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Choosing an Optimization Method for Water Resources Problems Based on the Features of Their Solution Spaces

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Abstract: One of the main challenges for solving complex water-resources optimization is choosing an appropriate solution method. An important feature of optimization problems is the convexity and extent of their solution spaces. The solution space is the set whose elements are all the reservoir releases that meet the optimization problem's constraints and are thus feasible. The solution space of optimization problems can be convex or nonconvex. This study presents a method for determining the convexity or nonconvexity of the optimization problem solution space. The convexity and the extent of the solution space for a water-supply and a hydropower-production reservoir operation problem are evaluated by the proposed method. It is shown that the solution spaces of the former and latter problems are convex and nonconvex, respectively. The dependence of the solution spaces of the two reservoir operation problems on changes in evaporation, water demand for the water-supply reservoir, power plant capacity (PPC) for the hydropower reservoir, dead storage, reservoir capacity, and reservoir inflow is evaluated. The results demonstrate that the generalized reduced gradient (GRG) method finds an optimal value faster and more accurately than does the genetic algorithm (GA) when solving the water-supply problem, and that the GRG search is trapped in a local optimum when solving the hydropower-production problem. **DOI:** 10.1061/(ASCE)IR.1943-4774.0001265. *© 2017 American Society of Civil Engineers.*

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Introduction

Optimization theory has found fertile ground for application in reservoir operation and design. Classical optimization methods that have been applied to reservoir-operation optimization include linear programming (LP), nonlinear programming (NLP), dynamic programming (DP), and stochastic dynamic programming (SDP). Houck (1979) and Kuczera (1989) proposed LP for reservoir operation. Chu and Yeh (1978) used duality and Lagrangian theory for a concave optimization problem that maximized daily energy production. Little (1955) and Stedinger et al. (1984) applied dynamic programming and stochastic dynamic programming to reservoir operation problems. Young (1967) reviewed LP, NLP, and DP methods for reservoir operation. Yeh (1985) reviewed NLP methods in reservoir problems.

Classic methods have proven effective in solving well-posed optimization problems. However, they have several limitations with respect to complex nonlinear problems with a large number of decisions. Most classic methods are gradient-based, which involves the approximation of partial derivatives numerically. They cannot solve discontinuous problems (Bozorg-Haddad et al. 2006; Jordehi and Jasni 2012). Classic methods may converge to local optima in problems with a nonconvex solution space (Ghosh and Dehuri 2004; Del Valle et al. 2009). Convergence to local optima may be caused by poor initial guesses of the solution (Bhattacharjya and Datta 2005).

Karatzas and Pinder (1993) applied the outer approximation method to solve groundwater management problems with a concave objective function and nonconvex feasible decision space, and argued that classical optimization algorithms for groundwater quality management cannot correctly solve nonconvex problems. Cai et al. (2001) relied on the generalized bender decomposition for nonconvex reservoir operation and salinity control optimizations. They concluded that, because of their scale and complexity, only specific algorithms can solve these problems.

More recently, evolutionary and metaheuristic algorithms have become effective tools in water resources systems research. East and Hall (1994), Oliveira and Loucks (1997), Wardlaw and Sharif (1999), and many others used genetic algorithms (GAs) to optimize reservoir operation. Mantawy et al. (2003) applied the simulated annealing (SA) algorithm for multireservoir operation. Bozorg-Haddad et al. (2008) introduced honey-bee mating optimization (HBMO) for calculating reservoir operational rules and assessed this algorithm's capabilities. Bozorg-Haddad et al. (2009) applied and evaluated the HBMO algorithm with the nonlinear, multimodal, irregular Fletcher-Powell function and a nonconvex hydropower optimization problem. Ostadrahimi et al. (2012) used multiswarm particle swarm optimization (MSPSO) for multireservoir system operation. Bozorg-Haddad et al. (2016) applied the biogeography-based optimization algorithm for optimal operation of the Karun 4 hydropower reservoir system in Iran. Giuliani et al. (2015) used the direct policy search method, nonlinear approximating networks, and a multiobjective evolutionary algorithm to design Pareto-approximate operating policies for a multipurpose water reservoir. Asgari et al. (2015) and Azizipour et al. (2016) reported the application of the weed optimization algorithm to reservoir operation. Garusi-Nejad et al. (2016) applied the firefly

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algorithm to irrigation supply and hydropower generation, and demonstrated the advantage of this algorithm.

A topic that has not been fully addressed in previous research concerning reservoir operation is the study of the characteristics of the solution space for the purpose of selecting appropriate solution methods. Yeh et al. (1980), Simonovic (1992), Wurbs (1993), and Labadie (2004) reported comprehensive reviews of optimization methods in water resources systems and concluded that there is not a single general optimization algorithm that works well for all problems. The choice of a suitable method depends on the problem's features, such as the type of objective function, the type of the solution space defined by the problem constraints (convexity or nonconvexity), and data availability (Bozorg-Haddad 2014).

This paper evaluates the convexity and extent of the solution spaces of two types of single-reservoir operation problems, one for agricultural and urban water supply and a second for hydropower generation. The evaluation of these two problems identifies patterns with which to assess whether or not the solution space is convex. Most of nonconvex optimization problems are complex and sometimes they are impossible to solve exactly in a reasonable time. Evolutionary algorithms are well suited to tackle nonconvex problems and are commonly implemented for that purpose. In contrast, most of the convex optimization problems can be solved with simpler, faster, and more-accurate methods. Therefore correct recognition of the solution space is helpful for determining suitable solution methods. In addition to determining convexity or nonconvexity of a solution space, this study evaluates the changes in state variables of selected reservoir operation problems by changing the values of key inputs such as evaporation, water demand, minimum and maximum storage, and power plant capacity (PPC), and assesses the effect of those changes on the convexity and extent of the solution space. Furthermore, the capacity of several optimization algorithms to solve the two example reservoir operation problems is evaluated with convex and nonconvex spaces.

Convexity

The feasible decision space of an optimization problem, which is called solution space, is the set of all the values that the decision variables can take without violating the constraints of the problem (Bozorg-Haddad 2014). The solution or solutions of an optimization problem are within the solution space. The solution space can be classified as convex or nonconvex. A convex set in two dimensions is a set of points such that, given any two points in that set, the line (or hyperplane in multidimensional spaces) joining them lies entirely within the set; otherwise, the set is nonconvex. Fig. 1 illustrates convex and nonconvex sets. Eq. (1) is the mathematical

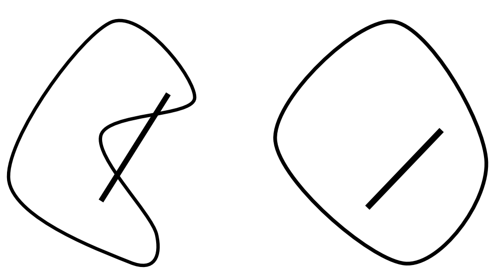

Fig. 1. Illustration of sets in two dimensions: (a) nonconvex; (b) convex

definition of a convex set A. If it is not satisfied, then the set is nonconvex

$$\forall\ X_1, X_2 \in A, \quad \forall\ \lambda \in [0,1], \qquad \lambda X_1 + (1-\lambda)X_2 \in A \tag{1}$$

where X_1 and X_2 = any two points of set A; $\lambda = 0-1$; and $\in$ = operator that establishes membership in a set; X_1 and X_2 are replaced by vectors in multidimensional spaces.

A convex optimization problem has convex constraint functions (the solution space is convex), and the objective function is a convex function under minimization or a concave function under maximization. The basic difference between convex and nonconvex optimization problems is that in convex optimization there are globally optimal solutions that can be found relatively straightforwardly without the solution algorithm converging to local optima. Nonconvex optimization may have multiple local optima, in which case it is difficult to ascertain the nature of any convergence point by a search algorithm. Many optimization methods do not perform well in nonconvex optimization. Linear functions are convex, so well-posed linear programming problems are convex problems that have globally optimal solutions.

Monte Carlo Method

The Monte Carlo method is a computational algorithm that implements a large number of iterations, each based on randomly generated numbers that initiate simulations of an arbitrary system. The system outputs derived from the many simulations are viewed as realizations of a stochastic process that can be analyzed with probabilistic methods. The Monte Carlo method is widely used for simulating many types of problems. The present study applies the Monte Carlo method for simulating and assessing convexity of the solution space in two types of reservoir operation problems explained in the "Methods and Materials" section.

Types of Optimization Problems

Optimization problems can be classified as linear or nonlinear. An optimization problem is linear when it involves objective functions, equations of the simulation model, and constraints that are linear and continuous. The problem is nonlinear if any constraint or the objective function(s) is nonlinear or discontinuous. Most water resources problems are nonlinear, such as the optimization of water quality, water distribution systems, and groundwater management. Nonlinear problems may be convex or nonconvex.

Implemented Optimization Methods

This study applies three optimization methods: the simplex method, the generalized reduced gradient (GRG) method, and the genetic algorithm. The simplex method was developed to solve linear programing problems (e.g., Hillier and Lieberman 2005). The GRG algorithm is a gradient-based optimization method for solving nonlinear optimization. The GRG method searches for optima close to the starting search point, whether local or global (Yeniay 2005; Lasdon et al. 1974). The GA is an evolutionary algorithm which is inspired by the process of natural selection of evolutionary biology. The GA starts with a generated initial population of possible solutions which are named chromosomes. These chromosomes are modified by genetic operators such as crossover and mutation that are applied to produce improved solutions. Those improved solutions with superior fitness are selected and modified (recombined and possibly randomly mutated) to generate a new population

(Fallah-Mehdipour et al. 2012). The algorithm generating populations of solutions and improving them is repeated until a specified search criterion is met, at which point the extant set of solutions is very near a global optimum. Genetic algorithms and other evolutionary algorithms have several advantages, such as local optimum avoidance, simplicity, and being derivation free. Because of their stochastic nature, they can escape local optima in contrast with classical methods. If an evolutionary algorithm is trapped in a local optimum, its stochastic operator helps the algorithm to escape the local optimum. In addition, evolutionary algorithms are generally inspired by natural concepts that are easy to understand and do not require the derivation of a mathematical model to reach a solution.

Reservoir System Operation Model

The mass balance equation for reservoir operation is

$$S_{t+1} = S_t + Q_t - R_t - SP_t - Loss_t t = 1, 2, 3, \ldots, T \quad (2)$$

where S_{t+1} = reservoir storage volume at the beginning of the $t+1$th period; S_t = reservoir storage volume at the beginning of the tth period; Q_t = inflow to the reservoir during the tth period; R_t = release from reservoir during the tth period; SP_t = volume of spill from the reservoir during the tth period; and $Loss_t$ = evaporative loss of volume during the tth period.

Losses are calculated by

$$Loss_t = Ev_t \times \bar{A}_t t = 1, 2, 3, \ldots, T \quad (3)$$

where Ev_t = depth of evaporation at the beginning and end of the tth period; and $\overline{A_t}$ = average reservoir surface area in the tth period

$$\bar{A}_t = \left(\frac{A_t + A_{t+1}}{2}\right) \quad t = 1, 2, 3, \ldots, T \quad (4)$$

where A_t and A_{t+1} = reservoir surface areas at the beginning of periods t and $t+1$, respectively; A_t depends on S_t and is obtained by

$$A_t = a_0 \times S_t + a_1 \times S_t^2 \quad t = 1, 2, 3, \ldots, T \quad (5)$$

where a_0 and a_1 = coefficients obtained from the water surface–volume curve.

The reservoir spill during the tth period is

$$SP_t = \begin{cases} S_t + Q_t - S_{\max} & \text{if } S_t + Q_t > S_{\max} \\ 0 & \text{if } S_t + Q_t < S_{\max} \end{cases} \quad t = 1, 2, 3, \ldots, T \quad (6)$$

where $S_{\max}$ = largest allowed reservoir storage.

Constraints on reservoir storage are expressed by

$$S_{\min} < S_t < S_{\max} \quad t = 1, 2, 3, \ldots, T \quad (7)$$

where $S_{\min}$ = minimum allowable reservoir storage.

The constraint on reservoir releases is

$$R_{\min} < R_t < R_{\max} \quad t = 1, 2, 3, \ldots, T \quad (8)$$

where $R_{\min}$ and $R_{\max}$ = smallest and largest releases allowed in the reservoir, respectively.

Eqs. (2)–(8) govern all reservoir systems regardless of their objectives.

Eq. (9) is a convex objective function for the reservoir system with the purpose of managing the water supply. It minimizes the sum of squared differences between reservoir releases and water demands. This paper uses Eq. (9) to test the GRG and GA methods

$$\text{Minimize } F(x) = \sum_{i=1}^{t} (De_t - R_t)^2 \quad t = 1, 2, 3, \ldots, T \quad (9)$$

where $F(x)$ = objective function; and De_t and R_t = water demand and reservoir release, respectively, during the tth period.

A linear objective function for the water-supply reservoir problem is defined by

$$\text{Minimize } F(x) = \sum_{i=1}^{t} |De_t - R_t| \quad t = 1, 2, 3, \ldots, T \quad (10)$$

Eq. (10) minimizes the sum of the absolute values of the differences between reservoir releases and water demands. If the reservoir storage does not satisfy its constraint [Eq. (7)], penalty functions defined in Eqs. (11) and (12) are added to the objective function to penalize the infeasible solution

$$P_{1,t} = \begin{cases} 0 & \text{if } S_{t+1} > S_{\min} \\ K_1(S_{\min} - S_{t+1})^2 & \text{Otherwise} \end{cases} \quad (11)$$

$$P_{2,t} = \begin{cases} 0 & \text{if } S_{t+1} < S_{\max} \\ (S_{t+1} - S_{\max})^2 & \text{Otherwise} \end{cases} \quad (12)$$

where $P_{1,t}$ and $P_{2,t}$ = penalty functions related to the violation of minimum storage in period t.

Reservoir Operation System Model for Hydropower Generation

Reservoirs with a hydropower supply function exhibit nonlinear properties that render optimization a complex task. This type of problem includes an equation for power generation in addition to the mass balance or continuity equation given previously

$$P_t = \frac{\gamma \times \eta \times \Delta H_t \times Qp_t}{Pf_t} \quad t = 1, 2, 3, \ldots, T \quad (13)$$

where P_t = hydropower generation in tth period; γ = specific weight of water (9.81 kN/m^3); η = efficiency of the powerhouse; ΔH_t = difference between the average water elevation at the penstock inlet and the average elevation of the powerhouse's tailwater during operating period t; Qp_t = discharge through the turbine in the tth period per m^3/s; and P_f = plant factor of the powerhouse during the tth period.

The released water is defined in units of volume and must be changed to a discharge in units of volume per second in Eq. (13). This conversion is

$$Qp_t = \frac{R_t}{CF_t} \quad t = 1, 2, 3, \ldots, T \quad (14)$$

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where R_t = volume of released water for hydropower generation in the tth period; and CF_t = unit conversion factor from million cubic meters to cubic meter per second during the tth period

$$CF_t = \frac{24 \times 3{,}600}{1{,}000{,}000} \times \text{day}_t \qquad t = 1, 2, 3, \ldots, T \tag{15}$$

where day_t = number of days during the tth operating period.

Eqs. (16)–(18) are used to calculate ΔH_t

$$\Delta H_t = \frac{H_t + H_{t+1}}{2} - EL_t \qquad t = 1, 2, 3, \ldots, T \tag{16}$$

$$H_t = b_0 + b_1 \times S_t + b_2 \times S_t^2 + b_3 \times S_t^3 + b_4 \times S_t^4 \qquad t = 1, 2, 3, \ldots, T \tag{17}$$

$$EL_t = m_0 + m_1 \times Q_{pt} + m_2 \times Q_{pt}^2 \qquad t = 1, 2, 3, \ldots, T \tag{18}$$

where H_t and H_{t+1} = water elevation at the penstock inlet at the beginning of the tth and $t+1$th periods, respectively; El_t = average elevation of the powerhouse's tailwater during the tth operation period; b_0, b_1, b_2, b_3, and b_4 = constant coefficients for converting reservoir storage to the corresponding elevation of the penstock inlet; and m_0, m_1, m_2, m_3, and m_4 = constant coefficients for converting discharge of water released from the reservoir to the tailwater elevation.

Eq. (19) is a constraint on hydropower generated in any operation period

$$0 < P_t < PPC \qquad t = 1, 2, 3, \ldots, T \tag{19}$$

where PPC = powerhouse plant capacity.

Eq. (20) is a convex objective function for the hydropower reservoir optimization problem

$$\text{Minimize } F(x) = \sum_{i=1}^{t} (P_t - PPC)^2 \qquad t = 1, 2, 3, \ldots, T \tag{20}$$

The penalty function for hydropower operation model is defined by

$$P_{3,t} = \begin{cases} 0 & \text{if } P_t < PPC \\ (P_t - PPC)^2 & \text{Otherwise} \end{cases} \tag{21}$$

where $P_{3,t}$ = penalty function that is added to the objective function to account for the violation of the hydropower constraint.

The objective functions of the two reservoir operation problems do not enter the analysis of the solution space. They serve solely as test cases for assessing the performance of the GRG and the GA algorithms.

Methods and Materials

The assessment of convexity or nonconvexity of the solution space involves simulating the decision space and the identification of the solution space. The decision space of two types of reservoir operation problems that are considered in this paper (reservoir operation problem for water supply and reservoir operation or hydropower generation) were simulated with the Monte Carlo method using *MATLAB*. Monte Carlo simulation was conducted by generating random values for 12 monthly reservoir releases based on each month's allowable range in a year in each iteration. The allowable bounds of release were defined between zero and demand. Each generated sequence of 12 releases (one for each of 12 months) constituted one point in the decision space. The decision space was simulated by executing a large number of Monte Carlo iterations. The solution space (or feasible decision space) was determined by selecting feasible sequences of releases from all of the generated sequences of releases. A generated point of the decision space (i.e., a sequence of 12 releases) was feasible if all of the reservoir system constraints were satisfied for the 12 generated release values [Eqs. (7) and (8) for the water-supply reservoir and Eq. (19) for the hydropower reservoir]. Thereafter, the values of the operational parameters of the two reservoir problems were modified and the convexity (or lack of it) of their solution spaces was evaluated. Lastly, the performance of the simplex, GRG, and GA optimization methods in convex and nonconvex solution spaces was tested. In summary, the methodology consisted of: (1) creation of solution spaces for the two reservoir operation optimization problems with Monte Carlo simulation; (2) analyzing the convexity or nonconvexity of the generated solution spaces, (3) generalizing the analysis of convexity by including the effect that the parameters of the reservoir operation problems had on the extent of the feasible solution space, and (4) assessment of the three chosen comparison optimization methods in solving problems with convex and nonconvex solution spaces.

Phase 1: Creation of Solution Spaces of Reservoir Operation Optimization Problems

The steps for developing the solution spaces of the two reservoir optimization problems are as follows:

1. Generate a sequence of random values (12 random numbers in each Monte Carlo iteration) using standard uniform distribution for the reservoir releases within an allowable range which is defined between zero and monthly demand values.
2. Enter the sequence of generated random releases as an input to the reservoir simulation process and claculate the storage values for each month (for hydropower simulation, the generated hydropower values are also calculated in each iterations).
3. Assess storage volumes (by verifying that the storage volumes are within the allowable range) for all 12 months of the year to ensure that the 12 generated releases are feasible, in which case the set of releases is feasible; otherwise, it is an infeasible set of releases (as for hydropower, the constraint of *PPC* is also verified).
4. Save the 12 values of each sequence of generated releases if they are all feasible; this sequence of releases is saved as a point of the solution space.
5. Calculate the ratio of the number of generated feasible sets of releases to the total number of generated sets of releases. This ratio produces the percentage of generated sets of feasible releases and measures the extent of feasible releases among all of the generated releases.
6. Repeat Steps 1–5 until increasing the number of Monte Carlo iterations does not appreciably lead to a change in the percentage of calculated feasible sets of releases. In other words, this procedure is repeated until the percentage of feasible sets of releases converges to a steady number. The solution space is identified after completing Phase 1.

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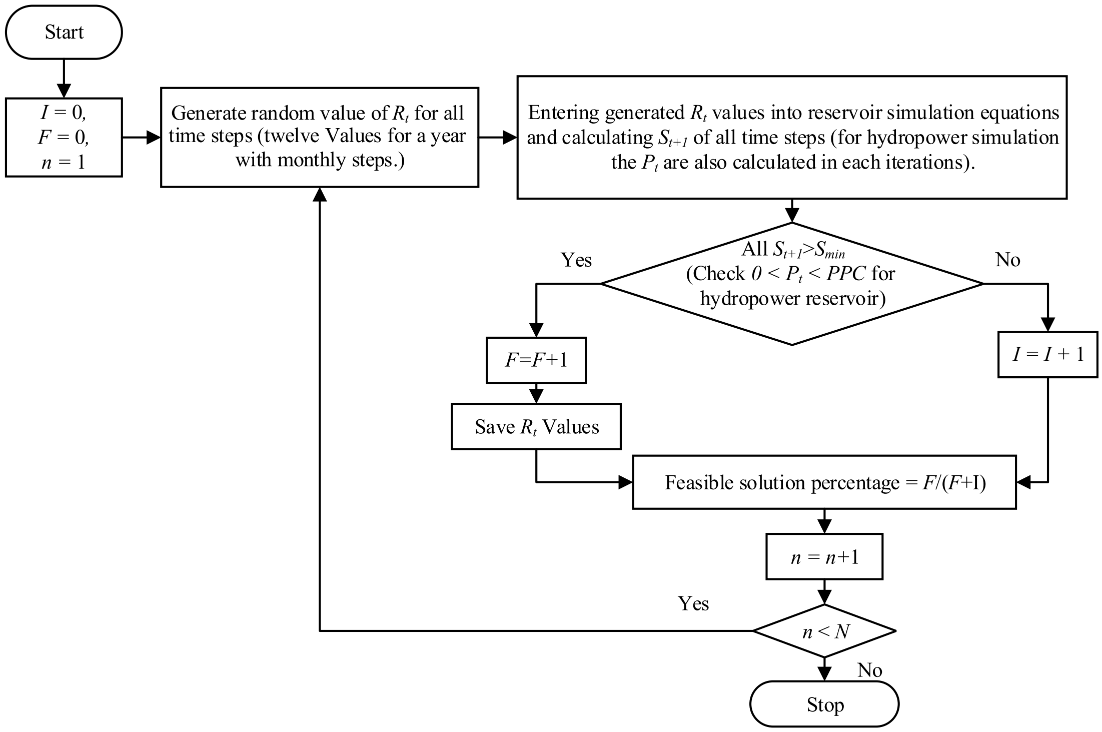

Fig. 2. Illustration of the generation of the decision space and the determination of the solution space

Phase 2: Analyzing Convexity or Nonconvexity of Solution Space

The solution space is convex if all the points located between any two points of the space lie within that space. The analysis of convexity follows these steps:

1. Randomly choose two feasible sets of releases from Phase 1, where each feasible sets comprises 12 monthly reservoir releases;
2. Interpolate, and generate a point (or set of solutions) on the line (or hyperplane) that connects the two feasible sets of releases [Eq. (1)];
3. Verify the feasibility of the interpolated value as explained in Phase 1;
4. Determine the percentage of interpolated feasible sets of releases among the total number of interpolated sets;
5. Repeat Steps 1–4 according to the number of Monte Carlo iterations;
6. If the final percentage calculated in Step 4 equals 100, all the interpolated sets of releases are feasible and the feasible solution space is convex; otherwise it is nonconvex.

Phase 3: Generalizing Analysis of Convexity by Including Effect of Parameters of Reservoir Operation Problems on Extent of Feasible Solution Space

Multiple states of the water-supply and hydropower-generation reservoir operation problems must be created to test the convexity or their solution spaces. This is accomplished by assigning different values to the problems' operational parameters, in which each value of the parameters defines a particular system state. The steps of Phases 1 and 2 are executed for each value of the parameters to cover all possible reservoir system states. This is a sensitivity analysis coupled with Monte Carlo simulation that allows a comprehensive assessment of the feasible solution spaces and their convexity for all possible system states. The parameters of the reservoir problem are the volume of evaporation, water demand for water supply, PPC for hydropower generation, dead storage, reservoir capacity, and reservoir inflow.

Fig. 2 shows how the decision space is generated and the solution space identified by selecting feasible values, where I = number of infeasible solutions, F = number of feasible solutions, n = iteration counter, and N = number of allowable iterations.

Fig. 3 illustrates the algorithm for assessing the convexity of a solution space, where M = number of infeasible solutions, L = number of feasible solutions, k = iteration counter, and K = number of allowable iterations.

Phase 4: Assessment of Different Optimization Methods and Comparison of Performance of GRG Algorithm and GA with Convex and Nonconvex Solution Spaces

The simplex method solves linear optimization problems, which in this case requires assuming that (1) evaporation can be ignored, because its value is small compared with releases, and (2) the volume of water in the reservoir does not exceed the reservoir capacity. Therefore, $Loss_t$ and SP_t were excluded from Eq. (2) and the water-supply reservoir operation problem was solved with a linear objective function [Eq. (10)]. Eliminating the $Loss_t$ and SP_t parameters is a simplification implemented to demonstrate the performance of the simplex method with a type of linear problem. The water-supply and hydropower-generation problems were solved with the convex objective function under minimization [Eqs. (9) and (20)] for 1 year (12 months) by means of the GRG and GA and their results were compared. The GRG method requires the specification of an initial starting guess of a solution, whereas the GA is initiated with a starting population of possible solutions. The GRG's initially guessed solution was specified in two ways: (1) generated randomly, and (2) set equal to zero for comparison purposes. The initial population of the GA's search was also specified in two ways, that is, randomly and by setting it equal to zero.

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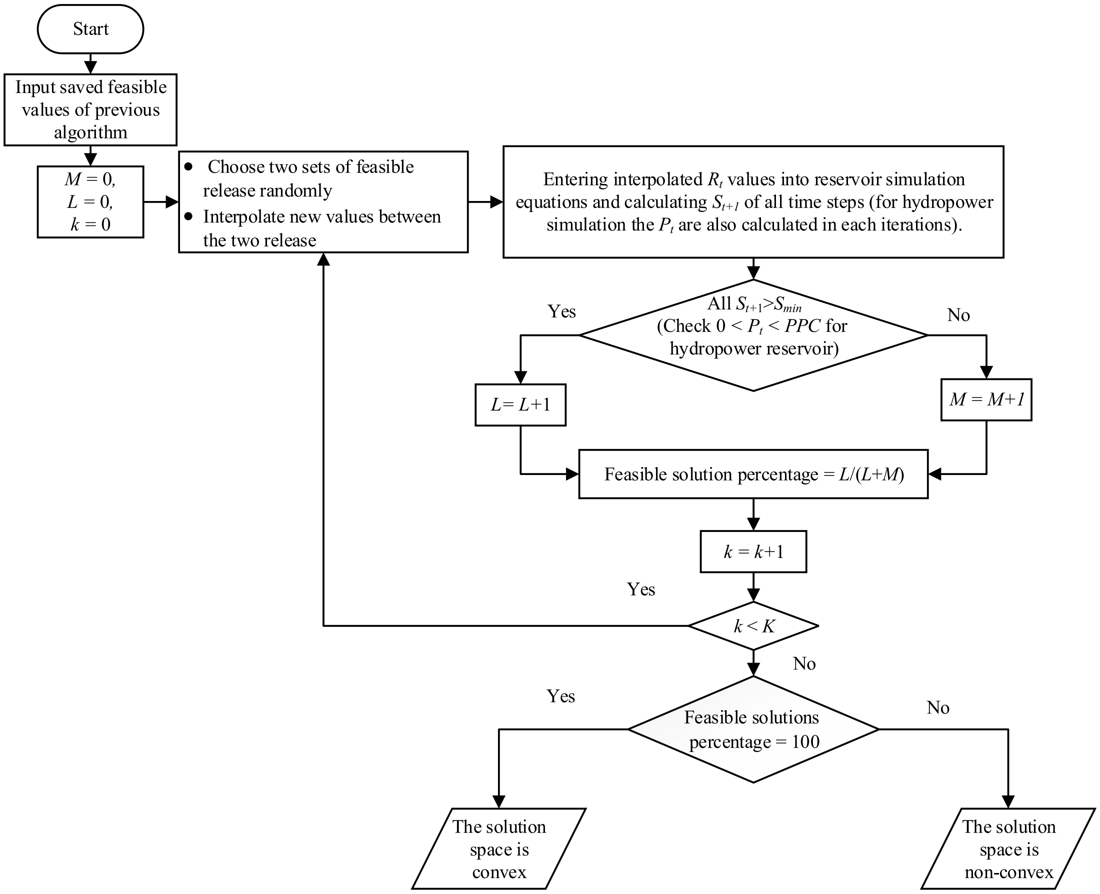

Fig. 3. Flowchart of the convexity assessment algorithm

Table 1. Reservoir Characteristics

Parameter	Value
Reservoir level above sea level (m)	800
Maximum water storage (1×10^6 m^3)	3,000
Minimum water storage (1×10^6 m^3)	400
The active storage (1×10^6 m^3)	2,600
The height of dam (m)	150
Total capacity of the powerhouse (1×10^6 W)	650
The efficiency of the powerhouse (%)	96
Plant factor (%)	35
Elevation of the turbine above mean sea level (m)	790
Elevation of the tailwater above sea level (m)	785

Benchmark Sample Reservoir

Table 1 lists the characteristics of the example reservoir. This reservoir is a hypothetical example of reservoir operation introduced by Seifollahi-Aghmiuni et al. (2016). All data required for modeling the system, such as inflow, evaporation, monthly water demand, and so on, are given by Seifollahi-Aghmini et al. (2016, Tables 2 and 5, Reservoir 3). The geometric characteristics of the reservoir include its storage volume [S (1×10^6 m^3)], lake area [A (square kilometers)], and water elevation [H (meters)]. The reservoir surface area and elevation are given by

$$A_t = 3.869 \times 10^{-2} \times S_t + 2.284 \times 10^{-6} \times S_t^2 \qquad t = 1, 2, 3, \ldots, T \tag{22}$$

Table 2. Value of the Objective Function for 10 Different Runs Calculated with GA and GRG for the Water-Supply Reservoir [Objection Function Given by Eq. (10)]

	Random initial population as the starting search point		Zero initial population as the starting search point	
Run number	GA	GRG	GA	GRG
1	3.18×10^6	3.15×10^6	3.23×10^6	3.15×10^6
2	3.20×10^6	3.15×10^6	3.17×10^6	—
3	3.17×10^6	3.15×10^6	3.18×10^6	—
4	3.17×10^6	3.15×10^6	3.16×10^6	—
5	3.17×10^6	3.15×10^6	3.17×10^6	—
6	3.19×10^6	3.15×10^6	3.23×10^6	—
7	3.17×10^6	3.15×10^6	3.16×10^6	—
8	3.19×10^6	3.15×10^6	3.19×10^6	—
9	3.18×10^6	3.15×10^6	3.16×10^6	—
10	3.25×10^6	3.15×10^6	3.20×10^6	—
Minimum	3.17×10^6	3.15×10^6	3.16×10^6	—
Average	3.19×10^6	3.15×10^6	3.19×10^6	—
Maximum	3.25×10^6	3.15×10^6	3.23×10^6	—
Standard deviation	2.5×10^4	1.22×10^1	2.81×10^4	—
Coefficient of variation	7.84×10^{-3}	3.89×10^{-6}	8.81×10^{-3}	—

$$H_t = 7.214 \times 10^{-2} \times S_t - 10.734 \times 10^{-6} \times S_t^2 + 30 \qquad t = 1, 2, 3, \ldots, T \tag{23}$$

The powerhouse discharge–elevation ($Q_{Pt} - El$) equation [EL_t (meters)] and Q_{pt} (cubic meters per second) is

$$EL_t = 0.2 \times 10^{-2} \times Q_p - 2.2 \times 10^{-7} \times Q_{pt}^2$$
$$t = 1, 2, 3, \ldots, T \qquad (24)$$

Results and Discussion

It was assumed that the initial reservoir storage volume was half its active storage. The number of Monte Carlo iterations was 10,000 and 100,000 for creating the solution spaces of the water-supply and hydropower generation problems, respectively. A sequence of 12 monthly reservoir releases was generated in each iteration. The number of iterations was determined from the convergence of the percentage of feasible solutions to a steady value. Fig. 4 depicts the calculated convergence curves.

Extent of Solution Spaces

The constraint considered in analyzing the extent of the solution space of the water-supply problem was that the storage volume may not be less than the dead storage volume. If this constraint is satisfied, the set of releases is feasible. It was found that 38.67% of the generated sets of releases were feasible.

The reservoir operation problem for hydropower generation imposed the same constraint on reservoir storage applied in the water-supply problem, and the generated power was constrained to the range from zero to the PPC. After checking these two constraints it was found that only 0.2% of the generated sets of releases were feasible, which implies a much more limited solution space than that of the reservoir operation problem for water supply.

Convexity of Solution Space

A solution space was created. New solutions were generated by interpolating between a large number of randomly selected pairs of points in the solution space. All the interpolating solutions using feasible solutions for the water-supply problem were feasible (located in the feasible solution space). Therefore this problem has a convex solution space. Regarding hydropower, it was found for the reservoir operation problem that many interpolated solutions were outside the feasible solution space, thus rendering its solution space nonconvex. The validity of these results was tested in a more general context by changing several reservoir parameters. The resulting problems were tested for convexity for each range of the values assigned to the parameters. The parameters were reservoir evaporation; water demand for the water-supply problem; PPC for the hydropower-generation problem; and dead storage, reservoir capacity, and inflow for both problems.

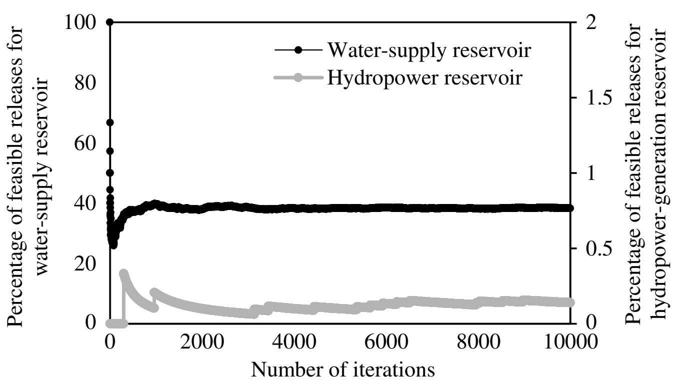

Fig. 4. Convergence curves for water-supply and hydropower-generation reservoirs

The results indicate that the feasible solution space of the water-supply problem is convex, whereas the solution space for the hydropower reservoir is nonconvex. Because the water-supply problem is convex, most optimization methods can solve this problem and find its global optimum efficiently. The hydropower generation-method, on the other hand, can be solved most effectively with evolutionary algorithms such as the GA. The GRG method converges to local optima frequently when solving nonconvex problems unless the starting initial guess of a solution is near the global optimum.

Assessing Effects of Operational Parameters on Extent of Solution Space

Each operational parameter value was varied and the change in the percentage of feasible solutions corresponding to each parameter value was assessed. Figs. 5 and 6 depict the results. The numbers on the horizontal axes in these figures are the values of the coefficients by which the values of the reservoir parameters are multiplied to vary the parameters. The next two sections explain the results.

Water-Supply Problem

The operational parameters in this problem were the volumes of evaporation, water demand, dead storage, reservoir capacity, and reservoir inflow. Fig. 5 shows the changes in the percentage of sets of feasible releases corresponding to changes in reservoir parameters.

The percentage of sets of feasible releases decreased only by 2% when the evaporation volumes were increased from a factor of 0.5 to a factor of 2, which is a relatively small change (Fig. 5). By increasing the values of water demand (Fig. 5) the solution space decreased sharply and that curve exhibited a downward trend. When values of water demand were decreased by a factor of 0.5, all the sets of releases were feasible, and when the water-demand values increased by a factor of 1.5, approximately 3% of the sets of

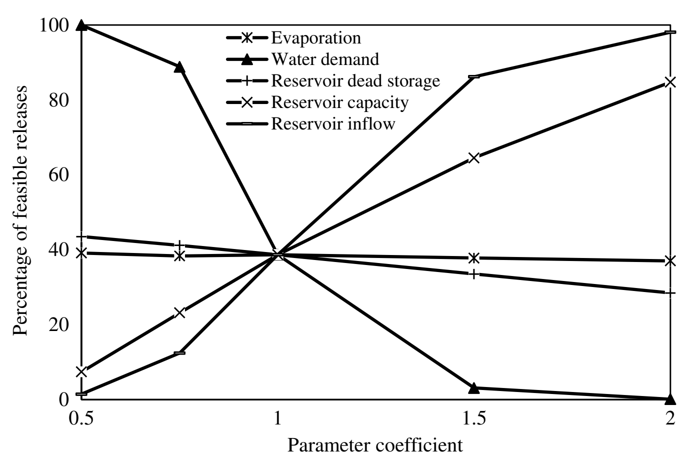

Fig. 5. Illustration of changes in the percentage of sets of feasible releases by variation of the coefficients that multiply the values of evaporation, water demand, reservoir dead storage, reservoir capacity, and reservoir inflow

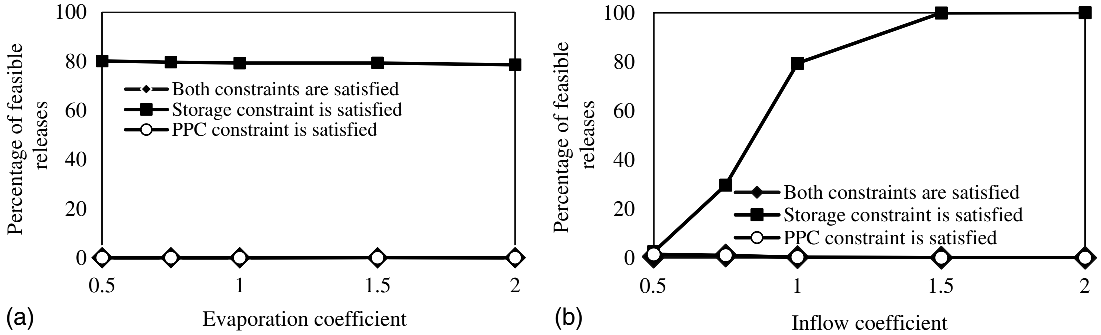

Fig. 6. Illustration of changes in the percentage of feasible sets of releases by variation of the coefficients that multiply: (a) evaporation, for which the PPC constraint, storage constraint, and both constraints are satisfied separately; (b) inflow, for which the PPC constraint, storage constraint, and both constraints are satisfied separately, for the hydropower-generation reservoir problem

releases were feasible. The percentage of the sets of feasible releases was nearly zero when their values increased by a factor of 2.

The graph of the dead storage (Fig. 5) shows that the solution space decreased by increasing the value of the dead storage. The percentage of sets of feasible releases calculated by changing the dead storage from a factor of 0.5 times to a factor of 2 decreased from 43 to 28%. When reservoir capacity was multiplied by factors of 0.5, 0.75, 1, 1.5, and 2, the percentage of set of feasible releases was 7.39, 23.17, 38.68, 64.54, and 84.75%, respectively (Fig. 5). The extent of the solution space increased dramatically by increasing the reservoir inflow values (Fig. 5). Under the initial values of inflow, 38% of the sets of releases were feasible; however, the percentage reached 86% when reservoir inflow increased by a factor of 1.5, and it became 98% when the reservoir inflow increased twofold.

The values of demand, capacity, and inflow significantly affected the span of solution space in the water supply reservoir.

Hydropower Reservoir

The impacts of changing evaporation, dead storage, reservoir capacity, river inflow, and PPC on the extent of the solution space were quantified. The hydropower-generation problem considered a constraint on reservoir storage and a constraint on the hydropower production ranging from zero to the PPC. The generated sets of releases were feasible if they satisfied both constraints. Figs. 6 and 7 graph the changes in the feasible solution space by considering both constraints together and each one separately.

The percentage of sets of feasible releases did not change significantly when the evaporation volume changed [Fig. 6(a)]. Changes in the percentage of sets of feasible releases by variation of reservoir inflows did not exhibit consistent trends in how they affected the percentage of sets of feasible releases [Fig. 6(b)]. This was due to the presence of two constraints in the reservoir operation problem for hydropower production that affect the feasibility of releases in a complex manner.

An increase in the volume of the dead storage led to a decrease in the number of sets of feasible releases. When dead storage decreased by 50% the percentage of feasible releases sets was 0.25%, whereas if dead storage increased to twice its value, this percentage decreased to 0.08% [Fig. 7(a)]. Increases in reservoir capacity led to decreases in the percentage of sets of feasible releases [Fig. 7(b)]. When reservoir capacity was multiplied by coefficient values between 0.5 and 2 the percentage of sets of feasible releases decreased from 3.3 to 0.08% [Fig. 7(b)]. Because the percentage of sets of feasible releases was very low (0.2%) in the hydropower-generation problem, a change from 3.3 to 0.08% is significant. In the reservoir problem for hydropower production, the PPC plays an important role in the extent of its solution space [Fig. 7(c)]. By increasing the PPC values, the allowable range of hydropower generation widened and the number of the sets of feasible releases increased.

Evaluating Performance of Solution Methods

Optimization methods were implemented in Microsoft *Excel* Solver with an Intel (Chandler, Arizona) core i5 processor with 32GB RAM. The simplex method obtained the global optimum of the linear water-supply problem [objective function Eq. (10)] in less than 1 s. This shows the efficient performance of linear optimization methods for solving linear problems, for which they are the obvious solution method.

It was established previously that the solution spaces of water-supply and hydropower-generation problems were convex and nonconvex, respectively. The GRG and GA, being popular gradient-based and evolutionary optimization methods, respectively, were implemented to solve the two reservoir problems. The reservoir operation optimization problems of water supply [with convex quadratic function Eq. (9)] and hydropower generation were solved. The objective functions of both problems involve convex functions under minimization, and only the convexity of the solution space influences the convexity of each problem. The initial solutions of the GRG and GA were generated in two ways: (1) random generation, and (2) by setting them equal to zero. The zero initial solution was chosen to conduct a comparison of the two methods under the same initial condition.

Tables 2 and 3 list the values of the objective functions in 10 different runs calculated with GA and GRG for the water-supply reservoir and the hydropower reservoir, respectively.

The number of GA populations was 1,000 and this algorithm was stopped by the specified convergence criteria [i.e., there is no significant difference (less than 0.000001) between the average values of the last 30 iterations]. The average computational time invested by the GA to find the solution of the water-supply problem was 40 s, and the average, maximum, and minimum values of objective function in 10 runs with a random initial population of solutions were 3.19×10^6, 3.25×10^6, 3.17×10^6, respectively (Table 2). The difference between the solutions found in the 10 runs was negligible given the scale of objective function values, thus providing strong evidence that the GA found a near-optimal value. The average computing time of the GRG was 1.5 s, and it

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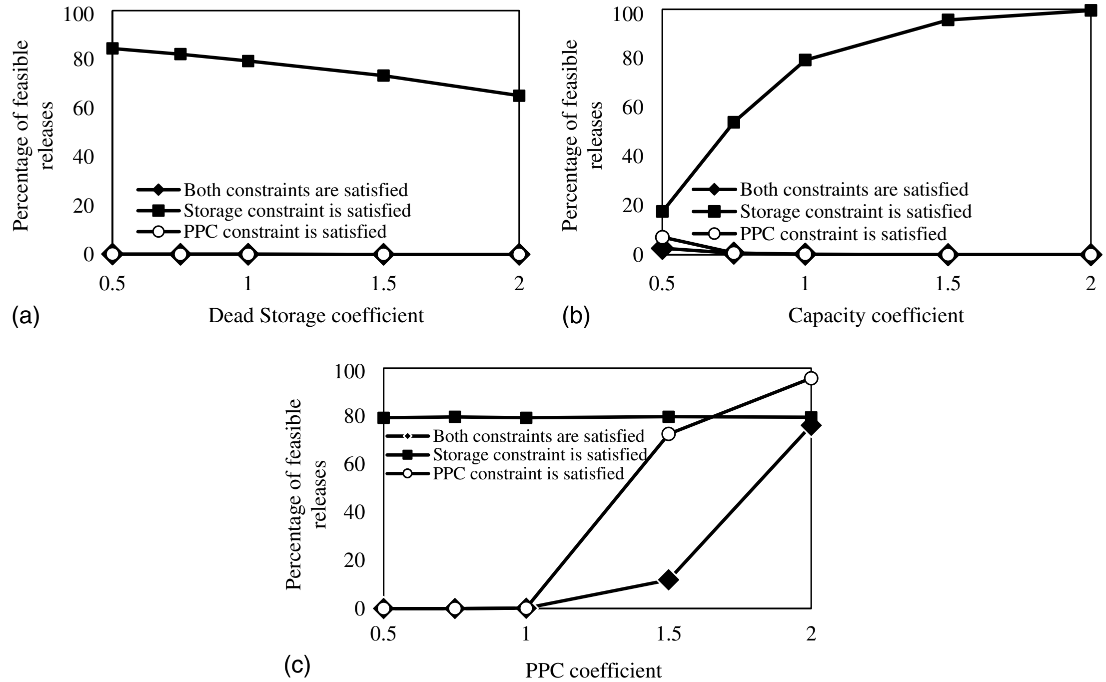

Fig. 7. Illustration of changes in the percentage of feasible sets of releases by variation of the coefficients that multiply: (a) dead storage, for which the PPC constraint, storage constraint, and both constraints are satisfied separately; (b) capacity, for which the PPC constraint, storage constraint, and both constraints are satisfied separately; (c) PPC for which the PPC constraint, storage constraint, and both constraints are satisfied separately, for the hydropower-generation reservoir problem

Table 3. Value of the Objective Function for 10 Different Runs Calculated with GA and GRG for the Hydropower Reservoir [Objective Function Given by Eqs. (20) and (21)]

	Random initial population as the starting search point		Zero initial population as the starting search point	
Run number	GA	GRG	GA	GRG
1	29.7	12.3	10.8	357
2	33.1	12.7	31.6	—
3	25.6	14.0	29.7	—
4	25.1	18.2	18.9	—
5	36.9	11.3	25.4	—
6	11.6	31.4	49.3	—
7	26.2	61.6	29.4	—
8	32.8	97.9	19.6	—
9	37.0	769	45.7	—
10	23.3	45.5	18.1	—
Minimum	11.6	12.3	10.8	—
Average	28.1	118	27.8	—
Maximum	37.0	769	49.3	—
Standard deviation	7.59	232	12.2	—
Coefficient of variation	0.27	1.97	0.438	—

converged to a value equal to 3.15×10^6 in all 10 runs starting the search at different initial points. This indicates that the GRG method is much faster and more accurate than the GA in solving the problem with a convex solution space. The GA and GRG performances when the initial solution guess equaled zero were very similar to those observed when the initial solutions were randomly generated.

The average, maximum, and minimum values of the objective function for the hydropower-generating problem associated with 10 runs that had random initial populations and zero initial populations equaled 2.81×10^1, 3.70×10^1, and 1.16×10^1 and 2.78×10^1, 4.93×10^1, and 1.08×10^1, respectively (Table 3). These values indicate variability among the solutions. This shows that convergence to a near-optimal solution is more difficult when solving nonconvex optimization problems. The GRG method found very different solutions to this problem in each run. The average, minimum, and maximum values of the objective function 10 runs of the GRG with a random initial solution and with a zero initial population were 1.18×10^2, 7.69×10^2, 1.23×10^1, and 3.57×10^2, respectively (Table 3), which differ substantially from each other. This indicates that the GRG failed to obtain an optimal value for the nonconvex solution space because of trapping at local optima. The computational times invested by the GRG and the GA in solving the hydropower-generation problem were 1.5 and 65 s, respectively. The GA found a near-optimal value, whereas the GRG converged to widely different local optima.

The results indicate superior performance of the simplex method for solving the linear optimization problem, efficient GRG for the nonlinear convex problem, and a superior GA for the nonlinear nonconvex optimization problems.

Conclusions

This study proposed a framework for assessing convexity and nonconvexity of optimization problem solution space. The proposed framework was applied to two types of reservoir operation optimization problems. Results indicated that the solution spaces of the water-supply reservoir operation and hydropower generation reservoir operation problems are convex and nonconvex, respectively.

It was determined that changing the values of water demand, river inflow, dead storage, and reservoir capacity significantly impacted the extent of the solution space of the water-supply problem.

However, the effect of changing evaporation was negligible. The impact of changing the reservoir inflow on the extent of the solution space of the hydropower generation problem exhibited an irregular pattern. The impacts of changes in dead storage and reservoir capacity on the extent of the solution space followed a pattern similar to that of the water-supply problem. The effect of changing the evaporation on the solution space of the hydropower-generation problem was minor.

The GA was successful in finding near-optimum solutions for the convex water-supply problem. Moreover, the GA performed well in solving the nonconvex hydropower-generation problem. The GRG, a gradient-based optimization method, found the global optimum of the convex water-supply problem in significantly shorter time than did the GA. The GRG converged to local optima in the nonconvex hydropower-generation problem.

This study's results suggest that the GRG should be a preferred method for solving nonlinear convex optimization problems. The GA, however, is a better choice for solving nonlinear nonconvex optimization problems.

References

Asgari, H. R., Bozorg-Haddad, O., Pazoki, M., and Loáiciga, H. A. (2015). "Weed optimization algorithm for optimal reservoir operation." *J. Irrig. Drain. Eng.*, 10.1061/(ASCE)IR.1943-4774.0000963, 04015055.

Azizipour, M., Ghalenoei, V., Afshar, M. H., and Solis, S. S. (2016). "Optimal operation of hydropower reservoir systems using weed optimization algorithm." *Water Resour. Manage.*, 30(11), 3995–4009.

Bhattacharjya, R. K., and Datta, B. (2005). "Optimal management of coastal aquifers using linked simulation optimization approach." *Water Resour. Manage.*, 19(3), 295–320.

Bozorg-Haddad, O. (2014). *Water resources systems optimization*, Tehran Univ., Tehran, Iran (in Persian).

Bozorg-Haddad, O., Afshar, A., and Mariño, M. A. (2006). "Honey-bees mating optimization (HBMO) algorithm: A new heuristic approach for water resources optimization." *Water Resour. Manage.*, 20(5), 661–680.

Bozorg-Haddad, O., Afshar, A., and Mariño, M. A. (2008). "Honey-bee mating optimization (HBMO) algorithm in deriving optimal operation rules for reservoirs." *J. Hydroinf.*, 10(3), 257–264.

Bozorg-Haddad, O., Afshar, A., and Mariño, M. A. (2009). "Optimization of non-convex water resource problems by honey-bee mating optimization (HBMO) algorithm." *Eng. Comput.*, 26(3), 267–280.

Bozorg-Haddad, O., Hosseini-Moghari, S. M., and Loáiciga, H. A. (2016). "Biogeography-based optimization algorithm for optimal operation of reservoir systems." *J. Water Resour. Plann. Manage.*, 10.1061/(ASCE)WR.1943-5452.0000558, 04015034.

Cai, X., McKinney, D. C., Lasdon, L. S., and Watkins, J. D. W. (2001). "Solving large nonconvex water resources management models using generalized benders decomposition." *J. Oper. Res.*, 49(2), 235–245.

Chu, W. S., and Yeh, W. (1978). "A non-linear programming algorithm for real-time hourly reservoir operations." *J. Am. Water Resour. Assoc.*, 14(5), 1048–1063.

Del Valle, Y., Harley, R. G., and Venayagamoorthy, G. K. (2009). "Comparison of enhanced-PSO and classical optimization methods: A case study for STATCOM placement." *Proc., 15th Int. Conf. on Intelligent System Applications to Power Systems*, IEEE, Piscataway, NJ.

Esat, V., and Hall, M. J. (1994). "Water resources system optimization using genetic algorithms." *J. Hydroinf.*, 94(1), 225–231.

Fallah-Mehdipour, E., Bozorg-Haddad, O., and Mariño, M. A. (2012). "Real-time operation of reservoir system by genetic programming." *Water Resour. Manage.*, 26(14), 4091–4103.

Garousi-Nejad, I., Bozorg-Haddad, O., Loáiciga, H. A., and Mariño, M. A. (2016). "Application of the firefly algorithm to optimal operation of reservoirs with the purpose of irrigation supply and hydropower production." *J. Irrig. Drain. Eng.*, 10.1061/(ASCE)IR.1943-4774.0001064, 04016041.

Ghosh, A., and Dehuri, S. (2004). "Evolutionary algorithms for multi-criterion optimization: A survey." *Int. J. Comput. Inf. Sci.*, 2(1), 38–57.

Giuliani, M., Castelletti, A., Pianosi, F., Mason, E., and Reed, P. M. (2015). "Curses, tradeoffs, and scalable management: Advancing evolutionary multiobjective direct policy search to improve water reservoir operations." *J. Water Resour. Plann. Manage.*, 10.1061/(ASCE)WR.1943-5452.0000570, 04015050.

Hillier Frederick, S., and Lieberman Gerald, J. (2005). *Introduction to operations research*, 7th Ed., McGraw-Hill, New York.

Houck, M. H. (1979). "A chance constrained optimization model for reservoir design and operation." *Water Resour. Res.*, 15(5), 1011–1016.

Jordehi, A. R., and Jasni, J. (2012). "Approaches for FACTS optimization problem in power systems." *Power Engineering and Optimization Conf. (PEDCO)*, IEEE, Piscataway, NJ.

Karatzas, G. P., and Pinder, G. F. (1993). "Groundwater management using numerical simulation and the outer approximation method for global optimization." *Water Resour. Res.*, 29(10), 3371–3378.

Kuczera, G. (1989). "Fast multireservoir multiperiod linear programing models." *Water Resour. Res.*, 25(2), 169–176.

Labadie, J. W. (2004). "Optimal operation of multireservoir systems: State-of-the-art review." *J. Water Resour. Plann. Manage.*, 10.1061/(ASCE)0733-9496(2004)130:2(93), 93–111.

Lasdon, L. S., Fox, R. L., and Ratner, M. W. (1974). "Nonlinear optimization using the generalized reduced gradient method." *Revue française d'automatique, d'informatique et de recherche opérationnelle. Recherche opérationnelle*, 8(V3), 73–103.

Little, J. D. (1955). "The use of storage water in a hydroelectric system." *J. Oper. Res.*, 3(2), 187–197.

Mantawy, A. H., Soliman, S. A., and El-Hawary, M. E. (2003). "An innovative simulated annealing approach to the long-term hydroscheduling problem." *Int. J. Electr. Power Energy Syst.*, 25(1), 41–46.

MATLAB [Computer software]. MathWorks, Natick, MA.

Oliveira, R., and Loucks, D. P. (1997). "Operating rules for multi reservoir systems." *Water Resour. Res.*, 33(4), 839–852.

Ostadrahimi, L., Mariño, M. A., and Afshar, A. (2012). "Multi-reservoir operation rules: Multi-swarm PSO-based optimization approach." *Water Resour. Manage.*, 26(2), 407–427.

Seifollahi-Aghmiuni, S., Bozorg-Haddad, O., and Loáiciga, H. A. (2016). "Development of a sample multiattribute and multireservoir system for testing operational models." *J. Irrig. Drain. Eng.*, 10.1061/(ASCE)IR.1943-4774.0000908, 04015039.

Simonovic, S. P. (1992). "Closing gap between theory and practice." *J. Water Resour. Plann. Manage.*, 10.1061/(ASCE)0733-9496(1992)118:3(262), 262–280.

Stedinger, J. R., Sule, B. F., and Loucks, D. P. (1984). "Stochastic dynamic programming models for reservoir operation optimization." *Water Resour. Res.*, 20(11), 1499–1505.

Wardlaw, R., and Sharif, M. (1999). "Evaluation of genetic algorithms for optimal reservoir system operation." *J. Water Resour. Plann. Manage.*, 10.1061/(ASCE)0733-9496(1999)125:1(25), 25–33.

Wurbs, R. A. (1993). "Reservoir system simulation and optimization models." *J. Water Resour. Plann. Manage.*, 10.1061/(ASCE)0733-9496(1993)119:4(455), 455–472.

Yeh, W. W. G. (1985). "Reservoir management and operations models: A state-of-the-art review." *Water Resour. Res.*, 21(12), 1797–1818.

Yeh, W. W. G., Graves, A. L., Toy, D., and Becker, L. (1980). "Central Arizona project: Operations model." *J. Water Resour. Plann. Manage. Div.*, 106(2), 521–540.

Yeniay, O. (2005). "A comparative study on optimization methods for the constrained nonlinear programming problems." *Math. Probl. Eng.*, 2005(2), 165–173.

Young, G. K. (1967). "Finding reservoir operating rules." *J. Hydraul. Div.*, 93(6), 297–322.