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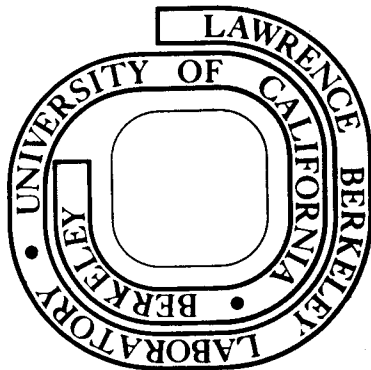
THREE DIMENSIONAL IMAGE
RECONSTRUCTION FROM AXIAL TOMOGRAPHY

L. T. Chang, B. Macdonald, and V. Perez-Mendez

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THREE DIMENSIONAL IMAGE RECONSTRUCTION FROM AXIAL TOMOGRAPHY †

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Introduction

In emission imaging as applied to Nuclear Medicine a number of different instruments and techniques which provide depth information have been devised such as the multiplane tomographic scanner with focussing collimator, the Tomocamera with rotating collimator, the coincidence positron camera, and coded aperture imaging.¹ These have in common that they give tomographic images of the object, that is, that images of a given object plane have that plane in focus with a background superimposed consisting of the blurred images of the other object planes. Various methods differ in how the out-of-focus images are blurred and considerable work has been done to determine blurring patterns which produce the fewest artifacts.

A tomographic image of a plane section through an object has a finite width slab of the object in focus, the thickness (depth of field) depending on geometry and detector resolution. If the out-of-focus background is removed from a collection of these tomographic planes which are separated by the system depths of field then the three dimensional object has been reconstructed to an accuracy determined by the lateral and longitudinal resolutions of the system.

We present here a specific method for producing transverse planes of tomographic images and a more general three dimensional image reconstruction method which removes the effect of off-plane activity from these axial tomograms. Although we use multiple single-pinhole images to make the tomograms, the reconstruction method is applicable to other type of tomographic images such as those from a scanner with focussing collimator or a positron camera.

Three Dimensional Imaging MethodFormation of the Tomographic Images

The tomograms needed in the reconstruction method are formed from a number of separate exposures of the object, using one pinhole at a time from a planar array of N_h pinholes and a planar detector a distance S_0 from the array. Using these N_h single-pinhole images in the following manner one can build up tomographic images in a computer on N_p transverse planes through the object parallel to the pinhole array and the detector and located at distances S_j , $j=1\dots N_p$, from the array. A given single-pinhole image in the detector is projected back through the known location of the pinhole onto each of the N_p tomographic planes. This is done for each of the pinhole images and the final tomographic images $t_j(\underline{r})$, $j=1\dots N_p$, are the sum of contributions from each of the N_h single-pinhole images. These are true tomographic images, $t_1(\underline{r})$, for example, having in focus all the points of the corresponding object plane $o_1(\underline{r})$ located at the same distance S_1 with all other object planes contributing out-of-focus backgrounds.

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Object Reconstruction from Tomographic Images

If the three dimensional object can be approximated by a finite number of planes of intensity distribution $o_i(\underline{r})$ located at distances S_i from the pinhole array plane, then, to reconstruct these o_i 's, one needs to consider the equations which describe the formation of the tomograms. We assume here that the pinholes in the array are small (delta functions). A further analysis shows that the reconstructions obtained with pinholes of finite width are the images one would get if one imaged each of the delta-function reconstruction planes $o_i(\underline{r})$ with a single pinhole of that width in the same geometry (S_i, S_0).

We can find the tomographic images $t_j(\underline{r})$ produced by the single-pinhole images by considering the response $h_{ij}(\underline{r}, \underline{r}')$ of the j th tomographic plane to a point source at \underline{r}' located in object plane i (Fig. 1). The figure shows that h_{ij} is a delta function of intensity N_h and that $h_{ij}(\underline{r}, \underline{r}')$ is just the pattern h of the N_h -hole array used, but displaced and with a size dependent on geometry. If $m_{ij} \equiv (S_i - S_j)/S_i$ is the size parameter and the pinholes in h are located at positions $\underline{r}_k, k=1, \dots, N_h$, we have

$$\begin{aligned} h_{ij}(\underline{r}, \underline{r}') &= h(\underline{r} - \underline{r}' S_j / S_i, m_{ij}) & i, j=1 \dots N_p & \quad (1) \\ &= \sum_{k=1}^{N_h} \delta(\underline{r} - (\underline{r}' S_j / S_i + m_{ij} \underline{r}_k)) \end{aligned}$$

The contribution of an object distribution o_i to tomographic plane j is just the convolution $o_i * h_{ij}$ and t_j is the sum of these contributions over the N_p object planes.

$$\begin{aligned} t_j(\underline{r}) &= \sum_i o_i(\underline{r}') h_{ij}(\underline{r}, \underline{r}') d^2 \underline{r}' & i, j=1, \dots, N_p & \\ &= \sum_i (S_i / S_j)^2 o_i(\underline{r}' S_i / S_j) h(\underline{r} - \underline{r}', m_{ij}) d^2 \underline{r}' & \\ &= \sum_i (S_i / S_j)^2 o_i(\underline{r}' S_i / S_j) * h(\underline{r}, m_{ij}) & \quad (2) \end{aligned}$$

For a given value of \underline{r} , position relative to the optic axis, equation (2) is a set of N_p equations in the N_p variables o_i . The t_j 's are combinations of single-pinhole image data and $h(\underline{r}, m_{ij})$ depends only on the pinhole locations in the array and the placement of the reconstruction planes.

Taking the Fourier transform of Eq.(2) and using the similarity theorem for Fourier transforms gives

$$T_j(\underline{u}) = \sum_{i=1}^{N_p} O_i(\underline{u} S_j / S_i) H_{ij}(\underline{u}) \quad j=1, \dots, N_p \quad (3)$$

where the quantities T_j, O_i, H_{ij} , Fourier transforms of the corresponding quantities of Eq. (2), are functions of the spatial frequency \underline{u} . To eliminate the j -dependence of the quantities O_i we let $\underline{u} = \underline{u}' / S_j$

$$T_j(\underline{u}' / S_j) = \sum_i O_i(\underline{u}' / S_i) H_{ij}(\underline{u}' / S_j) \quad i, j=1, \dots, N_p \quad (4)$$

For those (angular) spatial frequencies \underline{u}' for which the determinant $D(\underline{u}') \equiv |H_{ij}(\underline{u}' / S_j)|$ is not zero, Eqs. (4) can be solved for $O_i(\underline{u}' / S_i)$ and inverse Fourier transforms give the desired background-free images $o_i(\underline{r})$

The Determinant of the Reconstruction

For the determinant $D(\underline{u}')$ to be zero for some angular spatial frequency \underline{u}' means that the reconstructed transform images $O_i(\underline{u})$ are not determined at the spatial frequency \underline{u}' / S_i . From Eq. (1) we can get

$$H_{ij}(\underline{u}' / S_j) = \sum_{k=1}^{N_h} e^{-i2\pi \underline{u}' \cdot \underline{r}_k} [(S_i - S_j) / S_i S_j] \quad i, j=1, \dots, N_p \quad (5)$$

We find that the $N_p \times N_p$ determinant $D(\underline{u}')$ formed from these quantities is always zero for $\underline{u}' = 0$, since $H_{ij}(\underline{o}) = N_h$. This means that our reconstructions $o_i(\underline{r})$ are indefinite by an additive constant. This is not a problem if this is the only zero since this constant can be determined by some subsidiary condition, for instance, that $o_i(\underline{r})$ has no negative value.

A general solution for zeros of the determinant $D(\underline{u}')$ is complicated and has not been done. A fast Fourier transform calculation of $D(\underline{u}')$ for several arrays is shown in Fig. 2. As expected for all arrays $D(\underline{u}')$ is zero for $\underline{u}'=0$ and is small for small spatial frequencies. The regular array A (Fig. 2) has zeros elsewhere in the frequency plane but arrays B and C do not.

It is interesting to note that if a coded aperture exposure is used to make the tomographic images $t_j(\underline{r})$, instead of single-pinhole exposures, then $D(\underline{u}')$ vanishes identically and reconstruction by this method is not possible.²

Depth Resolution

The spacing of the reconstruction planes is determined by the tomogram depth of field δz (FWHM). This is given approximately by

$$\delta z = (d/r_m)(1 + S_i/S_0) S_i \quad (6)$$

where d is the diameter of the pinholes in the array and r_m is the distance in the aperture plane from the array center within which half the pinholes lies.

Results

The pinhole array of Fig. 2c was used in a computer simulation in order to test the reconstruction method. The object (Fig. 3a) was located in three planes and the computer generated the tomographic images in the same three planes, as shown. The reconstructions produced, using these tomographic images as input, appeared the same as the original object. The tomograms used, however, were made with no statistical variation in intensity of the object picture elements from one single-pinhole exposure to the next. In the more realistic case when the object picture elements vary statistically we find that there occurs a small component of background in the reconstructions in addition to the expected variation in intensity of the image elements. This is shown in Fig. 4 where an average of 400 events total from each object picture element has been collected, distributed statistically over the single-pinhole exposures, corresponding to a 5% fluctuation in object picture elements. The measured fluctuations in the reconstruction ran from 5.5% in o_3 to 8.5% in o_1 . Thus, the reconstruction method introduces a small amount of noise. If the total number of events collected is held fixed we find that the amount of introduced noise becomes larger as the number of pinholes is decreased. In addition to the relative insensitivity of the reconstruction to statistical noise, further work indicates that the method is not sensitive to errors in geometry, reconstructions of planes differing somewhat from actual object planes giving results close to the input object.

A xenon-filled multiwire proportional chamber (48x48cm, 2 mm resolution) was used with a radioactive source to make pinhole images. The object, a cross, circle, and triangle, was located on three planes, $S_1 = 20$ cm, $S_2 = 25$ cm, $S_3 = 30$ cm. S_0 was 39 cm. The array (Fig. 2b) was chosen to be circular (4 mm holes, 9 cm diameter) so as to maximize depth resolution with field of view. The tomograms and their reconstructions are shown in Fig. 5. There was 100,000 counts in each plane, bin width was 2.5mm, resolution was 6 mm. Although the choice of objects was such as to make their nature readily apparent from the tomograms alone, the reconstruction method has clearly removed artifacts and background successfully from the tomograms.

References

1. G.S. Freedman (ed.), Tomographic Imaging in Nuclear Medicine, (Society of Nuclear Medicine, New York, 1973).
2. L.T. Chang, B. Macdonald, and V. Perez-Mendez, Three Dimensional Image Reconstruction Using Pinhole Arrays, in Digest of Papers of the 1975 International Optical Computing Conference, Washington, D.C., April, 1975, pg. 61. Also- Lawrence Berkeley Laboratory report LBL 3388.

Figures

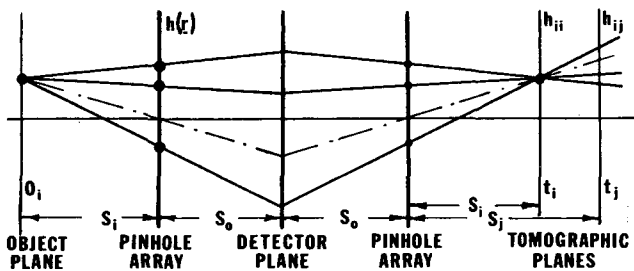


Fig. 1. Response of tomographic planes t_1, t_2 to a point source in plane O_i . The three left planes show the single pinhole exposures. The three right planes show the construction of the tomographic images from these exposures.

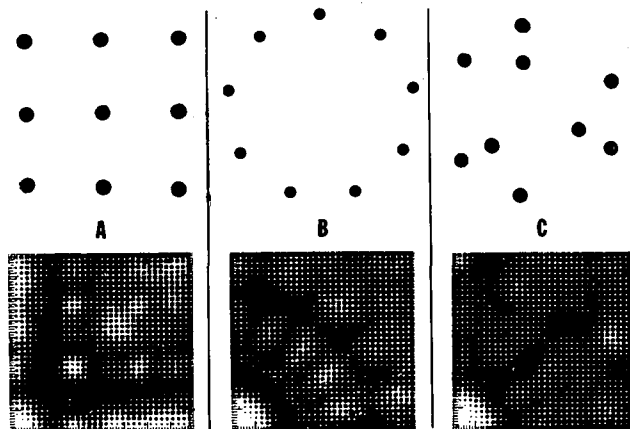


Fig. 2. Some pinhole arrays and the associated determinants $D(u')$ for three plane reconstructions ($S_0=10; S_1=8, S_2=10, S_3=12$).

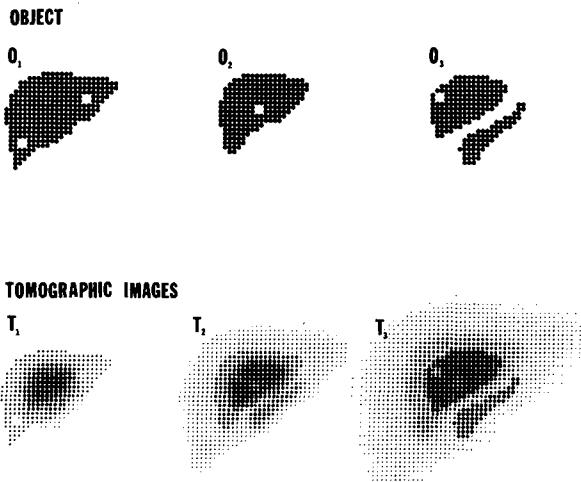


Fig. 3. Computer simulation of the reconstruction method. a) Object located in the 3 planes of Fig. 2 b) Tomographic images using the array of Fig. 2c.

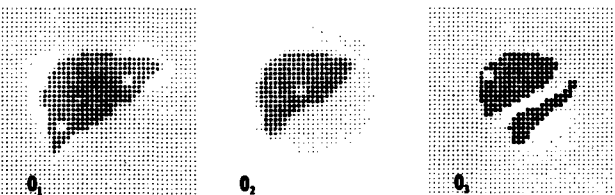


Fig. 4. Image reconstruction for the object of Fig. 3 when there has been a 5% statistical variation of object picture element intensity.

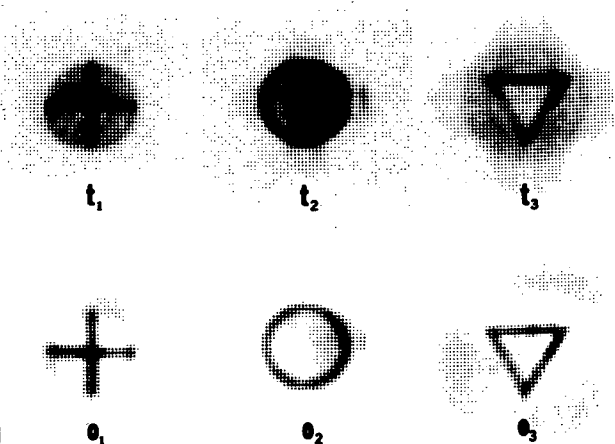


Fig. 5. Tomograms (t_1, t_2, t_3) and their reconstructions (O_1, O_2, O_3) using a wire proportional chamber and the pinhole array of Fig. 2b.

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