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INFLUENCE OF THE DISTRIBUTION FUNCTION ON EIGENOSCILLATIONS AND STABILITY OF A BEAM

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#### Abstract

Eigenfrequencies are calculated for the transverse oscillations of a round beam radially confined in a solenoidal magnetic field. Under the asumption of a linear restoring force in the equilibrium beam, with partial or total neutralization by an immobile background charge, the influence of different distribution functions on stability is investigated. It is found that the wellknown extended instabilities of a microcanonical (Kapchinskij-Vladimirsky) distribution are replaced by apparently insignificcant patches of instability if the distribution function is broadened, hence the loss-cone partially filled up. The waterbag distribution indicates transition to only stable eigenfrequencies and it is found that this transition is accompanied by suppression of negative energy oscillations, which are responsible for the instabilities of loss-cone or non-monotonically decreasing distributions. The method employed consists of infinite series expansion for the eigenfunctions and approximating the infinite determinant dispersion relation by rapidly converging finite order sub-determinants.

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#### I. Introduction

Stability of a plasma contained in an external potential well has been investigated in a variety of models and approximations in the attempt to meet requirements posed by different experimental situations. Such situations may occur in electrostatic or magnetic confinement schemes and also in connection with electron or ion beams focussed by a solenoidal magnetic field. In the latter . case opposite charges are often absent and one may speak of a non-neutral plasma if the beam intensity is sufficiently close to the space charge limit (assuming non-relativistic flow) to make collective effects dominate over single particle effects. In view of the recent interest in Heavy Ion Fusion stability of intense ion beams has received increased attention <sup>1)</sup>. If such a beam is launched from a shielded cathode and drifting along a solenoidal magnetic field there is a mean rotation with the Larmor frequency or half the cyclotron frequency

(1) 
$$\omega_{\rm L} = \frac{1}{2} \omega_{\rm C} = \frac{1}{2} \frac{eB_{\rm O}}{M \cdot c}$$

In a frame moving with the drift velocity of the ions and rotating with  $\omega_L$  the uniform solenoidal field is equivalent to a linearly varying radial electric field which gives rise to a harmonic potential well. The Hamiltonian in the Larmor frame is then

(2) 
$$H_0 = \frac{1}{2M} (p_r^2 + p_{\theta}^2) + \frac{M}{2} \frac{\omega_c^2}{4} r^2 + V_{sp.ch.}(r^2)$$

where V sp.ch. is the space charge induced potential if the beam is non-neutral in equilibrium. The question arises whether a given equilibrium distribution  $f_0(H_0)$  is stable, or how the choice of  $f_0$  affects the stability properties.

The above defined problem of Vlasov stability of a charge distribution in four-dimensional phase space is of rather general interest. For monotonic decreasing distributions  $f_0$  it is possible to prove stability on the basis of the Newcomb-Gardner theorem <sup>2,3)</sup>. While  $f_0' < 0$  is a sufficient stability condition for a neutral or a non-neutral case, the theorem does not permit any conclusion if the distribution is non-monotonic. In this case one has to investigate the spectrum of eigenoscillations. This problem was solved in connection with the stability of proton beams <sup>4)</sup>, assuming a microcanonic or Kapchinskij-Vladimirsky (K-V) distribution,  $f_0 \sim \delta(H_0 - E_0)$ , which results in a uniform unperturbed density and an unperturbed Hamiltonian

(3) 
$$H_0 = \frac{1}{2M} (p_r^2 + p_{\theta}^2) + \frac{M}{2} v^2 r^2$$

with the space charge depressed radial "tune"

(4) 
$$v^2 = v_0^2 - \frac{\omega_p^2}{2}$$

and  $v_0^2 = \omega_c^2/4$  and the plasma frequency defined by  $\omega_p^2 = 4\pi n_0 e^2/M$ . While the existence of an equilibrium requires  $0 \le \omega_p^2 \le \omega_c^2/2$  or  $0 \le v^2 \le v_0^2$ , stability imposes the more stringent condition  $\omega^2$ 

(5) 
$$\frac{\omega_p}{v^2} < 11.5$$

and the tune depression is limited to  $v/v_0 > 0.385$ . With an immobile neutralizing background of opposite charges the eigenoscillations of the K-V beam are unchanged and condition (5) is replaced by  $\omega_p^2/v_0^2 = 4 \omega_p^2/\omega_c^2 < 11.5$ .

It is clear that the unstable oscillations of a K-V beam are velocity space instabilities, which are absent in a macroscopic fluid model. One may suspect that the loss cone nature of the K-V distribution is responsible for these instabilities and the question arises how the eigenfrequency pattern changes if the distribution is broadened and the loss cone partially filled up. The problem of how to determine eigenfrequencies of a non K-V beam is complicated by the following circumstances:

(a) The beam is of finite transverse extent, comparable with the maximum gyroradius, as opposed to an extended plasma with physical dimension much larger than a gyroradius. In the latter case it is possible, for sufficiently short wave-lengths, to adopt a uniform (neutralized) plasma model and use plane waves as electric field perturbations. The resulting problem of cyclotron harmonic waves has been investigated by different authors 5,6 who found that a

 $\delta$ -function distribution  $\delta(v_{\perp} - \alpha_{\perp})$  was unstable beyond a threshold  $\omega_p^2/\omega_c^2 \approx 7.3$  (for comparison the threshold is  $\omega_p^2/\omega_c^2 \approx 11.5/4 \approx 2.88$  for a neutral  $\delta$ -function beam), while broadened distributions could have higher thresholds or be stable at all. For the beam case the strong spatial inhomogeneity requires infinite series expansion for the electric field perturbations. Since we know that for a K-V beam the eigensolutions can be expressed in terms of hypergeometric functions <sup>4)</sup>, we may use these as a convenient basis set for our expansion and obtain an infinite system of linear equations for the expansion coefficients. The eigenfrequencies then result from the roots of the determinant of the system.

(b) An additional difficulty arises if there is no neutralizing background, in which case the unperturbed space charge potential in (2) is non-quadratic (except for a K-V beam) and we obtain anharmonic zero-order orbits which prohibit straight forward integration of the Vlasov equation along zero-order orbits.

It is the purpose of this paper to study the effect on eigenfrequencies of a broadening of the distribution function and solve the problem described under (a). Such a broadening is the main stabilizing mechanism, because it reduces the strong positive slope of the radial velocity distribution, which acts as source of energy for the K-V beam instabilities. Since we are unable to deal with the difficulty in (b) our results pertain in a strict sense to either

1. a beam plasma with immobile neutralizing background in which case de-scaling of the numerical results expressed in the dimensionless intensity parameter  $1/R(0 \le 1/R < \infty)$ requires, through use of an averaged plasma frequency  $\omega_{n}$ ,

(6) 
$$\frac{1}{R} = \frac{\omega_p}{v_o} = 2 \frac{\omega_p}{\omega_c}$$

2. an unneutralized beam where only the nonlinear part of the unperturbed space charge force due to the non K-V distribution has been cancelled by fictitious sources. De-scaling requires

(7) 
$$\frac{1}{R} = \frac{\omega_{p}}{v} = 2 \frac{\omega_{p}}{(\omega_{c}^{2} - 2\omega_{p}^{2})^{1/2}}$$

There is some evidence, however, that the nonlinear part in the zero order space charge force and the resulting spread in  $\nu$  can act as a source for Landau damping and thus provide for additional stabilization. Hence, taking the full space charge force the distributions studied here are likely to be more stable rather than more unstable, if we apply the above de-scaling with averaged  $\omega_p$  and  $\nu$ .

We proceed, in section II, with a presentation of the basic equations within the Vlasov framework and, in section III, with the series expansion that results in an infinite determinant as dispersion relation. We limit the analysis on azimuthally symmetric modes, because they were the only unstable modes found for a K-V beam. In section IV we shall present eigenfrequencies for the following distributions: (A), a K-V distribution (derived elsewhere  $^{4,7)}$ ); (B), a broadened non-monotonic distribution; (C), a water-bag distribution, which is uniform in four dimensional phase space and (D), a center-peaked monotonic distribution. The results will be discussed in section V in terms of coupling of positive and negative energy oscillations induced by the positive slope of the distribution function.

We remark that in the existing literature eigenfrequencies of bounded charge distributions have been calculated for the K-V beam in two dimensions, while several distributions have been examined in one dimension: a water-bag model  $^{8,9)}$ , a Gaussean distribution  $^{10)}$  and the one-dimensional analogue to the KV-distribution with uniform density in the unperturbed beam  $^{11)}$ .

#### II. Basic Equations

For the unperturbed beam in the Larmor frame we employ the quadratic Hamiltonian from Eq.(3) with M = 1 and adopt an arbitrary zero order distribution which we write as superposition of a continuum of K-V beams with radius a

(8) 
$$f_0(H_0) = \frac{N}{\pi^2 \langle a^2 \rangle} \int g(a^2) \, \delta[p_r^2 + p_\theta^2 + v^2(r^2 - a^2)] \, da^2$$

and average beam radius

(9) 
$$\langle a^2 \rangle \equiv \int g(a^2) a^2 da^2$$

With  $\int g(a^2) da^2 = 1$  the energy distribution  $g(a^2)$  is given by (10)  $g \equiv \frac{\langle a^2 \rangle \pi^2 v^2}{N} f_0\left(\frac{v^2 a^2}{2}\right)$ .

For azimuthally symmetric electrostatic perturbations the linearized Vlasov equation can be written as

(11) 
$$\frac{df_1}{dz} = \frac{\omega_p^2}{2e\pi^2} p_r \frac{\partial V}{\partial r} \int g(a^2) \delta' da^2$$

with z the longitudinal coordinate  $z = v \cdot t$  and  $\omega_p^2 \equiv \frac{4 \text{ Ne}^2}{\text{mv}^2 < a^2 >}$ and Poisson's equation

(12) 
$$\frac{1}{r} \frac{\partial}{\partial r}$$
 (r  $\frac{\partial}{\partial r}$  V) =  $4\pi en_1 = 4\pi e \iint f_1 dp_r dp_{\Theta}$ 

We integrate Eq.(11) along zero-order orbits, which we conveniently express in angle-amplitude variables  $\varphi$ , A and the canonical angular momentum  $P_{\alpha}$ 

$$r^{2} = \frac{A^{2}}{2} + \left[\frac{A^{4}}{4} - \frac{P_{\theta}^{2}}{\nu^{2}}\right]^{1/2} \sin \varphi$$

$$\frac{d\varphi}{d\varphi} = 2\nu$$

dz

and obtain, for  $f_1 \sim e^{i\omega z}$ , a perturbed density

(14)  

$$n_{1}(r) = -\frac{\omega_{p}^{2}}{2e\pi^{2}} \int g(a^{2}) \left[ \iint \delta' \left[ \nu^{2} (A^{2}-a^{2}) \right] (1-e^{-i\frac{\omega\pi}{\nu}})^{-1} + \int_{0}^{2\pi} e^{i\frac{\omega\mu}{2\nu}} \left[ \frac{\partial V}{\partial u} \right]_{u+\varphi} du dp_{r} dp_{\theta} da^{2}$$

If we take  $g(a^2) = \delta(a^2-1)$  the problem is reduced to that of a K-V beam as treated in Ref.4. Eqs. (12), (14) then have eigen-functions that can be expressed in terms of hypergeometric funtions. For azimuthally symmetric modes we obtain specifically

(15) 
$$V_j(r) = \theta(r^2-1)[P_j(1-2r^2) + P_{j-1}(1-2r^2)]$$

with P Legendre polynomials and  $\theta$  the step function. The eigenvalue  $\omega$  is found to satisfy the following dispersion relation 4,7

(16)  $1 = \frac{\omega_p^2}{v^2} N_j \left(\frac{\omega}{2v}\right)$ 

where  $N_{i}(s)$  are the following polynomials:

$$N_{1} = \frac{1}{4} \left[ \frac{1}{s^{2}-1} \right] \text{ (envelope oscillation)}$$

$$N_{2} = \frac{1}{32} \left[ 1-s^{2} \left( \frac{4}{s^{2}-1} - \frac{3}{s^{2}-4} \right) \right] \text{ ("fourth order", nonuniform)}$$
(17)
$$N_{3} = \frac{1}{96} \left[ s^{2} \left( \frac{3}{s^{2}-1} - \frac{6}{s^{2}-4} + \frac{5}{s^{2}-9} \right) - 2 \right] \text{ ("sixth order", nonuniform)}$$

$$N_{4} = \frac{1}{1024} \left[ 9-s^{2} \left( \frac{24}{s^{2}-1} - \frac{20}{s^{2}-4} + \frac{40}{s^{2}-9} - \frac{35}{s^{2}-16} \right) \right] \text{ etc.}$$

## III. Expansion of Eigenfunctions

For general  $g(a^2)$  with  $g(a^2) = 0$  for  $a^2 > 1$  we can expand the eigenfunctions as infinite series

(18) V(r) = 
$$\sum_{j=1}^{\infty} \xi_j [P_j(1-2r^2) + P_{j-1}(1-2r^2)]$$

which satisfies, term by term, the correct boundary condition V'(r) = 0 for r > 1. The momentum integration in (14) can be

done most elegantly if we expand (18) in terms of the solutions for a K-V beam with boundary at r = a and then use the same procedure as for the K-V beam problem, where the addition theorem for Legendre polynomials can be applied. Hence, we take

(19) 
$$V(r) = \sum_{j=1}^{\infty} \xi_j \sum_{m=1}^{\infty} \alpha_{m,j} (a^2) \left[ P_m (1 - 2\frac{r^2}{a^2}) + P_{m-1} (1 - 2\frac{r^2}{a^2}) \right]$$

With (12), (14) we obtain the following equation

$$\sum_{j=1}^{\infty} \xi_{j} j [P_{j}(1-2r^{2})-P_{j-1}(1-2r^{2})] = \frac{\omega_{p}^{2}}{\nu^{2}} \sum_{j=1}^{\infty} \sum_{m=1}^{\infty} \xi_{j} m \cdot \frac{1}{2}$$
(20)  

$$\cdot N_{m} \left(\frac{\omega}{2\nu}\right) \cdot [P_{n}(1-2r^{2})-P_{n-1}(1-2r^{2})] \cdot \int_{0}^{1} g(a^{2}) \alpha_{m,j}(a^{2}) \tilde{\alpha}_{n,m}(a^{2}) \cdot da^{2}$$

with the coefficients  $\tilde{\alpha}_{n,m}$  defined by the expansion

(21) 
$$\theta(r-a) \left[P_{m}\left(1-2\frac{r^{2}}{a^{2}}\right)-P_{m-1}\left(1-2\frac{r^{2}}{a^{2}}\right)\right] = \sum_{n=m}^{\infty} \tilde{\alpha}_{n,m} \left[P_{n}\left(1-2r^{2}\right)-P_{n-1}\left(1-2r^{2}\right)\right]$$

Using the orthogonality properties of Legendre polynomials we can convert (20) into an infinite system of linear equations

(22) 
$$\begin{split} & \sum_{j=1}^{\infty} \xi_{j} \cdot S_{j\ell} = 0 \quad (\ell = 1, 2...) \\ & S_{j\ell} \equiv \frac{\omega_{p}^{2}}{\nu^{2}} \sum_{m=1}^{\infty} m N_{m} \left(\frac{\omega}{2\nu}\right) \int_{0}^{1} g(a^{2}) \alpha_{mj}(a^{2}) \tilde{\alpha}_{nm}(a^{2}) da^{2} - \ell \cdot \delta_{j\ell} \end{split}$$

and the eigenvalue  $\omega$  is determined by det{S<sub>il</sub>} = 0

#### IV. Numerical Evaluation of the Dispersion Relation

The functions  $\alpha$ ,  $\tilde{\alpha}$  can be determined from Eqs. (19), (21) with the help of the orthogonality properties of Legendre polynomials. If  $g(a^2) = \delta(a^2-1)$ , we use  $\alpha_{mj}(1) \equiv \tilde{\alpha}_{mj}(1) \equiv \delta_{mj}$  and  $\{S_{jl}\}$  is diagonalized with Eq.(16) as dispersion relation. Another special case arises for the water-bag distribution  $g(a^2) \equiv \theta(a^2-1)$  in which case  $\{S_{jl}\}$  is symmetric and  $S_{jl} = 0$  for |j-l| > 1. Approximate solutions of the system (22) can be obtained by replacing the infinite determinant by a finite sub-determinant  $(1 \le j, l \le T)$ . We found rapid convergence if  $g(a^2)$  was sufficiently smooth within  $0 < a^2 < 1$ , for instance a low order polynomial. Increasing T by 1 allows for new eigenmodes with one additional zero of the eigensolution dV(r)/dr in 0 < r < 1. In the subsequent examples we present the eigenmodes with zero, one and two zeroes which originate from matrices with rank T = 1, 2, 3. In some cases we have also calculated higher order modes to show their coupling with the lower order modes. We had excellent convergence, for these modes, if we went as far as T = 5. Clearly, one would have to go to larger T if  $g(a^2)$  had rapid variations in  $0 < a^2 < 1$ , and convergence would presumably break down if there were any discontinuities in  $0 < a^2 < 1$ .

In order to apply results to either case 1 or 2 (see Introduction) we use the dimensionless intensity parameter  $1/R^2$  defined according to Eqs.(6) or (7) and express eigenvalues in terms of  $\sigma$  as given by

(23)  $\sigma \equiv \begin{cases} \frac{\omega}{2v_0} & \text{case 1} \\ \frac{\omega}{2v} & \text{case 2} \end{cases}$ 

For zero intensity,  $1/R^2 = 0$ , the eigenvalues are simply harmonics of the cyclotron frequency, i.e.  $\sigma = 1, 2, 3$  ... The degeneracy at these values is removed for finite intensity. We denote the modes by  $\omega_{jn}$  rsp.  $\sigma_{jn}$ , where j - 1 stands for the number of zeroes of the corresponding dV(r)/dr in 0 < r < 1 and n = 1...jgives the harmonic of the cyclotron frequency in the limit  $1/R^2 + 0$ . The following distribution functions  $g(a^2)$ , defined in  $0 \le a^2 \le 1$ , have been investigated in more details (see Fig.1): (A) K-V distribution,  $g = \delta(a^2-1)$ , (Fig.2)

For completeness we present the results for the modes j = 1, 2, 3 which were calculated by other authors (see ref.7). The most striking feature is, for  $1/R^2 \gtrsim 11.5$ , the onset of an extended instability as a result of a confluence between the eigenvalues  $\omega_{31}$  and  $\omega_{32}$  belonging to the same electric field perturbation.

The instability persists for  $1/R \rightarrow \infty$  with considerable growth rate (Im  $\sigma$ ). The  $\omega_{21}$  branch gets unstable, with  $\omega_{21}^2 < 0$ , for  $1/R^2 > 32$ . For j > 3 there is a similar confluent instability between  $\omega_{j,j-1}$ ,  $\omega_{j,j-2}$  and  $\omega_{j,j-3}$ ,  $\omega_{j,j-4}$  etc., while for even j the lowest frequency branch  $\omega_{j,1}$  can become unstable through  $\omega_{j,1}^2 < 0$ . The limitation  $1/R^2 \lesssim 11.5$  or  $\nu/\nu_0 \gtrsim 0.39$  is sufficient to avoid all of these instabilities.

(B) Broadened loss-cone distribution,  $g \equiv 6(a^4 - \frac{2}{3}a^6)$ , (Fig.3) With a partially filled loss-cone there are still instabilities, yet of different origin and with substantially reduced growth rates. For the modes with n < j there is now a much weaker dependence of  $\sigma$  on 1/R and the previously observed confluences or  $\omega_{j,1}^2 < 0$  cases are no longer found. There is, instead, the possibility of confluence between the modes  $\omega_{11}$  with  $\omega_{32}$  or  $\omega_{43}$ etc. and similar for  $\omega_{22}$ ,  $\omega_{33}$  etc. This coupling of modes to different j-values is characteristic for non K-V distributions for which the electric field perturbation to a given frequency branch is no longer independent of intensity as in case (A).

(C) Water-bag distribution,  $g \equiv 1$ , (Fig.4)

No instabilities have been found for this case of a uniformly filled phase space volume. We note that the branches  $\omega_{21}$ ,  $\omega_{32}$ ,  $\omega_{43}$ , which were involved in instabilities in the previous cases are not found here. There are, instead, only branches for which  $\sigma_{1,n}$  is increasing monotonically with increasing intensity.

(D) Monotonic decreasing function,  $g \equiv \frac{4}{3} - a^4$ , (Fig.5)

Again no instabilities and only monotonically increasing  $\sigma_{j,n}$ . For completeness we note that the branches  $\omega_{21}$ ,  $\omega_{32}$ ,  $\omega_{43}$ , etc. (absent in (C)) are now also monotonically increasing, contrary to (A), (B).

#### V. Conclusion

Comparing results for different distribution functions we recognize peculiarities of the eigenmodes of the K-V distribution. While the unperturbed K-V beam has uniform density up to a sharp boundary, the round eigen oscillations are characterized by electric field perturbations which we write in terms of Legendre polynomials and which we label by an integer j, where j-1 gives the number of radial zeroes in the interior of the beam. A second integer n with n = 1...j indicates the harmonic of the cyclotron frequency at the zero intensity limit. We now observe that for j > 1 instability is connected with the occurrence of modes for which the normalized frequency  $\sigma_{jn}$  is decreasing with increasing intensity. One can show on the basis of the small signal energy of quasimonochromatic oscillations (see Ref.13) that a total energy density can be associated with this oscillation according to

(24) 
$$\overline{U} = \frac{1}{8\pi} \frac{d}{d\omega} [\omega \varepsilon(j, \omega)] \overline{E^2}$$

where  $\varepsilon(j,\omega)$  is the dielectric response function, the zeroes of which give the eigen frequencies. From Eq.(16) or (22) one can see that the sign of  $d\varepsilon/d\omega$  is given by the sign of  $d\sigma/d(1/R^2)$ , hence decreasing  $\sigma$  corresponds to negative signal energy. Coupling of positive and negative energy oscillations (for example  $\omega_{31}$  and  $\omega_{32}$ ) opens the possibility of growing modes leaving the total energy content of the beam unchanged.

For non K-V distributions the electric field eigen solutions are represented as infinite series of Legendre polynomials and truncation of the series is justified by the strong convergence for smooth distribution functions. Eigenmodes are again labelled by j, n as in the K-V case, though we find that the electric field perturbation of a given branch is no longer independent of intensity and coupling may occur now among modes belonging to different j. The broadened loss-cone distribution still has negative energy oscillations ( $\omega_{21}$ ,  $\omega_{32}$ ,  $\omega_{43}$  etc.) but with considerably weaker depression of  $\sigma$  by intensity. Instability therefore only arises if these modes couple with the positive energy oscillations  $\omega_{22}$ ,  $\omega_{33}$  etc., which we consider plasma oscillations since they are approaching  $\omega_p$  for  $1/R \rightarrow \infty$ . Though this additional coupling possibility is a new feature of the broadened distribution it appears harmless since growth is restricted to very short patches of instability with growth rates that are by an order of magnitude smaller than in the K-V case. We conclude that broadening of the K-V distribution suffices to obtain a practically stable situation.

The water-bag distribution is marginal in the sense that the modes with negative energy of the previous cases no longer exist. They turn out to be positive energy modes for the monotonic decreasing distribution, with  $\sigma_{jn}$  very close to n hence very small coherent frequency shift. It is the complete absence of negative energy modes which causes stability of distributions without loss-cone, i.e. distributions without energy inversion. This is in agreement with the stability of monotonic decreasing distribution functions as concluded from the theorem of Newcomb-Gardner, which can be applied to situations with or without neutralizing back-ground.

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#### Figure Captions

#### Fig.1

Energy Distribution profiles investigated subsequently

#### Fig.2

Plot of normalized mode frequencies  $\sigma_{jn}$  against intensity parameter 1/R for j = 1, 2, 3 and K-V distribution (A). The modes for j > 3 have been dropped; they lead to similar unstable situations.

#### Fig.3

Normalized mode frequencies  $\sigma_{jn}$  against 1/R for j = 1, 2, 3 and interaction with higher order modes  $\omega_{42}$ ,  $\omega_{43}$ ,  $\omega_{53}$  in case of a broadened loss-cone distribution (B).

#### Fig.4

Normalized frequencies  $\sigma_{jn}$  against 1/R for j = 1, 2, 3 and (stable) interaction with  $\omega_{42}$ ,  $\omega_{53}$  in case of a water-bag distribution (C).

#### Fig.5

Normalized frequencies  $\sigma_{jn}$  against 1/R for j = 1, 2, 3 and (stable) interaction with  $\omega_{42}$ ,  $\omega_{53}$  in case of a monotonic decreasing distribution (D); the branches  $\omega_{21}$  and  $\omega_{32}$  almost coincide with the lines n = 1 rsp. 2 and have been dropped.



Fig.1

Energy Distribution profiles investigated subsequently



Fig.2 Plot of normalized mode frequencies  $\sigma_{jn}$  against intensity parameter 1/R for j = 1, 2, 3 and K-V distribution (A). The modes for j > 3 have been dropped; they lead to similar unstable situations.



<u>Fig.3</u> Normalized mode frequencies  $\sigma_{jn}$  against 1/R for j = 1, 2, 3 and interaction with higher order modes  $\omega_{42}$ ,  $\omega_{43}$ ,  $\omega_{53}$  in case of a broadened loss-cone distribution (B).



Fig.4 Normalized frequencies  $\sigma_{jn}$  against 1/R for j = 1, 2, 3 and (stable) interaction with  $\omega_{42}$ ,  $\omega_{53}$  in case of a water-bag distribution (C).



Fig.5 Normalized frequencies  $\sigma_{jn}$  against 1/R for j = 1, 2, 3 and (stable) interaction with  $\omega_{42}$ ,  $\omega_{53}$  in case of a monotonic decreasing distribution (D); the branches  $\omega_{21}$  and  $\omega_{32}$  almost coincide with the lines n = 1 rsp. 2 and have been dropped.

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