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TESTS OF SU(3) IN PARTICLE REACTIONS*

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Abstract: It is shown that SU(3) symmetry is well satisfied in hadron and photoproduction reactions provided that both phase space and angular momentum barrier effects are introduced in the corrections made to take account of mass differences. Thus observed deviations from SU(3) symmetry are satisfactorily accounted for by the kinematic consequences of mass differences without the necessity of postulating additional dynamical corrections. Near the forward direction, these kinematic effects are substantial even at energies above 10 GeV.

1. INTRODUCTION

SU(3), one of the basic symmetries of strong interactions, has found only limited success in application to particle reactions. Thus, in the study of πN and $\overline{K}N$ interactions in the resonance region, whereas charge independence [SU(2)] is explicitly included in the partial wave analyses, SU(3) is generally only brought in at a late stage to group baryon states into appropriate multiplets. It is then tested quantitatively through the analysis of resonance widths [1]. Symmetry breaking as manifested by mass differences between members of a given multiplet is handled approximately by the introduction of angular momentum barrier and phase space corrections. This assumes that the effects of symmetry breaking manifest themselves almost exclusively via the external mass differences. The success of the procedure provides

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experimental justification to support this assumption.

Unfortunately the above tests of SU(3) symmetry are very incomplete in the following ways:

(1) They have been applicable to only the limited group of resonant states which have been classified unambiguously into multiplets.

(2) Resonances have been clearly identified only in the singlet, octet and decimet representations of SU(3). This leaves the question of SU(3) symmetry for amplitudes belonging to other representations, for example $\overline{10}$ or 27, an open one.

Presently experimental data on many different reactions, notably those of the form,

$$M(0^-) + B(1/2^+) \rightarrow M'(0^-) + B'(1/2^+) \quad (1a)$$

$$M(0^-) + B(1/2^+) \rightarrow M'(0^-) + B'(3/2^+) \quad (1b)$$

where $M(J^P)$, $B(J^P)$ indicate mesons and baryons of spin-parity J^P, and $B'(3/2^+)$ denotes members of the $3/2^+$ decimet, have been accumulating. In principle, were it not for the problem of how to handle symmetry breaking, reactions of the form (1a) and (1b) could be used to provide numerous tests of SU(3). Unfortunately such tests have, up to now, had only very limited success. The purpose of this paper is to examine past procedures for handling mass differences and to suggest that the invalidity of these procedures is in good part to blame for the apparent failures of SU(3). Alternative procedures based on analogies with the successful methods of compaing resonance widths are shown to remove many of the previously reported SU(3) violations, including some in photoproduction.

Before proceeding, we make a point of notation. We shall consider only reactions produced in $\pi^{\pm}p$, $K^{\pm}p$, $K^{\pm}n$, and γp collisions, and shall identify amplitudes and cross sections by giving the final-state particles. The incident state will then be obvious from charge and strangeness conservation. Thus $A(\pi^-\Sigma^+)$ and $\sigma(\pi^-\Sigma^+)$ will refer to amplitudes and cross sections for the reaction $K^-p \to \pi^-\Sigma^+$.

2. TWO BASIC EXAMPLES

In this section and the next we shall consider tests of relations whose validity depends only on SU(3) symmetry, without further assumption as to reaction mechanism.

Meshkov et al. [2] first attempted to compare with experiment the following striking predictions,

$$\frac{1}{3}\,\sigma(\pi^+\Delta^-) = \sigma(K^+\Sigma^-) = \sigma(\pi^+\Sigma^-) = \sigma(K^+\Xi^-) \quad , \qquad (2)$$

where Δ, Σ, Ξ represent members of the $3/2^+$ decimet. As defined earlier, the reactions to which Eq. (2) refers are the following,

$$\pi^-p \to \pi^+\Delta^-(1236) \qquad (3a)$$

$$\pi^-p \to K^+\Sigma^-(1385) \qquad (3b)$$

$$K^-p \to \pi^+\Sigma^-(1385) \qquad (3c)$$

$$K^-p \to K^+\Xi^-(1530) \quad . \qquad (3d)$$

The prediction (2), in the limit of exact SU(3), should hold true at all energies for each partial wave amplitude. At very high energies, all four cross sections in (2) are expected to be very small since the peripheral contributions require an exotic meson exchange and are therefore greatly suppressed. Very roughly one can therefore consider that in the high energy limit eq. (2) is trivially satisfied with all σ's equal to zero. Since SU(3) symmetry is known to apply to resonances in the 1-GeV energy region, relation (2) should also be satisfied at low energies where the cross sections are quite substantial.

The actual cross sections based on data from many experiments are shown in fig. 1 [3]. Only the errors for the reaction $K^-p \to \pi^+\Sigma^-$ are shown to avoid making the figure excessively confusing. Comparable uncertainties exist for the other reactions. It is clear from fig. 1 that, whereas $\frac{1}{3}\sigma(\pi^+\Delta^-) \approx \sigma(\pi^+\Sigma^-)$ and $\sigma(K^+\Sigma^-) \approx \sigma(K^+\Xi^-)$, there is a vast difference between these two groups, presumably caused by symmetry-breaking effects associated with mass differences. To remove in an approximate manner these symmetry-breaking effects, Meshkov et al. [2] suggested that data be compared through eq. (2) at the same Q value where,

$$Q = \sqrt{s} - \text{sum of final-state masses} \qquad (4)$$

and $\sqrt{s}$ is the center-of-mass energy. Furthermore they compared matrix elements obtained from

$$|M|^2 = \sigma s \frac{p_i}{p_f} \equiv \sigma F \qquad (5)$$

rather than cross sections. In eq. (5) p_i, p_f are initial and final center-of-mass momenta and F stands for the combination sp_i/p_f. A comparison of the values of $|M|^2 = \sigma F$ at fixed Q is given in the first figure of the paper of Meshkov et al. [2] and can be summarized by saying that the procedure implied by eq. (5) does not remove the enormous discrepancy, seen in fig. 1, between $\frac{1}{3}\sigma(\pi^+\Delta^-)$, $\sigma(\pi^+\Sigma^-)$ on the one hand and $\sigma(K^+\Sigma^-)$, $\sigma(K^+\Xi^-)$ on the other. Indeed the disagreements between the two groups of $|M|^2$ values amount to more than an order of magnitude.

The reason for this failure is the procedure implied by eqs. (4) and (5). Consider, for example, the reactions (3c) and (3d). The equality expressed in (2) would still hold if the only significant contribution came from a single resonant partial wave, for example, from an isospin-1 state belonging to a decimet. If we knew this to be the case, however, we would certainly not follow the procedure of eqs. (4) and (5); indeed, as is done in comparisons of resonance widths with SU(3) predictions, we would compare $\sigma(\pi^+\Sigma^-)$ and $\sigma(K^+\Xi^-)$

at the same <u>energy</u> (not the same Q value) and make a correction for centrifugal barrier and phase space effects,

$$\left(\frac{\sigma}{p_f^{2\ell+1}}\right)_{\pi^+\Sigma^-} = \left(\frac{\sigma}{p_f^{2\ell+1}}\right)_{K^+\Xi^-} , \qquad (6)$$

where p_f is the final center-of-mass momentum and ℓ is the orbital angular momentum in the resonance decay. Since, because of the mass differences, $p_f(K^+\Xi^-) \ll p_f(\pi^+\Sigma^-)$ at low energies, it is clear that (6) qualitatively accounts for the smallness of $\sigma(K^+\Xi^-)$ relative to $\sigma(\pi^+\Sigma^-)$. Precisely the same arguments can account for the smallness of $\sigma(K^+\Sigma^-)$ relative to $\frac{1}{3}\sigma(\pi^+\Delta^-)$. It remains to state how to compare the π^-p initiated reactions with those initiated by K^-p. This should in principle be done, not at precisely the same center-of-mass energy, but at energies which differ by the mass splitting between the $Y = 1$ and $Y = 0$ members of the multiplet to which the hypothesized resonance belongs.

These arguments, obviously sensible for resonances, should also apply to nonresonant amplitudes. From an SU(3) point of view, the only difference between resonant and nonresonant amplitudes is in their energy dependence. The relation (6), however, is independent of this dependence; to the extent that it validly represents barrier and phase-space effects, it applies at each s for each partial-wave amplitude. In principle, when one compares reactions like (3a) and (3c) with different initial states, there may also be barrier effects involving incident momenta, but these are sufficiently small that we have ignored them.

To give these ideas a somewhat more quantitative test requires in principle a partial-wave decomposition of the amplitudes. Rather than attempt this, we have considered whether an effective value of ℓ, ℓ_{eff} could be found which, when substituted into (6), would have an effect roughly equivalent to that of the actual partial waves. With the choice $\ell_{\text{eff}} = 1$, which is compatible with the angular distributions found for reaction (3c) by Huwe [3], the results shown in fig. 2 are obtained. The three curves drawn are the predictions for $\ell_{eff} = 1$ for reactions (3a, b and d) assuming for (3c) the hand-drawn curve shown in fig. 1. In relating (3a,b) to (3c) we have used the same incident laboratory momentum (which corresponds to differences in c.m. energy roughly corresponding to typical differences between strange and nonstrange baryons within a given multiplet in the momentum range under study). While the model is obviously very crude and the data for (3b,d) somewhat sparse, it is entirely clear that the apparent SU(3) violations found in the analysis of Meshkov et al. can be removed with the type of procedure we have followed.

A perhaps more interesting low-energy test of SU(3) is involved in the confrontation of the following relations with experiment,

$$\sigma(\pi^+\Sigma^-) = \sigma(K^o\Xi^o) \quad , \qquad (7a)$$

$$\sigma(K^+\Sigma^-) = \sigma(K^o\Xi^-) \quad , \qquad (7b)$$

where in (7a,b) the Σ and Ξ are members of the $1/2^+$ nucleon octet.

With Meshkov et al.'s method of comparison [eqs. (4), (5)], Berge et al. [4], and Harari and Lipkin [5] found that whereas (7b) was approximately satisfied, (7a) was violated by several orders of magnitude. It is immediately recognized that, in (7b), both reactions are highly endothermic and are inhibited by mass differences to about the same extent. On the other hand, in (7a), the left-hand side represents a somewhat exothermic reaction whereas the right-hand side contains again a highly endothermic process; it is therefore not surprising that in this case the mass differences mask the symmetry. Fortunately, in (7a) we are better off than in (2) in that detailed partial-wave analyses of the $K^-p \to \pi^+\Sigma^-$ reaction have been made [6]. Therefore, angular momentum barrier corrections can be made more precisely than in the case of the reactions (3). The solutions of Kane [6] corrected partial wave by partial wave for barrier and phase-space effects [via eq. (6)] have been compared with existing experimental data on the reaction $K^-p \to K^0\Xi^0$ [4,7]. This comparison, which takes Kane's solutions as he had found them with no attempt to change them to improve the fit to the $K^0\Xi^0$ final state, yields the following results.

Of Kane's seven acceptable solutions, one fits the $K^0\Xi^0$ data far better than any other. The χ^2 for this fit is 97 for 44 degrees of freedom to be compared with χ^2 values of 150 or more for the other solutions. The cross sections and one of the angular distributions predicted by that solution are compared with the data of refs. 4 and 7 in figs. 3 and 4. The cross-section fit is fair in that the magnitude is about right, but the detailed shape is not accurately reproduced. This is almost surely a consequence of the fact that the fit to $\pi^+\Sigma^-$ does not determine very precisely the low partial waves to which the $K^0\Xi^0$ system is particularly sensitive. The angular distribution comparison of fig. 4 is excellent.

It is worth pointing out that it may be useful to incorporate the $\pi^+\Sigma^-$ and $K^0\Xi^0$ states into the same phase-shift analysis. Although statistics on the latter are limited, they do provide information on low partial waves which may not be readily obtainable even from high statistics in $\pi^+\Sigma^-$.

3. THE ELASTIC SCATTERING TRIANGULAR SU(3) RELATIONS

It can be shown easily that in the limit of exact SU(3), one expects the following relations between amplitudes,

$$A(\pi^+p) + A(K^+\Sigma^+) = A(K^+p) \quad , \tag{8a}$$

$$A(\pi^-p) + A(\pi^-\Sigma^+) = A(K^-p) \quad , \tag{8b}$$

where Σ^+ refers to the nucleon octet.

These relationships are very similar to the SU(2) relations,

$$A(K^-p) + A(\overline{K}^0n) = A(K^-n) \quad , \tag{9a}$$

$$A(K^+n) + A(K^0p) = A(K^+p) \quad . \tag{9b}$$

Relations (8a) and (8b) have been previously subjected to tests of the form [5],

$$|A(K^+\Sigma^+)| \geq \Big| |A(\pi^+ p)| - |A(K^+ p)| \Big| \quad , \qquad (10a)$$

$$|A(\pi^-\Sigma^+)| \geq \Big| |A(K^- p)| - |A(\pi^- p)| \Big| \quad . \qquad (10b)$$

At t = 0, the inequalities (10a,b) can be somewhat sharpened via the optical theorem,

$$\left[\frac{d\sigma}{dt}(K^+\Sigma^+)\right]_{t=0} \geq 0.051 \, |\sigma_T(\pi^+ p) - \sigma_T(K^+ p)|^2 \quad , \qquad (11a)$$

$$\left[\frac{d\sigma}{dt}(\pi^-\Sigma^+)\right]_{t=0} \geq 0.051 \, |\sigma_T(\pi^- p) - \sigma_T(K^- p)|^2 \quad , \qquad (11b)$$

where σ_T are total cross sections for the systems in parentheses in mb, and $d\sigma/dt$ is in mb/(GeV)^2. Although these inequalities are relatively weak tests of SU(3), they are in fact strongly violated as is clear from figs. 5 and 6 where the solid lines represent the right sides of (11a,b) and the data points the left sides [8].

Meshkov and Yodh [9] suggested that this discrepancy could be fixed up by a procedure analogous to that described for the inelastic processes (3a-d) involving a comparison at fixed Q value and a correction to go from cross section to matrix element. This procedure however does not make much sense: it implies, for example for relation (8a), a comparison of the $\pi^+ p \to K^+\Sigma^+$ reaction near the 1950 MeV resonance with the $\pi^+ p \to \pi^+ p$ reaction in the neighborhood of the 1236 MeV resonance. Such a comparison obviously cannot have anything to do with SU(3). On the contrary it seems evident that these two reactions must be compared at the <u>same</u> energy and then related with possibly some slight shift in the energy scale to the reaction $K^+ p \to K^+ p$.

It is convenient to consider first the relation (8b) and the corresponding inequality (11b). As seen in fig. 6 the right side is larger than the left, in contradiction to the inequality (11b), but perhaps the most remarkable feature is the near-constancy of the right side. Indeed as noted by Barger and Phillips the difference $[\sigma_T(\pi^- p) - \sigma_T(K^- p)]$ changes very little between 3 GeV/c and 60 GeV/c [10]. The natural interpretation is that in the limit of exact SU(3), $\sigma_T(\pi^- p)$ and $\sigma_T(K^- p)$ are equal, and, consequently, if (8b) is satisfied the amplitude for $K^- p \to \pi^- \Sigma^+$ is real in the forward direction. The remarkable analogy to reaction (9b) is evident: in (9b) there are no mass-difference effects, and the $K^+ p$ and $K^+ n$ total cross sections are directly measured to be equal, from which it follows immediately that the forward K^+ charge-exchange amplitude is real.

This analogy is, of course, even more intimate. The reality of the $K^+ n \to K^o p$ amplitude is connected to the exchange degeneracy of the ρ and A_2 exchange and to the consequent cancellation of imaginary contributions to the forward amplitude. Similarly, the reality of the forward $K^- p \to \pi^- \Sigma^+$ amplitude arises from the exchange degeneracy of the K_V (vector K) and K_T (tensor K). Since the $K^- p$ or $\pi^- \Sigma^+$ systems are not exotic and hence do have resonances, it is not as obvious why this cancellation occurs. It can be understood theoretically from duality diagrams [11] or from simple arguments based on factorization [12].

Since $K^- p \to \pi^- \Sigma^+$ contains resonances, the reality of its forward amplitude implies that the couplings to baryon resonances must contain positive and negative contributions which cancel. That this actually occurs in the energy region where phase shift analyses have been made has been shown in detail by Schmid and Storrow [12]. Thus, the validity of SU(3) as exemplified by eq. (8b) is successfully tested by the direct demonstration that the reality of $K^- p \to \pi^- \Sigma^+$ in the forward direction implied by the combination of (8b) and the experimental equality [in the SU(3) symmetry limit] of $\sigma_T(\pi^- p)$, $\sigma_T(K^- p)$ is in accord with partial-wave analyses. One can, in principle, make a much more complete test, namely, use partial-wave analyses of all three reactions in (8b) to compare, partial wave by partial wave, the real and imaginary parts of these reaction amplitudes. The proper way of doing this would be to combine $A(K^-p)$ and $A(\pi^-\Sigma^+)$ at the same incident energy, making a phase space and barrier correction partial wave by partial wave in the latter reaction. The value of $A(K^-p) - A(\pi^-\Sigma^+)$ obtained thereby should according to (8b) be about the same, for each partial wave, as $A(\pi^-p)$ although the energy scale on the two sides of (8b) will differ by the SU(3) mass splitting. An analysis somewhat akin to this will be discussed in connection with the relation (8a).

Using the partial-wave analysis of Kalmus et al. [13], for $\pi^+p \to K^+\Sigma^+$, we have multiplied each partial-wave amplitude by $(p_i/p_f)^\ell$, added the imaginary parts, and combined with the values of $\sigma_T(\pi^+p)$ at the <u>same</u> momentum to compare with $\sigma_T(K^+p)$ according to (8a). Because $\sigma_T(K^+p)$ is nearly constant, the problem of relating the energy scales on the left and right sides of (8a) disappears. The results obtained with the Kalmus et al. [13] solutions 192B and 209B in the momentum range between 1.2 and 1.8 GeV/c are shown in fig. 7 [14]. The total π^+p cross section (solid curve) has a large peak produced by the 1950-MeV resonance. After combining with the imaginary part of $\pi^+p \to K^+\Sigma^+$ according to (8a), one is left with the squares and dots in fig. 7 corresponding to the two chosen solutions of Kalmus et al. These display a nearly constant cross section of magnitude around 22 mb. The constancy of the cross section is in agreement with the energy dependence of $\sigma_T(K^+p)$, and tests SU(3) principally through the couplings of the $\Delta(1950)$, which dominates the variation with energy of $\sigma_T(\pi^+p)$ in the region under study. The magnitude of the constant cross section is about 4 mb higher than the actual value of $\sigma_T(K^+p)$ (22 mb instead of 18 mb); this is precisely the cross-section difference seen between the "equal" π^-p and K^-p total cross sections in the discussion of (8b). Thus, in the same sense that $\sigma_T(\pi^-p) = \sigma_T(K^-p)$, we can consider $\sigma_T(K^+p) = 22$ mb instead of 18 mb, the difference of 4 mb coming from mass differences between π and K. It follows that not only does the $\Delta(1950)$ obey SU(3) in its couplings, but so do the background amplitudes which make up the K^+p elastic scattering.

We now consider high-energy tests of (8a). Using the exchange degeneracy noted to be true for K_V, K_T couplings to $K^-p \to \pi^-\Sigma^+$, we go to the line-reversed reaction $\pi^+p \to K^+\Sigma^+$ where the reversal of the K_V relative to the K_T coupling leads to an expected imaginary forward amplitude (just as for $K^-p \to \overline{K}^0n$). In this case, (11a) can be strengthened into an equality,

$$\left[\frac{d\sigma}{dt}\,(\pi^+p \to K^+\Sigma^+)\right]_{t=0} = 0.051[\sigma_T(\pi^+p) - \sigma_T(K^+p)]^2 \quad . \qquad (12)$$

As mentioned earlier, comparison of the solid line and data points in fig. 5 shows that (12) is not satisfied. However, our previous discussion suggests that we should replace $\sigma_T(K^+p)$ by its value corrected for the π, K mass differences; namely, the difference $\sigma_T(\pi^-p) - \sigma_T(K^-p) = 4.4$ mb should be added to the experimental values of $\sigma_T(K^+p)$. We now obtain for the right side of (13) the dash-dot curve in fig. 5, which still disagrees with the experimental values of the left side, although the two appear to merge at very high energies. This brings us to the angular momentum barrier effects. Although these effects might at first sight appear negligible because of the high energy, this is <u>not</u> true. As the energy increases, so does the leading angular momentum, and consequently the barrier factor changes very slowly. This point of view is in accord with the considerations of Harari [15] that the non-Pomeron-exchange contributions are highly peripheral.

To calculate the barrier factor, one can determine relevant angular momenta using a procedure similar to that of Davier and Harari [15]. Thus the contributions to amplitudes like $\pi^+p \to K^+\Sigma^+$ should be dominated by a peripheral term of the general form $J_0(a\sqrt{-t})$, which corresponds to an orbital angular momentum $\ell = pa$ where p is the center-of-mass momentum. We choose "a" so as to make the first zero of the Bessel function J_0 correspond to the cross-over between the π^+p and K^+p differential cross section, namely $-t = 0.3$ $(GeV)^2$, and ℓ follows correspondingly. Multiplying the dashed-dot line of fig. 5 by barrier factors $(p_{\Sigma K}/p_{\pi p})^{2\ell+1}$ with ℓ chosen as above, we obtain the dashed line, which agrees very well with the experimental data. It is striking to note that even at energies as high as 14 GeV, a barrier correction of about 50% is required.

We now consider in slightly greater detail the 4-mb total-cross-section difference associated in our previous analysis with just the mass difference between the kaon and the pion. Since SU(3)-violating effects have been interpreted in terms of the kinematic consequences of mass differences it is natural to ask if this effect could have a similar interpretation. Indeed mass differences in the inelastic channels might well be expected through phase space and angular momentum barrier effects to lead to this kind of reduction. As the incident energy increases, the correction factor in any one channel should tend to unity roughly as $(1 - \text{constant}/\sqrt{s})$, but since the number of channels increases, it is perhaps not unreasonable that this K-π total cross section difference be slowly varying as implied by the experimental data. If this approach is valid, the logarithmic energy dependence of multiplicities would eventually imply that the K and pion total cross sections should asymptotically tend to the same value.

4. t-CHANNEL SU(3) RELATIONS

The validity of the relations (2), (7), (8) discussed so far requires only that SU(3) be an exact symmetry. Many other such equations can be deduced, but they are generally triangular or even more complicated relations and hence are difficult to test quantitatively, even with a satisfactory prescription for taking care of symmetry breaking. However, if one adds the experimentally known fact that amplitudes corresponding to exotic t-channel exchanges drop rapidly toward zero near the forward direction, it is possible to deduce simplified predictions which can be subjected to experimental test. These predictions, the t-channel SU(3) relations, are most easily derived from direct application of SU(3) to the expected t-channel exchanges; but, by exhibiting their connection to the more complicated relationships which depend only on SU(3) symmetry (and not the vanishing of exotic exchange amplitudes), one can see more clearly the conditions for validity of these t-channel predictions and the methods of correcting for mass difference effects.

Thus consider as an example the following triangular relation, based only on SU(3) symmetry,

$$A(K^o\Delta^{++}) + \sqrt{3}\ A(\pi^-\Sigma^+) = \sqrt{3}\ \{A(K^+\Xi^-) - A(K^o\Xi^-)\} \quad , \tag{13a}$$

where Δ, Σ, Ξ all refer to members of the $3/2^+$ decimet. The amplitudes on the right side of (13a) involve exotic exchanges and hence go much more rapidly to zero at high energy and low t than each of the amplitudes on the left side. Consequently one can write,

$$\left[\frac{d\sigma}{dt}\ (K^o\Delta^{++}) = 3\ \frac{d\sigma}{dt}\ (\pi^-\Sigma^+)\right]_{\substack{s \text{ large} \\ t \text{ small}}} \quad . \tag{13b}$$

(13b) is a t-channel SU(3) relation which can also be easily derived by relating the ρ, A_2 exchanges of the left side to K_V, K_T exchanges of the right side. Other examples of relations based purely on SU(3), and the corresponding t-channel relations are the following,

$$A(\overline{K}^o\Delta^-) - \sqrt{3}\ A(K^+\Sigma^+) = -\sqrt{3}\ A(K^o\Xi^-) \tag{14a}$$

$$\left[\frac{d\sigma}{dt}\ (\overline{K}^o\Delta^-) = 3\ \frac{d\sigma}{dt}\ (K^+\Sigma^+)\right]_{\substack{s \text{ large} \\ t \text{ small}}} \tag{14b}$$

$$\frac{d\sigma}{dt}(K^o\Delta^{++}) + 3\frac{d\sigma}{dt}(K^+\Sigma^+) = \frac{d\sigma}{dt}(\pi^o\Delta^{++}) + 3\frac{d\sigma}{dt}(\eta\Delta^{++}) \qquad (15a)$$

$$\left[\frac{d\sigma}{dt}(K^o\Delta^{++}) + \frac{d\sigma}{dt}(\overline{K}^o\Delta^{-}) = \frac{d\sigma}{dt}(\pi^o\Delta^{++}) + 3\frac{d\sigma}{dt}(\eta\Delta^{++})\right]_{\substack{s\ \text{large} \\ t\ \text{small}}} \qquad (15b)$$

$$A(\pi^+p) - A(\pi^-p) + A(K^-p) - A(K^-n) + A(K^+n) - A(K^+p) = 2A(\pi^+\Sigma^-) - A(K^+\Xi^-) \qquad (16a)$$

$$\left[A(\pi^+p) - A(\pi^-p) + A(K^-p) - A(K^-n) + A(K^+n) - A(K^+p) = 0\right]_{\substack{s\ \text{large} \\ t\ \text{small}}} \qquad (16b)$$

$$\left[\sigma_T(\pi^+p) - \sigma_T(\pi^-p) + \sigma_T(K^-p) - \sigma_T(K^-n) + \sigma_T(K^+n) - \sigma_T(K^+p) = 0\right]_{s\ \text{large}} . \qquad (16c)$$

In relations (14a,b) and (15a,b), the symbols Δ, Σ, Ξ refer to members of the $3/2^+$ decimet, whereas in (16a) Σ and Ξ are evidently members of the $1/2^+$ nucleon octet. Relation (15b) follows from (15a) if one applies the result (14b).

Neglecting for the moment mass differences, we consider, in the limit of exact SU(3), under what conditions (13b), (14b), (15b) and (16b,c) are expected to be valid. Operationally these relations are satisfied when the terms on the right sides of (13a), (14a) and (16a) are negligible relative to each of the individual terms on the left sides. Roughly the fractional error in (13b), (14b), (15b) and (16b,c) is of the order of the ratio of the exotic exchange amplitude to the normal exchange amplitude at the same energy. For an error of say 10%, the exotic exchange cross section must then be down to less than 1% of the non-exotic exchange cross section. In very rough terms, this implies that t-channel relations will only be valid above 3-4 GeV/c over a t range which is sufficiently limited to maintain the large ratio of non-exotic to exotic exchange cross section. In the case of relations (16a,b), the left-hand side is essentially a superposition of πN, KN and $\overline{K}N$ charge-exchange amplitudes. Again (16b) is expected to be a good approximation when the exotic exchange amplitudes on the right side are of the order of a few percent of the charge-exchange amplitudes; thus 3 to 4 GeV also appears to be a reasonable lower limit here.

Since the t-channel relations are just approximations of the exact SU(3) predictions, it is clear that the procedures of correction for mass difference effects discussed earlier are equally applicable. First of all we note that relation (16b,c) might be expected to require negligible mass difference corrections. This follows from the fact that the left side involves the sum of three charge-exchange amplitudes for which the reaction products have just the same masses as the incident particles. Indeed it is already well known that at high energy (16c) is approximately satisfied [16]. We have attempted to make a somewhat more careful study of the left side of (16) using the following procedures:

(i) Below 3.5 GeV/c, precisely-known total cross-section data, with $\sigma_T(\pi^+ p)$, $\sigma_T(\pi^- p)$, etc. taken from the same experiment have been used to minimize the systematic errors which are the main source of uncertainty [8]. Not every measured datum has been used, but only enough to indicate clearly the trend.

(ii) At the higher momenta, we have used the well-established feature that the $K^- p$, $K^- n$ forward amplitudes are almost completely imaginary, as is their difference, and hence the optical theorem coupled to accurate experimental data on the charge-exchange reaction $K^- p \to \overline{K}^0 n$ near the forward direction permits the most precise determinations of $\sigma_T(K^- p) - \sigma_T(K^- n)$ [17]. We have taken $\sigma_T(K^+ p) = \sigma_T(K^+ n)$ at these higher momenta.

The results are shown in fig. 8. The principal uncertainties are systematic and are estimated to be less than ±0.5 mb. Figure 8 indicates a behavior just in accordance with the expectations from eqs. (16a,b,c): the left side of (16c) is essentially zero above 3 GeV/c, but deviates strongly at lower momenta because of the contributions of the exotic exchange amplitudes whose imaginary part at $t = 0$ gives the shape seen in the figure.

As mentioned earlier, the test of eq. (16c) is relatively simple because mass difference effects play little role. For the other relations (13, 14, 15) this is not true. Indeed corrections of the sort discussed earlier are necessary, both for the t-channel tests (13b, 14b, 15b) as well as in any attempt to estimate the corrections introduced by the contributions of the right-hand sides of (13a) and (14a).

Thus consider (14b). Experimental data from which a comparison of $K^- n \to \overline{K}^o \Delta^-$ and $\pi^+ p \to K^+ \Sigma^+$ can be made exist in the incident momentum range from 3 to 5 GeV/c [18]. The measured ratio of cross sections near $t = 0$ for these two reactions is close to 9/1 rather than 3/1 as predicted by (14b). However the $K^+\Sigma^+$ reaction is much more endothermic than the $\overline{K}^o\Delta^-$ reaction, and hence the experimental result is not unexpected. Indeed a barrier and phase space correction based on the interaction radius already used for the analysis of (12) gives an additional correction of a factor of 3 which just takes care of the observed discrepancy.

Going now to consideration of (13b), we note that unlike (14b) the K-π and Σ-Δ mass differences tend to cancel each other rather than reinforce each other; consequently the barrier and phase space corrections are estimated to be only 10-15% at a few GeV/c, and one might expect that (13b) is well satisfied. Experimentally, however, at least for momenta between 3 and 8 GeV/c, there is also about a factor of 9/1 rather than 3/1 between left and right side of (13b) [19]. This discrepancy is difficult to understand within the framework of what has been said, and the only simple possibility for its interpretation is that in (13a) the contribution of the exotic exchange amplitudes even at 8 GeV/c is not negligible. This is perhaps surprising but not at all ruled out. Thus if we assume that the forward exotic exchange amplitudes in (13a) are essentially real (hence in phase with the non-exotic exchange terms) and reinforce each other, a forward cross section for each exotic exchange term of just 2% of the $K^o\Delta^{++}$ cross section is enough to account for the discrepancy. Thus a forward cross section of a few microbarns/(GeV)^2 for the KΞ amplitudes would be needed. If the exotic exchange amplitudes were in fact mainly real here, they would not significantly contribute to (14a), where the other amplitudes are largely imaginary, explaining why (14b), with suitable mass difference corrections, works well while (13b) does not. If the above explanation of the discrepancy in (13b) is correct one would certainly expect that at higher energies this discrepancy should rapidly disappear. There is an indication of this at the highest measured energies. Thus, from the data of Berlinghieri et al. and Birnbaum et al. [19], one finds

$$\sigma(K^0\Delta^{++})\Big|_{|t|<0.4(\mathrm{GeV}/c)^2} \approx 38\ \mu\mathrm{b} \quad \text{at } 12.7\ \mathrm{GeV}/c$$

and

$$\sigma(\pi^-\Sigma^+)\Big|_{|t|<0.4(\mathrm{GeV}/c)^2} \approx 6\ \mu\mathrm{b} \quad \text{at } 16\ \mathrm{GeV}/c \quad .$$

Extrapolating $\sigma(K^0\Delta^{++})$ to 16 GeV/c gives an estimated cross section of 24 μb and a ratio $\sigma(K^0\Delta^{++})/\sigma(\pi^-\Sigma^+)$ of about 4/1 rather than the 9/1 seen at lower energies. This is of course far from conclusive, and more precise and extensive data at momenta above 12 GeV/c will be needed to resolve the question.

Finally we make a few comments concerning (15a) and (15b). As has been shown by Mathews [20], (15b) works fairly well without mass difference corrections. This is not surprising since all the reactions in (15b) involve the same baryon mass differences; the boson mass differences turn out to play a relatively small role. In view of the discussion of (14b), relation (15a) without mass difference corrections will fail whereas, with such corrections boosting the $\pi^+p \rightarrow K^+\Sigma^+$ contribution, it works. It should be noted that earlier tests of (15a) neglecting barrier effects but using the Meshkov et al. method of comparison also were successful [2]. However, in view of the clear evidence that that method of comparing is incorrect, the success of these tests is somewhat fortuitous.

5. PHOTOPRODUCTION

By taking the photon as a U-spin singlet it is possible to derive SU(3) relations for photoproduction reactions. Such a relation particularly susceptible to experimental test has been given by Levinson et al. [21],

$$-\sqrt{2}\ A(\pi^+n) = \sqrt{3}\ A(K^+\Lambda^0) + A(K^+\Sigma^0) \tag{17}$$

where the baryons all belong to the $1/2^+$ nucleon octet, and all three final states are produced in a photon-proton collision. In the absence of information about phases, the relation (17) can be tested by checking experimentally the inequalities,

$$\sqrt{2}\ |A(\pi^+n)| \leq \sqrt{3}\ |A(K^+\Lambda^0)| + |A(K^+\Sigma^0)| \tag{18a}$$

$$\sqrt{2}\ |A(\pi^+n)| \geq \sqrt{3}\ |A(K^+\Lambda^0)| - |A(K^+\Sigma^0)|\ . \tag{18b}$$

Elings et al. [22] tested relations (18) at 3-4 GeV and found no violation for momentum transfers $-t \geq 0.25\ (\text{GeV})^2$. Boyarski et al. [23,24] obtained extensive data on the three final states in (17) at energies 5, 8, 11 and 16 GeV over a range of angles which includes the forward direction, and found very substantial violations of (18a) for $-t < 0.1\ (\text{GeV})^2$. Unlike Meshkov et al., both Elings et al. and Boyarski et al. made their comparisons at the same energy rather than the same Q value, substituting for the $|A|$'s the square roots of the appropriate differential cross sections. Neither group made angular momentum barrier corrections, though Elings et al. did make a small (~ 10%) phase space correction to take account of mass difference effects.

In the spirit of our considerations of sec. 3 it seems appropriate to examine what happens to the apparent SU(3) violation reported by Boyarski et al. [24] when approximate angular momentum barrier corrections are introduced to take more accurate account of the effect of the mass differences.

To do so we consider the $\gamma p \rightarrow \pi^+ n$ angular distributions reported by Boyarski et al. [23] and correct the left side of eq. (17) by introducing the attenuation expected from the barrier factors relevant to the right side of that equation.

Specifically, the $\gamma p \rightarrow \pi^+ n$ data in this energy range exhibit an extremely sharp forward peak (for $-t \lesssim 0.01\ (\text{GeV})^2$) superposed onto a much more gentle distribution. The latter has roughly a behavior of the form e^{Bt} with $B \sim 2\text{-}3\ (\text{GeV})^{-2}$. In the forward direction the sharp peak and the gentle distribution make roughly comparable contributions to the cross section.

Considering first the very sharp peak and attributing it to the contribution of a few large angular momenta, we can, in the spirit already discussed in sec. 3, represent it roughly in the form $J_o(a\sqrt{-t})$ with $a \approx 14\ (\text{GeV})^{-1}$. The corresponding angular momenta for photon energies between 5 and 16 GeV go from 20 to 38, and the attenuations of the ΛK and ΣK final states due to the corresponding barriers vary from a factor of 35 to a factor of 6. In all cases the expected contribution of this high angular momentum amplitude to the ΛK and ΣK cross sections is less than 20% and is essentially in the noise of the measurements. This calculation accords with the experimental fact that no sharp forward peak is seen in the strange particle events. We then neglect this contribution to the left side of (17).

We now consider the contribution near $t = 0$ of the more slowly varying part of the $\gamma p \rightarrow \pi^+ n$ differential cross section. Boyarski et al. [23] quote values for this part of $d\sigma/dt$ at $t = 0$ which are given in the second column of table I. The third column of the same table shows the quantity

$$\frac{1}{2}\left[\sqrt{3(d\sigma/dt)(K^+\Lambda^o)} + \sqrt{(d\sigma/dt)(K^+\Sigma^o)}\right]^2$$

at $t = 0$ derived from Boyarski et al.'s study of strange particle photoproduction. It is clear from comparison of these two columns that without mass difference correction the right side of (18a) is substantially less than the left side, in violation of the SU(3) prediction. Thus even the removal of the contribution of the sharp forward peak in the $\pi^+ n$ final state does not eliminate the apparent SU(3) violation near the forward direction.

We now introduce mass difference corrections using precisely the same interaction radius as for the $\pi^+ p \to K^+ \Sigma^+$ reaction discussed in sec. 3. The resulting barrier and phase space attenuation factors range from 0.36 to 0.56 in going from 5 to 16 GeV/c incident momentum. These factors are almost the same for $K\Lambda$ as for $K\Sigma$, and we have just used an average of the two. With these corrections, the quantity $\frac{d\sigma}{dt}(\pi^+ n)$ at $t = 0$ is reduced to the values given in the fourth column of table I. Comparison of the third and fourth columns indicates that in the absence of mass differences,

$$\left\{2 \frac{d\sigma}{dt}(\pi^+ n) = \left[\sqrt{3 \frac{d\sigma}{dt}(K^+\Lambda^o)} + \sqrt{\frac{d\sigma}{dt}(K^+\Sigma^o)}\ \right]^2 \right\}_{t=0} \quad . \tag{19}$$

Thus not only is SU(3) satisfied in the forward direction, but the $K^+\Lambda^o$ and $K^+\Sigma^o$ amplitudes appear to be essentially in phase. This is reminiscent of the situation for the reactions $\pi^+ n \to K^+\Lambda^o$, $\pi^+ n \to K^+\Sigma^o$ which by arguments of duality and exchange degeneracy similar to those made in sec. 3 for the reaction $\pi^+ p \to K^+\Sigma^+$ are both expected to have imaginary amplitudes in the forward direction.

6. CONCLUSIONS

From the analysis presented in this paper, we draw the following conclusions.

(1) SU(3) symmetry seems to be well satisfied in high energy reactions, provided one applies mass difference corrections of the same sort as those made in the comparison of resonance widths. With such corrections, previously noted discrepancies disappear. The one failure which appears to remain is the

t-channel relation (13b); it may be explained by a large contribution from the exotic exchange amplitudes in (13a).

(2) It follows that there is direct experimental evidence that SU(3) symmetry is satisfied for the 27 and $\overline{10}$ "exotic" representations as well as for the singlet, octet and decimet representations. It also follows that the observed violations of SU(3) can largely be interpreted in terms of kinematic effects of external mass differences without having to introduce further dynamical symmetry breaking.

(3) In the peripheral region near the forward direction, angular momentum barrier factors play an important role in modifying SU(3) predictions even at quite high energies. Thus reactions of the form $\pi N \to (\Sigma,\Lambda)K$ and $\gamma N \to (\Sigma,\Lambda)K$ are attenuated relative to reactions $\pi N \to \pi N$ and $\gamma N \to \pi N$ by factors of 2 to 3 over and above the Clebsch-Gordan coefficients in the incident momentum range between 5 and 15 GeV/c.

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Table I. SU(3) test in photoproduction.

Photon momentum (GeV/c)	$[\frac{d\sigma}{dt}(\pi^+n)]_{t=0}$ mb/GeV^2 (a)	$\frac{1}{2}\left[\sqrt{3\,\frac{d\sigma}{dt}(K^+\Lambda^o)}+\sqrt{\frac{d\sigma}{dt}(K^+\Sigma^o)}\right]^2_{t=0}$ mb/GeV^2 (b)	$[\frac{d\sigma}{dt}(\pi^+n)]_{t=0}$ mb/GeV^2 (c)
5	1.98±0.09	1.0	0.71±0.04
8	0.66±0.03	0.28	0.28±0.013
11	0.31±0.014	0.20	0.15±0.007
16	0.143±0.008	0.10	0.08±0.005

(a) Extrapolated from fit in region $0.07 < |t| < 0.6$ GeV^2.

(b) Errors of ±20% are estimated from uncertainties given in ref. [24].

(c) Quoted errors are just measurement errors from ref. [23]; no uncertainties for correction procedure are included.

FIGURE CAPTIONS

Fig. 1. Cross sections for the reactions $\pi^- p \to \pi^+\Delta^-(1236)$, $K^+\Sigma^-(1385)$, $K^- p \to \pi^+\Sigma^-(1385)$, $K^+\Xi^-(1530)$. The solid curve is a hand-drawn fit through the $\pi^+\Sigma^-(1385)$ data. Errors are shown only for $\pi^+\Sigma^-(1385)$ to minimize confusion.

Fig. 2. Predicted and experimental cross sections using the curve of fig. 1 and setting $\ell_{eff} = 1$. See text for details.

Fig. 3. Comparison of $K^- p \to \pi^+\Sigma^-$ and $K^- p \to K^0\Xi^0$ cross sections. Curves are from fit B2 of Kane [6].

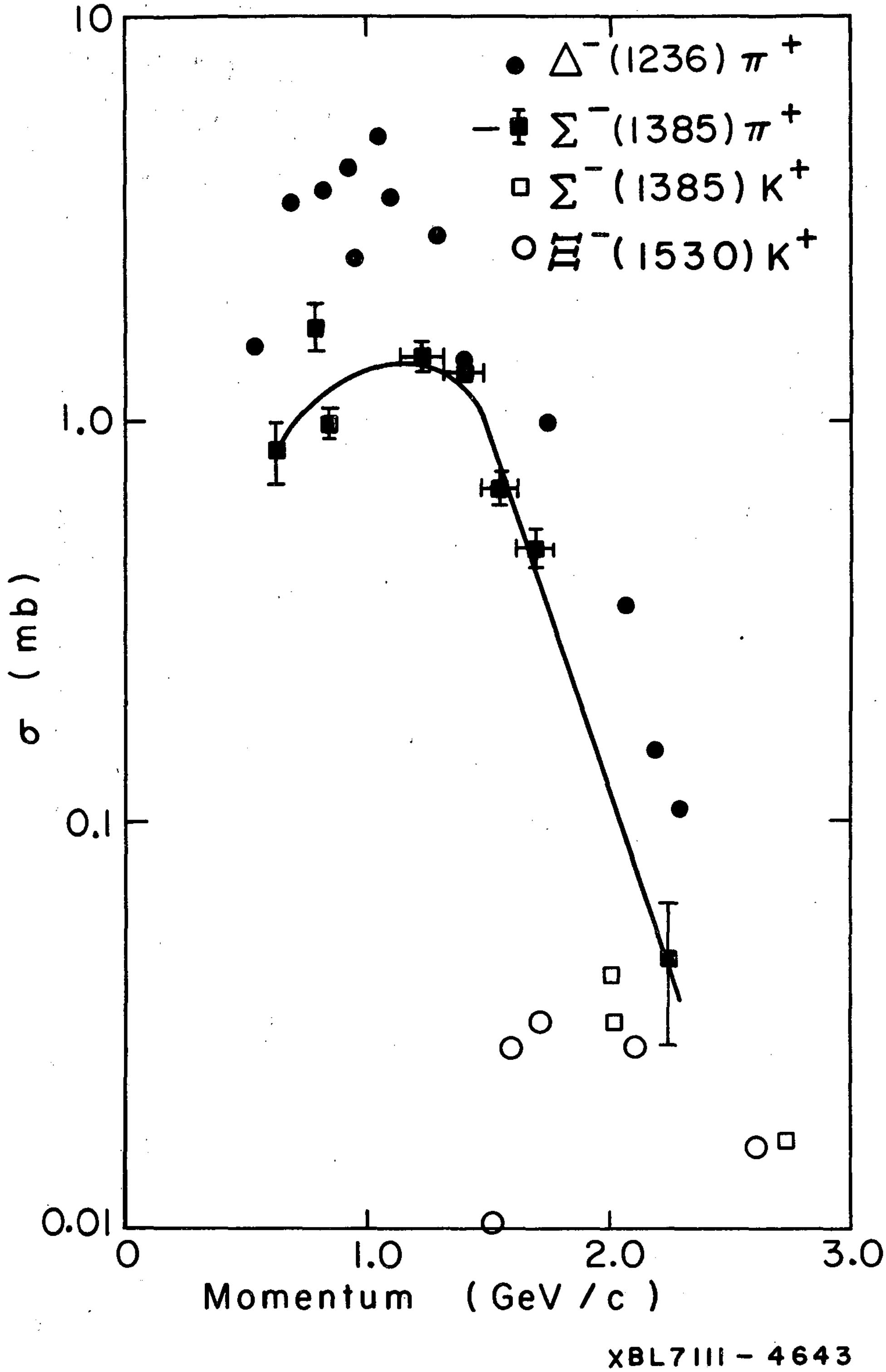
Fig. 4. Comparison of $K^- p \to \pi^+\Sigma^-$ and $K^- p \to K^0\Xi^0$ angular distributions at 1.5 GeV/c. Curves are from fit B2 of Kane [6].

Fig. 5. Test of relations (11a) and (12). Solid curve is right side of (12) and data points are left side of (12). Other curves are explained in the text.

Fig. 6. Test of relation (11b). Solid curve is right side of (11b) and data points are left side of (11b).

Fig. 7. Test of relation (8a). The solid curve is $\sigma_T(\pi^+ p)$ and the dashed curve is $\sigma_T(K^+ p) + 4.4$ mb. The squares and dots are values of the $K^+ p$ total cross section predicted by (8a) using solutions 209B (dots) and 192B (squares) of Kalmus et al. [13].

Fig. 8. Test of relation (16c). The ordinate $\Delta\sigma$ is the left side of (16c).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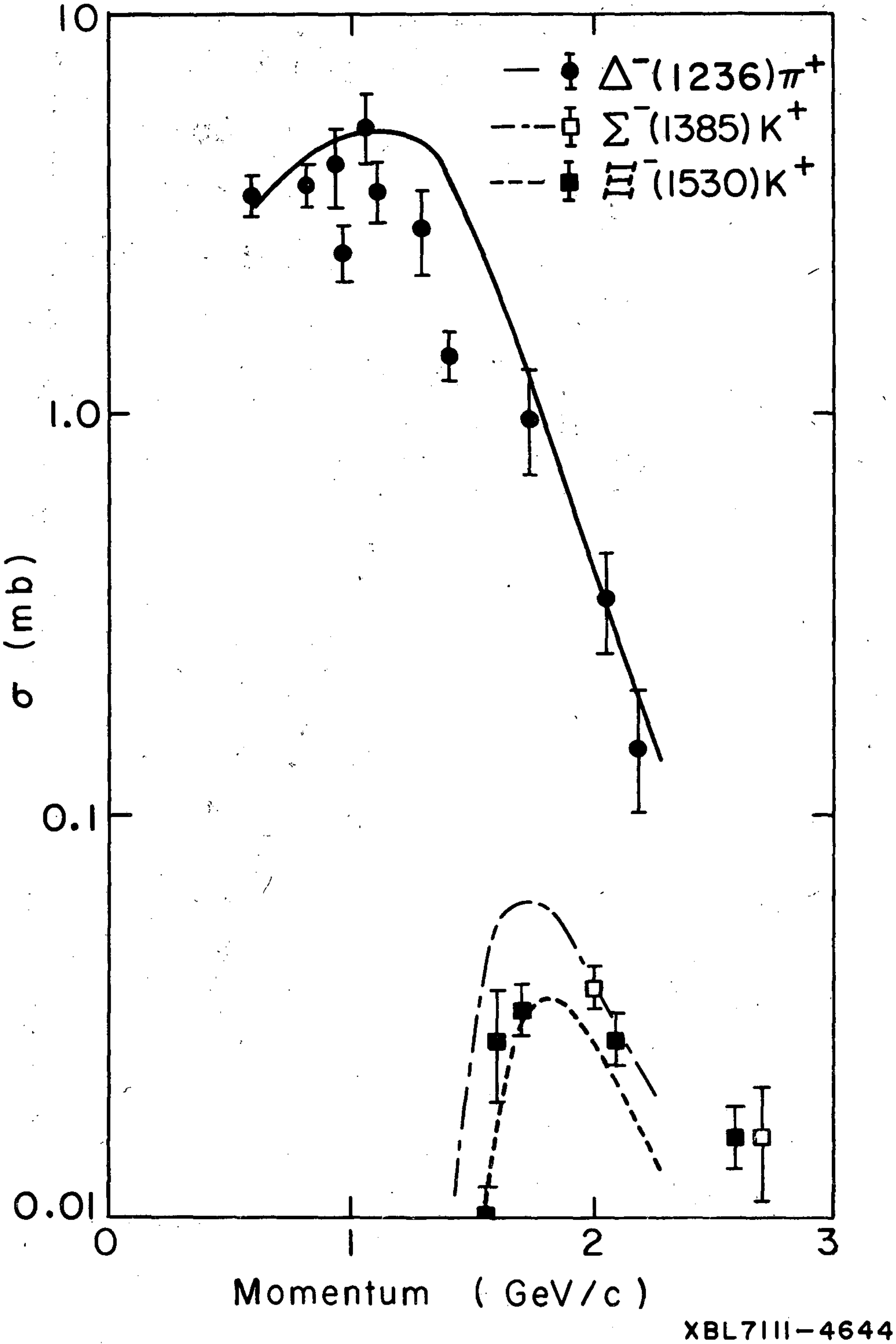
0.01
σ (mb)
0
1.0
2.0
3.0
Momentum (GeV/c)
Δ⁻(1236) π⁺
Σ⁻(1385) π⁺
Σ⁻(1385) K⁺
Ξ⁻(1530) K⁺
XBL7111-4643

Fig. 1



10
1.0
0.1
0.01
0
1
2
3
σ (mb)
Momentum (GeV/c)
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Σ⁻(1385)K⁺
Ξ⁻(1530)K⁺
XBL7111-4644

Fig. 2

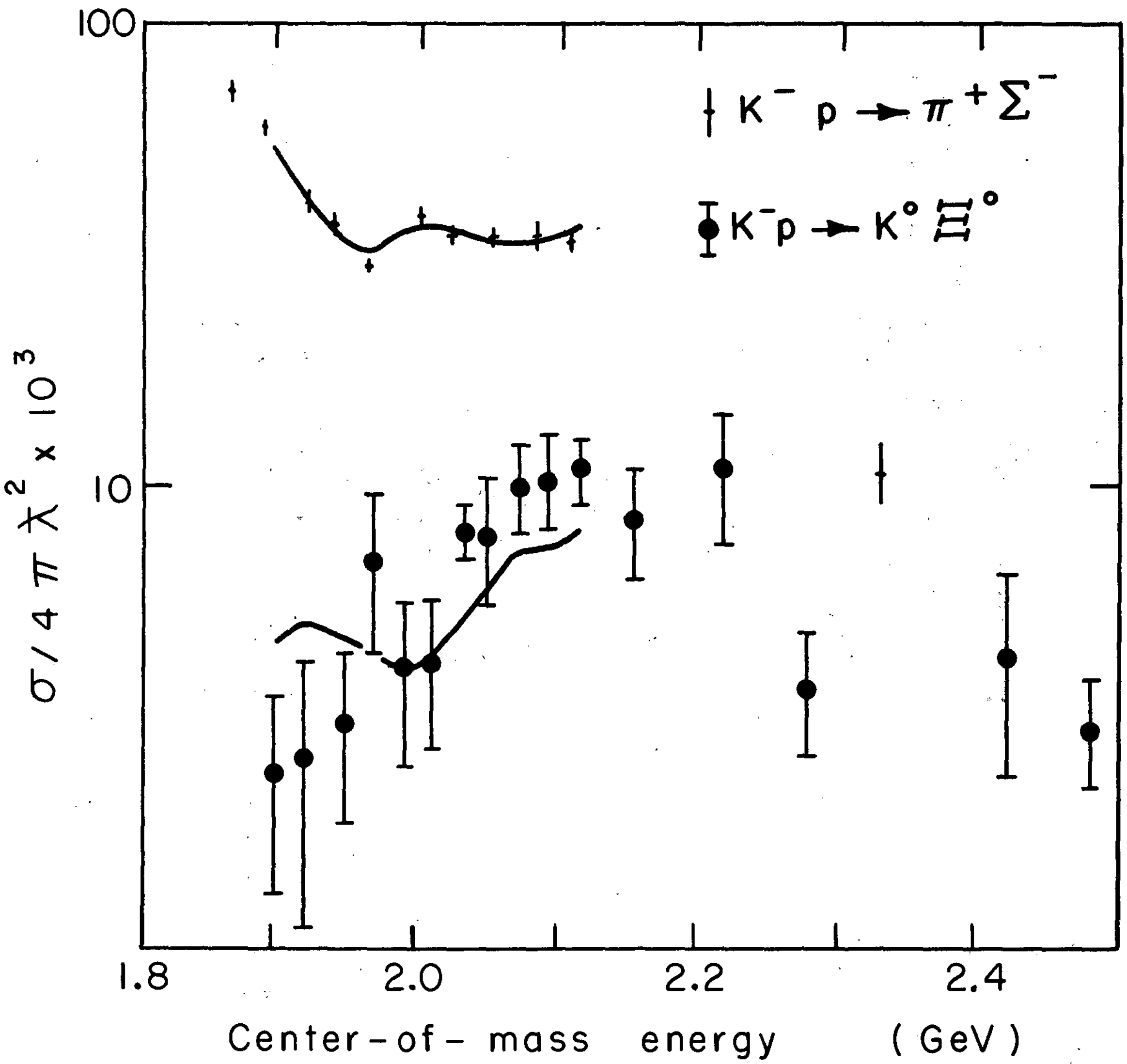

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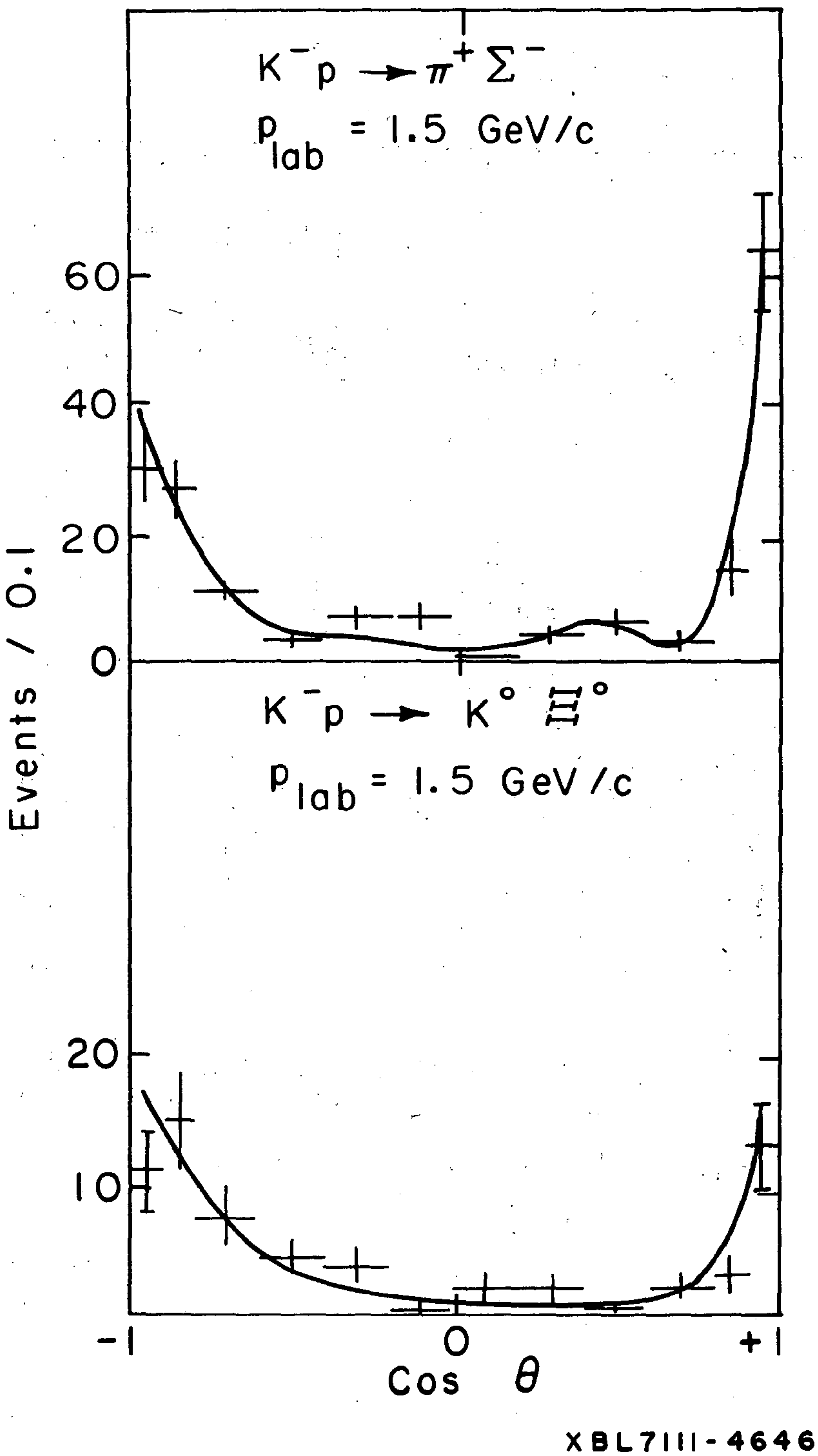
Fig. 3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p_lab = 1.5 GeV/c
K⁻p → K° Ξ°
p_lab = 1.5 GeV/c
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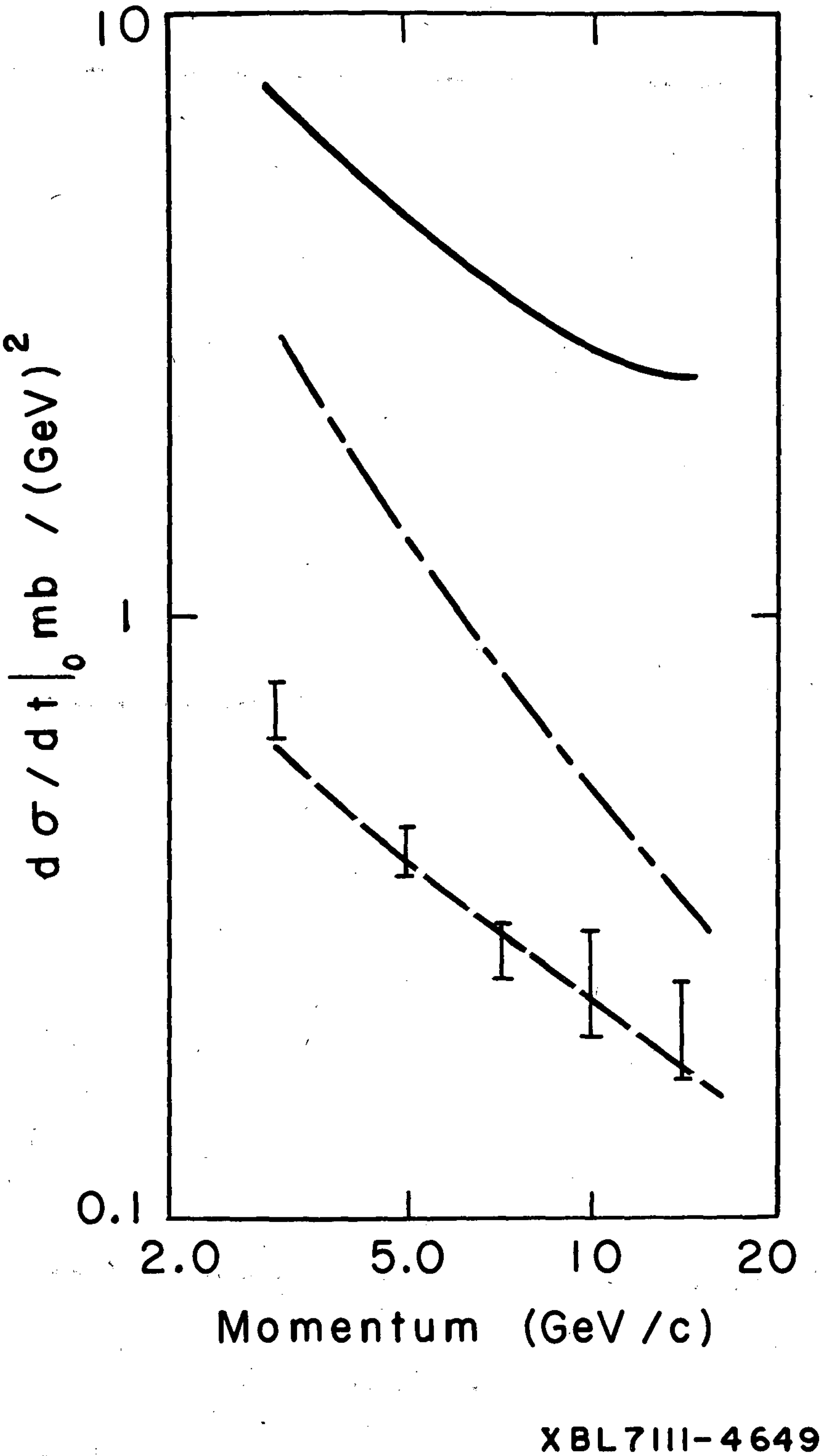
40
20
0
20
10
-1
0
+1
Cos θ
XBL7111-4646

Fig. 4



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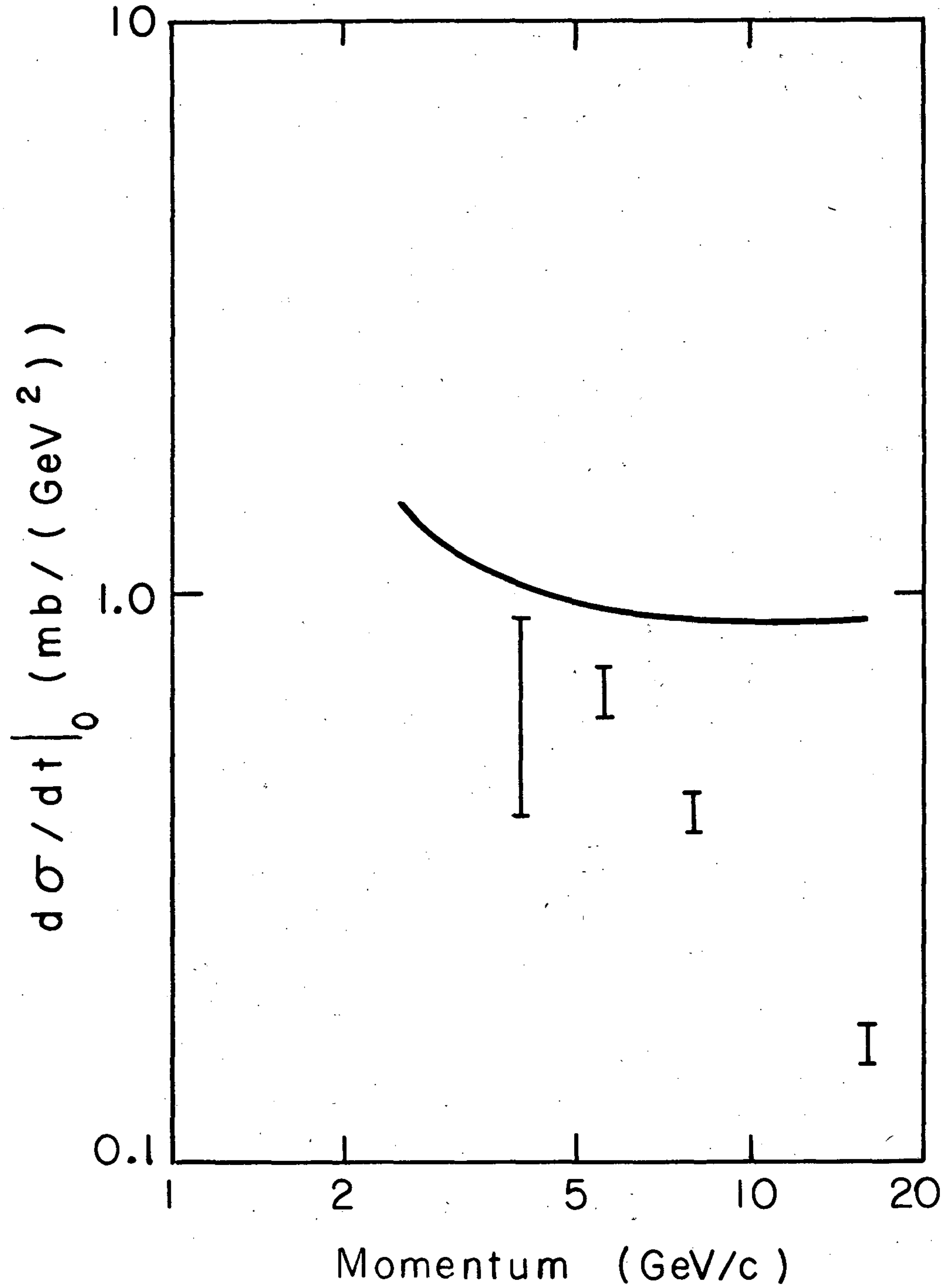

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Fig. 6

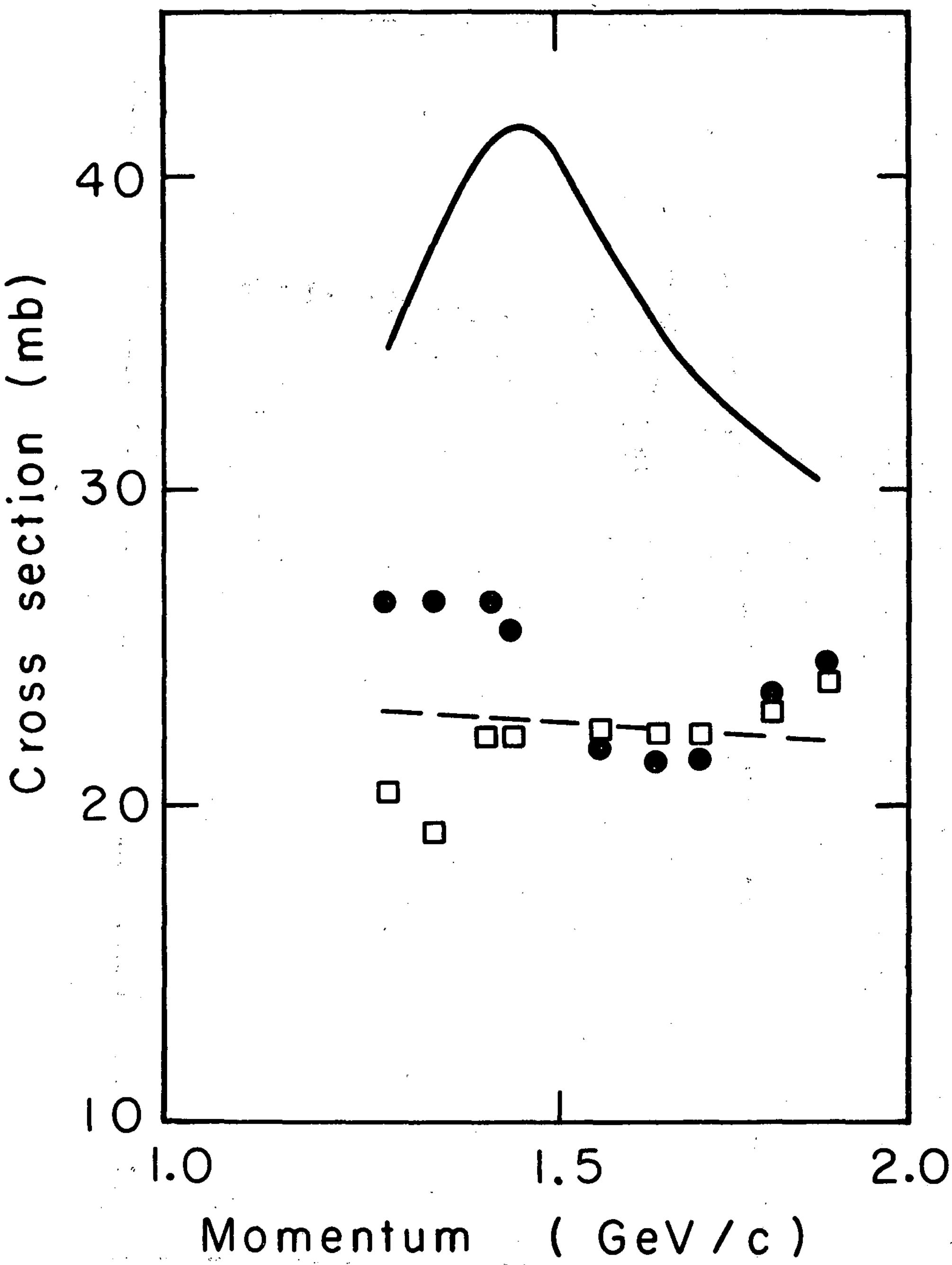

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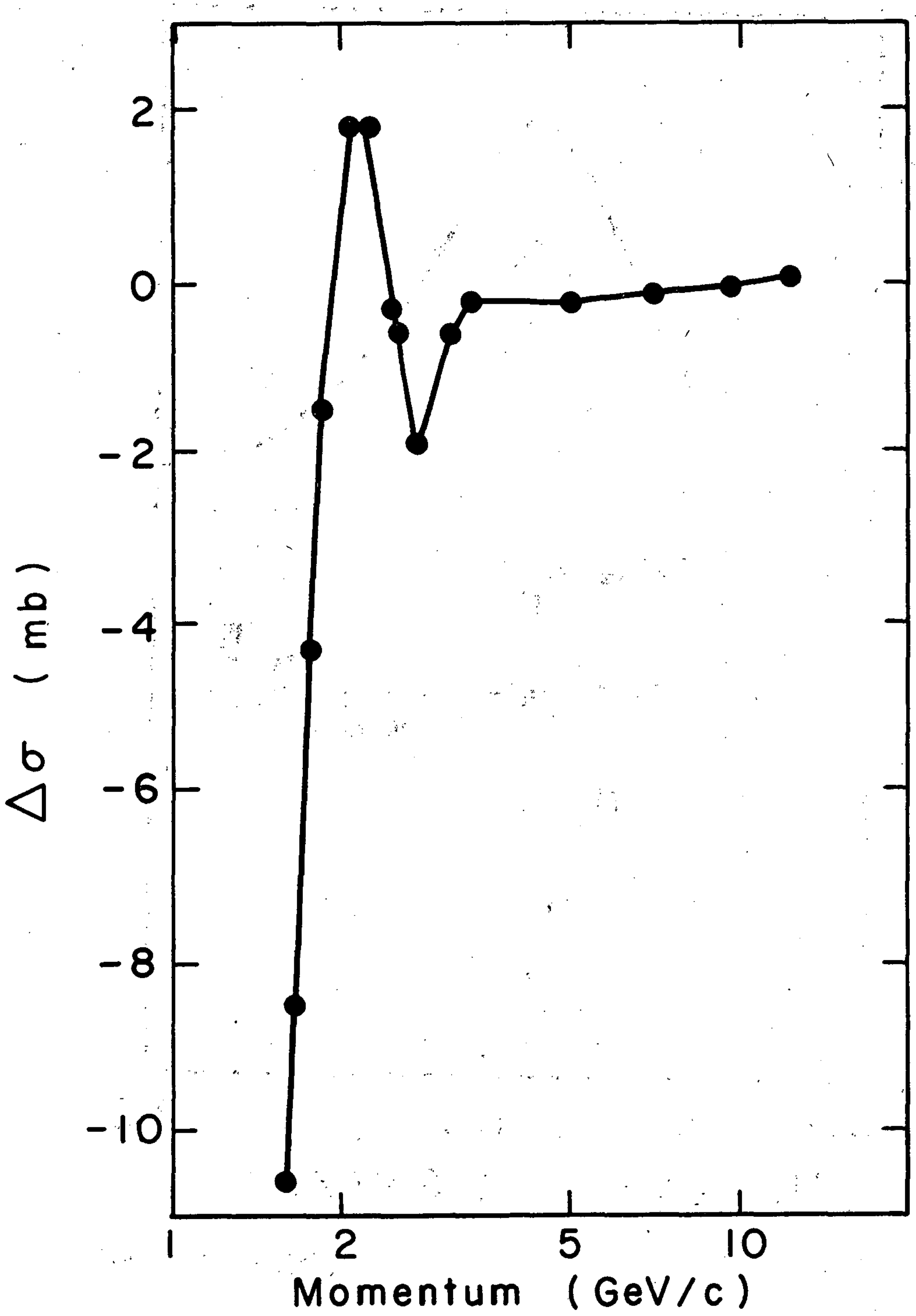
Fig. 7



XBL7111-4650

Fig. 8

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