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Supplementing the effective number of parties

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Abstract

The effective number of parties, $N = 1/\Sigma p_i^2$ (where p_i is the fractional share of the ith party), usually suffices to describe adequately a constellation of parties of different strengths. Difficulties arise when disparity in party sizes is such that the largest share (p_1) surpasses 0.50 (meaning absolute dominance), while N still indicates a multi-party constellation. In such cases $N_\infty = 1/p_1$ is proposed as a supplementary indicator: a value less than 2 indicates absolute dominance. An 'NP' index proposed earlier is a combination of N and N_∞; its values are close to those of N_∞, but NP sometimes falls below 2 even when many parties are relevant for coalition formation. Appendix A offers an alternative approach based on indices of deviation from a norm, but it proves cumbersome.

Keywords: Effective number of parties; Disparity in party sizes

1. Introduction

This paper proposes a way to characterize a party constellation parsimoniously and yet more completely than is done by the 'effective number of parties' (Laakso and Taagepera, 1979) alone. It does so by introducing a second, supplementary index that can be specified when the effective number is deemed insufficient—which is the case, in particular, when one component is larger than 50% and hence dominates absolutely a crowd of smaller parties.

The broad problem is the following. For many purposes, we wish to indicate the number of parties in a polity. When parties are of unequal size, their total number may tell us little. For instance, when seat distribution in a 100-seat assembly is 40–

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30–11–9–5–1–1–1–1–1, it would hardly be considered a 10-party system. Rather than impose an arbitrary cutoff, the effective number of parties uses a self-weighting approach, meaning that each party's fractional share of seats or votes (p_i) is multiplied by itself, before adding the contributions of all parties (and independents) and taking the inverse: $N = 1/\Sigma p_i^2$. In the above case $N = 3.66$, reflecting approximately the number of parties relevant for majority coalition formation.

The use of effective number N has become widespread (Lijphart, 1994, p. 70; Cox, 1997, p. 29), because it usually tends to agree with our average intuition about the number of serious parties (Taagepera and Shugart, 1989, p. 80). Most often, it also usually comes close to the estimates of Sartori (1976) of the number of 'relevant' parties—as close as any operational index based on seat (or vote) shares alone can come, without detailed knowledge about the given country.

However, N does not always tell the whole story. Table 1 shows various constellations of party vote shares, all leading to $N = 3.00$. Also shown is the 'physical' number of parties, designated as N_0 (for reasons to be explained soon), and the

Table 1
Different party constellations at the same effective number of parties

Party constellation (fractional shares)		N_0	N	N_∞	NP	Relevant parties	
A	0.3333–0.3333–0.3333	3	3.00	3.00	3.00	3	Balance
B	0.35–0.33–0.32	3	3.00	2.86	2.90	3	Balance
C	0.39–0.32–0.28–0.01	4	3.00	2.56	2.63	3	Balance
D	0.45–0.29–0.21–0.05	4	3.00	2.22	2.17	3	Semi-balance
E	0.47–0.22–0.22–0.07	4	3.01	2.13	2.01	4	Semi-balance
F	0.48–0.23–0.21–0.08	4	3.00	2.08	**1.93**	4	Semi-balance
G	0.48–0.30–20 at 0.01	22	2.99	2.08	1.93	1 or 2	Practical hegemony
H	0.53–0.15–0.10–0.10–0.10–0.02	6	**3.00**	1.89	1.48	1	Hegemony
I	0.55-6 at 0.07-0.03	8	**3.00**	1.82	1.28	1	Hegemony
J	0.57–21 at 0.02–0.01	23	**3.00**	1.75	1.06	1	Hegemony
K	0.57735 and large number at $\epsilon \rightarrow 0$	indet.	**3.00**	1.732	1.00	1	Hegemony

number of parties that might be considered relevant for formation of a majority coalition in a 100-member assembly. Disregard for the moment the N_∞ and NP columns.

The constellations at the top of Table 1 represent a fair balance among three major parties, as $N = 3$ suggests. As the largest share increases (and the others decrease, so as to preserve $N = 3.00$), the largest party obviously has more coalition power (or blackmail power), but a semi-balance is preserved in the sense that a majority coalition that excludes the largest party is feasible in principle. Case G becomes debatable in this respect. In principle, either the 0.48 party or the 0.32 party could forge a majority coalition out of the field of 20 independents (assuming a 100-seat assembly). But it is clearly much easier for the 0.48 party to attract three independents than for the 0.32 party to corral 19 of them, so that there is practical hegemony. Consequently, the number of relevant parties might be seen as either 2 or 1. A line separates the remaining cases (H to K), where $p_i > 0.50$ in an otherwise highly fractionalized field. In these cases the largest party can form a majority cabinet single-handedly, so that the other parties become largely irrelevant (at least until the next election). The value $N = 3.00$ in these cases is printed in bold type, so as to highlight the discrepancy, compared with the actual situation.

This variety of constellations hidden behind the value $N = 3.00$ is not surprising. One single number contains perforce less information than many numbers do. Take for instance our usual ways to characterize a distribution. Its most important characteristic is its mean (or median). The mean tells us quite a lot, but it isn't the whole story. We also like to have a second measure to reflect the typical divergence from this mean (e.g., standard deviation). We may add further information by including a third number, a measure of asymmetry. Even then we do not have the full information contained in the original data set (if there are more than three items), but we have most of the information we need for most purposes.

Our specific question here is how to supplement the effective number of parties (the vague analog of the mean of a distribution) with a second number (vaguely analogous to standard deviation), so as to characterize the starkest differences among same-N constellations such as those presented in Table 1. This paper recommends the fractional share of the largest component, p_1 (or rather its inverse), as such a supplementary measure. It will be shown that it cannot replace N but only supplements it. Also considered is an interesting complex index, NP, that has been proposed as an alternative to N (Molinar, 1991). It will be shown that its values tend to be close to those of $1/p_1$, but that NP intimates less than two parties in some cases where three or even four are relevant. Appendix A investigates a completely different approach derived from measures of deviation from some norm; this approach has some merit but is deemed overly cumbersome and non-intuitive.

It should be stressed that for most purposes N alone will do, just as we often deal with the means of distributions, without the concomitant standard deviations. We should not clutter our data set by including the supplementary index unless it serves a purpose. However, the secondary index should be available when the need arises.

2. The largest component approach

The effective number of parties, N, is actually part of a wider family of possible measures, with a core Σp_i^a, where the power index (a) can be varied (Laakso and Taagepera, 1979; Taagepera and Shugart, 1989, pp. 259–260): $N_a = [\Sigma p_i^a]^{1/(1-a)}$. Some members of this N_a family have special meanings. N_0 is simply the total number of parties (and independents) involved, as shown in Table 1. Most often neglected in practice, it actually has its uses in construction of theoretical models of representation (Taagepera and Shugart, 1993). As $a \rightarrow 1$, N_1 is the exponential of entropy. Though occasionally used (cf. Taagepera and Shugart, 1989, p. 260), N_1 has the disadvantage of being overly sensitive to the sizes of the smallest components. N_2 is the usual effective number N, where we drop the subscript 2. Finally, as $a \rightarrow \infty$, N_∞ is $1/p_1$, the inverse of the largest share.

Thus, as the power index increases from 0 to infinity, N_a decreases from the 'physical' number of parties down to $1/p_1$. As an example, for case D in Table 1 (0.45–0.29–0.21–0.05) some values of N_a are the following: $N_0 = 4$, $N_1 = 3.31$, $N_2 = N = 3.00$, $N_3 = 2.82$, $N_4 = 2.71$ and $N_\infty = 1/0.45 = 2.22$. At low a values the size differences of parties matter very little, while at large a values they matter so much that, in the extreme case, only the largest component has an impact. Apart from striking a balance between these extremes, there is also a practical problem. The number and size of the smallest components, on which the low-a indices ($a = 0$ and $a \rightarrow 1$, in particular) depend, may not be known because data sources lump them into an 'Others' category. This problem of indeterminacy becomes manageable only when $a = 2$ is reached, and even then it presents problems at times (Taagepera, 1997). Table 1 shows the values of N_0 and N_∞, in addition to N.

One could characterize the number of parties by using two values of N_a at different a values. The aforementioned difficulties with the lumped 'Others' make $a = 2$ the lowest advisable choice, leading to $N = N_2$. As the second number, N_∞ has several advantages:

1. among the indices with $a > 2$, its values contrast most with those of N;
2. it is simple to calculate; and
3. it very explicitly signals one-party hegemony: $N_\infty < 2$ tells us that one party has more than 50% of the votes or seats, while $N_\infty \geq 2$ tells us that no such absolute majority exists.

Except for this latter point, N_∞ tells us less than N does, given that the impact of all but the largest component is nil. For instance, $N_\infty = 2$ alone could mean a two-party balance (0.50–0.50, $N = 2$) or a large party facing splintered opposition (0.50–0.10–0.10–0.10–0.10–0.10, $N = 3.33$). Thus it would be inadvisable to use N_∞ as the sole measure of the number of parties. However, when joined to N, N_∞ adds information. In our previous example D (0.45–0.29–0.21–0.05), $N = 3.00$ and $N_\infty = 2.22$. The '2.22' denotes appreciable deviation from the picture of three *equal* components, while also assuring us that no component has absolute majority.

One shortcoming of using the pair N plus $N_\infty = 1/p_1$ is that the two correlate considerably. For given N_∞, N is restricted to the range $N_\infty \leq N \leq (N_\infty)^2$. Conversely,

for given N, N_∞ is restricted to the range $N^{0.5} \leq N_\infty \leq N$. For instance, with $N = 3$, N_∞ can range only from 1.73 to 3 (cf. Table 1). With $N_\infty = 3$ ($p_1 = 0.33$), N can range only from 3 to 9. In contrast, the mean and the standard deviation of a distribution are essentially independent of each other, and hence they pack relatively more information. In this sense, the approach indicated in Appendix A might be more efficient, but N plus N_∞ has the advantages of simplicity and intuitiveness.

The degree of information added by the second index is illustrated by the following example, which considers the largest and the second-largest components.

1. With $N = 3.00$ alone, we know that $0.33 < p_1 < 0.57$ and $0 < p_2 < 0.33$.
2. With $N_\infty = 2.00$ alone, we know of course that $p_1 = 0.50$, and also that $0 < p_2 < 0.50$.
3. With both $N = 3.00$ and $N_\infty = 2.00$, we know that $p_1 = 0.50$ and $0.166 < p_2 < 0.289$.

The addition of the second-order index N_∞ is seen to reduce appreciably the possible range of the second-largest component. In the present case, the limited values of p_2 suggest that the largest party is likely to remain near-hegemonic even in the case of serious losses in the next election.

3. Comparison with NP

An alternative index to N has been proposed (Molinar, 1991), under the name of NP. The formula given involves N, p_1 and also Σp_i^2. When one realizes that $\Sigma p_i^2 = 1/N$ and replaces p_1 by N_∞, Molinar's formula simplifies into:

$$\mathrm{NP} = 1 + N[1/N - (1/N_\infty)^2]/[1/N] = 1 + N - (N/N_\infty)^2.$$

An important implication follows: *when two constellations have the same* N *and the same* N_∞ *then they also have the same NP*. The values of NP are shown in Table 1. They are always closer to N_∞ than to N. At a closer look the following can be noted. When the components are equal, $\mathrm{NP} = N_\infty = N$ (case A). At slight unevenness (cases B and C) NP exceeds N_∞: $N > \mathrm{NP} > N_\infty$. With more unevenness in the size of parties (from case D on) NP falls below N_∞: $N > N_\infty > \mathrm{NP}$. The critical cases to consider are F and G. And there are some disappointing surprises.

Problems are few when we deal with non-party concerns such as the effective number of religious or ethnic groups in a country where these cleavages are not politicized (e.g., Switzerland). But when political coalition building enters, then cases F and G look quite different, despite having the same N and also the same N_∞ (and hence the same NP). In F, even the smallest of the four parties has coalition potential. Ideology permitting, it could be the largest party's preferred partner, or it could clinch a majority coalition that excludes the largest party. Here even $N = 3.00$ understates the number of relevant parties. N_∞ and NP fare even worse, but with one crucial difference: $N_\infty > 2$, correctly suggesting that more than two parties are competing, while $\mathrm{NP} = 1.93$ suggests that fewer than two parties are relevant. This is a serious strike against NP, as compared with N_∞. In contrast, case G represents

practical largest-party hegemony, as discussed earlier, with the runner-up (0.32) a potential challenger. Here $N = 3$ clearly overstates the number of parties, while N_∞ and NP both come close. The disappointing surprise is that the same combination of N and N_∞ (and NP) can hide coalition-building implications as different as those of cases F and G. What it means is that even the two indicators (N and N_∞) jointly cannot always convey all the information we would like to have.

Still, N_∞ draws at least a clear line ($N_\infty = 2$) at the point where total hegemony begins (largest share more than 50%, cases H to K). NP fails to do so, and not only in case G (which could be said to mean practical hegemony) but also in case F, where clearly four parties are relevant. The only situations where NP has advantages over N_∞ occur when hegemony is overwhelming (cases H to K). Here N_∞ intimates close to two players, while NP sensibly approaches 1. However, once one party has more than 50%, how much does it matter whether it has 53 or 57%? (Once it goes beyond 57%, not only N_∞ but also N is bound to decrease.)

In sum, in most cases the values of NP are quite close to those of N_∞ so that the extra effort to calculate NP is not worthwhile. In the cases of absolute hegemony, NP tells it better, but N_∞ does not mislead either. In the other direction, case F offers a situation where NP suggests absolute hegemony when this is not the case at all. Even more striking is the contrast between 0.51–0.49 ($N = 1.999$, $N_\infty = 1.96$, NP = 1.96) and 0.49–0.26–0.25 ($N = 2.70$, $N_\infty = 2.04$, NP = 1.95). In the former case, all three indices agree on a value slightly less than 2, reflecting absolute majority plus strong opposition. The latter case (where the 0.51 party has split into two) clearly has more than two relevant parties, as expressed by N and quite marginally by N_∞. Yet NP actually decreases and implies that there still are less than two parties. The conclusion is that N_∞ is preferable to NP—and not only because of simplicity of calculation.

If we are restricted to a single indicator, then N tends to convey more information than N_∞ (as noted earlier)—and N_∞, in turn, is preferable to NP. When the distribution is lopsided, two indicators should be reported: either N plus N_∞ or N plus NP—and the combination N plus N_∞ is preferable.

4. Discussion

We could of course include even more information by adding a third and a fourth index, but then we might as well just reproduce the original data. The purpose of general indices is to gain in ability to compare different constellations, always at the cost of losing some information on each of them. For most purposes, the effective number of parties (N) alone may suffice to characterize a party constellation. If information on disparity in party sizes is desirable, then adding the inverse of the largest party's share (p_1) is the most efficient: the gap between $1/p_1$ and N tells us about deviation from equal shares and p_1 itself informs us about the degree of largest-party predominance.

What do we gain by keeping track of another index? The supplementary index may explain some apparent anomalies or at least make us more cautious. For a

country that uses single-seat plurality, India 1952–1984 did have unusually many electoral parties, but the average $N = 4.2$ (based on votes) exaggerates it. The average share of the largest party was 44.3%, leading to $N_\infty = 2.26$ (which, in turn, understates India's multipartism). More generally, in comparative studies it is worth looking into N_∞ when a value of N looks out of step with the general pattern.

When should N_∞ be reported along with N? A simple absolute recipe is hard to come by. If one deals with the effective number of non-politicized ethnic or religious groups, then N will probably do. In the case of assembly seats problems arise chiefly when N_∞ is less than 2 while N is more than 2. This is the case for India's parliament in 1952–1984. An average of $N = 2.14$ suggests two-party balance, which was close to true only in 1977, while the addition of $p_1 = 66.7\%$ and hence $N_\infty = 1.50$ indicates heavy largest-party hegemony in the face of splintered opposition. (NP = 1.10 makes the same point, reached in a more complex way.)

A look at Table 1 further suggests that when the gap between the largest and next-largest shares exceeds 0.20 one might wish to report N_∞. This is the case for Japan in 1958–1990, Sweden in 1932–1994 and Italy in 1948–1958. However, cases F and G in Table 1 serve as a warning. N_∞ would be superfluous in case F, although the gap is as high as 0.25, while in case G even the addition of N_∞ would not tell the entire story, although the gap is only 0.18. Fortunately, we often deal with averages of many elections, and the rare paradoxical cases tend to be ironed out.

One may harbor the illusion that by judicious combination of N and N_∞ (plus possibly something else) one might achieve a single super-index that satisfies all desiderata. This is about as wishful as hoping to combine the mean and the standard deviation of a distribution into a single measure. Two numbers are inherently able to transmit more information than a single one.

Appendix A

Deviation from the expectation of equal shares

This approach makes rational sense but leads to practical complications. The expression $N = 3$ conjures the image of three equal parties; yet this is the case only for the first of the many constellations sampled in Table 1. To express divergence from such equality, we can use standard indices of deviation from an expected norm, which in the present case is that $p_1 = 1/N$ for the first N parties, and 0 thereafter. We could use the well-known Schutz coefficient, $S = 1/2\Sigma|p_i - 1/N|$. Alternatively, in analogy with the measure proposed by Gallagher (1991) for deviation from proportional representation (PR), we could also use $\mathrm{Gh} = [1/2\Sigma(p_i - 1/N)^2]^{0.5}$. As an example, for case D in Table 1, the picture is the following:

Actual:	0.45	0.29	0.21	0.05
Equality:	0.333	0.333	0.333	0.000
Difference:	+0.117	−0.043	−0.123	+0.050

The deviation is characterized by $S = 0.167 = 16.7\%$ or by $\mathrm{Gh} = 0.129 = 12.9\%$. The maximum possible deviation from equality in the case of $N = 3$ ($S = 66.7\%$,

Gh = 37.5%) occurs when $p_1 = 1/N^{0.5} = 0.577$ and all other components are infinitesimal (but add up to $1 - 0.577 = 0.423$).

So far, so good. Computations become more complex, however, when N is not an integer—which is usually the case. Consider the constellation 0.40–0.30–0.20–0.10, for which $N = 3.333$ is between 3 and 4. If we follow the previous logic, a fraction 0.333 of the fourth party should be expected to have equal representation ($1/N = 0.30$), while the remaining 0.667 should be unrepresented:

Actual:	0.40	0.30	0.20	0.10
Equality:	0.30	0.30	0.30	0.30(0.333)+0.00(0.667)
Difference:	+0.10	0.00	−0.10	−0.20(0.333)+0.10(0.667)

Hence $S = 0.167 = 16.7\%$ and $Gh = 0.141 = 14.1\%$. While this is logical in terms of deviation from equal shares, it risks getting rather confusing.

Moreover, the values of S or Gh obtained lack intuitive meaning. In particular, we cannot tell offhand which combinations of N plus S (or N plus Gh) correspond to an absolute majority of the largest party. A further theoretical concern is that at maximum deviation from equality (for given N) the indices do not reach 100%. One can correct for that, but then the computations become even more complex. In sum, what looks like a sensible approach conceptually bogs down in practice.

Is there a connection between the values of N_∞ and S or Gh? Indirectly, there is. Consider the reduction in N_a as one shifts from $N = N_2$ to N_∞: $r = (N - N_\infty)/N = 1 - (N_\infty/N)$. This measure has the same form as the reduction in the effective number of parties as one goes from vote shares to seat shares (Taagepera and Shugart, 1989, p. 273). The values of r tend to be somewhat larger than those of S, but they follow the same basic pattern. This is not surprising. Both transmit information regarding the degree of deviation from the constellation with three equal shares. Gh follows the same pattern with somewhat lower values. I do not suggest that r is a useful index to calculate in general, but the comparison with S and Gh shows that supplementing N with N_∞ has about the same information content as adding a formal measure of deviation.

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