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ABSTRACT

Generalized beta equilibrium involving nucleons hyperons and isobars is examined for neutron star matter. The hyperons produce a considerable softening of the equation of state. It is shown that the observed masses of neutron stars can be used to settle a recent controversy concerning the nuclear compressibility. Compressibilities less than 200 MeV are incompatible with observed masses.

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The large scale features of neutron stars, mass, radius and gravitational red-shift, for example, depend upon the equation of state of the matter of which they are composed, and that is why they are of interest to this conference. However I want to be precise at the outset in how the equation of state controls the structure of the star. The matter of a star will be arranged in accord with the condition of hydrostatic equilibrium, that is to say the pressure of the matter at every point balances the force of gravity. In a neutron star the concentration of energy is high enough that the metric of space time is curved, and the condition of equilibrium has to be framed in terms of the general theory of relativity, which was done long ago for static spherically symmetric stars by Oppenheimer and Volkoff. The coupled equations are,

$$(4\pi r^2)dp(r) = -\frac{GM(r)dM(r)}{r^2}\left(1 + \frac{p(r)}{\epsilon(r)}\right)\left(1 + \frac{4\pi r^3 p(r)}{M(r)}\right)\left(1 - \frac{2GM(r)}{r}\right)^{-1} \quad [1]$$

$$dM(r) = 4\pi r^2 \epsilon(r) dr \quad [2]$$

These equations express the balance of net pressure, dp, acting on a spherical shell of thickness dr, with the force of gravity acting on the mass dM in the shell. With a knowledge of the equation of state $p = p(\epsilon)$, they can be integrated outward for an arbitrary choice of ϵ at $r = 0$, to the point $r = R$ where the pressure becomes zero. Then M(R) is the star's gravitational mass and R its radius for the assumed central energy density. You see therefor that the structure of the star is an *integral* property of the equation of state. The connection of the large scale features and the equation of state, while being quite precise, cannot be inverted with the kind of data that are accessible to us. Nonetheless we can get a handle on the problem in the sense of distinguishing widely different equations of state through their predicted star structure.

The known masses of neutron stars provides a constraint on the theory, since for any particular equation of state, there corresponds a limiting star mass above which the star is unstable and collapses into a black hole. An acceptable theory must yield a limiting mass as large or larger than the largest observed mass (about 1.5 solar masses).

Now I turn to some of the demands that seem reasonable to make of the extrapolation of theory from the known properties of matter at sub-nuclear densities to the domain of super-nuclear densities. Although one requires the equation of state from super-nuclear densities down to zero pressure to solve the structure equations, almost all of the matter of neutron stars is in the unknown domain of high density, as shown in Fig 1 . Except for the very lightest of neutron stars, this figure shows that 95% or more of the mass of the star is in the nuclear or super-nuclear regime. Since a neutron star is a relativistic object, it is of course highly desirable that the theory of matter should be relativistically covariant. The non-relativistic Breuckner-Bethe theory for example when extrapolated to the high density domain will in general violate causality. The equation of state will become manifestly incorrect. It is usually argued in such cases that if this occurs above the density of the cores of neutron stars, it doesn't matter. This is false. The best non-relativistic calculation of nuclear matter to date, that of Day and Wiringa [1], saturates at twice the empirical density! Baron, Brown, Cooperstein and Prakash [2] calculate relativistic corrections amounting to 100 % to bring the calculation into agreement with the empirical value. Relativity is therefor already important at nuclear density and non-relativistic theories will therefor become increasingly worse with increasing density. Since they ultimately violate causality, their equation of state is stiffer than the underlying relativistic theory (were it known) and therefor the masses of neutron stars is overpredicted. Since observed neutron star masses provide a lower limit on the theory, it becomes clear that finding a non-covariant theory to be compatible with neutron star masses is empty of meaning. Moreover since the relativistic corrections to the non-relativistic Breuckner-Bethe theory are so large, it raises serious questions as to whether the traditional approach is valid in any density regime of interest in nuclear physics.

Aside from the general requirement of covariance, the theory should be able to account for the bulk properties of matter where known, namely at saturation density. Of particular importance is the symmetry energy, since neutron stars are highly isospin asymmetric. This last constraint seems not to have been imposed in earlier work. We require therefor,

1. relativistic covariance

2. bulk nuclear properties (binding, saturation density, compressibility, isospin (charge) symmetry energy)

A theory that satisfies these requirements is the relativistic nuclear field theory based on nucleons interacting with the scalar, vector and isovector mesons and having ϕ^4 interactions. The latter can be exploited to bring the compressibility into line. The rho meson couples to the isospin density, and so tends to drive matter to isospin symmetry. This theory is also known to reproduce a large number of single-particle properties of finite nuclei [3].

What about other baryons and mesons? From the vast zoology in the particle data tables what others can be involved in the structure of neutron stars?

GENERALIZED BETA EQUILIBRIUM [4]

Stars are essentially charge neutral because the repulsive Coulomb force is so much stronger than the gravitational one. A star composed solely of neutrons satisfies this condition but is unstable against beta decay. The neutron at the top of the fermi sea has enough energy to decay into a proton and electron. So pure neutron stars cannot exist. As the density increases other baryon thresholds will be reached and hyperons also will be present, and perhaps the delta. Therefor we should allow for a generalized beta equilibrium in dense neutron star matter, allowing whatever baryons to participate as dictated by their masses and the interactions. There is no essential difficulty introduced into the theory connected with the increase in the number of baryon species.

MESONS AND PHASE TRANSITIONS

Unlike the baryons, mesons can introduce new levels in complexity in the theory according to the quantum numbers that they carry. To see why this is so recall that the lagrangian must be a scalar. Therefor the interaction terms between baryons and mesons must involve the meson field and a bilinear baryon current of such a form that it can be contracted with the meson to form a scalar. Table I lists a few of the mesons in ascending order of their quantum number, and the form of an interaction term with any baryon B. The operator appearing in the bilinear form, will appear in the Dirac equation for the baryons and as the source in the Klein Gordan equation for the meson, and so the complexity of the theory is increased with each new type meson. The omega and (neutral) rho meson couple to familiar baryon currents, the expectation value of the time-like components are the baryon density and isospin density respectively. The scalar meson couples to the scalar density. All three densities are finite in neutron star matter and so drive to finite amplitude the meson to which they couple.

The other currents vanish in normal matter. For example the charged rho mesons change the charge of the baryon that absorbs or emits them, and so there can be no diagonal matrix element, and the expectation value of the baryon currents to which they are coupled vanish in the ground state. Analogously, the pion changes the isospin and moreover, because of its parity requires that spatial isotropy be broken. The Kaon changes the strangeness quantum number and so also its source current vanishes.

When the source in the Klein-Gordan equation for a meson vanishes, its amplitude can vanish, and since that is a lower energy state, all such mesons as have vanishing baryon source currents will decay.

The characteristics of normal matter are that it is isotropic and that the fermion eigenstates of the system have the same quantum numbers as they do in vacuum. In the normal state of neutron star matter the sigma, omega and neutral rho mesons are therefor the only ones that have finite mean amplitudes. However it is conceivable that a phase transition could occur as the density of matter is increased, a transition to a new state in which one or both of the characteristics of normal matter are violated. In the new state one of the previously absent mesons will now be driven to finite amplitude by a new non-vanishing source. It is clear that for such a transition to occur there must be lowering of the energy. The interaction energy must be attractive, and more than enough to compensate the mass and momentum of the meson.

We must now discuss specific phase transitions which would introduce additional mesons into the theory (and the star). In principle, a succession of phase transitions corresponding to each of the mesons whose sources vanish in the normal ground state can occur as the density of neutron star matter is increased. The transition to a state in which the pion acquires a finite amplitude is the first that might be encountered. It is a state that was much discussed for symmetric matter, and as precursor phenomena in finite nuclei. It seems that it is too distant from normal density to produce such phenomena. We have also investigated it in neutron star matter [5]. There the charge asymmetry of the system encourages the pion condensate. In addition of course the attractive p-wave interaction, as for symmetric matter, is a driving force. I will not discuss this rather complicated topic in detail here, but wish only to introduce some general notions that will be useful in discussing the plausibility of phase transitions after this one, say a kaon condensate.

First of all it is the negative pion that can be a candidate for a phase transition, since when the fermi energy of the electron exceeds the effective mass of the pion in the medium, it is energetically favorable for any additional negative charge to be carried by pions rather than electrons, because they are bosons, and can all condense in the lowest energy state. What this means more precisely, is that the electron chemical potential, which is an increasing function of density for low density neutron star matter, saturates at the effective mass of the pion. Let us rewrite the momentum transform of the Klein-Gordan equation of the pion,

$$\left(-k_\mu^2 + m^2\right) <\pi> = J \qquad [3]$$

where J is the baryon source current for the pions, in the form,

$$\left(-\mu_e^2 + k^2 + \Pi_\pi(\mu_e,k)\right) <\pi> = 0 \qquad [4]$$

in the vicinity of the hypothesized phase transition. Remember that J vanishes in the normal phase. In this equation I have replaced the energy of the pion by the electron chemical potential, since as I remarked, this is the energy that the pions would have. Π is the polarization operator,

$$\Pi_\pi = - \lim_{<\pi>\to 0} \frac{<J>}{<\pi>} \qquad [5]$$

Now it is apparent that for the pion to have a finite amplitude, the equation

$$-\mu_e + k^2 + m_\pi^2 + \Pi_\pi(\mu_e,k) = 0 \qquad [6]$$

(for k a real number) must be satisfied. It is clear that as the electron chemical potential increases it becomes more likely that eq. 6 can be satisfied. If the pion interactions were vanishingly small it would be satisfied at the pion mass. In a related work, the attractive p-wave interaction was taken into account in the presence only of the nucleons, and the equation was satisfied for $\mu_e = k_0 = 170$ MeV, somewhat higher than the mass, because the p-wave attraction can be gained only at some cost of momentum [5]. Even if the pion-baryon interaction were strongly repulsive, forbidding the condensation of pions, the chemical potential would eventually saturate because of the appearance of negatively charged hyperons. This gets a little ahead of the story, but needs to be mentioned here. It would saturate then at about 200 MeV [4]. The behavior of the electron chemical potential under these three scenarios is depicted in Fig. 2.

Now an analogous discussion can be carried out for the kaon and all more massive mesons that require a phase transition to acquire a finite amplitude in the star. But since the electron chemical potential will saturate with increasing density at a value of 200 MeV or less, according to the above remarks, such massive mesons cannot condense, for it would require an attractive interaction on the order of 10 times that experienced by pions, and this is implausible.

The above, rather long line of reasoning leads to an important conclusion, namely that in addition to the three mesons that condense in normal matter, the only additional one that can do so through a change in phase is the pion. This conclusion allows us to limit the zoology of mesons in the theory, and we can now write down the effective lagrangian as the sum of sigma, omega, rho, and pion, as well as the sum of baryon terms and their interactions with the mesons named [4].

$$\mathscr{L} = \sum_{B} \overline{B}(i\gamma_{\mu}\partial^{\mu} - m_{B} + g_{\sigma B}\,\sigma - g_{\omega B}\,\gamma_{\mu}\,\omega^{\mu})\,B - g_{\rho}\,\rho_{\mu 3}\,J_{3}^{\mu}$$

$$+ \mathscr{L}_{\sigma}^{0} + \mathscr{L}_{\omega}^{0} + \mathscr{L}_{\rho}^{0} + \mathscr{L}_{\pi}^{0} - \frac{1}{3}bm_{n}(g_{\sigma}\sigma)^{3} - \frac{1}{4}c(g_{\sigma}\sigma)^{3} + \sum_{\lambda}\Psi_{\lambda}(i\gamma_{\mu}\partial^{\mu} - m_{\lambda})\Psi_{\lambda} \qquad [7]$$

Here B denotes a baryon spinor and is summed over all baryons p, n, Λ, Σ, Ξ ... , the rho meson is coupled to the isospin current, J, the $\mathscr{L}^{0}$ denotes field free lagrangians, the cubic and quartic terms are scalar self-interactions and Ψ_{λ} in a lepton spinor summed over electrons and muons.

We shall simplify the treatment of pions by considering two limiting cases, as concerns their effect on the equation of state. In the one case, we shall assume that the effective pion mass is so large compared to the electron chemical potential (see eq. 6) that they do not condense. The other limiting case will produce a maximum softening of the equation of state due to pions. We neglect the pion-nucleon interaction, but include pions in the conditions of chemical equilibrium. In this case, negative pions will condense when the electron chemical potential attains the value of the pion mass. This will produce a maximum softening, because according to our calculation of pion condensation with pseudo-vector coupling to neutrons and protons in neutron star matter, condensation will occur when the chemical potential reaches a somewhat higher value than the pion mass [5]. (This is because the pion must have momentum to exploit the attractive p-wave interaction.)

The field equations that follow from eq. 7 must be supplemented by the conditions for chemical equilibrium and charge neutrality. This yields a set of N+8 non-linear equations that determine the meson amplitudes, chemical potentials and the fermi momenta of the leptons and baryons, of which there are, say N. When the solution is obtained, the energy density and pressure can be computed as a function of density, in other words the equation of state. If instead, the field equations are solved subject to the suplemental condition of isospin symmetry, we get the solution for uniform nuclear matter. This solution is used to fit the constants of the theory to the bulk properties of nuclear matter.

EQUATION OF STATE

The energy density and pressure are given in this theory by the expressions,

$$\varepsilon = \frac{1}{3} b m_n (g_\sigma \sigma)^3 + \frac{1}{4} c (g_\sigma \sigma)^4 + \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{2} m_\omega^2 \omega_0^2 + \frac{1}{2} m_\rho^2 \rho_{03}^2 + n_\pi m_\pi$$

$$+ \sum_B \frac{2J_B + 1}{2\pi^2} \int_0^{k_B} \sqrt{p^2 + (m_B - g_{\sigma B}\sigma)^2}\, p^2 dp$$

$$+ \sum_\lambda \frac{1}{\pi^2} \int_0^{k_\lambda} (p^2 + m_\lambda^2)^{1/2} p^2 dp \quad . \qquad [8]$$

$$p = -\frac{1}{3} b m_n (g_\sigma \sigma)^3 - \frac{1}{4} c (g_\sigma \sigma)^4 - \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{2} m_\omega^2 \omega_0^2 + \frac{1}{2} m_\rho^2 \rho_{03}^2$$

$$+ \frac{1}{3} \sum_B \frac{2J_B + 1}{2\pi^2} \int_0^{k_B} \frac{p^4}{\left[p^2 + (m_B - g_{\sigma B}\sigma)^2\right]^{1/2}} dp$$

$$+ \frac{1}{3} \sum_\lambda \frac{1}{\pi^2} \int_0^{k_\lambda} \frac{p^4}{(p^2 + m_\lambda^2)^{1/2}} dp \quad . \qquad [9]$$

The equation of state showing pressure as a function of energy density is shown in fig. 3 for the full theory and in the absence of hyperons and pion condensate, and these are compared with the causality limit, the ultrarelativistic gas, and an ideal neutron gas. One can see here the softening due to the additional degrees of freedom, that are usually omitted.

It is interesting to see the composition of dense neutron star matter, and this is shown in fig. 4 for four cases, (a) the full theory, (b) absent the pions, (c) absent the rho coupling, which shows the important role of the isospin symmetry in dictating populations within the constraint of charge neutrality, and (d) absent hyperons, isobars, and pion condensate. The last case corresponds to the more usual scenario of a neutron star where beta equilibrium between neutrons, protons and leptons only is assumed. What we see in the full theory is a sequence of thresholds being reached after which the populations rise rapidly. At high density, the neutron population has fallen significantly and there is an abundance of other baryons. In fact charge neutrality for high density is achieved mainly among baryons, and the lepton and pion populations are quenched.

Most of a neutron stars mass is composed of matter at nuclear or super-nuclear densities, but the surface itself depends on the equation of state of matter at lower density and still exotic conditions. Table II shows three broad density regions, the nature of matter in each and the source of the equation of state. It is shown over the wide density range in fig. 5.

Integrating the Oppenheimer-Volkoff equations for star structure we can find the mass of a star as a function of its central density. In fig. 6, this is shown for cases a,b,d of fig. 3. I mention that for pure neutron matter, such as Walecka and Chin calculated, the limiting mass is 2.5 solar masses. However such matter is unstable and far too stiff. Here we see a limiting mass for the full theory of 1.8 solar masses. Since this exceeds the maximum known neutron star mass, the theory is compatible with the data.

Let us now look at the contents of the star. This is shown in fig. 7 for two cases (a) the full theory and (b) the conventional theory having as baryons only the neutron and proton. The distance between lines represents the populations. What we see here is that the central core has a higher density of hyperons than nucleons. It should also be noted that the isobars are completely absent. This is because of the most favored charge state is the negative one, but it is unfavored by its isospin.

CONSTRAINT ON COMPRESSIBILITY

All of the foregoing calculations were for parameters of the theory that yield a compressibility for symmetric nuclear matter of K=285 MeV. There is some debate about what the compressibility really is. For a long time a careful analysis of the giant monopole resonance by two French groups has been generally accepted. They place K above 200 MeV [6]. More recently Brown and Osnes has claimed that it may be as small as 100 MeV [7]. Keeping the other properties of nuclear matter fixed, I have investigated the effects of varying the parameters of the theory so as to vary the compressibility from 100 MeV upward. I have two sets of calculations, with and without a pion condensate. The equations of state are shown in fig. 8. The weak structure that appears in the equation of state and the mass curve of the star reflects the various thresholds that occur as the density changes. While the differences produced by varying the compressibility at saturation density of nuclear matter do not appear to be great in the equation of state of neutron star matter, the scale is logarithmic, and the neutron star masses show considerable variation, as depicted in fig. 9. Moreover since neutron stars of 1.5 solar mass are known, it appears that compressibilities less than 200 MeV can be ruled out.

SUMMARY

We have solved a relativistically covariant field theory of nuclear matter in the mean field approximation, for both symmetric nuclear matter and neutron star matter, involving a generalized beta equilibrium between nucleons, hyperons, isobars and leptons and in two limiting cases in which pions condense at an effective mass equal to the vacuum value and in which they do not condense because of an assumed effective mass which is too large compared to the electron chemical potential. The cores of stars near the limiting mass have large hyperon populations, and these populations integrated over the star amount to 15-20% of the baryon population. The paramters of the theory were fitted to the bulk properties of nuclear matter except that the compressibility was treated as unknown. The mass curves of neutron stars were calcu-

lated for various assumed values of the compressibility, and it was found that values less than about 200 MeV are incompatible with observed neutron star masses.

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Table I.
Partial list of mesons ordered as to quantum numbers and the corresponding part of the interaction Lagrangian. The parentheses enclose the various baryon currents to which the mesons are coupled.

Meson	J^{π}	I	$\lvert S\rvert$	$\mathscr{L}^{int}$
σ	0^+	0	0	$g_\sigma \sigma(\bar{B}B)$
ω	1^-	0	0	$g_\omega \omega_\mu(\bar{B}\gamma^\mu B)$
π	0^-	1	0	$g_\pi \underset{\sim}{\pi} \bullet (\bar{B}\gamma_5 \underset{\sim}{\tau} B)$
ρ	1^-	1	0	$g_\rho \underset{\sim}{\rho}_\mu \bullet (\bar{B}\gamma^\mu \underset{\sim}{\tau} B)$
K	0^-	1/2	1	$g_K K(\bar{\Lambda}\gamma_5 N)\ldots$
K*	1^-	1/2	1	$g_K K^*_\mu (\bar{\underset{\sim}{\Sigma}}\cdot\underset{\sim}{\tau}\gamma^\mu N)\ldots$

Table II.
Density regions needed to describe the neutron star surface (I and II) and the star interior (III).

Region	Density (g cm^{-3})	Character of Matter	Reference
I..........	$2 \times 10^3 < \rho < 1 \times 10^{11}$	crystalline: light metals, electron gas	Harrison and Wheeler 1965
II..........	$1 \times 10^{11} < \rho < 2 \times 10^{13}$	crystalline: heavy metals, relativistic electron gas	Negele and Vautherin 1973
III..........	$2 \times 10^{13} < \rho < 5 \times 10^{15}$	relativistic hyperons: pions, leptons	This work

Fig. 1. Mass profiles of stars of several masses. The dot on each curve marks the point interior to which 95 % of the stars mass is contained. The Roman numerals refer to three density rgions of the equation of state charcacterized in table II.

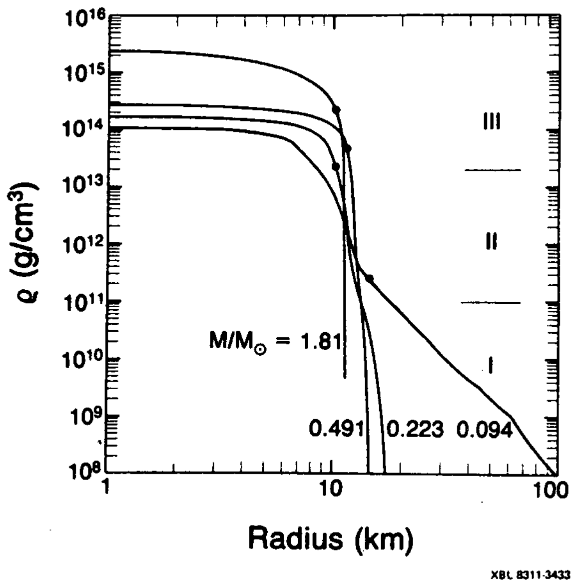
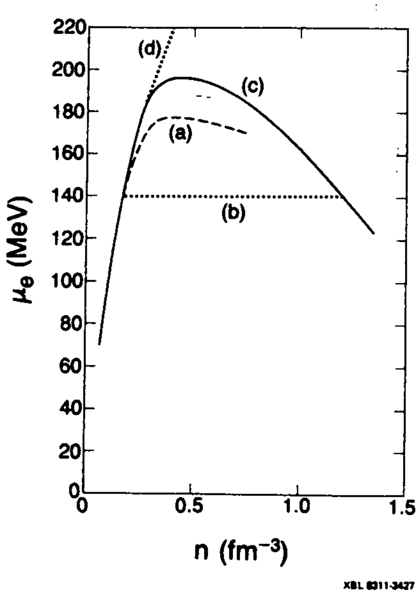


Fig. 2. Behavior of the electron chemical potential in neutron star matter in four cases, (a) pion condensation through the p-wave interaction with nucleons, no hyperons or isobars present [5], (b) free pions condense with their vacuum mass, (c) generalized beta equilibrium among all baryons, but no pion condensate, (d) no hyperons isobars or pion condensate.



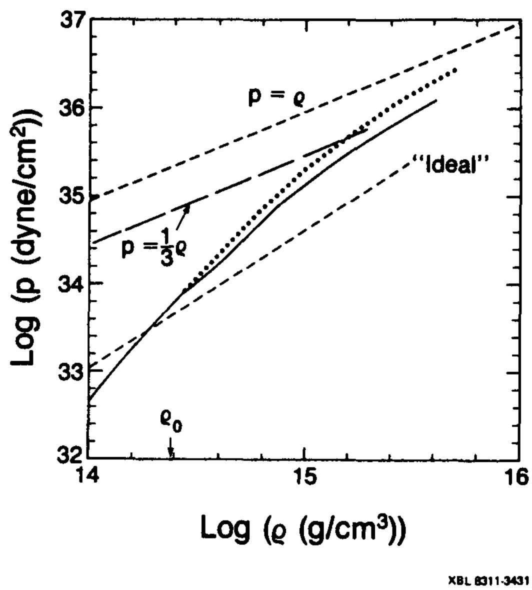


Fig. 3. Equation of state for full theory including nucleons, hyperons, isobars and pion condensate (solid line), absent the hyperons, isobars and pion condensate (dotted line), and for the special cases marked, the causality limit, the ultrarelativistic gas, and the ideal gas of neutrons.

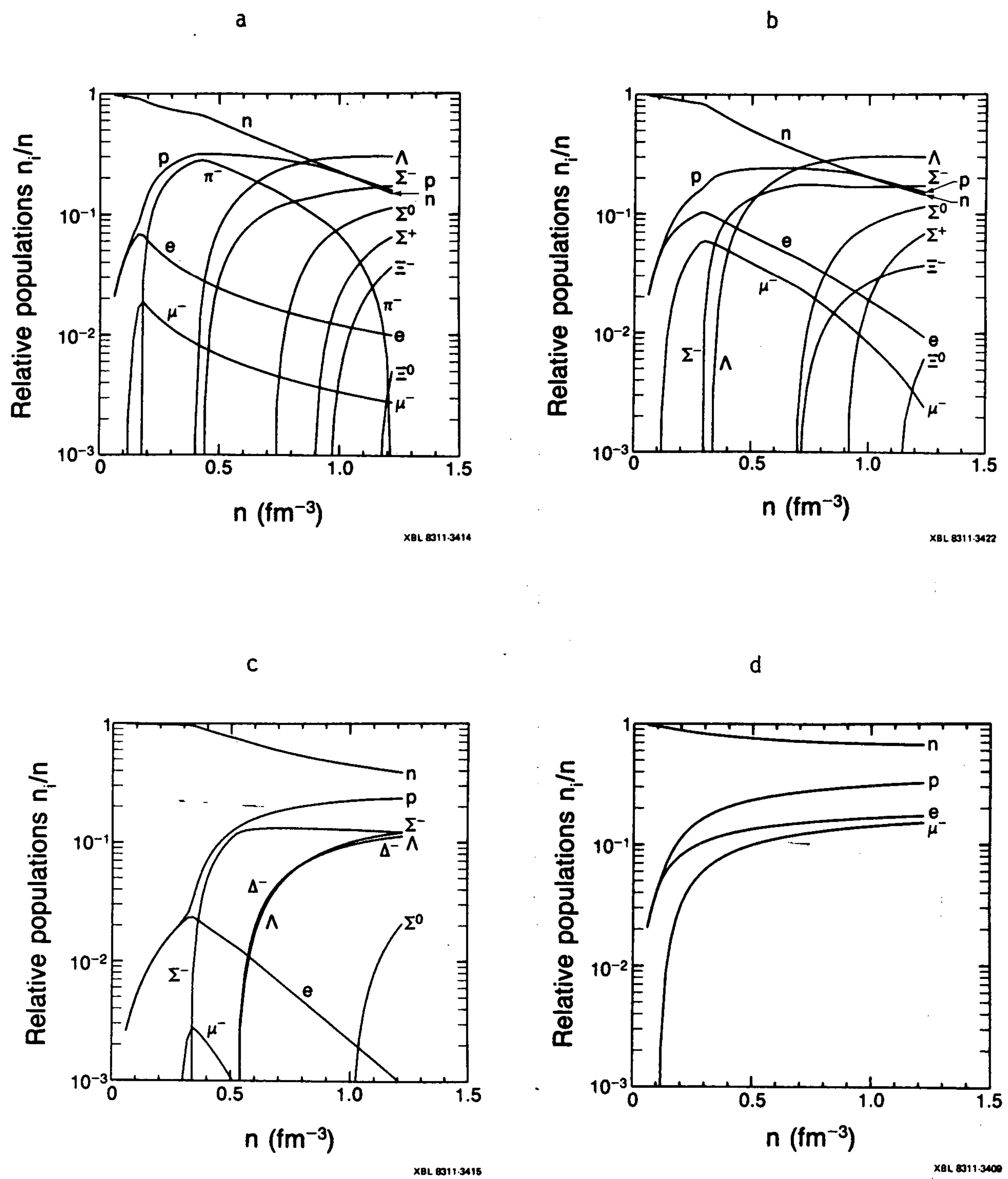


Fig. 4. Composition of neutron star matter as a function of baryon density for (a) the full theory including all baryons and pion condensate, (b) absent the pions, (c) absent the rho coupling, (d) absent the hyperons, isobars and pions.

Fig. 5. Equation of state over wide density range.

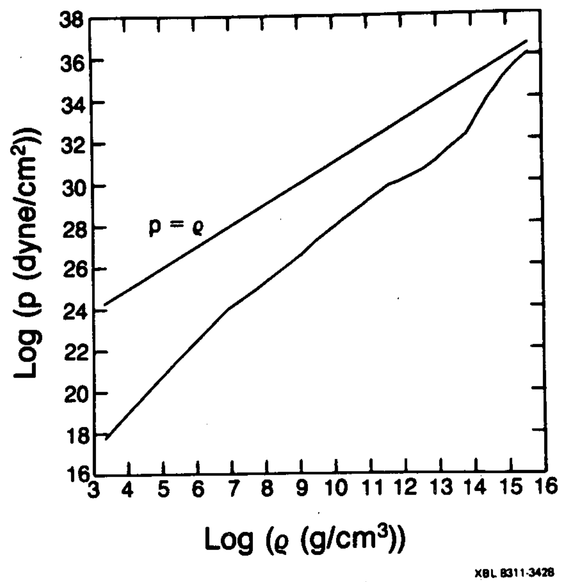
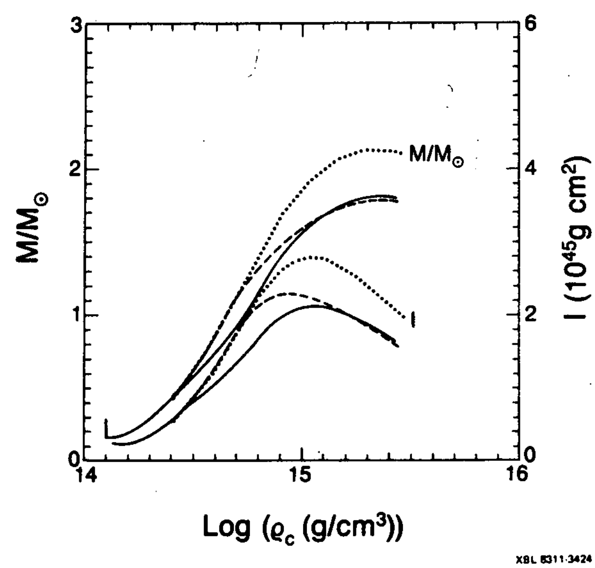


Fig. 6. Masses and moments of inertia of neutron stars in the cases of Fig 4, (a) the full theory including all baryons and pion condensate (solid line), (b) absent the pions (dashed line), (d) absent the hyperons, isobars and pions (dotted line).



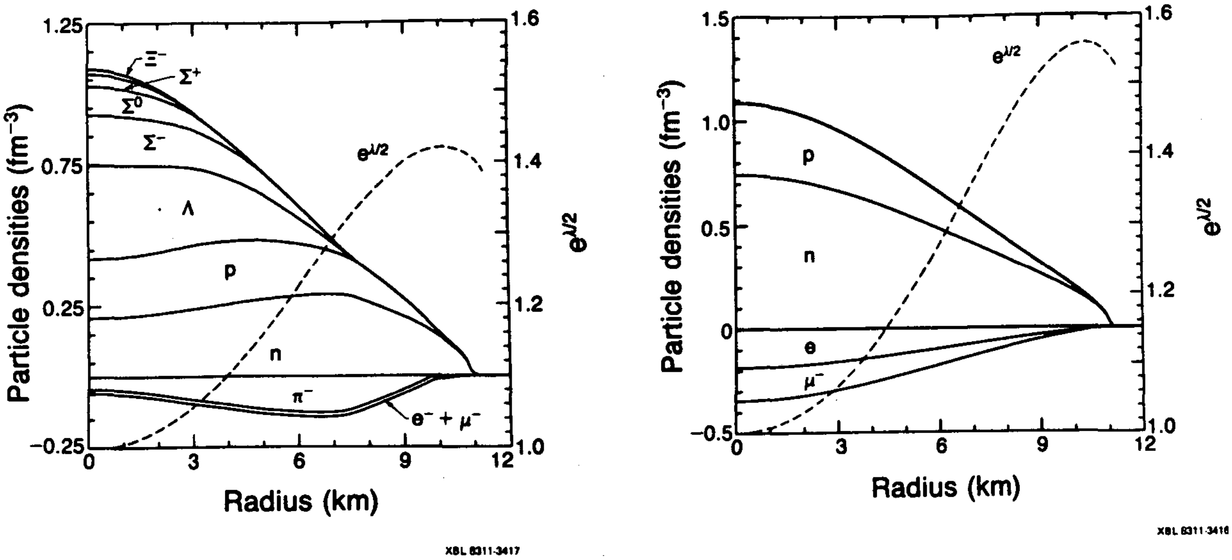


Fig. 7. Proper number densities in neutron stars at the limiting mass in an onion skin depiction (distance between curves denotes population), for two cases, (a) full theory, and (b) absent the hyperons and pions.

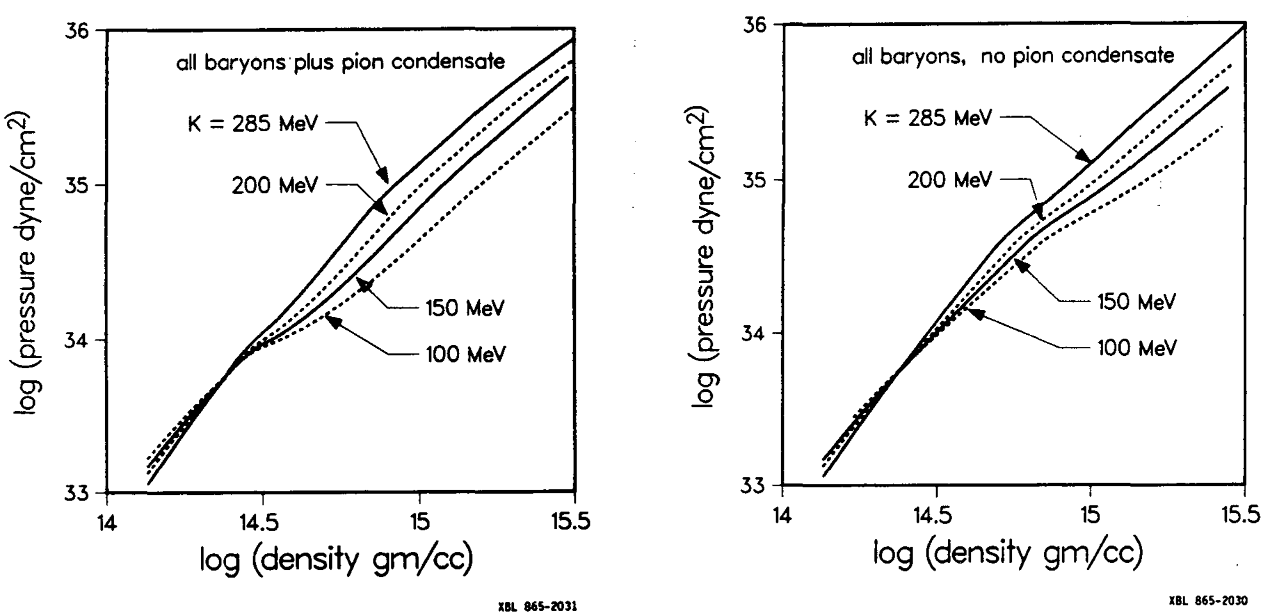
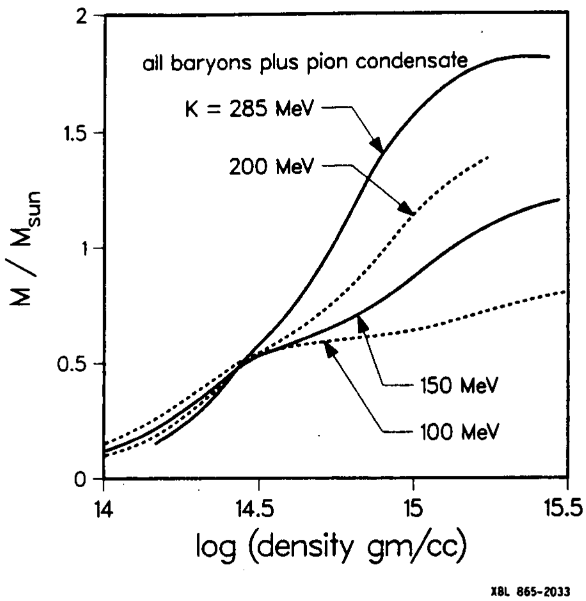


Fig. 8. In part (a) the equation of tate for various compressibilities when free pions condense, and (b) when they do not. These are limiting cases for the role of pions.



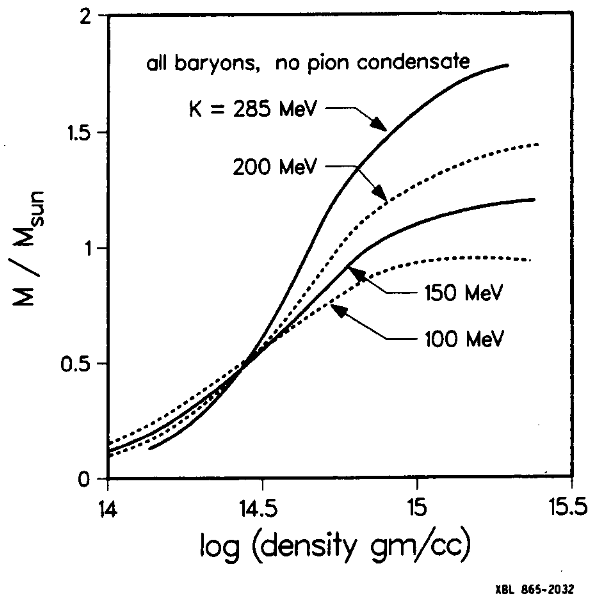


Fig. 9. The Masses of neutron stars as a function of their central density for various nuclear compressibilities, in the two cases (a) free pions condense and (b) pions do not condense.