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The isotope and isotone shifts in the nuclear charge radius and the thickness of the neutron skin are calculated for nuclei along beta stability by means of the Droplet Model.

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A refinement of the Liquid Drop Model--called the "Droplet Model"--has recently been formulated.[1] This model is based on a comprehensive inclusion of all effects which enter when the statistical treatment of average nuclear properties is extended. One of the consequences of this approach is a nuclear mass formula which gives new insights into the relationships between various nuclear properties. For example, it provides us with a means for calculating both the isotope and isotone shifts in the nuclear charge radius and relating them to the existence and thickness of the neutron skin.

The Droplet Model is based on an expansion of nuclear properties about their Liquid Drop Model values in terms of the two small quantities ε and δ, where

$$\varepsilon = -\frac{1}{3}(\rho - \rho_0)/\rho_0 \quad , \tag{1}$$

$$\delta = (\rho_n - \rho_z)/\rho \quad . \tag{2}$$

Here ρ_n and ρ_z are the neutron and proton particle number densities. The quantity ρ is the total density (the sum of ρ_n and ρ_z) and ρ_0 represents the density of standard nuclear matter. The quantity ρ_0 is related to the nuclear radius constant r_0 by the expression

$$\rho_0 = (\tfrac{4}{3}\pi r_0{}^3)^{-1} \quad . \tag{3}$$

If the average values of the quantities ε and δ over the central region of the nucleus are specified by $\bar{\varepsilon}$ and $\bar{\delta}$ then the radii of the spheres corresponding to the proton and neutron density distributions are given by

$$R_z = r_0 \left[\frac{2Z}{(1-3\bar{\varepsilon})(1-\bar{\delta})}\right]^{1/3} \quad \text{and} \quad R_n = r_0\left[\frac{2N}{(1-3\bar{\varepsilon})(1+\bar{\delta})}\right]^{1/3} \tag{4}$$

For a nucleus with N neutrons, Z protons and a total of A particles we can define the quantity I by the expression

$$I = (N-Z)/A \quad , \tag{5}$$

and then the neutron skin thickness t (equal to $R_n - R_z$) is given approximately by

$$t = \frac{2}{3} r_0 A^{1/3}(I-\bar{\delta}) \quad . \tag{6}$$

The above relations are discussed in more detail in Sec. II of Ref. 1.

In order to calculate the charge radius R_z or the neutron skin thickness t we must know the values of $\bar{\varepsilon}$ and $\bar{\delta}$ as functions of N and Z. The appropriate expressions, which are derived in Ref. 1, are

$$\bar{\varepsilon} = \left(-2a_2 A^{-1/3} + L\,\bar{\delta}^2 + c_1 Z^2 A^{-4/3}\right)/K \quad , \tag{7}$$

$$\bar{\delta} = [I + (3c_1/8Q)Z^2 A^{-5/3}]/[1 + (9J/4Q)A^{-1/3}] \quad . \tag{8}$$

The various coefficients appearing in these expressions are listed below with estimates of their values:

$$\begin{aligned}
a_1 &= 15.677 \text{ MeV, volume energy coefficient,} \\
a_2 &= 22.0 \text{ MeV, surface energy coefficient,} \\
J &= 35 \text{ MeV, symmetry energy coefficient,} \\
K &= 300 \text{ MeV, compressibility coefficient,} \\
L &= 99 \text{ MeV, density-symmetry coefficient,} \\
M &= 4.5 \text{ MeV, symmetry anharmonicity coefficient,} \\
Q &= 25 \text{ MeV, effective surface stiffness,} \\
c_1 &= 0.745 \text{ MeV, Coulomb energy coefficient,} \\
\text{where } c_1 &= 3e^2/5r_0 \text{ and} \\
r_0 &= 1.16 \text{ fm, the nuclear radius constant.}
\end{aligned} \tag{9}$$

The values given here reproduce nuclear sizes better than the values given in Ref. 1, which were related to earlier adjustments of parameters to nuclear masses. Work is in progress toward determining the single set of parameters best suited for predicting all average nuclear properties.

The coefficients in the list above, some of which may be unfamiliar, are defined in the following way: The expansion of the energy per particle $\bar{e}$ of infinite nuclear matter in powers of ε and δ^2 is written as

$$\bar{e} = -a_1 + J\delta^2 + \frac{1}{2}(K\varepsilon^2 - 2L\varepsilon\delta^2 + M\delta^4) \quad . \tag{10}$$

Similarly the expansion of the surface tension coefficient σ in powers of the neutron skin thickness t is written as

$$4\pi r_0^2 \sigma = a_2 + Q(t/r_0)^2 \quad . \tag{11}$$

To illustrate the nature of the results which one obtains using the Droplet Model let us consider nuclei for which

$$I = 0.4\ A/(200 + A) \quad . \tag{12}$$

This expression--due to Green[2]--provides an approximation to the valley of beta stability.

Figure 1 is a plot of the neutron skin thickness t as a function of the particle number A for nuclei along beta stability. This curve was obtained from Eqs. (5), (6), (8), and (12). It shows, for example, that in the mass region $A \approx 200$ the radius of the neutron distribution is expected to be, on the average, 0.3 fm larger than that of the proton distribution.

There is a vast literature concerning theoretical and experimental discussions of the possible existence of a neutron skin for neutron rich, heavy nuclei. This literature ranges from the early speculations of Johnson and Teller[3] through the review of Wilkinson[4] (see also the references listed there) to the more recent papers, one by Burhop et al.[5] which discusses the interpretation of recent experiments with K^- mesons and another by Auerbach et al.[6] which determines the distribution of the excess neutrons from a study of isobaric analog resonances. To date both theoretical and experimental investigations have given a variety of results. Sometimes the neutron skin is said to be large and other times it is said to be small or zero. The neutron skin predicted in the present work is relatively small when compared with the spatial extent of the surface itself. In the mass region of $A \approx 200$ we find that the neutron skin is only 0.3 fm thick as compared with the 2.4 fm thickness of the diffuse surface region. However, this 0.3 fm neutron skin thickness is by no means negligible when compared with the 1.0 fm skin thickness that would result if the <u>entire</u> neutron excess of these nuclei were in the surface (thus making the neutron and proton bulk densities equal). It should be stressed that the Droplet Model predicts the neutron skin thickness to be expected on the basis of statistical considerations alone. One expects that the skin thickness of any particular nucleus will also depend on the particular single particle states which are occupied.

The anomalous behavior of the isotope and isotone shifts in the nuclear charge radius is related to the existence of a neutron skin. The quantity usually considered in these discussions is

$$\delta\langle r_z^2\rangle / \delta\langle r_z^2\rangle_{standard} \quad . \tag{13}$$

The quantity $\delta\langle r_z^2\rangle$ is the change in the expectation value of r^2 for the proton distribution when a neutron or proton is added. In our case

$$\delta\langle r_z^2\rangle = \frac{3}{5}\left[R_z^2\,(N+1) - R_z^2(N)\right] \quad . \tag{14}$$

The quantity $\delta\langle r_z^2\rangle_{standard}$ is defined in exactly the same way except that one uses for R_z the standard expression

$$R_z = 1.2\ A^{1/3}\ \mathrm{fm} \quad . \tag{15}$$

The quantity of interest (13) is plotted in Fig. 2 as a function of the mass number A. The dashed line in the lower part of the figure is the Droplet Model prediction for nuclei along beta stability for the case where $\Delta N = 1$. It is seen to be almost constant at a value of about one half. The circles are experimental points referring to atomic isotope shifts from the compilation of Brix and Kopfermann[7] and the triangles refer to experimental points given in Elton's[8] compilation of isotope shifts in mu-mesic atoms. The upper part of

Fig. 2 is again a plot of the quantity (13) but this time for the case $\Delta Z = 1$. The dashed line which is approximately constant at a value of about one and one half, is the Droplet Model prediction for nuclei along beta stability and the experimental points are from a review paper by Quitmann.[9] The experimental points show considerable scatter which is presumably due to single particle and deformation effects. An extensive discussion of the microscopic origin of the isotope and isotone shift anomalies is contained in a paper by Uher and Sorensen[10] and a related work has recently appeared by Brueckner et al.[11] In a more detailed treatment than the one given here the large values in the mass region $A \approx 150$ could probably be reduced by correcting for the deformation of these nuclei. The experimental points shown in the lower part of the figure (the case $\Delta N = 1$) have the average value 0.67±0.52, where the ±0.52 is not a measure of experimental uncertainty but of the spread in the actual values. The average of the predictions in this plot is 0.56. In the upper part of the figure (the case $\Delta Z = 1$) the average value of the experimental points is 1.66±0.92 and the average predicted value is 1.41.

One of the most interesting aspects of treating isotope shifts by means of the Droplet Model was the discovery that previous explanations[7,12] of this effect in terms of the nuclear compressiblity are probably incorrect. In fact, in the calculations shown here nuclear compressibility tends to reduce the size of the isotope shift anomaly rather than be the cause of it. This is shown in Fig. 3 where the quantity (13) is plotted as a function of A for nuclei along beta stability. Both the cases $\Delta N = 1$ and $\Delta Z = 1$ are shown, each for three values of the compressibility K. As can be seen from the dot-dashed lines in the figure a substantial isotope and isotone shift anomaly exists even for incompressible ($K = \infty$) nuclei. In fact, allowing finite values of the compressibility tends to <u>decrease</u> the size of the anomaly. Thus, as nuclei become softer the value of the quantity (13) becomes more nearly unity for nuclei along beta stability.

The actual origin of the isotope shift anomaly and the role of compressibility in determining its size can be seen by analyzing the appropriate Droplet Model equations. Equation (4) may be expanded in terms of $\bar{\varepsilon}$ and $\bar{\delta}$ to read

$$R_z = r_0 A^{1/3} [1 + \bar{\varepsilon} - \frac{1}{3}(I-\bar{\delta})] \quad , \tag{16}$$

where use has been made of the fact that

$$Z = \frac{1}{2} A (1-I) \quad . \tag{17}$$

Equation (6) may then be used to give

$$R_z = r_0 A^{1/3} (1+\bar{\varepsilon}) - \frac{1}{2} t \quad . \tag{18}$$

This equation for R_z consists of two parts. The first part, $r_0 A^{1/3} (1+\bar{\varepsilon})$, is simply the matter radius of the nucleus in question. The value of the $\bar{\varepsilon}$ which appears here may be calculated from Eq. (7). It represents the dilatation of the nucleus under the influence of three specific effects which correspond to the terms in the parenthesis of this equation (see Ref. 1). The first term, $-2a_2 A^{-1/3}$, describes the squeezing of the nucleus by the surface tension. Since this term does not differentiate between the addition of neutrons and protons it does not contribute to the isotope and isotone shift anomalies. The second term, $L\bar{\delta}^2$, describes the dilatation of the nucleus caused by the neutron excess. The last term, $c_1 Z^2 A^{-4/3}$, describes the dilatation due to Coulomb repulsion. The earlier explanations of the isotope shift anomaly considered only this last term and it is true that if this term dominated then a finite compressibility would tend to increase the size of the isotope shift anomaly. If the effects of the second and third terms canceled then the size of the anomaly would be independent of the value of K. In the work presented here we find that the second term in the parentheses is the most important and consequently we have the interesting result shown in Fig. 3 that a finite compressibility actually tends to reduce the size of the anomaly. Clearly the origin of the isotope and isotone shift anomaly lies not in the first term of Eq. (18) which gives the effect of finite compressibility but rather in the last term which simply represents the fact that the charge radius is smaller than the matter radius by half the thickness of the neutron skin. To see how the neutron skin thickness gives rise to the isotope and isotone shift anomalies we must examine the appropriate Droplet Model equations. If we solve Eq. (8) for the quantity $(I-\bar{\delta})$ and substitute this into (6) we find that

$$t = r_0[(3J/2Q)\bar{\delta} - (c_1/4Q)Z^2 A^{-4/3}] \quad . \tag{19}$$

It can be seen from this equation that adding a neutron to a nucleus increases $\bar{\delta}$ and A which causes an increase in t and consequently a reduction in R_z. Adding a proton has the opposite effect since this reduces $\bar{\delta}$ and increases Z^2 more than it increases $A^{4/3}$. Thus, the anomalous behavior of both the isotope and isotone shifts is explained by this one simple expression which shows how the neutron skin thickness changes when a neutron or proton is added to a nucleus.

In concluding it seems useful to point out that in so far as the isotope and isotone shifts calculated here agree with the experimental results they provide indirect support for the validity of the neutron skin thickness calculations plotted in Fig. 1. In addition, Ref. 1 contains a discussion of how the neutron skin enters into the determination of the binding energy of nuclei. Indeed, the existence of a neutron skin as calculated here is essential to the understanding of both the sign and magnitude of the coefficient of the term proportional to $I^2A^{2/3}$ in the nuclear mass formula. The accumulating evidence in support of the results shown here makes more accurate experimental determination of the neutron skin thickness itself more important than ever before. Such experiments should be carried out throughout the periodic table in order to make it possible to distinguish between shell effects and the average behavior which is predicted.

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FOOTNOTES AND REFERENCES

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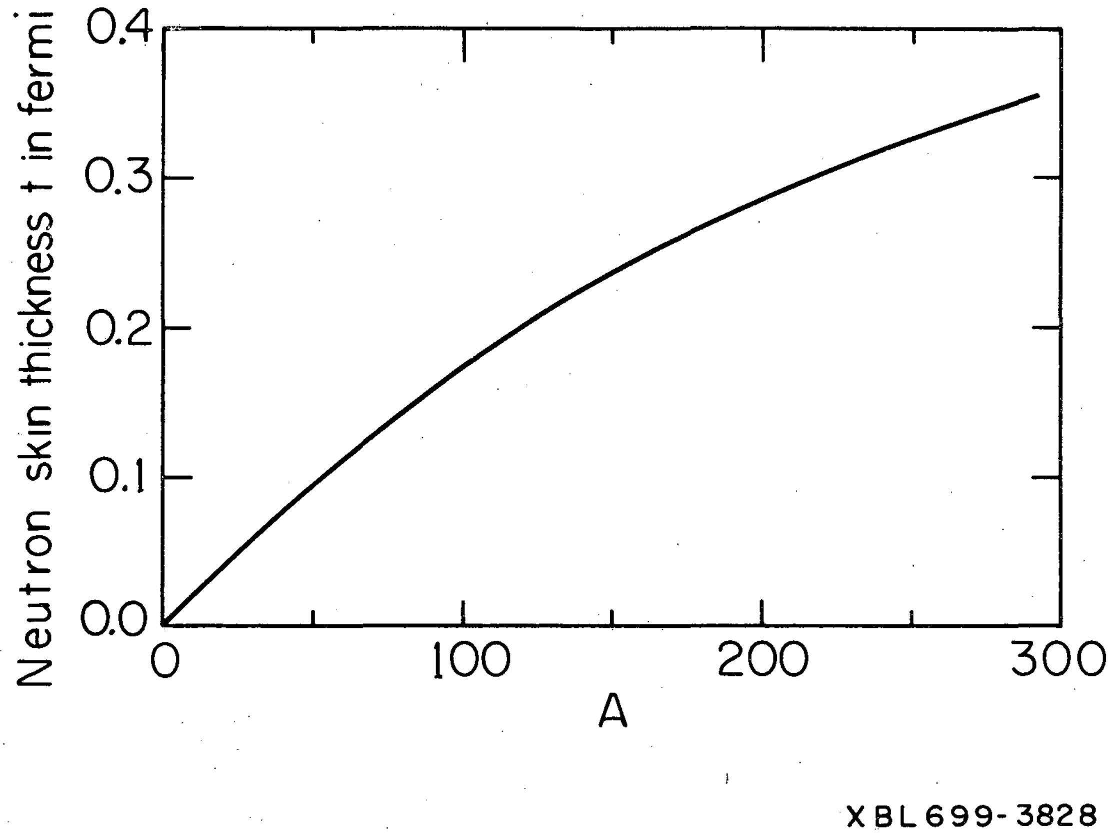
FIGURE CAPTIONS

Fig. 1. Neutron skin thickness t in fermi plotted as a function of the mass number A for nuclei along beta stability.

Fig. 2. The ratio of the change in $\langle r_z^2 \rangle$ to the change expected from the Liquid Drop Model is plotted against the mass number A. The upper part of the figure is for the case $\Delta Z = 1$ and the lower part is for the case $\Delta N = 1$. In both cases experimental points are shown as well as dashed lines representing the predictions of the Droplet Model for nuclei along beta stability.

Fig. 3. The ratio of the change in $\langle r_z^2 \rangle$ to the change expected from the Liquid Drop Model is plotted against the mass number A for nuclei along beta stability. Curves for $\Delta Z = 1$ and $\Delta N = 1$ are shown for three different values of the compressibility coefficient K.


Neutron skin thickness t in fermi
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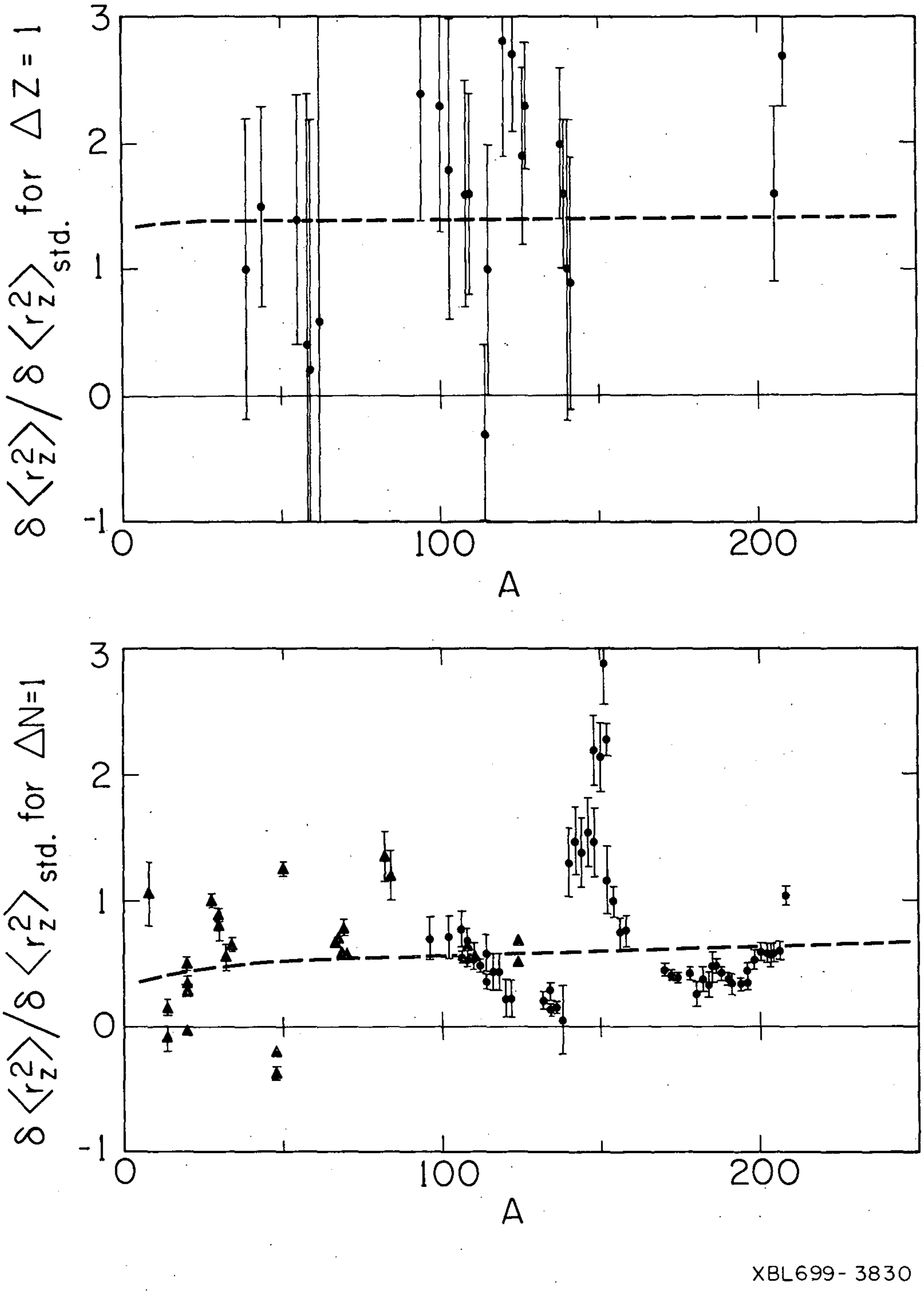
0.3
0.2
0.1
0.0
0
100
200
300
A
XBL699-3828

Fig. 1



$\delta\langle r_z^2\rangle/\delta\langle r_z^2\rangle_{std.}$ for $\Delta Z=1$
3
2
1
0
-1
0
100
200
A
$\delta\langle r_z^2\rangle/\delta\langle r_z^2\rangle_{std.}$ for $\Delta N=1$
3
2
1
0
-1
0
100
200
A
XBL699-3830

Fig. 2

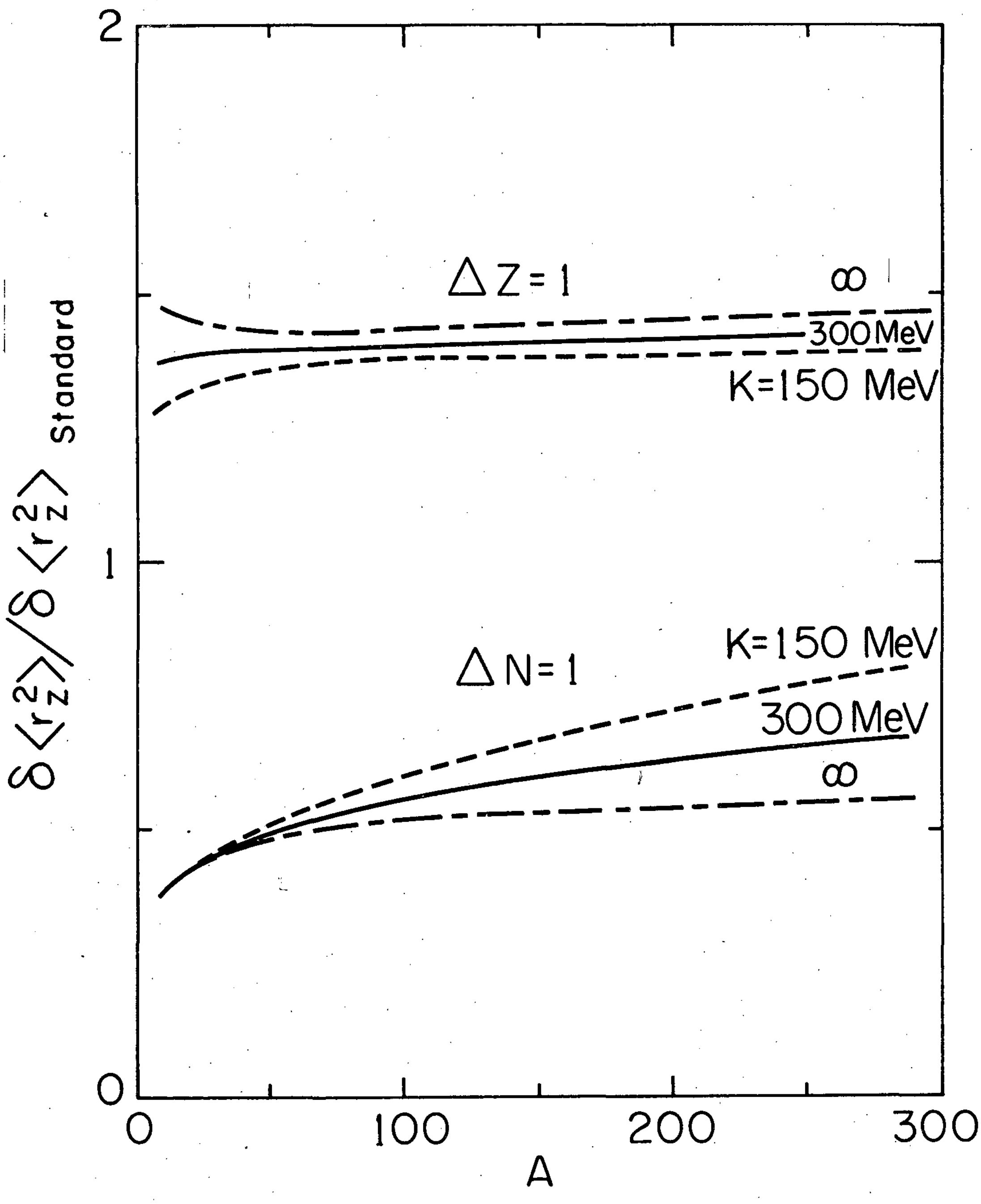


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Fig. 3

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