LOCATION
AND SPATIAL PRICING THEORY
WITH NONCONVEX TRANSPORTATION SCHEDULES

by

Konrad Stahl*

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UNIVERSITY OF CALIFORNIA, BERKELEY

AND UNIVERSITY OF DORTMUND, WEST GERMANY

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ABSTRACT

A model of consumer behavior is developed in which, due to a nonconvexity in the transportation cost of obtaining the desired commodity bundle, consumers are attracted to marketplaces offering a larger variety of commodities. Impacts on marketeers' equilibrium locations and pricing strategies are discussed in two examples. "Agglomeration economies" naturally emerge, leading sellers to associate spatially and charge higher (noncooperative) equilibrium prices than under locational monopoly, despite possibly increased competition. In general, equilibrium prices at one marketplace depend on the structure of the commodity bundle offered there. Differences in that bundle across marketplaces lead to a purely demand induced equilibrium price dispersion for the same commodity.

1 INTRODUCTION

It is often claimed by microeconomic theorists that location theory is excessively partial. Indeed, the notion of "spatial competition" so prominently figuring in that theory involves marketeers of physically identical commodities only. In view of the rich body of theory developed for multicommodity economic systems, this restriction to a one commodity world appears unsatisfactory.

More importantly, a casual observation of the spatial arrangement of markets reveals a considerable concentration of marketing activities for different commodities and services, relative to the spatial distribution of consumers. This suggests that oligopolistic interactions, leading marketeers to sell imperfect substitutes or complements jointly at one point in space are very intense, and possibly more important than the interactions between sellers marketing identical commodities at different points in space. Curiously enough, only the latter interactions have been concentrated upon in location theory.[1]

However, what brings sellers of different commodities and services to join each other is a nonconvexity in consumers' transportation schedules which so far has been disregarded in spatial economics: The typical consumer's outlays to haul a bundle of goods from the marketplace to the location of consumption is not proportional to the quantities of each and every commodity hauled, but decreasing in these quantities.[2] Similarly, transportation costs are decreasing in the quantity of services consumed on one shopping trip, and/or in the number of stores patronized in search for the best deal, provided that marketeers of these commodities and services are located sufficiently close to each other. All of this induces consumers to follow the "one stop shopping" pattern so well known to the business world but so far disregarded by the economist.[3]

In the present paper I shall bring out some impacts of this nonconvexity in transportation outlays in a fairly general model of consumers' behaviour in space and in two examples of marketeers' reactions to this: one in terms of their locational choices, and one in terms of their pricing decisions. All of this is done within a standard world of certainty, in which, however, the consumer's cost of hauling a bundle of commodities from some marketplace is considered invariant in the size of the commodity bundle.[4] Within this paradigm, it is very straight-forward to demonstrate such empirically appealing facts as that the attractivity of marketplaces to consumers increases with their size, defined by the number of commodities offered there, and that in consequence consumers may be willing to travel longer distances to larger marketplaces. Because of the surplus a consumer obtains when patronizing the latter, his decision may be upheld even if the prices charged in large marketplaces are higher than those charged for the same commodities in smaller ones. All of this provides incentives for marketeers to cluster in space, despite the increased competition they may face with a closer spatial association to marketeers selling substitutes. In fact, I will show that the Cournot equilibrium prices charged by sellers of substitutable commodities are higher in large marketplaces than in small ones, as long as the substitutes offered are not too close. In that case, the sellers' incentive to absorb some of the consumers' surplus is stronger than the competitive forces tending to drive prices down. This obviously inefficient behavior is reinforced when sellers of complementary commodities enter the marketplace.

Yet another result emerging from the simple paradigm is that noncooperative equilibrium prices for the same commodity will persistently differ across marketplaces at which different commodity bundles are offered. This implies an equilibrium price dispersion for the typical commodity, provided that such differences in the bundles supplied are upheld in equilibrium. There are several very plausible reasons, however, for this to be the case. It seems worth emphasizing that so far there has been no plausible explanation for the observation that some stores *persistently* sell their product at other prices than other stores. (For a most recent statement on this, see Varian [13].)

2 A MODEL OF CONSUMER BEHAVIOR IN SPACE.

Consider a consumer endowed with income R and a preference ordering over $N + 1$ physically different commodities i, $i \in \mathcal{N}$, $\mathcal{N} = \{0,\ldots,N\}$, $N \geq 2$. Let these preferences be representable by a utility index $u(x)$ where $x = (x_0,\ldots,x_N)$, $x \geq 0$.[5] That index is assumed to satisfy the usual properties, namely differentiability, strict monotonicity and strict quasiconcavity. In addition, suppose that only a proper subset $\mathcal{E}$ of the commodities is essential in the sense that whenever $i \in \mathcal{E}$, our consumer cannot survive without consuming a strictly positive quantity of i, whereas he will do so if he is unable to consume any positive quantity of an inessential commodity $i \in N \setminus E$.[6] Thus if we choose to represent graphically our consumer's utility index and select i to be an essential and j an inessential commodity, then indifference curves will cut the x_i, but not the x_j axis.

Let us now introduce space. Fix our consumer's location at y. Suppose that at y he can purchase and consume a proper subset of the $N + 1$ commodities, such as housing services, or commodities and services delivered to y by some other economic agent. For simplicity of exposition, let that subset consist of the first commodity only. In order to purchase other commodities the consumer must patronize some marketplace k, $k = 1,\ldots,K$ located at v^k. Again for expositional purposes, assume that at k, commodities $1,\ldots,k$ are offered. This implies a strict hierarchy of marketplaces, in terms of the bundles of commodities available there, with k referring to the size of the marketplace. Prices at y and v^k are denoted by $p^k \in \mathbb{R}^{N+1}_{++}$, $p^k = [p_0, p_1^k, \ldots, p_k^k, \infty, \ldots, \infty]$, and quantities available at y and v^k by $x^k \in \mathbb{R}^{N+1}_{+}$, $x^k = [x_0^k, x_1^k, \ldots, x_k^k, 0, \ldots, 0]$. If our consumer decides to patronize k and to purchase x^k, he incurs a transportation cost T which we assume to be strictly increasing in distance but invariant in x^k. This is but a simple specification of the idea that the consumer's cost of hauling his consumption bundle over a given distance is decreasing in the quanity of commodities hauled at a time. While this assumption would be questionable if applied to all possible consumption bundles $x^k \geq 0$, it does realisticly reflect typical consumer behavior at today's incomes, and transportation and consumption technologies.[7] In any case the customary assumption of proportionality of transportation rates in the quantity of each commodity hauled is, while preserving convexity, entirely unrealistic. We thus write $T = T(y,v^k)$.

Our consumer behaves as a price taker and utility maximizer. For expositional purposes, he is supposed to patronize at most one marketplace k per period of time. If he chooses to visit a marketplace he decides for that one at which he obtains the utility maximizing consumption bundle. Let C^k denote the consumer's consumption set conditional upon a visit of k. Then

$$C^k \equiv C[p^k, R-T(y,v^k)] = \\ = \{x^k \in \mathbb{R}_+^{N+1} \mid p^k \cdot x^k \leq R - T(y,v^k)\}, \ k = 1,\ldots,K .$$

Let $k = 0$ denote no visit of any marketplace. Let $p^k \equiv [p_0,\infty,\ldots,\infty]$, $k = 0$ and $x^k \equiv [x_0,0,\ldots,0]$, $k = 0$. The consumer's global consumption set C is then defined as

$$C \equiv \bigcup_{k=0}^{k} C^k .$$

We now can describe the consumer's behavior by the following two stage maximization process: $K + 1$ maximization decisions

$$\max_x u(x) \text{ st. } x \in C^k \quad k = 0,\ldots,K , \tag{1}$$

and

$$\max_k \{u(\hat{x}^k), k = 0,\ldots,K\} \tag{2}$$

where $\hat{x}^k$ denotes his optimal decison conditional upon a visit of k. This concludes the description of the model.

Observe that, while the conditional consumption sets C^k are convex, the global one, C is not, owing to the nonconvexity in the consumer's transportation outlay. The existence of solutions to problems (1) is guaranteed by the conditions that $R - T(y,v^k) > 0$, $k = 1,\ldots K$, and that prices are strictly positive, both of which are assumed to hold furtheron. Uniqueness follows from the strict convexity of preferences. We therefore move directly to a characterization of generic solutions to (2).[8]

<u>Proposition</u>: Suppose that $T(y,v^k) = T(y,v^\ell)$ for some $\ell = k+1,\ldots,K$ and that $p_i^\ell = p_i^k$ $i = 1,\ldots,k$. Then under the assumptions made above, $u(\hat{x}^\ell) \geq u(\hat{x}^k)$ and $u(\hat{x}^\ell) > u(\hat{x}^k)$ provided that $p_i^\ell < \tilde{p}_i^\ell$ for some i , $i = k+1,\ldots,\ell$ with

$$\tilde{p}_i^\ell \equiv p_j^k \frac{\frac{\partial u}{\partial x_i}(\hat{x}^k)}{\frac{\partial u}{\partial x_j}(\hat{x}^k)} \qquad \text{for some } j = 1,\ldots k . \qquad (3)$$

<u>Proof</u>: $T(y,v^k) = T(y,v^\ell)$ and $p_i^\ell = p_i^k$, $i = 1,\ldots,k$ guarantee that $\hat{x}^k \in C^\ell$. Hence $u(\hat{x}^\ell) \geq u(\hat{x}^k)$. Furthermore, $\hat{x}^\ell = \hat{x}^k$ would violate the relevant Kuhn-Tucker-Conditions for $p_i^\ell < \tilde{p}_i^\ell$. Hence $\hat{x}^\ell \neq \hat{x}^k$. But the uniqueness of solutions for each k guarantees that $\hat{x}^\ell$ is unique for ℓ . Hence $u(\hat{x}^\ell) > u(\hat{x}^k)$ whenever $p_i^\ell < \tilde{p}_i^\ell$ for some i , $i = K+1,\ldots,\ell$.

The proposition states that our consumer, faced with an identical transportation cost to any two marketplaces and identical prices for commodities offered in both, will prefer to visit the one offering a higher variety of commodities. He will definitely do so under the mild condition that the price of one commodity exclusively offered there is not so high as to drive down to zero his demand for that commodity. The result is easily demonstrated in a graphical example (cf Fig.1).

Fig 1: Illustration of the proposition

The following corollary is quite immediate:

Corollary:

(i) Let $p_i^{\ell} = p_i^k$, $i = 1,\ldots,k$ and $p_i^{\ell} < \tilde{p}_i^{\ell}$ for some i, $i = k+1,\ldots,\ell$, with $\tilde{p}_i^{\ell}$ as defined in (3). Then there is a $\tau > 0$, $\tau \equiv T(y,v^{\ell}) - T(y,v^k)$ such that $u(\hat{x}^{\ell}) > u(\hat{x}^k)$.

(ii) Let $T(y,v^k) = T(y,v^{\ell})$, and $p_i^{\ell} = \varepsilon\, p_i^k$, $i = 1,\ldots,k$ as well as $p_i^{\ell} < \tilde{p}_i^{\ell}$ for some i, $i = k+1,\ldots,\ell$, with $\tilde{p}_i^{\ell}$ as defined in (3). Then there is an $\varepsilon > 1$ such that $u(\hat{x}^{\ell}) > u(\hat{x}^k)$.

Proof: Both statements follow from the proof of the proposition and the observation that by the assumptions on the consumer's preferences, $\hat{x}(p,R-T)$ is continuous in all variables.

Thus the corollary ascertains that, provided that prices for the commodities offered only in the larger market places are not so high as to drive our consumer's demand down to zero, our consumer may end up patronizing the larger marketplace even if this involves a higher transportation outlay, or if prices for commodities offered in both the smaller and the larger marketplace are higher in the latter. This again is easily demonstrated in a graphical example.

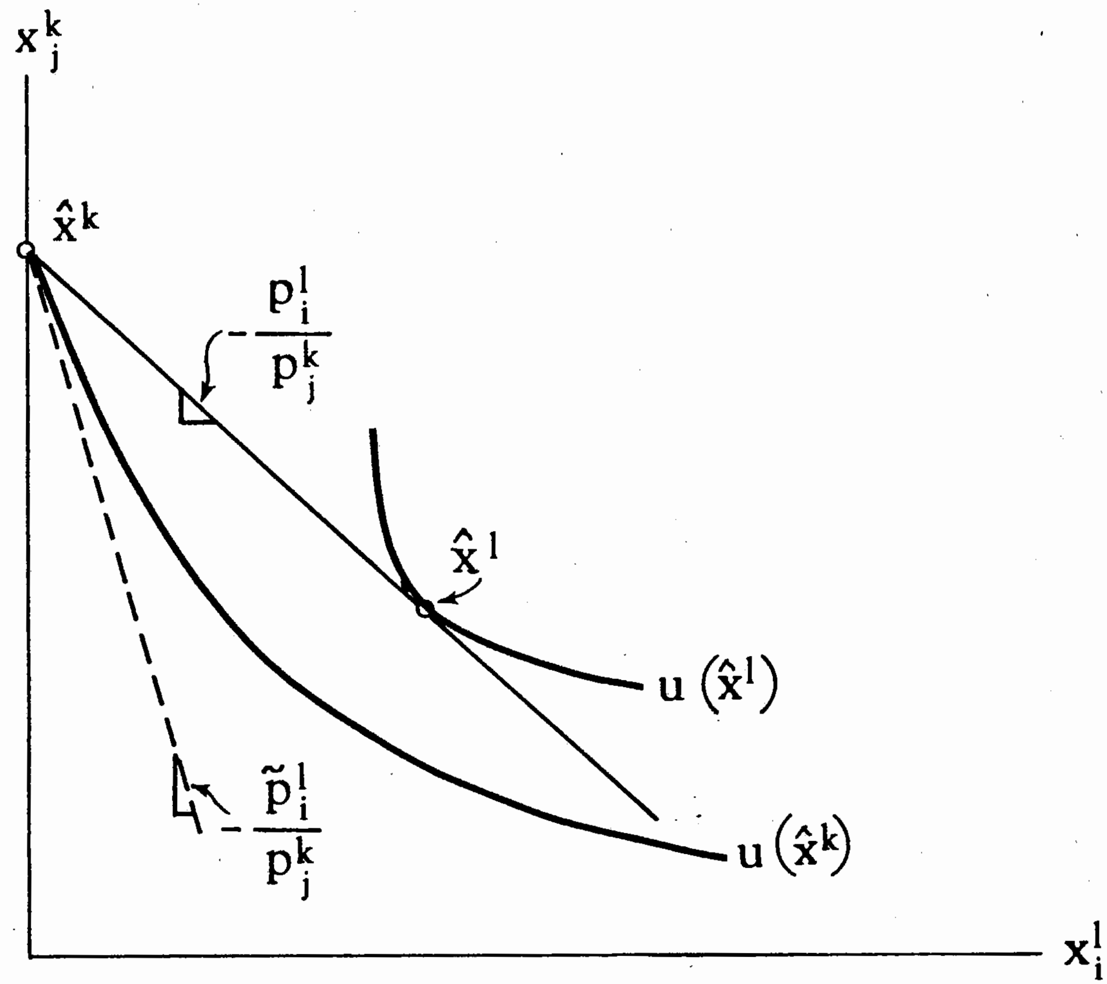

Fig. 1: Illustration of the Proposition

Figure 2: Illustration of the Corollary

In Fig. 2, the magnitude $\tilde{\tau}$ refers to the maximal difference (in terms of p_j^k) in transportation costs the consumer is willing to bear, in order to be availed of the larger marketplace's consumption opportunities. Similarly, $(\tilde{p}_j^k - p_j^k)$ refers to the maximal price increase the consumer would be willing to bear while continuing to patronize marketplace ℓ . We may conclude that under very mild conditions *large marketplaces are more attractive to consumers than smaller ones*.

All of this suggests immediate consequences on the locational and pricing behavior of marketeers of commodites and services.: A most direct one is that in turn, *large marketplaces may be attractive locations for marketeers* not only of commodities that are "complementary", in terms of consumers' preferences, to the ones already traded there, but also of those that are "substitutes". Another consequence is that in a large marketplace, *sellers may have an incentive to absorb the consumers' surplus generated from spatial association,* by increasing their asking prices at least if competitive forces in the marketplace are not strong enough. Both of these consequences may be realized, although in a different way, by a monopolistic seller of these commodities.

These and other conclusions are made more precise within the examples given in the next two sections. The treatment there should be considered preliminary. It is quite obvious that the conclusions hold under much more general conditions.

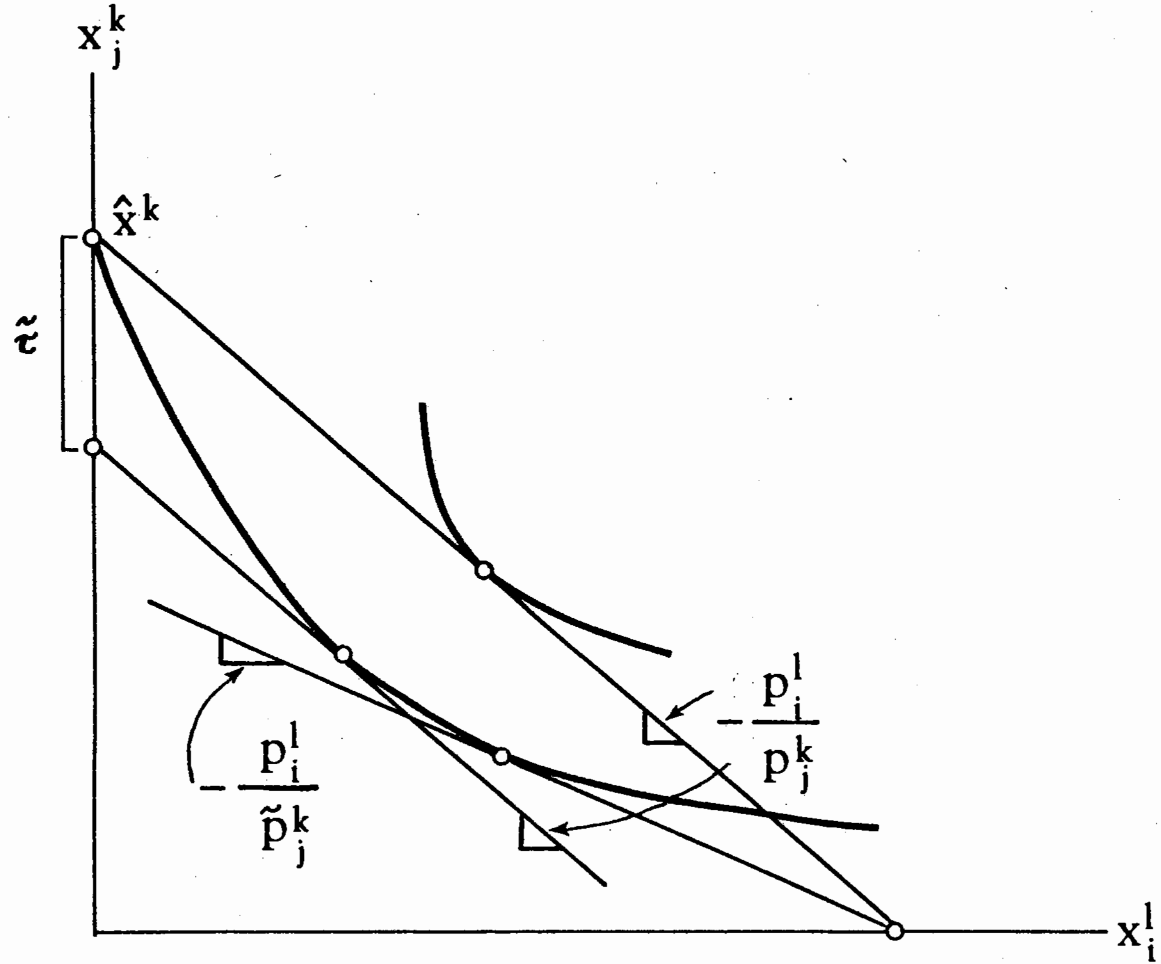

Fig. 2: Illustration of the Corollary

3. EXAMPLE I: SELLERS' LOCATIONS AT FIXED PRICES

The economy considered here involves, in addition to a numeraire good $i = 0$, two goods i, $i = 1,2$. Consumers are identical in preferences and income, R. The typical consumer's preferences are approximated by the utility index

$$u(x) = x_0 + \alpha(x_1 + x_2) - \frac{1}{2}[\beta x_1^2 + 2\gamma x_1 x_2 + \beta x_2^2] ,$$

$$\alpha > 0 , \beta > 0 , \beta^2 \leq \gamma^2 .$$

Commodities 1 and 2 are called complements if $\gamma < 0$ and substitutes if $\gamma > 0$. They are perfect substitutes if $\gamma = \beta$.[9]

The geographical space considered is linear and extends along an unbounded line. An infinite population of consumers is distributed at unit density along the line. The numeraire commodity may be purchased everywhere. The location problem involves two firms i, $i = 1,2$, with firm i selling commodity i at location i at constant (zero) cost and unit price.

Consumers' behavior is as in section 2. Their transportation outlays for a visit of some marketplace k are specified as $T(y,v^k) = c|y - v^k|$, $c > 0$. The firms are assumed to exhibit Cournot behavior. Let $\pi_i(\ell_i,\ell_{-i})$ denote firm i's payoff (sales) as a function of its and the other firm's locations. Then i selects the location $\ell_i^C \in (-\infty,+\infty)$ maximizing π_i for given ℓ_{-i}. A "Cournot equilibrium in locations" is a vector $\ell^C = (\ell_1^C , \ell_2^C)$ such that

$$\pi_i(\ell_i^C , \ell_{-i}^C) \geq \pi_i(\ell_i,\ell_{-i}^C), \quad i = 1,2 .$$

The example mainly serves to demonstrate that due to the attractivity of larger marketplaces to consumers, even sellers of substitutable commodities may consider profitable a joint location. The assumptions facilitate this aim without being unduly restraining. For example, the linearity of u in x_0 does away with income effects on x_1 and x_2; incorporating these would complicate matters without adding much insight. The same holds for the assumed symmetry of u in x_1 and x_2. Furthermore, assuming an unbounded geographical space is a convenient way of getting rid of "spatial competition" in the sense of Hotelling[6]: Any marketeer may establish a "spatial monopoly" if he finds it profitable to do so. It is obvious that conditions under which in this context firms will cluster will imply the same behavior in a market with effective spatial boundaries. The only assumption that appears to be effectively constraining is that location decisions are made at fixed prices. That is commented upon in the next section on pricing.

The unboundedness of the geographical space also relieves us of considering explicitely the firms' locations. Let X_i^k denote firm i's market demand (at unit price) if locating in a marketplace of size k, $k = 1,2$. Then a Cournot equilibrium in locations will involve a spatial merger of the two firms if and only if $X_i^2 \geq X_i^1$, $i = 1,2$. These quantities are obtained as follows: Let $\hat{x}^k(z)$ denote the typical consumer's optimal bundle if his best choice is visiting a marketplace of size k at distance z. Then, ignoring corner solutions,[10]

$$X_i^k = 2 \int_0^{\tilde{z}^k} \hat{x}_i^k(z)\,dz$$

where

$$\tilde{z}^k \equiv \{z \mid u(\hat{x}^k(z)) = u(\hat{x}^0)\}$$

refers to the distance at which consumers are indifferent between visiting a marketplace of size k and staying put. In the present example, the respective magnitudes are $\hat{x}_i^1 = (\alpha - 1)/\beta$, $i = 1,2$ and $\tilde{z}^1 = (\alpha - 1)^2/2\beta c$, and furthermore $\hat{x}_i^2 = (\alpha - 1)/(\beta + \gamma)^2$, $i = 1,2$ and $\tilde{z}^2 = (\alpha - 1)^2/(\beta + \gamma)c$. It follows that $X_i^2 \geq X_i^1$ if $(\beta + \gamma)^2 \leq 2\beta^2$. This inequality holds *always* if $\gamma \leq 0$, i.e. *if commodities* 1 and 2 *are complements*. However, *it may also hold if* $\gamma > 0$, i.e. *if the commodities are substitutes*, provided that γ is sufficiently small relative to β, i.e. substitutes are not too close.

These results may be rationalized at ease as follows: Observe first that, when joining the other seller, the demand $\hat{x}_i^k$ the typical seller faces from an individual consumer increases if the commodities are complements, and decreases if they are substitutes. However, the market area, and with it the number of consumers partronizing the larger marketplace always increases as long as the commodities involved are not perfect substitutes, i.e. as long as $\gamma < \beta$. While both effects work in the same direction if the commodities are complements, the latter positive *market area effect* may more than outweigh the negative *substitution effect* generated when sellers compete about the typical consumer's demand. In any case, a marketeer, in attracting a certain number of consumers, provides an *"externality"* to other marketeers if they choose to locate in the same marketplace. The very presence of that externality suggests a possible source of inefficiencies in marketeers' locational behavior.

Observe finally that, as long as commodites 1 and 2 are not perfect substitutes, the typical consumer's share of income spent at the marketplace increases in k . Thus, the more attractive the consumption opportunities in the marketplace, the less is spent locally on housing, or delivered commodities. While this observation is interesting in itself, it is due to an assumption implicit in the present example, namely that both, commodities 1 and 2 are pairwise substitutable to the numeraire commodity.

There are obvious generalizations of this example. A particularly attractive one is an increase in the number of commodities sold. In that case, sellers will concentrate under similar conditions, as long as the corresponding extensions in the typical consumer's consumption possiblities are not too much constrained by income. Another generalization could allow for an internalization of the externality generated from spatial association, and a welfare comparison of the monopolistic behavior induced by that, with the behavior of the noncooperative sellers.[11]

4. EXAMPLE II: SELLERS" PRICING AT FIXED LOCATIONS

Using the model developed in the last section, we shall compare sellers' pricing policies under the following different arrangements: At one marketplace

(i) supply of one commodity by one seller

(ii) supply of two commodities by two noncooperative sellers

(ii) supply of both commodities by one seller.

As it will turn out, the pricing policies will differ between these arrangements. Particularly intriguing will be the observation that, despite selling substitutes, competitors may end up charging higher prices than a monopolist selling both commodities. This, and other results are obtained at ease as follows (again, I neglect corner solutions):

Case (i): The typical consumer's demand, if going to a marketplace where i is sold, is $\hat{x}_i^1(p_i) = (\alpha - p_i)/\beta$. The market area $\tilde{z}^1$ is given by $\tilde{z}^1(p_i) = (\alpha - p_i)^2/2\beta c$. Thus seller i's revenues are $p_i(\alpha - p_i)^3/\beta^2 c$. Maximization with respect to p_i results in an optimal price $p_i^{1M} = \alpha/4$.

Case (ii): Here, the typical consumer's demand is given by $\hat{x}_i^2(p_i,p_{-i}) = [\beta(\alpha - p_i) - \gamma(\alpha - p_{-i})]/(\beta^2 - \gamma^2)$, and the sellers' market area $\hat{z}^2$ by

$$\tilde{z}^2(p_1,p_2) = [\beta(\alpha - p_1)^2 + 2\gamma(\alpha - p_1)(\alpha - p_2) + \beta(\alpha - p_2)^2]/2(\beta^2 - \gamma^2)c$$

Seller i's revenues are given by $p_i\hat{x}_i^2(p_i,p_{-i})\cdot \tilde{z}^2(p_1,p_2)$. Again, both sellers are assumed to exhibit Cournot behavior. Thus seller i chooses that p_i maximizing his revenues, conditional upon a given p_{-i} . A "Cournot equilibrium in prices" is a vector $p^{2C} = (p_1^{2C}, p_2^{2C})$ such that

$$\pi_i(p_i^{2C}, p_{-i}^{2C}) \geq \pi_i(p_i^2, p_{-i}^{2C}) , \ i = 1,2$$

where π_i refers to firm i's payoff (revenues) as a function of its and other firm's prices charged. Even in this simple example, it is not possible to compute all Cournot equilibrium price vectors (if there is more than one such vector at all). I therefore will restrict myself to the quite sensible case where $p_1^{2C} = p_2^{2C}$. In that case,

$$p_i^{2C} = (\beta - \gamma)\alpha/(3\beta - 2\gamma) .$$

<u>Case (iii)</u>: $\hat{x}_i^2(p_i, p_{-i})$ and $\tilde{\pi}^2(p_1, p_2)$ are as in case (ii). The monopolist's revenues, however, are $[p_1\hat{x}_1^2(p_1,p_2) + p_2\hat{x}_2^2(p_1,p_2)]\ \tilde{\pi}^2(p_1,p_2)$. Maximization of these with respect to p_1 and p_2 , assuming that $p_1 = p_2$ results in $p_1^{2M} = p_2^{2M} = \alpha/4$.

Comparing these results, we observe first that $p_i^{1M} = p_i^{2M}$, $i = 1,2$, i.e. that the prices charged by a "one commodity spatial monopolist" are identical to those charged by a "two commodity spatial monopolist", and this, possibly owing to the symmetry assumption on consumers' preferences, irrespectively of the value γ takes on. Thus in the present example a *spatial monopolist offering a larger range of commodities at one and the same marketplace does not absorb the consumers' surplus generated thereby,* by increasing the prices charged for these commodities.

Second, we observe that $p_i^{2C} \gtrless p_i^{2M} = p_i^{1M}$ if $\beta \gtrless 2\gamma$. As one would expect, *Cournot oligopolists will always charge higher prices than the one, or two commodity spatial monopolist* if the two commodities sold are *complementary. But they may also do so, if the two commodities are substitutable to each other, provided that they are not too close substitutes.*

The latter result can again be given a straight-forward explain-ation: The impact of a seller's price change on his market demand may again be decomposed into the two familiar effects: a substitution effect, and a market area effect. Both effects are observed by firms, at least in a small marketplace. While the duopolist gives due account to the fact that the typical consumer, when faced with a price increase, rearranges his consumption bundle, he does not take into consideration that marginal consumers ceasing to patronize the marketplace will not only negatively affect his, but also his competitor's demand. Putting things differently, we may consider the number of consumers patronizing the marketplace or, what amounts to the same, the market area a public commodity to the oligopolists assembled in the marketplace. (Note that $z_i^k = z_j^k = z(p^k)$ for all firms i in k.) Decreases in any one seller's asking price increase the value of that commodity to all sellers. Only the impact on his own revenues is accounted for by the noncooperative seller, however. Although it is quite dangerous to draw welfare implications from such a simple model, one nevertheless might observe that in an appropriately closed version of the model the duopolists' pricing behavior may turn out to be not only less efficient, but also less egalitarian than the two commodity monopolist's one.

There is yet another quite intriguing conclusion to be drawn from this simple model: If there are incentives to offer identical commodities in market places of different size, (or commodity bundles offered), then there *are purely demand induced reasons for charging different prices for the same commodity in different marketplaces.*[12] Thus in an otherwise "homogeneous" world we will observe prices for identical commodities not to be uniform anymore.

Observe finally that the results derived for cases (i) and (ii) link up to the main one derived in the last section: The condition $\gamma^2 < \beta(\beta - 2\gamma)$ under which the two sellers join implies the condition $2\gamma < \beta$ under which the sellers, if joined in one marketplace charge higher prices than if locating on their own. It follows that the former condition can be weakened, if in the location problem we give due account to price adjustments.

5. RELATIONSHIPS TO EXISTING THEORY

The paradigm most obviously relates to Losch's [7] central place theory. In that theory, a hierarchy of marketplaces is generated by superimposing patterns of sellers' locations determined separately for each commodity. In the development of each single pattern, spatial competition in Hotelling's [6] sense figures prominently. The generation of the central place hierachy is strange, however.[13] Presently relevant assumptions implicit in this are first, that consumer demands for each commodity are independent of the availability of, and the prices charged for other commodities, and second, that demands are functions of delivered prices per unit of each one commodity, rather than a function of f.o.b. prices and the transport cost of obtaining an entire consumption bundle. These assumptions imply that the market demands realized by sellers of different commodities are unrelated and therefore that these sellers have no incentive to agglomerate in space.[14] By contrast, such an incentive exists in the present model, even if the first of the above assumptions is maintained: It is easy to check that if $\gamma = 0$, i.e. even if the typical consumer's demand for one commodity is unrelated, in terms of his preferences, to the demand for the other one, the Cournot equilibrium in locations involves a spatial association of sellers (and the Cournot equilibrium in prices higher prices charged under spatial association than under spatial monopoly).

We are left with the problem of incorporating spatial competition into the present paradigm, and with the question of how a hierarchy of central places is generated that makes Losch's paradigm empirically so appealing. The former problem awaits further research. There is a straight-forward answer to the latter question , however. In Losch's model the "equilibrium spacing" of marketeers varies with the typical consumer's demands and the typical marketeer's cost of providing the commodity in question. In the present model variations, across commodity bundles, in the consumer's optimal frequency of shopping trips emerge as another reason for this. These frequencies do not only vary with variations in the consumer's demands for the different commodities, but also with his physical and capital cost of storing, as well as the transportation cost of obtaining them.[15] Yet other rationalizations can easily be given with slight modifications of the model.[16]

In any case, the hierarchy of marketplaces naturally obtained in such modifications will exhibit properties quite distinctive to the ones derived by Losch. The principal ones are that *larger marketplaces will command larger market areas, and that prices for identical commodities will persistently differ between marketplaces offering differing bundles of commodities*. Both results are empirically plausible and theoretically novel. It should finally be emphasized that, in contrast to the results derived from Losch's paradigm, the locational and price equilibrium patterns emerging from the present one have a sound behavioral foundation, and thus can be subjected to a welfare evaluation. That in turn is a necessary condition for the development of optimal policies influencing these equilibrium patterns.

6. CONCLUDING REMARKS

Although the model developed in the last sections is deceptively simple, we were able to derive quite a number of empirically appealing conclusions on the behavior of consumers and sellers in a spatial economy. Prominently figures a specification (out of many possible) of the notion of "*agglomeration economies*" to both, consumers and marketeers, associated with the spatial concentration of marketing activities. These economies are solely due to a nonconvexity in consumers' transportation outlays.[17] There are further conclusions also of some theoretical appeal: given that nonconvexity, relations between aggregate demands for different commodities cease to be based only on individual consumers' preference relations: commodities that are substitutes, in terms of the typical consumer's preferences, may appear as complementary in the aggregate demands observed at a location. Another conclusion is that even in the simplest of all spatial economies involving identical consumers and a uniform distribution of these consumers in space, noncooperative equilibrium prices and demands for physically identical commodities will differ across marketplaces offering different commodity bundles. The latter differences arise naturally in realistic modifications of the model.

Many extensions of the paradigm are possible. Some were specified in the preceeding sections. There is yet another important deficiency of the present model. It should be extended to incorporate the fact that marketing is a decreasing cost operation, together with free entry (and exit) of marketeers. It should be noted, however, that none of the essential results mentioned above would then have to be dismissed. It is hopefully not exaggerated to say in conclusion, that the approach suggested here opens an interesting avenue for positive and normative research on economies whose participants feel a bit uneasy staying on a pin's head for the rest of their lives.

FOOTNOTES

[1] Central place theory only appears to account for the former type of interactions. Yet only the latter type is developed formally. In particular, the forces leading marketeers to cluster in space are not endogenous in the model. See section 5 of this paper.

[2] These "economies of scale" were fairly quickly exhausted as long as consumers used to visit markets by foot. They are definitely not at today's mode of shopping by car.

[3] I recently have become aware, however, that Eaton and Lipsey [5] also pursue this idea, although with a different emphasis. See the discussion in section 5.

[4] In [11], I treat the case where consumers are imperfectly informed and marketeers react, in terms of their locational choices, to consumer's search for the best deal.

[5] In terms of the consumer's preferences, I do not distinguish between commodities differing only by location of availability. Although this is done customarily in the general equilibrium literature since Debreu [3], it is not very helpful. As is well known, such a differentiation may lead to nonconvexities in consumption sets and preferences (as well as technologies) that are inconsistent with standard assumptions of convex analysis. Conversely, in my approach convexity is preserved. Furthermore it falls out naturally that locations, being distinguished by the commodity bundles available, can be ordered in terms of their attractiveness to consumers or firms, once prices are known.

[6] A formal specification of that assumption is given by

$$i \in E \ , \quad \hat{x}_i = 0 \Rightarrow \nexists \ x \text{ such that } u(x) < u(\hat{x}) \ , \ \hat{x} = (x, \ldots, \hat{x}_i, \ldots x_N) .$$

[7] It should be mentioned, however, that the cost of collecting commodities in the marketplace is neglected here.

[8] Observe that the solutions to problem (2) were trivial in the case $E = N$.

[9] The formulation is due to Dixit [4].

[10] They are avoided if R is large relative to α , $\frac{1}{\beta}$ and $\frac{1}{\gamma}$.

[11] Two modes of internalization can be observed in reality: one is done by the seller himself: an example in point is the department store; another one is exercised by the shopping center developer who enforces sellers' collusive behavior. Yet the observed presence of many noncooperative sellers at most marketplaces suggests that there are definite limits to both modes of internalizing that externality: the possible inefficiency appears not to be fully removed in the marketplace.

[12] A discussion of such incentives is relegated to the next section.

[13] As mentioned before, only the spatial arrangement of marketplaces for one commodity and the related pricing questions have been subjected to a more thorough investigation, by Mills and Lav [8], Stern [12], Bolobás and Stern [1], or Capozza and Van Order [2], among many others.

[14] While the geometric rotation procedure used by Losch to generate agglomerations of sellers is motivated by consumers' (and producers') savings in transportation costs, neither consumers', nor sellers' cost vary in the implementation of this procedure, not to speak of variations in the equilibrium prices charged. For a more detailed critique of these and other problems of the central place model, see [10].

[15] This phenomenon is studied by Eaton and Lipsey [5], and Stahl and Varaiya [9]. In the former paper, the emphasis is on firms' entry decisions in a bounded market, with consumers' shopping frequencies held exogenous; in the latter, shopping frequencies are determined endogenously, but the number of marketeers is held fixed.

[16] For instance, let the commodities be realisticly provided at decreasing cost and consider variations in the spatial distribution of consumers together with an asymmetry in the typical consumer's preferences for the two commodities. Then at sufficiently low population densities only the more preferred commodity (or the one supplied at lower cost) will be offered.

[17] In my view, the notion of "agglomeration economies" so extensively used in the literature on urban and regional economics so far lacks sufficient specification to be operational: any welfare evaluation of equilibrium location patterns emerging in the presence of these economies or their counterpart, of diseconomies of agglomeration, must be based on a more precise notion of these in order to be useful for policy development.

REFERENCES

[1] Bollobás, B.; Stern, N.: The Optimal Structure of Market Areas, Journal of Economic Theory, Vol. 4 (1972), 174-179.

[2] Capozza, D.R.; Van Order, R.: A Generalized Model of Spatial Competion, American Economic Review, Vol. 68, #5 (Dec. 1978), 896-908.

[3] Debreu, G,: Theory of Value, N.Y.: Wiley, 1959.

[4] Dixit, A.: A Model of Duopoly Suggesting a Theory of Entry Barriers, The Bell Journal of Economics, Vol 10, #1 (Spring 1979), 20-32.

[5] Eaton, B.C.; Lipsey, R.G.: An Economic Theory of Central Places, Kingston/Ontario: Queen's University, manuscript 1980.

[6] Hotelling, H.: Stability in Competition, Economic Journal, Vol. 39, (March 1929), 41-57.

[7] Losch, A.: The Economics of Location, New Haven: Yale U. Press, 1954.

[8] Mills, E.S.; Lav, M.: A Model of Market Areas with Free Entry, Journal of Political Economy, Vol. 15 (1964), 278-288.

[9] Stahl, K.; Varaiya, P.P.: Spatial Arrangement of Markets with Varying Shopping Frequencies, University of Dortmund, manuscript, 1978.

[10] Stahl, K.: A Critical Note on Industrial and Retail Location Theory, University of Dortmund , Working Papers in Economic Theory and Urban Economics, 1980.

[11] Stahl, K.: Oligopolistic Location under Imperfect Consumer Information, Berkeley: University of California, Center for Real Estate and Urban Economics, 1980.

[12] Stern, N.: The Optimal Size of Market Areas, Journal of Economic Theory, Vol. 4 (1972), 154-173.

[13] Varian, H.: A Model of Sales, American Economic Review, Vol. 70, #4 (Sept. 1980), 651-659.