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Thai Secondary School Students' Probability Misconceptions: The Impact of Formal Instruction

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Thai Secondary School Students' Probability Misconceptions:

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by

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ABSTRACT

Thai Secondary School Students' Probability Misconceptions:

The Impact of Formal Instruction

by

Puttoei Talawat

Probability is an important mathematics topic and is required for all Thai secondary school students. However, previous research found that both students and teachers often held several probability misconceptions, students had difficulties learning the topic, and teachers did not have sufficient knowledge to teach the topics. The purpose of this study was to investigate the types of probability misconceptions held by Thai secondary school students and their mathematics teachers, how the teachers implemented a nationwide inquiry-based curriculum, and how instruction impacted the students' understanding of the concepts. The participants were two grade 9 mathematics teachers and their students in four classes, 204 students in total. The data collection included lesson observations, a probability misconception pretest and posttest, and teacher interviews. The results showed that both the teachers and the students held several types of probability misconceptions. Even though the students' performance on the posttest was significantly improved after instruction, they still did not do well. The students learned knowledge on probability as evidenced in their responses on the posttest, but they were still having trouble appropriately applying the

knowledge. Teachers' probability misconceptions and their understanding of probability topics affected how their students learned and understood probability.

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Chapter 1 - Introduction

Probability and statistics are the science of learning from data. Probability theory is applied in everyday life to assess situations involving risk, while statistical procedures are used to make conclusions and to determine which data and conclusions are trustworthy. The world today produces more data than ever before, therefore probability and statistics are even more important in order to make use of these data. Probability and statistics are used in many fields such as science, industry, economic, public health, and public policy. It is important for students to learn and understand probability and statistics as they will encounter them in many aspects of their lives.

As the role of probability and statistics increases in society, they are also becoming an important part of school mathematics curriculum as well (Common Core State Standards Initiatives, 2010; National Council of Teachers of Mathematics, 2000). Many researchers have given these disciplines more attention in recent years. Several studies found that students have difficulties in learning probability and statistics (e.g., Amir & William, 1999; Batanero & Serrana, 1999; Fischbein, Nello, & Marino, 1991; Fischbein & Schnarch, 1997; Konold, 1989; Konold, Pollatsek, Well, Lohmeier, & Lipson, 1993; Lecoutre, Rovira, Lecoutre, & Poitevineau, 2006; Morsanyi, Primi, Chiesi, & Handley, 2009; Nilsson, 2009; Quinn, 2004; Rubel, 2007). Other studies made efforts to identify the characteristics of instruction that promote student understanding of the topics (e.g., Aspinwall & Tarr, 2001; Fischbein & Gazit, 1984; Jones, Langrall, Thornton, & Mogill, 1999; Jun & Pereira-Mendoza, 2002; Kafoussi, 2004; Konold, Madden, Pollatsek, Pfannkuch, Wild, Ziedins, et al., 2011; Polaki, 2002). However, probability and statistics still receive less attention compared to other areas of mathematics in educational research, teacher education, and professional development and there have been calls for more research in these areas (Shaughnessy, 1992; Stohl, 2005).

In Thailand, probability and statistics have been included in the school mathematics curriculum for over 30 years. The topics are covered throughout the 12 years of school, but are more of a focus during grades 9, 11, and 12. However, the topics, especially probability, are still viewed as less important, by teachers and students, compared to other mathematics content such as algebra or geometry. The Institute for the Promotion of Teaching Science and Technology (IPST), the main organization whose responsibility is to support the teaching and learning of science, mathematics, and technology in Thailand, has been making efforts to support teachers in teaching these topics. IPST has created teaching materials and supplements as well as organized several teacher development programs. Unfortunately, there is not yet enough scientific evidence to understand the obstacles of teaching and learning the topics in real classroom settings.

Stohl (2005) pointed out in her review that students' probability and statistics reasoning and understanding depend greatly on teachers' probability reasoning and understanding as well as the teachers' deeper understanding of students' misconceptions. However, a review of research studies on teacher knowledge showed that both primary and secondary school teachers lacked sufficient knowledge to teach these topics (e.g., Batanero, Godino, & Roa, 2004; Begg & Edward, 1999; Jacobbe & Horton, 2010; Liu & Thompson, 2007; Makar & Confrey, 2005; Watson, 2001). Many teachers had little exposure to these topics prior to the adoption of new mathematics standards and curricula and they are now expected to teach these disciplines. I argue that knowledge and understanding about what kinds of probability misconceptions students hold and how to help them overcome these misconceptions would greatly improve the teaching and learning of probability topics.

In summary, research studies to date have found that: students are struggling to learn probability and statistics, they often have several misconceptions and current ways of instruction do not always to help eliminate these misconceptions, teachers have inadequate understanding of these topics and of students' concepts of these topics, and teachers need more training in order to effectively teach the topics.

Need for Study

There has been interest in children' probability understanding even before probability and statistics became a part of the mathematics curriculum and standards. Since Piaget and Inhelder's (1975) book, *The Origin of the Idea of Chance in Children* (original work published in 1951), there have been debates about what is the best way to teach probability to children, and what is the appropriate age to begin formal instruction. Children usually have experienced many situations that involve uncertainty earlier in their everyday lives, such as playing games that involve tossing dice, listening to a weather forecast, or drawing raffle tickets for prizes. Children therefore already have some probability conceptions, or misconceptions, before any formal instruction on the topics takes place.

Many studies (e.g., Fischbein & Schnarch, 1997; Kahneman & Tversky, 1972; Konold, Pollatsek, Well, Lohmeier, & Lipson, 1993; Lecoutre, Rovira, Lecoutre, & Poitevineau, 2006; Morsanyi, Primi, Chiesi, & Handley, 2009; Rubel, 2007; Shaughnessy, 2003) have confirmed that students as early as 10 years-old hold a variety of probability misconceptions before receiving any probability lessons. For example, students may have a misconception that a die is more likely to show heads on the next toss if it had shown mostly tails in several previous tosses, or that the weather forecast of 90% chance of rain is incorrect if it does, in fact, not rain. According to Fischbein and Schnarch (1997), some of these misconceptions decrease with age, while some increase with age or instruction, and some remain stable across ages. Formal instruction on probability topics, hence, should be carefully designed to correct these misconceptions as well as to prevent new misconceptions from forming.

According to Shaughnessy (1992), in the line of research on probability misconceptions, there were very few studies that investigated the impact of formal instruction on students' probability misconceptions, and few studies with secondary school students have been undertaken compared to primary school or college students. After Shaughnessy's review, more studies have focused on secondary school students. However, most of these studies focused on the students' probabilistic reasoning prior to instruction and there is still the need for classroom research that investigates the effect of instruction on secondary school students' probability concepts and learning (Jones, Langrall, & Mooney, 2007). Shaughnessy (1992) also emphasized the need for research on instruction related to students' concepts of probability, such as research that would trace changes in students' probabilistic concepts before and after instruction.

In summary, to extend and deepen the knowledge in the field, more research on secondary school students' probability concepts and how their concepts change over a course of instruction is needed. Therefore, a sample of four classrooms of grade 9 Thai secondary school students was selected for this research study. The samples received their first formal instruction on probability topics as a part of their second semester mathematics course. Testing these students before instruction began revealed what kinds of probability misconceptions, if any, students held. Also, observing the nature of classroom instruction provided insight into how the lessons promoted students' correct understanding and influenced their misconceptions. Then, the students' posttests provided knowledge about how instruction impacted students' understanding and misunderstanding.

Purpose of Study

There have not been many reports on teaching probability. Some of these reports were about experienced teachers and educators sharing their lessons. Other reports were studies using specially designed lessons or programs. However, there is not much research on how probability is taught in real classrooms using existing curriculum. This research study explored Thai secondary school students' probability misconceptions, how Thai secondary school mathematics teachers implemented an inquiry-based curriculum, and how formal instruction influenced the students' understanding and concepts about the topics.

The curriculum used in this study was developed by IPST. Even though the curriculum was not mandatory, it was widely used throughout the country. The curriculum also met national standards, was intended to be constructivist, and would meet recommendations of the NCTM standards. The research was conducted at a secondary school in Bangkok, Thailand during late 2011 to early 2012, using both qualitative and quantitative methods. The participants were two grade 9-mathematics teachers and four groups of their students, 204 students in total. The research questions were:

RQ1. How did Thai secondary school mathematics teachers understand and teach probability topics? What were the differences between how the curriculum was intended and how the teachers implemented it?

RQ2. What were Thai secondary school students' probability misconceptions before and after formal instruction? How did their responses on the tests change after instruction?

RQ3. What happened in the classrooms that influenced students' misconceptions? What impact did this have on their probability misconceptions?

In summary, this research study explored the understudied area about the impact of classroom instruction on secondary school students' probability understanding. The results on how students' conceptions and misconceptions changed before and after instruction can be used to improve teachers' pedagogical knowledge of the topics. The results can also be useful for teachers, educators, and curriculum developers to better support students' learning of the topics.

Chapter 2 - Literature Review

This chapter presents a review of research studies related to probability misconceptions, teaching and learning of probability topics, and teacher knowledge of probability and statistics. The review consists of three main parts. The first part reviews research studies on probability misconceptions, as well as efforts to help eliminate students' probability misconceptions. The second part reviews research studies on the teaching and learning of probability and how instruction of the topics could promote students' learning. The last section focuses on research on teacher knowledge of probability and statistics.

Probability Misconceptions

In order to understand the effects of instruction upon students's probabilistic reasoning, it is important to distinguish between different approaches to define probability. Probability can be defined in many different ways, but the three common approaches, as described by Barnes (1998), are:

- **Subjective probability** is the degree of belief a person holds that an event will happen.
- **Experimental probability** is the frequency with which an event occurs in a large number of identical trials.
- **Theoretical probability** is based on a theoretical analysis of the outcomes of a random experiment, and usually involves counting equally likely outcomes. (p.17)

Experimental probability is sometimes called "frequentist probability" and theoretical probability is sometimes called "classical probability". Several authors, such as Jones, Langrall, and Mooney (2007) and Jun and Pereira-Mendoza (2002), also mentioned these three approaches to probability in their papers, although they did not specifically define them. Other researchers, when referring to non-normative responses, also used the term subjective probability.

The theoretical approach to probability is the main approach taught in formal instruction, while the experimental approach was recently gained more attention in school as well as from researchers and educators (Konold et al., 2011). However, the subjective approach is rarely mentioned during formal instruction of probability, despite the likelihood that it is the approach that is most likely to create probability misconceptions. Note that the difference between the three approaches to probability is not in a number as a result, but in how a person derives that number. The three approaches influence how people think about probability and all three of them can lead to misconceptions.

Before probability instruction, students usually hold a variety of probability misconceptions and these misconceptions do not necessary grow weaker with age, as confirmed in Fischbein and Schnarch's (1997) study. They investigated students' probability misconceptions for several age groups. Their participants were 80 students in grades 5, 7, 9, and 11 along with 18 students who were prospective teachers specializing in mathematics. All students in this research had received no previous instruction in probability. They found that the probability misconceptions that the students held did not always decrease with age or after regular mathematics instruction. In order to better address these misconceptions during formal instruction of probability, the nature of these misconceptions should be taken into consideration.

Probability misconception in the literature. In this study, the school adopted the Institute for the Promotion of Teaching Science and Technology's (IPST) mathematics curriculum and standards (2001). Based on the curriculum, there were five probability misconceptions related to the grade 9 course: Representativeness, Negative and Positive Recency Effects, Compound and Simple Events, Conjunction Fallacy, and Effect of the Time

Axis. Students at other ages may also have the same misconceptions, but these five misconceptions were likely to be found in 9th graders. Although more types of probability misconceptions have been documented in the literature (e.g., Fischbein & Schnarch, 1997; Kahneman & Tversky, 1972; Konold et al., 1993; Lecoutre et al., 2006; Morsanyi et al., 2009; Rubel, 2007; Shaughnessy, 2003) such as the effect of sample size[1] and the heuristic of availability[2] misconceptions, this review will only focus on the five misconceptions above. Since there were not many studies specific to 9th graders, the review also includes studies with participants in other age groups. Readers should keep in mind that an answer in term of quantity alone should not be interpreted as a specific misconception, rather the determination should be based on the reasoning that underlies the answer. The same answer for the same situation, if resulting from different reasoning, may come from different misconceptions. These five misconceptions will be described in turn, including related research.

Representativeness. A representativeness misconception happens when a person estimates the likelihood of an event on the basis of how well it represents the parent population (Kahneman & Tversky, 1972; Shaughnessy, 2003). The two common tasks used to investigate this misconception are 1) lotto game, when a person believes that "random" numbers like " 39, 1, 17, 33, 8, 27" are more likely to win than "pattern" numbers like "1, 2, 3, 4, 5, 6" (Fischbein & Schnarch, 1997, p. 98); and 2) coin tossing, when a person believes that a sequence of five coin tosses is more likely to be "THHTH" than "HTHTH" (Konold et al., 1993; Rubel, 2007). Fischbein and Schnarch (1997) investigated students' probability

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[1] Happens when a person neglects the influence of the magnitude of a sample when estimating probability (Fischbein & Schnarch, 1997).

[2] Happens when a person estimates frequency or probability by the ease with which instances can be brought to mind (Fischbein & Schnarch, 1997).

misconceptions for several age groups. Their participants were 80 students in grades 5, 7, 9, and 11 along with 18 students who were prospective teachers specializing in mathematics. They found that this misconception, though it grew weaker with age, persisted through all age groups. In their study, 22 percent of college students still had this misconception. However, Morsanyi et al. (2009) found that this misconception was mostly unaffected by education and also unrelated to participants' cognitive abilities. Their participants were 185 psychology and biology students from the University of Plymouth. Even though older participants were less likely to have a representativeness misconception, education (not specific to mathematics or probability) did not help eliminate the misconceptions. Konold et al. (1993) did a study with 88 high school and undergraduate students. They also found that participants who based their responses on representativeness misconceptions in one context may or may not give the same reasoning in other contexts or tasks. The lotto situation is closely related to the gambler's fallacy (details follow), while the coin situation is more likely to link to the notion of randomness.

Lecoutre et al. (2006) conducted an experiment to explore subjective beliefs about randomness and probability using stochastic items[3] and real items. The participants were 20 students (age 14 – 16 year-old) and 40 researchers from universities. They found that, for the stochastic items, a large majority of individuals were in agreement that an event is random or not with the reason that because it is easily possible to compute a probability. On the other hand, for the real items, there was no large majority of answers about whether an event is random and two main conceptions have been observed. Either randomness is involved because probabilistic reasoning is involved, or randomness is not involved because causal

[3] Items either involve a repeatable process or consist of events produced via a mechanism.

factors can be identified. An interesting finding is that "although the concept of probability has been introduced to formalize randomness (randomness implies probability), a majority of individuals appear to consider probability as a primary concept (probability implies randomness)" (p. 31). Lecoutre et al.'s finding implied that when dealing with a stochastic situation, like the coin situation above, participants are more likely to view the situation as a random event and might expect the results to appear "random". Hence, the sequence "THHTH" is more likely to occur than "HTHTH".

Batanero and Serrana (1999) conducted a study with 277 secondary school students on their notions of randomness. They found that students overemphasized unpredictability and luck to justify their attribution of randomness. Even though the researchers did not use the term "misconception", the underlying concept was in line with the representativeness misconception. They also found that the tendency to have the misconception seemed greater with older students.

Negative and positive recency effect. Negative and positive recency effect misconceptions happen when a person believes that a specific outcome of a sequence of independent events is more likely (positive recency effect) or less likely (negative recency effect) to occur due to the lack of that outcome in the previous results (Fischbein & Schnarch, 1997). For example, if a person were to toss a coin five times and it showed heads the first four times, he would think the coin is more likely (positive recency effect) or less likely (negative recency effect) to show heads the fifth time. Fischbein and Schnarch (1997) also called the "negative recency effect" the " gambler's fallacy". The gambler's fallacy seemed to arise from people's interaction with gambling. People often recognize that someone wins a lottery everyday, despite a very low chance of winning. Fischbein and Schnarch investigated students' probability misconceptions with 80 students in grades 5, 7, 9, and 11 along with 18 students who were prospective teachers specializing in mathematics. They found that the negative recency effect decreased with age (20% for 9th graders), where as the positive recency effect was almost absent.

Quinn (2004) studied 113 secondary education major students at the University of Nevada, Reno, a town where gambling is legal. He found that most of the participants responded correctly to the coin problem, that the chance of a coin landing heads or tails is equal, after a sequence of three heads. On the other hand, fewer than half of the participants responded correctly to the card problem, that the chance of dealing two specific sets of five cards from a standard deck is equal. Quinn found both correct reasoning with an incorrect answer, and a correct answer with incorrect reasoning. This finding emphasized that the justification is as important as the answer itself. Quinn also offered two explanations why there was a big difference in the responses between the two problems: 1) the coin problem, or one very similar to it, is mentioned in almost every class involving probability or statistics, and 2) one of the two set of cards in the card problem is called a "royal flush" in a poker game, and since the participants had experience with poker, they might have related the experience of never or very few times being dealt a royal flush, but having been dealt the hand that looks "normal" most of the time.

In contrast, the reasoning of the positive recency effect is that the person assumes (implicitly or explicitly) that the condition is not fair (Fischbein & Schnarch, 1997). For example, if a coin was tossed four times and always shows heads, a person may believe that the coin was biased. However, Nilsson (2009) argued that this notion seemed to be more in line with experimental probability. A person might consider the experimental data that they have available, in this case, the four heads showing compared to none of the tails, in making a decision.

The representativeness misconception and negative and positive recency effect misconceptions are closely related. Reasoning that shows these misconceptions may result from an *Outcome Approach* - answering a question of uncertainty based on the prediction of the outcome of an individual trial (Konold, 1989). According to Konold's definition, individuals who reason according to the outcome approach believe that they were asked to predict the outcome of an individual trial. Hence, they tend to give inconsistent answers for different questions in the same context. Usually, their predictions are in the form of yes or no on whether an outcome will occur on a particular trial. Konold conducted an interview study with 16 undergraduate students and also found that individuals who employ the outcome approach in a situation may or may not employ the same concept in other situations. Hence, the outcome approach is not a belief system that individuals either do or do not hold but a set of beliefs that individuals hold to differing degrees.

Although the outcome approach is inconsistent with formal theories of probability, its components are logically consistent and reasonable in the context of everyday decision-making. There are two features of the outcome approach:

1. Evaluating probabilities as either right or wrong after a single occurrence by first predicting outcomes of single trials and then interpreting probabilities as predictions.

2. Basing probability estimates on casual characteristics rather than on distributional information.

As mentioned earlier, the outcome approach, representativeness, and the negative and positive recency effect misconceptions are closely related. It is not always clear which misconceptions a person holds and he or she could be switching among the three. Konold et al. (1993) investigated participants' responses on several probabilistic situations. They found that several participants responded to some items based on a representativeness misconception, while they answered the other items based on an outcome approach. Rubel (2007) conducted a research study with 173 middle and high school students using similar tasks. She found more consistency among the participants' responses, and overall results showed fewer misconceptions in the participants than those of Fischbein and Schnarch (1997) and Konold et al. (1993). However, her efforts to use cognitive conflict as a mean to correct the participants' misconceptions and their inconsistency during supplemental clinical interviews did not work well. This failure led to the same conclusion as Konold et al. that the inconsistency in the responses appeared because participants employed different approaches to different situations. They might not see the conflict because they do not see the tasks similarly. Both Konold et al. and Rubel also emphasized that the correct answer alone is not enough evidence to conclude that participants have correct understanding. More evidence than paper and pencil responses is needed in order to judge the participants' understanding. Morsanyi et al. (2009) explained another source of inconsistency, the cognitive process that participants employ in solving the problem. They found that, when their participants (185 psychology and biology students from the University of Plymouth) were asked to think logically versus rely on their intuitions, they gave different responses.

Compound and simple events. A compound and simple events misconception happens when a person does not take order into account when comparing a compound event with a simple event (Fischbein & Schnarch, 1997). For example, when tossing 2 dice simultaneously, a person would think that the probability that each of the two dice showing 6 is equal to the probability that one die shows 5 and the other die shows 6. Jones et al. (1999) did not define this type of response as a misconception; rather, they defined the participant who gives this response as in the transitional level (from the subjective level to the informal quantitative level. If this is the case, the misconception should improve with age or instruction. However, Fischbein and Schnarch (1997) found that this misconception was frequent and stable across ages. Their participants were 80 students in grades 5, 7, 9, and 11 along with 18 students who were prospective teachers specializing in mathematics. This finding is very interesting and might be counter-evidence for Jones and his colleagues' (1999) Probabilistic Thinking Framework. In my earlier research with in-service teachers, 31% of the participants gave responses that showed the compound and simple events misconception, even after a three hour lecture on probability (Talawat, 2011). A closer look at the participants' reasoning pointed to other issues that might cause the misconception: the language used and how the question was asked. It is possible that (my and Fischbein & Schnarch's) participants interpreted the pair 5-6 as different from 6-5 (taking order into account), based implicitly on the problem wording.

Fischbein et al. (1991) investigated the compound and simple events misconception with 618 elementary and junior high school students in two situations. They found that the participants were more likely to respond correctly on the question, "One rolls two dice. Which is more probable: to obtain the same number with both dice, or different numbers?" (p. 536) than the question, "Considering the sum of the points obtained when rolling a pair of dice, will you bet on 3 or 6?" (p. 540). The participants' justifications for the first situation usually referred to the corresponding sample space. For the second situation, many participants seemed to be able to relate the estimations of probabilities to the magnitude of sample space. However, the description of the sample space, in this case, was incorrect, and hence led to the wrong answer. The authors concluded that the notion of compound and simple events "is a topic which deserves more attention, considering its intuitive complexity and the variety of situations, which it may generate" (p.547).

When addressing the three misconceptions described above, research studies usually used tasks that involve considering whether or not two situations are equally likely to happen, either equal as correct answer or incorrect answer. However, the answer "equally likely to happen" itself could be considered as another type of misconception: *Equiprobability* or believing that all outcomes from a probability experiment have the same chance of happening (Shaughnessy, 2003). "Anything can happen" and "50-50 chance" are equiprobability statements students tend to say. Morsanyi et al.'s (2009) study with 185 psychology and biology students from the University of Plymouth found that this misconception increased with statistics education, and it was negatively correlated with students' cognitive abilities.

Conjunction fallacy. A conjunction fallacy misconception happens when a person views the probability of an event to be smaller than the probability of the intersection of the same event with another (Fischbein & Schnarch, 1997; Shaughnessy, 2003). For example, imagine a scenario where a woman goes food shopping for herself at a local market. A person with a conjunction fallacy misconception would think that the chance of the woman buying her favorite fruit is higher than the chance that she buys fruit of any kind. Fischbein and Schnarch (1997) found that this misconception was very strong through grade 9 but less strong (by about half) for high school and college students. Because conjunction questions usually employ rich language, students often encounter meaning making and reasoning challenges. As a result, students tend to use familiar information to judge a conjunction of two events as more likely to occur than either one of the two events itself (Watson, 2005).

Effect of the time axis. An effect of the time axis misconception happens when a person believes that an event cannot act retroactively on its cause (Fischbein & Schnarch, 1997). For example, envision drawing two marbles, one at a time, from an urn that contains three yellow marbles and three green marbles. A person with a time axis misconception who did not know the color of the first marble he drew but did know the color of the second one would think that knowing the color of the second drawn marble does not affect the probability of the color of the first marble he drew. This misconception is also called the "the falk phenomenon." Fischbein and Schnarch (1997) investigated students' probability misconceptions from several age groups. Their participants were 80 students in grades 5, 7, 9, and 11 along with 18 students who were prospective teachers specializing in mathematics. They found an increase with age of this misconception, except in the case of college students. They explained that the apparently causal order of the story hides the stochastic structure of the problem. In addition, a similar finding was found in Watson and Kelly's (2007) empirical research conducted with 69 students in grades 3-13 using the same problem.

Efforts to help eliminate students' probability misconceptions. The five probability misconceptions described above arise because participants employed different approaches to probability. Though researchers still cannot be certain about how these misconceptions arise, teachers can still address and try to eliminate them during instruction. However, very few studies have made an effort to help students eliminate their misconceptions. Two such studies are described here.

Rubel (2007) used cognitive conflict as a mean to correct the participants' misconceptions and their inconsistencies. She had 173 students in grades 5, 7, 9, and 11 took a probability misconception test and did one-on-one interventions during interviews with 33 of the participants in which she asked them questions, based on their reasoning, that would lead to a conflict (i.e., impossible situations). For example, a participant believed that when tossing two coins, there was a 50% chance that both coins land on tails. Rubel asked the participant what the probability of getting all tails when tossing 3, 100, and 100,000 coins are, hoping to lead the participant to a cognitive conflict. She used several kinds of questions based on the participants' reasoning. However, her method did not work well. The participants did not see the situation as conflict.

Several researchers described how instruction could help improve students' probabilistic understanding. However, instruction that focuses on one concept**,** if not carefully developed, can sometimes lead to a misconception in other concepts. Another study was from Sharma's (2007) article about 24 pre-service teachers' understanding about statistical variation and Farmer's (2008) response to it. Sharma's (2007) results indicated that pre-service teachers have inadequate concepts of variability. One of the problems mentioned in the article is shown in Figure 2-1.

(a) Imagine you threw a die 60 times. Fill in the table below to show how many times each number might come up.

Number on die	How many times it might come up?
1	
2	
3	
4	
5	
6	
Total	60

(b) Why do you think these numbers are reasonable?

Figure 2-1. A problem used in Sharma's (2007) article (p. 36).

Two of the pre-service teachers' responses provided were 1) "12, 11, 9, 10, 8, 9 – because they are around the expected but you can't really tell", and 2) "10, 10, 10, 10, 10, 10 because each number has the same chance of being rolled" (p. 37). The first answer was coded as "statistical". The second answer was coded as "partial-statistical" and was regarded as showing less understanding about variation. There were only 2 responses coded as statistical and 18 responses coded as partial-statistical. Sharma concluded that his participants were unable to integrate centers and variation, and that they relied on expectation in their explanation. However, Farmer (2008) argued that suggesting to students that "12, 11, 9, 11, 8, 9"[4] is a good answer can create another type of misconception, namely, the gambler's fallacy[5].

Farmer's arguments included 1) "12, 11, 9, 11, 8, 9" has, in fact, a lower chance of occurring than "10, 10, 10, 10, 10, 10" (approximately 4.2×10^{-5}, compared to 7.5×10^{-5}), and

[4] Farmer (2008) changed the 4th number from 10 to 11 to make the sum equal to 60.

[5] Farmer (2008) did not describe "the gambler's fallacy" in his article. I found that he used this term quite differently from what I described earlier.

2) provided that the question does not require an assessment of probability, any responses with the six numbers summing to 60 is equally valid. By suggesting a particular response as more "correct" than a "special pattern" response can lead students to a misconception that a "non-special pattern" (in general) is more likely to occur. Participants illogically view the "non-special pattern" as representative of many of those "non-special" patterns, and the probability that they obtain is actually from the large set of "non-special" patterns. Of course, this probability exceeds the probability of one special pattern like "10, 10, 10, 10, 10, 10".

Farmer did not suggest how the item could be improved to avoid leading to the misconception, rather, he simply concluded that "it is certainly important that students and teachers have a good understanding of the effects of variation, … We do however need to be careful that discussion of variation effects do not lead to other misunderstandings" (p.62).

In summary, research showed that students at various age groups hold a variety of probability misconceptions. Some of these misconceptions increase with age, while some decrease with age. However, 9th graders are likely to have these five misconceptions: representativeness, negative and positive recency effects, conjunction fallacy, compound and simple events, and effect of the time axis. Even though there were not many studies that examined 9th graders' probability misconceptions, these five misconceptions existed in both younger and older students. Most of these misconceptions persist through all age groups and instruction or education does not always eliminate students' misconceptions. To investigate how students understand probability and to promote correct conceptions of probability, the knowledge of students' probability misconceptions and how the misconceptions obstruct their learning of the topics is necessary. The next section discusses research on the teaching and learning of probability.

Research on the Teaching and Learning of Probability

There are very few research studies on the teaching and learning of probability that focus on middle school students. Many of them either study younger children (kindergarten and elementary school students) or college level students. This section includes research on younger students even though the focus of this study is on 9th graders. Research on college level students is not included since the content investigated was typically beyond the scope of this study.

Constructivism as a paradigm for learning. The idea of constructivism theory is that students construct their own knowledge through experience. When applying the theory in the classroom, there are several teaching practices that promote constructivism, such as encouraging students to do experiments, solve real-world problem, and reflect on their understanding. Constructivism is often associated with pedagogic approaches that promote learning by doing (Tobias & Duffy, 2009). Review of research in this section shows that researchers are encouraging teachers to teach probability using constructivism theory.

Intended curriculum and implemented curriculum. Curriculum is one of the most important elements of teaching and learning, not only which curriculum to use but also how to use it. Porter and Smithson (2001) gave a definition of an intended curriculum as "policy tools [such] as curriculum standards, framework, or guidelines that outline the curriculum teachers are expected to deliver (p.2)" including textbooks. They also defined an implemented curriculum (enacted curriculum) as "the actual curricular content that students engage in the classroom" (p. 2). It is important to distinguish the intended curriculum from the implemente curriculum as they may referring to different contents, concepts, or teaching styles.

When to teach probability? The question that has often been asked is: "What is the appropriate age to begin probability instruction?" There have been debates about what is the best way to teach probability and statistics to students, and when is the best time to begin. The Common Core State Standards for Mathematics (National Governors Association Center for Best Practices, 2010) indicate that one critical area that instruction should focus on in grade 6 is developing understanding of statistical thinking. However, the Principles and Standards for School Mathematics (National Council of Teachers of Mathematics, 2000) recommend instructional programs to include data analysis and probability beginning in grades K-2. Both sets of standards emphasize the statistics and probability domain more at the high school level (grades 9-12). Several researchers (e.g., Jun & Pereira-Mendoza, 2002; Konold et al., 2011; Quinn, 2004) suggested that instruction of probability should integrate both experimental probability and theoretical probability, while others, such as Amir and Williams (1999), Barnes (1998), Hawkins (1984), and Jones et al. (2007), suggested that all three approaches to probability (subjective, experimental, theoretical) should be integrated.

Piaget and Inhelder's (1975) book *The Origin of the Idea of Chance in Children* (original work published in 1951) was cited in many documents (e.g., Falk and Wilkening, 1998; Hawkins, 1984; Konold, 1989) and was regarded as the earliest work on probability teaching. Piaget and Inhelder (1975) claimed that most concepts of probability were not available until the level of formal operations, which usually occur at age 12 or older. Therefore, they suggested that formal instruction in probability should wait until children reach that age. Similarly, Falk and Wilkening (1998) conducted probability experiments with

6 – 14 year-old children. They found that only at around the age of 13 did most children proportionally integrate the two dimensions of the events. Falk and Wilkening's results agreed with Piaget and Inhelder. However, several subsequent researchers found contradictory results.

Fischbein and Gazit's (1984) exploratory research study analyzed the effects of a teaching program in probability for grades 5, 6, and 7 students. They found that most of the concepts were too difficult for the fifth graders. In contrast, the majority of sixth graders and most of the seventh graders were able to understand and correctly use the concepts of probability taught in the program. Fischbein and Gazit did not describe their teaching program in detail, but stated that the lessons included practical activities and emphasized the relation between calculated probabilities and empirical frequencies. Due to the fifth grade students' poor performance after the implementation of the instructional program, the researchers suggested not to teach this concept prior to sixth grade. However, others researchers (see for example, Shaughnessy, 2003) suggest to begin to teach probability at a young age, and continue this throughout the school-age-years. Several teaching experiments with younger students (e.g., Kafoussi, 2004; Polaki, 2002; Tatsis, Kafoussi, & Skoumpourdi, 2008) showed that students as young as kindergarten age could be successful in learning probability.

For example, Kafoussi (2004) conducted a teaching experiment with kindergarten school students. In summary, Kafoussi's teaching intervention focused on 1) distinguishing events that always, sometimes or never happen in their daily life and in random experiments, 2) discussion about all the possible events in a one or two stage random experiment, both ordered and unordered pairs, 3) discussion about probability of an event and the probability comparison, and 4) discussion about conditional probability. The results showed that "the children managed to overcome their subjective interpretations and seemed to develop a primitive quantitative reasoning in probabilistic tasks" (p. 29).

Tatsis et al. (2008) observed kindergarten school students and their teachers while discussing the concept of two chance games. They found that the students were able to overcome their primary intuitions concerning the fairness of the games and to comprehend the important role of materials. Based on their framework, which focused on the creation of a primary discursive community, the authors suggested that teachers' verbal acts should be made to assure four aspects:

> a) Most – if not all – children will have the chance to talk and express their opinion.
> b) Most – if not all – children will comprehend the concepts involved in (the situation).
> c) The correct view will be accepted at least by the majority of children.
> d) The practical tasks involved will be completed successfully and on time. (p. 225)

Polaki (2002) conducted two teaching experiments with fourth and fifth grade students in Lesotho, South Africa. There were six students in each group. Students in the first group focused on analyses of small-sample experimental data and sample space composition as strategies for tackling probability problems. Students in the second group were challenged to make connections between large-sample experimental data and sample space composition after looking at small sample data and sample space symmetry. The researcher found no significant differences in probabilistic thinking levels of students between the two versions of teaching experiments, which meant that asking Group II students to make connections between analysis of large-sample experimental data (drawn from simulations) and sample space symmetry did not give them an advantage over their group I counterparts. Polaki gave two explanations for this observation: 1) the law of large numbers, which is the underlying concept in making the connection, may not have been easily accessible to the young students, and 2) analyzing the data generated from a computer was not part of students' experiences, and they had difficulty making sense of it.

There were mixed results on whether the students, specifically elementary school students, could be successful in learning probability. It should be considered how the successful lessons differed from the unsuccessful ones. As mentioned earlier, there are several approaches to probability and each of them can lead to misconceptions or misunderstandings, making it more difficult for students to develop a correct understanding of probability and statistics. Instruction in probability should provide experiences in which students are allowed to confront their misconceptions and develop understanding based on mathematical reasoning (Jones et al., 2007).

Integration of experimental and theoretical approaches. Teaching probability without using empirical experiences to enhance the theory presented could result in incomplete or inaccurate perceptions. Quinn's (2004) results of his investigation suggest that the converse of this is also true. He studied 113 secondary education major students at the University of Nevada, Reno using two probabilistic tasks. He found that empirical experiences without the appropriate discussion and analysis of the theory behind these experiences were insufficient to result in a strong understanding of the topics.

Jun and Pereira-Mendoza (2002) conducted a teaching experiment with grade 8 students to investigate whether focused instruction can help overcome misconceptions. They defined the teaching experiment as "activity-based combined with whole class discussion … focused on the misconceptions related to identification of impossible, possible, and certain events and the frequentist (experimental) definition of probability" (pp. 1-2). They found that students' performance improved after the intervention and their misconceptions were decreased. Since the intervention used experimental probability and did not give the classical (theoretical) definition, they also found that some students could compare chance events but could not give the real chance values. The authors concluded that experimental probability does not necessarily contribute to students' knowledge of classical (theoretical) probability and introducing probability in the frequentist (experimental) approach or in the classical (theoretical) approach cannot replace each other.

Integration of all three approaches and beyond. Could probability misconceptions be eliminated by instruction that focuses only on experimental and theoretical approaches? Hawkins (1984) argued that "it seems to be an over-simplification to assume that subjective probability will just 'melt into' formal probability as an individual matures" (pp. 356-357). Fischbein et al.'s (1991) study, with 618 elementary and junior high school students in two situations, found that their participants did not have the full understanding of the terms "possible", "impossible", "certain" and "rare." Hence, teachers should not take for granted that students understand these terms in one (correct) way. Teachers should learn to recognize the heuristics and approaches that are commonly applied by students. Teachers should take as a starting point for teaching not only students' formal knowledge, but also their informal knowledge, which includes their specific beliefs (subjective probability) (Amir & Williams, 1999). Jones et al. (2007) suggested that notions that involve probability, like chance variation and stability, independence and co-occurrence, figuring likelihood from multiple sources (classical, frequentist, and subjective), language of chance, critical questions, and contexts, need to be examined more deeply when future curriculum documents are developed.

Barnes (1998) suggested that, before beginning to teach probability, a teacher should discover some of the intuitive ideas (misconceptions) that students hold. She also provided an effective sequence, based on her experience, in lessons involving probability experiments.

> 1. Engage the students' interest by posing a question.
> 2. Get the students to commit themselves by writing down an answer. This prevents them from sliding out of controversy later.
> 3. Ask students to discuss their answers in groups and try to explain to one another why they chose the answers they did.
> 4. Help the class to devise experiments to test the answers they have given.
> 5. Analyze the combined data and compare this with the students' initial ideas.
> 6. Where appropriate, discuss a theoretical model of the problem and work out theoretical probabilities. Compare this with the experiment results and discuss both similarities and differences. (p. 19)

Furthermore, a few researchers considered that using an integration of the three approaches in teaching probability as insufficient. The activities used in the probability lesson also need to be designed to capture the special notions of probability that differ from other topics in mathematics. Probability, usually taught as a part of the mathematics curriculum, requires a way of thinking that is different from that required by most school mathematics. A main difference between probability and other areas of mathematics is that probability involves interpretations of data and it has a notion of uncertainty. This means that students need to learn how to interpret data and deal with the variation of data, as well as the theoretical aspects of the events. However, the nature of uncertainty plays a role that, despite how "good" or "accurate" an answer is, no one could be certain of what the outcome will be. To deal with this issue, Konold et al. (2011) gave this suggestion:

> An alternative approach to probability instruction that we think holds promise is for students to start with explorations of a situation where there is no clear theoretical model. In exploring these situations, students would see their objective as estimating a probability by collecting data and that the data gave them some information about the tendency of the object to land in particular orientations. These activities would serve as the basis for students later coming to understand that while the data we collect can inform our estimates of probability, we can never know exactly what that probability is

> for the same reason that we never know the exact length of a table we measure, no matter how finely calibrated our ruler. (pp. 83-84)

Similar to Konold et al.'s suggestion, a probabilistic situation where there are multiple correct answers should be brought into classroom discussion. Consider this situation taken from Liu and Thompson (2007):

> At the Cobb County fair a clown is sitting at a table with three cards in front of him/her. (S)he shows you that the first card is red on both sides, the second is white on both sides, and the third is red on one side and white on the other. (S)he picks them up, shuffles, hides them in a hat, then draws out a card at random and lays it on the table in a manner such that you can see only one side of the card. (S)he says: "This card is red on the side we see. So it is either the red/red card or the red/white card. I'll bet you one dollar that the other side is red." What is the probability that you would win this bet were you to take it? (p.143)

This problem, unlike a typical math problem, does not have one correct answer. The probability of the event differed depending on how one conceived of the stochastic process. Some possible answers, among others, are 1/2, 1/3, and either 0 or 1. Instruction needs to include this type of problem as well as a detailed discussion of why different answers could be acceptable provided that they are accompanied by a valid reason.

Finally, Hawkins (1984) suggested coherence in teaching probability:

> Coherence is an important concept which needs to be taught to children. They need to be made overtly aware of subjective probability, as we believe that this is closer to the intuitions that they try to apply in formal probability situations. Frequentist and 'a priori' approaches also have an important role to play, but the three approaches need to be blended together to provide children with an appropriate framework for formal probability, rather than merely focusing on frequentist and/or 'a priori' perspectives which may well conflict with the children's expectation and intuitions. (p. 372)

In summary, research on teaching and learning of probability has shown that students as young as kindergarten could be successful in learning probability. The keys to the success are the appropriate content and approaches. The theoretical approach to probability, when taught alone, does not always eliminate students' misconceptions. Combining a theoretical approach with an experimental approach, if not done properly, is likely to create new misconceptions. Researchers, therefore, suggest combining the three approaches. However, there is not yet any evidence to confirm its effectiveness.

Teacher Knowledge on Probability and Statistics

Students' probability reasoning and understanding depend greatly on teachers' probability reasoning and understanding as well as the teachers' deeper understanding of students' misconceptions (Stohl, 2005). Probability and statistics have received less attention compared to other areas of mathematics in teacher education and professional development and there have been calls for more research in these areas (Shaughnessy, 1992; Stohl, 2005). With statistics and probability becoming an increasingly important part of the mathematics curriculum (e.g., National Governors Association Center for Best Practices, 2010; National Council of Teachers of Mathematics, 2000), many teachers who have had little exposure to these topics are now expected to teach them. Though this dissertation studied in-service teachers of probability, the research in this area is very limited and, therefore, the review of research is extended to include pre-service teachers in probability and statistics as well. This section discusses some issues related to teacher knowledge in probability, statistics, and related topics. Most of the research results presented in this section show that both elementary and secondary school teachers have inadequate knowledge of probability and statistics.

There is not much empirical research that studied teachers' content knowledge of probability and statistics. Of this research, studies focused on a wide range of topics, such as conceptions and misconceptions of probability, variation, distribution, and analysis and interpretation of data, etc. (e.g., Begg & Edward, 1999; Jacobbe & Horton, 2010; Liu &

Thompson, 2007; Watson, 2001), which makes it difficult to draw comparisons and conclusions on the basis of content. However, most of the research results indicated that teachers lacked sufficient knowledge of these topics.

According to his experiences with teachers, Pereira-Mendoza (2002) argued that primary teachers do not have sufficient statistical knowledge to teach statistics in the primary school. He stated that the problem derives from the teachers' limited statistical exposure. The teachers may know the theories and the formalized use of these theories, but they may not fully comprehend the situations in which a theory is appropriate. Two empirical research studies, Begg and Edward (1999) and Jacobbe and Horton (2010), support his argument that primary teachers lack sufficient knowledge.

First, Begg and Edward (1999) studied 22 primary teachers from 14 schools and 12 pre-service primary school teachers in New Zealand, using unstructured, semi-structured, and clinical interviews, and surveys. They found that very few participants had formal training in statistics in school and had weakly developed concepts of probability, showing several misconceptions such as the representativeness misconception and the effect of the sample size misconception[6]. Unfortunately, the main researcher, Roger Edwards, passed away shortly after the completion of data collection, and only partial findings were reported.

Second, Jacobbe and Horton (2010) investigated elementary school teachers' comprehension of data displays, which is a topic that is often taught as part of statistics. The researchers found that teachers performed well on reading the data, computation, and comparison. However, they did not perform as well on trend, and selection and construction of data displays. Finally, the researchers concluded that the participants could benefit from

[6] The tendency to neglect the influence of the magnitude of a sample when estimating probabilities (Fischbein & Schnarch, 1997).

professional development training that focuses on the development of statistical content knowledge.

Similarly, the empirical research on secondary teachers also indicated that the teachers have limited and often inadequate content knowledge of probability, statistics, and related topics. A study by Makar and Confrey (2005) focused on the use of language in describing variation, which is considered part of statistics, while Liu and Thompson's (2007) and Batanero, Godino and Roa's (2004) studies focused on the concepts of probability. Details of these three studies follow.

Standard statistical language refers to the standard terminology such as mean, maximum, minimum, sample size, outlier, range, and standard deviation, while non-standard language refers to the words or phrases used by the participants that are not the standard terminology, such as clustered, spread out, clump, and chunk. Makar and Confrey (2005) examined the participants' use of standard and non-standard statistical language when they describe variation. The participants were 17 pre-service secondary mathematics and science teachers. The researchers reported the combination of the interview results, before and after the intervention. They found the preservice teachers used both standard and non-standard language. When they used standard language, the participants' concept of variation and variability were related. This finding could be interpreted as the participants' incomplete concept of variation and variability. They should have been able to distinguish between the two concepts.

Stochastic reasoning or probabilistic reasoning is one of the big ideas in probability. Probabilistic reasoning is different from logical reasoning because in logical reasoning a proposition is always true or false but there is no certainty on a proposition concerning a random event (Borovcnik & Peard, 1996 as cited in Batanero et al., 2004). Two research studies by Liu and Thompson (2007) and Batanero, et al. (2004) investigated teachers' conceptions of this idea. Setting out as professional development programs, the researchers drew from their participants' responses on tasks and discussions their conceptions and understanding of stochastic reasoning.

Liu and Thompson (2007) conducted an 8-day seminar using modified constructivist teaching with eight high school statistics teachers in Singapore. Their purpose was to develop a theoretical framework for describing teachers' understanding of probability. The data collected included videotapes of sessions, interviews, teachers' written work, and fieldnotes. The article reported findings about teachers' understanding about probability, rather than describing the professional development program implemented. The main finding was that there was a complex mix of conceptions and understandings of probability, within and across the teachers. Five out of eight teachers had a situational conception of probability, which meant their interpretations of probability varied according to the particularity of the context. The researchers seemed to view this situational concept as a low level of understanding. Another three teachers had a stochastic conception of probability (viewing an observed outcome as one expression of a repeatable process). The researchers seemed to view this stochastic concept as a higher level of understanding. One of the teachers in this latter group also had a nonsituational conception of probability, meaning he was able to distinguish a situation from the underlying concepts and offer multiple interpretations of a situation. His conception of probability, unlike the other teachers, was not situationally triggered. He was able to correctly understand the problems' concepts regardless of the problems' context. This latter type of conception is important. It enables the teacher to approach probability more systematically and less subjectively. The researchers concluded that teachers need help in order to gain a better conception (i.e., a stochastic conception) of probability.

Similar results were found by Batanero et al. (2004) during the implementation of their teacher education course. Their participants were 47 pre-service teachers, both primary and secondary level at the University of Granada, Spain. They were 4th- and 5th- year statistics majors. The authors found that their participants lacked adequate knowledge about randomness and stochastic process. The participants provided inappropriate and insufficient reasoning both with correct and incorrect answers.

In summary, research results indicated that teachers have limited knowledge of probability and statistics, often hold several misconceptions, and use non-standard terminology when explaining their ideas. However, the existing research is not sufficient to meaningfully contrast elementary and secondary teachers or pre-service and in-service teachers. Researchers and teachers educators need to help teachers to be better prepared to teach these topics.

Research Questions

My review of research on probability misconceptions, the teaching and learning of probability topics, and teachers' knowledge of probability and related topics reveals that teachers have inadequate knowledge of these topics and of how to teach these topics, and that students hold a variety of probability misconceptions. However, there is still a lack of research on how formal instruction impacts students' understanding of these topics and how teachers could help students eliminate their misconceptions. This study investigated the above under-studied area.

More specifically, the participants, Thai grade 9 mathematics teachers and their students, provided an opportunity to investigate students' understanding both before and after their first formal instruction in probability topics. The inquiry-based curriculum was in line with the researchers' suggestions on combining the three approaches, giving an opportunity to evaluate the impact of the method. The content also related to multiple well-known probability misconceptions, which provided a rich context of discussion. Here are the research questions.

RQ1. How did Thai secondary school mathematics teachers understand and teach probability topics? What were the differences between how the curriculum was intended and how the teachers implemented it?

RQ2. What were Thai secondary school students' probability misconceptions before and after formal instruction? How did their responses on the tests change after instruction?

RQ3. What happened in the classrooms that influenced students' misconceptions? What impact did this have on their probability misconceptions?

Chapter 3 - Methodology

This chapter explains how the research study was conducted, including details about location, participants, data collection, and data analysis.

Location

This research was conducted at Ramathip School[7] in Bangkok, Thailand during the second semester of 2011. Ramathip School[8] is a large size secondary school (grades 7 – 12), serving about 5,000 students. It is located in the urban area of Bangkok. The school admits students using two main systems: 1) Lottery system, about 70%, for students who live within the school district, and 2) Examination system, about 30%, for any students who live outside the school district and those who did not get accepted by the lottery system (personal communication with a teacher participant). Ramathip School ranked seventh in the top ten best schools in Thailand list as ranked by the website toptenthailand.com in 2014. The ranking factors used were college admission test scores, Ordinary National Education Test scores (ONET), students' fellowships from various organizations, and teachers' and students' sciences, mathematics, and technology related awards.

Ramathip School employs an academic semester system with two 16-week long semesters per year. Usually, the first semester starts around mid May and ends in late September. The second semester starts at the end of October and ends around mid March. Unfortunately, due to the major flood that occurred throughout the center part of Thailand near the end of 2011, the school could not start until the second week of December 2011. The school made up for the time lost by adding one extra period of instruction every school

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[7] Pseudonym.

[8] Compared to other schools in Bangkok.

day in which students received a lesson on one of the four main subjects: mathematics, science, or language arts (Thai and English). However, students still received significantly fewer periods of instruction compared to the normal schedule. For example, students received only 4 - 6 periods of probability instruction instead of 14 periods as described in the curriculum.

Ramathip School was selected for this study because it adopted the IPST's mathematics curriculum (2001). There were three teachers teaching 16 classes of grade 9 students, providing a large sample size. The teachers also claimed that they followed that curriculum in their lessons.

Participants and Recruitment

Two grade 9-mathematics teachers and 204 students participated in this study. One of the teacher participants was recruited during an unrelated meeting about 5 months prior to the data collection process. The participants in the meeting were IPST's staff and mathematics teachers from several secondary schools in the Bangkok area. This first teacher volunteered to be in the study during the meeting and she recruited another teacher from the same school. Due to the limited resources and time constraint, only two out of three teachers and four out of 16 classes of students were in the study. The teachers were given the time frame of the research and the types of data that would be collected. They were informed that the research was about their students' understanding of probability topics and how they teach the topics, but no other specific details. They were not required to prepare or teach their lessons differently.

The first teacher, Aj[9]. Kim[10], was a woman in her 50s. She had a bachelors degree in Teaching Mathematics and a masters degree in Developmental Psychology. She had been teaching mathematics for 35 years. Her last 20 years had been at Ramathip School, where she had experienced teaching every grade level from 7 - 12 and had been teaching grade 9 every school year. She had taken a course in probability and statistics during her college years and had participated in some professional development programs on probability topics. During the school year when the research was conducted, Aj. Kim was teaching three classes of grade 9 students.

The second teacher, Aj. Nan[11], was a woman in her 50s. She had a bachelors degree in Teaching Mathematics from the same college as Aj. Kim. She has been teaching mathematics for more than 30 years. She had taught in another school for two years and then moved to Ramathip School. She had taught grades 7 - 10 and had been teaching grade 9 for more than 10 years. She too had taken a course in probability and statistics during her college years and had participated in some professional development programs on probability topics. During the school year when the research was conducted, Aj. Nan was teaching six classes of grade 9 students.

The two teachers each chose two classes, at different achievement levels, of their students to participate in the study, with the condition that none of the class periods overlapped. There were 53 – 58 students in each class. The student participants were recruited at the beginning of their first probability lesson and 204 students agreed to

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[9] Short for ar-jarn, a Thai word for teacher.

[10] Pseudonym.

[11] Pseudonym.

participate, with parental consent. Table 3-1 gives details on the students' basic demographics.

Table 3-1

Students' Demographics Details

Class	Teacher	# of students	Male/Female	Level	# of prob. lessons
A	Aj. Kim	52	25/27	higher track	4
B	Aj. Kim	51	22/29	lower track	4
C	Aj. Nan	55	33/22	lower track	6
D	Aj. Nan	46	19/27	lower track	6

As a common practice at Ramathip School, and most secondary schools in Thailand, students were assigned into classes according to their prior achievement and school placement test scores at the beginning of 7th grade. They would remain in these classes throughout their lower secondary school years (grades 7 – 9). Students in the same class took most of their courses together including mathematics, science, social science, language arts (Thai and English), religious study, and physical education. Therefore, based on their achievement, students in the same class were quite similar to each other but different from students in other classes.

The school also ranked the classes of students into four tiers. The first, second, and third tiers were each composed of two classes of students. The fourth tier was composed of 10 classes of students. The students in the first tier classes had the highest achievement, while the students in the fourth tier classes had the lowest achievement. Among the four participating classes of students, class A was in the third tier, while classes B, C, and D were in the fourth tier.

In addition to the school's ranking, Aj. Kim described class A students as smarter than class B students. They knew how to "survive" and moved along with school. They were faster learners and hence did not pay much attention to the lessons. They also still had quite childish behaviors. As for class B students, Aj. Kim said they paid more attention to teachers and lessons, which gave her a better opportunity to elaborate and get into more detail in her lessons. They were also more responsible and put more thought and effort into their school work.

Aj. Nan described class C students as a little above average and class D students as a little below average. Class C had more higher achieving students than class D. Class C students were more focused, paid more attention to the lessons and were more responsible than class D students.

Data Collection

Data collected for this study included video-taping and observing of classroom lessons, student testing on probability misconceptions (pre and post), and interviews with teachers.

Classroom lessons observation and video-taping. According to the curriculum, students were supposed to receive 14 periods of probability lessons as part of their mathematics course. However, due to the natural disaster, students only received 4 - 6 periods of instruction. Each period was scheduled to be 50 minutes long and took place in the students' regular classrooms. All sessions of probability lessons were videotaped using one video camera. The camera was set up near the back of the classroom and focused on the teacher and participating students (the ones who were asking questions, answering questions, demonstrating how to solve a problem, etc.). There was a total of 20 sessions of probability lessons. The total class time recorded for Classes A, B, C, and D were 140.04, 127.17, 195.59, and 208.31 minutes, respectively, 671.51 minutes combined. Fieldnotes were written after the lessons.

Probability misconception tests. Students were asked to take a pretest on probability misconceptions before their probability lessons and a posttest after the lessons. Each test took approximately 20 minutes. Both tests took place during regularly scheduled class periods, under usual classroom conditions, except for class D's posttest. Aj. Nan decided to have class D students take their posttest in a cafeteria where students had more room to spread out than in a regular classroom. Every student who was present at the time was asked to take the tests, but only the data from students who completed and returned the consent forms were used. Class A students took their posttest on the same week of their last probability lesson. The other three classes took their posttest about ten days after their last probability lesson, because they had a school-wide midterm examination for one week and then regularly scheduled instruction resumed a week later. Ideally, the posttest was supposed to be administered at the end of the last probability lesson. However, the teachers needed more time for their lessons in order to prepare students for their midterm exam. Classes B, C, and D's posttests were postponed until after the midterm exam. The students did not receive grades on either the pretest or the posttest. There were no incentives given to the teachers or the students.

The probability misconceptions pretest and posttest were parallel and each consisted of 10 items. Each item was designed to examine the participants' probability misconceptions. They required minimal calculation. Most items could be answered correctly without carrying out a procedure. The probability misconceptions investigated in the tests were representativeness (items 1, 2, and 4), positive and negative recency effect (item 3), conjunction fallacy (items 5 and 6), compound and simple events (items 7 and 8), and effect of the time axis (items 9 and 10). Some other misconceptions, such as effect of sample size and availability, were not included because they required knowledge beyond the scope of the intended curriculum. Most items on the pretest (i.e., items 1 - 4 and 7 - 10) were adjusted from the items used in previous research (Fischbein & Schnarch, 1997; Konold 1989; Shaughnessy 1992). Items 5 and 6 were adapted from Fischbein & Schnarch (1997). They were designed to better suit the students' ages and the scope of the curriculum. Since the two tests were administered within one month of each other, a new set of test items were used to avoid the possibility that the students would absorb knowledge just from taking the test. The posttest items were written to be parallel to those on the pretest, using similar situations with the same level of difficulty. Students were asked to choose an answer and explain their reasoning for each test item. Tables 3-2 and 3-3 show the pretest and the posttest items and the solutions. Appendix A shows the comparability between the pretest and the posttest in terms of contexts and difficulty levels.

Table 3-2

Pretest Items and Solutions

Item	Pretest	Answer	Explanation
1	Let H stands for the event of getting a head in tossing a coin and T stands for the event of getting a tail. Toss one coin five times. Which of the following is most likely to happen? Explain. a. HTHTH b. HHHTT c. THHTH d. THTTT e. All are equally likely	e.	Each choice is one possible outcome out of 32 possible outcomes.
2	From 1. Which of the following is least likely to happen? Explain. a. HTHTH b. HHHTT c. THHTH d. THTTT e. All are equally likely	e.	Each choice is one possible outcome out of 32 possible outcomes.
3	In tossing a coin five times, the coin showed heads the first four times. Which is more likely to happen the fifth time? Heads or tails? Explain.	Both are equally likely.	Each toss is independent. The outcomes of the previous tosses do not affect the outcome of the next toss.
4	In a 2-digit lottery game, Tor buys 21, 22, 23, 24, and 25. Tan buys 17, 38, 62, 59, and 84. Who is more likely to win this game? Explain. a. Tor b. Tan	Both are equally likely.	Each person buys an equal number of five lotteries. Each number has the same chance of winning.
5	43 year-old Sonny is a very heavy smoker. Lately, he has been suffering from constant chest pain and cough. So, he decides to see a doctor. Which of the following events has the higher probability? Explain. a. Sonny has lung cancer. b. Sonny has cancer.	b.	Lung cancer is a type of cancer. Sonny may have other types of cancer.

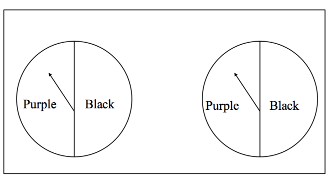
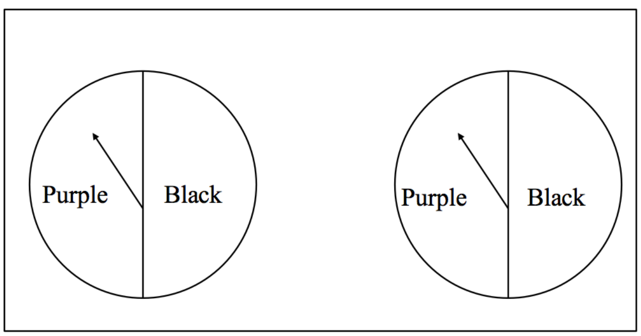
Item	Pretest	Answer	Explanation
6	Jane likes sweet fruit. She walks to a fruit store with 50 *bahts* (Thai unit of money). The store sells three kinds of fruit, guava, longan, and mandarin orange, for 40, 45, and 60 *bahts* per kilogram, respectively. If the minimum amount of fruit to be bought is one kilogram, which of the following events has the highest probability? Explain. a. Jane buys guava. b. Jane buys longan. c. Jane buys mandarin orange. d. Jane buys fruit.	d.	Guava, longan, and mandarin orange are types of fruit.
7	In tossing 2 dice once, which outcome is more likely to occur? Explain. a. One die shows 5 and the other die shows 6. b. Both of the dice show 6. c. Both outcomes are equally likely.	a.	Choice a. has two outcomes, (5,6) and (6,5). Choice b. only has one outcome, (6,6).
8	A game consists of spinning two fair spinners (see diagram). A player wins only when both arrows land on purple, otherwise he or she loses. Does a player have a 50-50 chance of winning this game?  a. Yes. Why? b. No. Why?	No	The event that the player wins has only one outcome, (purple, purple), while the event that the player loses had three outcomes, (purple, black), (black, purple), (black, black). The player is more likely to lose than win.
9	An urn only contains two yellow marbles and two green marbles. Randomly pick one marble out of the urn. What is the probability that the marble is green? Show your work.	1/2	There are 2 green marbles from the total of 4 marbles.
10	Use the same urn from 9 (with all four marbles in it). You picked out one marble and put it aside without checking the color. Then, you picked another marble and found that this second marble is green. What is the probability that the first marble picked is also green? Show your work.	1/3	There is 1 green marble left from the total of 3 marbles.

Table 3-3

Posttest Items and Solutions

Item	Posttest	Answer	Explanation
1	Let H stands for the event of getting a head in tossing a coin and T stands for the event of getting a tail. Toss one coin five times. Which of the following is most likely to happen? Explain. a. THTHT b. TTTHH c. HTTHT d. HTHHH e. All are equally likely	e.	Each choice is one possible outcome out of 32 possible outcomes.
2	From 1. Which of the following is least likely to happen? Explain. a. THTHT b. TTTHH c. HTTHT d. HTHHH e. All are equally likely	e.	Each choice is one possible outcome out of 32 possible outcomes.
3	A token was painted red on one side and blue on the other side. If I toss the token four times and red came up all four times, which is more likely to happen the fifth time? Red or blue? Explain.	Both are equally likely.	Each toss is independent. The outcomes of the previous tosses do not affect the outcome of the next toss.
4	An urn contains 100 small cards with number 00 – 99 written on them. If you pick 5 cards out of the urn, which group of numbers is more likely to come up? Explain. a. 11, 22, 33, 44, and 55 b. 03, 49, 67, 81, and 92	Both are equally likely.	Each choice has an equal number of five cards. Each number has the same change of being picked.
5	Ben had just graduated from college, majoring in accounting. He applies for a job at two different accounting companies. Company A has 3 opening jobs. Company B has 5 opening jobs. Which of the following events has the higher probability? Explain. a. Ben gets a job at company B. b. Ben gets a job.	b.	Getting a job means getting a job, anywhere.

Item	Posttest	Answer	Explanation
6	May likes to drink iced milk. She goes to a beverage store that sells four kinds of beverage; iced milk, hot milk, iced chocolate, and hot chocolate. Which of the following events has the highest probability? Explain. a. May buys iced milk. b. May buys hot milk. c. May buys iced chocolate. d. May buys hot chocolate. e. May buys a beverage.	e.	Each choice is a kind of beverage.
7	In tossing 2 dice once and considering the sum of the numbers, which outcome is more likely to occur? Explain. a. The sum equal 11. b. The sum equal 12. c. Both outcomes are equally likely.	a.	Choice a. has two outcomes, (5,6) and (6,5). Choice b. only has one outcome, (6,6).
8	A game consists of spinning two fair spinners (see diagram). A player wins only when the arrow on the left spinner land on purple and the arrow on the right spinner land on black, otherwise he or she loses. Does a player have a 50-50 chance of winning this game?  a. Yes. Why? b. No. Why?	No	The event that the player wins has only one outcome, (purple, black), while the event that the player loses had three outcomes, (purple, purple), (black, purple), (black, black). The player is more likely to lose than win.
9	A box only contains three red chips and three blue chips. Randomly pick one chip out of the box. What is the probability that the chip is blue? Show your work.	1/2	There are 3 blue chips from the total of 6 chips.
10	Use the same box from 9 (with all six chips in it). You picked out one chip and put it aside without checking the color. Then, you picked another chip and found that this second chip is blue. What is the probability that the first chip picked is also blue? Show your work.	2/5	There are 2 blue chips left from the total of 5 chips.

Teacher interviews. Two semi-structured interviews were conducted with each of the teachers, before and after they taught the probability lessons. Teachers were asked to take the probability misconceptions test (post only) after they taught the probability lessons and to return it during the post-lesson interview. They saw neither the pretest nor the posttest before instruction. The interviews took place in the Department of Mathematics conference room. The pre-lesson interviews took 18 minutes with Aj. Kim and 12 minutes with Aj. Nan. The post-lesson interviews took 40 minutes with Aj. Kim and 32 minutes with Aj. Nan. All interviews were audiotaped. The objective of the pre-lesson interview was to gather the teachers' background information, conceptual understanding of the topics, understanding of their students' knowledge, teaching plan, and value of teaching, while the post-lesson interview was designed to have them reflect on their practice and to check their understanding and misconceptions on the probability topics. The interview protocol was designed to gather the above information. Teachers were also asked to predict how their students would perform on the posttest. See Appendix B for the teacher interview guide.

Language and Translation Issue. Thai was the only language used in collecting data for this research. All the lessons were taught in Thai. All the interviews were in Thai. All the participants, teachers and students, and the researcher were Thai native speakers. The tests were written first in English before translated into Thai. The tests were approved by IPST and the school before administration. Only the Thai version of the materials were used in collecting data.

Data Analysis

First, the videotapes and fieldnotes were used to explain (a) how the teacher implemented the IPST curriculum, (b) the content actually taught as compared to that in the curriculum, (c) the specific examples that the teachers used that may have affected students' conceptions and misconceptions, and (d) the similarities and differences between the two teachers' lessons and how they could lead to differences in students' understanding.

Second, the teacher interviews give insight into the teachers' understanding about the topics and about their students.

Third, the probability misconception test results were used to describe the nature of the teachers' and the students' probability misconceptions. The changes from the pretest to the posttest were used to explain the effect of instruction on students' understanding of the concepts. When coding the test responses, both the answers and the reasoning were taken into account. The code for an item's answer consisted of one correct answer and multiple incorrect answers. The code for an item's reasoning consisted of one to three types of correct reasoning and multiple types of incorrect reasoning. For example, Table 3-4 shows the code for the reasoning part of pretest item 1. More details on the coding of the test responses are given in the results (Chapter 5). Table 3-5 summarizes how the data were used to answer the research questions.

Table 3-4

Code for Pretest Item 1 Reasoning

Code	Reasoning
Proc. (correct)	Correct and complete use of procedure or diagram, probability of each choice is 1/32.
Incor.	Incorrect or incomplete use of procedure or diagram.
Equal Prob.	Equal probability of each toss/choice.
Represent.	Representativeness misconception. Pick one sequence and give explanation (easy, hard, impossible, rare) use with answer a.- d.
Exper.	Experiment/trial and error.
Uncert.	Uncertainty or uncontrollability of the situation. Everything is possible. Do not know what will happen. Physical factors.

Table 3-5

Research Questions, Data, and Analyses

Research Question	Data Source	Data Analysis
RQ1. How did Thai secondary school mathematics teachers understand and teach probability topics? What were the differences between how the curriculum was intended and how the teachers implemented it?	1. Intended curriculum (student textbook and teacher manual). 2. Lesson observation (fieldnotes and videotaped lessons). 3. Teachers' pre and post interviews.	1. Lesson maps were made from videotaped of the lessons and fieldnotes. 2. The interviews were transcribed. 3. Comparisons were made among the curriculum materials, the lessons, and the interviews.
RQ2. What were Thai secondary school students' probability misconceptions before and after formal instruction? How did their responses on the tests change after instruction?	1. Teachers' and students' responses on the pretest and posttest.	1. The test responses were graded using a 20 points-scale. 2. The test responses were coded based on misconceptions.
RQ3. What happened in the classrooms that influenced students' misconceptions? What impact did this have on their probability misconceptions?	1. Intended curriculum (student textbook and teacher manual). 2. Lesson observation (fieldnotes and videotaped lessons). 3. Teachers' pre and post interviews. 4. Teachers' and students' responses on the pretest and posttest.	1. Comparisons were made among the lessons, the students' misconceptions, and the teachers' misconceptions.

Chapter 4 - Results: The Probability Curriculum and Lessons

This first results chapter attempts to answer the first set of research questions: How did Thai secondary school mathematics teachers understand and teach probability topics and what were the differences between how the curriculum was intended and how the teachers implemented it? The chapter describes the probability curriculum and lessons in detail. The chapter is divided into 3 sections. Section 1, about the intended curriculum, explains the curriculum and the curriculum materials that were adopted by the school. It also explains how the probability misconceptions investigated in the tests were related to the content of the curriculum. Section 2, about the implemented curriculum, describes the actual lessons that took place and how they were different from the intended curriculum. Section 3, about the teachers, describes the teachers' own understanding about probability topics and their reflections on the lessons.

The Intended Curriculum

Ramathip School adopted the Institute for the Promotion of Teaching Science and Technology's (IPST) mathematics curriculum (2001). The school was required to teach according to IPST's 2001 national standards and decided to use the IPST's curriculum. The curriculum required all 7th – 9th graders to take one 2-units mathematics course every semester, for a total of six courses. The sixth mathematics course, and the last one in the series, was for 9th graders, second semester. The topics included inequality (12 periods), probability (14 periods), statistics (20 periods), and process and skill in mathematics (14 periods). The curriculum required four 50-minute periods of lessons each week for 15 weeks. The students usually took a midterm exam in week 8 and a final exam in week 16.

The curriculum and the curriculum materials were developed by IPST's secondary mathematics department staff and were designed to be aligned with the standards using an inquiry-based approach. During the developmental process, the developers received comments and suggestions from in-service school teachers, university professors, and expert educators. The curriculum materials included:

- *The Student Textbook* (IPST, 2008) contained the presentation and explanation of concepts, examples, activities, and practice exercises. The concepts were presented based on students' prior knowledge or experiences before moving to the new knowledge. For example, at the beginning of the probability chapter, the textbook listed several situations that involve uncertainty in everyday life. (See next section for main concepts represented in the probability chapter.) Experiments and activities were often included to foster students' understanding of the concepts. There were several types of examples and practice exercises included in the textbook, such as calculation, problem solving, analyzing, reasoning, and making predictions and conclusions.
- *The Teacher Manual* (IPST, 2008) gave supports that the teachers may need in using the curriculum. It contained three main sections for each unit: (1) *Objectives* reminded teachers of what students should be able to perform after learning the unit, (2) *Recommendation* told teachers how the unit was intended to be represented, and (3) *Explanation for Teachers* provided teachers with extra knowledge they may need to understand. (See next section for the recommendations given in the probability chapter.) It also provided solutions or solution ideas to all the exercises and activities at the end of each chapter.

According to the teacher manual, to use the curriculum in the most effective way, the teachers are recommended to (translated from Thai):

> 1. Study in detail the content and teaching approaches.
> 2. Do the practice exercises and find the best possible way to solve the problem especially those with multiple solutions.
> 3. Plan lessons for the semester in advance so that the lessons would cover all content within the time period.
> 4. When teaching specific content, do not "tell" the students the concept; rather teachers should use activities and class discussion to have the students obtain the concepts themselves as much as they can.
> 5. Use real or local situations, problems, and questions that conform to the content to extend from the questions provided in the textbook. This method would support the students to better understand the content and be able to apply and integrate among concepts. (IPST, 2008, pp. b-c)

Moreover, about the solutions for the exercises and activities, the teacher manual emphasized that (translated from Thai):

> Solutions or solution ideas are provided for every question in the activities and exercises. For some questions with multiple solutions, at least one solution is provided as an example. Because these activities or exercises provide the students with the opportunities to search, observe, collect data, analyze, make conjectures, and compose simple proof, in response to student' answers, teachers must take into account students' age and prior knowledge. Student' answers may differ from those in the solutions provided. A teacher should consider students' responses carefully and accept all the answers that are correct and possible, even though they differ from those in the Teacher Manual. (p. b)

The probability chapter was recommended to take 14 periods and contained four units. Table 4-1 shows the objectives of each unit and the recommended time as listed in the teacher manual. The curriculum materials emphasized having students do activities, practice skills of observing patterns, and systematically write results of random events. In addition, students should have an opportunity to use knowledge about probability as a tool to make decisions and apply to real situations (e.g., gambling and investing). During the teaching period, teachers were recommended to insert the advantages of using knowledge about probability in analyzing and reasonably predicting situations. In addition, expected value, which was included in the last unit, Probability and Decision Making, was only supplemental content. Teachers were recommended not to assess students about this content directly.

Table 4-1

List of Probability Units, Objectives, and Number of Periods

Unit	Objective	Number of Periods
1. Probability: Meaning and Uses	Students can use common sense to tell whether an event is likely or unlikely to happen.	2
2. Random Experiments and Events	Students can list all the possible outcomes of a random experiment. Students can list all outcomes of an event.	4
3. Probability of an Event	Students can find the probability of an event. Students can use knowledge about probability in making logical predictions. Students can use knowledge about probability in making decisions.	5
4. Probability and Decision Making	Students can use knowledge about probability in making decisions.	3

To understand the content of the probability units and how the content was intended to be represented, the following details explain the probability content as represented in the corresponding student textbook (ST) and the recommendations given in the teacher manual (TM) (IPST, 2008).

Unit 1. Probability: Meaning and uses. The ST began the unit with a list of situations in everyday life that involve uncertainty. The textbook introduced the term probability as "a number that indicates the likelihood of an event" (p. 33). It explained that the study of probability began from about 1654, when Pascal helped Chevalier de Mere solve his problem in gambling. Students then did activities that involved discussing several situations in which probability could be used (e.g., waiting time for buses and predicting the weather). The unit led to the conclusion that knowing the probability of an event can help in making decisions regarding the event. Figure 4-1 shows one of the activities from the textbook.

> *Situation 3*. Awe is about to take an entrance exam for secondary school. She applied at School A and School B. When it is close to the exam day, Awe checks the number of applicants at each school. School A has 405 applicants and 120 spaces. School B has 492 applicants and 180 spaces. On the exam day, Awe goes and takes her exam at School B. What does Awe think?
>
> *Answer:* Awe thinks that she has more chance of getting in School B than School A.

Figure 4-1. An activity from the student textbook unit 1.

The TM gave these recommendations for the teachers: (1) The teachers should lead a discussion on situations that students have experienced that involve predictions. Teachers could assess students' number sense and common sense from their ability to tell how likely an event may be to occur. (2) The story about the origin of probability, described above, was presented for the students to realize that figuring out how to solve a problem is how mathematicians discover new knowledge. Teachers may tell the story themselves or have the students read it.

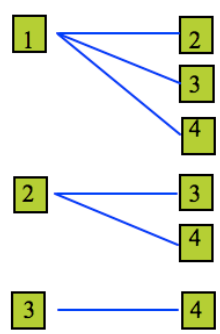
The objective of this unit was for the students to be able to use common sense to tell whether an event is likely or unlikely to happen. This unit gave teachers an opportunity to discuss subjective probability and learn about students' probability misconceptions. The TM emphasized having students discuss several situations and using common sense in making decisions. Teachers could address and correct the conjunction fallacy misconception during this unit. For example, when discussing the above situation, students with the conjunction fallacy misconception might think that Awe is more likely to get in School B than in any schools.

Unit 2. Random experiments and events. The ST gave the definitions of random experiments and events, and demonstrated several activities such as tossing coins and die, picking marbles from urns, spinning spinners, etc. The explanation employed the uses of tree diagrams, permutations, and combinations without mentioning these names. Figure 4-2 shows an example from the textbook. A typical example in the textbook usually first stated the problem and listed all the questions or instructions and then showed all the work. For example, this problem contained three parts. After stating the problem, instructions, and the figure depicting the situation, the textbook showed the work for each part. The example used tree diagrams to represent the actions of sampling in each scenario, followed by the lists of the results and the answers for each part.

Example 3. Randomly pick 2 cards from a box that contains cards number 1, 2, 3, and 4. Find the results of an event such that the summation of the numbers on both cards is equal to 5. When:

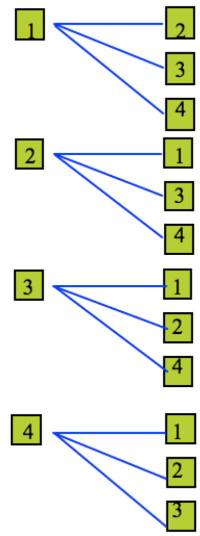
1) Pick both cards at once.
2) Pick one card at a time without replacement.
3) Pick one card at a time with replacement.

1) Pick both cards at once.



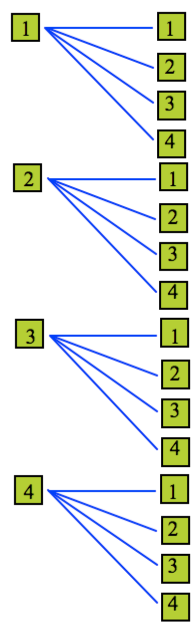
There are 6 possible results from this random event:
(1, 2), (1, 3), (1, 4), (2, 3), (2, 4), and (3, 4).
There are 2 events that the summation of the numbers on both cards equal 5:
(1, 4), and (2, 3).

2) Pick one card at a time without replacement.



There are 12 possible results from this random event:
(1, 2), (1, 3), (1, 4), (2, 1), (2, 3), (2, 4), (3, 1), (3, 2), (3, 4), (4, 1), (4, 2), and (4, 3).
There are 4 events that the summation of the numbers on both cards equal 5:
(1, 4), (2, 3), (3, 2), and (4, 1).

3) Pick one card at a time with replacement.



There are 16 possible results from this random event:
(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (3, 4), (4, 1), (4, 2), (4, 3), and (4, 4).
There are 4 events that the summation of the numbers on both cards equal 5:
(1, 4), (2, 3), (3, 2), and (4, 1). (pp. 40-43)

Figure 4-2. An example from the student textbook unit 2.

The TM gave these recommendations for the teachers: (1) The teachers were recommended to do the experiments with the students. However, they should carefully select the instructional materials. For example, when picking objects out of an urn, the urn should be opaque and the action should be random. The dice should be tossed in a way that they roll independently. (2) When giving examples of random experiments, the teachers should make sure that the examples were truly random experiments (i.e., not an experiment that the result was always predictable, such as a football match between a school team and a national team). (3) Teachers should explain why the order of the results mattered in some situations (sampling one object at a time) and did not matter in other situations (sampling more than one object at a time). (4) Teachers should explain why, when tossing three coins at once, the outcomes HTT, THT, and TTH were considered different (they were from different coins).

The teachers could address and correct the compound and simple events misconception during this unit. For example, as in number 4 above, students might think the outcomes HTT, THT, and TTH were similar, since all of them are composed of one head and two tails. Moreover, the teachers should give examples of various situations, such as tossing coins, tossing dice, spinning a spinner, randomly picking objects from a container, etc. This is because each situation has a different way of producing outcomes and students might employ a subjective approach to probability specific to the situation. For example, when dealing with a spinner, students might think that a starting position of the spinner mattered or that the person who made the spin could manipulate the outcomes.

Unit 3. Probability of an event. The ST defined probability as the proportion of the number of the results of an event to the number of all possible results[12], when each result has an equal chance of occurring. Three conclusions based on prior examples were stated:

1. Probability of an event is a number from 0 to 1.
2. Probability of a certain event is 1.
3. Probability of an event that cannot happen or will not happen is 0.

The textbook also explained the differences between a theoretical probability and an experimental probability and how they were related. Figure 4-3 shows one of the practice exercises from the textbook.

> *Situation 3*. Wonpen is a student. She lives in Bangkok. Everyday she gets up early in the morning, getting ready while listening to the television news. Usually, she leaves the house for school around 6:30 am. One day, the weather forecast announces that there will be rain scattered throughout 80% of the area. Wonpen looks outside and sees that the sky is dark and cloudy. How do you think Wonpen could use this information to help prepare for her day? Explain.
>
> *Answer:* From the forecast and the cloudy sky, it could be concluded that there was a high chance of rain that day. Therefore, Wonpen should bring an umbrella or a raincoat with her. She should also leave home earlier than usual because it may take longer to commute.

Figure 4-3. An exercise from the student textbook unit 3.

The TM gave these recommendations for the teachers: (1) Teachers should have the students consider results from a random experiment and how likely they were to occur, before making a conclusion of the formula for finding probability. (2) Teachers should have the students observe the possible range of probability value and explain what it means when the probability was 0 or 1. (3) When calculating a theoretical probability, there usually were assumptions that each result was equally likely and that the experiment was unbiased. When

[12] The term sample space was not used here.

finding an experimental probability, the experiment should be performed a large number of times, so that the probability would be closer to the theoretical probability. (4) For activities that have students make predictions, some students might predict the situations differently from others. Teachers should let the students present their idea and discuss the logic of the predictions using probability knowledge. (5) For the situations that connect probability and social concerns, teachers should insert the idea of citizens' responsibility to support projects that would help improve the problem, such as campaigns for protecting the forest, etc.

The teachers could address and correct the representativeness misconception, positive and negative recency effect misconceptions, and the effect of the time axis misconception during this unit. The first two misconceptions could be addressed when dealing with the experimental approach to probability. For example, students might think that, when repeatedly doing a random experiment, the outcomes must always represent the ideal population or that an outcome that occurred recently was less likely or more likely to occur again. The effect of the time axis misconception could be addressed when calculating probability of events that involve more than one step. For example, when sampling objects out of a container without replacement, the outcome of the first pick does affect the outcome of the second pick.

Unit 4. Probability and decision making. The ST employed expected value as a tool in making decisions. The expected value was defined as the summation of the products between the possible payoffs and their probabilities. Examples were drawn from several types of gambling and led to a conclusion that, in the long run, gamblers were much more likely to lose than win. Figure 4-4 shows one of the practice exercises from the textbook.

2. A county wants to build a road between cities and has qualified construction companies auction for the job. Chan-Chai Construction Company has considered the situation and found that if they participate in the auction, they have the probability of 0.6 to get the job with a profit of 300,000 *bahts* and the probability of 0.4 not to get the job and losing 200,000 *bahts* in getting ready for the auction. Answer the following questions:

1) What is the expected value of Chan-Chai Construction Company in auctioning for the job?

2) If you were the owner of Chan-Chai Construction Company and you wanted to make income for the company and create jobs for your employees, do you think you should participate in the auction and why?

Answer: 1) Expected value is 100,000 *bahts*.

2) Chan-Chai Construction Company should participate in the auction because they can expect to get 100,000 *bahts* in profit and so that the employees can have a job. Otherwise, the company may have to lay off the employees, which could create more cost. The possibility of losing 200,000 *bahts* is a business risk they should take. (This is only one possible answer.)

Figure 4-4. An exercise from the student textbook unit 4.

The TM gave these recommendations for the teachers: (1) Before introducing the students to the expected value, teachers should have a conversation with them about their experiences of payoff value for various situations. (2) Teachers should point out that the probability and the possible payoff are the main components in finding an expected value. When these numbers change, the expected value would also change. (3) Some students might make different decisions than others, therefore teachers should let the students discuss their ideas. (4) Teachers should make sure students are aware that it is almost impossible to gain income from gambling. There was no probability misconception investigated in this study directly related to this unit.

This section explained the curriculum, the contents of the probability chapter, how the contents were expected to be represented, and the probability misconceptions related to each unit. The curriculum used an inquiry-based approach and emphasized having students do experiments, discuss their ideas, and make decisions and predictions. In summary, the curriculum materials focused on having students 1) be aware of their own subjective approach to probability, 2) write all possible outcomes of a random experiment or an event, 3) understand the difference between theoretical probability and experimental probability, and 4) use probability knowledge in making decisions and predictions. The next section describes what happened in the actual lessons.

The Implemented Curriculum

During the interviews, both teachers claimed that they followed the curriculum materials in their lessons. This section first describes the classroom setting and a typical lesson to give readers an idea of the physical context of a classroom and the general aspects of a lesson. Then the details of each lesson follows.

The classroom setting. Ramathip School had six 4-story interconnected classroom buildings. The mathematics teachers' common room was located on the first floor of the third building. Grade 9 students' classrooms were on the third and fourth floor of the fifth and sixth building. Teachers walked to their students' classrooms to give lessons. Each period was 50 minutes long and signaled by the central school bell. Even though the teachers did not have their own rooms, they were responsible for a class of students as a homeroom teacher (i.e., advisor). Aj. Kim was the homeroom teacher for Class A, but Aj. Nan was not the homeroom teacher for either of her classes in this study, Class C or Class D. The school usually had a short homeroom period before the first regular period started in which the homeroom teachers met with their students in their classroom. However, for the semester where this study was conducted, the homeroom period was canceled. An extra period was scheduled every school day in the morning for instruction instead. As a result, during Class

A's mathematics periods, Aj. Kim needed to spend time addressing issues other than probability lessons.

Regular classrooms had a blackboard in the middle of the front wall and two small bulletin boards on each side. One side wall had a row of windows and the opposite side wall had two doors, one near the front and another near the back. Students had their own seat (usually unassigned), which consisted of a wooden chair and a wooden table with a book compartment. Two tables were put side by side and arranged into four columns between the two side walls, leaving a small gap between columns. The teacher's desk was in the front of the room near the front door. At the back of the room near the back door were a trash can and cleaning supplies. Most classrooms also had a speaker for which the teachers brought their own microphone to plug in. Since there were more than 50 students in each class, the teachers needed to use the microphone in their lectures so that all students could hear them. Students took turns cleaning their room.

The teachers shared a common room based on their department (there were eight departments in total). All mathematics teachers (approximately 30 of them) used the common room on the first floor of the third building. Each teacher had his or her own desk. Most teachers usually did their work here during the non-class periods.

The typical lesson. A lesson map for each of the videotaped lessons was created in order to analyze the implemented curriculum. The maps listed the times, the activities, the contents, and the participant structures of each section of the lessons. One of the lesson maps is provided in Appendix C as an example. The observation of instruction and the lesson maps revealed that most periods were constituted in a similar style. In general, the teachers stayed in their common room until the bell rang, then they made their way to the students' classrooms. The students took most of their lessons in the same classroom, so they usually remained in the room waiting for the teacher. A student in each class maintained an attendance report where he or she kept track of the classmates who were absent and the teachers signed the form for each period. Once the teacher arrived at the classroom, the student head called out, “Students, greet.” Then all students said, “Good morning/afternoon, teacher,” while putting their palms together in salutation.

Most teachers required the students to have a specific notebook for the subject. Students used the notebook to copy the teachers’ notes from the board and do their practice exercises and homework. The teachers usually assigned homework from textbook, but the students were expected to copy the problem and show their work in the notebook. From time to time, the teachers might ask the students to submit their notebooks for grading. If the notebooks were submitted, the students were responsible to take them back before the next period.

Analysis of the video data shows that Aj. Kim and Aj. Nan often began their lessons by giving out the previous homework solutions, orally or in writing on the board. Usually, only the final answers were given, unless the students asked for elaboration. During this time, students checked their own work and made notes. From the observation, Aj, Kim and Aj. Nan did not seem to have a specific lesson plan for each period. Rather, they followed one long lecture plan for the chapter, paused at the end of a period, and continued in the next one. This observation was confirmed during the teacher interviews.

The lecture itself was carried out in four parts: the content, the example, the exercise, and the solution. First, the content part was where the teacher explained the concepts and gave definitions of the new terms and formulas. They did so orally while writing the main ideas and formulas on the board. From the video data, this content part took between 2 - 5 minutes. Second, the example part was when the teacher showed how to solve problems, both orally and in writing on the board. Students were expected to copy everything from the board to their notebooks, unless told otherwise. Third, the exercise part was when the teacher gave the students problems to work on. They usually wrote them on the board or had the students look them up from the textbook. Students copied the problems onto their notebook and solved them, mostly individually but sometimes with a partner. The teacher gave students a few minutes to work on their own. They sometimes circled the classroom to answer questions or maintain students' attention on the task, or stayed at the desk preparing for the next step. Fourth, the solution part was when the solution to the exercises was given. The teachers usually did this orally and in writing on the board. They sometimes had the students answer in unison. In rare cases, students might be asked to go up to the board to show their work. Then, either more examples or exercises were given or they moved on to the next content.

Near the end of each period, the teacher usually assigned the homework from the textbook. Some other logistics, non-lesson related problems or concerns were usually addressed here, especially in the homeroom teacher's class (i.e., Aj. Kim in Class A). A bell ring marked the end of the period where the teacher moved on to his or her next class or returned to the common room.

The actual lessons. Since the natural disaster had a major impact on the operation of the school, the available class periods for the probability topic were greatly reduced. As a result, class A and class B each only received four periods of probability lessons. Class C and class D each received six periods. Table 4-2 summarizes the content addressed in each

Table 4-2

List of Content Addressed in Each Period

Class	Period	Content
A (Aj. Kim)	1	Intro to probability: examples of events involve probability. Random experiment: difference between scientific and mathematical experiment, definition of random experiments, examples, tree diagram, definition of outcome of a random experiments. Sample space.
	2	Probability of events: definition of events, calculating probability, examples.
	3*	Exercises: calculating probability
	4	Probability of events: examples, sampling with and without replacement, sampling of two samples at the same time.
B (Aj. Kim)	1	Intro to probability: examples of events involve probability. Random experiment: difference between scientific and mathematical experiment, definition of random experiments, examples, tree diagram.
	2	Outcome of random experiments: sample space, sampling with and without replacement, sampling of two samples at the same time.
	3	Probability of events: calculating probability, examples.
	4	Exercises and solutions: calculating probability
C (Aj. Nan)	1**	Intro to probability: what probability is, examples of events involve probability. Random experiment: examples and non-examples, definition of random experiments, examples. Outcome of random experiment: sample space, examples, tree diagram.
	2	Event and probability: definition of events, examples, calculating probability, examples, comparing probability.
	3	Probability: examples, sampling with and without replacement, sampling of two samples at the same time.
	4	Card and spinner
	5	Probability examples: counting principles. Probability and decision making: expected value.
	6	Exercises and solutions: calculating probability
D (Aj. Nan)	1	Did not observe
	2	Probability: examples, calculating probability, examples.
	3	Probability: examples, sampling with and without replacement, sampling of two samples at the same time.
	4	Card and spinner. The meanings of “and” and “or”.
	5	Probability examples: sport matching. Probability and decision making: expected value.
	6	Probability examples: counting principles. Exercises: calculating probability

Notes. *Substitute teacher. ** The first 14 minutes were spent addressing the previous topic, inequality. The final 26 minutes were about probability.

period. The details of each period follow. Since most lessons were constructed in a similar style, this section focuses on the content aspect of the lesson. The readers can assume that the teaching aspect of the lessons was as explained above unless otherwise stated.

Class A, period 1. Aj. Kim started the first period with a short introduction to probability. She gave brief verbal examples of situations that involved probability, such as the chance of rain, winning a lottery, and tossing a coin. She pointed out that these situations were random experiments which could be considered "mathematical experiments." She assumed that students were familiar with a scientific experiment and explained that "when doing a scientific experiment, you have no knowledge of what could happen", whereas "when doing a mathematics experiment (e.g., tossing a coin), we already know what could happen, heads or tails, before the coin was tossed."

Next, she gave two more examples of random experiments along with possible outcomes: 1) Tossing a coin once, the possible outcomes are heads or tails. 2) A family has a child, the possible outcomes are boy or girl. Then she had the students work on two exercises: 1) tossing a coin twice, and 2) a football match among four teams, Red, Yellow, Blue, and Green. Then, she asked three students to come out and write their answers on the board.

Aj. Kim then explained about tree diagrams and how using them could help find solutions to the previous exercises. She gave students another exercise: a family with three children. She asked students who had two siblings (who came from a family with three children) to come up and write their genders on the board. After five students, she had the class answer the rest of the combinations in unison. Then, she showed how to use a tree diagram to write this problem's solution. The next exercise was to write all the possible outcomes of tossing a dice once, and twice. She had the students answer in unison, while she wrote on the board. Next, Aj. Kim gave a description of sample space (S) and the number of elements in the sample space (n(S)). Then, she assigned students to do Exercise 2.2 (four problems with multiple questions). Students were to start in class and finish them as homework.

Class A, period 2. Aj. Kim started this period by orally giving out the homework solutions, except for problem 2 which she wrote on the board instead. Then, she started a lecture on probability of events by first reciting the definitions of events and sample spaces and gave the formula for the probability of an event (p(E)) as n(E)/n(S). She gave an example by pointing out the number of students in the class, n(S) = 55, and that the event of female students, n(E) = 28, had the probability of p(E) = 28/55. A similar example with students who wear glasses was also demonstrated. Next, Aj. Kim showed an example of tossing a die twice, during which she orally stated that the results were the same as tossing two dice once. The example included finding the probability of three different events of the same experiment. Then, she assigned Exercise 2.3 items 1 and 2 for the students to do for the rest of the period.

Class A, period 3. Aj. Kim was not available for this period and a substitute teacher was in charge. The students were assigned to work on Exercise 2.3 items 3, 4, and 5 individually.

Class A, period 4. This period's lecture was on finding probability when sampling two objects from a pool in three methods: both at the same time, one at a time with replacement, and one at a time without replacement. Aj. Kim used a marker and an eraser to demonstrate the experiment. Then, she set up an example of a box containing five balls: two red, two yellow, and one white. The period was spent finding the sample space and the probability of getting two balls of the same color with each sampling method. When writing all the possible outcomes, Aj. Kim did it in a table form.

Class B, period 1. This period was similar to Class A, period 1, both the content and the teaching style.

Class B, period 2. Aj. Kim started this period by orally giving out the homework solutions. Then, she gave an example on taking a 10-item exam. Next, she demonstrated sampling two objects from a pool in three methods: both at the same time, one at a time with replacement, and one at a time without replacement. She used students' colored pens and a pencil case to demonstrate. Then, she set an example of a box that contained five balls: two red, two green, and one blue. She showed the students how to find the sample space with each sampling method, writing in table form. The last example of the period was about a deck of cards. Aj. Kim explained to the students what a deck of cards looked like and what types of cards were in it.

Class B, period 3[13]. Aj. Kim started a lecture on the probability of events by first reciting the definitions of events and sample spaces and giving the formula for the probability of an event (p(E)) as n(E)/n(S). She also gave these three examples: 1) Guessing which day of the week a friend was born, find the probability of guessing correctly. 2) Tossing a die, find the probability that the number shown is prime. 3) Tossing a die, find the probability that the number shown is divisible by 3. Next, referring to the previous example on sampling, Aj. Kim showed the students how to find the probability of getting two balls of the

[13] This period took place right after period 2.

same color with each sampling method. At the end of the period, she assigned Exercise 2.3, items 1 - 5 as a homework.

Class B, period 4. This period was devoted to practicing exercises. Students individually worked on exercises, which were from the prior year's midterm exam. Aj. Kim circled the room, maintaining students' attention on task.

Class C, period 1. The first part of this period was spent on the previous topic, inequality. When it was time to move to the probability topic, Aj. Nan asked the students to use the textbook to follow her lecture. She started by telling what probability means and what kinds of situations involve probability. Then she explained what a random experiment is along with its definition and examples of situations that are and are not random experiments. She then explained the outcomes of a random experiment, sample space, and a short lecture on set and set notation[14]. She gave an example of tossing a coin and showed how to write the outcomes of tossing a coin once, twice, and three times, using tree diagrams. She also observed the pattern of the number of results as two powers the number of coin or the number of toss. The students were to find the outcome of tossing a coin four times as a homework.

Class C, period 2. Aj. Nan started this period by writing the homework solution on the board. Then, she started a lecture on the meaning of events. She gave a definition of events as "the outcomes that we consider to occur from all the possible outcome of a random experiment." She explained this using an example of tossing a coin twice and showed what the events were when taking an interest in the coin to show (1) the same face, and (2) show

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[14] Set and set notation are outside the scope of this course. None of the notation was used in the textbook.

heads more than tails. Another example was to find the event that the coins show the same number of heads and tails when tossing three coins once. (This event has no element.)

Next, Aj. Nan gave the formula for calculating probability of events as the proportion of the number of outcomes of the event to the number of all possible outcomes. Then orally she added "when each outcome has the same chance of occurring." She then gave several exercises on tossing coins, pausing for a few minutes for students to work before giving out the solutions.

After examples on coins, Aj. Nan moved on to dice. She had brought a pair of giant dice to the class to show to the students. She then gave multiple examples on tossing one and two dice, finding all possible outcomes and outcomes of different events, and the probability of events. At one point a student asked if a pair (1, 2) and (2, 1), from tossing two dice once, were different. Aj. Nan explained that they are different, even though both dice are similar. She recommended the students to think of the two similar dice as different. She then assigned Exercise 2.2, item 2 and Exercise 2.3, items 1, 3, and 4 as homework.

Class C, period 3. Aj. Nan started this period by writing the homework solutions on the board. She observed the pattern between the sum of two dice and the numbers of its elements. Then, she started a new example on sampling two objects from a pool in three methods: both at the same time, one at a time with replacement, and one at a time without replacement. The example was about a box containing two red balls and two white balls. Aj. Nan wrote the solutions in terms of tree diagrams. The questions were to find the probability of (1) getting the same colored balls and (2) getting different colored balls. She also observed the pattern of the numbers of all possible outcomes of each sampling technique.

The students were assigned to practice with a situation including three red balls and two white balls, along with Exercise 2.3, item 2 as homework.

Class C, period 4. Aj. Nan started this period by writing the homework solutions on the board. She then explained to the students what a deck of cards looked like and what types of cards were in it. The students were instructed to use the textbook to follow her explanation. Several examples were shown: 1) the probability of getting an ace card, and 2) the probability of getting a king or a heart. Next, Aj. Nan showed a spinner problem from the textbook, Exercise 2.3, item 3. She showed how to write the sample space and answered the first question. Students were to do the rest of the problems as homework.

Class C, period 5. Aj. Nan started this period by orally giving out the homework solutions. She then showed an example of a combinatoric problem. Then, she gave a lecture on probability and decision making by reading the examples from the textbook. She summarized by writing the formula for expected value on the board and assigned three activities and exercises from the textbook as homework.

Class C, period 6. Aj. Nan started this period by shortly reviewing the expected value and wrote the homework solutions on the board. She then gave an example on taking a 20-item exam. Next, she gave out the solutions to a worksheet, which was the prior year's midterm exam, orally and in writing on the board.

Class D, period 1. Due to a schedule conflict, this period was not observed. However, the post lesson interview with Aj. Nan revealed that the content covered and the teaching style were the same as Class C, period 1.

Class D, period 2. Aj. Nan started this period by writing the homework solutions on the board. Then, she started a lecture on probability of events. She gave the formula for calculating probability of events as the proportion of the number of outcomes of the event to the number of all possible outcomes. Then she orally added “when each outcome is equally likely”. She then gave several exercises on tossing coins, pausing for a few minutes for students to work before giving out the solutions.

After examples on coins, Aj. Nan moved to dice. She had brought a pair of giant dice to the class to show to the students. She then gave multiple examples of tossing one and two dice, finding all possible outcomes and outcomes of different events, and the probability of events. She then assigned an in-class exercise of finding the probability of an event that, when tossing two dice, the numbers shown have a difference of three.

Class D, period 3. Aj. Nan started this period by demonstrating sampling two objects from a pool in three methods: both at the same time, one at a time with replacement, and one at a time without replacement. She used students’ colored pens and a pencil case to demonstrate. Then, she gave an example about a box containing five balls, three red balls and two blue balls, and showed the students how to find the sample space with each sampling method, using tree diagrams. She also observed the pattern of the number of all possible outcomes of each sampling technique. Students practiced finding the probability of various events under the same situations. The homework assigned were Exercise 2.3, items 1-4.

Class D, period 4. Aj. Nan started this period by writing the homework solutions on the board. She then showed a spinner problem from the textbook, Exercise 2.3, item 3. She showed how to write the sample space and answered the first question. The next example was a combinatoric problem; a person randomly picked two candies from a box that contained four different colored candies. Next, she explained to the students what a deck of cards looked like and what types of cards were in it. Several examples were shown: (1) the probability of getting an ace card, and (2) the probability of getting an ace or a heart. The homework assigned was an extra worksheet (the prior year's midterm exam).

Class D, period 5. Aj. Nan brought a regular size deck of cards to show to the students. She then wrote the homework solutions on the board. A new example on matching football teams was given. Next, she gave a lecture on probability and decision making by reading the content from the textbook. She summarized that the expected value can be calculated from the summation of the probability of the event times the returned value of the event. She had the students follow her explanation by looking at the textbook. She also assigned three activities and exercises from the textbook as homework.

Class D, period 6. This period was devoted to reviewing content and practicing exercises. The exercises were from the worksheet (the prior year's midterm exam). Aj. Nan shortly solved each one, then paused for a few minutes for students to catch up.

The day by day descriptions of the lessons above revealed that (1) the teachers' teaching styles were mostly lecture, while the students had no opportunity to discuss and do experiments, and (2) the teachers focused on finding outcomes of random experiments and events and calculating probability, rather than making predictions or decisions. The differences between the intended curriculum and the implemented curriculum will be discussed next.

Comparisons between intended curriculum and implemented curriculum. Even though both teachers claimed that they followed the curriculum materials, the implemented curriculum was quite different from the intended one. This section discusses two aspects of the lesson comparison: (A) content comparison with the student textbook, and (B) teaching comparison with the teacher manual.

Content comparison. Aj. Kim covered slightly different content in her two classes (A and B). They both included a very short (less than 80 seconds) introduction to probability (unit 1). In unit 2, they both included limited examples of different situations: Class A did not learn about a spinner and cards, and Class B did not learn about a spinner and tossing two dice. Both classes learned how to calculate probability, but did not get to discuss the meaning of the numbers that represent probability (e.g., what probability of 0, 0.25, or 1 mean) (unit 3). Both classes also did not learn about experimental probability (unit 3). For class A, Aj. Kim combined units 2 and 3 together (finding the outcome of an event and its probability at the same time). However, for class B, she taught units 2 and 3 separately. Aj. Kim did not cover unit 4 at all in both of her classes. It was possible that she decided not to cover this unit because of the time constraint, however, the post-lesson interview with her revealed that it was her intention not to teach the unit.

Aj. Nan had covered similar content in both of her classes (C and D). They both included a short (about five minutes) introduction to probability (unit 1). Both classes received at least one example of each of the different situations (coin, dice, card, spinner, etc.) (unit 2). Both classes learned how to calculate probability, but did not get to discuss the meaning of the numbers that represent probability (unit 3). Both classes also did not learn about experimental probability (unit 3). They both learned how to find the expected value, but did not get to practice making predictions or decisions (unit 4). On the other hand, Aj. Nan had inserted some extra curricular ideas such as set, notation of set, counting principle, observing pattern, etc.

Aj. Kim only used the textbook as a source for practice exercises and homework. Aj. Nan, in contrast, had students follow the textbook during her lecture in units 1 and 4. She also assigned exercises from the textbook as homework. However, both teachers only assigned textbook exercises on finding outcomes and calculating probability or expected value. None of the experiments, activities, or exercises on making predictions or decisions from the textbook were assigned.

In summary, both teachers focused on units 2 and 3 in all four classes. Class C and Class D were given examples on all various random experiments as presented in the textbook, while the examples given to Class A and Class B were limited. Unit 1 was briefly mentioned in all four classes but without students' discussion and activities or exercises. Class C and Class D learned the content of unit 4 while Class A and Class B did not. Table 4-3 shows probability units that were addressed in each period.

Table 4-3

Intended Curriculum as Implemented in Each Class

Period	Class			
	A	B	C	D
1	1,2	1,2	1,2	(1,2)**
2	3	2	2, 3	3
3	3	3	3	3
4	3	3	3	3
5	-	-	3,4	3, 4
6	-	-	2, 3, 4	2, 3

Notes. The numbers 1-4 represent the probability units; 1. Probability: Meaning and Uses, 2. Random Experiments and Events, 3. Probability of an Event, and 4. Probability and Decision Making. ** This period was not observed. The data came from an interview with the teacher.

Teaching comparison. The actual teaching in the lessons was compared to the recommendations given in the teacher manual, as described in the previous section, and was categorized into 3 categories: (a) addressed as recommended, (n) not addressed, and (p) partially addressed or inappropriately addressed. The recommendations from the unit that was not mentioned during some of the classes were marked as n/a (not applicable). Table 4-4 shows the results of this comparison. The details of how the teacher manual's recommendations were addressed in the lessons are discussed next.

Unit 1. Probability: Meaning and uses. As mentioned earlier, both teachers spent very little time on this unit. They did not address any of the points given in the TM.

Unit 2. Random experiments and events. The TM gave four recommendations for teaching this unit. The teachers addressed them all to some degrees. Below are the details:

(1) The teachers were encouraged to do experiments with their students. However, they should carefully select the instructional materials. Both teachers did a short demonstration when explaining the three sampling methods of picking objects from a container: two at a time, one at a time with replacement, and one at a time without replacement. (In Class C, Aj. Nan did not do this demonstration.) However, they did not use appropriate materials. In order to simulate a random experiment of this situation, the objects should be similar in size and shape. The container should be opaque. And, the action should be random and done without looking. However, both teachers used objects they could grab from around the classroom (e.g., pens, pencils, markers, and erasers). The container used was a small pencil bag or, in Class A, no container at all. Hence, the demonstration that the teachers showed the students was not a true random experiment.

Table 4-4
Intended Teaching Aspects as Implemented in the Lessons

Recommendation	Class			
	A	B	C	D
Unit 1: Probability: Meaning and Uses				
1. Lead a discussion on situations that involve prediction.	n	n	n	n
2. Explain that figuring out how to solve a problem is how mathematicians discover new knowledge.	n	n	n	n
Unit 2: Random Experiments and Events				
1. Do the experiments with the students, be careful with the instructional materials.	p	p	n	p
2. Make sure that examples were truly random experiments.	a	p	p	p
3. Explain why the orders of the results mattered in some situations and did not matter in other situations	a	a	a	a
4. Explain why the outcomes HTT, THT, and TTH were considered different.	n	n	a	a
Unit 3: Probability of an Event				
1. Have the students consider results from a random experiment and how likely they were to occur, before making a conclusion of a formula for finding probability.	n	n	n	n
2. Have the students observe the possible range of probability and explain what it mean when the probability was 0 or 1.	n	n	n	n
3. Explain assumptions of theoretical probability and experimental probability.	n	n	n	n
4. Let the students present their idea and discuss the logic of the prediction using probability knowledge.	n	n	n	n
5. Insert the idea of citizens' responsibility to support projects that would help improve social problems.	n	n	n	n
Unit 4: Probability and Decision Making				
1. Have a conversation with students on their experiences of payoff value for various situations.	n/a	n/a	n	n
2. Point out that the probability and the possible payoff are the main components in finding an expected value.	n/a	n/a	a	a
3. Let the students discuss their ideas.	n/a	n/a	n	n
4. Make students aware that it is almost impossible to gain income from gambling.	n/a	n/a	n	n

Notes. a = addressed as recommended; n = not addressed; p = partially or inappropriately addressed; n/a = not applicable.

(2) When giving examples of random experiments, the teachers should make sure that the examples were truly random experiments (i.e., not an experiment in which the result was always predictable). All the examples given to Class A were truly random experiments. However, the other three classes received at least one example that was not truly a random experiment, such as the score of an exam or the result of a sport game (more details in the next section). It is worth noting that Class A also received relatively fewer examples than the other three classes.

(3) Teachers should explain why the orders of the results mattered in some situations and did not matter in other situations. Both teachers correctly addressed this issue in all classes.

(4) Teachers should explain why, when tossing three coins at once, the outcomes HTT, THT, and TTH were considered different. Aj. Kim did not explain this point, while Aj. Nan did.

Unit 3. Probability of an event. The TM gave five recommendations for teaching this unit. Even though both teachers spent the majority of their class periods on this unit, they did not address any of these points.

Unit 4. Probability and decision making. The TM gave four recommendations for teaching this unit. Aj. Kim did not cover this unit, therefore none of the points were addressed. Aj. Nan addressed the second point, teachers should point out that the probability and the possible payoff are the main components in finding an expected value, in her classes. However, the other three points were not addressed.

The comparisons between the intended curriculum and the implemented curriculum showed that the actual lessons focused on different content from the textbook and the teachers used different teaching approaches from the teacher manual. The contents of the lessons were focused on writing the outcomes of random experiments and events and calculating probability, rather than making predictions and decisions as presented in the textbook. The teachers mainly used lecture in presenting the concepts, rather than experiments and discussion as recommended in the teacher manual.

Teachers' Probability Understanding and Reflection

The interviews with the teachers before and after the lessons provided insight into the teachers' understanding of the concepts and of their students. During the interviews, teachers were also asked to reflect on their lessons. This section provides details of the interview results, followed by the teachers' misunderstandings on the topics as made visible during the lessons.

Aj. Kim's reflection. Aj. Kim had been continuously teaching grade 9 mathematics courses for almost 20 years. From her experiences, she found that students who did not usually pay much attention in class had had more experiences in situations involving probability (e.g., gambling) than "nerdy" students. She thought that she could teach the probability topics in a more concrete way, compared to other mathematics topics. She also thought it was more convenient to find teaching materials for a probability topic. She said, "I can just take whatever are around in the classroom as examples." When asked how learning the topic would benefit the students, she said the knowledge could help students in making better predictions and decisions. For example, students should be able to predict that gambling would result in a loss and decide not to gamble. The main concept that she wanted to address in her lessons was for the students to be able to apply their knowledge in solving problems.

Even though Aj. Kim perceived students in Class A as higher achievers than students in Class B, she reported using the same teaching plan for both classes, with some adjustment when needed. However, she did not show her lesson plans during the interview nor in the classroom. Aj. Kim said she assessed students by grading their notebooks, having students take midterm and final exams, and assigning students a mathematics project. However, she did not assign a project for the probability units. She reported that many students in Class A had a tutor and "do not pay much attention in class because they have already learned the topic", while Class B students "do not usually have a tutor and are still waiting for teachers to give them knowledge". During the post-lesson interview, she reported that she thought the lessons went well and would like to have given the students more examples if she had had more time.

Aj. Nan's reflection. Aj. Nan had been continuously teaching grade 9 mathematics courses for longer than a decade. She viewed the content of the probability units to be appropriate for the students, given that the students would have an opportunity to study the topics again in the higher secondary levels[15]. However, when asked how learning the topics would benefit the students, she hesitated. She said the benefit that the students would receive from the lessons would be to apply it in their everyday lives, such as making better decisions, but could not give further explanation. From her experiences, she found that 9th graders had had more difficulties with these topics than other topics in the course. Most students had trouble finding the outcomes of random experiments and considering which outcome belonged to an event.

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[15] In Thailand, the compulsory educations are grades 1 to 9. However, almost all of Ramathip school 9th graders would continue their education in grades 10 to 12.

Since Aj. Nan viewed the two groups of her students, class C and class D, as quite similar, she planned to use the same teaching plan for both classes. However, she seemed reluctant to give details of her teaching plan. She briefly stated that some instruction materials and experiments would be used. When asked what the big ideas she wanted to address in her lessons were, she said they were all equally important and that the students should "know" them all. As for the student assessment, she claimed that she used exercises and homework as formative assessment. Looking at students' responses would give her understanding of students' difficulties and she would address them before moving on to the next lesson. She also gave a quiz after the lessons as summative assessment, in addition to the midterm and final exams. She also thought the lessons went well and would like to have given the students more examples if she had had more time.

In summary, both teachers were satisfied with their lessons. They did not seem to realize that their lessons were different from how the curriculum was intended. It was possible that the teachers used only the students textbook but not the teacher manual. Lesson observation found no used of the teacher manual and the teachers never mentioned using the manual during the interviews.

Teachers' misunderstanding. As mentioned in the implemented curriculum section, both teachers had given at least one inappropriate example during their lessons. The examples that showed the teachers' mistakes were the ones they added to the lessons (i.e., not taken from the textbook). This section explains three of the examples in two situations.

1) Exam situation. Aj. Kim gave an example on taking a 10-item exam in Class B, period 2. Figure 4-5 shows the summary of the example. In an exam situation, the problem could be viewed as a random experiment if the test taker randomly selected an answer for each item. However, in normal circumstances, a test taker would not randomly answer test items, but uses his or her knowledge to answer them. Aj. Kim did not point this out to her students.

Example 1	An exam contains 10 multiple-choice items, one point each. All possible outcomes are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. n(S) = 11. The event of passing the exam is {5, 6, 7, 8, 9, 10}. n(E) = 6. The event of failing the exam is {0, 1, 2, 3, 4}. n(E) = 5.

Figure 4-5. One of Aj. Kim's examples from Class B, period 2.

Similarly, Aj. Nan gave an example on taking a 20-item exam in Class C, period 6. Figure 4-6 shows the summary of the example. Aj. Nan stated the condition of randomly selecting the answers, however, the possible outcomes (score) of 0, 1, 2, ..., 20 are not equally likely. A test taker is more likely to get a score in the middle than a very low or very score. (There was only one way to get a score of zero, by answering all item wrong. However, there were 20 different ways to get a score of 1, by answering one item out of 20 items correctly.) Hence, the formula p(E) = n(E)/n(S) could not be used in this situation.

Example 1	An exam contains 20 true-or-false-items, one point each. A student passes the exam if he or she gets a score of 60% or higher. If a student randomly answers all 20 items, what is the probability that he or she passes the exam? The passing score is 60% x 20 = 12 points All possible scores are 0, 1, 2, 3, ... , 20. n(S) = 21. The probability of passing the exam is 9/21 or 3/7.

Figure 4-6. One of Aj. Nan's examples from Class C, period 6.

2) Sport situation. Aj. Nan gave an example on a sporting event in Class D, period 5. Figure 4-7 shows the summary of the example. Normally, when a sport team plays against another team, the probabilities that they will win, lose, or tie are not equally likely, depending on how good the team and the players are, and could not be used in the formula p(E) = n(E)/n(S).

Example 2	Team A plays two matches. Find the probability that team A win both matches. All the possible outcomes are (lose, lose), (lose, win), (lose, tie), (win, lose), (win, win), (win, tie), (tie, lose), (tie, win), and (tie, tie). p(E) = 1/9.

Figure 4-7. One of Aj. Nan's examples from Class D, period 5.

From the above examples, Aj. Kim seemed to have trouble identifying a random experiment, while Aj. Nan repeatedly used the formula for calculating the probability incorrectly. Both of this observations were confirmed during the interview: Aj. Kim incorrectly gave examples of random experiments, and Aj. Nan insisted that the solutions she gave students as in Figures 4-6 and 4-7 were appropriate for 9th graders.

The analysis of the videotaped lessons and the interviews revealed that the teachers had some misunderstandings about probability topics. They used the curriculum materials in their lessons, but did so selectively. Even though they both believed that it was very important that their students could apply this knowledge in everyday life, they had not addressed this issue in their lessons. They were also unaware of students' probability misconceptions or how students would approach probability subjectively.

This chapter has shown that (1) the curriculum was implemented differently from how it was intended to be, in both the content aspect and the teaching aspect, and (2) the teachers still had misunderstandings about the topics despite many years of experience. The next chapter, chapter 5, presents the results of the pretest and the posttest. A discussion of the teachers' test results is also provided. Chapter 6, the last results chapter, discusses how the implemented curriculum described here impacted students' understanding of the topics and their performances on the probability misconception posttest.

Chapter 5 - Results: Students' Probability Knowledge Before and After Instruction

This second results chapter examines the probability misconception pretest and posttest to understand students' knowledge of probability topics before and after formal instruction. The chapter attempts to answer the second set of research questions: What were Thai secondary school students' probability misconceptions, before and after formal instruction? How did their responses on the tests change after instruction? The students' probability conceptions and misconceptions were analyzed using their responses from the tests. The purposes of the tests were to gain insight into students' understanding of the topics and to evaluate the impact of the probability instruction on students' ability to solve problems specially designed to capture their misconceptions. Similarly, the teachers' responses on the test (post only) will also be analyzed in order to gain more knowledge into their probability understanding. This chapter presents the results of the analysis of the tests responses in three main parts; (1) students' test scores, (2) students' responses on the tests and their probability misconceptions, and (3) teachers' probability misconceptions.

Students' Tests Scores

Students' responses on the tests were graded using a 20-point scale (one point for each correct answer and one point for giving the correct reasoning) for the 10 items on the tests. The analyses in this chapter were made with 195 students who completed both the pretest and the posttest. Table 5-1 shows the summary of students' mean pretest and posttest scores by class and for the total sample.

Table 5-1

Students' Average Test Scores (and Standard Deviations) by Class and Total

Class	N	Pretest			Posttest		
		Mean Correct Answer	Mean Correct Reasoning	Mean Total Score	Mean Correct Answer	Mean Correct Reasoning	Mean Total Score
A	50	5.16 (1.33)	2.50 (1.71)	7.66 (2.64)	6.76 (1.59)	4.18 (2.22)	10.94 (3.46)
B	48	3.08 (1.62)	1.10 (1.23)	4.19 (2.46)	5.02 (1.58)	2.79 (2.07)	7.81 (3.31)
C	54	5.22 (1.83)	2.59 (1.61)	7.81 (3.25)	6.57 (1.98)	3.85 (1.99)	10.43 (3.68)
D	43	3.67 (1.82)	1.33 (1.55)	5.00 (3.19)	5.47 (2.09)	2.88 (2.01)	8.35 (3.57)
Total	195	4.34 (1.90)	1.92 (1.67)	6.26 (3.30)	5.99 (1.95)	3.46 (2.15)	9.46 (3.73)

Notes. N = number of students.

Table 5-2 shows t-test statistics when comparing the pretest and the posttest scores. The students' probability knowledge was significantly improved after the lessons. Their average total score improved from 6.26 to 9.46 out of the possible score of 20 (t = 14.42, p < .001, effect size = 1.043). When the responses were separated into two parts, answers and reasoning, both parts also showed significant improvement. The average score on the answer part improved from 4.34 to 5.99 out of the possible score of 10 (t = 13.89, p < .001, effect size = 0.990). The average score on the reasoning part improved from 1.92 to 3.46 out of the possible score of 10 (t = 10.89, p < .001, effect size = 0.798).

Table 5-2

Paired Samples t-test Statistics

Pair	MD	SD	*t*	df	*p*	Cohen's *d*
Posttest total - Pretest total	3.20	3.09	14.422	194	0.001	1.043
Posttest answer - Pretest answer	1.66	1.67	13.890	194	0.001	0.990
Posttest reasoning - Pretest reasoning	1.54	1.97	10.894	194	0.001	0.798

Notes. MD = Mean difference; SD = Standards deviation; t = t-value; df = degree of freedom; p = p-value.

Since there were four classes of students, Multivariate Analysis of Variance (MANOVA) was used to determine whether there was a difference among classes in their pretest and posttest scores. Table 5-3 shows the MANOVA statistics for the pretest scores (pretest total score, pretest answer score, and pretest reasoning score). Table 5-4 shows the MANOVA statistics for the posttest scores (posttest total score, posttest answer score, and posttest reasoning score). The results showed that the difference on the pretest scores among the four classes were significant ($p < .01$), and the difference on the posttest scores among the four classes were significant ($p < .01$). Therefore, the four classes of students started the lessons with a difference on their probability knowledge, and they also finished the lessons differently with a significant improvement.

Table 5-3

Multivariate Analysis of Variance Among Classes for Pretest Scores (N=195)

Effect		Value	F	Hypothesis *df*	Error *df*	*p*
Group	Pillai's Trace	0.253	9.225	6	382	0.001
	Wilks' Lambda	0.748	9.903	6	380	0.001
	Hotelling's Trace	0.336	10.581	6	378	0.001
	Roy'd Largest Root	0.332	21.143	3	191	0.001

Notes. F = F-value, df = degree of freedom, p = p-value.

Table 5-4

Multivariate Analysis of Variance Among Classes for Posttest Scores (N=195)

Effect		Value	F	Hypothesis *df*	Error *df*	*p*
Group	Pillai's Trace	0.149	5.121	6	382	0.001
	Wilks' Lambda	0.852	5.289	6	380	0.001
	Hotelling's Trace	0.173	5.455	6	378	0.001
	Roy'd Largest Root	0.168	10.721	3	191	0.001

Notes. F = F-value, df = degree of freedom, p = p-value.

Further, a post hoc test was used to examine how the classes were different. The results showed that class A and class C were different from class B and class D based on their pretest total score, pretest answer score, and pretest reasoning score at a 0.01 significance level. Similarly, class A and class C were different from class B and class D based on their posttest total score and posttest answer score at a 0.05 significance level. For the posttest reasoning score, only class A was different from class B and class D at a 0.05 significance level. See Table 5-5 for statistical details.

Table 5-5

Multiple Comparisons of Each Score Components Between Classes (N=195)

Scores	Class		Mean Difference	SE	*p*
Pretest Total	A	B	3.472	0.587	0.001
		C	-0.155	0.570	0.993
		D	2.660	0.604	0.001
	B	C	-3.627	0.576	0.001
		D	-0.812	0.610	0.544
	C	D	2.815	0.594	0.001
Pretest Answer	A	B	2.077	0.336	0.001
		C	-0.062	0.326	0.998
		D	1.486	0.346	0.001
	B	C	-2.139	0.330	0.001
		D	-0.591	0.349	0.329
	C	D	1.548	0.340	0.001
Pretest Reasoning	A	B	1.396	0.311	0.001
		C	-0.093	0.302	0.990
		D	1.174	0.320	0.002
	B	C	-1.488	0.305	0.001
		D	-0.221	0.323	0.902
	C	D	1.267	0.314	0.001
Posttest Total	A	B	3.127	0.709	0.001
		C	0.514	0.689	0.878
		D	2.591	0.730	0.003
	B	C	-2.613	0.696	0.001
		D	-0.536	0.737	0.886
	C	D	2.077	0.717	0.022
Posttest Answer	A	B	1.739	0.367	0.001
		C	0.186	0.356	0.954
		D	1.295	0.378	0.004
	B	C	-1.533	0.360	0.001
		D	-0.444	0.381	0.650
	C	D	1.109	0.371	0.017
Posttest Reasoning	A	B	1.388	0.420	0.006
		C	0.328	0.408	0.852
		D	1.296	0.432	0.016
	B	C	-1.060	0.412	0.053
		D	-0.092	0.436	0.997
	C	D	0.968	0.425	0.106

Notes. SE = Standards error, *p* = *p*-value.

Since the four classes were different based upon their pretest scores and posttest scores, Multivariate Analysis of Covariance (MANCOVA) was used to determine if there was a significant difference in how much each of the different classes improved between the pretest and the posttest. The pretest scores were used as the covariate. Table 5-6 shows the MANCOVA statistics. For all three score components (total, answer, and reasoning), the results showed that the differences in improvement between the four classes were not significant when controlling for the pretest scores ($p > 0.05$). In other words, there is no evidence that some classes benefited more than others from instruction.

Table 5-6

Multivariate Analysis of Covariance Among Classes for Posttest Scores When Controlling for Pretest Scores (N=195)

Effect		Value	F	Hypothesis *df*	Error *df*	*p*
Group	Pillai's Trace	0.017	0.534	6	378	0.782
	Wilks' Lambda	0.983	0.532	6	376	0.784
	Hotelling's Trace	0.017	0.530	6	374	0.786
	Roy'd Largest Root	0.012	0.750	3	189	0.523

Notes. F = F-value, df = degree of freedom, *p* = *p*-value.

Quantitative analyses of the students' tests responses showed that students' test scores improved from pretest to posttest. The improvement was significant for every component (total test score, answer score, and reasoning score). Each teacher had one class that performed significantly better on the pretest (classes A and C). These two classes also performed significantly better on most posttest measures except that Class C did not perform significantly better than the other classes (B and D) on reasoning. However, these differences among the four classes on their posttest scores were because of the differences on their pretest scores. Moreover, the students' mean posttest score of 9.46 out of 20 suggests that there was still room for improvement. The next section examines students' responses on the tests in term of their probability misconceptions.

Students' Responses on the Tests and Their Probability Misconceptions

To further examine students' probability understanding, the students' responses on the pretest and the posttest were also coded in terms of probability misconceptions. There were five probability misconceptions investigated in the tests: representativeness, positive and negative recency effects, conjunction fallacy, compound and simple events, and effect of the time axis. This section provides results of the analysis of the students' responses for these five main misconceptions. In addition, extra misconceptions evidenced in the students' responses are also discussed. Since there were no important differences by class, the results reported here is of the total group of students. The results by class are provided in Appendix D.

Table 5-6 shows the percentage of students' correct responses (answers and reasoning) for each item on the pretest and the posttest. For items 1-9, more students responded correctly on the posttest than on the pretest. However, for item 10, fewer students responded correctly on the posttest. Similarly, for items 1-2, and 4-9, more students gave correct reasoning on the posttest than on the pretest, and for item 3 and 10, fewer students gave correct reasoning on the posttest. Even though students showed improvement from pretest to posttest, more than half of the students still gave wrong answers and reasoning on most items.

Table 5-6

The Percentage of Students' Correct Responses on Pretest and Posttest (N=195)

Item	Correct Answer			Correct Reasoning		
	Pre	Post	Post - Pre Difference	Pre	Post	Post - Pre Difference
1	85.1	97.0	11.9	10.3	21.5	11.2
2	66.7	90.3	23.6	5.1	11.3	6.2
3	56.9	72.3	15.4	31.8	30.3	-1.5
4	27.7	42.1	14.4	13.8	23.6	9.8
5	19.0	39.5	20.5	13.8	34.4	20.6
6	29.2	45.6	16.4	15.9	29.2	13.3
7	15.4	48.2	32.8	7.7	40.5	32.8
8	28.7	40.5	11.8	20.5	42.6	22.1
9	86.7	92.8	6.1	39.0	61.0	22.0
10	20.5	14.4	-6.1	16.4	6.2	-10.2
Total	43.6	58.3	14.7	17.4	30.1	12.7

Representativeness misconception. Items 1, 2, and 4 examined the representativeness misconception. The items are shown in Figure 5-1. "According to the representativeness heuristic, people estimate the likelihood of an event on the basis of how well it is "representative" of the parent population from which it is drawn or on how well it represents the process that generates it" (Shaughnessy, 2003, p. 219). Though all three items examined the same misconception, item 2 was intended to inspect the consistency of participants' reasoning when compared to item 1 (Konold et. al., 1993; Rubel, 2007), while item 4 used a different context for the same concept. Tables 5-7, 5- 8, 5-9, 5-10, 5-11 and 5-12 show the percentage of students' answers and reasoning for items 1, 2, and 4, respectively.

Pretest	Posttest
1. Let H stands for the event of getting a head in tossing a coin and T stands for the event of getting a tail. Toss one coin five times. Which of the following is most likely to happen? Explain. a. HTHTH b. HHHTT c. THHTH d. THTTT e. All are equally likely	1. Let H stands for the event of getting a head in tossing a coin and T stands for the event of getting a tail. Toss one coin five times. Which of the following is most likely to happen? Explain. a. THTHT b. TTTHH c. HTTHT d. HTHHH e. All are equally likely
2. From 1. Which of the following is least likely to happen? Explain. a. HTHTH b. HHHTT c. THHTH d. THTTT e. All are equally likely	2. From 1. Which of the following is least likely to happen? Explain. a. THTHT b. TTTHH c. HTTHT d. HTHHH e. All are equally likely
4. In a 2-digit lottery game, Tor buys 21, 22, 23, 24, and 25. Tan buys 17, 38, 62, 59, and 84. Who is more likely to win this game? Explain. a. Tor b. Tan	4. An urn contains 100 small cards with number 00 – 99 written on them. If you pick 5 cards out of the urn, which group of numbers is more likely to come up? Explain. a. 11, 22, 33, 44, and 55 b. 03, 49, 67, 81, and 92

Figure 5-1. Probability misconception test items 1, 2, and 4: Representativeness.

Table 5-7

Item 1: Percentage of Students' Answers on Pretest and Posttest (N=195)

Answer	a.	b.	c.	d.	e. (Correct)	No Answer	Other
Pretest	7.2	1.0	6.2	0	85.1	0	0.5
Posttest	1.5	1.0	0.5	0	97.0	0	0

Table 5-8

Item 1: Percentage of Students' Reasoning on Pretest and Posttest (N=195)

Reasoning	Correct	Incorrect					None	Other
		Incor.	Equal Prob.	Represent.	Exper.	Uncert.		
Pretest	10.3	12.8	14.4	1.0	4.1	17.9	38.5	1.0
Posttest	21.5	21.5	4.1	0.5	0	10.3	38.5	3.6

Notes. Incor. = Incorrect or incomplete use of procedure or diagram; Equal Prob. = Equal probability; Represent. = Representativeness misconception; Exper. = Experiment or trial and error; Uncert. = Uncertainty.

Table 5-9

Item 2: Percentage of Students' Answers on Pretest and Posttest (N=195)

Answer	a.	b.	c.	d.	e. (Correct)	No Answer	Other
Pretest	5.6	6.7	0.5	20.5	66.7	0	0
Posttest	1.0	0.5	0	8.2	90.3	0	0

Table 5-10

Item 2: Percentage of Students' Reasoning on Pretest and Posttest (N=195)

Reasoning	Correct	Incorrect					None	Other
		Incor.	Equal Prob.	Represent.	Exper.	Uncert.		
Pretest	5.1	1.0	12.8	10.8	0.5	12.3	55.4	2.1
Posttest	11.3	3.1	3.1	2.6	0	10.8	65.0	4.1

Notes. Incor. = Incorrect or incomplete use of procedure or diagram; Equal Prob. = Equal probability; Represent. = Representativeness misconception; Exper. = Experiment or trial and error; Uncert. = Uncertainty.

Table 5-11

Item 4: Percentage of Students' Answers on Pretest and Posttest (N=195)

Answer	a. (Incorrect)	b. (Misconception)	Equally likely (Correct)	Both	No Answer	Other
Pretest	35.4	29.2	27.7	0	6.2	1.5
Posttest	5.1	41.5	42.1	4.1	6.2	1.0

Table 5-12

Item 4: Percentage of Students' Reasoning on Pretest and Posttest (N=195)

Reasoning	Correct		Incorrect				None	Other
	Proc.	Eq Ch.	Incor.	Represent.	Uncert.	50:50		
Pretest	1.0	12.8	0	53.9	7.7	1.0	19.5	4.1
Posttest	1.0	22.6	1.0	35.4	9.2	0.5	26.2	4.1

Notes. Proc. = Correct use of procedure or diagram; Eq Ch. = Equal change; Incor. = Incorrect or incomplete use of procedure or diagram; Represent. = Representativeness misconception; Uncert. = Uncertainty.

More students were able to answer correctly on the posttest than on the pretest on all three items investigating the representativeness misconception. The overall changes from pretest to posttest were 11.9% (85.1% to 97.0%), 23.6% (66.7% to 90.3%), and 14.4% (27.7% to 42.1%), for items 1, 2, and 4, respectively. Though all three items tested the same misconception, students seemed to have more difficulties with item 4 than items 1 and 2. Moreover, for item 4, 12.3% (from 29.2% to 41.5%) more of the students selected the answer that showed the misconception on the posttest than on the pretest.

Based just upon their ability to choose a correct answer on the pretest items 1 and 2, most students did not demonstrate the representativeness misconception on the pretest with a correct response rate of 85.1% and 66.7%. However, very few were able to demonstrate the reasoning behind their correct responses, as shown by the low percentages of 10.3 and 5.1. Similarly, on the posttest items 1 and 2, 97.0% and 90.3% of the students did not demonstrate the representative misconception, while only 21.5% and 11.3% were able to demonstrate correct reasoning. For both pretest and posttest, more students correctly answered item 1 than item 2, even though the two items were similar. All students who answered item 2 correctly also answered item 1 correctly. Students with the representativeness misconception incorrectly picked a sequence (choices a., b., c., or d.) as an answer. Therefore, students were less likely to show the misconception when asked to pick the most likely outcome (item 1) than the least likely outcome (item 2). For both pretest and posttest, the most popular incorrect answer for item 2 was d. (20.5% in pretest and 8.2% in posttest), which is the only choice that no one chose as the answer for item 1. Therefore, the question (item 2) itself may not have elicited the misconception, but the choices did. This finding might not be suitable to compare to those of Konold et. al. (1993) and Rubel (2007) due to a much higher percentage of correct answers for both items, however it supports the claim that the inconsistency exists.

When considering their reasoning for item 1, students seemed to move from subjective types of reasoning (e.g., equal probability and uncertainty) on the pretest toward using a procedure or diagram (both correctly and incorrectly or incompletely) on the posttest. Only 29 (14.9%) students gave subjective types of reasoning on the posttest compared to 73 (37.4%) on the pretest. However, there were still more than one third of students (38.5%) who did not give reasoning on the posttest. When comparing item 1 and item 2 in terms of the consistency in the students' reasoning, the results showed fewer inconsistency responses on the posttest (4.1%) than on the pretest (12.8%). However, 65.1% of students did not provide reasoning for item 2 on the posttest, higher than on the pretest (55.4%) and on item 1 (38.5%). The students who provided reasoning for item 1 but not for item 2 may have thought that another explanation was unnecessary because their answers were the same for both items.

Another interesting rationale was from students who gave "equal probability" reasoning (item 1: 14.4% on the pretest and 4.1% on the posttest, item 2: 12.8% on the pretest and 3.1% on the posttest). The responses with this reasoning are usually similar to “they (the choices/the faces of the coin) are all equally likely because it’s a random experiment.” This explanation alone was not entirely correct because there were three possible meanings: 1) each outcome (choice) was equally likely with the probability of 1/32 (correct), 2) each face (heads or tails) was equally likely (correct but not sufficient for the situation), and 3) each outcome (choice) was equally likely with the probability of 1/2 or 1/4 (incorrect).

Even though most students did not demonstrate the representativeness misconception when tested with items 1 and 2, they demonstrated the misconception when tested with item 4. There were improvements both in terms of answer and reasoning (14.4% improvement for answer and 9.7% improvement for reasoning), but there were more than one half incorrect answers and more than three quarters incorrect reasoning. According to Fischbein and Schnarch (1997), those who possess the representativeness misconception would choose the second set of numbers ({17, 38, 62, 59, 84} on the pretest and {03, 49, 67, 81, 92} on the posttest) as their answer because it seemed more random. Interestingly, there were more students who chose the second set of numbers (the representativeness misconception answer) on the posttest than on the pretest (41.5% on the posttest and 29.2% on the pretest), but there were fewer students whose explanation showed the representativeness misconception on the posttest (35.4% on the posttest and 53.8% on the pretest). However, about one fourth (26.2%) of students did not provide any explanation for their answer. Though there appears to be almost no other research investigating the inconsistency across problem contexts, this finding suggests that some participants shift their reasoning when solving the same type of problem in different contexts.

Results from the representativeness misconception items showed that there were inconsistencies in both students' answers and reasoning among items. Students showed more inconsistency when the random experiments were generated differently (coin tossing versus randomly selecting numbers from a finite set) than within the same random experiment. Students also had more difficulties dealing with randomly selected numbers from a finite set experiment than a coin tossing experiment.

Positive and negative recency effect misconceptions. Item 3 examined the positive and negative recency effect misconceptions. The items are shown in Figure 5-2. Negative and positive recency effect misconceptions happen when a person believes that a specific outcome of a sequence of independent events is more likely (positive recency effect) or less likely (negative recency effect) to occur due to the lack of that outcome in the previous results (Fischbein & Schnarch, 1997).

Pretest	Posttest
3. In tossing a coin five times, the coin showed heads the first four times. Which is more likely to happen the fifth time? Heads or tails? Explain.	3. A token was painted red on one side and blue on the other side. If I toss the token four times and red came up all four times, which is more likely to happen the fifth time? Red or blue? Explain.

Figure 5-2. Probability misconception test item 3: Positive and negative recency effect.

Tables 5-13 and 5-14 show the percentage of students' answers and reasoning for item 3. There was a 15.4% improvement for the correct answer (from 56.9% to 72.3%). In terms of misconception answers, both positive recency effect misconception answers and negative recency effect misconception answers were lower on the posttest (from 14.9% to 7.7% and from 12.8% to 5.1%, respectively). However, the percentage of students who gave correct explanations on the posttest was a little lower than on the pretest (31.8% in pretest and 30.3%

in posttest). Even though the number of students who gave reasoning that showed positive and negative recency effect misconceptions was lower in posttest than in pretest (from 16.4% to 7.7%), more students gave "uncertainty" reasoning (from 13.8% to 20.4%). This uncertainty reasoning could be considered as another type of probability misconception and is discussed more later.

Table 5-13

Item 3: Percentage of Students' Answers on Pretest and Posttest (N=195)

Answer	Equally likely (Correct)	Heads/Red (Positive Recency Effect)	Tails/Blue (Negative Recency Effect)	Both	No Answer	Other
Pretest	56.9	14.9	12.8	12.3	2.6	0.5
Posttest	72.3	7.7	5.1	10.3	3.1	1.5

Table 5-14

Item 3: Percentage of Students' Reasoning on Pretest and Posttest (N=195)

Reasoning	Correct		Incorrect				None	Other
	Proc.	Indep.	Incor.	Represent.	Pos/Neg RE	Uncert.		
Pretest	28.2	3.6	1.5	2.1	16.4	13.8	28.2	6.2
Posttest	19.0	11.3	6.2	0	7.7	20.4	31.3	4.1

Notes. Proc. = Correct use of procedure or diagram; Indep. = Independency of each toss; Incor. = Incorrect or incomplete use of procedure or diagram; Represent. = Representativeness misconception; Pos/Neg RE = Positive recency effect and negative recency effect misconceptions; Uncert. = Uncertainty.

Similar to items 1 and 2, most students were able to choose the correct answers for item 3, but failed to provide correct reasoning. These three items, 1, 2, and 3, were the only items (except item 9) that more than 50% of students gave a correct answer on the pretest and more than 70% of the students gave a correct answer on the posttest. One explanation is that these items deal with tossing a coin/token which students are familiar with and also was used most often as examples in their lessons.

Conjunction fallacy misconception. Items 5 and 6 examined the conjunction fallacy misconception. The items are shown in Figure 5-3. A conjunction fallacy misconception happens when a person views the probability of an event to be smaller than the probability of the intersection of the same event with another (Fischbein & Schnarch, 1997; Shaughnessy, 2003). Tables 5-15, 5-16, 5-17 and 5-18 show the percentage of students' answers and reasoning for items 5 and 6, respectively.

Pretest	Posttest
5. 43 year-old Sonny is a very heavy smoker. Lately, he has been suffering from constant chest pain and cough. So, he decides to see a doctor. Which of the following events has the higher probability? Explain. a. Sonny has lung cancer. b. Sonny has cancer.	5. Ben had just graduated from college, majoring in accounting. He applies for a job at two different accounting companies. Company A has 3 opening jobs. Company B has 5 opening jobs. Which of the following events has the higher probability? Explain. a. Ben gets a job at company B. b. Ben gets a job.
6. Jane likes sweet fruit. She walks to a fruit store with 50 *bahts* (Thai unit of money). The store sells three kinds of fruit, guava, longan, and mandarin orange, for 40, 45, and 60 *bahts* per kilogram, respectively. If the minimum amount of fruit to be bought is one kilogram, which of the following events has the highest probability? Explain. a. Jane buys guava. b. Jane buys longan. c. Jane buys mandarin orange. d. Jane buys fruit.	6. May likes to drink iced milk. She goes to a beverage store that sells four kinds of beverage; iced milk, hot milk, iced chocolate, and hot chocolate. Which of the following events has the highest probability? Explain. a. May buys iced milk. b. May buys hot milk. c. May buys iced chocolate. d. May buys hot chocolate. e. May buys a beverage.

Figure 5-3. Probability misconception test items 5 and 6: Conjunction fallacy.

Table 5-15

Item 5: Percentage of Students' Answers on Pretest and Posttest (N=195)

Answer	a. (Misconception)	b. (Correct)	No Answer	Other
Pretest	79.5	19.0	1.0	0.5
Posttest	60.5	39.5	0	0

Table 5-16

Item 5: Percentage of Students' Reasoning on Pretest and Posttest (N=195)

Reasoning	Correct			Incorrect		None	Other
	Proc.	Set & Subs.	Mult. Occur.	Incor.	Ment. Factor		
Pretest	0	5.6	8.2	0	64.2	20.5	1.5
Posttest	0.5	33.9	0	8.2	32.3	24.1	1.0

Notes. Proc. = Correct use of procedure or diagram; Set & Subs. = Choice a. is a subset of choice b.; Mult. Occur. = Multiple occurrences; Incor. = Incorrect or incomplete use of procedure or diagram; Ment. Factor = Mentioned Factor.

Table 5-17

Item 6: Percentage of Students' Answers on Pretest and Posttest (N=195)

Answer	a.	b.	c.	d. (Pretest Correct)	e. (Posttest Correct)	No Answer	Other
Pretest	23.6	46.2	0	29.2	N/A	0.5	0.5
Posttest	53.3	0	1.0	0	45.7	0	0

Table 5-18

Item 6: Percentage of Students' Reasoning on Pretest and Posttest (N=195)

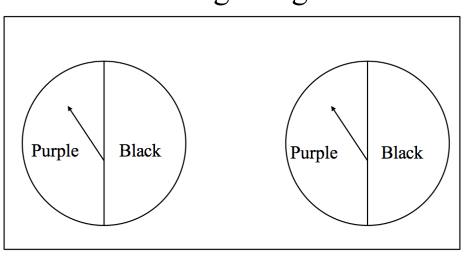
Reasoning	Correct			Incorrect		None	Other
	Proc.	Set & Subs.	Mult. Occur.	Incor.	Ment. Factor		
Pretest	0	9.7	6.2	0.5	54.4	29.2	0
Posttest	0	29.2	0	5.1	47.3	17.9	0.5

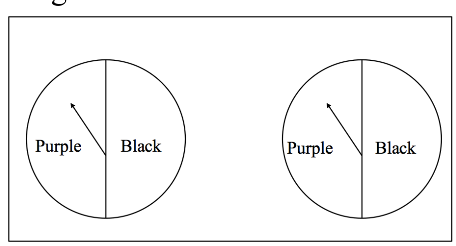
Notes. Proc. = Correct use of procedure or diagram; Set & Subs. = Choice a. is a subset of choice b.; Mult. Occur. = Multiple occurrences; Incor. = Incorrect or incomplete use of procedure or diagram; Ment. Factor = Mentioned Factor.

There were improvements in terms of correct answers for both items 5 and 6: from 19.0% to 39.5% for item 5, and from 29.2% to 45.6% for item 6. There was also improvement in terms of correct reasoning: from 13.8% to 34.4% for item 5, and from 15.9% to 29.2% for item 6. Even though more students responded correctly on the posttest on both items, the improvement was a little better on item 5 than on item 6. Item 5 involved a situation where the person does not make his own decision, while item 6 involved a situation where the person makes her own decision. The results indicated that students were less likely to apply probability knowledge to the latter situation, where the person appeared to be in control. The students might have thought that the factors that relevant to the problem, such as prices and preferences, were more important than the probability concept.

Even though fewer than half of the students were able to choose a correct answer for items 5 and 6, most of the students who chose a correct answer were also able to provide correct reasoning for their correct answer, which was the opposite of items 1 and 2. On the pretest, there was only one student who attempted to use a procedure or diagram in answering item 6, and none did that for item 5. On the posttest, 17 students attempted to use a procedure or diagram in answering item 5, and 10 for item 6. Even though most of these responses were incorrect or incomplete, it showed that students were trying to apply the knowledge they learned in solving the problems.

Compound and simple events misconception. Items 7 and 8 examined the compound and simple events misconception. The items are shown in Figure 5-4. The compound and simple events misconception happens when a person does not take order into account when comparing a compound event with a simple event (Fischbein & Schnarch, 1997). Tables 5-19, 5-20, 5-21, and 5-22 show the percentage of students' answers and reasoning for items 7 and 8, respectively.

<table>
<tr><th>Pretest</th><th>Posttest</th></tr>
<tr><td>7. In tossing 2 dice once, which outcome is more likely to occur? Explain.
a. One die shows 5 and the other die shows 6.
b. Both of the dice show 6.
c. Both outcomes are equally likely.</td><td>7. In tossing 2 dice once and considering the sum of the numbers, which outcome is more likely to occur? Explain.
a. The sum equal 11.
b. The sum equal 12.
c. Both outcomes are equally likely.</td></tr>
<tr><td>8. A game consists of spinning two fair spinners (see diagram). A player wins only when both arrows land on purple, otherwise he or she loses. Does a player have a 50-50 chance of winning this game?


a. Yes. Why?
b. No. Why?</td><td>8. A game consists of spinning two fair spinners (see diagram). A player wins only when the arrow on the left spinner lands on purple and the arrow on the right spinner lands on black, otherwise he or she loses. Does a player have a 50-50 chance of winning this game?


a. Yes. Why?
b. No. Why?</td></tr>
</table>

Figure 5-4. Probability misconception test items 7 and 8: Compound and simple events.

Table 5-19

Item 7: Percentage of Students' Answers on Pretest and Posttest (N=195)

Answer	a. (Correct)	b.	c. (Misconception)	No Answer	Other
Pretest	15.4	2.1	81.0	0	1.5
Posttest	48.2	1.5	49.3	1.0	0

Table 5-20

Item 7: Percentage of Students' Reasoning on Pretest and Posttest (N=195)

Reasoning	Correct		Incorrect					None	Other
	Proc.	Comp. Outc.	Incor.	Eq. Prob.	Uncert.	Represent.	Comp.		
Pretest	2.6	5.1	5.6	13.9	17.4	1.0	0	51.3	3.1
Posttest	19.5	21.0	4.6	5.6	13.3	0	8.2	24.2	3.6

Notes. Proc. = Correct use of procedure or diagram; Comp. Outc. = Compare outcomes of each event; Incor. = Incorrect or incomplete use of procedure or diagram; Eq. Prob. = Equal probability; Uncert. = Uncertainty; Represent. = Representativeness misconception; Comp. = Compound and simple events misconception.

Table 5-21

Item 8: Percentage of Students' Answers on Pretest and Posttest (N=195)

Answer	a. (Misconception)	b. (Correct)	No Answer	Other
Pretest	64.1	28.7	3.6	3.6
Posttest	46.2	40.5	13.3	0

Table 5-22

Item 8: Percentage of Students' Reasoning on Pretest and Posttest (N=195)

Reasoning	Correct		Incorrect			None	Other
	Proc.	Comp. Outc.	Incor.	Eq. Prob.	Uncert.		
Pretest	18.5	2.1	2.1	42.5	13.3	13.8	7.7
Posttest	42.5	0	3.6	31.8	7.7	11.3	3.1

Notes. Proc. = Correct use of procedure or diagram; Comp. Outc. = Compare outcomes of each event; Incor. = Incorrect or incomplete use of procedure or diagram; Eq. Prob. = Equal probability; Uncert. = Uncertainty.

Similar to items 5 and 6, students did better on the posttest than on the pretest in terms of both answer and reasoning on items 7 and 8. Their performance improved from 15.4% to 48.2% on item 7 answer, from 28.7% to 40.5% on item 8 answer, from 7.7% to 40.5% on item 7 reasoning, and from 20.6% to 42.5% on item 8 reasoning. Students showed more improvement on item 7 than on item 8. Before instruction, students might have found item 8 easier with only four elements in the sample space compared to 36 for item 7. On the other hand, after instruction, which spent more time on dice and not much attention to spinners, students might have found item 7 more familiar. Item 7 also was the item that students had the most improvement (32.8%) in terms of both the answer and reasoning. However, 51.8% of students still answered item 7 incorrectly, even after having experienced a similar context in the classroom. Item 8 was also the only item in which fewer students gave an answer on the posttest than on the pretest. Table 5-23 shows the percentage of students who did not answer each item on the pretest and the posttest. Students might have felt that they were not prepared to solve a spinner problem since they did not have enough experience of it in class.

Table 5-23

The Percentage of Students Who Did Not Give an Answer in the Probability Misconception Tests (N=195)

Item	1	2	3	4	5	6	7	8	9	10	Total
Pre	0	0	2.6	6.2	1.0	0.5	0	3.6	7.7	7.2	2.9
Post	0	0	3.1	6.2	0	0	1.0	13.3	1.5	3.6	2.9

Items 7 and 8 were also the items with the highest improvement on correct explanations. Even though more than half of the students could not answer the items correctly after instruction, those who did were also able to explain it.

Effect of the time axis misconception. The effect of the time axis misconception, or the Falk phenomenon, happens when people reason based on the principle that an event cannot act retroactively on its cause (Shaughnessy, 1992). Item 10 examined this misconception, while items 9 was intended to prepare the participants for item 10. The items are shown in Figure 5-5. Tables 5-24, 5-25, 5-26 and 5-27 show the percentage of students' answers and reasoning for items 9 and 10, respectively.

Pretest	Posttest
9. An urn only contains two yellow marbles and two green marbles. Randomly pick one marble out of the urn. What is the probability that the marble is green? Show your work.	9. A box only contains three red chips and three blue chips. Randomly pick one chip out of the box. What is the probability that the chip is blue? Show your work.
10. Using the same urn from 9 (with all four marbles in it). You picked out one marble and put it aside without checking the color. Then, you picked another marble and found that this second marble is green. What is the probability that the first marble you picked is also green? Show your work.	10. Using the same box from 9 (with all six chips in it). You picked out one chip and put it aside without checking the color. Then, you picked another chip and found that this second chip is blue. What is the probability that the first chip you picked is also blue? Show your work.

Figure 5-5. Probability misconception test items 9 and 10: Effect of the time axis.

Table 5-24

Item 9: Percentage of Students' Answers on Pretest and Posttest (N=195)

Answer	1/2 (Correct)	Other Numbers (Incorrect)	No Answer	Other
Pretest	86.6	3.1	7.7	2.6
Posttest	92.9	4.1	1.5	1.5

Table 5-25

Item 9: Percentage of Students' Reasoning on Pretest and Posttest (N=195)

Reasoning	Correct		Incorrect		None	Other
	Proc.	Equal No.	Incor.	Uncertainty		
Pretest	11.3	27.7	5.6	2.1	52.8	0.5
Posttest	48.3	12.8	4.6	0	33.3	1.0

Notes. Proc. = Correct use of procedure or diagram; Equal No. = Equal number of marbles or chips; Incor. = Incorrect or incomplete use of procedure or diagram.

Table 5-26

Item 10: Percentage of Students' Answers on Pretest and Posttest (N=195)

Answer	1/3 (Pretest) 2/5 (Posttest) (Correct)	1/2	1/4 (Pretest) 1/6 (Posttest)	1/6 (Pretest) 1/5 (Posttest)	2/6	No Answer	Other
Pretest	20.5	35.3	20.0	6.2	0	7.2	10.8
Posttest	14.4	30.8	4.1	18.4	10.3	3.6	18.4

Table 5-27

Item 10: Percentage of Students' Reasoning on Pretest and Posttest (N=195)

Reasoning	Correct		Incorrect							None	Other
	Proc.	Count #.	Incor.	Disreg.	Not consid.	Samp. w/o repl.	Both poss.	Simp. E. w incor. S	Incor #.		
Pretest	1.5	14.9	8.7	9.2	7.2	3.6	2.6	1.0	3.6	27.2	20.5
Posttest	1.0	5.1	25.6	1.0	10.3	7.7	0	2.6	1.0	36.0	9.7

Notes. Proc. = Correct use of procedure or diagram; Count #. = Count the correct number of marbles or chips left; Incor. = Incorrect or incomplete use of procedure or diagram; Disreg. = Disregard the second marbles or chips; Not consid. = Not consider the second marbles or chips; Samp. w/o repl. = Sampling without replacement; Both poss. = Both colors are possible; Simp. E. w incor. S = Simple event with incorrect sample space; Incorr #. = Use incorrect number of marbles or chips.

For this group of students, item 9 seemed to be easy, with the highest percentage of correct answers on the pretest, and most students either correctly explained their reasoning or did not give an explanation at all. Item 10, on the other hand, seemed to be the item that students had the most difficulty with. It was the only item with no improvement from pretest to posttest in terms of both answer and reasoning.

Even though the students' responses indicated that the effect of the time axis misconception was worse after instruction, a closer look at the posttest responses showed that the students had attempted to apply probability knowledge in solving the problem. The probability concepts that the students demonstrated, inappropriately, when solving item 10 were:

a. Sampling without replacement (picking two items out and getting both items in the preferred color).

b. Retroactive conditional event (picking one item out, then another, and knowing the color of the second item, calculating the probability of the first item being the same color).

In addition, there were still many answers and reasoning that differed from what was described in the Tables 5-26 and 5-27 above (18.5% of the answers and 9.7% of the reasoning). These "other" responses were not informative enough to understand the responders' ideas on the item.

Other misconceptions that emerged from the students' responses. Other than the five main misconceptions discussed above, there were other types of probability misconceptions that were not the main focus of this study, but emerged from the students' responses on the tests. These misconceptions are (1) uncertainty and (2) outcome approach. They are discussed next.

Uncertainty. Uncertainty was one type of reasoning that the students used in the tests across most items. The reasoning that was coded as uncertainty included responses like "it's not certain", "everything is possible", "anything could happen", "we can't tell what will happen", "we can't control the coin/token", etc. It is true that, when dealing with probability, there is and always will be uncertainty involved. But with some knowledge of probability in a situation, one can also make good predictions about likely outcomes. The students should have been able to give an explanation that showed this understanding after instruction. The total percentage of uncertainty reasoning on all 10 items on the posttest was 7.2%, a little lower than on the pretest (8.5%). Table 5-28 gives the itemize numbers of uncertainty reasoning. On most items, the results were as expected, fewer students gave uncertainty reasoning responses on the posttest than on the pretest. For items 3 and 4, on the other hand, more students gave uncertainty reasoning on the posttest than on the pretest. However, more students were able to answer both items 3 and 4 correctly on the posttest than on the pretest. The results indicate that the improvement on correct answers did not guarantee improvement on reasoning.

Table 5-28

The Percentage of Students Who Gave Uncertainty Reasoning in the Probability Misconception Tests (N=195)

Item	1	2	3	4	5	6	7	8	9	10	Total
Pre	17.9	12.3	13.8	7.7	0	0	17.4	13.3	2.1	0	8.5
Post	10.3	10.8	20.5	9.2	0	0	13.3	7.7	0	0	7.2

Outcome approach. Why were several students able to answer some items (especially items 3 and 4) correctly but could not give appropriate reasoning? One explanation is that these students were answering the items based on the Outcome Approach. Participants use an outcome approach when they believe that their task is to predict the outcome of a single probability experiment (Konold, 1989). An item is likely to encourage students to answer based on an outcome approach if the task is to select an outcome from a repeatable probability experiment (items 1, 2, 3, 4, 7, and 8). Items 5 and 6 were not a repeatable process, while items 9 and 10 asked for a probability rather than for the students to make a selection. Hence, after instruction, students had a better idea of how the situation should turn out and were able to select a correct answer, but failed to give an appropriate explanation.

In summary, the analyses of the students' responses on the probability misconceptions pretest and posttest showed that students held several probability misconceptions before and after instruction. Among the five misconceptions investigated in the tests, the conjunction fallacy, the compound and simple event, and the effect of the time axis misconceptions were common among the participants. After instruction, students showed an overall improvement in their probability knowledge. Four misconceptions, representativeness, positive and negative recency effects, conjunction fallacy, and compound and simple events misconceptions, were made less common. The effect of the time axis misconception became more common. However, students showed an attempt to use the knowledge learned during instruction to solve the problems, but sometimes did so incorrectly or inappropriately. Moreover, when the students answered an item correctly, they could not always give an appropriate reasoning to support their correct answer.

Teachers' Probability Misconceptions

In order to better understand why the students' posttest results turned out as described above, the teachers' responses on the posttest were also examined. The teachers were asked to take the test (post only) after their lessons. During the posttest interviews, teachers showed their responses on the test, explained their reasoning (if necessary), and discussed how they thought their students would respond to the test. This section describes the teacher participants' probability conceptions and misconceptions as based on the probability misconception posttest. Teachers' perceptions of how their students would perform on the test are also discussed.

Aj. Kim's probability misconceptions. At the time of the interview, the teachers were asked to bring a finished copy of the posttest. However, Aj. Kim had not done the test yet. She said she would do it during the interview, but she seemed reluctant to write down her answers. Hence, the interview questions and process were modified to help her relax.

She, therefore, was asked to predict whether the students would be able to answer the questions and how they would explain them.

For items 1 and 2, Aj. Kim predicted that most students should be able to correctly choose choice d. as an answer. They should also be able to explain that it was because each choice was one outcome out of 32 possible outcomes. For item 3, after reading the question, Aj. Kim said "some students would be confused and answered equally likely." [16] She predicted that about 75% of students should be able to solve the problem, but she did not say what she meant by the correct answer. When asked how she would explain to students how to solve the problem, she drew a tree diagram to represent the situation and said "have the students look at the diagram." She did not provide any further explanation, but it was obvious that she thought "equally likely" was an incorrect answer, even after drawing the tree diagram. Aj. Kim correctly answered item 4 and she predicted that 75% of the students should be able to give a correct answer, but many of them might not be able to explain their reasoning. Other students who give incorrect answers would not be able to give any explanation. For item 5, Aj, Kim predicted that most students would answer, "Ben gets a job at company B" (incorrect). However, she could not decide for herself which choice was the correct answer and argued that there was not enough information to make the decision. For item 6, Aj. Kim predicted that 70% of the students would answer, "May buys iced milk" (incorrect) and 30% would answer "May buys a beverage" (correct). For item 7, Aj. Kim said the students should be able to solve the problem, but did not provide any explanation. For item 8, she correctly answered that the player does not have a 50-50 chance of winning this game. However, her reasoning was on the physical factor of the spinner,

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[16] Equally likely was the correct answer.

rather than the probability factor. She also predicted that students would answer this item incorrectly. Aj. Kim predicted that students would be able to answer item 9 correctly because of the short wording. For item 10, she gave the incorrect answer of 1/6, and predicted that students would answer 1/3 or 1/2 (both incorrect).

Based on the interview, it seemed that Aj. Kim answered five out of ten items correctly (i.e., items 1, 2, 4, 7, and 9). She demonstrated three misconceptions, positive and negative recency effects (item 3), conjunction fallacy (items 5 and 6), and the effect of the time axis (item 10). For the compound and simple events misconception, it seemed that she answered item 7 (dice item) correctly and item 8 (spinner item) incorrectly. She was able to answer all three items (1, 2, and 4) on the representativeness misconception correctly.

Aj. Nan's probability misconceptions. At the interview, Aj. Nan brought a finished copy of the posttest. However, she needed sometimes to refresh her memory before answered the interview questions. She predicted that an average student should be able to answer seven out of ten items correctly. The three items that she thought would be challenging for the students were items 3, 4 and 10.

For item 3, Aj. Nan originally answered "blue" (incorrect) and demonstrated the positive and negative recency effects misconception. However, during the interview, she said the item could be answered as "equally likely" (correct). She thought both answers were correct, depending on how a person interpreted the situation. She also demonstrated the representativeness misconception in answering item 4. For item 5, Aj. Nan predicted that more students would answer "Ben gets a job" (correct) than "Ben gets a job at company B" (incorrect). However, she also thought that there was not enough information to make the decision. Similarly, for item 6, she predicted that most students would answer, "May buys iced milk" (incorrect) and some students would answer, "May buys a beverage" (correct). She originally chose the correct answer for this item, however, she changed her mind and chose an incorrect answer during the interview. For item 10, Aj. Nan originally gave the misconception answer of 1/2, however, during the interview, she was able to correctly redo the problem. For items 1, 2, 7, 8, and 9, Aj. Nan predicted that students should be able to correctly answer them, provided that they understood the lessons.

Based on her responses on the test, Aj. Nan answered six out of ten items correctly (i.e., items 1, 2, 7, 8, 9, and 10). She demonstrated two misconceptions, positive and negative recency effects (item 3), and the conjunction fallacy (items 5 and 6). For the representativeness misconception, she answered items 1 and 2 (coin item) correctly and item 4 (card item) incorrectly. She was able to demonstrate a correct understanding on items 7, 8, and 10, which investigated compound and simple events and the effect of the time axis misconceptions.

Table 5-29 shows the itemized comparison between the students' and the teachers' probability misconceptions. The two teachers had quite similar understanding of probability topics as their responses on seven items were similar. In most cases, more students seemed to be able to answer a posttest item correctly when both teachers answered that item correctly as well. The connection between the teachers' and the students' understanding of the topics is discussed in the next chapter.

Table 5-29

The Percentage of Students Who Gave Correct Answer in the Probability Misconception Tests (N=195) and Whether the Teachers Answered the Posttest Items Correctly

Item	Student		Teacher	
	Pretest	Posttest	Aj. Kim	Aj. Nan
1	85.1	96.9	Yes	Yes
2	66.7	90.3	Yes	Yes
3	56.9	72.3	No	No
4	27.7	42.1	Yes	No
5	19.0	39.5	No	No
6	29.2	45.6	No	No
7	15.4	48.2	Yes	Yes
8	28.7	40.5	No	Yes
9	86.7	92.8	Yes	Yes
10	20.5	14.4	No	Yes
Total	43.6	58.3	5	6

Notes. Yes = the teacher answered the item correctly; No = the teacher answered the item incorrectly.

This chapter discussed the teachers' and the students' responses on the probability misconception tests. The results showed that (1) the students' test scores were significantly improved from the pretest to the posttest, (2) however, they still did not perform well after instruction, (3) each teacher had one class (Classes A and C) that performed significantly better than the other class (Classes B and D), (4) on the five misconceptions investigated, students improved on four misconceptions and did worse on one misconception, (5) even when they were able to choose a correct answer, students were not always able to give correct reasoning, (6) teachers' perceptions of how their students would perform on the test was better than they actually were, and (7) teachers also demonstrated several misconceptions. The next chapter discusses how instruction influenced students' understanding of the probability topics and their responses on the tests.

Chapter 6 - Results: Lessons and the Misconceptions

This last results chapter discusses the connections between what happened in the probability lessons and the students' misconceptions as evidenced in their posttest responses. The previous two chapters, Chapters 4 and 5, described the curricula, both intended and implemented, and the teachers' and students' probability misconceptions. This chapter, in general, attempts to make connections between Chapters 4 and 5 in order to answer the third set of research questions: What happened in the classrooms that influenced students' misconceptions? and What impact did this have on their probability misconceptions? There are five issues discussed here. The first four issues are aspects of the lessons. The last issue discusses how the five misconceptions tested on the tests, plus the two emergent misconceptions, were addressed during the lessons.

Lack of a Subjective Approach to Probability

Several authors suggest that effective instruction should help students deal with their ideas about subjective probability (e.g., Amir & Williams, 1999; Barnes, 1998; Hawkins, 1984; Jones et al., 2007; Konold et al., 2011). Subjective probability is the degree of belief a person holds that an event will happen (Barnes, 1998). For example, when tossing a coin, using theoretical probability, we can explain that the heads and the tails are equally likely to come up with the equal probability of 1/2 (assuming the coin is fair). However, a person may believe that heads is more likely to come up than tails with any probability between 1/2 and 1. Subjective probability does not always have to be different from theoretical probability. A person may come up with the probability of 1/2 using a subjective approach. The solution to this simple example is easily resolved and explained, but for more complex situations, this is not always the case. For example, which is more likely? Getting one double heads in tossing two coins four times or getting two double heads in tossing two coins eight times?[17] In general, a person finds it easier to come up with a subjective probability than a theoretical or experimental probability in a given situation, especially in a complex situation. Students, however, should recognize which approach they employ when giving an answer.

Being self-aware of how they approach probability supports students' understanding, students' reasoning and could help students avoid mistakes. Take item 3 from the posttest (Figure 6-1) as an example:

Item 3: A token was painted red on one side and blue on the other side. If I toss the token four times and red came up all four times, which is more likely to happen the fifth time? Red or blue? Explain.

Figure 6-1 Probability misconception posttest item 3.

Three common answers to this item are red, blue, and both are equally likely. With the theoretical approach, the answer is both red and blue are equally likely, because the token has two sides, one red and one blue, and the outcome of the previous toss does not affect the outcome of the current one. With the experimental approach, the answer is red because from previous results red has a probability of 1 (4/4) and blue has probability of 0 (0/4). With the subjective approach, the answer could be anything, as long as the students provide reasoning to support it. For example, blue is more likely to come up than red, because red already came up many times, blue should come up to even it out.

The intended curriculum instructed the teachers to have a conversation with their students about the students' experiences in situations that involve probability at the beginning of the probability lessons, which was the opportunity for students to discuss and be aware of their subjective probability. The curriculum also emphasized that students practice making

[17] The probabilities are 0.421875 and 0.3114824.

predictions and decisions based on probability knowledge throughout the chapter. Teachers were also recommended to let students discuss their different ideas and to accept any answer students came up with, provided that the students give appropriate reasoning, even though the answer was different from those given in the answer keys.

The lessons (the implemented curriculum), on the other hand, did not provide students an opportunity to experience any of these concepts. Neither teacher spent time in having conversations with the students, having the students practice making predictions or decisions, or having the students discuss their ideas. Therefore, after the lessons, students were unaware of their subjective approach to probability and had difficulties giving reasoning for their answers.

The analysis of the students' tests responses had a result that supports this claim. Many students were unable to provide reasoning for their answers to the probability test items. Most students could figure out an answer (correctly or not) as the rate of unanswered items was very low, 2.9% for both pretest and posttest. However, 33.6% and 30.8% of the items on the pretest and the posttest, respectively, were unexplained. There was only a 2.8% difference in how often students gave their reasoning from pretest to posttest, which showed almost no improvement. Moreover, when the students did provide reasoning for their answers, much of these reasoning was incorrect or inappropriate, 49% on the pretest and 39.1% on the posttest. Similarly, the teachers were also unaware of their subjective approach to probability as evidence in their test responses and during the interview. The teachers were able to provide reasoning for their answers (correct and incorrect), but sometimes, unknowingly, gave subjective reasoning.

Emphasizing Too Much About Finding Probability

One of the main objectives of the intended curriculum was for the students to be able to use knowledge about probability in making predictions and decisions. According to Unit 3 of the students' textbook, students should learn how to calculate probability of several situations and then practice making decisions using probability knowledge. The textbook provided multiple activities giving students opportunities to practice making decisions. When doing these activities, students did not always have to calculate the probability of the situations. For example, situation 3 (Figure 4-3, page 57) did not require a calculation of probability, but students would need to use probability knowledge in making the decision.

On the other hand, when addressing Unit 3, both teachers only taught the students to calculate probability. They did not provide any examples or exercises for students to practice making decisions, even though they had spent a lot of time on this unit. Table 6-1 shows the amount of time spent on each of the four probability units. Classes A and B spent about half of their class time on unit 3, while Classes C and D spent about three quarters of their class time on unit 3. However, all of the examples, exercises, and homework from all four classes on this unit were on calculating probability. As a result, students seemed to perceive that, when answering questions about probability, they needed to find "the probability" of the events in question. The analysis of the students' test responses supports this interpretation.

Table 6-1

The Amount of Time, in Minutes, Spent on Each of the Probability Units by Class

Unit	Class			
	A	B	C	D
1. Probability: the Meaning and Uses	1.14	1.18	5.45	5.45*
2. Random Experiment and Event	48.02	66.23	20.56	20.56*
3. Probability of an event	90.48	59.36	149.04	159.39
4. Probability and Decision Making	0	0	20.14	22.11
Total	140.04	127.17	195.59	208.31

Notes. *Estimated using Class C's lessons.

The probability misconception test items 1-8 asked students to choose an outcome or event but did not require the calculation of probability. Among these eight items, items 5 and 6 did not provide sufficient information to calculate the probability, but the correct answer could be chosen without knowledge "the probability". Therefore, students should not be able to calculate "the probability" when solving items 5 and 6, and any number they came up with would be considered inappropriate. On the pretest items 5 and 6, only one student attempted to calculate "the probability" for the event; on the posttest, there were a total of 26 such responses (0.7%). The percentage of 0.7 may not seemed like a number of any significance. However, if the responses that resulted from a subjective approach and the responses that did not come with an explanation were not included, these 26 responses were equal to 17.3%. In other words, after the lesson, students were more likely to calculate probability of an event in an inappropriate situation when attempting to solve a probability problem using probability knowledge. This result indicated that students did learn how to calculate probability but did not understand how to apply the knowledge in making decisions. On the other hand, the teachers did not seem to have this misunderstanding. They did not attempt to calculate the probability for items 5 and 6.

For items 1-4, and 7-8, the percentages of responses that showed calculations of probability also increased (from 7.4% on the pretest to 12.9% on the posttest). However, reasoning in terms of "the probability" on these six items was appropriate since the students had enough information to calculate the probability. Moreover, items 9 and 10 were removed from this discussion because they directly asked students to calculate probabilities.

Use of Sample Spaces with Unequally Likely Elements

As described in Chapter 4, the teachers demonstrated two main mistakes during their lessons: (1) they inappropriately demonstrated experiments using un-identical objects, and (2) they incorrectly used sample spaces with unequally likely elements when giving students examples. The first mistake could not be linked to the students' performance on the posttest since the test was not designed to investigate this concept and there was no student' response that showed this mistake. However, several students' responses on the posttest showed the second mistake, as described next.

One objective of the probability chapter was that students should be able to write all the possible outcomes of random experiments (sample spaces). Even though the intended curriculum did not recommend using the term sample space, both teachers used it throughout their lessons. They first described the sample space (S) as the set of all possible outcomes of a random experiment and used the term and the notation on every occasion afterward. They also used the rather simple formula of probability, $p(E) = n(E)/n(S)$, which was the same formula used in the textbook, only the textbook did not use any notation. The main condition in using this formula is that the elements in the sample space have to be equally likely. Both teachers failed to emphasize this condition: Aj. Kim did not mention the condition at all, while Aj. Nan orally stated the condition once when she first gave the formula. In a few instances, both teachers also incorrectly demonstrated the uses of the formula with sample spaces that did not meet the condition (e.g., Figures 4-5, 4-6 and 4-7, pages 81-82).

To better understand why this condition for the formula is critical, consider this example: tossing three identical coins once. Four possible sample spaces are:

S = {HHH, HHT, HTT, TTT} (as when considering which side of the coin is up);

S = {0, 1, 2, 3} (as when considering the number of heads shown),

S = {Yes, No} (as when considering whether there is any heads shown), and

S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT} (as when considering all possible permutations, assuming the coins were somehow different).

The above sample spaces are all possible for the given situation, however, only the last one contains equally likely elements and hence was the only one that could be used in calculating probability with the formula p(E) = n(E)/n(S). It is also the sample space that commonly, and usually the only one, taught in the classrooms. Since the students had only been taught to write one sample space for each situation (random experiment), no matter what the consideration was, they were not aware that there could be other sample spaces for the given situation and the sample space they wrote may not have been appropriate. They also did not realize the necessity of using the sample space with equally likely elements when employing the formula p(E) = n(E)/n(S). Moreover, the students were instructed to always write the sample space, even before reading the question. Every example from all four classes started with the teachers writing down the sample space, before the questions or the considerations were given.

The evidence that the students lacked understanding of this specific condition of the formula p(E) = n(E)/n(S) could be seen in their responses to the posttest items 5 and 7. When solving posttest item 5, 16 students (equal to 8.2% of the students or 10.8% of all the responses) wrote, “The probability of Ben getting a job at company B is 5/8.” The students wrongly assumed the eight opening jobs at both companies as a sample space. However, they failed to notice that this sample space did not satisfy the “equally likely element” condition. Similarly, when solving posttest item 7, 11 students (equal to 5.6% of the students or 7.4% of all the responses) answered that “both 11 and 12 are equally likely as they are both one outcome of the sample space {2, 3, 4, ..., 11, 12}”. They also failed to notice that the elements in this sample space were not equally likely.

The percentages (10.8 % and 7.4%) of students who showed this mistake during the posttest might not be very high, since the test was not designed to capture this misunderstanding. Nevertheless, the fact that the teachers made this mistake in their lessons could imply that many students could make the same mistake, especially when solving a problem with similar situations (a sports situation or an exam situation).

Theoretical and Experimental Probabilities

Other than the subjective approach to probability discussed earlier, there are two other approaches to probability: (1) Experimental probability is the frequency that an event occurs in a large number of identical trials. (2) Theoretical probability is based on a theoretical analysis of the outcomes of a random experiment, and usually involves counting equally likely outcomes (Barnes, 1998). The intended curriculum focused more on theoretical probability, however, it also encouraged teachers to do experiments with the students. The textbook contained a section that discussed how to find probability using an experimental approach and how it is compared to a theoretical approach. The implemented curriculum, on the other hand, only taught the theoretical approach and did not mention the experimental approach at all.

Why is it important for the students to know an experimental approach to probability? Theoretical probability offers an unbiased objective approach to probability that the subjective approach cannot. However, it is not always possible to find a theoretical probability of a given situation. Experimental probability offers another objective way to find probability, but it also has it own conditions; the experiment has to be performed naturally (e.g., spinner has to be freely spun, dice have to be tossed in a way that they independently roll, etc.), and the trial number has to be large.

Since the test was not designed to capture whether the students understood the experimental probability and its conditions, there was no evidence from the test to support claims about student understanding. However, the incidents in the classroom where the teachers demonstrated sampling methods using the un-identical objects showed that even the teachers were not aware of the conditions of experimental probability. Also, the interview with Aj. Kim showed that she did not understand these conditions. As when solving item 8, she said the probability that the arrow would land on either side of the spinner is not 1/2 but depends on where the arrow starts and how the person makes the spin.

Misconceptions as Addressed During the Lessons

The previous four issues discussed were aspects of probability topics and how they were addressed during the lessons. This last section will directly discuss how the five main misconceptions tested on the tests and the two emerging misconceptions were addressed during the lessons and the impacts they made on the students' test performances.

Representativeness. There were three items (items 1, 2, and 4) in two contexts (coin and number) that tested the representativeness misconception. Before the lesson, students seemed to be more familiar with the coin context than the number context as more students answered items 1 and 2 correctly than item 4 on the pretest. During the lesson, coin context was used repeatedly throughout the lessons as each teacher used coin contexts as both examples and exercises. On the other hand, students did not get to experience the number context as it was not a context of any example or exercise during the lessons. Moreover, the concept of representativeness itself had not been discussed, since the two opportunities to discuss the idea (during Unit 1 and the experimental probability section of Unit 3) as presented in the intended curriculum were both skipped. As a result, students improved more on the coin situation than the number situation. On the posttest, almost all of the students (96.9%) were able to answer item 1 correctly and 90.3% (23.6% improvement) were able to answer item 2 correctly. On the other hand, fewer than half of the students (42.1%) were able to answer item 4. Similarly, the teachers were also more familiar with the coin context as they both answered items 1 and 2 correctly. However, only Aj. Kim was able to correctly answer item 4.

Positive and negative recency effects. Item 3 tested the positive and negative recency effects misconceptions and it was also in a coin context. 56.9% of the students answered this item correctly on the pretest. Similar to items 1 and 2, the context was repeatedly used in the lessons but the misconception itself was not. As a result, students made some improvement on this item as 72.3% of the students answered this item correctly on the posttest. Interestingly, both teachers were not able to answer this item. However, the teachers' misconceptions did not seem to transfer to the students, since the misconceptions itself was not addressed during the lessons.

Conjunction fallacy. Items 5 and 6 tested the conjunction fallacy misconception. Students seemed to have difficulties with these two items as only 19.0% and 29.2% of the students were able to correctly answer items 5 and 6, respectively, on the pretest. Students had more difficulties with the item 5 context (where the person did not make his own decision) than the item 6 context (where the person made her own decision). The lesson failed to address these contexts and the misconception as Unit 1 was barely addressed. However, students made some improvement on these items, 20.5% and 16.4% respectively. Both teachers also demonstrated this misconception in both contexts. There was no evidence that the teachers' conjunction fallacy misconception impacted the students' misconception.

Compound and simple events. Items 7 and 8 tested the compound and simple events misconception. Most students came to the class with difficulties with this misconception. Only 15.4% and 28.7% of the students correctly answered items 7 and 8, respectively, on the pretest. Students seemed to have less understanding of the dice context (item 7) than of the spinner context (item 8). During the lesson, the dice context was used repeatedly as both teachers used it in many examples and exercises. The spinner context, on the other hand, was briefly discussed in Aj. Nan's classes and was not mentioned at all in Aj. Kim's. The misconception was discussed in all classes as the teachers gave examples of both simple events and compound events in several context (tossing coins, tossing dice, picking objects from a container, etc.). However, both teachers made some mistakes regarding this misconception when they gave an example in the exam context (see Figures 4-5 and 4-6, page 81). Even though the teachers' mistakes were on a different problem context than those on the tests, some students had shown the same mistake in their test response as discussed earlier in section 3 of this chapter.

As a result, students made their biggest improvement on item 7 (32.8%). However, there was still more than half (51.8%) of the students who did not get this item right. Item 8, on the other hand, only had an 11.8% improvement, which was on the smaller side compared to other items. However, the fact that Aj. Nan's students had had more experience than Aj. Kim's students dealing with the spinner during their lessons did not seem to have an effect on the students' improvement. Class C's and Class D's improvement on item 8 was not better than that of Class A and Class B.

Aj. Nan demonstrated a correct understanding of this misconception as she was able to correctly answer both item 7 and item 8. However, the mistake she made during her lessons showed this misconception (see Figures 4-5 and 4-6, pages 81). This finding means that the inconsistency in participants' answer and reasoning across context does exist in the compound and simple events misconception, other than the representativeness misconception.

Effect of the time axis. Item 10 tested the effect of the time axis misconception. Most students came to the class with a difficulty with this misconception (20.5% correct on the pretest). During the lessons, both teachers spent quite a lot of time (roughly one period for each class) addressing the context of picking objects from a container. Even though the misconception itself was not discussed, the items could be solved using the knowledge learned in class. Interestingly, students did worse on this item on the posttest than on the pretest (14.4% correct). One reason could be because the posttest item was more difficult than the pretest item. However, students' responses on this item showed that they tried to solve this item using the knowledge learned, only incorrectly. Aj. Kim did not provide an explanation for her item 10 answer. Aj. Nan first gave a misconception answer and reasoning before changing to a correct answer and reasoning during the interview. It seemed that the teachers and the students both have difficulties with this misconception.

Uncertainty. Students usually gave uncertainty reasoning when they approached probability subjectively. On the pretest, students gave an uncertainty explanation as 8.5% of the total responses. The lessons did not address this misconception as students naturally should be less likely to use the uncertainty reasoning as they move toward approaching probability theoretically. Unfortunately, students did not have an opportunity to learn and be aware of their subjective probability, as discussed in the first section of this chapter. Therefore, many students still gave subjective probability answers and uncertainty reasoning remained virtually unchanged (7.2% of the total response on the posttest). There was no evidence of the teachers' uncertainty misconception.

Outcome approach. Similar to the uncertainty reasoning, students usually used the outcome approach when they could not (or did not realize they should) solve the problem theoretically. The answers that resulted from an outcome approach, in most cases, came with incorrect reasoning or no reasoning at all. The number of responses on the test that resulted from an outcome approach was unattainable since there is no sure way to tell if an answer resulted from an outcome approach. However, students should be able to give more appropriate explanations for their answer as they move toward approaching probability theoretically. However, the 58.3% correct answers on the posttest only came with 30.1% correct explanations, indicating that many students may still have used an outcome approach on the posttest. Aj. Kim demonstrated using the outcome approach twice during the interview (when solving items 3 and 8). There was no instance that Aj. Nan used an outcome approach during the interview or in the lessons.

This chapter discusses how the lessons impacted the students' performances on the test, noting four aspects of the lessons: 1) they lacked the subjective approach to probability, 2) they emphasized too much about finding probability, 3) they contained the use of sample spaces with unequally likely elements, and 4) they did not contain an experimental approach to probabilities. The chapter also discusses how each misconception was or was not addressed during the lessons. The results showed that the lessons had both positive and negative impacts on the students' performances. Students' overall test scores improved, which meant that they gained more probability knowledge during the lessons. However, their low posttest scores indicated that the lesson may not have been as effective in helping students to deal with their misconceptions. Moreover, student still had a lot of difficulties applying their knowledge and giving appropriate reasoning for their answers.

Chapter 7 - Discussion

This research study was conducted with 204 grade 9 students and two mathematics teachers at a secondary school in Bangkok, Thailand. The purposes of the study were to explore Thai secondary school students' probability misconceptions, the implementation of a nationwide inquiry-based curriculum, and the impact instruction had on the students' understanding and concepts on probability topics. The results showed that both the teachers and the students held several probability misconceptions. Among the five probability misconceptions investigated, the effect of the time axis misconception was the most common among the participants. The teachers claimed they followed the intended curriculum, but they did so selectively, using only lecture and skipping many critical parts of the curriculum. Teachers also had some misunderstandings of the topics and made mistakes during the lessons. After instruction, the students' probability knowledge was improved, but they still held several misconceptions and were not able to appropriately explain their reasoning. The lessons had both positive and negative impacts on the students' performance on the test.

Results as Compared to Prior Research

Probability misconceptions. The students' misconceptions found in this study were quite similar to other studies such as Fischbein and Schnarch (1997), Konold et al. (1993), and Rubel (2007). The choices of the misconceptions investigated were a little different. This study did not investigate the effect of the sample size and the heuristic of availability misconceptions due to the scope of the curriculum; whereas, Konold et al. (1993), and Rubel (2007) did not investigate the conjunction fallacy and the effect of the time axis misconceptions. It was difficult to compare the percentages of the students who had these misconceptions in each study since the participants were varying in ages and education levels. In general, fewer students in this study had representativeness misconceptions than other research and, after instruction, fewer students also had compound and simple event misconceptions. On the other hand, more students in this study had the effect of the time axis misconception, both before and after instruction. Hence, Thai students may share similar probability misconceptions as students from other countries, but the percentages of students who have each misconceptions were different.

As measured by the pretest and the posttest, even though students' probability knowledge improved after instruction, the students' average scores were still low. This result suggests that Thai secondary school mathematics teachers' and students' probability knowledge still needed to be improved. There was very little previous research focused on the relationship between teachers' and students' probability misconceptions; however, the similarities between the teachers' and the students' responses on the test suggest that a relationship may exist. Both the teachers and the students seemed to have similar difficulties, misunderstandings, and misconceptions, whether or not they were made explicit during the lessons.

Probability misconceptions, variations across contexts. Probability misconceptions are not something that a person either holds or not, but can depend on context. When participants responded to probability misconception problems, their reasoning varied. Evidence from participants' responses on the probability misconception tests showed that a participant may use theoretical probability to solve one problem, and then use the subjective approach to solve another problem, whether each problem is testing the same misconception or not. There was evidence that participants still used the subjective approach in various situations even after instruction. Konold et. al. (1993) explained this variation using the term "availability of multiple frameworks" (p. 405). They defined three general frameworks for making probability judgments as: (a) the normative or formalized framework used by experts to compute probabilities, (b) the informal judgment heuristic used in everyday situations to arrive at quick assessments of probabilities, and (c) the single-trial-prediction framework of the outcome approach[18]. The researchers also stated "[i]nconsistencies would result ... if a subject switched among these frameworks in thinking about different aspects of the same situation (p. 405)". Konold et. al. (1993) only investigated these inconsistencies in one context (tossing coins). However, the results of this study show that the inconsistencies also exist when investigating across contexts (tossing coins and randomly picking objects from a container). Before instruction, the inconsistency may result from the participants shifting how they approach the problem. However, after instruction, the inconsistency, especially the across contexts one, could result from the fact that the students and the teachers were not able to appropriately apply knowledge in other situations.

Implementation of the curriculum. IPST's curriculum had several aspects that would promote students' learning and help them to address their misconceptions (Amir & Williams, 1999; Barnes, 1998; Hawkins, 1984; Jones et al., 2007). It integrated the three approaches to probability, encouraged experiments and discussion, and emphasized making predictions and decisions. Grade 9 students were also old enough according to Falk and Wilkening's (1998) and Fischbein and Gazit's (1984) studies to be able to effectively learn these topics.

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[18] I used the term subjective approach for both (b) and (c).

Despite the IPST's intention to promote inquiry-based learning, all the probability lessons in this study were taught in lecture format. The aspects of the curriculum that were intended to promote students' learning and help them to eliminate their misconceptions were skipped. Even though the class times were greatly reduced because of the natural disaster, the short lessons did not affect how the teachers implemented the curriculum. The teachers both said during their interviews that they were satisfied with their lessons and they only would have given students more examples if they had had more time. Therefore, the results should not be used to evaluate the effectiveness of the curriculum, since it was implemented differently than intended.

Contribution

This research study found that Thai students had similar probability misconceptions as students from other countries and that the teachers, even with many years of teaching experience on probability topics, had similar misconceptions as their students. There was also a connection between the teachers' misconceptions and the students' misconceptions. After instruction, the students improved more on the concepts where the teachers had correct understanding and had smaller improvement or negative improvement on the concepts in which the teachers had incorrect understanding.

The subjective approach played an important role in the participants' responses to the probability misconceptions tests. There was evidence that the students (and teachers) still used the subjective approach to answer the test items after instruction. This subjective approach was varied in degree across problems and problem contexts. The teachers and the students seemed to be unaware that they sometimes used the subjective approach, which made it more difficult to address their misconceptions.

Students had difficulties not only with answering probability questions, but also with explaining their reasoning. Instruction improved both the students' answers and reasoning on the test, but students were still more likely to answer an item correctly than to give appropriate reasoning. After instruction, more students attempted to use a theoretical approach in solving the problems, but many still could not apply their knowledge properly. Instruction of probability topics needs to better help students to be able to explain their reasoning and correctly apply their knowledge in solving problems.

This study also examined how a real curriculum was implemented in regular classrooms. The participants received no special treatment except from the tests, hence the results offered a picture of what real teaching of probability topics in Thailand looked like. Even though the students received fewer periods of instruction than what should have been, the lesser amount of instruction time did not seem to affect how the lessons were taught.

In summary, this study made four major contributions to the literature. It showed that; (1) there was a connection between the teachers' and the students' probability misconceptions, whether or not the misconceptions were addressed during the lessons, (2) after instruction, both teachers and students still sometimes used a subjective approach, and they were unaware of it, (3) students struggled in explaining their reasoning and applying their knowledge, and (4) the inquiry-based curriculum was not currently implemented as intended.

Limitations and Future Research

Due to the nature of the research and uncontrollable aspects of it, there were several limitations to this study. First, the natural disaster greatly reduced instruction time and might have had an impact on how much content the teachers could cover in their lessons. Second, the test responses might not be accurate. Because the students took the tests in their regular classroom under a usual classroom setting, they might have looked at each others' responses. Third, the teachers were unfamiliar with being studied. They seemed uncomfortable during the interviews, especially when the questions were probing their own understanding or the understanding of their students. As a result, the interviews were not as informative as they could have been.

This study showed that teachers and students held several probability misconceptions. Instruction that focused only on the theoretical approach to probability and procedural understanding made insufficient improvement on students' understanding of probability topics. There was also a connection between the teachers' and the students' misconceptions. Future research needs to examine this relationship in more detail. Knowing how the teachers' misconceptions influence the students' learning would help educators to design more effective curriculum and more effective professional development materials. A better understanding about how the participants' inconsistencies in their answers and reasoning play a role when solving probability related problems within and across contexts is also necessary. Researchers need to find better ways to help students deal with their misconceptions, gain more conceptual understanding of the topics, support teachers to implement a curriculum as intended and raise the teachers and students' awareness of their own probability misconceptions.

Implications

Curriculum redesign. Since the curriculum was not implemented as intended, this study shuld not be used to evaluate the effectiveness of the IPST's curriculum. However, understanding how the teachers implemented the curriculum could help in redesigning the curriculum. The teachers followed parts of the curriculum, but they chose to teach some topics or parts of topics and skipped others. It seemed that the teachers only looked at the textbook but not the teacher manual, since they missed most of the issues discussed in the teacher manual. They also assigned homework from the textbook, but only the parts that were labeled as "exercise". Therefore, situations and activities where students get to practice making predictions and decisions were not used. The curriculum may be implemented more effectively if the teachers had a manual that combined the current student textbook and the current teacher manual. Teachers may also be more likely to assign those activities to the students if they all were labeled as exercises.

Teacher education and professional development. The results showed that the teachers had several misconceptions and misunderstandings on probability topics and these misconceptions and misunderstandings were transferred to the students during instruction, or at least did not facilitate deeper students' understanding of probability. Teacher education and professional development programs need to be designed to address such problems. This study identified several kinds of probability misconceptions and misunderstandings that the teachers had. Teacher education programs should be redesigned so that pre-service teachers examine their misconceptions and gain deeper understanding of probability topics and professional development programs could help in-service teachers to (1) eliminate persistent misconceptions and (2) deepen their understanding as well.

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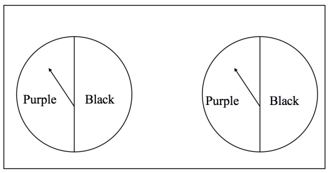
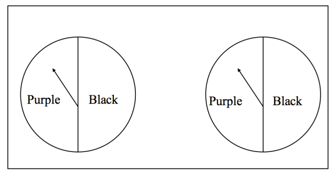
Appendix A

The Pretest, the Postest, and Their Comparability

Pretest	Posttest	Comparability
1. Let H stands for the event of getting a head in tossing a coin and T stands for the event of getting a tail. Toss one coin five times. Which of the following is most likely to happen? Explain. a. HTHTH b. HHHTT c. THHTH d. THTTT e. All are equally likely	1. Let H stands for the event of getting a head in tossing a coin and T stands for the event of getting a tail. Toss one coin five times. Which of the following is most likely to happen? Explain. a. HTHTH b. HHHTT c. THHTH d. THTTT e. All are equally likely	Same level of mathematical difficulty. Questions vary in selection of multiple choices response. Both test Representativeness misconception.
2. From 1. Which of the following is least likely to happen? Explain. a. HTHTH b. HHHTT c. THHTH d. THTTT e. All are equally likely	2. From 1. Which of the following is least likely to happen? Explain. a. HTHTH b. HHHTT c. THHTH d. THTTT e. All are equally likely	Same level of mathematical difficulty. Questions vary in selection of multiple choices response. Both test Representativeness misconception.
3. In tossing a coin five times, the coin showed heads the first four times. Which is more likely to happen the fifth time? Heads or tails? Explain.	3. A token was painted red on one side and blue on the other side. If I toss the token four times and red came up all four times, which is more likely to happen the fifth time? Red or blue? Explain.	Same level of mathematical difficulty. Superficial different in coin versus token. Both test positive and negative recency effect misconceptions.
4. In a 2-digit lottery game, Tor buys 21, 22, 23, 24, and 25. Tan buys 17, 38, 62, 59, and 84. Who is more likely to win this game? Explain. a. Tor b. Tan	4. An urn contains 100 small cards with number 00 – 99 written on them. If you pick 5 cards out of the urn, which group of numbers is more likely to come up? Explain. a. 11, 22, 33, 44, and 55 b. 03, 49, 67, 81, and 92	Both involve randomly pick 5 numbers out of 100. Lottery game versus numbered cards in an urn. Different selections of choices, both with one pattern and one non-pattern. The lottery situation was remove to accommodate participants who not familiar with lottery. Both test Representativeness misconception.

Pretest	Posttest	Comparability
5. 43 year-old Sonny is a very heavy smoker. Lately, he has been suffering from constant chest pain and cough. So, he decides to see a doctor. Which of the following events has the higher probability? Explain. a. Sonny has lung cancer. b. Sonny has cancer.	5. Ben had just graduated from college, majoring in accounting. He applies for a job at two different accounting companies. Company A has 3 opening jobs. Company B has 5 opening jobs. Which of the following events has the higher probability? Explain. a. Ben gets a job at company B. b. Ben gets a job.	Cancer versus job application. In both case the person does not make his own decision. Selections of choices compose of one specific and one general. Both test conjunction fallacy misconception.
6. Jane likes sweet fruit. She walks to a fruit store with 50 bahts (Thai unit of money). The store sells three kinds of fruit. The types of the fruit and prices per kilogram are; guava 40 bahts, longan 45 bahts, and mandarin orange 60 bahts. If the minimum amount of fruit to be bought is one kilogram, which of the following events has the highest probability? Explain. a. Jane buys guava. b. Jane buys longan. c. Jane buys mandarin orange. d. Jane buys fruit.	6. May likes to drink iced milk. She goes to a beverage store that sells four kinds of beverage; iced milk, hot milk, iced chocolate, and hot chocolate. Which of the following events has the highest probability? Explain. a. May buys iced milk. b. May buys hot milk. c. May buys iced chocolate. d. May buys hot chocolate. e. May buys a beverage.	Buying fruit versus buying beverage. The person makes her own decision in both items. Selections of choices compose of multiple specifics and one general. The items' prices were removed to make the item less complicated. Both test conjunction fallacy misconception.
7. In tossing 2 dice once, which outcome is more likely to occur? Explain. a. One die shows 5 and the other die shows 6. b. Both of the dice show 6. c. Both outcomes are equally likely.	7. In tossing 2 dice once and considering the sum of the numbers, which outcome is more likely to occur? Explain. a. The sum equal 11. b. The sum equal 12. c. Both outcomes are equally likely.	Same situation, different points of interest; the numbers shown on the dice versus sum of the numbers. Selection of multiple choices changed based on the point of interest. Both test compound and simple events misconception.

Pretest	Posttest	Comparability
8. A game consists of spinning two fair spinners (see diagram). A player wins only when both arrows land on purple, otherwise he or she loses. Does a player have a 50-50 chance of winning this game?  a. Yes. Why? b. No. Why?	8. A game consists of spinning two fair spinners (see diagram). A player wins only when the arrow on the left spinner lands on purple and the arrow on the right spinner lands on black, otherwise he or she loses. Does a player have a 50-50 chance of winning this game?  a. Yes. Why? b. No. Why?	Same situation, different points of interest; both arrows land on purple versus the arrow on the left spinner lands on purple and the arrow on the right spinner lands on black. Both test compound and simple events misconception.
9. An urn only contains two yellow marbles and two green marbles. Randomly pick one marble out of the urn. What is the probability that the marble is green? Show your work.	9. A box only contains three red chips and three blue chips. Randomly pick one chip out of the box. What is the probability that the chip is blue? Show your work.	Superficial different in marbles and urn versus chips and box. The number of item increased from two each to three each.
10. Using the same urn from 9 (with all four marbles in it). You picked out one marble and put it aside without checking the color. Then, you picked another marble and found that this second marble is green. What is the probability that the first marble you picked is also green? Show your work.	10. Using the same box from 9 (with all six chips in it). You picked out one chip and put it aside without checking the color. Then, you picked another chip and found that this second chip is blue. What is the probability that the first chip you picked is also blue? Show your work.	Superficial different in marbles and urn versus chips and box. The number of item increased from two each to three each. Both test the effect of the time axis misconception.

Appendix B

Interview Protocol for Teachers

The teacher participants were asked the following questions during the interview. All interviews followed the guide but the questions might had been modified when necessary. The questions did not need to be asked in the same order and more probing questions were generated during the interview process.

Interview 1. Before Probability Lessons

Background information.

1. Please tell me about your teacher education and past experience as a teacher.

Probing Questions: Where did you study your teacher education program? What was your emphasis? Had you taught at other school before? Where? How long? What level? How long have you been teaching in this school? What other level do you teach? What are your professional development experiences?

Experience about probability as a learner.

2. Please tell me about your learning experience on the probability topics.

Probing Questions: Where did you first learn about probability? What do you remember about how your teacher(s) taught the topics? How did it support or hinder your learning? How confident are you about your own knowledge?

Experience about probability as a teacher.

3. Please tell me about your teaching experience on the probability topics.

Probing Questions: How long have you been teaching probability topics for this level? Have you taught the topics at other level? Which one? How long? What are your professional development experiences on probability topics? How did the experiences impact your teaching? How would you describe the value of teaching probability?

Concepts and understanding about students.

4. Please describe the students in the two classes that will participate in the research.

Probing Questions: What are their learning behaviors? How do their learning behaviors compare to another class? How different is your teaching plans between the two classes?

Lessons and activities plan.

5. Please tell me about your teaching plan for this semester.

Probing Questions: How do you prepare yourself in teaching the topics? What do you think is the most important aspect of your lesson plan? What do you expect the students to receive from the lessons? What are the main ideas/concepts that you want to pass on to the students? How will you assess your students' learning?

Interview 2. After probability lessons

Lesson/activity reflection.

1. Overall, what do you think about the probability lesson?
2. What do you think work well? What do you think did not work well? What are the differences between your two classes?
3. The next time you teach this topic, how would you adjust/improve the lessons?
4. Please describe what you think the students had learned from the lessons compare to what you expected?

Probability misconceptions.

5. What do you think are challenging in the problem?
6. How do you predict your students would response on the test?

Appendix C

Example of Lesson Map

Class: A Period: 2 of 4 Time: 35:00 minutes

Time	Activity	Content and Structure
0:00	Homework solutions	The teacher orally gave out homework solutions, from textbook exercise 2.2, items 1- 4. The teacher stood in front of the class with a copy of textbook in her hand, read the homework problems out loud, then gave the answer of item 1. Students sat in their seat, looked at their notebook, checked their work. The teacher wrote a tree diagram associates with item 2 on the board, then marked the answer for each question. Teacher then orally gave answers to items 3 and 4.
7:53	Lecture on probability of events	The teacher stood in front of the class, lecture while writing on the board. Students sat in their seat, listened and copied notes from the board to their notebook. *Probability of an event, p(E).* *The number of all possible outcomes, n(S).* *The number of outcomes, n(E).* $p(E) = n(E)/n(S)$.* The teacher gave examples about the number of students in the class. There were 55 students in the class, therefore $n(S) = 55$. There were 28 female students, therefore $n(E) = 28$ and $p(E) = 28/55$. <u>Note: The event was not clear. There was no explanation of what really was p(E) in this case (e.g., the teacher could had explained that if someone randomly selects a student from this class, n(S) = 55, the probability that the selected student was a female student, n(E) = 28, was p(E) = 28/55).</u>** Then the teacher asked what if we interested in students who wore glasses, and the class counted the amount of 13 students. The teacher concluded that the probability of finding a student who wear glasses in this class was 13/55. Then the teacher gave an example on dice. *Tossing 1 dice 2 times* (Orally - same as 2 dice once) *The possible outcomes were:*

Time	Activity	Content and Structure
		(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), *(2,1), (2,2), (2,3), (2,4), (2,5), (2,6),* *(3,1), (3,2), (3,3), (3,4), (3,5), (3,6),* *(4,1), (4,2), (4,3), (4,4), (4,5), (4,6),* *(5,1), (5,2), (5,3), (5,4), (5,5), (5,6),* *(6,1), (6,2), (6,3), (6,4), (6,5), (6,6).* *n(S) = 36.* *1) Probability of getting the same number.* *(1,1), (2,2), (3,3), (4,4), (5,5), (6,6).* *n(E) = 6.* *p(E) = n(E)/n(S) = 6/36.* *2) Probability of getting the sum of even number.* The teacher paused for the students to catch up on the note and attempt to solve the problem. The teacher marked the ordered pairs with even sum in the above sample space. *n(E) = 18.* *p(E) = 18/36 = 1/2.* *3)Probability of getting the sum of 13.* *n(E) = 0.* *p(E) = 0/36 = 0.*
19:20	Individual exercises	The teacher instructed students to open the textbook to page 54 and work on exercise 2.3 items 1 and 2. She then circled the classroom to maintain students' focus on work.
35:00	End of period	

*The teacher' notes from the board are in italic.
**The researcher' notes are underlined.

Appendix D

The Probability Misconceptions Tests Results by Class

Table D-1

Item 1: Number (and Percentage) of Students' Answers and Reasoning on Pretest and Posttest

	Pretest					Posttest				
Class (N)	A (50)	B (48)	C (54)	D (43)	Total (195)	A (50)	B (48)	C (54)	D (43)	Total (195)
Answer										
a.	2 (4.0)	4 (8.3)	3 (5.6)	5 (11.6)	14 (7.2)	0	1 (2.1)	1 (1.9)	1 (2.3)	3 (1.5)
b.	1 (2.0)	1 (2.1)	0	0	2 (1.0)	0	0	1 (1.9)	1 (2.3)	2 (1.0)
c.	1 (2.0)	7 (14.6)	4 (7.4)	0	12 (6.2)	0	1 (2.1)	0	0	1 (0.5)
d.	0	0	0	0	0	0	0	0	0	0
e. (Correct)	45 (90.0)	36 (75.0)	47 (87.0)	38 (88.4)	166 (85.1)	50 (100)	46 (95.8)	52 (96.3)	41 (95.3)	189 (96.9)
No Answer	0	0	0	0	0	0	0	0	0	0
Others	1 (2.0)	0	0	0	1 (0.5)	0	0	0	0	0
Reasoning										
1 (Correct)	9 (18.0)	2 (4.2)	7 (13.0)	2 (4.7)	20 (10.3)	17 (34.0)	5 (10.4)	7 (13.0)	13 (30.2)	42 (21.5)
2 (Incorrect or Incomplete)	14 (28.0)	5 (10.4)	5 (9.3)	1 (2.3)	25 (12.8)	8 (16.0)	15 (31.2)	9 (16.7)	10 (23.3)	42 (21.5)
3 (Equal Probability)	6 (12.0)	7 (14.6)	11 (20.4)	4 (9.3)	28 (14.4)	3 (6.0)	2 (4.2)	1 (1.9)	2 (4.7)	8 (4.1)
4 (Representative)	0	1 (2.1)	1 (1.9)	0	2 (1.0)	0	0	0	1 (2.3)	1 (0.5)
5 (Experiment)	1 (2.0)	7 (14.6)	0	0	8 (4.1)	0	0	0	0	0
6 (Uncertainty)	8 (16.0)	11 (22.9)	8 (14.8)	8 (18.6)	35 (17.9)	5 (10.0)	3 (6.2)	8 (14.8)	4 (9.3)	20 (10.3)
No Explanation	12 (24.0)	15 (31.2)	20 (37.0)	28 (65.1)	75 (38.5)	14 (28.0)	20 (41.7)	28 (51.9)	13 (30.2)	75 (38.5)
Others	0	0	2 (3.7)	0	2 (1.0)	3 (6.0)	3 (6.2)	1 (1.9)	0	7 (3.6)

Table D-2

Item 2: Number (and Percentage) of Students' Answers and Reasoning on Pretest and Posttest

	Pretest					Posttest				
Class (N)	A (50)	B (48)	C (54)	D (43)	Total (195)	A (50)	B (48)	C (54)	D (43)	Total (195)
Answer										
a.	1 (2.0)	4 (8.3)	1 (1.9)	5 (11.6)	11 (5.6)	0	1 (2.1)	0	1 (2.3)	2 (1.0)
b.	1 (2.0)	6 (12.5)	1 (1.9)	5 (11.6)	13 (6.7)	0	1 (2.1)	0	0	1 (0.5)
c.	0	1 (2.1)	0	0	1 (0.5)	0	0	0	0	0
d.	6 (12.0)	8 (16.7)	14 (25.9)	12 (27.9)	40 (20.5)	1 (2.0)	1 (2.1)	8 (14.8)	6 (14.0)	16 (8.2)
e. (Correct)	42 (84.0)	29 (60.4)	38 (70.4)	21 (48.8)	130 (66.7)	49 (98.0)	45 (93.8)	46 (85.2)	36 (83.7)	176 (90.3)
No Answer	0	0	0	0	0	0	0	0	0	0
Others	0	0	0	0	0	0	0	0	0	0
Reasoning										
1 (Correct)	2 (4.0)	2 (4.2)	5 (9.3)	1 (2.3)	10 (5.1)	9 (18.0)	2 (4.2)	5 (9.3)	6 (14.0)	22 (11.3)
2 (Incorrect or Incomplete)	0	1 (2.1)	1 (1.9)	0	2 (1.0)	1 (2.0)	4 (8.3)	0	1 (2.3)	6 (3.1)
3 (Equal Probability)	6 (12.0)	7 (14.6)	8 (14.8)	4 (9.3)	25 (12.8)	3 (6.0)	1 (2.1)	1 (1.9)	1 (2.3)	6 (3.1)
4 (Representative)	3 (6.0)	7 (14.6)	5 (9.3)	6 (14.0)	21 (10.8)	0	1 (2.1)	2 (3.7)	2 (4.7)	5 (2.6)
5 (Experiment)	0	1 (2.1)	0	0	1 (0.5)	0	0	0	0	0
6 (Uncertainty)	8 (16.0)	6 (12.5)	7 (13.0)	3 (7.0)	24 (12.3)	6 (12.0)	3 (6.2)	8 (14.8)	4 (9.3)	21 (10.8)
No Explanation	31 (62.0)	23 (47.9)	25 (46.3)	29 (67.4)	108 (55.4)	27 (54.0)	43 (70.8)	38 (70.4)	28 (65.1)	127 (65.1)
Others	0	1 (2.1)	3 (5.6)	0	4 (2.1)	4 (8.0)	3 (6.2)	0	1 (2.3)	8 (4.1)

Table D-3

Item 3: Number (and Percentage) of Students' Answers and Reasoning on Pretest and Posttest

	Pretest					Posttest				
Class (N)	A (50)	B (48)	C (54)	D (43)	Total (195)	A (50)	B (48)	C (54)	D (43)	Total (195)
Answer										
a. (Correct)	36 (72.0)	12 (25.0)	42 (77.8)	21 (48.8)	111 (56.9)	47 (94.0)	20 (41.7)	42 (77.8)	32 (74.4)	141 (72.3)
b. (Positive Recency Effect)	1 (2.0)	11 (22.9)	5 (9.3)	12 (27.9)	29 (14.9)	0	9 (18.8)	4 (7.4)	2 (4.7)	15 (7.7)
c. (Negative Recency Effect)	7 (14.0)	10 (20.8)	3 (5.6)	5 (11.6)	25 (12.8)	0	7 (14.6)	3 (5.6)	0	10 (5.1)
d.	5 (10.0)	13 (27.1)	3 (5.6)	3 (7.0)	24 (12.3)	3 (6.0)	10 (20.8)	2 (3.7)	5 (11.6)	20 (10.3)
No Answer	0	2 (4.2)	1 (1.9)	2 (4.7)	5 (2.6)	0	0	2 (3.7)	4 (9.3)	6 (3.1)
Others	1 (2.0)	0	0	0	1 (0.5)	0	2 (4.2)	1 (1.9)	0	3 (1.5)
Reasoning										
1 (Correct)	20 (40.0)	5 (10.4)	19 (35.2)	11 (25.6)	55 (28.2)	9 (18.0)	4 (8.3)	11 (20.4)	13 (30.2)	37 (19.0)
2 (Incorrect or Incomplete)	1 (2.0)	0	2 (3.7)	0	3 (1.5)	3 (6.0)	2 (4.2)	3 (5.6)	4 (9.3)	12 (6.2)
3 (Correct - Independence)	2 (4.0)	0	4 (7.4)	1 (2.3)	7 (3.6)	14 (28.0)	2 (4.2)	6 (11.1)	0	22 (11.3)
4 (Representativeness)	2 (4.0)	2 (4.2)	0	0	4 (2.1)	0	0	0	0	0
5 (Positive/Negative RE Misc.)	6 (12.0)	9 (18.8)	5 (9.3)	12 (27.9)	32 (16.4)	0	11 (22.9)	4 (7.4)	0	15 (7.7)
6 (Uncertainty)	8 (16.0)	8 (16.7)	3 (5.6)	8 (18.6)	27 (13.8)	12 (24.0)	6 (12.5)	15 (27.8)	7 (16.3)	40 (20.5)
No Explanation	10 (20.0)	21 (43.8)	13 (24.1)	11 (25.6)	55 (28.2)	11 (22.0)	19 (39.6)	13 (24.1)	18 (41.9)	61 (31.3)
Others	1 (2.0)	3 (6.2)	8 (14.8)	0	12 (6.2)	1 (2.0)	4 (8.3)	2 (3.7)	1 (2.3)	8 (4.1)

Table D-4

Item 4: Number (and Percentage) of Students' Answers and Reasoning on Pretest and Posttest

	Pretest					Posttest				
Class (N)	A (50)	B (48)	C (54)	D (43)	Total (195)	A (50)	B (48)	C (54)	D (43)	Total (195)
Answer										
a. (Incorrect)	24 (48.0)	14 (29.2)	12 (22.2)	19 (44.2)	69 (35.4)	3 (6.0)	2 (4.2)	2 (3.7)	3 (7.0)	10 (5.1)
b. (Misconception)	14 (28.0)	17 (35.4)	14 (25.9)	12 (27.9)	57 (29.2)	24 (48.0)	25 (52.1)	17 (32.5)	15 (34.9)	81 (41.5)
c. (Correct)	9 (18.0)	13 (27.1)	27 (50.0)	5 (11.6)	54 (27.7)	20 (40.0)	13 (27.1)	32 (59.3)	17 (39.5)	82 (42.1)
d.	0	0	0	0	0	2 (4.0)	1 (2.1)	1 (1.9)	4 (9.3)	8 (4.1)
No Answer	3 (6.0)	3 (6.2)	1 (1.9)	5 (11.6)	12 (6.2)	0	6 (12.5)	2 (3.7)	4 (9.3)	12 (6.2)
Others	0	1 (2.0)	0	2 (4.7)	3 (1.5)	1 (2.0)	1 (2.1)	0	0	2 (1.0)
Reasoning										
1 (Correct)	0	0	2 (3.7)	0	2 (1.0)	0	1 (2.1)	1 (1.9)	0	2 (1.0)
2 (Incorrect or Incomplete)	0	0	0	0	0	0	0	1 (1.9)	1 (2.3)	2 (1.0)
3 (Correct - equal chance)	5 (10.0)	3 (6.2)	15 (27.8)	2 (4.7)	25 (12.8)	12 (24.0)	6 (12.5)	17 (31.5)	9 (20.9)	44 (22.6)
4 (Representativeness)	32 (64.0)	27 (56.2)	22 (40.7)	24 (55.8)	105 (53.8)	25 (50.0)	20 (41.7)	16 (29.6)	8 (18.6)	69 (35.4)
5 (Compound and Simple Event Misc.)	3 (6.0)	0	1 (1.9)	0	4 (2.1)	0	0	0	0	0
6 (Uncertainty)	2 (4.0)	6 (12.5)	6 (11.1)	1 (2.3)	15 (7.7)	4 (8.0)	5 (10.4)	7 (13.0)	2 (4.7)	18 (9.2)
7 (Equal Probability - 50:50)	0	0	0	2 (4.7)	2 (1.0)	0	0	0	1 (2.3)	1 (0.5)
No Explanation	8 (16.0)	10 (20.8)	8 (14.8)	12 (27.9)	38 (19.5)	7 (14.0)	13 (27.1)	11 (20.4)	20 (46.5)	51 (26.2)
Others	0	2 (4.2)	0	2 (4.7)	4 (2.1)	2 (4.0)	3 (6.2)	1 (1.9)	2 (4.7)	8 (4.1)

Table D-5

Item 5: Number (and Percentage) of Students' Answers and Reasoning on Pretest and Posttest

	Pretest					Posttest				
Class (N)	A (50)	B (48)	C (54)	D (43)	Total (195)	A (50)	B (48)	C (54)	D (43)	Total (195)
Answer										
a. (Posttest-Correct)	39 (78.0)	41 (85.4)	42 (77.8)	33 (76.7)	155 (79.5)	34 (68.0)	27 (56.2)	40 (74.1)	17 (39.5)	118 (60.5)
b. (Pretest-Correct)	11 (22.0)	7 (14.6)	12 (22.2)	7 (16.3)	37 (19.0)	16 (32.0)	21 (43.8)	14 (25.9)	26 (60.5)	77 (39.5)
No Answer	0	0	0	2 (4.7)	2 (1.0)	0	0	0	0	0
Others	0	0	0	1 (2.3)	1 (0.5)	0	0	0	0	0
Reasoning										
1 (Correct)	0	0	0	0	0	1 (2.0)	0	0	0	1 (0.5)
2 (Incorrect or Incomplete)	0	0	0	0	0	2 (4.0)	2 (4.2)	9 (16.7)	3 (7.0)	16 (8.2)
3 (Correct - set and subset)	5 (10.0)	1 (2.1)	5 (9.3)	0	11 (5.6)	21 (42.0)	21 (43.8)	20 (37.0)	4 (9.3)	66 (33.8)
4 (Correct - Multiple occurrences)	4 (8.0)	4 (8.3)	6 (11.1)	2 (4.7)	16 (8.2)	0	0	0	0	0
5 (Incorrect - Mentioned Factor)	30 (60.0)	34 (70.8)	36 (64.8)	26 (60.5)	125 (64.1)	16 (32.0)	17 (35.4)	11 (20.4)	19 (44.2)	63 (32.3)
No Explanation	11 (22.2)	7 (14.6)	8 (14.8)	14 (32.6)	40 (20.5)	10 (20.0)	8 (16.7)	13 (24.1)	16 (37.2)	47 (24.1)
Others	0	2 (4.2)	0	1 (2.3)	3 (1.5)	0	0	1 (1.9)	1 (2.3)	2 (1.0)

Table D-6

Item 6: Number (and Percentage) of Students' Answers and Reasoning on Pretest and Posttest

	Pretest					Posttest				
Class (N)	A (50)	B (48)	C (54)	D (43)	Total (195)	A (50)	B (48)	C (54)	D (43)	Total (195)
Answer										
a.	3 (6.0)	16 (33.3)	15 (27.8)	12 (27.9)	46 (23.6)	28 (56.0)	26 (54.2)	22 (40.7)	28 (65.1)	104 (53.3)
b.	24 (48.0)	26 (54.2)	17 (31.5)	23 (53.5)	90 (46.2)	0	0	0	0	0
c.	0	0	0	0	0	0	1 (2.1)	0	1 (2.3)	2 (1.0)
d. (Pretest-Correct)	23 (46.0)	5 (10.4)	21 (38.9)	8 (18.6)	57 (29.2)	0	0	0	0	0
e. (Posttest-Correct)	-	-	-	-	-	22 (44.0)	21 (43.8)	32 (59.3)	14 (32.6)	89 (45.6)
No Answer	0	0	1 (1.9)	0	1 (0.5)	0	0	0	0	0
Others	0	1 (2.1)	0	0	1 (0.5)	0	0	0	0	0
Reasoning										
1 (Correct)	0	0	0	0	0	0	0	0	0	0
2 (Incorrect or Incomplete)	0	0	1 (1.9)	0	1 (0.5)	1 (2.0)	3 (6.2)	5 (9.3)	1 (2.3)	10 (5.1)
3 (Correct - set and subset)	6 (12.0)	2 (4.2)	10 (18.5)	1 (2.3)	19 (9.7)	15 (30.0)	15 (31.2)	19 (35.2)	8 (18.6)	57 (29.2)
4 (Correct - Multiple occurrence)	2 (4.0)	2 (4.2)	5 (9.3)	3 (7.0)	12 (6.2)	0	0	0	0	0
5 (Incorrect - Mentioned Factor)	25 (50.0)	34 (70.8)	25 (46.3)	22 (51.2)	106 (54.4)	25 (50.0)	26 (54.2)	20 (37.0)	21 (48.8)	92 (47.2)
No Explanation	17 (34.0)	10 (20.8)	13 (24.1)	17 (39.5)	57 (29.2)	9 (18.0)	4 (8.3)	9 (16.7)	13 (30.2)	35 (17.9)
Others	0	0	0	0	0	0	0	1 (1.9)	0	1 (0.5)

Table D-7

Item 7: Number (and Percentage) of Students' Answers and Reasoning on Pretest and Posttest

	Pretest					Posttest				
Class (N)	A (50)	B (48)	C (54)	D (43)	Total (195)	A (50)	B (48)	C (54)	D (43)	Total (195)
Answer										
a. (Correct)	14 (28.0)	3 (6.2)	5 (9.3)	8 (18.6)	30 (15.4)	29 (58.0)	13 (27.1)	27 (50.0)	25 (58.1)	94 (48.2)
b. (Incorrect)	0	2 (4.2)	1 (1.9)	1 (2.3)	4 (2.1)	0	1 (2.1)	1 (1.9)	1 (2.3)	3 (1.5)
c. (Misconception)	36 (72.0)	40 (83.3)	48 (88.9)	34 (79.1)	158 (81.0)	21 (42.0)	33 (68.8)	25 (46.3)	17 (39.5)	96 (49.2)
No Answer	0	0	0	0	0	0	1 (2.1)	1 (1.9)	0	2 (1.0)
Others	0	3 (6.2)	0	0	3 (1.5)	0	0	0	0	0
Reasoning										
1 (Correct)	3 (6.0)	1 (2.1)	1 (1.9)	0	5 (2.6)	17 (34.0)	3 (6.2)	13 (24.1)	5 (11.6)	38 (19.5)
2 (Incorrect or Incomplete)	6 (12.0)	0	5 (9.3)	0	11 (5.6)	3 (6.0)	0	5 (9.3)	1 (2.3)	9 (4.6)
3 (Correct - Compare events)	3 (6.0)	0	2 (3.7)	5 (11.6)	10 (5.1)	7 (14.0)	12 (25.0)	8 (14.8)	14 (32.6)	41 (21.0)
4 (Incorrect - Equal Probability)	8 (16.0)	6 (12.5)	12 (22.2)	1 (2.3)	27 (13.8)	4 (8.0)	2 (4.2)	5 (9.3)	0	11 (5.6)
5 (Incorrect - Uncertainty)	6 (12.0)	10 (20.8)	8 (14.8)	10 (23.3)	34 (17.4)	8 (16.0)	8 (16.7)	5 (9.3)	5 (11.6)	26 (13.3)
6 (Incorrect - representativeness)	1 (2.0)	1 (2.1)	0	0	2 (1.0)	0	0	0	0	0
7 (Incorrect - Comp. &Simple Events)	0	0	0	0	0	2 (4.0)	5 (10.4)	7 (13.0)	2 (4.7)	16 (8.2)
No Explanation	21 (42.0)	28 (58.3)	26 (48.1)	25 (58.1)	100 (51.3)	9 (18.0)	14 (29.2)	9 (16.7)	15 (34.9)	47 (24.1)
Others	2 (4.0)	2 (4.2)	0	2 (4.7)	6 (3.1)	0	4 (8.3)	2 (3.7)	1 (2.3)	7 (3.6)

Table D-8

Item 8: Number (and Percentage) of Students' Answers and Reasoning on Pretest and Posttest

	Pretest					Posttest				
Class (N)	A (50)	B (48)	C (54)	D (43)	Total (195)	A (50)	B (48)	C (54)	D (43)	Total (195)
Answer										
a. (Misconception)	29 (58.0)	37 (77.1)	30 (55.6)	29 (67.4)	125 (64.1)	21 (42.0)	31 (64.6)	23 (42.6)	15 (34.9)	90 (46.2)
b. (Correct)	15 (30.0)	9 (18.8)	23 (42.6)	9 (20.9)	56 (28.7)	27 (54.0)	10 (20.8)	26 (48.1)	16 (37.2)	79 (40.5)
No Answer	2 (4.0)	1 (2.1)	1 (1.9)	3 (7.0)	7 (3.6)	2 (4.0)	7 (14.6)	5 (9.3)	12 (27.9)	26 (13.3)
Others	4 (8.0)	1 (2.1)	0	2 (4.7)	7 (3.6)	0	0	0	0	0
Reasoning										
1 (Correct)	10 (20.0)	3 (6.2)	17 (31.5)	6 (14.0)	36 (18.5)	31 (62.0)	6 (12.5)	24 (44.4)	22 (51.2)	83 (42.6)
2 (Incorrect or Incomplete)	0	2 (4.2)	2 (3.7)	0	4 (2.1)	0	4 (8.3)	3 (5.6)	0	7 (3.6)
3 (Correct - Compare events)	0	0	3 (5.6)	1 (2.3)	4 (2.1)	0	0	0	0	0
4 (Incorrect - Equal Prob.)	24 (48.0)	18 (37.5)	23 (42.6)	18 (41.9)	83 (42.6)	17 (34.0)	22 (45.8)	11 (20.4)	12 (27.9)	62 (31.8)
5 (Incorrect - Uncertainty)	2 (4.0)	15 (31.2)	2 (3.7)	7 (16.3)	26 (13.3)	2 (4.0)	3 (6.2)	8 (14.8)	2 (4.7)	15 (7.7)
No Explanation	9 (18.0)	5 (10.4)	5 (9.3)	8 (18.6)	27 (13.8)	0	8 (16.7)	7 (13.0)	7 (16.3)	22 (11.3)
Others	5 (10.0)	5 (10.4)	2 (3.7)	3 (7.0)	15 (7.7)	0	5 (10.4)	1 (1.9)	0	6 (3.1)

Table D-9

Item 9: Number (and Percentage) of Students' Answers and Reasoning on Pretest and Posttest

	Pretest					Posttest				
Class (N)	A (50)	B (48)	C (54)	D (43)	Total (195)	A (50)	B (48)	C (54)	D (43)	Total (195)
Answer										
c. (Correct)	47 (94.0)	33 (68.8)	52 (96.3)	37 (86.0)	169 (86.7)	50 (100)	40 (83.3)	52 (96.3)	39 (90.7)	181 (92.8)
i. (Incorrect)	2 (4.0)	2 (4.2)	2 (3.7)	0	6 (3.1)	0	5 (10.4)	1 (1.9)	2 (4.7)	8 (4.1)
No Answer	0	12 (25.0)	0	3 (7.0)	15 (7.7)	0	1 (2.1)	1 (1.9)	1 (2.3)	3 (1.5)
Others	1 (2.0)	1 (2.1)	0	3 (7.0)	5 (2.6)	0	2 (4.2)	0	1 (2.3)	3 (1.5)
Reasoning										
1 (Correct)	9 (18.0)	3 (6.2)	8 (14.8)	2 (4.7)	22 (11.3)	30 (60.0)	20 (41.7)	27 (50.0)	17 (39.5)	94 (48.2)
2 (Incorrect or Incomplete)	7 (14.0)	1 (2.1)	3 (5.6)	0	11 (5.6)	2 (4.0)	2 (4.2)	2 (3.7)	3 (7.0)	9 (4.6)
3 (Correct - equal #of chips)	11 (22.0)	11 (22.9)	13 (24.1)	19 (44.2)	54 (27.7)	4 (8.0)	12 (25.0)	5 (9.3)	4 (9.3)	25 (12.8)
4 (Uncertainty)	1 (2.0)	2 (4.2)	1 (1.9)	0	4 (2.1)	0	0	0	0	0
No Explanation	22 (44.0)	30 (62.5)	29 (53.7)	22 (51.2)	103 (52.8)	14 (28.0)	12 (25.0)	20 (37.0)	19 (44.2)	65 (33.3)
Others	0	1 (2.1)	0	0	1 (0.5)	0	2 (4.2)	0	0	2 (1.0)

Table D-10

Item 10: Number (and Percentage) of Students' Answers and Reasoning on Pretest and Posttest

	Pretest					Posttest				
Class (N)	A (50)	B (48)	C (54)	D (43)	Total (195)	A (50)	B (48)	C (54)	D (43)	Total (195)
Answer										
b.	7 (14.0)	0	5 (9.3)	0	12 (6.2)	16 (32.0)	1 (2.1)	7 (13.0)	12 (27.9)	36 (18.5)
c. (Correct)	16 (32.0)	2 (4.2)	17 (31.5)	5 (11.6)	40 (20.5)	7 (14.0)	9 (18.8)	7 (13.0)	5 (11.6)	28 (14.4)
d.	0	0	0	0	0	0	6 (12.5)	9 (16.7)	5 (11.6)	20 (10.3)
i.	10 (20.0)	13 (27.1)	10 (18.5)	6 (14.0)	39 (20.0)	2 (4.0)	0	3 (5.6)	3 (7.0)	8 (4.1)
m. (Misconception)	13 (26.0)	20 (41.7)	17 (31.5)	19 (44.2)	69 (35.4)	17 (34.0)	13 (27.1)	21 (38.9)	9 (20.9)	60 (30.8)
No Answer	0	5 (10.4)	2 (3.7)	7 (16.3)	14 (7.2)	0	3 (6.2)	2 (3.7)	2 (4.7)	7 (3.6)
Others	4 (8.0)	8 (16.7)	3 (5.6)	6 (14.0)	21 (10.8)	8 (16.0)	16 (33.3)	5 (9.3)	7 (16.3)	36 (18.5)
Reasoning										
1 (Correct)	1 (2.0)	0	1 (1.9)	1 (2.3)	3 (1.5)	0	0	1 (1.9)	1 (2.3)	2 (1.0)
2 (Incorrect or Incomplete)	9 (18.0)	2 (4.2)	4 (7.4)	2 (4.7)	17 (8.7)	11 (22.0)	14 (29.2)	9 (16.7)	16 (37.2)	50 (25.6)
3 (Correct)	6 (12.0)	5 (10.4)	13 (24.1)	5 (11.6)	29 (14.9)	3 (6.0)	3 (6.2)	4 (7.4)	0	10 (5.1)
4 (Incorrect - Disregard 2nd chip)	5 (10.0)	3 (6.2)	8 (14.8)	2 (4.7)	18 (9.2)	1 (2.0)	0	1 (1.9)	0	2 (1.0)
5 (Incorrect - not consider the 2nd chip)	3 (6.0)	2 (4.2)	2 (3.7)	7 (16.3)	14 (7.2)	10 (20.0)	2 (4.2)	7 (13.0)	1 (2.3)	20 (10.3)
6 (Incorrect - Sampling without replacement)	2 (4.0)	0	4 (7.4)	1 (2.3)	7 (3.6)	14 (28.0)	0	0	1 (2.3)	15 (7.7)
7 (Inc. - both possible)	0	5 (10.4)	0	0	5 (2.6)	0	0	0	0	0
8 (Incorrect - Simple event with incorrect S)	2 (4.0)	0	0	0	2 (1.0)	0	2 (4.2)	2 (3.7)	1 (2.3)	5 (2.6)
9 (Incorrect - incorrect number of chip)	4 (8,0)	3 (6.2)	0	0	7 (3.6)	0	2 (4.2)	0	0	2 (1.0)
No Explanation	8 (16.0)	17 (35.4)	14 (25.9)	14 (32.6)	53 (27.2)	10 (20.0)	16 (33.3)	24 (44.4)	20 (46.5)	70 (35.9)
Others	10 (20.0)	11 (22.9)	8 (14.8)	11 (25.6)	40 (20.5)	1 (2.0)	9 (18.8)	6 (11.1)	3 (7.0)	19 (9.7)