Title
The implementation process and impact of a six-week number talk intervention with sixth-grade middle school students in a large urban middle school

Author
Okamoto, Yoshiko

Publication Date
2015

Peer reviewed|Thesis/dissertation

UNIVERSITY OF CALIFORNIA

Los Angeles

The Implementation Process and Impact of a Six-Week Number Talk Intervention with Sixth-Grade Middle School Students in a Large Urban School District

A dissertation submitted in partial satisfaction of the

requirements for the degree Doctor of Education

by

Yoshiko Okamoto

2015

ABSTRACT OF THE DISSERTATION

The Implementation Process and Impact of a Six-Week Number Talk Intervention

with Sixth-Grade Middle School Students in a Large Urban School District

by

Yoshiko Okamoto

Doctor of Education

University of California, Los Angeles, 2015

Professor Megan Loef Franke, Chair

This study employed a mixed-methods design to examine the implementation process and impact of a six-week number talk intervention during the advisory period with sixth-grade middle school students in a large urban school district. The sample consisted of 22 students in one sixth-grade advisory class. The quantitative portion of the study utilized a one-group pretest and posttest design to measure the effects of the intervention on the development of students' number sense. The qualitative portion of the study, which included classroom videos, student work, and a teacher journal, examined process data on the implementation of number talks to determine in what ways number talks can be implemented with middle school students to support the development of number sense. There were three teacher practices identified as being important in implementing number talks in a way that supported student learning: 1) examining students' written work during and after the number talks as a means of formative assessment, 2) using visual representations to support students' understanding of mathematical concepts and relationships, and 3) focusing on one mathematical idea for a set of number talks. Results of the quantitative analysis showed that there were statistically significant increases in the overall mean score on the number sense assessment and the mean score on the equivalent expressions subtest of the assessment which was closely aligned to the content of the intervention.

The dissertation of Yoshiko Okamoto is approved.

Christina A. Christie

Linda P. Rose

James W. Stigler

Megan Loef Franke, Committee Chair

University of California, Los Angeles

2015

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

VITA

1999	Bachelor of Arts, Economics University of California, Berkeley
2003	Multiple Subject Clear Teaching Credential
2004 – present	Mathematics Teacher Hoover Middle School Long Beach Unified School District
2005 – present	Mathematics Department Chair Hoover Middle School Long Beach Unified School District
2005	Single Subject Clear Teaching Credential in Mathematics, Japanese, and Economics
2007	Master of Arts, Educational Administration California State University, Dominguez Hills

CHAPTER 1: STATEMENT OF THE PROBLEM

Introduction

Number sense as McIntosh, Reys, and Reys (1992) describe is a "person's general understanding of number and operations along with the ability and inclination to use this understanding in flexible ways to make mathematical judgments and to develop useful strategies for handling numbers and operations" (p.3). Over the past twenty-five years, many national and international reports have highlighted the importance of number sense in school mathematics (Australian Education Council, 1991; National Council of Teachers of Mathematics, 2000; National Mathematics Advisory Panel, 2008; National Research Council, 1989, 2001). In *Everybody Counts,* the National Research Council (1989) considered number sense to be the major objective of elementary school mathematics. In *Principles and Standards for School Mathematics*, the National Council of Teachers of Mathematics (2000) identified the development of number sense as central to the numbers and operations strand which forms the core of mathematics education for the elementary grades. In the recently adopted Common Core State Standards for Mathematics (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010) the development of the understanding of numbers and operations is a major component of the elementary school standards.

Despite the importance of number sense, research shows that many students complete elementary school not having developed an understanding of basic concepts associated with numbers and operations (Z. Markovits & Sowder, 1994; National Mathematics Advisory Panel, 2008; National Research Council, 2001; Parrish, 2010; Verschaffel, Greer, & De Corte, 2007). Researchers attribute this problem to various factors including a lack of focus and coherence in the traditional U.S. curriculum (Schmidt, McKnight, Houang, & Wang, 2001) and the teaching of mathematics in the U.S. as a collection of rules, facts, and procedures to be memorized instead of a system of relationships to investigate and understand (Carpenter, Franke, & Levi, 2003; Hiebert, Gallimore, Garnier, & Givvin, 2003). The TIMSS video studies (Hiebert et al., 2003; Stigler & Hiebert, 1999) showed that in high-achieving countries, the instructional methods that are used help students to make connections between concepts and related procedures to develop deep understanding. In the U.S., most lessons tend to focus almost entirely on rote practice of procedures with almost no emphasis on developing core mathematics concepts that would help students to make connections among topics taught in the curriculum.

Number sense serves as a critical foundation for higher level mathematics. The Numbers and Operations and Operations and Algebraic Thinking progressions, which comprise number sense, form the base for middle school mathematics and algebra in the Common Core State Standards for Mathematics (Zimba, 2011). To successfully learn mathematics with understanding in middle and high school, students need a strong foundation of number sense to which new learning can be connected.

Although there are many interventions in place at the K to 8 level and beyond to help students having difficulty with reading, there has been significantly less in the way of "mathematics recovery" that provides students with opportunities to develop the foundational skills and knowledge needed to be successful in later mathematics (National Research Council, 2001). The few interventions that are in place at the middle school level usually involve a second period of mathematics, in lieu of an elective class, with time spent on additional rote learning and practice of grade-level topics. Data show that these interventions are not productive. What students need is not more time on rote learning and practice of grade-level topics, but the opportunity to develop number sense so that they have the conceptual base to learn middle and high school mathematics with understanding.

One promising method that can be used to develop students' number sense is an approach called "number talks." Number talks are ten- to fifteen-minute classroom discussions around purposefully crafted mental computation and number of the day problems designed to engage students together in exploring and sharing their ideas of number relations. Number talks engage students in productive struggle and in making connections among important mathematical ideas which the TIMSS 1999 Video Study showed are the two key features of instruction that help to develop mathematical understanding and were missing from the teaching in most U.S. classrooms (Hiebert et al., 2003).

Number talks are closely aligned to the recently adopted Common Core State Standards for Mathematics which were benchmarked to the standards of high-performing countries in the TIMSS studies. The Common Core Standards for Mathematical Practice call for students to be able to "justify their conclusions, communicate them to others, and respond to the arguments of others" and to be able to use "clear definitions in discussions with others and in their own reasoning" (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010, p. 7). Sharing and discussion of strategies is a key component of number talks. By explaining their thinking and engaging in other students' ideas, students have the opportunity to clarify their thinking, develop understanding of mathematical relationships, build a repertoire of efficient strategies, and consider and test strategies to see if they are logical (Parrish, 2011). Another key component of number talks is mental computation. Various reports have highlighted the importance of mental computation as a vehicle for promoting the development of number sense (National Mathematics Advisory Panel, 2008; National Research

Council, 2001) and mental computation standards are included as explicit goals in the Common Core State Standards for Mathematics. Phil Daro, one of the core authors of the Standards, identifies mental computation as one of the key areas needing attention in the Common Core (Daro, 2011). Mental computation encourages students to think about number relationships and to use these relationships to develop efficient and flexible strategies instead of relying on memorized rules. It involves decomposing and recomposing of numbers, applying the basic properties and the meaning and effects of operations, and developing understanding of place value by looking at numbers as whole quantities instead of discrete columns of digits, all of which are emphasized in the Common Core State Standards.

Number talks have been increasingly used in classrooms across the U.S. as part of the implementation of the Common Core State Standards. They are a promising approach that can be used to enrich students' understanding of numbers and operations and support the implementation of the Common Core State Standards for Mathematics. Number talks are also practical because they are only ten- to fifteen-minutes long and can be easily incorporated into the regular school day.

Research Questions

The purpose of this study was to explore the implementation process and impact of a six-week number talk intervention during the advisory period with sixth-grade students in a middle school in a large urban school district. The study explored the following research questions:

1. In what ways can number talks be implemented with 6^{th} grade middle school general mathematics students to support the development of number sense?
2. What are the effects of a 6-week number talk intervention on sixth-grade middle school general mathematics students' number sense?

Research Design

A mixed-methods design which incorporated both quantitative and qualitative approaches was used to examine the research questions. I chose to use a mixed-methods approach because it would provide a more complete understanding of the research problem than either approach alone (Creswell, 2013). The quantitative portion of the study utilized a one-group pretest and posttest design to measure the effects of the intervention on the development of students' number sense. The qualitative portion of the study, which included classroom videos, student work, and a teacher journal, examined the implementation process of the number talks to determine in what ways number talks can be implemented to support the development of students' number sense.

Research Site

The study was conducted at Eastside Middle School in Central Unified School District. Eastside has an enrollment of about 850 students in grades six through eight. It is a Title I designated school with about 60% of students being eligible for free or reduced lunch. The racial makeup of the school is about 45% Hispanic, 25% African-American, 12% Asian, and 11% White. The site was chosen for several reasons: 1) the school was typical of many middle schools in urban districts with a racially diverse student population and a high percentage of low SES students, 2) the school had been struggling to increase the proficiency of students in its general mathematics program which had been around ten percent based on the California State Test, and 3) site administration and teachers were interested in exploring the use of instructional interventions during the school's advisory class period to provide targeted support to students in the general mathematics program.

Significance of the Research

The results of the study are of interest to middle school teachers who want to enrich students' understanding of numbers and operations and to support the implementation of the Common Core Standards in Mathematics. To inform others about my findings and recommendations, I plan to share the results with staff at Eastside Middle School and Central Unified School District's Math Curriculum Office.

CHAPTER 2: LITERATURE REVIEW

This study explored the implementation process and impact of a six-week number talk intervention with sixth-grade students in a middle school in a large urban school district. In this literature review, I will first examine research on number sense, the targeted learning outcome of number talks. Next, I review research on teaching practices that promote mathematical understanding. Then, I consider research on mental computation which is considered to be a hallmark of number sense. Finally, I make connections between number talks, teaching practices, mental computation, and the development of number sense.

Research on Number Sense

Components of number sense. The term number sense is often discussed in mathematics education, but has been difficult to define. McIntosh, Reys, and Reys (1992) describe number sense as a "person's general understanding of number and operations along with the ability and inclination to use this understanding in flexible ways to make mathematical judgments and to develop useful strategies for handling numbers and operations" (p.3). Greeno (1991) describes number sense as follows: "The term number sense refers to several important but elusive capabilities including flexible mental computation, numerical estimation, and quantitative judgment" (p.170). Reys et al. (1999) write that number sense "results in a view of numbers as meaningful entities and the expectation that mathematical manipulations and outcomes should make sense. Those who view numbers in this way continually utilize a variety of internal 'checks and balances' to judge the reasonableness of numerical outcomes" (p.61). According to the National Council of Teachers of Mathematics (1989), indicators of number sense include well-understood number meanings, understanding of multiple numerical relationships, recognition of relative magnitudes of numbers, and use of referents for measure of common objects and situations in their environment.

McIntosh et al. (1992) developed a detailed framework for number sense based on the literature associated with the topic. The framework included three major interconnected components: knowledge of and facility with numbers, knowledge of and facility with operations, and applying knowledge of and facility with numbers and operations to computational settings.

To operationalize number sense, the National Math Advisory Panel (2008) provided the following description:

> In its most fundamental form, number sense entails an ability to immediately identify the numerical value associated with small quantities, a facility with basic counting skills, and a proficiency in approximating the magnitudes of small numbers of objects and simple numerical operations…A more advanced type of number sense that children must acquire through formal instruction requires a principled understanding of place value, of how whole numbers can be composed and decomposed, and of the meaning of the basic arithmetic operations of addition, subtraction, multiplication, and division. It also requires understanding the commutative, associative, and distributive properties and knowing how to apply these principles to solve problems. (p.27)

The Panel explains that this more advanced type of number sense should extend to numbers written in fraction, decimal, percent and exponential forms.

Sowder (1992) describes number sense in terms of nine dimensions or behaviors that would demonstrate some presence of number sense based on previous research. The nine dimensions include: (1) the ability to compose and decompose numbers, to move flexibly among different representations, and to recognize when one representation is more useful than another, (2) the ability to recognize the relative magnitudes of numbers including the ability to compare and order numbers, (3) the ability to deal with the absolute magnitude of numbers, (4) the ability to use benchmarks (e.g. using 1 as a benchmark, the sum of $\frac{7}{8} \text{ and } \frac{9}{10}$ should be a little under 2 since each fraction is a little under 1), (5) the ability to link numeration, operation, and relation symbols in meaningful ways, (6) understanding the effects of operations on numbers, including recognizing how to compensate if necessary when one or more operands are changed in computation (e.g. if 348-289 = 59, then what is 358-289?) and recognizing when a result of computation remains the same after changing the original numbers operated on (e.g. 123-59 is the same as 124-60), (7) the ability to perform mental computation through "invented" strategies that take advantage of numerical and operational properties, (8) being able to use numbers flexibly to estimate numerical answers to computations, and to recognize when an estimate is appropriate, and (9) a disposition toward making sense of numbers (pp. 5-6). Sowder (1992) explains that number sense can expand to reflect new experiences. In addition, it is dependent on the number system within which one is working. For example, it is possible for a person to have good number sense for whole numbers, but not for fractions.

Why the development of number sense is important. For the past twenty-five years, national and international reports and curriculum documents have highlighted the importance of number sense in school mathematics (Australian Education Council, 1991; National Council of Teachers of Mathematics, 1989, 2000; National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010; National Mathematics Advisory Panel, 2008; National Research Council, 1989, 2001). In *Everybody Counts*, the National Research Council (1989) considered number sense to be the major objective of elementary school mathematics. In *Principles and Standards for School Mathematics*, the National Council of Teachers of

Mathematics (2000) identified the development of number sense as central to the numbers and operations strand which forms the core of mathematics education for the elementary grades. Studies, however, show that many students demonstrate little understanding of number sense (Z. Markovits & Sowder, 1994; National Mathematics Advisory Panel, 2008; Parrish, 2010; Richland, Stigler, & Holyoak, 2012; Verschaffel et al., 2007).

Researchers attribute the lack of number sense to instruction in which rules and procedures are memorized without connections made to the numerical relationships that provide the foundations for these rules (e.g., Carpenter et al., 2003; Stigler & Hiebert, 1999). A lack of understanding of what numbers and operations means can become a barrier to learning mathematics (National Council of Teachers of Mathematics, 2000). To students who are not taught in a way that develops number sense, mathematics is a set of isolated, disconnected facts which must be memorized and practiced. It has little meaning, relevance and application (National Council of Teachers of Mathematics, 2000). Students who have learned mathematics with understanding have developed connections between mathematical ideas, procedures and facts and these connections help them to retain what they have learned and ransfer their knowledge to new situations (Hiebert & Carpenter, 1992). Learning mathematics with understanding also helps students develop more flexibility in their learning and use of mathematics (National Council of Teachers of Mathematics, 2000).

A study by Gray and Tall (1994) supports the importance of developing flexibility, an important component of number sense. Gray and Tall (1994) studied seventy-two students between the ages of seven and twelve. They asked teachers to nominate students they thought of as above average, average and below average. Then they gave the students various addition and subtraction problems to solve and they categorized their strategies. In addition, for example, there were four common strategies:

1) Counting all (13 + 5 is simply counted: 1, 2, 3, …)

2) Counting on (13 + 5 can be counted by starting at 13 and counting on 5 more…14, 15, … or by starting at 5 and counting on 13 more)

3) Using known facts

4) Using derived facts by decomposing and recomposing numbers to make easier number combinations (e.g. $8 + 6 = 8 + 2 + 4 = 10 + 4$).

The low-performing students often used tedious strategies such as counting all for addition and counting back for subtraction. In contrast, the high-performing students consistently used derived facts from the early grades. Their flexibility in thinking about numbers led them to simply knowing facts or being able to calculate them so quickly that they appeared to be known facts.

Gray and Tall (1994) explain that the low-achieving students had developed only procedural thinking in which numbers are seen as entities to be manipulated through a counting process. The high-achieving students exhibited what they call "proceptual thinking" which is characterized by "the ability to compress stages in symbol manipulation to the point where symbols are viewed as objects which can be decomposed and recomposed in flexible ways" (p. 132). The low-achieving students had only the procedure of counting, which became more tedious as problems grew more complex. The students who had developed flexible strategies were able to develop new known facts from old, creating a "built-in feedback loop that acts as an autonomous knowledge generator" (p.132). Gray and Tall (1994) found that the higher achieving students engaged in what is called compression, which Thurston (1990) described as follows:

> Mathematics is amazingly compressible: you may struggle a long time, step by step, to work through some process or idea from several approaches. But once you really understand it and have a mental perspective to see it as a whole, there is often a tremendous mental compression. You can file it away, recall it quickly and completely when you need it, and use it as just one step in some other mental process. The insight that goes with this compression is one of the real joys of mathematics (p.847).

Unfortunately, for low-achieving students, the learning of mathematics does not involve flexibility or compression, but a never-ending ladder of rules. Children across a wide spectrum of performance face the challenge of climbing this hierarchical ladder that becomes more and more difficult to climb as mathematics increases in complexity.

Number sense is critical for the learning of algebra, which is often referred to as the “gatekeeper” to higher mathematics. As Parrish (2010) describes, for many students “their foundation based on memorization crumbles when they are called to generalize arithmetic relationships in algebra courses” (p. 4). Boaler (2013) explains that the high rate of failure in algebra in the U.S., which is often viewed as the “algebra crisis,” is actually a symptom of students’ lack of foundational mathematics, particularly number sense.

The National Mathematics Advisory Panel (2008) identified the essential concepts and skills that should be learned in preparation for algebra which they called the “Critical Foundations of Algebra.” They concluded that by the end of Grade 5, students should have a “robust sense of number” which includes an understanding of place value and the ability to compose and decompose numbers; a grasp of the meaning of the four basic operations of addition, subtraction, multiplication and division; the use of the commutative, associative, and distributive properties; computational facility; and knowledge of how to apply the operations to problem solving (National Mathematics Advisory Panel, 2008, p. 17).

Similarly, Carpenter, Franke, and Levi (2003) identify the understanding of fundamental properties of number and number operations, which are key components of number sense, as critical foundations for algebra. They explain that the fundamental properties that are used in carrying out arithmetic calculations provide the basis for most of the symbolic manipulation in algebra:

> The procedures we use to add, subtract, multiply, divide, and compare numbers are based on a small number of fundamental properties of number and number operations, and much of algebra is based on the same basic properties. When students clearly understand these properties and how they apply to the mathematics they learn, they have acquired the basis for understanding arithmetic and algebra. (p.4)

For example, the distributive property serves as a common basis for multiplying multidigit numbers in arithmetic calculations (e.g. 78 x 5 = (70 + 8) x 5 = 70 x 5 + 8 x 5 = 390). It is also the basis for addition in algebraic calculations (e.g. $5y + 3y = (5+3)y = 8y$). When students understand this connection, they are less likely to make common errors (e.g. $5(y+8) = 5y + 8$) (Jacobs, Franke, Carpenter, Levi, & Battey, 2007, p. 261).

Carpenter et al. (2003) explain that as children learn arithmetic, they implicitly use these fundamental properties, but these properties must be made explicit so that:

- All students have access to basic mathematical properties;
- Students understand why the computational procedures they use work the way they do;
- Students apply their procedures flexibly in a variety of contexts; and

- Students recognize the connections between arithmetic and algebra and can use their understanding of arithmetic as a foundation for learning algebra with understanding. (p.5)

Unfortunately, in many U.S. classrooms, arithmetic is taught as a series of arbitrary rules and steps to be memorized. As a result, students complete elementary school unaware of the fundamental properties of number operations and fail to recognize that arithmetic and algebra are based on the same fundamental ideas (Jacobs et al., 2007, p. 264).

The long-term effects of the failure to develop students' number sense can be seen in the high percentage of students entering the nation's community colleges unable to perform basic arithmetic, pre-algebra, and algebra (Givvin, Stigler, & Thompson, 2011; Richland et al., 2012; Stigler, Givvin, & Thompson, 2010). Stigler et al. (2010) and Givvin et al. (2011) found in their studies of community college developmental mathematics students that students were lacking the fundamental concepts that would be required to reason about mathematics and often viewed mathematics as a collection of rules and procedures to be remembered. They hypothesized that students entered school with basic intuitive ideas about mathematics, but through school experiences with teaching that focused on procedures disconnected from concepts, intuitive concepts that supported their thinking and reasoning when they were younger atrophied. For these students, mathematics and in particular algebra, serves as a gatekeeper that prevents them from pursuing particular professions or from completing their degrees. But as Carpenter et al. (2003) suggest, through teaching that develops mathematical thinking, students can be put on a path to "learning mathematics with understanding so that algebra is a gateway to opportunity, not a gate that blocks their way" (p.6). What then are the key features of teaching that develop mathematical understanding? In the following section, I explore research to answer this question.

Teaching That Develops Mathematical Understanding (Number Sense)

Hiebert and Grouws (2007) define teaching as “classroom interactions among teachers and students around content directed toward facilitating students’ achievement of learning goals” (p.372). It is widely agreed upon that teaching has a significant effect on student learning, but because of the complexity of teaching, detecting which instructional methods are most effective for producing student learning and identifying useful theories of classroom teaching are a challenge. Different teaching methods might be effective for different goals and the system nature of teaching makes the effects of individual features difficult to isolate. In addition, there may be factors–both inside and outside the classroom–that mediate the effects of teaching. Because too many factors affect the results, no single study can prove that one method or feature of teaching is better than another, but by examining patterns across studies, researchers can begin to identify features of teaching that seem to produce similar effects related to particular learning goals (Hiebert & Grouws, 2007).

In this section, I present studies that examine patterns of empirical connections between teaching and student learning. The first is Hiebert and Grouws’ (2007) synthesis of studies in which features of teaching were empirically related to student learning outcomes. The second is the TIMSS 1999 Video Study, which examined nationally representative samples of teaching in seven different countries. Both studies revealed that there were two key features of instruction that were consistently linked to developing conceptual understanding of mathematics. I also examine research on classroom discourse, an area in which there are emerging patterns (Hiebert & Grouws, 2007) relating to its effects on learning.

Making explicit connections & engaging students in productive struggle. Hiebert and Grouws (2007) conducted a literature review of all studies that examined empirically-based connections between teaching and student learning and identified two key features of instruction that were likely to promote students' conceptual understanding of mathematics: attending to mathematical connections in an explicit way and engaging students in struggling with important mathematical ideas. According to Hiebert and Grouws (2007), the first feature, attending to mathematical connections in an explicit way, could include:

> discussing the mathematical meaning of underlying procedures, asking questions about how different solution strategies are similar to and different from each other, considering the ways in which mathematical problems build on each other or are specific (or general) cases of each other, attending to the relationships among mathematical ideas, and reminding students about the main point of the lesson and how this point fits within the current sequence of lessons and ideas. (p.383)

The second feature, *struggling* or wrestling with important mathematical ideas, is defined as follows:

> We define *struggle* to mean that students expend effort to make sense of mathematics to figure something out that is not immediately apparent. We do *not* use *struggle* to mean needless frustration or extreme levels of challenge created by nonsensical or overly difficult problems. We do not mean the feelings of despair that some students can experience when little of the material makes sense. The struggle we have in mind comes from solving problems that are within reach and grappling with key mathematical ideas that are comprehendible but not yet well formed. (p. 387)

Engaging students in productive struggle means that students are not simply presented with information to be memorized or asked only to practice what has been presented, but must expend effort to make connections between facts, ideas and procedures in order to make sense of problems. Hiebert and Grouws (2007) explain that in order to engage students in struggle that is productive, it is important to consider students' entry knowledge. Tasks that are selected need to be "within reach but present enough challenge so there is something new to figure out" (p.388).

The finding that attending explicitly to mathematical connections helps students to develop conceptual understanding was supported across a wide range of research designs and instructional treatments. The effects appeared in studies of varying designs from tightly controlled short-term studies (Brownell & Moser, 1949; Hiebert & Wearne, 1993) to long-term studies lasting two to three years (Boaler, 1998; Fawcett, 1938); in studies conducted in the primary grades (Brownell & Moser, 1949; Fuson & Briars, 1990; Hiebert & Wearne, 1993) to the middle grades (Good & Grouws, 1977; Good, Grouws, & Ebmeier, 1983) to high school (Boaler, 1998; Fawcett, 1938); and in classrooms where the teachers played an active role in demonstrating mathematical relationships (Brownell & Moser, 1949; Fuson & Briars, 1990; Good et al., 1983) to classrooms where the teachers took less active roles (Fawcett, 1938; Fuson & Briars, 1990; Hiebert & Wearne, 1993). The methods of teaching used to make mathematical relationships explicit also varied from study to study, from those that focused on connections between concrete and symbolic representations (Fuson & Briars, 1990; Hiebert & Wearne, 1993) to those that highlighted teacher explanations of why mathematical procedures worked as they did (Brownell & Moser, 1949) to those that highlighted student invention of solution strategies (Fawcett, 1938; Hiebert & Wearne, 1993).

Hiebert and Grouws (2007) explain that although there has been little, if any, research to isolate the effects of struggle on the development of students' conceptual understanding, the finding that engaging students in productive struggle can help to develop conceptual understanding can be inferred from studies in which students were presented with challenging problems and asked to work out new solution methods on their own (e.g. Carpenter, Fennema, Peterson, Chiang, & Loef, 1989; Hiebert & Wearne, 1993; Inagaki, Hatano, & Morita, 1998; Lampert, 2001; Silver & Stein, 1996; Stein & Lane, 1996).

It is important to note that Hiebert and Grouws (2007) do not justify a single or "best" method of instruction to facilitate conceptual understanding. As they explain:

> …conceptual development of mathematics can take many pedagogical forms. Concepts can be developed through teacher-centered and highly structured formats as in some of Fuson and Briar's (1990) classrooms and in Good et al.'s (1983) classrooms or through student-centered and less structured formats as in Fawcett's (1938) classroom. (p.387)

Hiebert and Grouws (2007) also point out that the findings from several studies suggest that teaching features that promote conceptual understanding also promote skill fluency (e.g. Fawcett, 1938; Fuson & Briars, 1990; Good et al., 1983; Hiebert & Wearne, 1993).

Hiebert and Grouws' (2007) findings are corroborated by the TIMSS 1999 Video Study (Hiebert et al., 2003), which was the largest cross-national video study of classroom teaching ever conducted. It examined teaching in seven different countries: Australia, the Czech Republic, Hong Kong, Japan, the Netherlands, Switzerland and the United States. The study collected a national representative probability sample of approximately 100 lessons from each country, which were then coded by an international team of researchers to gather data about average teaching practices in each country.

One of the purposes of the study was to compare eighth-grade mathematics teaching in different countries to try to identify features that might differentiate the high-achieving countries (all countries included in the study except for the United States and Australia) from the low-achieving countries. What the researchers found was that teaching in each of the high-achieving countries looked quite different from each other. Many superficial features of teaching, which reformers often focus on, such as whether teachers lectured or whether students worked in groups, or whether real-world situations were or were not used, did not explain the cross-national differences in achievement. For example, among high-achieving countries there were countries that emphasized teacher lecture as the primary mode of instruction, and countries that tended to have students working independently or in small groups on assignments.

A closer look at the high-performing countries revealed that beneath the superficial differences in teaching there were deeper similarities. Although teachers in high-achieving countries used a variety of instructional strategies and different types of problems in their lessons, the commonality among the high-performing countries was that teachers made explicit connections among mathematical ideas, facts, and procedures and engaged students in active struggle with core mathematical concepts and procedures. These two key features of teaching which are associated with the development of conceptual understanding, were largely absent in lessons in the United States. For example, while 37% to 52% of problems in the lessons in high-performing countries were worked on so that connections were made, in the United States, so few problems were worked on in this way that the percentage rounded to 0% (Hiebert et al., 2003). This finding helps to explain why so many U.S. students do not develop conceptual understanding of mathematics and view math as simply a series of disconnected facts and procedures to be memorized.

Mathematical discourse. Hiebert and Grouws' (2007) research synthesis on teaching and learning and the TIMSS Video Study (2003) showed that engaging students in "productive struggle" was one of the key features of teaching associated with developing mathematical understanding. One way that students can engage in productive struggle is through mathematical discourse in which students explain their reasoning about mathematical concepts and processes and engage in other students' reasoning processes (Webb et al., 2014). In the Common Core State Standards for Mathematics (CCSS) which were recently adopted by most states, the Standards for Mathematical Practice call for students to be able to "justify their conclusions, communicate them to others, and respond to the arguments of others" and be able to "communicate precisely to others" by using "clear definitions in discussion with others and in their own reasoning" (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010, p. 7).

As Webb et al. (2014) explain, there are a number of reasons that students can benefit from communicating their own reasoning to others and engaging in other students' reasoning. First, when communicating their ideas, students must think about how they can explain their reasoning in a way that is coherent and complete so that others can understand it. In the process of formulating their ideas or explaining their ideas to others, students may recognize their own misconceptions or incompleteness of their ideas. Second, engaging in other students' ideas can expose students to new ideas and different ways of thinking, and encourages students to re-examine their own ideas which may solidify or clarify their thinking.

Despite the importance of engaging students in mathematical discourse in order to develop mathematical understanding, evidence from the TIMSS 1999 Video Study (Hiebert et al., 2003) indicates that in many classrooms in the United States, the type of classroom discourse that would help students to make connections among mathematical ideas and engage in productive struggle is not taking place. Although teachers in the U.S. often have their students engage in group and whole-class discussions as advocated in reform-documents such as NCTM's (1991) *Professional Standards for Teaching Mathematics*, research shows that the types of discussions occurring in classrooms often do not help to move the class forward mathematically (e.g., Stein, Engle, Smith, & Hughes, 2008; Stigler & Hiebert, 1999). Many teachers mistakenly believe that in order for classroom discussion to focus on student thinking, teachers must avoid providing any guidance at all (Stein et al., 2008). Stein et al. (2008) describe many classroom discussions as "show and tells" in which students are first given a "cognitively demanding task" to work on in groups then during the "discuss and summarize" phase present their solution to the class. There is limited teacher or student commentary and no assistance in drawing connections among methods or ideas. Students are often told to simply "pick the way they liked best" (Stein et al., 2008, p. 319). As Stein et al. (2008) explain, this type of "show and tell" discussion does not help to develop mathematical thinking.

Research shows that supporting productive classroom discourse requires teachers to provide appropriate tasks and to structure discussions to focus on important mathematical ideas (Franke, Kazemi, & Battey, 2007). In recent years, recognizing the importance of the teacher's role in guiding classroom discussions and the need to provide clear guidance to teachers on how to facilitate productive discussions, researchers have begun to detail the ways in which teachers can support student communication. Chapin, O'Connor, & Anderson (2009) created a set of "talk moves" that teachers can use to support students in engaging with mathematics and with each other. Teachers can use specific talk moves to help students clarify and share their own thoughts, orient to others' thinking, deepen their reasoning, and engage with others' thinking. Stein et al.

(2008) identified five practices for facilitating mathematical discussions around cognitively demanding tasks. These practices include: (1) anticipating likely student responses, (2) monitoring students' responses, (3) selecting particular students to present their mathematical responses, (4) purposefully sequencing the student responses that will be displayed, and (5) helping the class make mathematical connections between different students' responses and between students' responses and the key ideas (p.321). Stein et al. (2008) explain that these practices give teachers a roadmap of things that they can do in advance and during whole-class discussions to orchestrate discussions that both build on student thinking and also advance important mathematical ideas.

To support classroom discourse, teachers also need to establish norms for creating mathematics classroom communities. Norms proposed in the National Research Council's (2001) report *Adding It Up* include value placed on ideas and methods, student autonomy in sharing and choosing problem solving methods, appreciation of the value of mistakes for learning for everyone, and renegotiation of authority for whether something is correct or sensible so that authority lies in the logic and structure of the subject and not the status of the teacher or the popularity of the person making the argument.

Research also shows the importance of the role of tasks in promoting productive classroom discussions. Starting with a good task for students is critical in providing opportunities for students to engage in sharing their thinking, comparing different approaches, making conjectures and generalizing (Silver & Stein, 1996). Silver and Stein (1996) describe meaningful and worthwhile tasks as those that are connected to core mathematical ideas and have multiple solution strategies, involve multiple representations, and require students to explain and justify their procedures and understandings in oral and/or written form. There are a variety of tasks that can help students to engage in productive discourse and make connections in order to develop mathematical understanding. In the next sections, I examine tasks that can help to engage students in this type of discourse and mathematical thinking.

Mental computation. Various reports have highlighted the importance of mental computation as a vehicle for promoting the development of number sense (National Council of Teachers of Mathematics, 1989; National Research Council, 2001; Sowder, 1990). In addition to its practical uses in daily life, mental computation, which is the process of carrying out arithmetical operations without using external aids to arrive at an exact answer, is increasingly thought to be a vehicle for promoting thinking, conjecturing and generalizing based on conceptual understanding (B. J. Reys & Barger, 1994; Thompson, 1999). Reys (1985) cites the following as the benefits of mental computation:

> 1) Mental computation promotes an understanding of the base ten number system as well as of basic number properties.
>
> 2) Mental computation rewards flexibility in dealing with various forms of numbers.
>
> 3) Mental computation nurtures the development of a keen number sense.
>
> 4) Mental arithmetic utilizes visual thinking skills. (p.45)

The National Research Council (2001) in its report *Adding It Up* explains that "mental arithmetic places a premium on flexible procedures that take advantage of mathematical structure and rely on well-known operations". They add that:

> Beyond its many practical uses in the modern world, mental arithmetic can promote mathematical proficiency by bringing together the various strands [of proficiency]. Mental arithmetic should be taught to encourage children to reason about the problem situation and the numbers involved, to take advantage of their conceptual understanding

of the properties and rules of arithmetic, and to strategically select and adapt procedures to simplify a computation and calculate the answer. (National Research Council, 2001, p. 216)

Various studies show that mental computation encourages students to think about number relationships and to use these relationships to develop efficient and flexible strategies instead of relying on memorized rules (Blote, Klein, & Beishuizen, 2000; Hope & Sherrill, 1987; Maclellan, 2001; Markovits & Sowder, 1988, 1994; B. J. Reys, 1985; Sowder, 1992b).

Phil Daro (2010), one of the core authors of the Common Core State Standards for Mathematics (CCSS), points out that the most important ideas in the CCSS for mathematics that need attention are "properties of operations: their role in arithmetic and algebra," "mental math and [algebra vs. algorithms]," and the "mathematical practices." The CCSS contain explicit goals for mental computation as students learn about the four basic operations in elementary school and as they learn about expressions in Grade 7. In Grade 2, for example, under Operations and Algebraic Thinking, students are expected to be able to "fluently add and subtract within 20 using mental strategies" such as making ten (e.g., $8 + 6 = 8 + 2 + 4 = 10 + 4 = 14$); decomposing a number leading to a ten (e.g., $13 – 4 = 13 – 3 – 1 = 10 – 1 = 9$); using the relationship between addition and subtraction (e.g., knowing that $8 + 4 = 12$, one knows $12 – 8 = 4$); and creating equivalent but easier or known sums (e.g., adding $6 + 7$ by creating the known equivalent $6 + 6 + 1 = 12 + 1 = 13$). Also, the Grade 4 standards state: "depending on the numbers and the context, [students] select and accurately apply appropriate methods to estimate or mentally calculate products…and quotients". As students learn to use the four operations with whole numbers to solve problems in elementary school and solve mathematical problems using numerical and algebraic expressions and equations in middle school, they are expected to "assess the reasonableness of answers using mental computation and estimation strategies including rounding" (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010, pp. 23, 29, and 49).

Threlfall (2002) writes that when mental computation is approached through teaching that focuses on number knowledge and understanding, students can develop flexibility in their thinking, which Gray and Tall (1994) found in their research was a key characteristic of high-achieving mathematics students. Thompson (1999) identifies four "attributes" that assist in the development of flexible mental computation: 1) Good knowledge of number facts (appropriate to age), 2) Clear understanding of what can be done with numbers, such as when the order can be changed, how numbers can be decomposed, the behavior of zeros, 3) Well developed skills (appropriate to age) including automaticity with mental calculations at the level below, and 4) Positive disposition towards mathematics (p.171). Beishuizen (2001) argues for "verbalizing and discussing alternative mental calculations" and "recording the procedural steps of number operations" so that children are able to follow the process of transforming numerical expressions to carry out mental computation (as cited in Threlfall, 2002, p.44).

Number Talks. One pedagogical approach that is being used in some U.S. classrooms to enrich students' number sense is a method called "number talks." Number talks are ten- to fifteen-minute classroom conversations around purposefully chosen mental computation or number of the day problems that are designed to engage students together in exploring and sharing their ideas of number relations. Number talks engage students in productive struggle and in making connections among important mathematical ideas which the TIMSS 1999 Video Study showed are the two key features of instruction that help to develop mathematical understanding and were missing from the teaching in most U.S. classrooms (Hiebert et al., 2003). Number talks are also closely aligned to both the content and process standards in the Common Core State Standards for Mathematics.

Key components of number talks include the role of mental computation in developing fluency and flexibility; classroom discussions in which students communicate their own ideas and engage in other students' ideas; and purposeful computation problems that guide students to focus on mathematical relationships to build mathematical understanding and knowledge (Parrish, 2010). Solving the problems through mental computation encourages students to think about number relationships and to use these relationships to develop efficient and flexible strategies instead of relying on memorized rules. In order to carry out computations mentally, students need to understand place value, be able to compose and decompose numbers to create equivalent problems that are easier to solve, apply the commutative, associative, and distributive properties, and understand the meaning and effects of operations as well as the relationships between operations. As students share and discuss their strategies they develop mathematical understanding. By explaining their thinking and engaging in other students' ideas, students have the opportunity to clarify their thinking, develop understanding of mathematical relationships, build a repertoire of efficient strategies, and consider and test strategies to see if they are logical (Parrish, 2011). Providing a set of purposeful problems, as opposed to a mixture of random problems such as 28 x 5, 45-17, 128 + 248, is also a key factor in guiding students to focus on mathematical relationships.

Number talks have been increasingly used in classrooms across the United States as schools and districts implement the Common Core. They are a promising approach that teachers can use to enrich students' understanding of numbers and operations and support the implementation of the Common Core State Standards for Mathematics. Number talks are also practical because they are only ten- to fifteen-minutes long and can be easily incorporated into the regular school day.

Conclusion

The research reviewed suggests that the development of number sense, or understanding of numbers and operations and the ability to use this understanding flexibly, is an important and central goal of elementary school mathematics. However, studies show that many U.S. students do not develop number sense and this becomes a barrier to students' success in higher level mathematics. Empirical research on teaching and learning indicates that making connections among mathematical ideas and engaging students in productive struggle are two key features of teaching that develop mathematical understanding, but studies show that these features are absent in most U.S. classrooms. Research also shows that engaging students in mathematical discourse and mental computation are two ways of developing students' mathematical thinking. Based on this research, I propose the use of number talks in middle schools to help students develop conceptual understanding numbers and operations. The next chapter describes the research design for studying the implementation process and impact of a six-week number talk intervention on middle schools students' development of number sense.

CHAPTER 3: METHODOLOGY

The purpose of this study was to examine the implementation process and impact of a six-week number talk intervention with sixth-grade middle school students in a large urban school district. The study explored the following research questions:

1. In what ways can number talks be implemented with 6th grade middle school general mathematics students to support the development of number sense?
2. What are the effects of a 6-week number talk intervention on sixth-grade middle school general mathematics students' number sense?

Research Design

A mixed-methods design which incorporated both quantitative and qualitative approaches was used to examine the research questions. I chose to use a mixed-methods approach because it would provide a more complete understanding of the research problem than either approach alone (Creswell, 2013). The quantitative portion of the study utilized a one-group pretest and posttest design to measure the effects of the intervention on the development of students' number sense. The qualitative portion of the study, which included classroom videos, student work, and a teacher journal, examined the implementation process of the number talks to determine in what ways number talks can be implemented to support the development of students' number sense.

Methods

Site and population. The study was conducted at Eastside Middle School, where I currently teach Grade 6 Accelerated Mathematics. The site was chosen for several reasons: 1) the school was typical of many middle schools in urban districts with a racially diverse student population and a high percentage of low SES students, 2) the school had been struggling to increase the proficiency of students in its general mathematics program which had been around ten percent based on the California State Test, and 3) the administration and staff were interested in exploring the use of instructional interventions during the school's advisory class period to provide targeted support to students in the general mathematics program.

Eastside has an enrollment of about 850 students and serves students in grades six through eight. It is a Title I designated school with about 60% of students being eligible for free or reduced lunch. The racial makeup of the school is about 45% Hispanic, 25% African-American, 12% Asian, and 11% White. Sixth-grade students are placed into one of two math courses based on fifth-grade assessment scores: Math 6 ("general mathematics") or Math 6 Accelerated. For both classes, the intended curriculum includes the sixth-grade Common Core State Standards for Mathematics; however, the curriculum for the Math 6 Accelerated class also includes about half of the seventh-grade standards which are selected by the district's math office. The school has a twenty-minute advisory class period at the beginning of the school day which is used for school announcements, completing homework, silent reading, and seventh- and eighth-grade club activities. The site was interested in exploring different ways of using the advisory class period to provide academic support to students. In one sixth-grade general advisory class, number talks were piloted.

The population for the study included approximately 140 sixth-grade students in the general mathematics classes at Eastside Middle School. Assessment data showed that the majority of students in these classes enter middle school not having developed number sense and consequently, have difficulty learning the sixth-grade content. In 2013, only eleven percent of students in the general mathematics classes earned a passing score on the sixth-grade state exam. This is a significant concern because studies show that students' performance in mathematics by late elementary/early middle school is a strong predictor of their ultimate educational success

(Finkelstein, Fong, & Tiffany-Morales, 2012; National Research Council, 2001; Siegler et al., 2012).

The treatment.

Participants. The sample included 22 students in one sixth-grade general advisory class. The students' advisory teacher was interested in exploring ways to use the advisory period more productively to provide academic support to students in the general program. As part of a pilot, for six weeks, I taught number talks to the participating advisory teacher's students. I chose to teach the number talks myself in order to gain an in-depth understanding of the implementation process that can only occur through teaching the number talks personally. Although 34 students participated in the number talks as part of the regular class activities, there were eleven students who did not turn in consent forms and one student who was absent on the second day of the post-test and did not complete the assessment. Therefore, written work was analyzed for only 23 of the 34 students and pre-and post-assessment data came from 22 of the 34 students who both consented to participating in the study and completed the pre-and post-assessments.

Curriculum for number talks. The content of the intervention focused on multiplication of whole numbers. The book *Number talks: Helping children build mental math and computation strategies* by Parrish (2010) and San Diego City School's (2004) "Mathematics routine bank" were used as resources in developing the curriculum. I also received guidance from Professor Megan Franke of UCLA.

Implementation of number talks. Students participated in fifteen-minute number talks during advisory class, two to three times per week for six weeks. The number talks focused on developing students' understanding of multiplication to support the development of number sense. Each number talk included a series of purposefully selected mental computation problems or a number of the day problem in which students were asked to make as many equivalent expressions as possible. In a number talk with mental computation problems (which I will refer to as "mental math number talks" hereafter), I presented a problem on the board, for example, 5 x 18, and asked the students to work it out mentally, in their heads, without paper and pencil. When students found an answer, they signaled privately with a thumb up in front of them. After most students indicated that they had an answer, I asked students to share their strategy with a neighbor then with the class. I facilitated the discussion, recorded students' strategies on the board, and provided visual representations as needed to illustrate concepts and to help students make connections among mathematical ideas. In some number talks, students were asked to write out their strategies using mathematical notation and visual representations. Student participation during the number talks and written student work served as formative assessments that enabled me to monitor student learning and make adjustments to instruction as needed.

Data collection methods.

Written pre-/post-number sense assessment. A pre-/post-written assessment was created to measure the effects of the intervention on students' development of number sense. The assessment had seventeen items overall and was divided into three subsections: 1) algorithms, 2) equivalent expressions, and 3) mental computation. The algorithm section included three items intended to measure students' abilities to solve problems using the subtraction, multiplication, and division algorithms. The purpose of these items was to gather additional information about students' competencies with typical "school-type" mathematics. The equivalent expressions section assessed students' abilities to apply their understanding of place value, the distributive property, and doubling and halving to write equivalent expressions. The mental computation section was intended to measure students' abilities to apply concepts about numbers and operations to solve multiplication problems using mental computation.

Classroom video recordings. All of the number talk sessions were video recorded using an iPod Touch which was placed on a small tripod in the back corner of the classroom. The video recordings provided a detailed record of the interventions that helped me to recall what I said and did as a teacher, what students said and did, what was successful, and what was challenging.

Teacher journal. I kept a reflective journal to document the problems that I used, what went well, what the challenges were, possible changes and ideas for future number talks, and what I learned. The journal provided a record of my perspectives on the implementation process as it was occurring. For most of the number talks, I wrote the journal entries as I watched the videos. Starting in the middle of the intervention, I also began doing audio journals which I later listened to as I wrote the journal entries.

Student work. Written student work from the number talks was collected and used as formative assessment.

Data analysis methods.

Statistical analysis. Quantitative student outcome data from the pre-and post-number sense assessments were analyzed with descriptive statistics and statistical tests. Paired *t* tests were performed to determine whether there were statistically significant changes in the overall mean and the means for each subtest.

Content analysis. My qualitative findings are based on a combination of my analysis as I implemented the intervention and my analysis as I reflected back on what happened across all of the number talks. As I implemented the intervention, I analyze what worked and what was challenging and continuously made adjustments to my number talks. After the intervention ended, I reflected back through all of the number talks by examining the reflective journals, reviewing video recordings as needed, and examining students' written work. I also reexamined some of the number talk resources including Parrish's (2010) book to gain additional insights about the implementation process.

Ethical Considerations

Ethical protocols were followed through both data collection and data analysis. Before beginning data collection, I submitted an application to do research through UCLA's IRB process and Central Unified School District's IRB process. Once I received approval to do the study, I followed protocols for obtaining informed parent consent and student assent. Ethical procedures were followed during the data analysis phase. All records related to the study were destroyed at the end of the study to protect participants' privacy. Video recordings were seen only by me and were destroyed at the end of the study. The data used for the study did not have any impact on students' grades.

Validity and Reliability

To ensure internal validity, or the extent to which the research findings match reality, data was collected through multiple sources to allow for triangulation (Creswell, 2013). To ensure external validity I provide a detailed description of the intervention and the findings of the study to enable someone reading the study to assess the transferability of this study to their context (Merriam, 2009). I used triangulation or multiple methods of data collection to strengthen reliability as well as internal validity.

For the quantitative portion of the study, possible threats to internal validity included history and selection. History is an extraneous variable in the form of events that influence the outcome beyond the treatment. The major potential threat of history in this study was from instruction during the regular mathematics class, but this threat was minimal because the content that was taught in the mathematics class did not overlap with the content assessed on the pre-/post-assessments. The threat of selection was minimal because students were not assigned to one advisory class based on ability, behavior, gender or any other predetermined criterion and it can be assumed that the each sixth-grade general advisory class was representative of students in the sixth-grade general program at Eastside Middle School.

To control for the threat of instrumentation, I used the same instrument for both the pre-test and post-test. Using the same instrument, however, increased the threat of testing, or improvements from the pre-test to the post-test that are due to students remembering responses for later testing. To control for this threat, the results of the pre-test were not shown to students and there was a period of six weeks between the pre-test and post-test.

Summary

A mixed-methods design which incorporated both quantitative and qualitative approaches was used to examine the implementation process and impact of a six-week number talk intervention with sixth-grade middle school students. The quantitative portion of the study utilized a one-group pretest and posttest design to measure the effects of the intervention on the development of students' number sense. The qualitative portion of the study, which included classroom videos, student work, and a teacher journal, examined process data on the implementation of number talks to determine in what ways number talks can be implemented with middle school students to support the development of number sense. In the next section, findings from the study are reported.

CHAPTER 4: FINDINGS

This chapter examines the implementation and impact of a six-week number talk intervention with sixth-grade students in a middle school in a large urban school district. The chapter is organized into four parts. The first section gives a description of the six-week intervention. The second section describes what I found I needed to do in implementing number talks productively with middle school students. The third section describes the challenges that persisted. The final section presents data on the impact of the intervention on students' number sense as measured by the pre- and post-number sense assessments.

Description of Intervention

The goal of this section is to describe the intervention. I begin with an overview and then I provide more detailed information about what the work looked like on a day-to-day basis. The data for this section comes from my written and audio reflections, videorecordings of the number talks, and written student work. My findings are based on a combination of my analysis as I implemented the intervention and my analysis as I reflected back on what happened across all of the number talks.

Overview. I implemented the intervention from November to December 2014 with a sixth-grade general advisory class of thirty-four students. I set out to do six weeks of number talks, three times a week with the students, but during several of the weeks, I ended up doing only two number talks a week because of time needed for pre- and post-testing and a school assembly that occurred during the advisory period. In total, I did fifteen number talks each on average fifteen to eighteen minutes in length.

The goal of the intervention was to help students develop number sense in the domain of whole number multiplication. I chose this focus because proficiency with multiplication is critical for learning the sixth-grade Common Core Mathematics Standards and assessments had shown that many students entered sixth-grade lacking proficiency in this domain. To prepare to teach the intervention, I studied the Common Core learning progressions for Operations and Algebraic Thinking (K-5), and Number and Operations in Base Ten (K-5), and an elementary school textbook series to get a picture of how concepts and skills are developed in the domain of whole number multiplication. I also researched various sources to gather sample problem sets for multiplication number talks.

There were two types of number talk structures that I planned to try, number of the day problems in which students are asked to write a number in as many ways as possible and mental math number talks which are a set of related problems that students work out using mental computation. I planned to use number of the day problems to help students learn how to flexibly decompose and compose numbers and to deepen their understanding of place value and the base-ten structure of the number system. The mental math number talks would be designed to provide students with opportunities to decompose numbers and develop and apply their understanding of place value, operations, and the commutative, distributive, and associative properties as they solved multiplication problems. I also hoped to develop students' abilities to use visual representations and record their strategies using algebraic notation.

My number talks typically followed a routine. In a number of the day number talk, I would write the number on the board, and then give students several minutes to write the number in as many ways as possible. If I wanted to focus student thinking on a particular concept, I added constraints to the way the number could be written, such as using only addition or only powers of 10. While students were writing, I walked around to monitor student work. After students were given a few minutes to work independently, I had students share their expressions with their neighbor, and then asked for volunteers to share their expressions with the class and gathered ideas from all students who volunteered. I recorded each student's response on the board or under the document camera and then simplified it together with the class to check if it was equal to the given number. For example, if the number of the day was 24 and the expression shared by a student was 12 + 8 + 4, I would lead an oral move through the problem saying "12 + 8 is 20 and 4 more is 24." If the expression that was shared by a student did not equal the given number, I would ask the student to think about a way to revise the expression so that it would equal 24. To guide the discussion, I asked questions to help students make connections between different expressions, to generate an equivalent expression, or to encourage students to derive new numerical expressions by systematically modifying prior ones. For example, if 4 + 4 + 4 + 4 + 4 + 4 was given as a way to write 24, I would ask students if there was a way to quickly check if this was equal to 24 to encourage them to think about the relationship between repeated addition and multiplication. Also, if 10 + 10 + 1 + 1 + 1 + 1 was given as a way to write 24, I would ask students if they could write any of the repeated addends as multiplication to help them to learn about writing numbers in expanded notation (2 x 10 + 4 x 1).

In a mental math number talk, I would write the first problem on the board, such as 2 x 25, ask students to work out the problem in their heads and signal with a thumb up in front of them when they had an answer. When I saw that most students were ready, I had students share their answer with a partner, and then asked for volunteers to share their answer and strategies, which I recorded on the board. I collected different methods that students had used and recorded them. I then repeated the procedure with the other problems. For some problems, if it was possible for students to show me the digits of the answer on their fingers, I asked students to show the answer in front of them before any strategies were shared. This allowed me to get an idea of how many students were able to get the correct answer. To help students to make connections among the various problems and strategies, I asked guiding questions such as if anyone had used a strategy similar to one that had been used for a previous problem, prompted students to think about how they could use the previous problems to help them solve the next problem, and used visual representations to illustrate students' strategies. During the last two weeks of the intervention, I modified the procedures for the number talks, placing greater emphasis on written work to support students' participation in the number talks and to develop their ability to use multiple representations. After a problem was given, I had students write out the answer on their paper first, and then had them draw a diagram and record their strategy before sharing their work with a partner and asking for volunteers to share with the class.

Daily/weekly descriptions. The goal in this section is to provide more detailed information about what the work looked like on a day-to-day and weekly basis.

Week 1. For my first number talk with the students, I did a number of the day problem in which I asked students to write the number 24 in as many ways as possible. My goals for the first number talk were to increase students' abilities to decompose numbers in different ways and to get students to feel comfortable participating. On the pre-test, I saw that there was a wide variation in the scores, so in order to engage all students I would need to find ways to support the students who started out demonstrating less knowledge while challenging the higher performing students. I had been using number of the day problems with my own math classes to teach students about decimal concepts and found that the problems were flexible and could engage students who were at different levels of mathematical understanding and get a lot of students participating.

For the second number talk, I did a mental math number talk with the problems 2 x 25, 4 x 25, 6 x 25, 8 x 25, and 9 x 25. I had used a similar set of problems in my first number talk with my math classes last year and it had gone really well, probably because working with 25's is easy for students since they can think of quarters or skip count. On the pre-test, over two-thirds of the students had correct answers for 2 x 25, 4 x 25, and 8 x 25, but about half of the students had completed the problems by writing out the traditional algorithm instead of using mental math strategies. This seemed like a good set of problems to start out with, would be doable and could help students to build concepts about multiplication.

Overall, my perception as a teacher was that the first number talks went well. Students were really cooperative and seemed engaged (they were on task, participated in pair shares and appeared to be attending to the class discussion), and many different students volunteered to share their expressions and strategies with the class. There was also evidence of students building on each others' ideas. For example, in the first number talk, the expressions that were shared at first all involved addition of two numbers, but after one student shared 4 + 4 + 4 + 4 + 4 + 4, which then led to a discussion about repeated addition and multiplication (6 x 4), the next two students came up with expressions that included multiplication and division or a combination of operations such as (3+3) x 4 and 10 x 2 + 4. There was a little less participation in the second number talk compared to the first number talk based on the number of different students who volunteered to share a strategy and I wondered if this was because some of the problems in the second number talk may have been too difficult for some students. Number of the day problems were nice because they were completely open ended, making it possible for all students to participate, but I was concerned that there was not a clear mathematical objective. I wondered how I could design number talks that would be accessible to all students and help to move students forward towards a specific mathematical content goal.

Week 2. During the second week, I decided to try using number of the day talks to help students develop understanding of the base-ten structure of the number system, since on the pre-test, most students missed the problem that asked them to write 645 in expanded notation (645 = 6 x __ + 4 x ___ + 5 x __). To help students to focus on developing concepts about place value and the base-ten system, I added the constraints of writing the numbers using only 1's, 10's, and 100's and using only addition. The number 24, for example, could be written as 10 + 10 + 1 + 1 + 1 + 1 using the least number of values or it could be written with 24 ones or 1 ten and 14 ones. If I saw students using numbers other than 1's, 10's, or 100's as I walked around, I reminded them about the constraint and asked them to think about how they could decompose the numbers they were using into 1's, 10's, or 100's. To help student develop an understanding of expanded notation, after a number such as 24 was written as 10 + 10 + 1 + 1 + 1 + 1, I would ask students to think about how to generate an equivalent expression by using multiplication for any repeated addition (2 x 10 + 4 x 1). I also tried to develop students' understanding that to compose a higher unit of place value, ten of the next smaller place value is needed (i.e. 10 is composed of ten 1's, and 100 is composed of ten 10's, etc.) and to develop students' ability to unitize or see a number in different units (e.g., 250 can be thought of as 250 ones or 25 tens and 1.4 could be seen as 1 one and 4 tenths or 14 tenths). To guide students toward these understandings, after a number such as 250 was written as 100 + 100 + 10 + 10 + 10 + 10 + 10, I asked students to think about another way to write the number with the same constraints in place (changing the order of the numbers did not count). Because of the restrictions on the numbers that could be used, students would have to exchange at least 1 hundred for 10 tens or 100 ones, or 1 ten for 10 ones.

Students had difficulty figuring out how to do this at first, but I suggested thinking about how they could exchange a $100 bill, for example, for $10 bills. I also explained to students that if a number was repeatedly added many times, instead of writing out all of the numbers they could use an ellipse (e.g. 10 + 10 + 10 + … + 10) and put a brace under the numbers indicating how many of the number were being added together.

Students seemed less engaged in this week's number talks than the first week's number talks. I had tried out similar number talks with my math classes to develop concepts about decimal numbers and the structure of the number system and my students appeared to really enjoy and learn from the number talks, so I was surprised that these number talks did not go equally well. I realized that I would need to figure out a way to gather more information about students' current understanding and knowledge and do some additional research to figure out how to teach the structure of the number system with number talks.

Week 3. For week three, I decided to take my number talks in a different direction and focus on mental math with one-digit by two-digit multiplication problems. My goal for these number talks was to have students explore how to solve multiplication problems in a variety of ways. I selected problems that I thought would be doable for students (2 x 15, 3 x 15, 4 x 15 and 2 x 16, 3 x 16, 4 x 16) and would build upon each other. When we started the week's number talks, many students used the algorithm mentally to find the answer, but I challenged students to think about the 1 in 15, for example, as 1 ten or 10, and by the second number talk, the students who volunteered to share their strategies appeared to be using number sense, not just attempting the algorithm in their heads. A variety of strategies were shared by the students who volunteered to participate in the whole class discussions, including repeated addition, factoring and using the associative property, repeated addition using a known fact (e.g., I know 2 x 16 = 32, so 3 x 16 is

32 + 16 which is 48), and breaking the numbers into place value and using the distributive property (e.g. to find 3 x 16, 3 x 10 = 30 and 3 x 6 = 18 and 30 + 18 = 48). Also, some of the strategies that were shared by students indicated that they had made connections to previous problems in the set (e.g. I know 2 x 16 = 32, so 4 x 16, is 32 + 32) or used problems from the previous day (e.g. I know 2 x 15 = 30, so 2 x 16 would be 30 + 2 or 32). I was pleased with the variety of strategies that were shared by the students who volunteered during the number talks and hoped that students would be able to learn from their classmates' ideas; but at the same time, I was concerned that without a written assessment it was not possible for me determine to what extent the collective strategies presented during the number talks reflected the understanding of individual students in the class.

Since students seemed comfortable with the mental math problems from the first two number talks of the week and with the circle diagrams, I decided to try introducing the area model and algebraic notation to students. I had seen dot arrays being used to introduce concepts of multiplication in an elementary textbook that I had studied, so I decided to try it out as a bridge to the open array or area model which is useful for modeling the distributive property. Since students were probably not familiar with the dot array, I started by modeling how a problem like 4 x7 would be shown with a dot array as four rows of 7. I demonstrated how the 7 or the size of the group could be split up in different ways and the partial products could be added to find the product (see Figure 1).

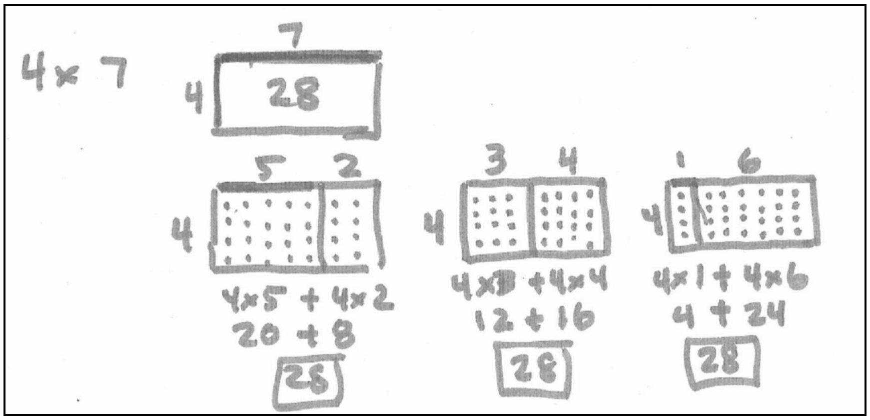

Figure 1. Dot Arrays

I could see many students drawing the diagrams in their notebooks without prompting and when I asked students to give an expression to represent each diagram (e.g. 4 x 1 + 4 x 6), most students participated in the choral response. I decided to have students try the problem 6 x 15 by drawing an open array and showing the steps using algebraic notation. I explained to students that the dot array could be drawn more simply with an open array by simply labeling the sides of a rectangle, but I saw that many students were drawing in the dots inside the area model for 6 x 15 and some students were not sure how to decompose 15. I had to suggest to students to decompose 15 into 10 and 5. I later realized that I should have had students spend more time exploring different ways of representing multiplication problems using the dot array and one-digit numbers (e.g. 3 x 8 is the same as 3 x 4 + 3 x 4 and 3 x 7 + 3 x 1) before moving to the open array or area model which did not show individual units.

Week 4. During the fourth week, I continued with one-digit by two-digit multiplication mental math number talks, but I added a new element to my number talks. My goal for these number talks was to have students continue to explore how to solve multiplication problems by decomposing the numbers in a variety of ways and applying the distributive property, but I also wanted students to see that decomposing the two-digit number or the size of the group into tens and ones would make the resulting numbers easy to multiply mentally and was a generalizable method. To guide students toward breaking up the two-digit number into place value then applying the distributive property, I included “helper” problems in the multiplication number strings. For the problems 3 x 22 and 6 x 22, for example, I had students do the helper problems 3 x 20 and 6 x 20 first.

Weeks 5 and 6. After week four, I went back and reviewed some of the number talk videos and wrote down students’ names next to the strategies they had shared. I noticed that although a variety of strategies were shared in each number talk some students were using only one type of strategy. I noticed, for example, that there was one student who only used repeated addition. There were several students who were able to use multiple strategies, but I realized that to help more students, I would probably need to provide some additional focus. The distributive property can be applied in a couple of different ways when multiplying numbers. The number of groups can be decomposed (e.g., 3 x 15 = 2 x 15 + 1 x 15 = 30 + 15 = 45) or the size of the group can be decomposed (e.g., 3 x 15 = 3 x 10 + 3 x 5 = 30 + 15 = 45). The goal would be for students to be able to understand and use both types of strategies, but I realized that for students who did not understand either strategy, seeing a variety of ways to decompose the number of groups and the size of groups could be confusing. I decided to have students focus on breaking down the size of the group by place value then applying the distributive property, since this method is an efficient and generalizable way of multiplying one-digit by two-digit numbers using mental math and would help students to think of two-digit numbers as tens and ones, rather than as concatenated digits.

To introduce this approach to students, I referred back to a poster that I had created showing various strategies that students had shared during past number talks. I told students that we would be focusing on a strategy that two students had previously suggested, which was breaking up the size of the group by place value. I referred to the poster to review the strategy and the diagram. I then asked students to think about how 2 x 35 could be solved using the same idea. I had students begin by drawing the circle diagrams to represent how the problem would be done in a similar way (two circles with 30 and 5 in each circle), then asked students to solve the problem mentally first, then write the answer and the steps on their paper before asking them to share with their neighbor and then with the class.

I began requiring all students to do written work and gave students one problem to do independently as an "exit problem" at the end of each number talk so that I could assess what individual students had learned. I also focused on having students use multiple representations for each problem, which included the circle diagrams and steps and the area model and algebraic notation. I did four number talks focused on these concepts and the problems that students did own their own at the end of each number talk indicated that most students had learned the intended content.

Before I knew it, I had just one number talk left before the end of the intervention. I considered various options for the last number talk including having students explore other ways of applying the distributive property and providing more practice with multiplying one-digit by two-digit numbers by decomposing the size of the group by place value. After considering various options, I decided to introduce doubling and halving to students for the last number talk. I had tried this strategy with my students last year and used the area model to demonstrate it, but most students had difficulty understanding it. My idea this year was to try to model it using the circle diagrams. I started the number talk by diagnosing students on whether they knew how to divide numbers by two and saw that some students had difficulty with problems such as 48 ÷ 2, which is a pre-requisite to being able to use the doubling and halving strategy. I also noticed that some students were not able to fluently multiply two-digit numbers by two mentally (e.g. 35 x 2), which is a pre-requisite skill for the doubling part of the doubling and halving strategy. I realized that although I had focused on these types of multiplication problems during the last four number talks, the focus had become more about using multiple representations and writing out the strategies and less on doing the math mentally. I was concerned that some students might have difficulty with the doubling and halving since they did not have the component skills, but I decided to go ahead with the strategy anyway since this was the last number talk. I modeled how 1 x 48 (one circle with 48 in it), was equivalent to 2 x 24. I made the circle for 24 half as wide as the one for 48 to show that when the number of groups doubled, the size of the group got divided by two. I then showed how 2 groups of 24 could be broken down further into 4 groups of 12 and 8 groups of 6. I then modeled two additional sets of similar problems (see Figure 2).

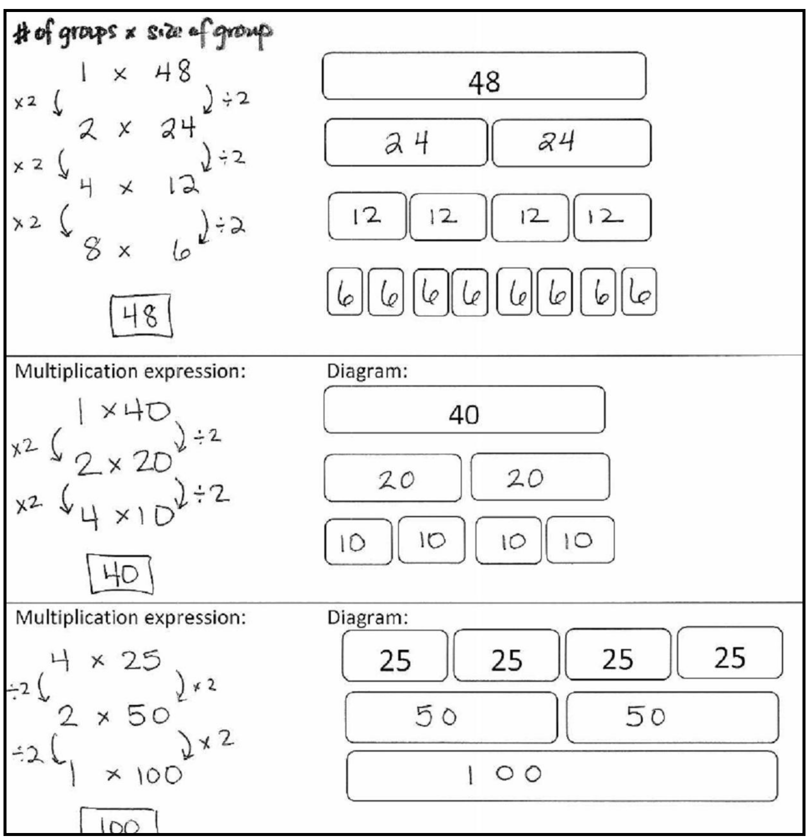

Figure 2. Visual models for doubling and halving

I showed students the models pretty quickly because I wanted to get to the two-digit-by two-digit multiplication problems, but when I had students attempt the problems, I could see that most were lost. Since this was the last number talk, I wanted to fit in as much as possible, but in the end only a few students were able to understand the strategy. I had tried to do too much in one number talk. I realized that with one number talk left, it would have been better to continue with the one-digit by two-digit multiplication number talks to help students develop more fluency with those types of problems, instead of bringing in new concepts.

In Table 1, on the pages that follow, the number talks for the six week intervention are summarized. The table includes the problems used for each number talk, the focus or goal, and the types of visual representations used.

Table 1.

Summary of Number Talks

Week. Day	Problem(s)	Focus/Goal	Visual Represen-tations
1.1	Number of the Day: How many ways to make 24?	▪ Decomposing numbers in different ways	
1.2	Mental Computation: 2 x 25, 4 x 25, 6 x 25, 8 x 25, 9 x 25	▪ Understanding the meaning of multiplication as the total number of objects in a given number of equal groups (e.g. 2 x 25 means “2 groups of 25”) ▪ Decomposing the number of groups to find the product (e.g. 9 x 25 = 8 x 25 + 1 x 25)	▪ Circle diagrams
2.1	Numbers of the Day: 243, 120 Make the numbers using only 1’s, 10’s, and 100’s and using addition	▪ Developing understanding of the base-ten structure of the number system: o Composing powers of 10 (e.g. ten 1’s = 10, ten 10’s = 100…) o Writing numbers using expanded notation o Thinking about numbers in units other than 1’s. (e.g. 250 is 25 tens or 25 x 10, 20 is 2 tens)	
2.2	Numbers of the Day: 24, 250, 12.4, 24.2 Make the numbers using only 0.1’s, 1’s, 10’s, and 100’s and using addition		
3.1	Mental Computation: 2 x 15, 3 x 15, 4 x 15	▪ Exploring a variety of ways to solve multiplication problems using mental math: o using repeated addition o decomposing the number of groups or the size of the group and applying the distributive property o using the associative property	▪ Circle diagrams
3.2	Mental Computation: 2 x 16, 3 x 16, 4 x 16		▪ Circle diagrams
3.3	Mental Computation: 6 x 15, 5 x 18, 5 x 12 (4 x 7 used to initially model the dot array)	▪ Teacher modeling of dot arrays (e.g. 4 x 7 can be 4 x (5+2), 4 x (3 +4), 4 x (1+6)) ▪ Using open arrays (area model) to model multiplication using partial products when the size of the group is decomposed.	▪ Dot arrays and open array/area model
4.1	Mental Computation: Multiplying by 10’s, 3 x 22	▪ Diagnosing understanding/skill in multiplying by multiples of 10; Providing input/modeling as needed (e.g. 2 x 30 is 2 x 3 tens = 6 tens or 60) ▪ Exploring a variety of ways to solve multiplication problems, including decomposing the size of the group by place value	▪ Circle diagrams
4.2	Mental Computation and Dot Arrays: 3 x 22, 6 x 22	▪ Using dot arrays to explore different ways of decomposing the number of groups and size of groups to find the product	▪ Dot arrays
4.3	Mental Computation: 6 x22 using circle diagrams; 4 x 31, 5 x 31, 8 x 31	▪ Exploring a variety of ways to solve multiplication problems by decomposing numbers and applying the distributive property	▪ Circle diagrams

Week. Day	Problem(s)	Focus/Goal	Visual Represen-tations
5.1	Mental Computation: 2 x 35, 2 x 65, 2 x 85, 2 x 75 (2 x 30, 2 x 40, 2 x 50, 2 x 80)	▪ Solving multiplication problems by decomposing the size of the group by place value and applying the distributive property	▪ Circle diagrams
5.2	Mental Computation: 4 x 15, 4 x 35, 4 x 65 (4 x 30, 4 x 40, 4 x 50)	▪ Teacher modeling of distributive property with rectangular array and area model along with algebraic notation (NT 5.2 and 5.3) ▪ Solving multiplication problems by decomposing the size of group by place value and using multiple representations including circle diagrams with steps and area model with algebraic notation	▪ Circle diagrams ▪ Rectan-gular array and open array/area model
5.3	Mental Computation: 6 x 16, 5 x 38, 3 x 45		
6.1	Mental Computation: 2 x 245, 2 x 256, 2 x 235		
6.2	Doubling and halving 1 x 48 to 8x6 (model); 15 x16, 25 x 48	▪ Teacher modeling of doubling and halving using circle diagrams ▪ Applying doubling and halving to solve 2-digit by 2-digit multiplication problems	▪ Circle diagrams

What's Important When Implementing Number Talks

In this section, I describe three teacher practices that were most important for me in implementing number talks in a way that supported student learning: 1) examining students' written work during and after the number talks as a means of formative assessment, 2) using visual representations to support students' understanding of mathematical concepts and relationships, and 3) focusing on one mathematical idea for a set of number talks. My findings are from a combination of my analysis as I implemented the number talks and as I reflected back across all of the number talks to identify the practices that improved my ability to gather information on student thinking and learning, and helped students to participate and move forward mathematically based on evidence from written formative assessments and my observations as a teacher. As I thought about what was most important, the areas that immediately came to mind were a focused and coherent curriculum, visual representations, and formative assessment, but a challenge in describing these categories was the overlap. As I tried to write about the importance of formative assessment, I had difficulty because it encompassed so many aspects of implementing number talks including selecting the goals, diagnosing students' current understanding, designing appropriate tasks, and making adjustments to instruction based on evidence of student learning. Some of these aspects were not only what I considered to be important in implementing number talks, but also what was most challenging. Therefore, for this section, I decided to highlight the importance of written work in formative assessment since it noticeably improved my ability to gather information about student learning and make adjustments to my instruction. I also had the category of a focused and coherent curriculum, but as began to write about its importance, I realized it would fit better under the category of challenges that persisted. I also considered including the category "making adjustments to instruction based on evidence of student learning" because I believe that it is so central to effective teaching, but I decided not to create a separate category for it since I discuss it in the other categories.

Examining students' written work during and after the number talks as means of formative assessment. Although mental math number talks are typically done without pencil-and-paper, I found that having students record their thinking on paper was helpful for formative assessment and for increasing student engagement. After I gave students a problem to do, as students recorded their strategies, I walked around the classroom to check students' answers and work. This enabled me to observe how students were working on problems to see if they were able to do a problem easily or if they were struggling to complete a problem, diagnose areas of difficulty for students or misconceptions that I may not have been able to uncover during class discussions, and determine whether students were engaged by examining their work. As I walked around to monitor student work, I was also able to quickly provide assistance to students as needed. This had a couple of benefits. First, it helped me to differentiate instruction and provide additional support to individuals who may not have been able to understand a concept from the whole group discussion. Second, I found that interacting with students individually, even for just 15 to 20 seconds, could make a difference in understanding and help to increase the student's engagement. There were two students sitting in the back row that did not seem very engaged in the number talks and who I had caught off-task a couple of times, but after I spent about 20 seconds helping them with a problem, they completely changed their attitudes. Another student who had not participated in the whole class discussions began to volunteer after I gave him positive reinforcement on his written work.

I also realized that giving students a problem to work on individually at the end of the number talk, which was a sort of an "exit slip," would give me an effective way to do formative assessment. After the number talk, I could quickly look through the sheets that students turned in to see what they had learned so that I could determine how effective a number talk was and decide on the next steps. I could also see where individual students were in their understanding and analyze their errors. For example, in Number Talk 5.2 where students worked on problems involving one-digit by two-digit multiplication such as 4 x 35 by decomposing the size of the group by place value, I reintroduced the area model and algebraic notation after we had worked on using circle diagrams and showing the steps. When I checked the problem that students did on their own at the end of the number talk which asked them to solve 4 x 65 and show their work using multiple representations, I saw that most students were able to show the steps and the circle diagrams correctly, but only some students were able to also use the area model and algebraic notation. Some students had incomplete or incorrect area models that indicated that they were having trouble finding the partial products using the model because they did not know that the height of a rectangle is the same throughout. This helped me to see that I needed to spend more time on modeling the rectangular array, which shows individual unit squares, before having students use the open array. I also noticed that some students seemed to get the concepts that were being addressed in the number talks – they could decompose the size of the group, draw the circle diagrams, and show the steps for finding the partial products, but got an incorrect answer because they made mistakes with basic multiplication facts (e.g. 4 x 60 = 260), added incorrectly (e.g. 240 + 20 = 280), or transposed digits (2 x 60 = 210).

Having students do the “exit slip” problem also gave me a way to differentiate for advanced students, which was important because there was a very wide range of levels in the class. For students who were able to finish the “exit slip” problem quickly, I included an optional challenge problem. This was usually an extension of the topic of the number talk. For example, if the number talk was on one-digit by two-digit multiplication problems, the challenge problem might be a one-digit by three-digit multiplication problem. This enabled me to provide an additional challenge for students who had already mastered the concepts that were being addressed in a number talk and engage them in applying what they had learned to figure out something new.

Having students write out their strategies and diagrams on paper also helped to support their participation in the number talks. In the beginning, students’ explanations were sometimes incomplete or not easy for other students to understand and I often had to ask questions to help them to elaborate on their thinking, but after I began requiring that students write out their steps and diagrams, students began to give more complete explanations. After awhile, the same seven to eight students tended to volunteer to share their strategies during the whole class discussions, and I was reluctant to call on students randomly because I was not sure if students were not volunteering because of a lack of understanding. But by examining students' written work as I walked around the classroom, I could see what individual students understood. I could see that most of the students sitting in the fourth row who rarely volunteered to participate had a strong grasp of the concepts based on their written work, so I asked some of the students in that row if they would volunteer to share their answers. Also, after I began requiring students to record their steps and diagrams, students were able to engage in more productive pair-shares. I observed students helping each other more and acting as resources for each other after I required students to record their work. I could see students referring to the written work as they helped their partners.

Using visual representations to support students' understanding of mathematical concepts and relationships. I found that visual representations were useful for illustrating students' strategies and for providing scaffolding to help students solve problems during the mental math number talks. For example, in Number Talk 1.2, which included the problems 2 x 25, 4 x 25, 8 x 25 and 9 x 25, I used circle diagrams to help students make connections among the problems in the set. After writing the problem 8 x 25 on the board, I reminded students that we had seen earlier that 4 groups of 25 was 100 referring to the problem and the diagram of the 4 circles with 25's in them. I then asked what 8 groups of 25 would be and drew two rows of 4 circles of 25's. Right away, many students indicated with a thumb up that they had an answer and many students volunteered to share their strategy. Interestingly, the strategy that was shared matched the diagram that I had drawn on the board. When I polled students to see how many of them used the same strategy, many students raised their hands. To help students calculate 9 x 25, I reminded students that we had just seen that 8 x 25 or 8 groups of 25 was 200 and asked if they could they now find 9 x 25. Students seemed eager to share their answers. The student who I called on explained that she knew that 8 x 25 = 200 and then added 25 to get 225. If this problem had been given in isolation, it may not have been easy for most students to do, but the circle diagrams seemed to help students by guiding them to think about how they could use the previous problem to solve this problem.

The area model or the open array was also useful for representing strategies and scaffolding student thinking. The area model is not as intuitive as the circle diagrams, but it has several advantages over the circle diagrams. It can be used to represent multiplication of large numbers easily as well as division, and it is a representation that students will encounter when they learn about how to use the distributive property to multiply algebraic expressions such as $5 \cdot (2n+4)$ and $(n+5)\cdot(n+4)$. The area model also lends itself to representing the partial products strategy using algebraic notation, which I considered an important goal for the intervention, since students generally show their work using algebraic notation in middle school.

The circle diagrams required little to no explanation for the students to understand, but developing students' ability to understand and use the area model required careful sequencing. Before students could understand the area model, they first needed to understand dot arrays or rectangular arrays. With the dot array, the equal groups are arranged in rows (or columns, depending on how the rows and columns are defined), so a 4 x 7 dot array would represent 4 groups of 7. Instead of the 7 dots being enclosed in four separate circles to represent 4 groups of 7, they are now arranged in four equal rows of seven. When I introduced the dot array in a number talk to show how 4 x7 could be decomposed in different ways and that sum of the partial products (e.g. 4 x 6 + 4 x 1) was the same as 4 x 7, I found that students could understand it easily, but I made the mistake of moving too quickly to the area model which is much more abstract because it does not show each unit. I also jumped from modeling a one-digit by one-digit number to asking students to use the area model to solve 6 x 15 which had a two-digit number. I saw that many students were drawing in the dots inside the area model for 6 x 15 and some students were not sure how to decompose 15. I had to suggest to students to decompose 15 into 10 and 5. I realized that students needed more experiences working with single-digit numbers and dot arrays which showed individual units before moving to the area model with a two-digit number.

The rectangular array, like the dot array, has equal-sized rows and columns, except instead of a dot representing one unit, a square represents one unit. I introduced the rectangular array to students during week five, after students were familiar with the circle diagrams and decomposing two-digit numbers into tens and ones and using the distributive property. I figured out that it was helpful to first show students two separate rectangular arrays when modeling the distributive property, as shown in Figure 3.

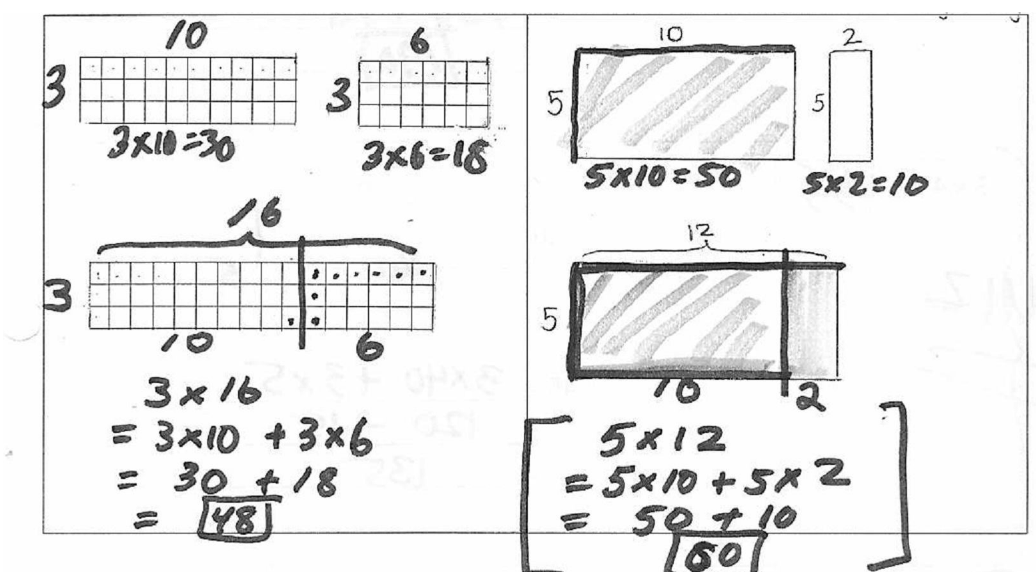

Figure 3. Rectangular array and area model

I started by showing students the 3 x 10 and 3 x 6 rectangular arrays and asking them if there was a way to use a calculation to find the total number of squares in each array. After students came up with the multiplication problems 3 x 10 and 3 x 6, I asked students to identify the multiplication problem that could be used to find the total number of square units in the larger rectangle (3 x 16). To demonstrate the distributive property, I took the 3 x 10 rectangle, which I had colored green and the 3 x 6 rectangle, which I had colored yellow and placed both of them on top of the large rectangle so students could see that 3 x 16 was the same as 3 x 10 + 3 x 6. I then showed the algebraic notation under the diagram. From there, I modeled the same concept using the area model or open array, which does not show individual squares. I then had students use the model to solve 6 x 16 using the distributive property. As I walked around, I could see that most students were able to complete the algebraic notation. Students who normally did not volunteer to participate raised their hands to share the steps. At the end of the number talk, I had students draw the area model and algebraic notation to solve 3 x 45 by decomposing 45 by place value and applying the distributive property. About 80 percent of the students were able to do the exit problem. I think one reason that I had more success in introducing the area model during this number talk was that students had already developed some proficiency with decomposing numbers by place value and applying the distributive property and using circle diagrams, so they were not learning a new concept along with a new representation, which would have been more difficult. Also, first showing the separate rectangles and then putting the rectangles together to show that area or rectangular arrays are additive seemed to be helpful. If there was more time in the intervention, I would have spent more time on the rectangular array before moving to the area model so that students would be able to develop a more solid understanding of the area model and all students in the class would be able to understand it.

Focusing on one mathematical idea for a set of number talks. Figuring out how much to focus the number talks and what to focus on during the number talks was also important in supporting student learning. My initial goal for the mental math number talks was for students to explore a variety of ways to solve multiplication problems using mental math, including repeated addition, decomposing the number of groups or decomposing the size of the group and applying the distributive property. In the beginning, I considered having a variety of strategies presented by students to be one indication of a successful number talk because it was my hope that students in the class would be able to develop greater understanding of number and operation concepts from the strategies presented by their classmates. But as mentioned earlier, during the fourth week of the intervention, after I went back and reviewed some of the number talk videos and wrote down students' names next to the strategies they had shared, I noticed that although a variety of strategies were shared in each number talk some students were using only one type of strategy. There was one student, for example, who only used repeated addition. Though there were several students who were able to use multiple strategies flexibly, I was concerned that there might be more students who were not developing understanding of multiplicative concepts, so I decided to narrow the focus of the number talks.

After some consideration, I decided to have students focus on decomposing by the size of group using place value because it was a generalizable and efficient method for multiplying numbers using mental math and would help students to think of two-digit numbers as tens and ones, rather than as concatenated digits. It would also help to lay the foundation for students to develop a conceptual understanding of the multiplication algorithm. At first, I was concerned that I was narrowing the focus of the number talks too much, but after narrowing the focus I was able to help more students. The exit slips showed that the majority of students were able to draw the circle diagrams and solve the problems using the distributive property. Once I saw that most students were able to use the circle diagrams, I was able to bring in the area model and algebraic notation successfully. Although I had focused a set of my number talks on only one way of decomposing the numbers which I worried at first might be too narrow, I found that I was able to help a larger number of students learn from the number talks and help students progress toward other important goals which included decomposing numbers by place value, multiplying by multiples of ten mentally (e.g. 3 x 40), and using multiple representations.

Challenges that Persisted

Although I was able to figure out ways to address most of the challenges that I encountered as I implemented the intervention, a persistent challenge was figuring out how to design a focused and coherent curriculum for the number talks. This included figuring out what the goals would be, selecting the "right problems," and chunking and sequencing the problems. To design a curriculum, it is first necessary to have clear goals, but it was difficult to figure out what the "right goals" were for the intervention. The goals would depend largely on where students currently were and the amount of time the students would need to progress towards a particular goal, but it was difficult to gather enough detailed information about students based on the pre-test and their oral participation in the number talks to consistently design number talks that would effectively build upon students' current knowledge and understanding. It was also difficult to figure out how focused the goals should be and what the specific focus should be.

It was also a challenge to select the "right problems' for the number talks. If the problems were too easy or too difficult students would not be engaged and would not learn. The problems also needed to guide students towards developing the concepts that I wanted them to focus on, but I was not always sure how to design problems in this way. For some number talks, the problems that I selected seemed to be "just right" based on where students were and the intended goals, but it was difficult to figure out how to select problems for the next number talks that would be equally effective. I referred to various sources such as Parrish's (2010) book and district booklets which had problems for addition, subtraction, multiplication, and division number talks, but I found that most of the multiplication problem sets did not address the goals of my intervention or they required components skills that the students in the intervention had not yet developed. I also looked at various resources on the internet, but found that most of the advice focused on instructional strategies such as providing adequate think time, providing a safe environment, and asking questions to guide student thinking, all of which I was already doing. Any advice that I found on designing problems for number talks tended to be too general to be helpful. The instructional strategies that were suggested were part of any good teaching practice, but they would not help students to participate in the number talks and move forward mathematically *unless* the tasks that the teacher selected were within reach, based on students' current understanding and were designed to progressively move students toward the desired goals. Designing multiplication number talks that met these conditions was a challenge.

Impact of the Intervention on Students' Number Sense

In this section, I present data on the impact of the intervention on students' number sense as measured by the pre- and post-number sense assessments. Although 34 students participated in the intervention, there were eleven students who did not turn in consent forms and one student who was absent on the second day of the post-test and did not complete the assessment; therefore data is included for 22 of the 34 students.

The assessment had seventeen items overall and was divided into three subsections: algorithms, equivalent expressions, and computation/mental math. In most cases, the identical item was given on the pre- and post-assessments (see Table 3). To determine whether there were statistically significant changes in the mean scores for each subtest and the overall test, I conducted matched pair t-tests (one-tailed, alpha = 0.05) using Microsoft Excel. The results showed that there were statistically significant increases in the overall mean score (t-value=2.48, p-value = 0.01) and the mean score for the equivalent expressions section (t-value=4.28, p-value = 0.00), which was closely aligned to the intervention. The results are summarized in Table 2.

Table 2. *Means and SDs for Pre/Post-Test (n=22)*

Category	Max Score	Pre-Test Mean (SD)	Post-Test Mean (SD)	Change Mean (SD)	t-value (1-tail)	p-value (1-tail)
All items (#1-17)	17	9.7 (3.3)	12.0 (3.4)	2.2 (4.2)	2.48 *	0.01 *
Algorithm (#1-3)	3	2.4 (0.7)	2.1 (0.7)	-0.2 (0.9)	-1.15	0.13
Equivalent Expressions (#4-10)	7	2.1 (1.8)	4.3 (2.0)	2.1 (2.3)	4.28 *	0.00 *
Computation/MentalMath (#11-17)	7	5.2 (1.8)	5.5 (1.5)	0.3 (2.5)	0.60	0.27

*Statistically significant increase: p<0.05

Table 3. *Pre/Post Number Sense Assessment Items*

Category	Item #	Item Description
Algorithm	1	subt algorithm: 3005-246
Algorithm	2	mult algorithm: 645 x 80
Algorithm	3	div algorithm: 9045÷15
Equiv Expressions	4	8x56 = 8 x__ + 8 x 6
Equiv Expressions	5	6x24 = (6 x __) + (__ x 4)
Equiv Expressions	6	4x36 = 4 x (__ + 6)
Equiv Expressions	7	645 = 6 x__ + 4 x__ + 5 x __
Equiv Expressions	8	420=___x10
Equiv Expressions	9	Double & Halve 24 x 25 = 12 x __, 6 x __
Equiv Expressions	10	T/F; justify: 3x28 = 3x20 + 3x8
Comp/MentalMath	11	28 x 10
Comp/MentalMath	12	28 x 100
Comp/MentalMath	13	2.8 x 10
Comp/MentalMath	14	2x25
Comp/MentalMath	15	4x25
Comp/MentalMath	16	8x25
Comp/MentalMath	17	3x18 (pre); 4x15 (post);justify

The problems in the computation/mental math section were intended to be done using mental math, but it was difficult to monitor the actual process used by students because this was a written test given in a large group setting. On the pre-test, in particular, there was evidence of students writing out the work using the standard algorithm. On at least half of the tests, some work was shown using the algorithm or there were traces of work that had been erased. Since it was difficult to determine what process was used in all cases, as some students could have erased their work more completely, an answer was counted as correct if the correct answer was written, regardless of the process used. On the post-test, students tended not to use the algorithm for these items and tried to do the problems using mental math. For example, one student earned 6 points on the mental math section on the pre-test, but all work was shown using the algorithm. On the post-test, the student attempted mental math, but was not able to do any of the problems. The student's score for this section dropped from 6 points to 0 points and her overall score dropped from 8 points on the pre-test to 1 point on the post-test. Because some of the correct responses on the pre-test did not necessarily indicate students' ability to do the problem using mental math, the results of this section are difficult to interpret.

Item #17, however, which asked students to solve 3 x 18 on the pre-test and 4 x 15 on the post-test using mental math does provide us with some insight into student thinking since it asked students to justify their answers. Although the change in the number of students answering the item correctly from pre-test to post-test was very small (73% to 77%), there were changes in the methods that students used. On the pre-test, all but two students justified their answers using the algorithm (one student used the distributive property and one student used the associative property), but on the post-test, 16 out of 22 or 72% of students successfully used strategies that demonstrated number sense, including the distributive property (11 students), associative property (1 student), and doubling and halving (4 students). It was interesting to note that three of the four students who used doubling and halving on Item #17 on the post-test, missed Item #9 which assessed students' ability to apply doubling and halving to write equivalent expressions (24 x 25 = 12 x __, 6 x ___, 3 x ___). This may have been because the numbers in Item #9 were both two-digit numbers and doubling and halving was more difficult to apply or because the students had difficulty applying their understanding of doubling and halving in the format in which the question was asked.

On the post-test, I added a supplemental item to assess students' abilities to solve a one-digit by two-digit multiplication problem using the concepts and representations that I had focused on during the last two weeks of the intervention in Number Talks 5.1 to 6.1. Students were asked to solve the problem 3 x 45 by decomposing the numbers by place value and applying the distributive property. They were asked to show their work using the circle diagrams and steps, and area model and algebraic notation. About 80% of students were able show their work using algebraic notation and the circle diagrams and get the correct answer. However, only 64% of students were able to use the area model to represent the solution, which was a drop from 80% on the exit slips for Number Talk 5.3. This was probably because I did not spend enough time developing the rectangular array.

Although I do not have pre-test data on students' entry level knowledge of the concepts assessed on this item, I believe that the knowledge that students demonstrated on this item was most likely attributable to the number talks because the content was closely aligned to what was addressed in Number Talks 5.1 to 6.1 and information that I had gathered during the intervention through formative assessment indicated that students were learning the content through those number talks. Also, it is unlikely that students learned the content of Number Talks 5.1 to 6.1 in their math classes because the content being taught in students' math classes was different from what was addressed in the number talks.

CHAPTER 5: DISCUSSION

In this study, I set out to examine the implementation process and impact of a six-week number talk intervention during the advisory period with sixth-grade students in a middle school in a large urban school district. My findings showed that there were three teacher practices that were important for me in implementing number talks in a way that I felt supported student learning: 1) examining students' written work during and after the number talks as a means formative assessment, 2) using visual representations to support students' understanding of mathematical concepts and relationships, and 3) focusing on one mathematical idea for a set of number talks. I identified one persistent challenge, figuring out how to design a focused and coherent curriculum. This included figuring out what the goals would be, selecting the "right problems," and chunking and sequencing the problems.

In this chapter, I first highlight key findings and make connections to relevant literature. I then discuss the limitations of the study and suggestions for future research.

What's Important When Implementing Number Talks

Examining students' written work during and after the number talks as a means of formative assessment. Although number talks are typically done without pencil-and-paper (Boaler, 2008, 2013; Math Perspectives, 2007; San Diego City Schools, 2004), I found that having students record their thinking on paper and examining students' written work both during and after the number talks was helpful for formative assessment. Research shows that formative assessment, which is any activity that provides sound feedback on student learning, is one of the most powerful tools a classroom teacher might use to support student learning (Black & Wiliam, 1998).

Three processes are considered essential to formative assessment: 1) figuring out where learners are in their learning, 2) figuring out where they are going, and 3) figuring out how to get there (Heritage, 2010; Wiliam, 2011; Wiliam & Thompson, 2007). Wiliam (2011) writes that "all teaching really boils down to [these] three processes" (p.45). Teachers typically gather data about students for formative assessment in several ways: by listening to what students say, by observing students, and by examining students' written work. All three are important sources of information. However, in a typical mental math number talk, because only the teacher records the strategies and the students do not use any pencil or paper (Boaler, 2008, 2013; Math Perspectives, 2007), information about student thinking and learning can only be gathered by listening to and observing students. Not being able to gather data through written work during a number talk limited both the amount of information that I could gather about students and the number of students from whom I could gather information. Since each number talk was just fifteen minutes long, typically I would only be able to gather strategies from six to eight students out of thirty-four students, and often, the same students volunteered to share their strategies. I walked around to listen to student conversations as they shared their strategies with their neighbors and used strategies such as having students show their answers in front of them on their fingers, but this did not provide me with enough information about where students were and what they were learning.

Around the fifth week of the intervention I began requiring all students to record their work and I gave students an "exit problem" at the end of each number talk to assess individual student learning. Having students record their thinking and do the exit problem enabled me to gather information needed to make adjustments during the number talks and to determine afterwards how effective a number talk was and decide on what the next steps would be. I was able to make more informed decisions about instruction after I began examining students' written work because I could gather information from all students in the class, not just a self-selected sample of students. Having students do written work had the added benefit of increasing student engagement and supporting their participation in the number talks.

After I completed the intervention, I went back and reviewed Parrish's (2010) book on number talks and found that she recommends having students solve an exit problem using the discussed strategies and giving a weekly computation assessment as ways to check to see if students are "mentally participating and accessing the proposed strategies" (p.25). Parrish (2010) also suggests using students' written practice for formative assessment:

> Requiring students to solve five computation problems daily that mirror the number talk focus provides this practice and also gives the teacher a way to formatively assess student understanding and make instructional decisions for the next day. (p.326)

My finding about the importance of using written formative assessment in implementing number talks is consistent with Parrish's (2010) recommendations. Given how much of a difference it made in my ability to gather information about student thinking and learning, I found it surprising that this practice was not mentioned in other sources (e.g., Boaler, 2010; resources from Central Unified School District Math Office; Math Perspectives website; online videos of number talks) or in the videos included in the DVD that accompany Parrish's (2010) book.

Using visual representations to support students' understanding of mathematical concepts and relationships. Another practice that I found to be important was using visual representations to illustrate students' strategies and to provide scaffolding for solving problems during the number talks. My findings were consistent with current policy documents. In *Principles to Action*, The National Council of the Teachers of Mathematics (2014) recommends using mathematical representations to help students make connections and deepen their understanding of mathematical concepts and procedures. In the Common Core State Standards, the learning progression outlined in Numbers and Operations, Base Ten (K-5) calls for students to use strategies based on place value and the properties of operations when learning about multiplication and to illustrate and explain the calculations by using equations, rectangular arrays and/or area model (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010, p. 29). Parrish (2010) and Boaler (2013) also recommend using visual representations during number talks to illustrate students' strategies and help students make connections.

I found that several visual representations were helpful for multiplication number talks: circle diagrams, dot/rectangular arrays, and the area model. Circle diagrams are not mentioned specifically in the Common Core State Standards or in written resources for number talks, but they seemed intuitive for students and I found that they were useful for illustrating students' strategies and for illustrating the meaning of multiplication as the total number of objects in a given number of equal groups (e.g., 4 x 15 or 4 groups of 15 can be illustrated with four circles each containing the number 15). The circle diagrams required little explanation for the students to understand, but developing students' ability to understand and use the area model, which has some advantages over the circle model, required careful sequencing. Although the area model is often used as a visual model to represent the distributive property, it was difficult to find curricular supports that demonstrated *how* to develop students' understanding of the model; therefore, it took some trial-and-error to figure out how to develop this representation. I eventually figured out that it was critical to first sufficiently develop students' understanding of dot arrays or rectangular arrays which show each individual unit before introducing students to the area model which is more abstract. I also learned that it is helpful to have students use the model with single-digit numbers first to develop understanding of the distributive property (e.g., 4 x 7 = 4 x 6 + 4 x 1), then to examine arrays with multiples of ten (e.g., 3 x 10, 3 x 20), before working with other one-digit by two-digit arrays (e.g. 3 x 24). Also, to demonstrate the distributive property it was helpful to show separate rectangular arrays first, then put them together over a larger rectangular array to show that area is additive. Most examples that I had seen only showed a large rectangle being split into smaller rectangles.

Results from the supplemental item on the post-test showed that about 80% of students were able to use place value concepts and the distributive property to solve one-digit by two-digit multiplication problems and could justify their work using circle diagrams and algebraic notation. However, only 64% of the students were also able to use the area model. By the time that I had figured out how to develop the area model, I had very little time left in the intervention so I was not able to spend sufficient time developing students' understanding the rectangular array. If I had been able to spend more time on the rectangular array, I think a larger percentage of students in the class would have been able to understand and use the area model. This would help to strengthen students' conceptual understanding of multiplication and the distributive property and provide them with a foundation for using the area model to solve two-digit by two-digit multiplication problems and division problems.

Focusing on one mathematical idea during a set of number talks. The third important teacher practice that I identified was selecting one mathematical idea to focus on during a set of number talks. My initial goal for the mental math number talks was for students to explore a variety of ways to solve multiplication problems and I considered having a variety of strategies presented by students to be one indication of a successful number talk because I hoped students would be able to learn from other students' strategies. But about two-thirds of the way through the intervention, after I reviewed videos of the number talks a second time, I realized that this was true only for students who had adequate prior knowledge. I realized that to support student learning, I would need to narrow the focus of the number talks to help students organize the knowledge and build up their conceptual understanding. This is not what Boaler (2013) and others might suggest, but I felt that it was important to make adjustments to instruction based on evidence of student learning. I found that by focusing on one important mathematical idea, my number talks helped to support more students in learning the content and helped students to make mathematical connections. Exit slips showed that the majority of students were able to apply the distributive property to solve one-digit by two-digit multiplication problems and use multiple representations to justify the strategy. After students were able to write out and draw the strategy, they were better able to explain it during the number talks. I was also able to address other goals which included decomposing numbers by place value and multiplying by multiples of 10.

Discussion of Challenges

One persistent challenge in implementing number talks was designing a focused and coherent curriculum that would build upon students' current understanding and progressively move students forward mathematically. Teaching is conceptualized as the interactions among teachers and students around content or tasks directed toward facilitating students' learning goals (Hiebert & Grouws, 2007; National Research Council, 2001). To support student participation and learning, it is critical to *start* with the right goals and tasks, but determining what the "right goals" were for students and selecting the "right tasks" for the number talks was a challenge.

One reason that selecting the “right goals” and “right tasks” was challenging was because this was an intervention. Math is comprised of logical progressions of concepts that increase in complexity and depth and it is essential for students to build a deep understanding of foundational concepts in order to reach more advanced levels (Clements & Sarama, 2009; Common Core Standards Writing Team, 2011; Daro, 2011; Heritage, 2008). Students in the intervention had learned about multiplication and other topics related to numbers and operations in elementary school, but they had only been taught procedures. The goal of the intervention was to help students to develop conceptual understanding of mathematics, which can be characterized as the formation of links or connections between pieces of knowledge to form a network or webs of knowledge (Hiebert & Carpenter, 1992), but it was difficult to know where to start in order to accomplish this goal. If learning mathematics was just a matter of memorizing disconnected procedures then learning something new would not depend too much on what students had learned before, it would just require more practice and more time to learn; but, if learning mathematics is about making connections, then it would be necessary to develop a foundation to which new knowledge could be connected.

It seemed that to really develop understanding it would be necessary to start at the beginning of a progression, but at the same time, I wondered if it would be possible to start at some other point and build students’ knowledge from there. After consulting a couple of people in my district, I decided to focus on whole number multiplication. I was concerned that students might not have the foundational understanding of addition and subtraction needed to learn multiplication with understanding, but I decided to follow the advice that I had received. I studied the Common Core progressions for whole number multiplication and saw that the content spanned several grades. Since my intervention was only six-weeks long, I would need to figure out where to start within the progression and what to focus on. I wanted to select goals that would be most helpful for students' further study of mathematics and would build on their current knowledge, but I did not have enough information about the students' current understanding of mathematics to know where that would be. Figuring this out would be critical if I wanted to engage my students in productive classroom discussions.

I ended up focusing on one-digit by two-digit multiplication for the mental math number talks since it seemed like a reasonable place to begin and would help students develop understanding of important mathematical concepts. However, there were a couple of challenges in implementing number talks with this content focus. First, trying to re-teach a topic conceptually that students have learned once without any understanding was much more difficult than teaching students who have not yet been exposed to the topic at all. This is consistent with the results of Wearne and Hiebert's (1988) study, which found that fifth and sixth grade students who had learned to add and subtract decimals by memorizing rote procedures were less likely than fourth graders with no such experience to acquire conceptual knowledge about decimals from instruction using base-ten blocks. When I presented multiplication problems for students to think about during the number talks, many students either thought of the mental algorithm or repeated addition. Because students had learned how to do multiplication problems involving one-digit by two-digit numbers once before, the ideas that most students suggested were based on rote procedures that they had learned in elementary school. This made picking the "right problems" for the multiplication number talks tricky because if the problems were too easy then students would simply use the mental algorithm which would not help them to develop any new concepts, but if the problems were not so easy that they could not be done with the mental algorithm (i.e. there was some regrouping), then the problems would be too difficult for the students who did not possess the component skill of adding the partial products mentally.

Another challenge was that although there were students who shared various strategies that demonstrated number sense, many students were not able to access these strategies. I tried various participation strategies recommended in the number talk resources such as using talk moves, but they did not seem to make much of a difference. As I was reflecting back on the number talks and examining the work that I had recorded for students, I realized that it would be difficult for students who did not have sufficient prior knowledge to organize these strategies and be able to apply them to new problems. I decided to narrow the focus of the number talks to help students build up their knowledge in a way that would help them to organize the information and make connections. I struggled with this decision at first because I was concerned that I would not be following the procedures for implementing number talks and I was taking out opportunities for productive struggle (which is where students make the connections for themselves); but I could not figure out a way to create the conditions needed to engage students in *productive* struggle with the content I had selected (one-digit by two-digit multiplication) and where students were in their current understanding. Therefore, I made the decision to help the students make the connections, since it would be better than having students not making connections at all. Hiebert and Grouws' (2007) literature review and the TIMSS 1999 Video Study showed that making mathematical connections explicit was a key feature of instruction associated with the development of conceptual understanding.

The number talks from the first week had shown me that if the problems were accessible then I could get a lot of students participating and implement the number talks in the ways described by Parrish (2010) and Boaler (2010). As I reflected on the intervention after it had ended, I wondered if starting with one-digit by one-digit multiplication problems would have made the difference. I had observed that when developing the dot array, students seemed to easily grasp the concept of the distributive property and partial products with one-digit numbers. If I had started there, then moved through a progression that included multiples of tens, then other two-digit numbers, then would I have been able to engage my students in productive struggle?

If I were to implement the intervention again, I would try starting with one-digit multiplication and dot arrays to develop multiplication concepts and then move through a step-by-step progression to two-digit numbers. I might first have students explore ways to represent, compose and decompose dot arrays involving one-digit by one-digit multiplication. I would then have students work with multiplication problems involving tens such as 1 x 10, 2 x 10, 3 x 10, then arrays such as 3 x 20, where students explore ways to calculate the total number of dots which might include 20 + 20 + 20 or breaking the rows of 20 into two 10's and using 3 x 2 to find the number of tens. Students could then explore how to think about problems such as 3 x 23. I believe that starting with one-digit by one-digit multiplication and dot arrays would help students to develop concepts related to multiplication. Developing students' understanding of multiplication involving tens would also be a critical step because it would deepen students' understanding of place value and provide them with the foundation for working with multi-digit numbers. Moving through this progression would hopefully help students to develop the foundation needed to be able to apply number and operation concepts rather than using the mental algorithm when doing one-digit by two-digit multiplication problems such as 3 x 23 and 4 x 15. If I were to find that there were only a few students participating and written formative assessments showed that students were not learning, I would make adjustments to the content of the number talks to ensure that they were accessible to all students but there was still something new to figure out, since experience showed that if the number talks were accessible, many students would participate and engage with other students' ideas. These adjustments might include using number of the day problems which are more open-ended or focusing on addition and subtraction to develop more foundational concepts.

As I reexamined Parrish's (2010) book on number talks after the intervention had ended, I found that Parrish actually recommends that teachers begin with "fluency number talks" or "kindergarten number talks" which include dot images and ten-frames before doing number talks that focus on computation in order to develop students' abilities to see numbers in a variety of ways, subitize and learn number combinations. Parrish mentions the importance of the fluency number talks several times in her book. In the introduction to the section on K-2 number talks Parrish writes:

> Although number talks are categorized by grade level, they should not be used as rigid structures but as fluid components based on student need. That said, fluency number talks should be used to build a strong foundation before moving into number talks that focus on computation. (p.67)

In the introduction to Chapter 8, which describes how to design multiplication and division number talks for grades 3-5, Parrish writes:

> Teachers should use fluency number talks to build a strong foundation before moving into number talks that focus on computation of larger numbers. (p.262)

Parrish (2010) also recommends starting with "small numbers that are age and grade-level appropriate" when doing multiplication and division number talks (p.263). She explains that using small numbers enables students to focus on the nuances of a strategy instead of the magnitude of a number and helps students to build confidence in their mathematical abilities.

Designing a focused and coherent curriculum for multiplication number talks was a challenge, but if I had started with the fluency number talks, it would have been fairly easy to select appropriate tasks and to create the conditions needed to engage all students in the types of number talks described by Parrish (2010) and Boaler (2013). There may be challenges, however, to implementing number talks that focus on dot cards and ten-frames in middle school. Students may be resistant, thinking that the dots cards are too easy and administrators, parents, and other teachers may also question the teacher's choice of content. Although the students would be building important foundational concepts about mathematics such as decomposing and composing numbers in various ways and making tens, people may not have the patience to allow students to move up through the number talk curriculum from the dot cards to addition and subtraction to multiplication and division. So the dilemma remains. Where should teachers start when doing a number talk intervention in a middle school?

I have focused my discussion here on selecting the right goals and tasks to design a focused and coherent curriculum that builds on students' current understandings, but I am not suggesting that other factors such as instructional strategies, classroom environment, teacher knowledge and teacher beliefs do not matter. Even with the right goals and tasks, a teacher could change the task and not implement it in the way intended. My point is not that tasks are the only factor, but rather, selecting the right goals and tasks based on students' current knowledge is a *starting point* for instruction that develops mathematical understanding. Instructional strategies are an important component of teaching, but in my experience, poor curricular design cannot be overcome with instructional strategies.

As part of my experience doing this study, I was able to learn about ways to use number talks to help students develop conceptual understanding and to learn new ways to engage students in classroom discussions. I found that although there were some challenges, students really enjoyed doing the number talks. I have been trying out number of the day problems with my own math classes when possible and students have been enjoying and learning from them. I believe that with additional guidance for teachers, number talks can be used productively in middle schools during the advisory period and the regular mathematics period to help students develop number sense. It may be difficult for middle school teachers to implement mental math number talks that address elementary school content during the regular mathematics period since there are only about fifty minutes in each period and teachers are required to teach grade-level content, but I believe teachers can incorporate number of the day problems into the mathematics period since they generally take less time and can easily be designed to address a range of skill levels and connect to the grade-level content. I also believe that number talks can be used effectively as part of an advisory-period intervention, as long as the teachers implementing the intervention have adequate pedagogical content knowledge and are provided with a curriculum and the tools needed to diagnose students' understanding of number sense to know where to start in the curriculum.

Next Steps

This study examined the implementation process and impact of a six-week number talk intervention with one class of sixth-grade middle school students. One finding in this study was that it was difficult to design number talks that were accessible to all students and could move students forward mathematically toward important goals; but this may have been because the initial focus area of whole number multiplication with one-digit by two-digit numbers may not have been the right focus. Parrish (2010) recommends beginning with fluency number talks for students regardless of grade, but there may be resistance from students, parents, and administrators if middle school teachers tried to follow this recommendation. One unanswered question I had was whether starting with one-digit by one-digit multiplication would have worked.

Future studies may want to examine the role of different tasks in teachers' abilities to implement number talks in the ways suggested in the literature and to examine what the learning outcomes are for students. Perhaps researchers can select a highly effective teacher who has the proven ability to implement number talks effectively with a set of students and then have him/her implement number talks using various tasks with a group of middle school students who are in need of an intervention. Researchers may want to examine if and how the teacher can successfully implement various tasks and goals with students. The quantitative portion of my study did not include a comparison group because I was not able to recruit enough students from a comparison advisory classroom to participate in the study, but in future studies, researchers may want to include comparison classrooms to examine the impact of the intervention on student learning outcomes.

Another recommendation is for researchers to develop widely available instruments that would help teachers and schools to diagnose students' understanding of number sense. There are free diagnostic assessments available through the Math Diagnostic Testing Project (MDTP) for pre-algebra and beyond, but MDTP does not offer assessments to test number sense. Research shows that many U.S. students complete elementary school without developing number sense which is critical for learning pre-algebra, algebra and beyond. Many of our students are struggling in algebra and other middle and high school courses because they lack number sense, but current interventions do not address this problem. If there were diagnostic assessments available to test number sense, schools would be more likely to offer students the types of interventions that would provide them with the opportunity to develop the necessary foundations for success in mathematics.

LIST OF APPENDICES

Appendix A: Parent Consent Letter

University of California, Los Angeles

PARENT PERMISSION FOR MINOR TO PARTICIPATE IN RESEARCH

Developing Sixth-Grade Students' Number Sense through Number Talks

Dear Parent/Guardian,

My name is Yoshiko Okamoto and I am a mathematics teacher at Hoover Middle School and a doctoral student in the Graduate School of Education at the University of California, Los Angeles (UCLA). I am working with my faculty advisor, Professor Megan Franke, on a research study for my doctoral dissertation.

Your child was selected as a possible participant in this study because he or she is a student in Ms. X's sixth-grade advisory class at Hoover Middle School. Your child's participation in this research study is voluntary.

Why is this study being done?

With the Common Core, students' mathematical number sense and understanding of the operations is critical. We are piloting an opportunity during advisory (15 minutes a day) to support students in developing their number sense and operations skills and understanding through Number Talks. Number Talks are an approach to engaging students together in exploring and sharing their ideas of number relations and are being used across the US.

As part of this pilot, I would like to collect data on how participation in these Number Talks supports student learning and learn more about how to implement them effectively with our middle school students.

What will happen if my child takes part in this research study?

I am asking that you allow your child's work in the class and beginning and ending written Common Core number sense and operations assessments be used as data in my dissertation study. I will also be video-recording the classroom Number Talks.

How long will my child be in the research study?

Number talks will occur during the advisory class period for 15 minutes a day.

Are there any potential risks or discomforts that my child can expect from this study?

There are no foreseeable risks or discomforts to your student from participating in this study.

What other choices do I/my child have if my child does not participate?

If you choose not to allow your child to participate in the study, s/he will still receive regular classroom instruction, but instead of the written Common Core number sense and operations assessments, s/he will be administered the district's math facts assessments and his/her work from class will not be included in the study. Your child will be seated in a section of the classroom where s/he will not appear in the video recordings. There will be no consequences for not participating in the study and no one will need to know who is and is not allowing me to use data.

Are there any potential benefits to my child if he or she participates?

There will be no direct benefit to your student from participating in this study. However, the information gained from this research may help education professionals better understand how Number Talks can be implemented in middle schools to help students develop number sense and support the implementation of the Common Core Standards for Mathematics.

Will information about my child's participation be kept confidential?

Any information that is obtained in connection with this study and that can identify your child will remain confidential. It will be disclosed only with your permission or as required by law. Confidentiality will be maintained by means of coding data without any personal identifiers. Assessments will be stored in a locked filed cabinet that is accessible only to me and computer files will be password protected. Video recordings will be used only for my research to help me recall what happened during the Number Talks and will only be seen by me and my university advisor. Study participants will not be able to review, edit, or erase the video recordings. All records including video recordings will be destroyed after data have been analyzed.

What are my and my child's rights if he or she takes part in this study?

- You can choose whether or not you want your child to be in this study, and you may withdraw your permission and discontinue your child's participation at any time.
- Whatever decision you make, there will be no penalty to you or your child, and no loss of benefits to which you or your child were otherwise entitled.
- Your child may refuse to answer any questions that he/she does not want to answer and still remain in the study.

Who can I contact if I have questions about this study?

- **The research team:**
  If you have any questions, comments or concerns about the research, you can talk to the one of the researchers. Please contact:

Yoshiko Okamoto, Principal Investigator	Dr. Megan Franke, Faculty Sponsor
562-421-1213	310-206-3511
yokamoto@ucla.edu	mfranke@ucla.edu

- **UCLA Office of the Human Research Protection Program (OHRPP):**
  If you have questions about your child's rights while taking part in this study, or you have concerns or suggestions and you want to talk to someone other than the researchers about the study, please call the OHRPP at (310) 825-7122 or write to:

  UCLA Office of the Human Research Protection Program
  11000 Kinross Avenue, Suite 211, Box 951694
  Los Angeles, CA 90095-1694

You will be given a copy of this information to keep for your records.

I agree to allow my child to participate in the research study described above.

_____ Yes ____ No

I give consent for my child to be videotaped during this study:

_____ Yes ____ No

SIGNATURE OF PARENT OR LEGAL GUARDIAN

____
Name of Child

____
Name of Parent or Legal Guardian

_______________________________________ _______________________
Signature of Parent or Legal Guardian Date

Appendix B: Student Assent Form

UNIVERSITY OF CALIFORNIA LOS ANGELES

ASSENT TO PARTICIPATE IN RESEARCH

Developing Sixth-Grade Students' Number Sense through Number Talks

1. My name is Yoshiko Okamoto. I am a sixth-grade math teacher at Hoover Middle School and a doctoral student at UCLA.

2. I am asking you to take part in a research study because I am trying to learn more about how participation in Number Talks supports sixth-grade students' learning of number sense and operations skills and understanding.

3. If you agree to be in this study your work in the class and beginning and ending written number sense and operations assessments will be used as data in my dissertation study. You will also be video-recorded during the classroom Number Talks.

4. There are no risks involved in being a participant in this study.

5. There are no direct benefits from participation in the study. However, the information gained from this research may help me better understand how Number Talks can be implemented in middle schools to help students develop number sense and support the implementation of the Common Core Standards for Mathematics.

6. Please talk this over with your parents before you decide whether or not to participate. We will also ask your parents to give their permission for you to take part in this study. But even if your parents say "yes" you can still decide not to do this.

7. If you decide not to participate in the study, you will still be able to participate in regular classroom activities, but instead of the Common Core number sense and operations tests you will take district math facts assessments and your classwork will not be used as data in my study. You will be seated in a section of the classroom where you will not be video-recorded.

8. Remember, being in this study is up to you and no one will be upset if you don't want to participate or even if you change your mind later and want to stop. Your decision to participate in the study or not participate will have no effect on your grade in class.

9. You can ask any questions that you have about the study. If you have a question later that you didn't think of now, you can call me at 562-421-1213 or stop by my classroom, Room 303.

10. Please indicate below whether you agree to participate in the study and whether you agree to be videotaped. Then sign below. You and your parents will be given a copy of this form after you have signed it.

I agree to participate in the research study described above. _____ Yes _____ No

I give consent to be videotaped during this study. _____ Yes _____ No

____ ____________________

Name of Student Date

REFERENCES

Australian Education Council. (1991). *A national statement on mathematics for Australian schools*. Melbourne, Austrailia: Curriculum Corporation.

Beishuizen, M. (2001). Different approaches to mastering mental calculation strategies. *Principles and Practices in Arithmetic Teaching*, 119–130.

Black, P., & Wiliam, D. (1998). *Inside the black box: Raising standards through classroom assessment*. London: Granada Learning Assessment.

Blote, A. W., Klein, A. S., & Beishuizen, M. (2000). Mental Computation and Conceptual Understanding. *Learning and Instruction, 10*(3), 221–247.

Boaler, J. (1998). Open and closed mathematics: Student experiences and understandings. *Journal for Research in Mathematics Education*, *29*, 41–62.

Boaler, J. (2008). *What's math got to do with it?*. New York, NY: Penguin Group.

Boaler, J. (2013). *EDUC115N: How to learn math*. Stanford, CA. Retrieved from https://www.class-central.com/mooc/917/stanford-university-educ115n-how-to-learn-math

Brownell, W. A., & Moser, H. E. (1949). *Meaningful vs. mechanical learning: A study in grade iii subtraction*. Durham, NC: Duke University Press.

Carpenter, T. P., Fennema, E., Peterson, P. L., Chiang, C.-P., & Loef, M. (1989). Using knowledge of children's mathematics thinking in classroom teaching: An experimental study. *American Educational Research Journal*, *26*(4), 499–531.

Carpenter, T. P., Franke, M. L., & Levi, L. (2003). *Thinking mathematically: Integrating arithmetic and algebra in elementary school*. Portsmouth, NH: Heinemann.

Chapin, S. H., O'Connor, M. C., & Anderson, N. C. (2009). *Classroom discussions: Using math talk to help students learn, grades K-6*. Sausalito, Calif.: Math Solutions.

Clements, D. H., & Sarama, J. A. (2009). *Learning and teaching early math: The learning trajectories approach*. New York: Routledge.

Common Core Standards Writing Team. (2011). *Progressions for the Common Core State Standards in Mathematics*. Tucson, AZ: Institute for Mathematics and Education, University of Arizona.

Creswell, J. W. (2013). *Research design: Qualitative, quantitative, and mixed methods approaches* (Fourth Edition). Thousand Oaks, CA: SAGE Publications, Inc.

Daro, P. (2010, August). *Curriculum standards and differences: Lessons learned in the U.S.* Retrieved from corestandards.org

Daro, P. (2011). Formative principles of the Common Core Standards [Video]. Retrieved from http://serpmedia.org/daro-talks/index.html

Fawcett, H. P. (1938). *The nature of proof: A description and evaluation of certain procedures used in a senior high school to develop an understanding of the nature of proof*. Teachers College, Columbia university. Retrieved from http://www.getcited.org/pub/101170140

Finkelstein, N., Fong, A., & Tiffany-Morales, J. (2012). *College bound in middle school & high school? How math course sequences matter.* Sacramento, CA: The Center for the Future of Teaching and Learning at WestEd. Retrieved from http://www.cftl.org/documents/2012/CFTL_MathPatterns_Main_Report.pdf

Franke, M. L., Kazemi, E., & Battey, D. (2007). Mathematics teaching and classroom practice. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 225–256). Charlotte, N.C.: Information Age Publishing.

Fuson, K. C., & Briars, D. J. (1990). Using a base-ten blocks learning/teaching approach for first-and second-grade place-value and multidigit addition and subtraction. *Journal for Research in Mathematics Education*, 180–206.

Givvin, K. B., Stigler, J. W., & Thompson, B. J. (2011). What community college developmental mathematics students understand about mathematics, Part II: The interviews. *The MathAMATYC Educator, 2*(3), 4–18.

Good, T. L., & Grouws, D. A. (1977). Teaching effects: A process-product study in fourth grade mathematics classrooms. *Journal of Teacher Education*, *28*(3), 49–54.

Good, T. L., Grouws, D. A., & Ebmeier, H. (1983). *Active mathematics teaching*. Longman New York. Retrieved from http://www.getcited.org/pub/102208063

Gray, E. M., & Tall, D. O. (1994). Duality, ambiguity, and flexibility: A "proceptual" view of simple arithmetic. *Journal for Research in Mathematics Education*, *25*(2), 116–140. http://doi.org/10.2307/749505

Greeno, J. G. (1991). Number sense as situated knowing in a conceptual domain. *Journal for Research in Mathematics Education*, *22*(3), 170–218. http://doi.org/10.2307/749074

Heritage, M. (2008). Learning progressions: Supporting instruction and formative assessment. *Washington, DC: Council of Chief State School Officers. Retrieved December*, *2,* 2009.

Heritage, M. (2010). *Formative assessment: Making it happen in the classroom*. Thousand Oaks, CA: Corwin Press.

Hiebert, J., & Carpenter, T. P. (1992). Learning and teaching with understanding. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics* (pp. 65–97). New York, NY, England: Macmillan Publishing Co, Inc.

Hiebert, J., Gallimore, R., Garnier, H., & Givvin, K. B. (2003). *Teaching mathematics in seven countries: Results from the TIMSS 1999 video study* (No. NCES 2003-013). Washington, D.C.: U.S. Department of Education, National Center for Statistics.

Hiebert, J., & Grouws, D. A. (2007). The effects of classroom mathematics teaching on students' learning. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 371–404). Charlotte, N.C.: Information Age Publishing.

Hiebert, J., & Wearne, D. (1993). Instructional tasks, classroom discourse, and students' learning in second-grade arithmetic. *American Educational Research Journal*, *30*(2), 393–425.

Hope, J. A., & Sherrill, J. M. (1987). Characteristics of unskilled and skilled mental calculators. *Journal for Research in Mathematics Education*, *18*(2), 98–111. http://doi.org/10.2307/749245

Inagaki, K., Hatano, G., & Morita, E. (1998). Construction of mathematical knowledge through whole-class discussion. *Learning and Instruction*, *8*(6), 503–526. http://doi.org/10.1016/S0959-4752(98)00032-2

Jacobs, V. R., Franke, M. L., Carpenter, T. P., Levi, L., & Battey, D. (2007). Professional development focused on children's algebraic reasoning in elementary school. *Journal for Research in Mathematics Education*, *38*(3), 258–288. http://doi.org/10.2307/30034868

Lampert, M. (2001). *Teaching problems and the problems of teaching*. New Haven, CT: Yale University Press.

Maclellan, E. (2001). Mental calculation: Its place in the development of numeracy. *Westminster Studies in Education*, *24*(2), 145–154.

Markovits, Z., & Sowder, J. T. (1988). Mental computation and number sense. In *Proceedings of the Tenth Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 58–64).

Markovits, Z., & Sowder, J. T. (1994). Developing number sense: An intervention study in grade 7. *Journal for Research in Mathematics Education, 25*(1), 4–29.

Markovits, Z., & Sowder, J. T. (1994). Developing number sense: An intervention study in grade 7. *Journal for Research in Mathematics Education,* 4–29.

Math Perspectives. (2007). Number talks. Retrieved from www.mathperspectives.com

Mcintosh, A., Reys, B. J., & Reys, R. E. (1992). A proposed framework for examining basic number sense. *For the Learning of Mathematics*, *12*(3), 2–44. http://doi.org/10.2307/40248053

Merriam, S. B. (2009). *Qualitative research: A guide to design and implementation*. San Francisco: Jossey-Bass.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: NCTM.

National Council of Teachers of Mathematics. (1991). *Professional standards for teaching mathematics*. Reston, VA: NCTM.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: NCTM.

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all*. Reston, VA: National Council of Teachers of Mathematics.

National Governors Association Center for Best Practices, Council of Chief State School Officers. (2010). *Common Core State Standards for Mathematics*. Washington D.C.:

National Governors Association Center for Best Practices, Council of Chief State School Officers. Retrieved from http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf

National Mathematics Advisory Panel. (2008). *Foundations for success: The final report of the National Mathematics Advisory Panel*. Washington, D.C.: U.S. Department of Education.

National Research Council. (1989). *Everybody counts: A report to the Nation on the future of mathematics education*. Washington, D.C.: National Academy Press.

National Research Council. (2001). *Adding it up: Helping children learn mathematics*. (J. Kilpatrick, J. Swafford, & Findell, B., Eds.). Washington, D.C.: National Academy Press.

Parrish, S. (2010). *Number talks: Helping children build mental math and computation strategies*. Sausalito, CA: Math Solutions.

Parrish, S. (2011). Number talks build numerical reasoning. *Teaching Children's Mathematics, 18*(3), 198–206.

Reys, B. J. (1985). Mental computation. *Arithmetic Teacher*, *32*(6), 43–46.

Reys, R., Reys, B., Emanuelsson, G., Johansson, B., McIntosh, A., & Yang, D. C. (1999). Assessing number sense of students in Australia, Sweden, Taiwan, and the United States. *School Science and Mathematics*, *99*(2), 61–70. http://doi.org/10.1111/j.1949-8594.1999.tb17449.x

Richland, L. E., Stigler, J. W., & Holyoak, K. J. (2012). Teaching the conceptual structure of mathematics. *Educational Psychologist*, *47*(3), 189–203.

San Diego City Schools. (2004). Middle level mathematics routine bank. Retrieved from http://www.sandi.net/cms/lib/CA01001235/Centricity/Domain/217/middle_level_bank.pdf

Schmidt, W. H., McKnight, C. C., Houang, R. T., & Wang, H. C. (2001). *Why schools matter: A cross-national comparison of curriculum and learning* (1st ed.). Jossey-Bass.

Siegler, R. S., Duncan, G. J., Davis-Kean, P. E., Duckworth, K., Claessens, A., Engel, M., … Chen, M. (2012). Early predictors of high school mathematics achievement. *Psychological Science*, *23*(7), 691–697.

Silver, E. A., & Stein, M. K. (1996). The Quasar Project: The "revolution of the possible" in mathematics instructional reform in urban middle schools. *Urban Education*, *30*(4), 476–521.

Sowder, J. T. (1990). Mental computation and number sense. *Arithmetic Teacher*, *37*(7), 18–20.

Sowder, J. T. (1992). Making sense of numbers in school mathematics. *Analysis of Arithmetic for Mathematics Teaching*, 1–51.

Stein, M. K., Engle, R. A., Smith, M. S., & Hughes, E. K. (2008). Orchestrating productive mathematical discussions: Five practices for helping teachers move beyond show and tell. *Mathematical Thinking and Learning*, *10*(4), 313–340. http://doi.org/10.1080/10986060802229675

Stein, M. K., & Lane, S. (1996). Instructional tasks and the development of student capacity to think and reason: An analysis of the relationship between teaching and learning in a reform mathematics project. *Educational Research and Evaluation*, *2*(1), 50–80.

Stigler, J. W., Givvin, K. B., & Thompson, B. J. (2010). What community college developmental mathematics students understand about mathematics. *MathAmatyc Educator*, *1*(3), 4–16.

Stigler, J. W., & Hiebert, J. (1999). *The teaching gap: What educators can learn from the world's best teachers*. New York: Free Press.

Thompson, I. (1999). Getting your head around mental calculation. In Thompson, Ian (Ed.), *Issues in teaching numeracy in primary schools* (pp. 145–156). Buckingham: Open University Press.

Threlfall, J. (2002). Flexible mental calculation. *Educational Studies in Mathematics*, *50*(1), 29–47.

Thurston, W. P. (1990). Mathematical education. *Notices of the American Mathematical Society*, 844–850.

Verschaffel, L., Greer, B., & De Corte, E. (2007). Whole number concepts and operations. In F. K. Lester, *Second handbook of research on mathematics teaching and learning* (pp. 557–628). Charlotte, N.C.: Information Age Publishing.

Wearne, D., & Hiebert, J. (1988). Constructing and using meaning for mathematical symbols: The case of decimal fractions. In J. Hiebert & M. J. Behr (Eds.), *Number concepts and operations in the middle grades* (pp. 220–235). Reston, VA: National Council of Teachers of Mathematics.

Webb, N., Franke, M. L., Ing, M., Wong, J., Fernandez, C., Shin, N., & Turrou, A. (2014). Engaging with others' mathematical ideas: Interrelationships among student participation, teachers' instructional practices, and learning. *International Journal of Educational Research*, *63*, 79–93. http://doi.org/10.1016/j.ijer.2013.02.001

Wiliam, D. (2011). *Embedded formative assessment*. Bloomington, IN: Solution Tree.

Wiliam, D., & Thompson, M. (2007). Integrating assessment with learning: What will it take to make it work? In C. A. Dwyer (Ed.), *The future of assessment: Shaping teaching and learning*. Mahwah, N.J.: Lawrence Erlbaum Associates. Retrieved from http://eprints.ioe.ac.uk/1162/

Zimba, J. (2011). *Examples of structure in the Common Core State Standards for Mathematical Content*. Retrieved from http://commoncoretools.me/2012/02/16/the-structure-is-the-standards/