function is misleading—White suggests that higher order polynomials are more complex than simpler ones. But there must be more to judging complexity than this, as simple trig functions are equivalent to infinite order polynomials. Which is more complex, $y=\sin x$ or $y=7.23x^6-882.293x^5+98x^4-7x^3+3.2x^2+1$?

He appeals to our intuitions in judging the complexity of functions. But what strikes us as intuitively simple can vary depending on the representational system used. An linear equation like $y=2x$ might seem simple, but if we transform this into an equation represented in terms of polar coordinates we get something more complicated. Likewise, a polar function like $r=\cos\theta$ might seem simple, but if we transform this into an equation represented in terms of rectangular coordinates, we get something more complicated. Thus our judgment of the complexity of a function can boil down to a choice of representational systems, and there's no obvious, non-arbitrary rationale for choosing one over another.

With regards to *machine* complexity, it's not even quite clear what White has in mind. He says that it is 'roughly a matter of how many different kinds of parts it contains'. This is ambiguous. Depending on how we understand it, we will end up distributing our credences in different ways.

He could mean 'number of parts'. He could mean 'number of kinds of parts', in other words, any number of identical parts will only be counted once. 'Kind' here could be understood in different ways. To

[3] Alan Baker makes a similar point in his (2008).

illustrate the problem, consider the following machines containing the listed parts (all of which compute the same function):

> Machine One: 1 small gear, 1 medium lever, 1 large spring
> Machine Two: 10 medium gears
> Machine Three: 2 large levers, 2 medium levers, 2 small levers
> Machine Four: 4 small springs, 4 small gears

It's not clear to me which one of these we should consider the simplest. But each choice would indicate a different complexity measure, and thus the result yielded by the SFP would be different.

To see why, suppose we think that machine three is the simplest, as it only contains one kind of part if we're counting the kinds as lever, spring, and gear (size doesn't matter on this measure). Thus for Machine One, $c = 3$, Machine Two, $c=1$, Machine Three, $c=1$, Machine Four, $c=2$. SFP would tell us that Machine Two and Machine Three are equally the simplest, and so we should choose one of them over the others. But suppose that size *does* matter. Then, for Machine One, $c=3$, Machine Two, $c=1$, Machine Three, $c=3$, and for Machine Four, $c=2$. Thus SFP tells us to choose machine two in this case.

Just as with functional complexity, there is no obvious, non-arbitrary rationale for choosing one measure of machine complexity over any other. In light of this, I claim that White's notions of complexity are not sufficiently well defined and there is no reason to suppose that when a stand is taken on alternatives such as the ones I have described that the choices would involve an assumptions in any way more modest than the SFP itself as he promised to show.

There is more trouble for White.

White claims it would be unwarranted bias on our part to have a credence distribution that is skewed towards higher possible machine complexity levels (which he claims is what we must do to avoid adopting the SFP); he's arguing that our credences should be constrained by what we think about machine complexity levels. But why should we divide up things according to complexity levels as opposed to other ways of dividing up the possibilities as a basis for saying something about probabilities?

White considers such an objection: why not distribute our credences over the possible functions rather than the possible machine complexity levels? If we're non-committal over which function it will turn out to be, why not just distribute our credences that way? In fact, if we do so, it will turn out that we get the converse of the SFP, since there are far more complex functions on White's setup than simpler ones.

White doesn't think such concerns can give us good reason to abandon (A). He responds with an analogy: suppose we have to guess how many children are in the Jones family moving in next door. Assuming that we don't have any significant information on the matter, he thinks we ought to be indifferent to at least several possibilities.[4] So we distribute our credences roughly evenly over them. He says that it wouldn't make sense to distribute our credences in some other way. We wouldn't, for example, distribute our credences over the possible number of child-arrangements in the house (his example). So if the Jones had three children, they could have two in the kitchen, one in the living room, or perhaps two in their bedrooms and one in the kitchen, or one on the stairs and two in the family room, and so on. The more children there are, the more ways there are to arrange them. So if we distributed our credences evenly over the number of possible child-arrangements rather than number of children, we get a similar result as we would if we distributed our credences over the possible functions rather than machine complexity levels. White claims that it's obvious we wouldn't distribute our credences over child-arrangements, so it's likewise obvious that we shouldn't distribute our credences over the possible functions rather than machine complexity levels.

White doesn't offer any argument as to why he thinks this situation is analogous in the relevant ways. Both cases are similar in that we're inclined to pick one distribution over another, but he hasn't made the connection clear. Depending on the example, distributing our credences over 'arrangements' *is* more plausible. For example, suppose we're throwing a pair of dice and want to gamble on the chance that the thrown total will be nine. Now we could distribute our credences evenly over the possible totals thrown (11 possibilities, 2-12), or we could distribute our credences over the *arrangements* of dice (36 possibilities). In this case it's clear we should pick arrangements, not possible totals. Why is the machine case to be analogized to the family rather than to the dice case? If the answer is that in *this* case, and the analogous

[4] We almost always have some significant information on the matter, and in this case, very familiar information about the ways families work might give us reason to be indifferent to a few of the possibilities.

case with the functions computed by the box, is that it just seems intuitively right, why is the appeal to this intuition any more modest that just appealing to our intuitions in favor of SFP to begin with?

I have now analysed several points in White's arguments that he thinks support a simplicity favoring principle.

First, he doesn't make it clear why adopting (A) supports the simplicity favouring principle. I offered one possible interpretation of what he might have in mind, but even if that interpretation is correct, (A) isn't relevant to the SFP.

Second, his argument that we're justified in adopting (A) because denying it would be arbitrary is fallacious. A judgment being arbitrary does not justify its negation.

Third, he claims that the complexity of the function output from the machine, generally speaking, bears a relationship to the "number of kinds of parts" a given machine has. The irregular gear counterexample shows this to be dubious.

Fourth, he supposes that there will be a unique way of measuring complexity. I showed that there are many ways to measure both functional and machine complexity, and that it is not clear why we should choose one over any of the others to support White's conclusions.

Finally, White appeals to intuitions about the Jones analogy in order to motivate why we should distribute our credences over one set of possibilities rather than another. White fails to make the case that the cases are analogous in the relevant respects.

White's argument that there are modest assumptions that support a simplicity favouring principle thus fails.[5]

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