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Preservice Mathematics Teachers’ Conceptualization of the Standards for Mathematical Practice: A Study Across Four Universities

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Preservice Mathematics Teachers' Conceptualization of the Standards for Mathematical Practice: A Study Across Four Universities

A dissertation submitted in partial satisfaction of the

requirements for the degree Doctor of Philosophy

in Education

by

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March 2021

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Preservice Mathematics Teachers' Conceptualization of the Standards for Mathematical Practice: A Study Across Four Universities

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McLean, M., Nation, J., **Spina, A**., Susko, T., Harlow, D., & Bianchini, J. (2020) The importance of community for narrowing the gender gap in engineering: An analysis of engineering identity development in elementary students. Submitted to *Journal of Pre-College Engineering Education Research* (J-PEER).

Aminger, W., Hough, S., Roberts, S., **Spina, A**., Meier, V., Pajela, H., McLean, M., & Bianchini, J. (2020). Preservice Secondary Science Teachers' Implementation of an NGSS Practice: Using Mathematics and Computational Thinking. *Journal of Science Teacher Education* (JSTE).

Spina, A., Macias, M., & Reimer, P. (2020) How Facilitators Define, Design, and Implement Effective Early Childhood Mathematics Professional Development. *Forty-Second Annual Meeting of PME-NA – The North American Chapter of the International Group for the Psychology of Mathematics Education*, Mazatlan, MX.

Spina, A., Mireles-Rios, R., & Roberts, S. A. (2019). Perceptions of Teachers as Attachment Figures at a Boarding School. *Journal of Education and Human Development*.

Pulgar, J., **Spina, A** ., Rios, C., & Harlow, D**.** (2019). Conextual Details, Cognitive Demand and Kinematic Concepts: Exploring Concepts and Characteristics of Student-Generated Problems in a University Physics Course. *Physics Education Research Conference*, Provo, UT.

Pulgar, J., McBeath, J., **Spina, A**., & Harlow, D. (2018). Resolución de problemas desde las ciencias como una actividad creativa: explorando un modelo de creatividad y efectividad grupal. (Science problem solving as a creative activity: exploring a model of group creativity and effectiveness). *Proceedings of the Congreso Internacional Interdisciplinariedad y Desarrollo.* Barranquilla, Colombia: Corporación Universitaria Americana.

Roberts, S. A. & **Spina, A**. (2017). Teachers' Developing Questioning to Support Linguistically Diverse Students in Junior High School Mathematics Classrooms. *Thirty-ninth Annual Meeting of PME-NA – The North American Chapter of the International Group for the Psychology of Mathematics Education*, Indianapolis, IN.

Peer Reviewed Book Chapters

Harlow, D., Skinner, R., Hansen, A., Nation, J., Barriault, C., Pulgar, J., McLean, M., **Spina, A.**, Prud'homme, A. Creating STEM learning opportunities through partnerships. In C. Johnson, M. Mohr-Schroeder, T. Moore, & L. English (Eds.), *Handbook of Research on STEM Education.*

Policy Reports

Roberts, S. A. & **Spina, A**. (2018). *ONDAS faculty professional development seminar 2016-2017 findings and recommendations*. Santa Barbara, CA: University of California, Santa Barbara.

Journal Articles Under Peer Review

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Kirksey, J.J., Hancock, K. & **Spina, A**. Instructional practices and academic outcomes in Australia's inclusive classrooms: Evidence from two nationally representative cohorts of primary school children. Submitted to *Teacher and Teacher Education.*

Macias, M., Caldwell, B., **Spina**, A., Rosenbaum, L., Gribble, J., & Reimer, P. Community walks: connecting to out of school experiences. Submitted to *Science and Children.*

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STEM Teachers for English Language Learners: Excellence & Retention (STELLER)
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- Study on preservice STEM teachers at UCSB, focusing on how they understand English learners, as well as the Common Core State Standards in Mathematics and the Next Generation Science Standards to ELLs in high-need schools.
- Responsible for collecting and analyzing data and creating papers for publication and conference presentations.

Santa Barbara Unified School District Evaluation
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- Study on teachers participating in the Santa Barbara chapter of the UC-wide Mathematics Project.
- Professional development offered to elementary and secondary teachers and administrators on mathematics education with a focus on teaching and supporting English learners.
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Teaching Fellows Research Initiative
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- Responsible for collecting and transcribing data.

CONFERENCE PRESENTATIONS

Spina, A., Macias, M., & Reimer, P. (2021, June). How Facilitators Define, Design, and Implement Effective Early Childhood Mathematics Professional Development. Forty-Second Annual Meeting of PME-NA – The North American Chapter of the International *Group for the Psychology of Mathematics Education*, Virtual Conference

Spina, A., Verma, A., Carpenter, S., & Bianchini, J. (2021). *Preservice Mathematics Teachers Conceptualization of the Standards for Mathematical Practice*. [Paper Session]. To be presented at the 2021 AERA Annual Meeting, Virtual Conference.

Spina, A., Macias, M., & Reimer, P. (2021) *Challenges and Adaptations in Early Childhood Mathematics Professional Development During the COVID-19 Pandemic.* [Paper Session]. To be presented at the 2021 AERA Annual Meeting, Virtual Conference.

Spina, A., Meier, V., Carpenter, S., & Bianchini, J (2020). Preservice Secondary Science and Mathematics Teachers' Understanding of How to Multilingual Learners: A Comparison Across Programs. Presented at the 2020 Gevirtz Graduate School of Education Research Symposium, Santa Barbara, CA.

Macias, M., **Spina, A.,** Rosenbaum, L. F., Reimer, P., Gribble, J., & Caldwell. B. (2020). Professional Development Aligned with Leaders' Goals in Early Childhood STEM Education: A Collaboration Between Researchers and Practitioners. Presented at the 2020 Gevirtz Graduate School of Education Research Symposium, Santa Barbara, CA.

Bennett, M., Galisky, J., Hyun, F., Macias, M., **Spina, A.,** Pattison, S., & Bianchini, J. Exploring How Secondary Preservice Teachers Implement Engineering. Presented at the 2020 Gevirtz Graduate School of Education Research Symposium, Santa Barbara, CA.

Spina, A. D., Meier, V. & Bianchini, J. (2020, Apr 17 - 21) *Preservice Secondary Science and Mathematics Teachers' Understanding of How to Teach English Learners: A Comparison Across Programs* [Roundtable Session]. AERA Annual Meeting San Francisco, CA http://tinyurl.com/qrhrf42 (Conference Canceled)

Spina, A. D., Macias, M., Rosenbaum, L., Gribble, J., Caldwell, B. & Reimer, P. (2020, Apr 17 - 21) *Professional Development for Leaders in Early Childhood STEM Education: A Collaboration Between Researchers and Practitioners* [Paper Session]. AERA Annual Meeting San Francisco, CA http://tinyurl.com/v8hcw37 (Conference Canceled)

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Spina, A., Macias, M., Iveland, A., & Britton, T. (2020). Teachers' Understanding and Implementation of Equitable Instructional Strategies with the NGSS. Paper to be presented at the 2020 National Association of Research in Science Teaching annual meeting, Portland, OR **Conference cancelled due to COVID-19*

White, M., **Spina, A**., Iveland, A., & Britton, T. (2020) NGSS Instructional Practice and Impact on Student Classroom Experience: A Comparative Case Study. Paper to be presented at the 2020 National Association of Research in Science Teaching annual meeting, Portland, OR **Conference cancelled due to COVID-19*

Macias, M., **Spina, A**., Iveland, A., & Britton, T. (2020) Student Opportunities to Enact Epistemic Agency Through Engagement with the NGSS Science and Engineering Practices. Paper to be presented at the 2020 National Association of Research in Science Teaching annual meeting, Portland, OR **Conference cancelled due to COVID-19*

Kirksey, J.J., Hancock, K.J., & **Spina, A.** (2019). Instructional Practices and Academic Outcomes in Australia's Inclusive Classrooms: Evidence from Two Nationally Representative Cohorts of Primary School Children. *Poster presentation at the annual conference of the Association for Public Policy Analysis and Management*, Denver, CO.

Spina, A., Pulgar, J., Nation, M., & Harlow D. (2019). Math and Making: How a knot typing makerspace activity can immerse children in the Standards for Mathematical Practice. (PERC) Summer Meeting, Provo, Utah.

Harlow, D., Skinner, R., Hansen, A., Muller, A., Macias, M., Nation, J., Marckwordt, J., Pulgar, J., **Spina, A**., Arevalo, A., Lucas, K. (2019, July). Museum-based physics education research through research-practice partnerships (RPPs). Parallel session at 2019 Annual Meeting of the Physics

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Pulgar, J., **Spina, A**., Rios, C. & Harlow D. (2019). Contextual Details, Cognitive Demand, and Kinematic Concepts: Exploring concepts and characteristics of student-generated concepts. (AAPT) Summer Meeting, Provo, Utah.

Spina, A., & Roberts, S. A. (2019). Investigating Coherence Among Mathematics Teachers & Administrators in Professional Development. Poster session presented at the 2019 American Educational Researcher Association annual meeting, Toronto, Canada.

Roberts, S.A. & **Spina, A**. (2019) Students' and Teachers' Responses to Questions Meant to Support Mathematical Discourse. Paper presented at the 2019 American Educational Researcher Association annual meeting, Toronto, Canada.

Meier, V., **Spina, A**., & Bianchini, J (2019) Investigating Preservice Secondary Science and Mathematics Teachers' Understanding of English Learners. Paper presented at the 2019 American Educational Researcher Association annual meeting, Toronto, Canada.

Meier, V., **Spina, A**., & Bianchini, J (2019) Investigating Preservice Secondary Science and Mathematics Teachers' Understanding of Multilingual Learners. Paper presented at the 2019 American Association for Applied Linguistics annual meeting, Atlanta, Georgia.

Aminger, W., Hough, S., Meier, V., McLean, M., Moon, S., Carpenter, S.L., Roberts, S**., Spina, A**., & Bianchini, J. A. (2019). Changes in Preservice Secondary Science Teacher' Understanding of Principles of Equitable Reform-Based Science Instruction. Paper presented at the 2019 meeting of the National Association for Research in Science Teaching, Baltimore, MD.

Roberts, S.A. & **Spina, A**. (2018). Changes in Questioning Practices to Support English Learners. Paper presented at the National Council of Teachers of Mathematics Research Conference annual meeting, Washington D.C.

Spina, A. (2018). The Influence of Teacher Gender on Female Students' Math Self Efficacy. Presented at the American Educational Researcher Association annual meeting, New York City, NY.

Aminger, W., Hough, S., Meier, V., McLean, M., Moon, S., Carpenter, S.L., Roberts, S., **Spina, A**., & Bianchini, J. A. (2018). Investigating pre-service science teachers' understanding of an NGSS practice: Using mathematics and computational thinking. Paper presented at the meeting of the National Association for Research in Science Teaching, Atlanta, GA.

Pulgar, J., **Spina, A**., & Harlow D. (2018). Assessing group effectiveness: A case study in physics education. *American Association of Physics Teachers* (AAPT) Winter Meeting, San Diego, CA.

Roberts, S. A. & **Spina, A**. (2017, October). Teachers' Developing Questioning to Support Linguistically Diverse Students in Junior High School Mathematics Classrooms. Thirty-Ninth Annual Meeting of PME-NA – The North American Chapter of the International *Group for the Psychology of Mathematics Education*, Indianapolis, IN.

Spina, A. (2017, April). The Influence of Teacher Gender on Female Students' Math Self Efficacy and Identity. Poster session to be presented at the American Educational Researcher Association annual meeting, San Antonio, TX.

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ABSTRACT

Preservice Mathematics Teachers' Conceptualization of the Standards for Mathematical Practice: A Study Across Four Universities

by

Alexis Deidre Spina

Preservice mathematics teachers today are charged with the challenging yet vital task of learning how to authentically engage their future students in the Standards for Mathematical Practice. These eight practices specify the various levels of competence that mathematics teachers of all levels should explore to see their student's flourish. While they are a critical component of the Common Core State Standards for Mathematics and are based on longstanding processes and mathematical proficiencies established in mathematics education, there is little research to date that has looked at how preservice mathematics teachers understand, implement, and engage their students in these practices.

This dissertation sought to understand how preservice mathematics teachers conceptualized the Standards for Mathematical Practice throughout their teacher education. Data collection consisted of initial and follow-up interviews and completed edTPA portfolios from 47 preservice mathematics teachers from three separate cohorts across four different university teacher education programs in California. I analyzed the data for this dissertation in three separate ways. First, I looked at which of the Standards for Mathematical Practice preservice mathematics teachers reported in interviews as the most important to teach, as well as which

ones they needed further help in understanding (Chapter 2). I looked to see how their responses compared across initial and follow-up interviews, as well across the four universities participants were from. Participants overwhelmingly considered Practices 1 and 3 to be the most important to teach regardless of pre- or post-interview and which university they attended. The same applied to which practices preservice mathematics teachers reported needing further support in understanding, which were Practices 4 and 8. Next, I investigated how the participants in this study incorporated the Standards for Mathematical Practice in their edTPA (Chapter 3). By coding preservice mathematics teachers' edTPA video clips according to the $MCOP^2$, I was able to create a correlation coefficient between participants' $MCOP^2$ scores with their edTPA instructional commentary and overall edTPA score, therefore understanding the extent to which preservice mathematics teachers incorporated the practices in their edTPA. Finally, I looked at how a subset of preservice mathematics teachers drew on the Standards for Mathematical Practice through the use of cognitively demanding tasks in their edTPA (Chapter 4). Through the correlation I created in Chapter 3, I selected six participants, three of who scored high and three low, and investigated the levels of cognitive demand of the tasks they incorporated into their edTPA planning section, as well as which Standards for Mathematical Practice these tasks incorporated. Participants who received high edTPA scores tended to include tasks that were of higher demand, which not only reflected a higher number of the practices, but also engaged students in Practices 1 and 3. Those who received low scores had a higher frequency of low demand tasks, which included less practices overall, with Practices 1 and 3 frequently absent. My work extends the literature on preservice mathematics teachers' understanding of the Standards for Mathematical Practice and supports the need for future research that will continue to support the successful education of our preservice mathematics teachers.

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Chapter 1: *Introduction*

Over the last 40 years, mathematics education in the United States has been driven by variations of practices and standards to help categorize the mathematical skills and knowledge that are crucial for students to be successful in K-12 mathematics. These mathematical skills and knowledge are based on processes such as problem solving, reasoning and proof, and modeling. The most recent of these attempts to standardize mathematics are the Standards for Mathematical Practice, which are at the beginning of the Common Core State Standards for Mathematics (CCSS-M; National Governors Association Center for Best Practices and the Council of Chief State School Officer [NGA & CCSSO], 2010). The aim of these eight practices (see Table 1 below) is to guide mathematics educators at all levels on how to develop the processes and proficiencies required of students in order to be successful in mathematics.

Table 1

CCSSM Standards for Mathematical Practice

Practice	*Practice Name*
1. Make sense of problems and persevere in solving them.	SMP 1: Problem-Solving
2. Reason abstractly and quantitatively.	SMP2: Reasoning
3. Construct viable arguments and critique the reasoning of others	SMP3: Argumentation
4. Model with mathematics	SMP4: Modeling
5. Use appropriate tools strategically	SMP 5: Tools
6. Attend to Precision	SMP 6: Precision
7. Look for and make use of structure	SMP 7: Structure
8. Look for and express regularity in repeated reasoning	SMP 8: Regularity

While the mathematics education community has made headway in developing and documenting practices for successful mathematics instruction, we are still facing the fundamental issue of developing rich, deep mathematical understanding in teacher candidates. By shifting away from a focus solely on the content knowledge necessary for teaching mathematics towards one based on mathematical practices that embrace content knowledge, the vision of mathematics teacher education needs to be reimagined (McDonald et al., 2013). Many researchers and mathematics teacher educators are turning their focus towards ways to better support preservice teachers in learning how to teach students both mathematics concepts and content by engaging them in practices, focusing specifically on the Standards for Mathematics Practice as a guiding path for this (McDonald et al., 2013; Zeichner, 2012). In light of this movement, teacher education programs are faced with the challenge of better preparing preservice mathematics teachers by providing them with the practices and tools they need in order to answer the call for high-quality mathematics teachers (Baldinger, 2014; Ball & Forzani, 2009).

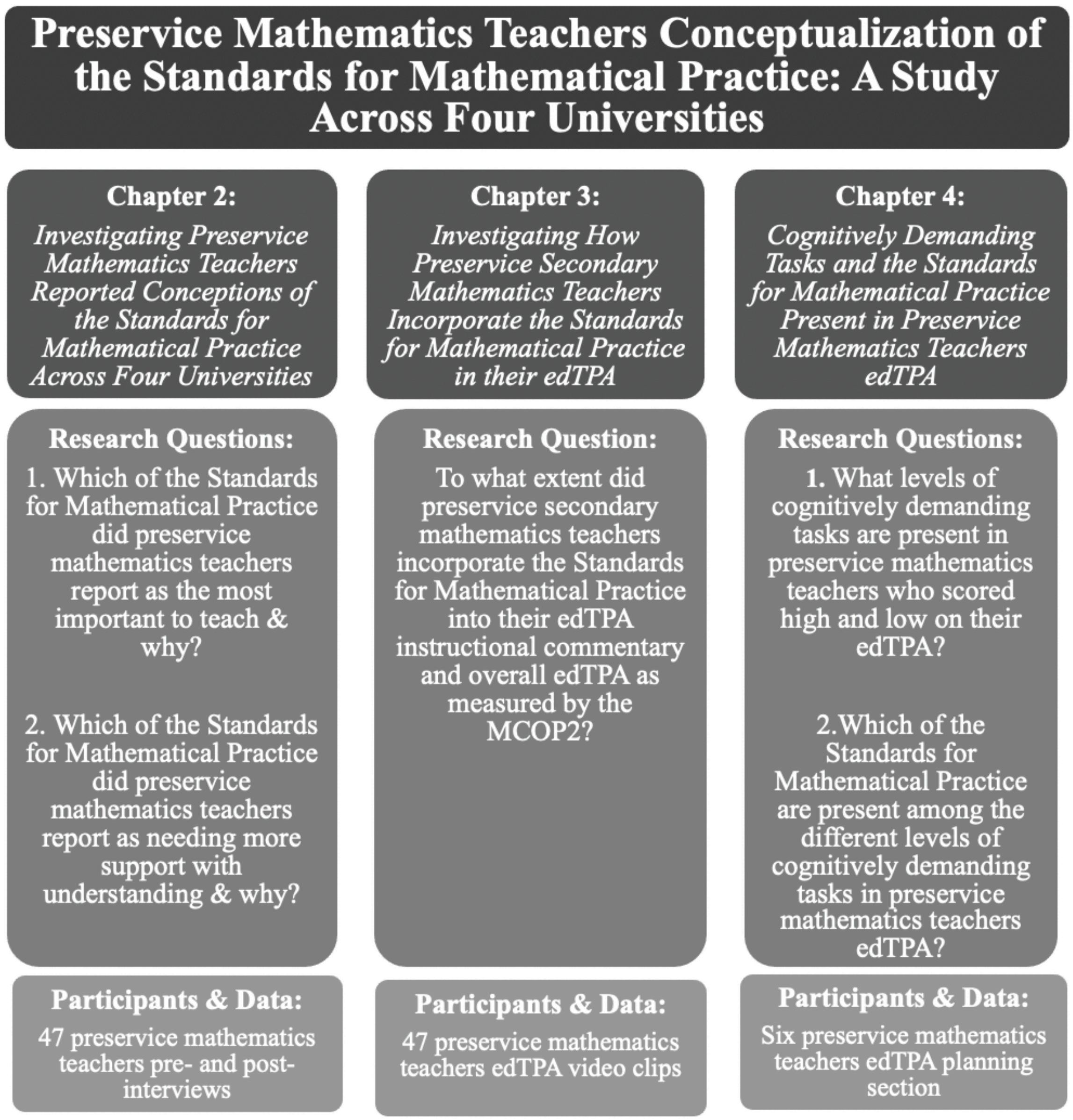
As teachers are the most important component of student learning, efforts to improve the quality of preservice mathematics teachers are essential to the mathematical development and proficiency of our students (Ball & Forzani, 2009). The Standards for Mathematical Practice allow for a balance of conceptual and procedural understanding of mathematics, problem solving, reasoning, the strategic use of mathematical tools, mathematical discussion, and sense making, all of which require our preservice mathematics teachers to have an in-depth understanding of the practices for both themselves and their students. However, little research has been done on preservice mathematics teachers and their conceptualization of the eight Standards for Mathematical Practice. Most studies either focus on preservice mathematics teachers' understanding of just one of the practices or focus on elementary preservice mathematics teachers (Bernander et al., 2020; Jung & Newton, 2018; Max & Welder, 2020). While a study by Baldinger (2014) did consider all of the practices when looking at preservice mathematics teachers, her focus was on how preservice mathematics teachers display their content knowledge through their engagement in the Standards for Mathematical Practice, and only looked at preservice teachers in one teacher education program. Thus, there is a gap in the literature when looking at not only how preservice mathematics teachers conceptualize all the practices, but how their understanding compares across several teacher education programs.

The goal of this dissertation was to look at how preservice mathematics teachers from four separate teacher education programs understood and implemented the Standards for Mathematical Practice. This was accomplished by looking at data from 47 preservice mathematics teachers from four different teacher education programs across the state of California. In addition to being from four separate teacher education programs, these 47 participants were also in three separate cohorts, the first from 2016-2017, the second from 2017-2018, and the third from 2018-2019. While each teacher education program differed in how they approached preparing their preservice mathematics teachers, all programs provided mathematics methods courses and field experiences where preservice mathematics teachers could learn about, engage in, and implement the Standards for Mathematical Practice. In addition, these 47 participants not only participated in pre- and post-interviews that asked them specific questions about the practices, but they all submitted completed edTPA portfolios (a teacher performance assessment). Both of these data points were used to analyze how teachers reported their understanding of the practices, how they planned to engage their students in the practices, and how well they actually succeeded in doing this.

In order to understand how preservice mathematics teachers conceptualized the practices before and after their teacher education programs, Paper 1 (Chapter 2) in this dissertation looked at initial and follow-up interviews from the 47 participants. In both of these interviews, participants were asked: 1. Which of the eight practices they felt were the most important to teach and why? and 2. What one or two do they need further help with to understand or implement and why? Through qualitative data analysis, I was able to look for what themes emerged in both of these two questions and compare them not only from pre to post interviews, but across the four teacher education programs as well. The second paper (Chapter 3) expanded on this by looking at how well these 47 preservice mathematics teachers incorporated and engaged their students in the Standards for Mathematical Practice as evident in their edTPA. I reviewed all of the video clips the 47 preservice mathematics teachers submitted for the instruction section of their edTPA and scored using the $MCOP^2$, a validated tool designed to assess how preservice mathematics teachers implement the Standards for Mathematical Practice through classroom observations (Gleason et al., 2017). Once scored, this allowed me to create a correlation coefficient between the participants $MCOP^2$ scores and both their edTPA instructional commentary and full edTPA scores. Creating this correlation coefficient allowed me to select preservice mathematics teachers who scored high and low on both their edTPA and $MCOP^2$ and look further into these specific cases in my third paper (Chapter 4). In this final paper, I focused on six preservice mathematics teachers, 3 high cases and 3 low cases, and looked at the various levels of cognitively demanding tasks they incorporated into the planning section of their edTPA. Once I identified the level of cognitive demand of each task based on the task rubric created by Smith and Stein (2011), I looked to see how many and which of the eight Standards for Mathematical Practice were visible across the various levels of tasks. I compared findings within and between the high and low cases to see how the level of cognitive demand of a task may support the presence of certain Standards for Mathematical Practice. Figure 1 below displays a visual representation of the three studies that constitute this dissertation.

Figure 1

Dissertation Overview



The three papers in this dissertation bring to light a more thorough understanding of how preservice mathematics teachers conceptualize and implement the Standards for

Mathematical Practice. In light of the emphasis on students learning mathematics skills and knowledge through the Standards for Mathematical Practice, it is critical that mathematics teachers successfully implement these practices in their daily classroom lessons. This starts with making sure our preservice mathematics teachers understand and are fully prepared to engage their future students in these eight practices. Thus, this dissertation has important implications for supporting preservice mathematics teachers in learning to engage in the Standards for Mathematical Practice in their teacher education programs.

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Chapter 2: Investigating Preservice Mathematics Teachers' Reported Conceptions of the Standards for Mathematical Practice Across Four Universities

Introduction

For years, researchers have called for preservice mathematics teachers to be afforded opportunities in their teacher education programs to engage with the mathematical standards and practices that they will be expected to engage their future students with (Association of Mathematics Teacher Educators [AMTE], 2017; Ma, 1999; Ferrini-Mundy, 2000). *The Common Core State Standards for Mathematics* (CCSSM; National Governors Association Center for Best Practices and the Council of Chief State School Officer [NGA & CCSSO], 2010) are the most recent set of mathematics standards and include the eight Standards for Mathematical Practice at their forefront. These eight practices "describe varieties of expertise that mathematics educators at all levels should seek to develop in their students" (NGA & CCSSO, 2010; p 6) and are essential for preservice mathematics teachers to develop an understanding of. Table 1 below provides a list of the eight Standards for Mathematical practice as they are found and worded in the CCSSM.

Table 1

CCSSM Standards for Mathematical Practice

Practice	Practice Name
1. Make sense of problems and persevere in solving them.	SMP 1: Problem-Solving
2. Reason abstractly and quantitatively.	SMP2: Reasoning
3. Construct viable arguments and critique the reasoning of others.	SMP3: Argumentation
4. Model with mathematics	SMP4: Modeling
5. Use appropriate tools strategically	SMP 5: Tools
6. Attend to Precision	SMP 6: Precision
7. Look for and make use of structure	SMP 7: Structure
8. Look for and express regularity in repeated reasoning	SMP 8: Regularity

Just as the Standards for Mathematical Practice are interwoven throughout the content, teacher understanding of the practices is connected to how they will implement them, making the practices a critical component for preservice mathematics teachers to learn about (McCallum, 2012). The practices were written as a set of skills, dispositions, and understandings that children should have in mathematics, although it is the teacher who must provide these experiences to their students (Koestler et al., 2013). How a preservice teacher understands and engages in the practices themselves can greatly affect how they implement and engage their own students in the practices (Baldinger, 2014). While we know that understanding the practices are a central component of a preservice mathematics teacher's education, we are unclear on what knowledge of the practices preservice mathematics teachers come into a teacher education program with and how, if at all, that knowledge changes.

In an effort to better understand how preservice mathematics teachers conceptualize the Standards for Mathematical Practice and how their conceptualization may change, this study looks specifically at preservice mathematics teachers' responses to questions about the practices from when they began and finished their teacher education program. Using interview data collected from four separate teacher education programs throughout California across three years, I compared their responses to questions about the practices at the start and end of their programs. I sought to investigate how their responses to questions about the practices changed from the start of their program to the end, and if there were any differences in these changes among programs. I asked the following research questions:

1. Which of the Standards for Mathematical Practice did preservice mathematics teachers report as the most important to teach and why?

2. Which of the Standards for Mathematical Practice did preservice mathematics teachers report as needing more support with understanding and why?

Conceptual Framework

For this study, I viewed my research through the lens of professional identity and how a mathematics teacher's development of their professional identity can be viewed as socially constructed. Below, I explain the background of this framework and how it can be used as a lens to view the Standards for Mathematical Practice.

Professional Identity

Previous research discusses how professional identity can be understood as socially constructed (Gee, 2000; Geijsel & Meijers, 2005). Peressini et al. (2004) argued that the professional identity of mathematics teachers shapes the ways in which they frame and address problems of practice as teachers. In addition, they explained that professional identity serves as a lens through which teaching is analyzed, understood, and experienced. In order to make instructional decisions and manage classroom issues, teachers draw on the ideas and interactions that define their professional identities. This idea of professional identity can be extended to preservice teachers, who form their identity based on their teacher education experiences. How and what preservice mathematics teachers learn in their teacher education programs is critical to shaping their knowledge and beliefs (Putman & Borko, 1996). The beliefs preservice mathematics teachers enter their teacher education programs with are essential factors that influence their teaching practices and the development of their identity as mathematics teachers (Connor et al., 2011). As these beliefs are usually dependent on how they were taught mathematics, preservice mathematics teachers need to develop new identities that are dependent on understanding and implementing the Standards for Mathematical Practice in order to engage their learners in mathematics content.

The beliefs that preservice mathematics teachers enter their teacher education programs with are challenged in their methods courses by both their instructors and classmates, as well as in their field placements by their cooperating teachers and the students who they are teaching. In addition, the Standards for Mathematical Practice themselves challenge the beliefs of preservice mathematics teachers. Such challenges are difficult for preservice mathematics teachers, as these beliefs may have developed many years ago when the preservice mathematics teachers were students themselves. In addition, preservice mathematics teachers' beliefs are usually about what mathematics is and how teaching should occur (Thompson, 1991). Many, if not most, preservice mathematics teachers were mathematics majors as undergraduates, and entered their teacher education programs with beliefs about mathematical teaching that were established during their undergraduate careers, which most likely did not draw on the Standards for Mathematical Practice (Leikin et al., 2018). Those who were mathematics majors as undergraduates were most likely taught by mathematicians, men and women trained in mathematics and not mathematics teaching, and therefore hold beliefs that they should teach mathematics the way they were taught by these mathematicians. In addition, as mathematics majors, preservice mathematics teachers enter their teacher education programs with the belief that they hold enough mathematical content knowledge to teach, not realizing the complexity of actually teaching. Therefore, the dangers in holding these beliefs are that they influence the way preservice mathematics teachers will teach their own students, the mathematical opportunities they will provide to their students, and the ways in which they will involve their students in mathematical discourse.

While the literature tells us that changing the beliefs of preservice mathematics teachers is not an easy feat (Cooney, 1999; Cross, 2009; Llinares, 2002; Oliveira & Hannula, 2008), providing constructivist environments for preservice mathematics teachers in their methods courses to learn about teaching mathematics, and involving preservice mathematics teachers in mathematical discovery (Llinares, 2002; Swan, 2007) are ways to help preservice mathematics teachers develop their professional identity. In addition, engaging preservice mathematics teachers in reflective practices, such as journaling, noticing, and video analysis, is essential to helping preservice mathematics teachers develop their sense of identity (Alger, 2006; Yesilbursa, 2011; Yost et al., 2000).

The component of professional identity in this larger framework serves as a way to trace preservice mathematics teachers' developing sense of themselves (Peresinni et al., 2004). In this study, the professional identities developed by the participants can be seen to include the goal of conceptualizing, engaging in, and implementing the Standards for Mathematical Practice, something that their former beliefs most likely did not draw on. While it is just a small part of what will eventually be their identities as mathematics teachers, it is essential that we understand how the prior beliefs and transformations of these preservice mathematics teachers lead to their conceptualization and implementation of the practices by participating in a community of learning.

Literature Review

Most of the current research on the Standards for Mathematical Practice focuses on in-service mathematics teachers' understanding and, making clear there is a gap in the literature on preservice mathematics teachers' conceptualization of the practices. While a study done by Baldinger in 2014 did look at preservice mathematics teachers, he focused on their engagement in the practices while in their mathematics methods courses. What is lacking from the literature is how preservice teachers report on the practices and how this may change over time as they complete their teacher education program. In this section, I review the literature that exists on the Standards for Mathematical Practice, bringing clarity to the gaps around research on preservice mathematics teachers and their reported conceptualization of the practices.

Origins of the Standards for Mathematical Practice

The authors of the CCSSM included the Standards for Mathematical Practice at the forefront of the standards because they wanted to provide teachers with guiding principles to follow as they enact the curriculum (NGA & CCSSO, 2010). (See Table in Appendix A for detailed descriptions of each practice.) The eight practices describe ways in which students should develop their understanding of mathematics, and how this should change as they grow in mathematical maturity from elementary school through high school. The creators of the CCSSM thought they needed practices that were a combination of process and understanding, with high expectations for students to understand how these practices connect to their mathematical thinking and work. The standards also justify the need for the practice as a tool to measure student understanding. If a student lacks understanding of a topic, he or she will not be able to engage in the practices and will resort to more procedural ways of doing the math (CCSSM, 2010). The practices are meant to be connected to the mathematics content and mathematical instruction in all ways possible. In addition, the Standards for Mathematical Practice are a way of providing coherence and rigor across the standards, which serve as key identifiers to show they are different than previous standards. The concepts are organized in a systematic way according to how the CCSSM believes students should learn and are clustered under the 11 content standards of the CCSSM.

The origins of the Standards for Mathematical Practice go back to the 1980s, when the National Council for Teachers of Mathematics (NCTM) took steps to establishing new standards that took a strong focus on routine skills, such as memorizing multiplication facts. In 1986, the NCTM established the Commission on Standards for School Mathematics, which sought input from classroom teachers across the country in order to help inform curriculum standards for school mathematics (Klein, 2003). The resulting document from this was finalized in 1989 and was referred to as the *NCTM Standards*. This set of standards called for an emphasis on conceptual understanding and problem solving informed by a constructivist understanding of how children learn (Klein, 2003). It also called for a deemphasis of rote learning, which would later cause critics to accuse the NCTM of eliminating basic skills. The NCTM received support for these standards from President George H. W. Bush and the National Science Foundation (NSF), both of whom endorsed the *NCTM Standards* as a benchmark to improving education in the United States. These partnerships resulted in textbooks and other curriculum that embodied the standards outlined by the NCTM, as well as two additional publications that further addressed reform efforts, *Professional Standards for Teaching Mathematics* (1991) and *Assessment Standards for School Mathematics* (1995) (Klein, 2003). However, parents and students strongly disliked the materials that the NCTM produced, stating that the changes were too radical, and the 1990s were filled with public backlash, resulting in this time period being dubbed the “Math Wars”. Because of this, the NCTM undertook the process of revising the standards into a more cohesive document that was supported by more thorough research and classroom experience. These efforts resulted in the publication of the *Principles and Standards for School Mathematics* (NCTM, 2000).

The Standards for Mathematical Practice are based on NCTM's document, the Process Standards (2000), and the National Research Council's (NRC) report, *Adding It Up* (NRC, 2001). The NCTM's Process Standards came out of their book *Principles and Standards for School Mathematics* (2000), and identified the standards of problem solving, reasoning and proof, communication, representation, and connections (NCTM, 2000). These five principles are the reasoning behind and formation of the first five practices. The other practices came from the NRC report *Adding It Up* (2001), which includes the Strands of Mathematical Proficiency: adaptive reasoning, strategic competence, conceptual understanding, procedural fluency, and product disposition. Together, these two documents served as the foundation for the eight practices. Based on the work from Koestler et al. (2013), Tables 2 and 3 show how the Standards for Mathematical Practice match up with NCTM's Process Standards and the NRC's Strands of Mathematical Proficiency.

Table 2

The Standards for Mathematical Practice Mapped onto the Process Standards

Standards for Mathematical Practice	**NCTM's Process Standards**
SMP 1, SMP 2, SMP 4, SMP 5	Problem Solving
SMP 1, SMP 3, SMP 8	Reasoning and Proof
SMP 1, SMP 2, SMP 4, SMP 6	Communication
SMP 1, SMP 2, SMP 4, SMP 7, SMP 8	Connections
SMP 1, SMP 2, SMP 4, SMP 5, SMP 6, SMP 7	Representation

Table 3

The Standards for Mathematical Practice Mapped onto the Strands of Mathematical Proficiency

Standards for Mathematical Practice	NRC's Strands of Mathematical Proficiency
SMP 1: Make sense of problems and persevere in solving them	Conceptual understanding; strategic competence; adaptive reasoning; productive disposition
SMP 2: Reason abstractly and quantitatively	Conceptual understanding; strategic competence; adaptive reasoning; productive disposition
SMP 3: Construct viable arguments and critique the reasoning of others	Conceptual understanding; strategic competence; adaptive reasoning; productive disposition
SMP 4: Model with mathematics	Conceptual understanding; strategic competence; productive disposition
SMP 5: Use appropriate tools strategically	Conceptual understanding; strategic competence; procedural fluency; productive disposition
SMP 6: Attend to precision	Conceptual understanding; procedural fluency; strategic competence
SMP 7: Look for and make use of structure	Conceptual understanding; strategic competence; productive disposition
SMP 8: Look for and express regularity in repeated reasoning	Conceptual understanding; procedural fluency; strategic competence; adaptive reasoning

Previous Research on the Standards for Mathematical Practice

A stated above, most of the current literature on the practices focuses on in-service teachers' implementation of them. Some of the literature takes on more of a practitioner lens in order to provide examples to mathematics teachers on how they can implement the practices both collectively and individually. Little research has been done on how preservice mathematics teachers come to understand and implement the practices. Below I discuss the current literature in these areas.

The Standards for Mathematical Practice as a Whole

When the practices were released in 2010, one of the biggest questions mathematics teachers had was concerning how to engage students in these practices while keeping the work meaningful and in line with the standards. In a sense, they became both a promising outlook on math and a problem for teachers at the same time. Some mathematics teachers viewed the practices as a way to engage students in coherent, deep, and meaningful math, while others saw it as pressure related to high stakes testing. Regardless, since the CCSSM had been adopted by states, strategies for engaging students in the eight practices needed to be investigated and addressed.

From the start, mathematics teachers needed support with understanding how to engage their students in these eight practices. The practices required mathematics teachers to realize that they were not separate from the mathematics content. To help with this, some researchers looked across grade levels to identify content in the curriculum where a teaching-learning emphasis could be placed on each practice (Russell, 2012). By taking the time to identify practices, it was argued that teachers would eventually start to see them come up in various content and contexts, which would allow teachers to engage their students in them more and more. Other researchers approached this by writing practitioner pieces that provided mathematics teachers with more of a "how to" model to engage students in the practices. Such pieces suggested that teachers should be more mindful of the practices in order to provide more opportunities for their students to engage in them (Billings et al., 2013). Recommendations on how to do this included making connections to students' past experiences and knowledge, focusing on the theme of the lesson, presenting students with a question to explore, providing mathematical activities that are interesting to students, and providing opportunities for students to reflect on their work and the work of others.

Mathematics teachers' explicit instruction of the Standards for Mathematical Practice to their students is another area that research has focused on. However, historically, this has been a struggle for teachers to do without turning the practices into procedural learning. This, in turn, could lead to lowering the cognitive demand of the task or lesson at hand. One study examined teachers explicitly addressing the practice after, rather than before, students engaged in the practices (Selling, 2016). By developing what they called a "reprising move," teachers were able to make the practices explicit to their students without the danger of making them overly prescriptive. The first five reprising moves were highlighting, naming, making evaluating statements, explaining the goal or rationale, and connecting to student engagement. All of these concepts supported students' mathematical discourse while also including reflective opportunities. The final three reprising moves, framing expansively, eliciting self-assessment, and making the teaching narrative explicit, provided students with opportunities to see how the practices can be applied beyond the classroom.

As I previously discussed, a study conducted by Baldinger (2014) looked at preservice mathematics teachers and how they learned to engage their students in the Standards for Mathematical Practice through their mathematics methods courses in their teacher education programs in one separate programs. Baldinger found all preservice mathematics teachers in the study connected the practices to a cognitively demanding task. In addition, most preservice mathematics teachers were able to engage their students in all of the practices with the exception of Practice 4, *modeling with mathematics*. Furthermore, when the preservice mathematics teachers were given opportunities to practice in their methods courses, specifically through cognitively demanding tasks, they reported feeling more confident in their levels of engagement with the practices themselves, and with engaging their students in them.

While Baldinger (2014) does highlight the importance of preservice mathematics teachers and the practices, my study differs in a number of ways. First, I looked across programs rather than across methods courses. Second, I included participants across multiple years and more programs. Finally, I focused on the challenges that preservice mathematics teachers faced to understanding the practice in addition to successes.

The Standards for Mathematical Practice as Individual Practices

In the past five years, several studies have been conducted that focus specifically on one of the Standards for Mathematical Practice. While looking at the practices individually has powerful implications for the field of mathematics education and is of great help to mathematics teachers, not all of the eight practices have been researched equally. The literature contains studies that look primarily at Practice 4, *modeling with mathematics*, and Practice 6*, attending to precision*, with the other practices woven throughout. This disparity may be because mathematics teachers are engaging students in certain practices more frequently than others, or they are struggling to engage their students in some of the practices at all.

In terms of Practice 4, *modeling with mathematics*, previous research has referred to modeling as finding an application of mathematics to solve problems that arise in everyday life (Schichl, 2004). Mathematical modeling has always presented teachers with a challenge, as modeling is not usually a course they are required to take (Phillips, 2016). In addition, modeling includes both working with open-ended problems and making and validating assumptions, all things mathematics teachers tend to struggle with (Blum & Ferri, 2009). Because of this, most of the literature around Practice 4 looks at ways to help teachers understand what modeling with mathematics means and how to engage their students in it. Mathematics teachers must understand what modeling means, both conceptually and as a process, as well as the mathematical content knowledge that goes with modeling (Anhalt & Cortez, 2016). Providing teachers with activities, such as lessons that guide students through the modeling process, dispels misconceptions and allows for explicit discussions and experiences with modeling to help improve their understanding of the practice (Stohlmann et al., 2015). Other research has focused on looking at how teachers define mathematical modeling, what problems and tasks they have chosen to engage their students in with regards to modeling, and then reviewing student work after they enacted the lesson (Jung & Newton, 2018).

Practice 6, *attending to precision*, is another practice that is frequently seen in the literature. While precision has long been a goal of the mathematics education community, it is a term that teachers and researchers struggle to define. Some think it refers to mathematical communication, while others understand it as the precision of calculations (Koestler et al., 2013). Therefore, the work around Practice 6 has examined both how to help teachers engage their students in attending to precision and in defining it more clearly for the mathematics education community. Some researchers looked at providing spaces for teachers where they could discuss what the practice means by building on each other's definitions and work. After teachers implemented a lesson that engaged their students in the practice, this space was used as a way for teachers to reflect on the lesson and examine student work (Otten et al., 2019). A study by Berger (2018) looked at how both preservice and in-service mathematics teachers developed definitions and their understanding of Practice 6 through trigonometry tasks. Specifically, the teachers were asked to develop Practice 6 around the definition of the sine function. By asking them to do this, Berger found that the more aware the teachers became of extending the definition of precision around sine functions, the more explicit they could be with their definitions and understandings for their students. Furthermore, the literature identifies Practice 6 as one that can connect with the other practices, especially Practice 7, *look for and make sure of structure*, and Practice 8, *look for and express regularity in repeated reasoning* (Berger, 2018; Otten et al., 2015). Combining Practice 6 with other practices may help students to see that precision is not about being nitpicky or difficult, but a practice that impacts all of their work.

Critiques and Concerns of the Standards for Mathematical Practice

The literature also contains research that focuses on the critiques and concerns around the Standards for Mathematical Practice. A pressing concern regarding the practices is the brevity of their definitions. While an essential element to the CCSSM, they are only briefly discussed and defined, leaving teachers with many questions on how their definitions can be expanded and what it means to engage students in them (Wiggins, 2011). Researchers have suggested that the practices were written this way so that teachers would have the freedom to define and enact them as they saw fit, without being held to strict guidelines. However, teaching mathematics is a difficult and complex task, and offering teachers more guidance on how to define, implement, and engage students in the practices, would greatly benefit not only our preservice and in-service mathematics teachers, but students themselves. In addition to their brevity, many feel that the practices are not a coherent set, and since coherence is one of the elements that the CCSSM advocates for, it does seem important that they reflect this. Finally, research has shown that many teachers have been concerned that parents are not aware of the practices (Walkowiak, 2015). With the establishment of the CCSSM, many parents have struggled with helping their children learn the perceived “new math” instead of the procedural math that they were taught as students. It is important to note that it is not the math that is new but the way in which it is being taught. The Standards for Mathematical Practice provide opportunities for students to persevere and justify their work and the work of others through written text and discourse, all of which parents struggle to understand (Otten & De Araujo, 2015). If parents were actively told about and supported in understanding the practices, they may be able to help their children more and understand the importance behind the work that is being done.

Contributions to the Literature

In summary, while there has been research conducted on the Standards for Mathematical Practice over the last 10 years, most of the work focuses on in-service teachers. In addition, most research has looked at how in-service teachers understand and implement specific practices, and not the practices as a collective. My study contributes to the literature because I focused on preservice mathematics teachers and their conceptions of all eight practices. By looking at which practices they reported as the most important and which ones they needed more help with, we can better structure our mathematics methods courses and prepare our preservice mathematics teachers to fully embrace the practices once they are in the classroom.

Methodology

Context and Participants

The context for this study was four teacher education programs from four large public universities across the state of California. Data were collected from these four programs over the course of three years (Year 1: 2016-2017, Year 2: 2017-2018, and Year 3: 2018-2019), resulting in three cohorts of preservice mathematics teachers from each of the four campuses. Of the four programs, three were graduate programs and one was an undergraduate program. Overall, there was a total of 73 mathematics candidates from all four universities across the three years. Of the 73 mathematics candidates, 70 agreed to participate in this study, and out of these, 47 had complete data sets, which consisted of both initial and follow-up interviews and completed edTPA portfolios. Table 4 shows the distribution of participants from the four universities over the three years of the study and Table 5 shows the demographics of participants across all three years of data collection.

Table 4

Distribution of Preservice Mathematics Teachers by University

	Year 1	Year 2	Year 3	Total
University 1	4	6	6	16
University 2	2	2	4	8
University 3	5	3	6	14
University 4	3	3	3	9
				47 Total

Table 5
Participant Demographics

Gender	
Female	72%
Male	28%
Race/Ethnicity	
White/European American	50%
Latinx	22%
Asian/Asian American	20%
Multiracial	4%
Other	2%

Pacific Islander	2%
First Language	
English	74%
Language(s) other than or in addition to English	26%
Undergraduate Major	
Mathematics	72%
Other	28%

*Note: All demographic data are self-reported
**Note: One participant did not report their gender, race/ethnicity, or first language

Data Collection

Participants in this study participated in two interviews, one near the beginning of their teacher education program (initial) and one towards the end (follow-up). Each audio-recorded interview lasted approximately one hour. The initial and follow-up interviews included questions on the following topics: 1. Background information, 2. Conceptions of mathematics teaching, 3. The Standards for Mathematical Practice, 4. Conceptions of learners, 5. Conceptions of effective practices for English language learners, 6. Practicum experience, and 7. edTPA information. Participants were also given a sheet with all eight practices listed when asked questions about the Standards for Mathematical Practice. For this study, I analyzed participants' responses to two of the three questions regarding the Standards for Mathematical Practice, which are displayed below in Table 6. Due to incomplete data, I did not analyze their response to the first question about the practices: Which two have you implemented most often in your current student teaching placement? Preservice mathematics teachers varied in their responses to this question, focusing on either which practices they saw their cooperating teachers implementing the most, or which practices they had tried to implement the most. Since the purpose of this study was to focus on what the preservice mathematics teachers' thoughts were, I excluded this question as many preservice mathematics teachers only discussed their cooperating teachers.

Table 6

Initial and Follow-up Interview Questions

These next few questions are about the Standards for Mathematical Practice: 1. Out of all eight, which one do you think is the most important to teach students? Why? 2. Which one or two practices do you think you need more help to understand or implement?

Data Analysis

Data analysis relied on the questions regarding the Standards for Mathematical Practice from the initial and follow-up interviews. As previously discussed, interviews were audio recorded and professionally transcribed, and then checked again for accuracy with all names replaced with pseudonyms. Next, another researcher and I met to qualitatively code the responses to interview questions from both the initial and follow-up interviews. For our tier 1 coding, we used a prior; (Saldana, 2015) coding based on the interview questions themselves. In other words, we coded to see which practices participants felt were the most important to teach students and which one or two practices they felt they needed more help to understand or implement. Each response was coded as one chunk. For tier 2 coding, we looked to see what themes emerged (Saldana, 2015) as to why preservice mathematics teachers reported certain practices in tier 1. The other researcher and I coded 30% of the interviews individually and met to discuss results and resolve disagreements until consensus was reached. This process continued until we reached an interrater reliability of .80 (Fleiss, 1971), at which point I continued to code the rest of the interviews myself. Table 7 below displays my coding for this analysis.

Table 7

Codes Used for Qualitative Analyses of Interview Data

Coding Round	Types of Coding	Code	Definition of Code
Tier 1	a priori	*Practice 1:* Problem-Solving *Practice 2:* Reasoning *Practice 3*: Argumentation *Practice 4:* Modeling *Practice 5:* Tools	Which one of the eight Standards for Mathematical Practice did preservice mathematics teachers report as the most important to teach?
		Practice 6: Precision *Practice 7:* Structure *Practice 8:* Regularity	Which one or two of the eight Standards for Mathematical Practice did preservice mathematics teachers report needing more help to understand or implement?
Tier 2	Emergent	*Strengths:*	
		Culture of perseverance	Preservice mathematics teacher explains that a practice promotes perseverance and resilience in students.
		Understanding mathematics at a deeper level	Preservice mathematics teacher explains that a practice supports students in developing a deeper understanding of mathematics

	Challenges:	
	Vagueness	Preservice mathematics teacher explains that a practice is worded and/or defined too vaguely for them to understand it.
	Lack of support and resources	Preservice mathematics teacher explains that a lack of support and resources contributes to their needing more help to understand a practice.
	Other	Preservice mathematics teacher explains additional reasons why they feel a practice is the most important and/or need further help in understanding.

To answer my first research question, I compared preservice mathematics teachers' responses to which practice they felt was the most important to teach. I looked at the total count for each of the practices that preservice mathematics teachers reported in both their initial and follow-up interviews within each university, and then I studied how they compared overall across all four universities. I reported the top two practices discussed within and between each university, for both the initial and follow-up interviews.

Furthermore, I looked at the reasons why preservice mathematics teachers reported certain practices more than others by looking at what themes emerged in their explanations.

To answer my second research question, I again compared preservice mathematics teachers' responses to which one or two practices they felt they needed further help in understanding. I again looked at the total count for each of the practices reported by the preservice mathematics teachers and compared them within and across the four universities. I reported the top two practices for both the initial and follow-up interviews. As with my first research question, I looked deeper at the reasons why preservice mathematics teachers reported certain practices more than others by looking at the emergent themes that developed from their explanations.

Finally, codes for tier 2 were not mutually exclusive, and therefore a preservice mathematics teacher could offer multiple reasons as to why they reported a certain practice. In addition, in order to compare results across the four universities, I converted responses to percentages. I did this because each university had a different number of preservice mathematics teachers in their programs.

Results

Practices That Preservice Teachers Reported as Most Important to Teach

In examining which Standards for Mathematical Practice preservice mathematics teachers reported being most important to teach and why, findings remained mostly the same between initial and follow-up interviews. I found that the major trends held across both initial and follow-up interviews for each university. In addition, I found that results were similar, but not identical, between universities as well. Below I discuss the practices preservice mathematics teachers reported as the most important to teach in their initial and follow-up interviews both as overall frequencies and compared by university, followed by a closer look at why participants reported certain practices over others.

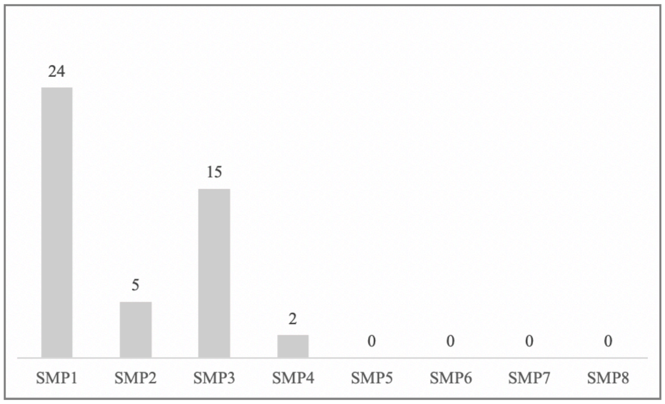
Most Important Practices to Teach Overall and by University

I first looked to see what the overall count was for each of the practices and how these findings compared from the initial to follow-up interviews. I coded for each practice and tallied the responses to get an overall picture of which practice preservice mathematics teachers reported as the most important from initial to follow-up interviews. I then broke down the findings further to see what these results looked like at the university level. I wanted to see not only how participants across the four universities reported the practices, but also how they compared from initial to follow-up interviews.

I found that preservice mathematics teachers overwhelmingly reported Practice 1 and Practice 3 as the most important to teach in their initial interviews. Figure 1 below shows these results, with Practice 1 reported a total of 24 times and Practice 3 reported a total of 15 times. These two practices combined accounted for over 80% of participants' responses across all four universities. The only other two practices reported by participants were Practice 2 (*n*=5) and Practice 4 (*n*=2). Practices 5, 6, 7, and 8 were not reported by any participants from any of the four universities in the initial interviews.

Figure 1

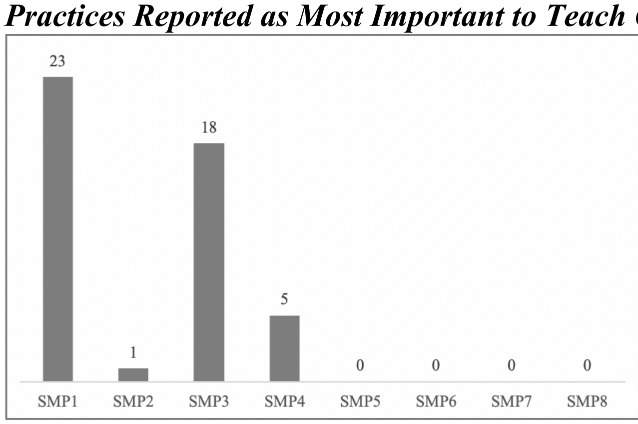
Practices Reported as Most Important to Teach Overall*: *Initial Interview



Findings for the follow-up interview mostly mirrored those of the initial interviews, with Practice 1 (*n*=23) and Practice 3 (*n*=18) again overwhelmingly reported the most by preservice mathematics teachers. These two practices combined accounted for almost 90% of all preservice mathematics teachers' responses to this question, which is a slight increase from the initial interviews. In terms of changes between the initial and follow-up interviews, Practice 1 only decreased by 1 response and Practice 3 increased by 3 responses. In addition, Practice 2 (*n*=1) decreased by 4 responses and Practice 4 (*n*=5) increased by 3 responses. Again, Practices 5, 6, 7, and 8 were not reported by any preservice mathematics teachers from any of the four universities as the most important practice to teach. Figure 2 below shows the findings from the follow-up interviews.

Figure 2

Practices Reported as Most Important to Teach Overall*: *Follow-up Interview



Looking further into these results, I was able to see how the responses from participants broke down according to the university they attended. Figure 3 below shows participants' responses to the question of which practice is most important to teach in their initial interviews broken down by university. Beneath this, Figure 4 displays the same results for the follow-up interviews. Because participant numbers were not the same for each of the four universities, I converted the results to percentages to get a clearer picture on how they compared.

Figure 3

Initial Responses of Practices Most Important to Teach by University

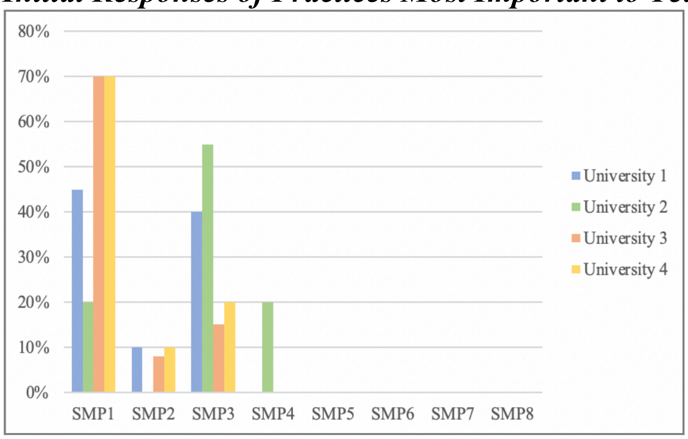
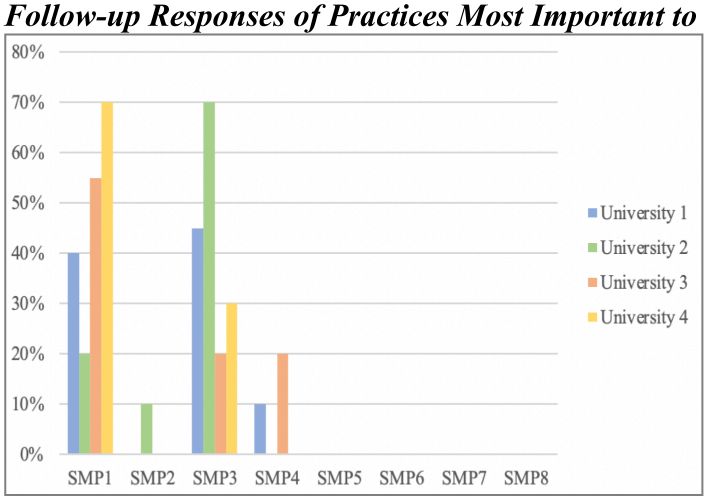


Figure 4

Follow-up Responses of Practices Most Important to Teach by University



To start, we can see that preservice mathematics teachers from University 1 only reported Practices 1, 2, and 3 in their initial interviews, and Practice 1, 3, and 4 in the final interviews. While not a major change, participants from this university did flip how many times they reported Practices 1 and 3, with Practice 1 reported slightly more than Practice 3 in the initial interviews and vice versa for the follow-up interviews. In addition to these two practices, Practice 2 was the only other practice participants responded with in the initial interviews, but was completely dropped in the follow-up interviews, with Practice 4 replacing it.

Participants from University 2 again reported Practices 1 and 3 the most in both the initial and follow-up interviews. While Practice 1 remained the same for both at 20%, Practice 3 was reported about 15% more from the initial to the final interviews. Again, the only other two practices discussed by participants were Practices 2 and 4. Practice 4 was the only other practice reported in the initial interviews, at 20% of the time, and Practice 2 was the only other practice reported in the follow-up interviews, discussed by 10% of participants.

University 3 and University 4 follow similar trends in responses from the initial interviews. Again, most of the preservice mathematics teachers from these universities responded with Practice 1 and Practice 3 as the most important to teach. Practice 1 was discussed by 70% of the participants at both universities and Practice 3 was discussed by about 15% of the participants from University 3 and about 20% of participants from University 4. While participants from both universities reported Practices 1 and 3 the most in their follow-up interviews, there were slight variations between the two. University 4 held at 70% of participants discussing Practice 1 the most, while University 3 decreased by about

15% in the final interviews. Participants from both University 3 and 4 increased how much they reported Practice 3 in their final interviews. Finally, while no participants from either university discussed Practice 2 in their follow-up interviews, 20% of preservice mathematics teachers from University 4 reported Practice 4 as the most important to teach in their follow-up interviews.

Explanations for Practices Reported as Most Important to Teach

The results above show that preservice mathematics teachers from all four universities overwhelmingly reported Practice 1 and Practice 3 as the most important to teach in both their initial and follow-up interviews. Looking more closely at participants' responses to this interview question, I was able to understand why preservice mathematics teachers reported these two practices the most. Overall, I found that the more commonly cited reasons behind participants reporting these practices had to do with building skills that could be extended beyond the mathematics classroom. Additionally, many preservice mathematics teachers felt Practice 1 and Practice 3 were foundational practices upon which the other practices . They felt that focusing on these two practices would provide a solid foundation from which a student could better tackle higher math.

Focusing specifically on Practice 1, participants expressed how implementing this practice the most created for their students a culture of resilience and dedication to mathematics in their classroom. I saw this with Anabel, a preservice mathematics teacher from University 3, who explained, "If they give up in math [if they do not persevere], then none of the other standards will matter, right?" Callen, a preservice mathematics teacher from University 2, echoed this with experiences from his field placement, stating, "A lot of students don't like making mistakes or thinking that whatever they're writing down is wrong.

[I try] to encourage my students to persevere in solving the problem even if the first method that they tried didn't work." This theme of seeing students give up when things become difficult was echoed by many preservice mathematics teachers, who felt that implementing Practice 1 the most was a first step to getting their students to explore math and develop grit. As Bertran, a preservice mathematics teacher from University 1 explained, it is about "finding ways to get a culture of persevering in class."

Furthermore, preservice mathematics teachers also felt that by emphasizing Practice 1, they were setting the tone for how students should approach problem-solving not only in their own classroom, but in future mathematics classes. Opal, a preservice mathematics teacher from University 4, spoke about how she felt Practice 1 to be preparatory, saying:

> That's a skill that I really try to embody with my students, so when they go on to future math so if they are struggling, they understand that that's just part of the process and they can get through and end up solving the problem.

Many preservice mathematics teachers emphasized that in their experience, struggle and working to find the correct answer is a key part of math, especially in higher mathematics. Their own experiences from college and beyond prompted many preservice mathematics teachers to suggest Practice 1 was critical to students learning mathematics, as Cecilia, a preservice mathematics teacher from University 2, explained:

> I was never the student to immediately understand a concept when it was being taught, and I think I have a lot of perseverance, which is how it got me through a lot of my math classes in high school and college, and I don't think that being good at math is necessarily because you're born with this amazing skill.

Practice 3 was also overwhelmingly reported by participants as the most important practice to teach in both the initial and follow-up interviews. Overall, preservice mathematics teachers described how focusing on Practice 3 allowed their students to demonstrate a deeper level of mathematical knowledge. Brandon, a preservice mathematics teacher from University 4, noted that "it really requires a good understanding of the material to be able to understand and respond to what someone else is saying." Other preservice teachers echoed what many of their counterparts who chose Practice 1 talked about: the practice's versatility in being applied to more than just math. Ella, a preservice mathematics teacher from University 2, explained:

> They should not be doing things because I tell them to do so…They should be doing things because they can justify why they're doing those things, and when other people have questions about it, they can defend themselves. I hope that those are skills that they can carry throughout their future math career, and also into real life situations, like if they see something wrong, they should be asking questions about it.

Many preservice mathematics teachers felt that critical thinking and being able to question a line of thought is paramount, and that Practice 3 emphasized skills that reinforce a way of thinking that will teach students to be perceptive and analytical. Timmy, a preservice mathematics teacher from University 1, articulated:

> I want my students to be able to think on their own and criticize what their classmates think is going on, so not just kind of go with the flow but realize, "Does this make sense?" or "Why is this person arguing this." Especially now with the era

> of all these fake articles that go around and stuff. I just think that's a really important skill that goes far beyond math.

It also seemed that preservice mathematics teachers wanted to emphasize Practice 3 more than ever in the age of technology, when students must build skills on how to evaluate the multitude of sources of information that they are bombarded with each day. This practice was commonly cited to be one that is becoming increasingly important in today’s day and age, contributing to how many preservice mathematics teachers believed it to be most important in teaching students. Danika, a preservice mathematics teacher from University 2, stated how even in the age of technology and automation, constructing and evaluating arguments is a uniquely human trait, and likely always will be.

> I think this is the thing that computers are least likely to be able to do. Right? And this is what I think ultimately, we need humans and just cognitive agents, in general. Right? So, to be critical and to be creative. And to make evaluations off of things. And I think that also carries on regardless of how much actual computation you're doing in your life.

Practices That Preservice Teachers Reported Needing Further Help Understanding

In examining which Standards for Mathematical Practice preservice mathematics teachers reported as needing further help to understand and why, findings were similar between the initial and follow-up interviews. It is important to note that for this question, participants were asked to identify one or two practices, with most reporting two. Overall, participants, regardless of their university, reported needing further help with mostly the same practices in their initial and follow-up interviews. When looking at the findings according to university, I again found that results were similar for the initial and follow-up interviews. Below I discuss the practices preservice mathematics teachers reported as needing further help to understand in their initial and follow-up interviews both as overall frequencies and compared by university, followed by a closer look at why participants reported certain practices over others.

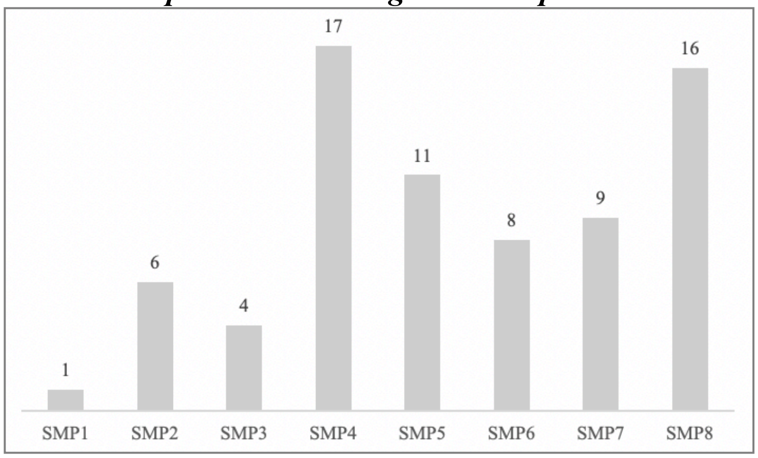
Practices Needing Further Help to Understand: Overall and by University

As with my first set of findings, I again looked to see what the overall count was for which practices participants discussed needing further help with and how these findings compared from the initial to follow-up interviews. I again coded for each practice that participants said they needed further help understanding and tallied the responses. I then broke down the findings further to see what these results looked like at the university level. I wanted to see not only how participants across the four universities reported the practices, but also how they compared from initial to follow-up interviews.

The top two practices preservice mathematics teachers reported as needing further help to understand in their initial interviews were Practice 4 (n=17) and Practice 8 (n=16). Figure 5 below shows these results. In addition, Practices 5, 6, and 7 were also reported by almost 10 participants each as needing further help to understand. While all of the practices were reported by at least one preservice mathematics teacher, Practices 1, 2, and 3 were reported the least. These three practices were the same in my first set of findings that preservice mathematics teachers reported as the most important to teach.

Figure 5

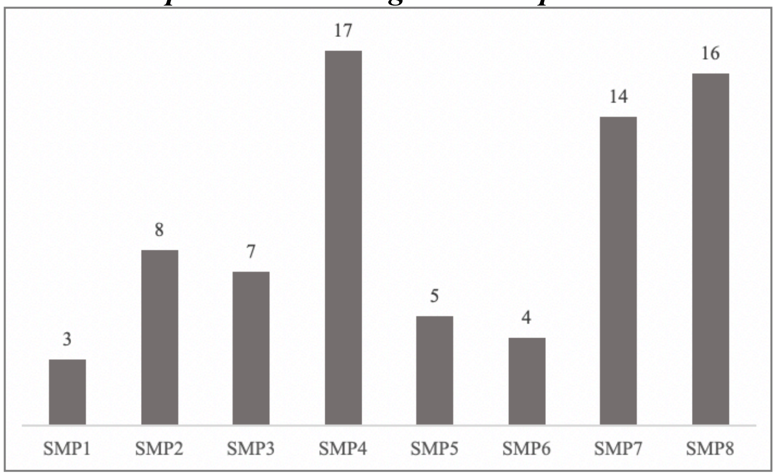
Practices Reported as Needing More Help With Overall*: *Initial Interview



Findings for the follow-up interviews (Figure 6) somewhat mirrored participants' responses from the initial interviews. As with the initial interviews, Practice 4 and Practice 8 were reported by preservice mathematics the most as the practices they needed further help in understanding how to teach. In fact, the exact count for each of these practices held, with 17 preservice mathematics teachers reporting Practice 4 and 16 preservice mathematics teachers reporting Practice 8. While Practice 1 was still the least reported practice that preservice mathematics teachers needed more help with understanding (n=3), there were changes within the other practices. Practices 2 (n=8), 3 (n=7), and 7 (n=14) were reported by more preservice mathematics teachers in their follow-up interviews, whereas Practice 5 (n=5) and Practice 6 (n=4) were reported less when compared to the initial interviews.

Figure 6

Practices Reported as Needing More Help With Overall*: *Follow-Up Interview



Looking further into these results, I was able to see how the responses from participants broke down according to the university they attended. Figure 7 below shows participants' responses to the question of which practices they need further help understanding in their initial interviews broken down by university. Beneath this, Figure 8 displays the same results for the follow-up interviews. Because participant numbers were not the same for each of the four universities, I converted the results to percentages to get a clearer picture on how they compared.

Figure 7

Initial Responses of Practices Needing Further Help by University

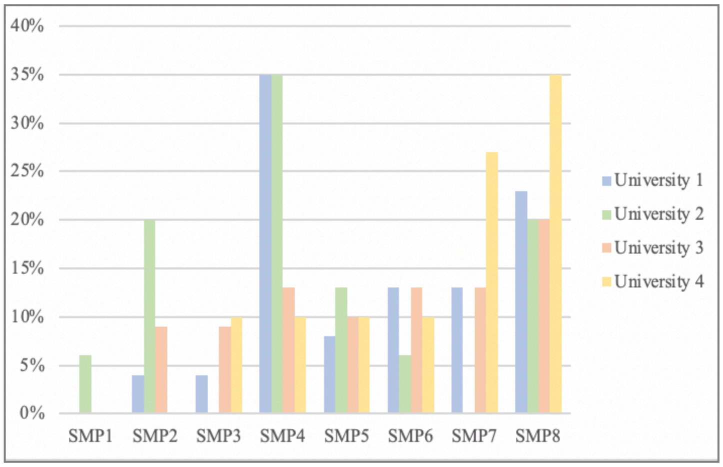
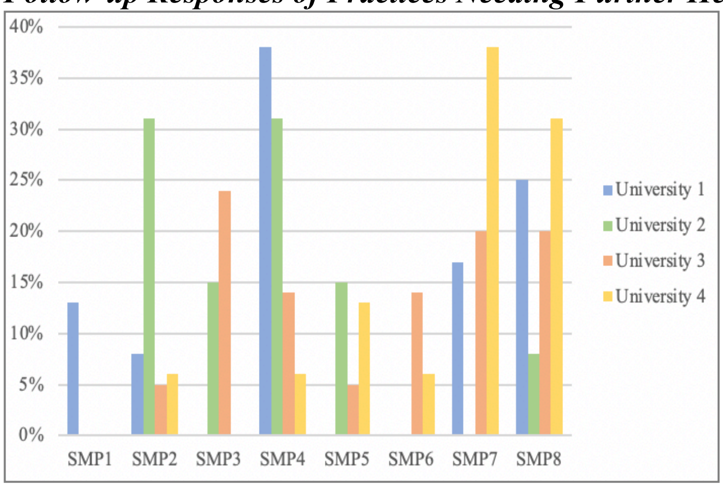


Figure 8

Follow-up Responses of Practices Needing Further Help by University



In their initial interviews, we can see that all of the practices were reported at least once from preservice mathematics teachers from University 1, with the exception of Practice 1. Most participants reported needing further help with Practice 4 and Practice 8 in their initial interviews, which mirrors the overall trend I saw in my findings above. These findings changed when looking at the follow-up interviews, where Practice 1 was reported by several preservice mathematics teachers, and Practices 3, 5, and 6 were not reported at all. Practice 4 and Practice 8 still remained the two practices reported the most, with both slightly increasing.

Preservice mathematics teachers from University 2 also reported Practices 4 and 8 the most in their initial interviews. However, Practice 2 was reported the same number of times as Practice 8, and Practice 3 and Practice 7 were not reported by any preservice mathematics teachers in their initial interviews. In their follow-up interviews, participants still reported Practice 4 the most, but it was tied with Practice 2, which increased in the number of times it was reported from initial to follow-up interviews. Practice 8 decreased by a little more than 10% in participants' follow-up interviews and was the least reported of the practices with the exception of Practices, 1, 6, and 7, which were not reported by any participants.

While trends held with Practice 8 for participants from University 3 in their initial interviews, Practice 4 was not reported in addition to Practice 8, which I saw in University 1 and University 2. Practices 2 through 7 were all reported around the same amount by participants from University 3, with none reporting needing further help with Practice 1. When looking at the findings for the follow-up interviews, there was about a 15% increase in participants reporting needing more help with Practice 3, the largest increase in any of the practices for this university. While most of the other practices did not change much, there was a slight increase in reporting for Practice 7, with Practice 8 staying consistent at 20%. Again, Practice 1 was not reported by any of the preservice mathematics teachers from University 3.

Finally, I saw similar trends in initial interviews with participants from University 4, who again reported Practice 8 as needing the most help with to understand further. The second most reported practice, however, was Practice 7, which had not been seen in other universities. These two practices encompassed the bulk of the practices reported by preservice mathematics teachers in their initial interviews, with Practices 3 through 6 reported much less, and Practices 1 and 3 not reported at all. In their follow-up interviews, trends held with participants again reporting needing further help with Practices 7 and 8 the most. However, Practice 8 slightly decreased in the follow-up interviews and Practice 7 slightly increased, making it the most reported practice. Finally, no preservice mathematics teachers reported Practices 1 and 3 as needing further help to understand in their follow-up interviews.

Explanations for Practices Reported as Needing Further Help to Understand

The results above show that preservice mathematics teachers from all four universities overwhelmingly reported needing further help in understanding Practices 8 and 4 the most, a trend that held for the initial and follow-up interviews. While there were fluctuations with the rest of the practices between the initial and follow-up interviews, preservice mathematics teachers consistently reported needing the least help in further understanding Practices 1 and 3. Looking more closely at the interview data, I was able to develop a deeper understanding of why Practices 8 and 4 were reported the most by participants.

To start, preservice mathematics teachers often reported needing further help with Practice 8 because they reported feeling confused by the vagueness of the wording and therefore struggled to understand what the actual intention of the practice was. Many preservice mathematics teachers also explained that they struggled with understanding the difference between Practice 8 and Practice 7. While participants highlighted Practice 8 as the one they needed further help with understanding, they spoke about Practice 8 and Practice 7 together in their reasoning. Janelle, a preservice mathematics teacher from

University 4, noted, “It's hard for me to tell the difference between Practice 8 and Practice 7. They seem very related to me.” Others reported similar sentiments of confusion, with Emmie, a preservice mathematics teacher from University 1, stating:

> I wouldn't even know, if someone asked me, "How do you use 'look for and make use of structure'?" I'm sure I do it somehow, but I don't really know...Seems just kind of something in the air. I don't really know too much about it. I feel like I also wouldn't really know what to say in regard to the last two practices.

Besides feelings of general confusion over the vagueness of Practice 8, many preservice mathematics teachers found it difficult to implement this practice without becoming too prescriptive. They wanted students to be able to grasp pattern recognition on their own but found that intervention or instruction on their part was necessary. As Jenny Lee, a preservice mathematics teacher from University 2, stated:

> [It] might be difficult to get students to do this or teach it without exactly directly telling them what to do or telling them that there's a pattern in this. Or without giving them a bunch of the same problems and having them realize that is a pattern within the problems.

In addition, many preservice mathematics teachers also reported that not only did they struggle to find ways to implement Practice 8, but they did not understand why this practice was important. This was well articulated by Kate, a preservice mathematics teacher from University 3, who said, “I'm not exactly sure...how to implement it at all. I mean, why is it important that they're repeating their reasoning?” Finally, preservice mathematics teachers also discussed not having the resources or support to help them understand how to implement Practice 8. They described how the curriculum often provided them with ideas on how to implement and engage their students in Practices 1 and 3, but rarely for Practice 8. Stassi, a preservice mathematics teacher from University 1, spoke to this struggle by saying:

> I think Practice 8 is the one I haven't had a lot of exposure to, or I just haven't realized, or I haven't been intentional in thinking about. CPM hasn't had a lot of ways for eighth grade to engage in them. I think it's really easy to be intentional about, "Okay, persevere and critique the reasoning of others' and the broader practices. But Practice 8 is pretty specific, and I just don't think I've been given the resources to understand how to implement it.

In addition to Practice 8, preservice mathematics teachers reported that they needed further help with understanding Practice 4. Overall, there was confusion associated with Practice 4. Modeling with mathematics, in general, was not something clearly defined and taught in a consistent way. As Tessa, a preservice mathematics teacher from University 1, explained:

> [It] is a math practice that is misunderstood a lot. Like teachers think one thing is modeling, but in one of our earlier classes saying modeling is not this it is actually this. And we were like, "Oh, that is not what we thought it was."

Kylie, a preservice mathematics teacher from University 4, expanded on this, explaining how modeling was not something she herself learned as a mathematics teacher, "It's something so detached from what my own learning experiences were, that developing those thoughts on my own is challenging." Furthermore, Danika, a preservice mathematics teacher from University 1, expressed how she could not tangibly grasp the meaning of Practice 4, saying, "I had an idea that modeling was just like any drawing pictures to help you understand, but I think there's something deeper than that that we're aiming for, and I don't think I completely understand that yet." These responses indicated an overall level of uncertainty regarding Practice 4 compared to their own experiences with modeling. The actual intent of the practice was not adequately clarified.

Another difficulty that preservice mathematics teachers reported with regards to Practice 4 was a lack of resources to implement mathematical models in a realistic and relatable way for students. This is best illustrated by Lennox, a preservice mathematics teacher from University 3, who reflected on modeling in trigonometry:

> I do use those book examples about applications or try to find them online, but I still don't always feel like they're exactly realistic even though you're saying they are, you know. It's always easy to find those very common examples. I'm teaching trig right now. It's easy to come up with that flagpole shadow example, but then are they really going to ever use that realistically.

This response highlights another difficulty associated with the practice: Even if preservice mathematics teachers felt that they had a grasp on modeling, they might struggle to implement the practice in a broader variety of scenarios across grade levels and topics in a way that is engaging and practical for students to learn. Many preservice mathematics teachers seemed to have difficulty with finding sources of classroom activities to implement that reflected Practice 4, further supporting the general sentiment that more clarification was needed with this practice, as well as strong examples of how to implement and engage students in modeling.

Discussion and Implications

Engaging students in the Standards for Mathematical Practice requires our mathematics teachers to have a strong understanding of what the practices are and how and when to implement them. Preservice mathematics teachers start to develop their understanding of the practices in their teacher education programs, specifically their mathematical methods courses. By looking at which of the eight practices preservice mathematics teachers reported as the most important to teach and which they needed more help to understand, I was able to take a closer look at how preservice mathematics teachers developed this understanding in their teacher education programs. This study, which looked at 47 preservice mathematics teachers across four universities to see which practices they felt were the most important to teach and which they needed further helping to understand, provides us with a lens to understand how prepared our preservice mathematics teachers are to implement and engage their future students in the Standards for Mathematical Practice.

Practices Most Important to Teach

Findings for my first research question show that Practices 1 and 3 are the two that preservice mathematics teachers found the most important to teach. This trend not only held for initial and follow-up interviews within each university, but also when I compared responses across universities. Overwhelmingly, preservice teachers discussed these practices more than any other as the most important to teach, supporting their responses with saying that these practices prepared students to be successful in mathematics the best.

Practice 1, *make sense of problems and persevere in solving them,* was reported the most by preservice mathematics teachers. Making sense of problems is not new to mathematics teaching and has been a central theme for learning mathematics for many years. NCTM has based their previous curriculum and standards on this idea of students making sense of problems in order to be successful in mathematics. For example, *NCTM's Principles and Standards for School Mathematics* (2000) clearly defines five process standards for learning mathematics, with *problem solving* being the first of the five. In addition, both *NCTM's Curriculum and Evaluation Standards* (1989) and Professional Standards for Teaching Mathematics (1991) both discuss making sense of problems as one of the central themes around students learning mathematics. In fact, many preservice mathematics teachers discussed Practice 1 as an umbrella over the remaining seven, and how it could be applied simultaneously with other practices. Because making sense of problems has always played such a dominant role in past mathematics standards and curriculum, preservice mathematics teachers are more aware of it in general and its importance to teaching mathematics. In addition, having it as the first Standards for Mathematical Practice may send a message to all mathematics teachers and students that it is the "most important" practice out of the eight.

In past NCTMs' standards and curriculum, constructing arguments was not referred to as "arguments", but instead referred to as "reasoning and proof". The word proof often implies proofs found in geometry or trigonometry classes and can be seen as more of a content focus than a lens to view all mathematics. Practice 3 asks students to support their reasoning with concrete arguments, while at the same time critiquing the reasoning of others in order to further their own learning. Preservice mathematics teachers found this to not only be clear, but to also hold great value. This is an opportunity for preservice teachers to reinforce all areas of mathematics through word problems and classroom discussions, where they can encourage students to make thoughtful answers and ask clarifying questions. Because students have to support their arguments, they are developing a deeper knowledge of mathematics. In addition, preservice mathematics teachers felt this practice was versatile, as they mentioned with Practice 1, in that it can be applied across all mathematics content and grade levels. Preservice mathematics teachers also spoke to how important Practice 3 is in today's world which has students facing fake news and false information daily and through multiple platforms. The overall idea of supporting their arguments and critiquing the arguments of others makes them more critical thinkers, which is an essential skill in today's world.

Preservice mathematics teachers also discussed Practices 1 and 3 as reflective of each other, which may be another reason as to why these were reported as the most important to teach. Students successfully persevering in problem solving can be seen as dependent on their ability to construct a successful argument and challenge the arguments of their peers. Both of these practices also promote a sense of resilience within students, which can be seen as a way for them to be successful in mathematics. In addition, neither practice is directly linked to specific content areas within mathematics and can be more easily applied across grade and content levels within mathematics. Other practices, such as Practice 4, *model with mathematics,* and Practice 5, *use appropriate tools strategically*, have been linked to specific content within mathematics, such as geometry and technological tools like graphing calculators. Preservice mathematics teachers may have found these two to be the most important to teach because they can be applied more generously, are clearly defined, and have a history in mathematics education.

Practices Needing Further Help to Understand

For my second research question, I found that preservice mathematics teachers reported Practice 4 and Practice 8 as the ones they needed further help understanding. Overall, Practice 4, *model with mathematics*, seemed to be the one reported most by preservice teachers as the one they needed further help with. This reflects previous research that discussed how mathematical modeling is a topic that many mathematics teachers need further professional development on because they are unsure of how to effectively teach it to their students (Jung & Newton, 2016). While modeling has been a part of past curriculum and standards developed by NCTM (1980, 1989, 2000), mathematical modeling is not something that most preservice teachers have experienced themselves as learners (Phillips, 2016). In addition, there is a difference between mathematical modeling and modeling mathematics, and the distinction between the two is not clearly defined. Mathematical modeling typically refers to translating a real-world problem into mathematics (Gravemeijer, 2004), while modeling mathematics is often seen as the use of tools to represent mathematics (Cirillo et al., 2016). The distinction between the two has usually not been made clear, not only in research but in the CCSSM as well (Jung & Newton, 2016). Furthermore, mathematical modeling has previously been taught as closely related to problem solving, and not necessarily as a separate practice (Lesh & Harel, 2003; Lester & Kehle, 2003). All of these points provide possible explanations as to why preservice mathematics teachers reported that Practice 4 was the one they needed most help with further understanding. The issue is that we do not have a clear definition of what modeling in mathematics looks like, which makes it a difficult practice to interpret. If in-service teachers struggle with this practice, then it can be assumed that preservice mathematics teachers will struggle as well. They may be receiving different definitions and understandings of this practice between their methods courses and their cooperating teachers, leaving them unsure of what the practice actually means.

Focusing on Practice 8, *look for and express regularity in repeated reasoning*, I saw that preservice mathematics teachers reported several reasons as to why they needed further help with understanding this practice. First, many preservice mathematics teachers were confused not only by the vagueness of the wording defining Practice 8, but why it was even important for students to learn. Unlike Practices 1 and 3, which have a clear history of why they are important to implement and engage students in, preservice mathematics teachers did not see the value in Practice 8. The wording in the definition of Practice 8 hints at more traditional mathematics skills, like memorizing and "drill and kill" calculations, both of which the standards and the practices have moved away from. Practice 8 also does not seem to be applicable to all content areas and grade levels in mathematics, whereas preservice mathematics teachers specifically said that they felt Practices 1 and 3 could be applied across the board in mathematics, making these practices more valuable to preservice teachers. In addition, some preservice mathematics teachers were confused as to how Practice 8 differed from Practice 7, *look for and make use of structure*. These two practices have similar words in their title, both starting with "look for" and the words "structure" and "repeated", which both imply a pattern. Both practices are also defined similarly, starting again with "look for" and making it difficult for preservice mathematics teachers to understand how they are independent of each other. This reflects the findings of Opfer et al. (2016), who looked to see what practices in-service mathematics teachers used the most. They found that 30% of the time, Practices 7 and 8 were never used in the classroom, which was greater than any of the other eight practices. If in-service mathematics teachers are struggling to define and use these practices, preservice teachers may not be provided with opportunities to see how they are implemented in their field experiences and thus develop a deeper understanding of them.

Implications

This research has implications for how to prepare preservice mathematics teachers to implement and engage their students in the Standards for Mathematical Practice. By understanding which practices preservice teachers find the most value in and the most challenging, we can better shape our methods courses and field experiences to support their needs. If preservice mathematics teachers find that Practices 1 and 3 are the most important to teach, they may implement and engage students in these practices more, with less value placed on other practices. Similarly, preservice mathematics teachers may shy away from practices they need further support in understanding and therefore not provide students with the opportunity to engage in these practices. The CCSSM intended for these practices to be implemented simultaneously and with equal value placed on all of them. However, these findings suggest that they are not valued and understood the same, which can lead to preservice teachers relying heavily on some practices and not others. Without equal representation of the practices, it is possible that students will not be provided with all of the necessary skill sets they need to be successful in mathematics.

Understanding which of the eight Standards for Mathematical Practice preservice teachers struggle with the most tells us which practices we need to focus on more during our methods courses and provide more opportunities for preservice teachers to engage in them. Future research should also look at in-service teachers and which practices they feel are the most important and which ones they struggle with implementing in order to provide better and more specific professional development opportunities for mathematics teachers. This also has implications for those who are cooperating teachers who work with preservice mathematics teachers. If a cooperating teacher is struggling with understanding, implementing, and engaging their students in certain practices, the preservice mathematics teacher will not have opportunities to witness and learn about certain practices either.

Limitations and Conclusion

One limitation of this study is that preservice mathematics teachers were asked for only one practice when discussing which one was most important to them, and one or two when discussing which they needed further help understanding. In addition, preservice mathematics teachers' responses may have been influenced by the thoughts and lessons of their cooperating teachers. As in-service mathematics teachers are still figuring out ways to implement and engage students in the Standards for Mathematical Practice, they may highlight ones they understand better over others, and those observing their classes may take note of this and form either the same value or confusion. Finally, focusing on the mathematics methods courses from these four universities would have provided further content into understanding how preservice mathematics teachers came to understand the Standards for Mathematical Practice.

The Conference Board of Mathematical Sciences (2012) notes that preservice mathematics teachers must be provided with opportunities in their teacher education programs to develop their understanding of the Standards for Mathematical Practice and how to engage their students in them to support the learning of mathematics content. The results of this study indicate that preservice mathematics teachers from four separate universities value Practice 1 and Practice 3 as the most important to teach among the eight practices. In addition, preservice mathematics teachers also identified Practice 4 and Practice 8 as ones they needed further help with understanding. These patterns did not change between initial and follow-up interviews. By providing preservice mathematics teachers with the opportunity to reflect on their understanding of the practices at the beginning and end of their teacher education programs, I was able to gain insight into how they conceptualized the practices, which can lead to changes in instruction of these practices in our mathematics methods courses.

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Chapter 3: *Investigating how preservice secondary mathematics teachers incorporate the Standards for Mathematical Practice in their edTPA*

Introduction

As of July 1, 2008, all teacher education programs (TEPs) in California were required to include a teacher performance assessment as part of the requirements preservice teachers must complete in order to earn a teaching credential (SB1209, 2006). The most widely used teacher performance assessment in the United States is the edTPA, with currently 41 states and over 800 TEPs using it as a requirement for teacher candidates to earn their credential (Kissau et al., 2019). This assessment was developed by the American Association of Colleges for Teacher Education (AACTE) and the Stanford Center for Assessment, Learning, and Equity (SCALE), and was the first standards-based assessment to become nationally available (SCALE, 2013). The assessment requires teacher candidates to submit a portfolio that demonstrates their readiness to teach through lessons designed to support the needs, strengths, and challenges of their students.

In general, the edTPA was designed as a teacher performance assessment to help determine whether preservice teachers, individuals in a teacher education program preparing to become in-service teachers, are ready to enter the profession. However, it is a complex exam consisting of many different sections and elements, and not all candidates pass the edTPA on the first try. Students submit unedited video footage of themselves teaching in their classroom and then answer commentary questions that are based on both the video footage and more general questions about teaching. The questions are divided into three sections: planning, instruction, and assessment. Each of these sections consist of five separate rubrics, which are each scored on a scale of 1-5, making the entire exam worth 75 points. While the passing score for the edTPA depends on the state, most preservice teachers must receive a score higher than a 40 to pass and receive their credential. In California, single-subject preservice teachers must receive a score of 41 or higher to receive their credential. With so many teacher candidates completing the edTPA in order to get their teaching credential, we need to look more closely at how well their portfolios demonstrate their preparedness to teach (Polly et al., 2020).

Equally as vital as preparing preservice teachers to successfully complete the edTPA is answering the call to produce effective, high-quality mathematics teachers with the knowledge and skills to successfully implement and engage students in the Standards for Mathematical Practice (Ball et al., 2005; Munter, 2014). The Standards for Mathematical Practice are a set of practices included in the Common Core State Standards for Mathematics that describe the variety of expertise mathematics educators should seek to develop in all their students (see Appendix A for full description of the Standards for Mathematical Practice). While the mathematics education community has produced large amounts of research in the last two decades concerning mathematical practices to engage students in, the paths to understanding teachers' learning and development of the Standards for Mathematical Practice have not been clarified (Munter, 2014).

The purpose of this study is to investigate the extent to which preservice mathematics teachers understood and implemented the Standards for Mathematical Practice as evident in the video clips they submitted with their edTPA. While there is previous research on the edTPA and the Standards for Mathematical Practice, there is a gap in the literature on combining these two areas when specifically looking at preservice secondary mathematics teachers (e.g., Bunch et al., 2015; Hildebrandt & Swanson, 2016; Kissau &

Algozzine, 2017; Lim et al., 2015). By filling this gap, we are taking steps to ensure that preservice mathematics teachers are not only prepared to engage in the Standards for Mathematics Practice in their edTPA, but as future in-service teachers. By using the validated tool the Mathematics Classroom Observation Protocol for Practices (MCOP^2) developed by Gleason et al. (2017) to assess how preservice mathematics teachers implement the Standards for Mathematical Practice in their edTPA videos, I was able to determine how well these preservice mathematics teachers in my study engaged their students in the Standards for Mathematical practice and if their edTPA scores were representative of their level of understanding. I posed the following research question to guide my study: To what extent did preservice secondary mathematics teachers incorporate the Standards for Mathematical Practice into their edTPA instructional commentary and overall edTPA as measured by the MCOP^2?

Conceptual Framework

This study is guided by the framework a community of learners (Rogoff et al., 1996). A community of learners is based on the idea that learning occurs as people simultaneously participate in a shared endeavor. This theory is in opposition to models of learning that focus on learning as the transmission of knowledge from teacher to student, with the learner playing a passive role. Successful implementation of and engagement in the Standards for Mathematical Practice can occur through the lens of a community of learners, where both the teacher and student are actively fulfilling their roles as facilitator of the mathematical material and engaged in the problem-solving process (Barker et al., 2004; NCTM, 2006, 2011; Stein et al., 2008; Stein et al., 2009). As the responsibility is shared between teachers and students within a community of learners, this framework is broken down into the components of teacher facilitation and student engagement.

When considering teacher facilitation, recent reform efforts call for teachers to develop student reasoning through high-quality tasks and interactions (Barker et al., 2004; NCTM, 1991, 2006, 2011; NCR, 2003; Stein et al., 2008, 2009). Teacher facilitation includes providing students with scaffolds in a lesson, supporting students' productive struggle, the level of questioning offered to students, and opportunities for students to produce mathematical discourse (Hiebert & Grouws, 2007; Nathan & Knuth, 2003; Reys et al., 2015; Schoenfeld, 1998). In particular, teachers must facilitate and support mathematical discourse that is respectful, student-centered, and occurs between students and students-to-teacher (Hiebert & Grouws, 2007; NCTM, 2000; Sherin et al., 2004).

In a community of learners, teacher facilitation is accompanied by student engagement, which requires an environment that supports students in social interactions to develop, learn, and understand (Bruner, 1960; Vygotsky, 1980). Being part of a mathematics community is essential for students to develop mathematical understanding, habits, and reasoning, and supports them in making connections where they can recognize patterns and use repeated reasoning (Ball, 1990; Ma, 1999). Through interacting with their peers and teacher, students are able to engage in mathematical questioning and assess their own strategies and arguments as well as those of their peers.

The Mathematics Classroom Observation Protocol for Practices ($MCOP^2$) is based on a community of learners framework as defined by teacher facilitation and student engagement (Gleason et al., 2017). The authors of the $MCOP^2$ based the creation of this instrument on the interactions among teachers, students, and the mathematical content.

Through internal structure analyses, they validated their items by placing them into the category of either teacher facilitation or student engagement. The MCOP^2 maps both teacher facilitation and student engagement directly onto the Standards for Mathematical Practice, as the authors believed that both these roles needed to be equally fulfilled in order for a mathematics classroom to successfully implement the Standards for Mathematical Practice. Therefore, I used this framework of community of learners broken down into teacher facilitation and student engagement to guide my study through the use of the MCOP^2. In addition, the MCOP^2 was also a valuable tool to use since the edTPA does not explicitly mention the Standards for Mathematical Practice.

Literature Review

This study is informed by two bodies of prior research based on the edTPA and the MCOP^2. While the edTPA is the most widely used teacher performance assessment in the United States, most of the previous literature focuses on ways to support preservice teachers in order for them to successfully pass the edTPA, as well as concerns and criticisms of the assessment. When looking specifically at preservice mathematics teachers and the edTPA, most studies do not focus on secondary preservice mathematics teachers. What research does exist concerns either the academic language component of the edTPA for preservice mathematics teachers or is focused on preservice elementary teachers.

When looking at academic language, both Lim et al. (2015) and Bunch et al. (2015) discussed their concerns for the ways the edTPA assesses academic language for preservice mathematics teachers. Lim et al. (2015) looked specifically at how the edTPA focuses on syntax, which is usually reserved for more advanced mathematics. They also addressed the area of classroom discourse, which is a challenge for preservice mathematics teachers when incorporating academic language and differentiated language demands, as they are simultaneously learning these components in their methods class and field placements. Bunch et al. (2015) extended this work, specifically on the edTPA's inclusion of supporting English learners. They pointed out that if a preservice mathematics teacher has little experience with English learners, they will not be able to understand or attend to their mathematical understandings during the edTPA. While strategies for English learners can be taught and discussed in preservice mathematics teachers mathematics methods courses, they are of little help to preservice mathematics teachers if they are unable to implement them in their field placements.

Two studies looked specifically at preservice elementary teachers who chose mathematics as the focus of their edTPA. Henry et al. (2013) and Santagata et al. (2018) both looked at whether preservice teachers' scores on their edTPA could predict how effective they were at teaching mathematics as in-service teachers. By conducting a multiple regression model, both studies found that preservice teachers' scores on their edTPAs could not be used to predict their effectiveness for teaching mathematics, pointing out that a single measure should not be the sole way to determine if a preservice teacher is ready to enter the workforce. However, it is important to note that while both of these studies focus on elementary preservice teachers, they slightly contradict the findings of Goldhaber et al. (2017) and Bastian et al. (2018), who found that edTPA scores could be used to predict teacher readiness. What we can take away from this is that, overall, more research needs to be done with the edTPA, not only when specifically looking at preservice mathematics teachers, but in all content-specific areas.

Most research on mathematics education is based on more well-known tools, such as the Instructional Quality Assessment in Mathematics (Boston & Wolf, 2006) and the Mathematical Quality of Instruction (Learning Mathematics for Teaching, 2006). However, none of these protocols were designed specifically to map onto the Standards of Mathematical Practice, which is unique to the MCOP^2. What research has been done involving the MCOP^2 does not include any based on preservice mathematics teachers, only in-service teachers. For example, a study conducted by Lambert et al. (2020) looked at how students with autism participated in the practices of mathematical reasoning, sense making, and discussion. In order to accomplish this, researchers used the MCOP^2 to code video lessons for Practice 1, *make sense of problems and persevere in solving them*, and Practice 3, *construct viable arguments and critique the reasoning of others*. Their work, a case study that focused on a fifth-grade student with autism, showed the influence that classroom activities have on a student's participation in mathematical reasoning, calling for more studies to be done that look at how to better include students with autism in the Standards for Mathematical Practice. In a more recent study, researchers used the MCOP^2 as a basis for developing their own instrument at the graduate student level. Rogers et al. (2020) sought to develop an observation protocol that could provide quality feedback to graduate student instructors, supporting them in developing into instructors who could engage students in mathematical meaning making. The MCOP^2 provided the foundation for developing this tool, which resulted in a 17-item mathematics Graduate Student Instructor Observation Protocol (GSIOP). Finally, another recent study used the MCOP^2 to measure the success of a mathematics professional development (Soto & Marzocchi, 2020). The larger study, based on the belief that teaching mathematics through active engagement leads to better learning outcomes for students, used the $MCOP^2$ to specifically measure student engagement in the Standards for Mathematical Practice. While the focus of this study was on designing effective professional development, the $MCOP^2$ was successfully used to measure how teachers implement the Standards for Mathematical Practice to engage their students in mathematical learning.

To summarize, while there is a large amount of previous research on the edTPA, very little of it is focused specifically on preservice secondary mathematics teachers. In addition, little research has been conducted based on the $MCOP^2$. My contribution to the literature combines these two areas, both the edTPA and the $MCOP^2$, and applies them to the context of preservice secondary mathematics teachers. This unique contribution to the literature can lead to better preparation of preservice mathematics teachers to be high-quality in-service mathematics teachers.

Methods

Context and Participants

The context for this study was four teacher education programs from four large public universities across the state of California. In addition, data were collected from these four programs over the course of three years (Year 1: 2016-2017, Year 2: 2017-2018, and Year 3: 2018-2019), resulting in three cohorts of preservice mathematics teachers from each of the four campuses. Of the four programs, three were graduate programs and one was an undergraduate program. Overall, there were a total of 73 mathematics candidates from all four universities across the three years. Of the 73 mathematics candidates, 70 agreed to participate in this study, and out of these, 47 had complete data sets, which consisted of pre- and post-interviews and completed edTPA portfolios. Table 1 shows the distribution of participants from the four universities over the three years of the study and Table 2 shows the demographics of participants across all three years of data collection.

Table 1

Distribution of Preservice Mathematics Teachers by Campus

	Year 1	Year 2	Year 3	Total
University 1	4	6	6	16
University 2	2	2	4	8
University 3	5	3	6	14
University 4	3	3	3	9
				47 Total

Table 2

Participant Demographics

Gender	
Female	72%
Male	28%
Race/Ethnicity	
White/European American	50%
Latinx	22%
Asian/Asian American	20%
Multiracial	4%
Other	2%
Pacific Islander	2%
First Language	
English	74%
Language(s) other than or in addition to English	26%
Undergraduate Major	
Mathematics	72%
Other	28%

*Note: All demographic data are self-reported
***Note: One participant did not report their gender, race/ethnicity, or first language

Data Collection

For this study, I examined participating preservice mathematics teachers' edTPA portfolios, specifically the video clips they submitted with their instructional commentary. As previously discussed, the edTPA is a teacher performance assessment that consists of three parts: planning, instruction, and assessment. As part of their instructional section, preservice teachers must submit video clips of their classroom teaching up to 15 minutes in length. Some preservice teachers decided to submit one longer clip, while others submitted two shorter clips. If they submit two separate clips, these clips could be from two different lessons. In addition to these video clips, preservice teachers submitted a commentary approximately 6 pages in length to describe what was going on in the video, answering specific questions about how they were engaging their students in learning, and analyzing their teaching effectiveness. Their instructional commentary was scored out of 25 points, broken down by 5 rubrics that are each rated on a scale of 1-5. The entire edTPA is out of a total of 75 points and scored by trained assessors using 15 5-point rubrics. In California, a score of 41 or higher was considered passing at the time of this study.

Data Analysis

Analysis for this study consisted of three phases. The first phase was the most extensive, as this phase consisted of watching all 47 edTPA video clips and scoring them using the MCOP^2, a validated K-16 mathematics classroom instrument designed to measure the degree of alignment of the mathematical classroom with the various standards (Gleason et al., 2017). This tool was originally designed as a classroom observation tool and measures both teacher facilitation and student engagement through a total of 16 items, which are mapped directly onto the eight Standards for Mathematical Practice. While the Standards for

Mathematical Practice consist of only eight practices, the MCOP^2 is designed to measure 16 items, because the creators believed most of the practices are interconnected and appear together, as do teacher and student interactions. The Teacher Facilitation subscale measures the role of the teacher as the one who provides the structure for the lesson and guides the problem-solving process through the practice. The Student Engagement subscale measures the role of the students and their engagement in the learning process. Items for both Teacher Facilitation and Student Engagement are scored on a scale of zero to three and added together for a final score. Tables 3 and 4 below provide a breakdown on how the MCOP^2 is structured. (See Appendix B for a complete breakdown of each item with a description of each of the possible levels it can be scored on.) I and one other graduate student individually coded 20% of the edTPA video clips using the MCOP^2, and then met to discuss results and resolve disagreements until we reached consensus. This process continued until we consistently reached interrater reliability greater than .80 (Fleiss, 1971), at which point I scored the rest of the videos independently.

Table 3

Student Engagement and Teacher Facilitation Items

Item	Student Engagement	Teacher Facilitation
1	*X*	
2	*X*	
3	*X*	
4	*X*	*X*
5	*X*	
6		*X*
7		*X*
8		*X*
9		*X*
10		*X*
11		*X*
12	*X*	
13	*X*	*X*
14	*X*	

15	*X*	
16		*X*

Table 4

MCOP² Items Mapped onto the Standards for Mathematical Practice

	The Standards for Mathematical Practice							
MCOP² Item	*SMP1*	*SMP2*	*SMP3*	*SMP4*	*SMP5*	*SMP6*	*SMP7*	*SMP8*
1	X						X	X
2	X				X			
3	X							
4	X		X					X
5	X	X	X		X			
6							X	X
7		X		X				
8							X	X
9	X							
10			X			X		
11	X							
12			X					
13			X					
14	X							
15			X					
16	X							

In my second phase of analysis, I ran a Multivariate Analysis of Variance (MANOVA) between the participants' MCOP² scores, their instructional commentary scores, and their full edTPA scores. I did this in order to see if there was a significant difference in scores among universities. By looking at the estimated marginal means for both instructional commentary and full edTPA scores across the four universities, I was able to determine if the program the preservice mathematics teachers attended had an impact on their scores or not.

Finally, I created a correlation coefficient and scatter plots in my third phase of analysis. I created the correlation coefficient to see if the participants' MCOP² scores were statistically significant predictors of their instructional commentary and full edTPA scores.

As participants' MCOP^2 scores did turn out to be statistically significant predictors of both their scores, I created two simple scatter plots in order to create a visual representation of this correlation. These scatterplots allowed me to visualize not only the high and low correlated scores, but also any outliers that were present.

Results

I considered a correlation between participants' MCOP^2 scores and their edTPA scores to be an indicator of the extent to which participants implemented and facilitated the Standards for Mathematical Practice in their edTPA video clips. I watched all the participants' edTPA video clips and scored them on the MCOP^2. Table 5 below shows the final MCOP^2 scores I found for all 47 participants.

Table 5

Participant MCOP^2 Scores

Participant	University	MCOP^2 Score
Henry	1	43
Kelly	1	43
Kylie	1	42
Timmy	1	39
Emmie	1	38
Isa	1	38
Danika	1	37
Tessa	1	36
Alexander	1	36
Tierney	1	33
Loralei	1	32
Bertran	1	31
Jordan	1	30
Christy	1	29
Stassi	1	26
Tyra	1	24
Lennox	2	43
Ella	2	43
Janet	2	41
Bethany	2	37
Jennylee	2	35
Jamie	2	32

Danica	2	30
Raechelle	2	22
Lily	3	43
Ellie	3	42
Cora	3	41
Kate	3	39
Kevin	3	38
Sabina	3	38
Cecilia	3	38
Anabel	3	36
Callen	3	35
Jeanette	3	35
Lala	3	35
Braxton	3	30
Liwei	3	27
Simon	3	24
Opal	4	45
Melanie	4	44
Brandon	4	41
Elle	4	37
Janelle	4	35
Reina	4	33
Sariah	4	33
Jemma	4	33
Jace	4	29

I than ran a MANOVA to examine the relationship between the predictors and variables. Results showed (see Table 6) that while campus was not a significant predictor for both the instructional commentary and full edTPA scores (p =.400), MCOP^2 scores were significant predictors of both scores. The estimated marginal means for both the instructional commentary and full edTPA scores were virtually identical across all four campuses, which is why campus was not a significant predictor (see Figures 1 and 2). Finally, I created a Pearson correlation coefficient to see if participants' instructional commentary and full edTPA scores were related to their MCOP^2 scores, which I suspected would be significant as they were in the MANOVA.

Table 6

Multivariate Tests

Effect		Value	F	Sig.
Intercept	Pillai's Trace	.823	95.252b	.000
	Wilks' Lambda	.177	95.252b	.000
	Roy's Largest Root	4.646	95.252b	.000
$MCOP^2$ Scores	Pillai's Trace	.583	28.640b	.000
	Wilks' Lambda	.417	28.640b	.000
Campus	Pillai's Trace	.109	.810	.565
	Wilks' Lambda	.893	.792b	.579

Figure 1

Estimated Marginal Means for Instructional Commentary Scores

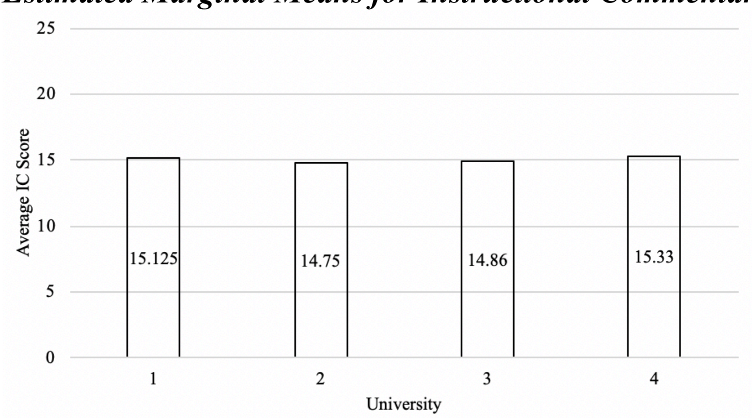
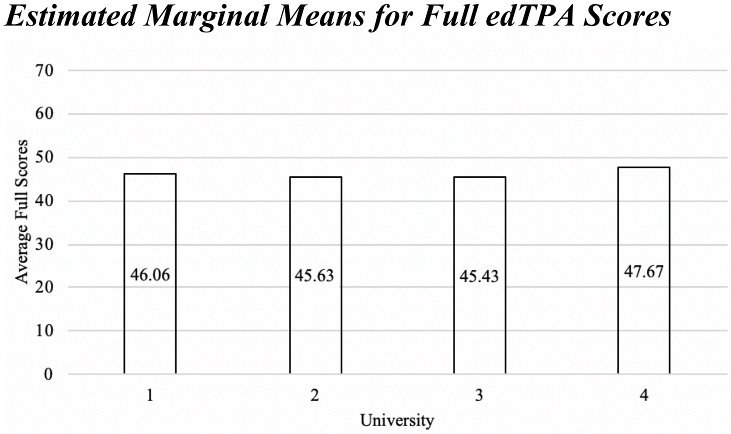


Figure 2

Estimated Marginal Means for Full edTPA Scores



In terms of participants' $MCOP^2$ and instructional commentary scores, there was a strong positive correlation between the two variables for both Pearson ($r = .75$, $p \leq .001$) and Spearman Rho ($r = .775$, $p \leq .001$). See Table 7 below for both correlations of the $MCOP^2$

and instructional commentary scores. For participants' MCOP^2 and full edTPA scores, there was again a strong positive correlation between the two variables for both Pearson (r = .76, $p \leq .001$) and Spearman Rho (r = .794, $p \leq .001$). See Table 8 below for both correlations of the MCOP^2 and full edTPA scores.

Table 7

MCOP^2 and Instructional Commentary Correlation

	IC Scores	Pearson's Correlation	Spearman's Rho Correlation
Correlation	1.000	.751**	.775**
Sig. (2-tailed)		.000	.000
N	47	47	47

**Correlation is significant at the 0.01 level (2-tailed).

Table 8

MCOP^2 and Full edTPA Correlation

	IC Scores	Pearson's Correlation	Spearman's Rho Correlation
Correlation	1.000	.763**	.794**
Sig. (2-tailed)		.000	.000
N	47	47	47

**Correlation is significant at the 0.01 level (2-tailed).

Finally, I created two simple scatterplots (see Figures 3 and 4) in order to create a visual map of the correlation coefficient between participants MCOP^2 scores and both their instructional commentary and the full edTPA scores. This allowed me to see which preservice teachers scored high, medium, and low on all three measures. For example, preservice mathematics teachers 43 (MCOP^2 =44, IC score=18, full edTPA score=53) and 46 (MCOP^2 =45, IC score=17, full edTPA score=51) both scored high across all three measures, making them good cases to investigate further for how their video clips incorporated the Standards for Mathematical Practice through both teacher facilitation and student engagement. Preservice mathematics teachers 19 (MCOP^2 =22, IC score=13, full edTPA score=42) and 34 (MCOP^2 =24, IC score=13, full edTPA score=24) scored low on all three components and could be further analyzed and compared to the high cases in order to see if there were missed opportunities for the preservice teachers to implement the practices in their lessons. Finally, these simple scatterplots also allowed me to see any cases that were outliers, in that their scores were not consistently in the high, medium, or low range across all three measures. While preservice mathematics teachers 2 and 23 both scored a 15 on their instructional commentaries and had similar full edTPA scores, they had very different MCOP^2 scores. Case 2 received an MCOP^2 score of 24 and case 23 received an MCOP^2 score of 43.

Figure 3

Simple Scatter of Instructional Commentary Scores by MCOP^2 Scores

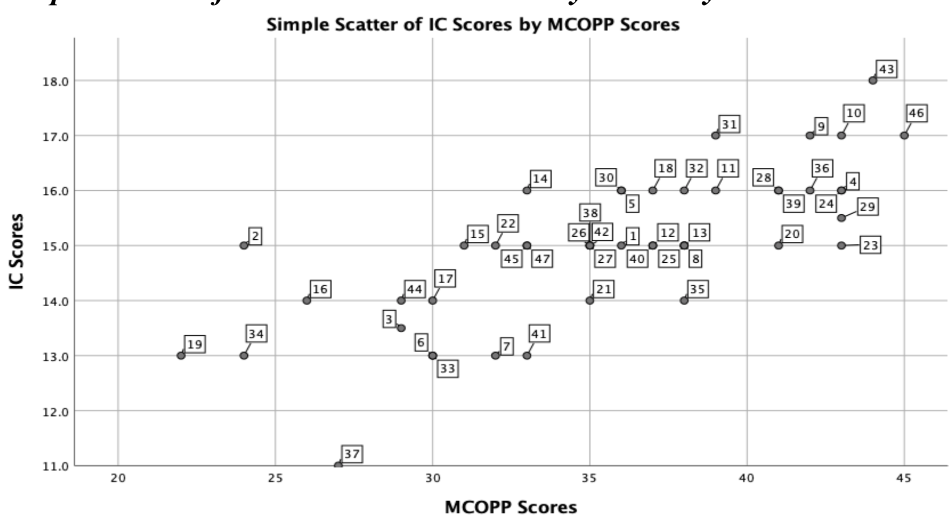
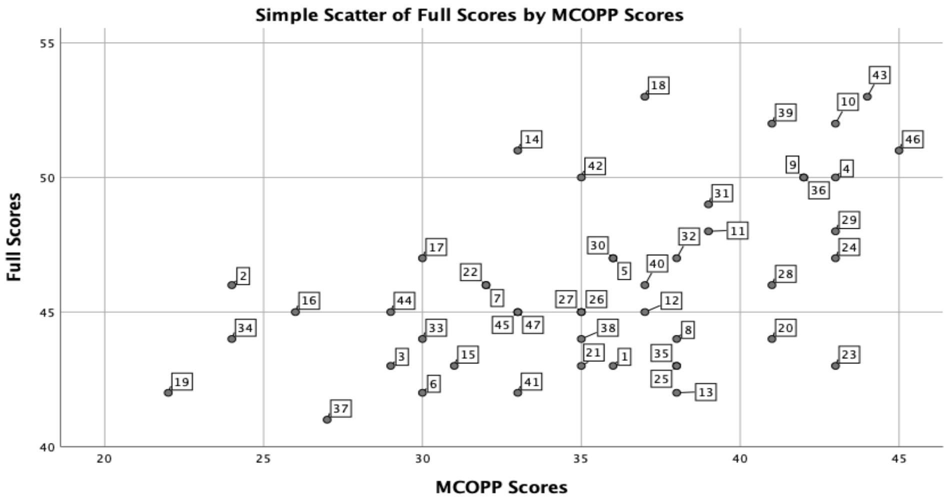


Figure 4

Simple Scatter of Full edTPA Scores by MCOP² Scores



Discussion and Implications

It is essential that preservice mathematics teachers develop a deep, rich understanding of how to facilitate and engage students in the Standards for Mathematical Practice in order to develop the processes and proficiencies in mathematics that are expected of K-12 students. As successful completion of the edTPA is required of all preservice teachers in order to receive their teaching credential, I examined the edTPA for evidence of preservice mathematics teachers' facilitation and engagement of their students in the practices. The purpose of this study was to measure the extent to which preservice mathematics teachers implemented and engaged their students in their edTPA video lessons as measured by the MCOP². By scoring all 47 preservice mathematics teachers' edTPA video clips with the MCOP², I was able to run a correlation with their edTPA scores to better understand the extent to which they incorporated the practices in their lessons, and therefore give us better insight into the depth of understanding preservice mathematics teachers have of the Standards for Mathematical Practice.

First, the results of the MANOVA clearly show that the university the preservice mathematics teacher attended was not a significant predictor for both their instructional commentary and full edTPA scores (p =.400). While each of the four teacher education programs in this study had many similar components, one major difference was with University 3, which was an undergraduate program. It is slightly surprising then, that there was not a difference in scores for this university when compared to the other three, as preservice teachers in the other three programs were older, had graduated from their undergraduate program, and possibly had more life experiences that could all add to higher edTPA scores. However, there was little fluctuation in either instructional commentary or full edTPA scores. Most instructional commentary scores fell between 12 and 18 and most full edTPA scores fell between 43 and 53. A larger range in scores may have led to the university being a significant predictor and would require a more critical lens to evaluate how the programs differed. In addition, each of the three sections of the edTPA include 5 detailed rubrics, and significant differences at this level might exist among universities.

Second, I found that $MCOP^2$ scores had a high, positive correlation with both the instructional commentary and full edTPA scores. This suggests that the $MCOP^2$ scores were not only significant predictors of each, but described the extent to which preservice mathematics teachers drew on the Standards for Mathematical Practice in their edTPA lessons. Participants who had high $MCOP^2$, instructional commentary, and full edTPA scores that correlated can be seen as those who successfully facilitated and engaged students in the practices in their lessons. On the other hand, those who scored low on all three suggests that these participants were not as successful in their implementation of the practices. We know from several previous studies (Lambert et al., 2020; Rogers et al., 2020;

Soto & Marzocchi, 2020) that the $MCOP^2$ has been used to successfully measure how mathematics teachers implement and engage their students in the practices. However, none of these studies applied the $MCOP^2$ to preservice mathematics teachers in order to understand the depth of their knowledge of the practices. In addition, while there is some previous literature that looked at preservice mathematics teachers and the edTPA (Bunch et al., 2015; Henry et al., 2013; Lim et al., 2015; Santagata et al., 2018), it was focused on academic language or elementary preservice teachers. Since the results of this study showed a high, positive correlation between the $MCOP^2$ and both the instructional commentary and full edTPA scores, further research should be done using the $MCOP^2$ in order to measure how well preservice mathematics teachers are able to implement and engage their students in the practices. In addition, preservice mathematics teachers' $MCOP^2$ scores could be broken down to see what specific practices they may be struggling with. This would allow researchers to build off of previous literature (Bunch et al., 2015; Lim et al., 2015) that focused on preservice mathematics teachers' use of classroom discourse and academic language to support their English learners and the depth of which they can display this understanding in their edTPAs.

Finally, this research has implications for preparing future preservice mathematics teachers to focus their students' mathematical learning on problem solving, reasoning, communication, representation, and connections, areas that are all addressed by the Standards for Mathematical Practice. By using the $MCOP^2$ to evaluate the extent to which these preservice mathematics teachers drew on the practices in their lessons, I was able to gain insight into how well they understood the practices themselves. As successful implementation of the Standards for Mathematical Practice is dependent on a community of learners, where both teacher facilitation and student engagement play a key role, using the MCOP^2 allowed the evaluation of preservice teachers' implementation of the practices from both aspects (Barker et al., 2004; NCTM, 2006, 2011; Stein et al., 2008; Stein et al., 2009). These findings can direct our mathematics methods courses and the curriculum we design to ensure that preservice mathematics teachers have a deep and rich understanding of the Standards for Mathematical Practice.

Conclusion

This study aimed at understanding the extent to which preservice mathematics teachers implemented and engaged their students in the Standards for Mathematical Practice in their edTPA video clips. Through the lens of a community of learners that involves both teacher facilitation and student engagement, I analyzed preservice mathematics teachers' edTPA video clips and scored them using the MCOP^2. By running a correlation, I was able to see if participants' MCOP^2 scores, their instructional commentary, and full edTPA scores were significant predictors of each other. I found a high, positive correlation between scores, which suggests that preservice mathematics teachers' edTPA video clips display their implementation of the Standards for Mathematical Practice. In addition, by running a MANOVA, I was able to determine that the university was not a significant predictor of edTPA scores for our participants.

There are a number of limitations that exist in this study. One of the main limitations was the sample size, which consisted of 47 participants. While it is above the recommended n=30 to look for significance, a larger sample size would shine more light on if there are significant differences in edTPA scores among campuses, as well as possibly produce a higher positive correlation between scores. In addition, the edTPA video clips themselves were a limitation, as they varied among participants. Some preservice teacher submitted two clips, each from a separate day, while other preservice teachers submitted one longer clip. Finally, the sample population for this study was generated from classrooms within one geographical region of the United States, and a larger, more diverse national sample should be the goal of subsequent work.

Teacher education programs are responsible for preparing future mathematics teachers, by supporting them in developing not only knowledge of their content, but the ability to connect concepts across lessons and present curriculum in multi-modal ways that are differentiated and scaffolded. We cannot have high expectations of our K-12 students without having high expectations of our teachers. Having high expectations of our students requires that teachers facilitate and engage them in the Standards for Mathematical Practice. While the edTPA is far from perfect, it does help us paint a picture of what our preservice mathematics teachers are prepared for and how we must support them in our teacher education programs. Teachers who are able to engage students in the Standards for Mathematical Practice will not only provide better outcomes for students, but an overall better understanding of what it means to be an effective teacher.

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Chapter 4: ***Cognitively Demanding Tasks and the Standards for Mathematical Practice Present in Preservice Mathematics Teachers edTPA***

Introduction

Mathematics education in the United States today is faced with the task of requiring instructional and curricular reforms that support how students must learn to think of math in new and demanding ways (Tekkumru-Kisa et al., 2020). Students are expected to be taught mathematics as a more authentic discipline as defined by the Common Core States Standards for Mathematics (CCSSM; National Governors Association Center for Best Practices, Council of Chief State School Offices, 2010). The CCSSM asks that students are provided with the appropriate mathematical tasks, allowing them opportunities to think about and learn mathematics on a deeper level. Mathematics teachers are the ones who decide on and assign students mathematical work, and therefore must implement tasks that demand high expectations of students, define the curriculum, and create mathematical meaning (Doyle, 1983). Past research on mathematics teachers' use of cognitively demanding tasks in their classrooms shows that while they attempted to embrace high-demand tasks, they actually strip tasks of their mathematical logic or ideas when put into practice and subsequently fail to support students to develop a deeper meaning of the content (Cohen, 1990; Minor et al., 2016; Spillane, 2000; Spillane & Zeuli, 1999). Because of this, it is essential that teacher education programs support their preservice mathematics teachers in learning how to combine high-quality curriculum materials that are aligned with student assessment to best prepare them for implementing cognitively demanding mathematical tasks when they are in-service teachers (Tekkumru-Kisa et al., 2020).

Previous mathematics education research has looked at the understanding and use of cognitively demanding tasks through the Task Analysis Guide (TAG) (Smith & Stein, 1998). This framework examines mathematical tasks using four different levels of thinking and levels of cognitive demand students engage in to solve them. *Doing mathematics* and *procedures with connections* are the two components of the TAG framework that elicit higher levels of cognitive demand from students, and the two low-level tasks are described as *procedures without connections* and *memorization*. As mathematical tasks impact students' perceptions of and opportunities to engage in and understand mathematics, selecting the appropriate task is one of the most vital decisions a mathematics teacher makes (Lappan & Briars, 1995). Therefore, it is essential that our teacher education programs help support preservice mathematics teachers in learning to differentiate between tasks on the basis of their cognitive demand (Boston, 2012) so that they are prepared to make this important decision to support their students' mathematical learning as in-service teachers.

One way for preservice mathematics teachers to successfully implement cognitively demanding tasks in their mathematics classrooms is through the use of the Standards for Mathematical Practice (Johnson et al., 2016). The Standards for Mathematical Practice map the way teachers should present and teach mathematics to their students; however, not all teachers fully understand the standards and are challenged to create engaging lessons for their students in order for them to have a deeper, more meaningful understanding of math concepts and their applications. These two essentials, cognitively demanding mathematical tasks and the Standards for Mathematical Practice, should be embedded together and reflective of each other in order to best support K-12 students in developing deep mathematical knowledge.

In order for our preservice mathematics teachers to be better prepared to support their future students in mathematical learning, we must offer our preservice mathematics teachers' opportunities to understand and engage in both cognitively demanding tasks and the Standards for Mathematical Practice. In this study, I looked to investigate the different levels of cognitively demanding tasks preservice mathematics teachers drew on and incorporated into their edTPA, as well as what Standards for Mathematical Practice are visible in these different levels of tasks. By focusing on both of these critical components in my study, I was able to develop a clearer understanding of not only the levels of tasks preservice mathematics teachers incorporated in their classroom lessons, but how these different levels of cognitively demanding tasks drew on and supported the eight Standards for Mathematical Practice. In this study, I posed the following research questions:

1. What levels of cognitively demanding tasks were present in preservice mathematics teachers' edTPA and how did this compare within and across high and low scoring cases?
2. Which of the Standards for Mathematical Practice were present among the different levels of cognitively demanding tasks in preservice mathematics teachers' edTPA lessons and how did this compare within and across high and low scoring cases?

Conceptual Framework

Designing and implementing cognitively demanding tasks for students in a mathematics classroom are primary responsibilities of a mathematics teacher. Cognitively demanding tasks are a catalyst for students to engage in higher level thinking and mathematical development and present an opportunity to engage students in the Standards for Mathematical Practice (Borko et al., 2000; Johnson et al., 2016). Understanding mathematics means understanding a complex system of relationships, which includes problem solving, conjecturing, and the use of metacognition (Stein & Smith, 1998). In order to “do mathematics”, students must be given appropriately challenging tasks to promote their mathematical knowledge of the topic at hand (Stein, 2000). However, identifying and designing cognitively demanding tasks for the mathematics classroom are a challenge for all teachers, and should therefore be an essential component of what preservice mathematics teachers learn in both their coursework and field experiences.

The importance of research on academic tasks in mathematics was investigated by Doyle (1983), who found that teacher expectations and student learning were both dependent on the types of tasks assigned. He categorized mathematics tasks as “familiar” and “novel”, where familiar problems are those that ask students to simply recall a solution or use rote memorization to answer a problem. Novel tasks, on the other hand, draw on students embracing both the structure and conceptual understanding of mathematics in order to problem solve. Mathematics teachers tend to rely heavily on familiar tasks, as this was how they were taught mathematics. Novel tasks require students to initially struggle with the content, but then create meaning from the tasks, which leads to a more organic understanding of mathematical principles and concepts (Carpenter & Lehrer, 1999; Hiebert et al., 2003; Plotnick & Gardner, 1991).

Based on Doyle’s previous work, Stein and Smith (1998) developed the Mathematical Tasks Framework (MTF) to provide consistency in understanding the complexity of mathematical tasks. They created a four-level framework to serve as a way to classify mathematical tasks dependent on their level of cognitive demand and cognitive opportunities provided to students. *Memorization* and *Procedures without Connections* are the two low-level types of tasks and *Procedures with Connections* and *Doing Mathematics* are the two high-level types of tasks. *Memorization* is the lowest level, where tasks are unambiguous for the student and present identical replications of previously known information and facts. For example, asking students to memorize and repeat multiplication facts is an example of a *Memorization* task. Next, *Procedures without Connections* are tasks that require students to recall previously learned algorithms for solutions, but do not ask students to make connections to the concepts that such algorithms are based on. The first of the two high-level tasks, *Procedures with Connections*, asks students to focus their attention on the required procedures without explicitly making such procedures evident from the task, providing an opportunity for students to engage with the underlying concepts and principles, in addition to raising the demand of the task. The highest-level task, *Doing Mathematics,* requires non-algorithmic thinking from students, with several possible solutions and no explicit predetermined pathway to get to those solutions. These types of tasks demand the highest levels of thinking from students, where they must actively engage in the concepts and use their previous knowledge to create an understanding of the content. Figure 1 below presents the two low-level cognitively demanding tasks and the two high-level cognitively demanding tasks, with descriptions of what constitute their categorization.

Figure 1

Levels of Cognitive Demand

LEVELS OF COGNITIVE DEMAND	
Lower Level	**Higher Level**
Memorization - Reproducing facts, rules, formulas or definitions with no connection to underlying meaning - Cannot be solved using procedures, because a procedure does not exist or because the time offered is too short to use a procedure - Not ambiguous	**Procedures with Connections** - Attention on purpose of procedure and understanding of mathematical concepts - Suggest pathway to follow that is connected to conceptual idea - Represented in multiple ways - Requires cognitive effort
Procedures without Connections - Algorithmic, use of procedure is specifically called for - Little ambiguity - No connection to meaning that underlies the procedure - Focus on correct answer, rather that developing mathematical understanding - No explanation required	**Doing Mathematics** - Requires complex and nonalgorithmic thinking - Requires exploration and understanding of mathematical concepts, processes or relationships - Requires considerable cognitive effort - Requires students to analyze the task and examine constraints that may limit the solution or strategies

Smith, M. S. & Stein, M. K. (2011). *5 practices for orchestrating productive mathematics discussions*. Reston, VA: National Council of Teachers of Mathematics.

While creating and selecting the appropriate mathematical tasks for classroom instruction are key first steps in ensuring the high-level of demand for students, it is the actual implementation of the task that has the most crucial impact on student learning (Boston, 2012). This is due to the fact that student learning occurs the most in the actual mathematics classroom, where students are continuously exposed to high-level cognitive demands and challenging problem-solving scenarios. Throughout their instruction, teachers tend to lose the high-level demand of a task, and it declines throughout the lesson (Henningsen & Stein, 1997). Often, when students struggle with reasoning and problem solving, they become disengaged and reach out to the teacher for help (Henningsen & Stein, 1997; Romanagno, 1994). Teachers become uncomfortable with this and tend to lower the demand of the task, providing step-by-step solutions for their students and not holding them to the level of accountability needed to develop deep, meaningful understanding of the mathematics content (Boaler & Staples, 2008; Henningsen & Stein, 1997).

The use and implementation of high-level cognitively demanding tasks has been and continues to be a frequent and central topic in mathematics education (e.g., Borko et al., 2000; Crespo & Nicol, 2003; Norton & Kastberg, 2012; Stein et al., 2008). High-level tasks are a central component to reform-based instruction in the mathematics education community (NCTM, 1991), and support students in developing meaningful mathematics content knowledge. As high-quality tasks are an essential part of mathematics teaching (Stein et al., 2001), teachers must focus on building conceptual understanding through problem solving, which requires providing an instructional environment to develop the behaviors presented in the Standards for Mathematical Practices (CCSSM, 2010). Furthermore, high-level tasks provide essential ways in which preservice mathematics teachers can learn to engage their students in the Standards for Mathematical Practice and must be a key component of any mathematics methods course. In the context of this study, I viewed high-level cognitively demanding tasks as a way for preservice teachers to successfully carry out and implement the Standards for Mathematical Practice in their classroom instruction. As the practices embrace student development of deeper levels of understanding and engaging in mathematical concepts, we can get a better understanding of preservice mathematics teachers' implementation of the Standards for Mathematical Practice by observing the level of the demand of mathematical tasks they implement in a lesson.

Literature Review

This study of preservice mathematics teachers' use of the Standards for Mathematical Practice and cognitively demanding tasks is informed by prior research done on mathematics teachers as designers and implementors of mathematics curriculum while upholding the skills and practices required by current standards. In other words, for this literature review, I focused on past research that looked at mathematics teachers' use of both cognitively demanding tasks and the Standards for Mathematical Practice. While there is a plethora of research on mathematics teachers' understanding and implementation of both areas, little research exists when looking at the interaction between the two. In addition, almost all studies that have been done have looked specifically at in-service mathematics teachers. This study seeks to fill a gap in the literature by not only looking at both cognitively demanding tasks and the Standards for Mathematical Practice, but also by focusing on preservice secondary mathematics teachers and their edTPA.

There are several existing studies that looked at in-service mathematics teachers' understanding and implementation of cognitively demanding tasks while upholding the skills and practices from the CCSSM and the Standards for Mathematical Practice (Boston & Smith, 2011; Boston, 2012; Conley, 2011; Johnson et al., 2016; Lambert & Stylianou, 2013; McDuffie et al., 2018; Wilhelm, 2014). These past studies recognize the challenge that mathematics teachers face with creating high-demand tasks that adhere to the CCSSM and the Standards for Mathematical Practice, with many studies focusing on professional development opportunities put into place to support mathematics teachers. As one example, Johnson et al. (2016) investigated how mathematics teachers made sense of the CCSSM and the Standards for Mathematical Practice and how this translated into mathematical tasks they implemented in their classroom. Through organizing and supporting teachers in a professional development experience where they analyzed mathematical tasks and looked for opportunities to engage students in the Standards for Mathematical Practice, researchers found that while teachers could be successful in this, external constraints that were not present in the professional development itself often prevented teachers from holding to the level of demand of the task. In other words, regardless of the professional development provided to mathematics teachers, they still reverted back to tasks of lower-level demand once in the classroom. While researchers concluded that cognitively demanding tasks are a promising approach to implementing the CCSSM and the Standards for Mathematical Practice, professional development for mathematics teachers must be approached as a collaborative effort, where teachers have a voice to express their goals and concerns. In a similar study, McDuffie et al. (2018) looked at how mathematics teachers incorporated the Standards for Mathematical Practice when they interacted with different types of curriculum. Researchers found that when teachers selected curriculum with mathematical tasks that asked students to *think* about the mathematics they were learning, the Standards for Mathematical Practice were prioritized in the classroom. However, when teachers selected curriculum that treated mathematical tasks as a *transfer* of knowledge, the practices were not nearly as visible in their daily lessons and, therefore, students engaged with them much less. By viewing mathematics curriculum as a way to integrate the practices into a classroom, this study provides insight into how vital designing and selecting curriculum is for teacher educators in preparing mathematics teachers to support student engagement in the Standards for Mathematical Practice.

Several studies specifically looked at preservice mathematics teachers and their use of cognitively demanding tasks. However, these studies looked mostly at the mathematical knowledge for teaching that is required by preservice mathematics teachers to successfully create and implement high-demand tasks, and not at the interaction of these tasks with the Standards for Mathematical Practice. For example, one study by Rocha (2020) looked at how the domains of KTMT-knowledge for teaching mathematics with technology-were taught by preservice mathematics teachers. Researchers found that the preservice teachers were most successful when teaching KTMT through cognitively demanding tasks and that the level of the tasks and how familiar the preservice teacher was with the task influenced their implementation of the domains of KTMT. Other recent studies have looked at enhancing preservice mathematics teachers' knowledge of meaningful problems through cognitively demanding tasks (Crespo & Sinclair, 2008; Guberman & Leikin, 2013; Lee, 2012; Norton & Kastber, 2012; Slavit & Nelson, 2010). These studies suggest that preservice mathematics teachers need to have knowledge of a variety of problems that are relevant and open-ended in order to successfully implement and hold to the level of demand of the task.

Finally, a study by Dogan (2020) examined how preservice mathematics teachers designed cognitively demanding tasks specifically around mathematical modeling. The 20 participants in this study designed their own tasks, which were evaluated based on four criteria for mathematical modeling: reality, openness, complexity, and model eliciting. Researchers found that most of the tasks the preservice mathematics teachers created only fulfilled the reality criterion and lacked in cognitive demand. While this study is similar to my research, I looked at how preservice mathematics teachers designed cognitively demanding tasks around all eight of the Standards for Mathematical Practice, not just modeling. My study explores the gap in the literature that exists by investigating how preservice mathematics teachers incorporated both the eight Standards for Mathematical Practice and cognitively demanding tasks into their edTPA. While there are a handful of studies that look at preservice mathematics teachers' knowledge of cognitively demanding tasks, this study fills a gap in the literature by looking to see not only what levels of demand they incorporated into their edTPA, but also if and where the presence of the Standards for Mathematical Practice were visible in these tasks.

Methodology

Study Context

This study is part of a larger, ongoing research project that is focused on assessing the impact of teacher education programs on preservice secondary mathematics and science teachers from six large public universities throughout the state of California. For this study, I focused on four of the six universities, as one had incomplete data sets and one did not use the edTPA as their teacher performance assessment. In addition, data were collected from these four programs over the course of three years (Year 1: 2016-2017, Year 2: 2017-2018, and Year 3: 2018-2019), resulting in three cohorts of preservice mathematics teachers from each of the four campuses. While all four teacher education programs shared similar attributes in their composition, goals, and enrolled students, they also had differences that are essential to understanding their programs. In particular, three of the four universities in this study were graduate teacher education programs, with the exception of University 3 which was an undergraduate teacher education program. Table 1 below displays similarities and differences among each of the four programs. It is important to note that the descriptions of each program were for students enrolled in the single-subject credential, which was what the preservice mathematics teachers in this study were also enrolled in.

Table 1

Teacher Education Program Descriptions

	University 1	University 2	University 3	University 4
Credential Type	Post-Baccalaureate Credential	Post-Baccalaureate Credential	Integrated Undergraduate Credential	Post-Baccalaureate Credential
Master's Option	Yes	Yes	No	Yes, after completing credential
Duration	12 months (13 with M.Ed)	13 months (23 if no undergrad minor)	4-5 years (with undergraduate coursework)	11 months (+2 quarters for M.Ed.)
Coursework	18 courses	18 courses	7 courses (addition to undergraduate coursework)	10 courses
Field Experiences	Year-Long	1 Quarter Tutor, 2 Quarters Teaching	1 Semester Long	Year-Long
Full-Takeover	18 Weeks	22 Weeks	15 Weeks	22 Weeks
edTPA Support	Yes	Yes	Yes	Yes
Mathematics Methods Courses	3 Courses	4 courses	4 courses	2 courses

Participants

Participant selection for this study consisted of two stages. Overall, there were a total of 73 mathematics candidates from all four universities across the three years. Of the 73 mathematics candidates, 70 agreed to participate in the larger study, and out of these, 47 had complete data sets, which consisted of pre- and post-interviews and completed edTPA portfolios. Table 2 shows the distribution of participants from the four universities over the three years of the study and Table 3 shows the demographics of participants across all three years of data collection.

Table 2

Distribution of Preservice Mathematics Teachers by Campus

	Year 1	**Year 2**	**Year 3**	**Total**
University 1	4	6	6	16
University 2	2	2	4	8
University 3	5	3	6	14
University 4	3	3	3	9
				47 Total

Table 3

Participant Demographics

Gender	
Female	72%
Male	28%
Race/Ethnicity	
White/European American	50%
Latinx	22%
Asian/Asian American	20%
Multiracial	4%
Other	2%
Pacific Islander	2%
First Language	
English	74%
Language(s) other than or in addition to English	26%

Undergraduate Major	
Mathematics	72%
Other	28%

*Note: All demographic data are self-reported
***Note: One participant did not report their gender, race/ethnicity, or first language

The second stage of participant selection was based on a previous research study that looked at the extent to which these preservice mathematics teachers incorporated the Standards for Mathematical Practice into their edTPA instructional commentary as measured by The Mathematics Classroom Observation Protocol for Practices ($MCOP^2$). This tool was originally designed as a classroom observation tool and measures both teacher facilitation and student engagement through a total of 16 items, which are mapped directly onto the eight Standards for Mathematical Practice (Gleason et al., 2017). I viewed all 47 preservice mathematics teachers edTPA video clips and scored them using the $MCOP^2$. I then created a correlation coefficient to see if the participants' $MCOP^2$ scores were statistically significant predictors of their instructional commentary and full edTPA scores. I created two simple scatterplots in order to create a visual of the correlation between the participants' $MCOP^2$ scores and their instructional commentary and full edTPA scores. Based on this analysis, I selected six preservice mathematics teachers, or cases, to further evaluate their use of the Standards for Mathematical Practice in their edTPA. Of these six cases, I selected the three that scored the highest on both the $MCOP^2$ and their edTPA, and the three that scored the lowest on both the $MCOP^2$ and their edTPA. I did this to determine if differences in the use of cognitively demanding tasks and the Standards for Mathematical Practice existed between preservice mathematics teachers who scored high and low on their edTPA. Table 4 below provides detailed information on the six participants included in this study.

Table 4

Breakdown of Six High and Low Preservice Mathematics Teachers

Pseudonym	Race/ Ethnicity	Gender	Undergrad Major	University	High/Low Case
Nellie	White/European American	Female	Cognitive Science	4	High
Emma	White/European American	Female	Statistics	4	High
Tierney	Asian/Asian American	Female	Mathematics	1	High
Janet	Hispanic or Latina/o	Female	Mathematics	2	Low
Leanette	Asian/Asian American	Female	Statistics	3	Low
Lea	Asian/Asian American	Female	Mathematics	3	Low

Data Collection

As previously discussed, data for the larger study were collected across four teacher education programs from four universities in three separate cohorts. Data collection for the larger study consisted of initial and follow-up interviews and surveys for both secondary science and mathematics preservice teachers, as well as their edTPA portfolios. Participants' edTPA portfolios consist of three sections: planning, instruction, and assessment. The planning section is made up of several different parts and is based on 3 to 5 lessons the preservice teacher plans to implement in their classroom, which are revisited during the instructional section of the edTPA. The planning section consists of the context for learning, lesson plans, and participants' planning commentary. Through these components, preservice mathematics teachers are asked to provide evidence of the focus of the central lesson, as well as the instructional strategies, materials and planned supports that are included. In addition, preservice mathematics teachers must support their instructional choices with an explicit rationale and justification through their planning commentary. For this study, I analyzed all components of participants' planning section. Together, these components provided an in-depth reflection of what types of cognitively demanding tasks participants provided to their students, as well as how they engaged them in the Standards for Mathematical Practice.

Data Analysis

For this study, I conducted two cycles of coding to analyze the data. For the first cycle, I first looked at the materials present in each of my participants' planning sections and identified the mathematical tasks they included. I identified each mathematical task as a single problem or set of problems that focused student attention on a mathematical idea (Stein et al., 1996). Figures 2 and 3 below are examples of two mathematical tasks based on the work of Stein et al. (2009) that I also used as a guide. In addition, Table 5 below shows the breakdown of lessons and mathematical tasks from each participant.

Figure 2

High-Level Mathematical Task

Create a real-world situation for the following problem: $\frac{2}{3} \times \frac{3}{4}$.

Solve the problem you have created without using the rule and explain your solution.

One Possible Student Response:

For lunch Mom gave me three-fourths of the pizza that we ordered. I could only finish two-thirds of what she gave me. How much of the whole pizza did I eat?

I drew a rectangle to show the whole pizza. Then I cut it into fourths and shaded three of them to show the part mom gave me. Since I only ate two thirds of what she gave me, that would be only two of the shaded sections.

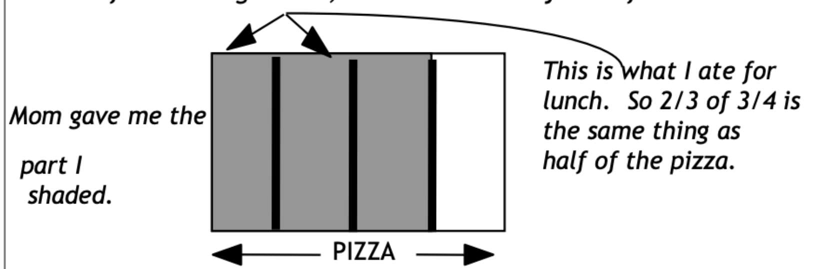


Figure 3

Low-Level Mathematical Task

Multiply: $\frac{1}{6} \times \frac{1}{2}$

$\frac{2}{3} \times \frac{3}{4}$

$\frac{4}{9} \times \frac{3}{5}$

Expected Student Response:

$$\frac{1}{6} \times \frac{1}{2} = \frac{1 \times 1}{6 \times 2} = \frac{1}{12}$$

$$\frac{2}{3} \times \frac{3}{4} = \frac{2 \times 3}{3 \times 4} = \frac{6}{12}$$

$$\frac{4}{9} \times \frac{3}{5} = \frac{4 \times 3}{9 \times 5} = \frac{12}{45}$$

Table 5

Number of Lessons and Tasks Present in Participants' Planning Section

	Number of Lessons	Number of Tasks
Nellie	5	14
Emma	4	8
Tierney	5	10
Janet	3	11
Leanette	4	15
Lea	3	12

Once I identified all the tasks present in my preservice mathematics teachers' materials, I used a priori coding to code the level of cognitively demanding tasks that the six preservice mathematics teachers discussed giving to their students throughout their lessons (Saldana, 2016). I placed each task the preservice mathematics teachers discussed in their planning commentary in one of the four cognitively demanding task categories defined by the TAGs Framework designed by Smith & Stein (2011) and looked to see how many tasks from each participant fell into which of the four categories. Because each participant had a different number of tasks, I converted these results into percentages so that I would be able to compare my results across all participants. Table 6 below provides the four coding levels for this cycle of analysis.

Table 6

Levels of Cognitive Demand

Code Level	*Code Name*	*Code description*
High-Level Task 1	*Doing Mathematics*	Tasks that require complex thinking, require students to explore and understand the nature of mathematical concepts, processes, or relationships, demand self-monitoring and self-regulation, require students to access relevant knowledge in working through the task and actively examine task

		constraints, and require considerable cognitive effort from students.
High-Level Task 2	*Procedures With Connections*	Tasks that focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas, are usually represented in multiple ways making connections among multiple representations to help develop meaning and require some degree of cognitive effort.
Low-Level Task 1	*Procedures Without Connections*	Tasks that are algorithmic and use procedure that is specifically called for, require limited cognitive demand, have no connection to the concepts or meaning that underlie the procedure being used, are focused on producing correct answers rather than developing mathematical understanding, and require no explanations or explanations that focus solely on describing the procedure that was used.
Low-Level Task 2	*Memorization*	Tasks that involve either producing previously learned facts, rules, formulae, or definitions or committing to these to memory, cannot be solved using procedures, are not ambiguous, and have no connection to the concepts or meaning that underlay the facts, rules, formulae, or definitions being learned or reproduced.

For the second level of analysis, I looked at each of the mathematical tasks I identified in my participants' planning section and coded for the presence of the Standards of Mathematical Practice in each task. In order to ensure that I accurately coded each level of task correctly, I looked at not only the task itself, but how the task was used as described by the preservice mathematics teachers in their lesson plans, planning commentaries, and context for learning. While my previous research used the MCOP^2 to identify which practices were present in these participants' edTPA instructional videos, this study looked to participants' planning section to see what their original intentions were for how they drew on the Standards for Mathematical Practice and how this compared across high and low cases. Again, I used a priori codes constructed from the eight Standards of Mathematical Practice, as seen below in Table 7. A fellow graduate student and I coded 20% of the data, and then met to discuss results and resolve disagreements until we reached consensus. This process continued until we consistently reached interrater reliability great than .80 (Fleiss, 1971), at which point I coded the rest of the data independently. It is important to note that I coded for the presence of a practice in each task, and not how many times that practice may or may not have been present within a task. Once coding was complete, I created a table for each of my six participants in order to show which practices were present in which of the four levels of cognitively demanding tasks. I identified the presence of a practice in a task by placing a dot in the table, with no dot meaning that there was no practice visible in that task. By analyzing and presenting the data in this way, I was able to see which practices were present in which tasks and compare results not only within my high and low cases, but across all six cases.

Table 7

The Standards for Mathematical Practice

Practice Name	*Practice Definition*
SMP 1: Problem-Solving	Preservice mathematics teacher described opportunities for students to make sense of problems and persevere in solving them.
SMP 2: Reasoning	Preservice mathematics teacher described opportunities for students to reason abstractly and quantitatively.

SMP 3: Argumentation	Preservice mathematics teacher described opportunities for students to construct viable arguments and critique the reasoning of others.
SMP 4: Modeling	Preservice mathematics teachers described opportunities for students to model with mathematics.
SMP 5: Tools	Preservice mathematics teachers described opportunities for students to use appropriate tools strategically.
SMP 6: Precision	Preservice mathematics teachers described opportunities for students to attend to precision.
SMP 7: Structure	Preservice mathematics teachers described opportunities for students to look for and make use of structure.
SMP 8: Regularity	Preservice mathematics teachers described opportunities for students to look for and express regularity in repeated reasoning.

Results

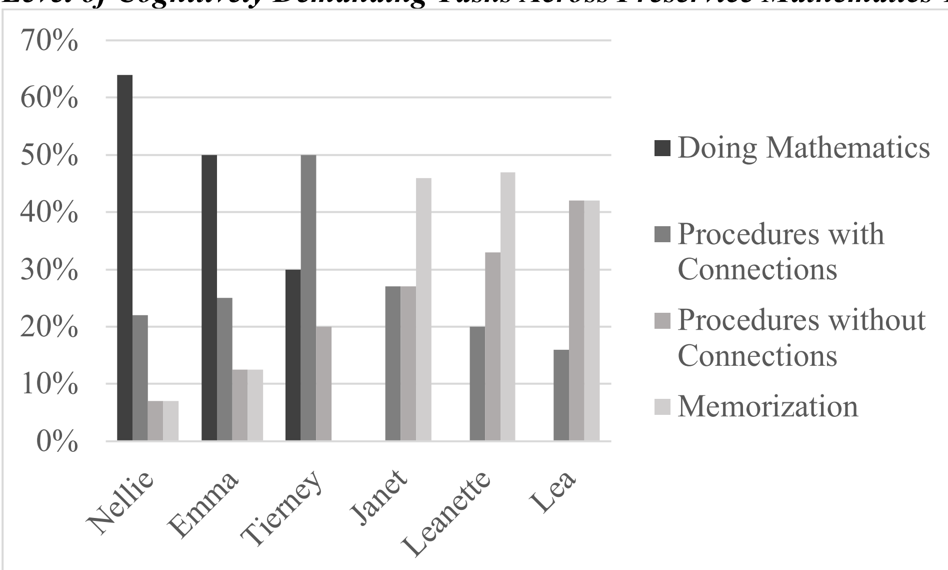
Level of Cognitively Demanding Tasks in Preservice Mathematics Teachers' edTPA

For my first research question, I identified the mathematical tasks present in each of the six participants' planning sections of their edTPA, and then investigated the level of cognitive demand of each task. I did this by placing each task in one of the four cognitively demanding task categories defined by the TAGs Framework designed by Smith and Stein

(2011). This allowed me to not only see the level of cognitively demanding tasks used by each participant, but also to compare within and across high and low cases. Table 8 below shows each of the six participants and the percentage of their tasks that were coded into the four levels of cognitive demand.

Table 8

Level of Cognitively Demanding Tasks Across Preservice Mathematics Teachers



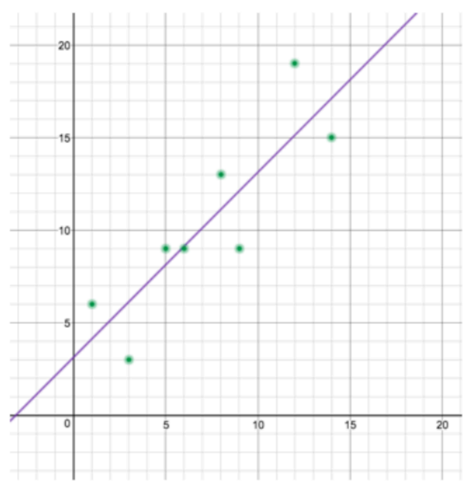
**Note: Percentages based on number of tasks coded for each level*

Looking first at the percentage of high-level cognitively demanding tasks these six preservice mathematics teachers incorporated into their planning commentary, only half incorporated tasks that fell into the *doing mathematics* category. This category is the highest level of demand for mathematical tasks and requires students to think at a complex level with considerable cognitive effort. Nellie, Emma, and Tierney, the three high cases, all had tasks that were coded as *doing mathematics*. The three low cases, Janet, Leanette, and Lea, had no tasks coded at this level of cognitive demand. In addition, both Nellie and Emma had more tasks that I coded as *doing mathematics* than any other level, making it the level of cognitive demand that most of their tasks came from. Figure 4 below is an example of a task coded as *doing mathematics* included as an assessment in Nellie's planning commentary. Not only does this task require her students to use high cognitive effort in order to decide which correlation coefficient is correct, but she also asked them to explain their reasoning which allows them to explore and display their own understanding of the mathematical relationship.

Figure 4

Doing Mathematics Task From Nellie's edTPA

Which correlation coefficient matches the scatterplot best? Explain why.

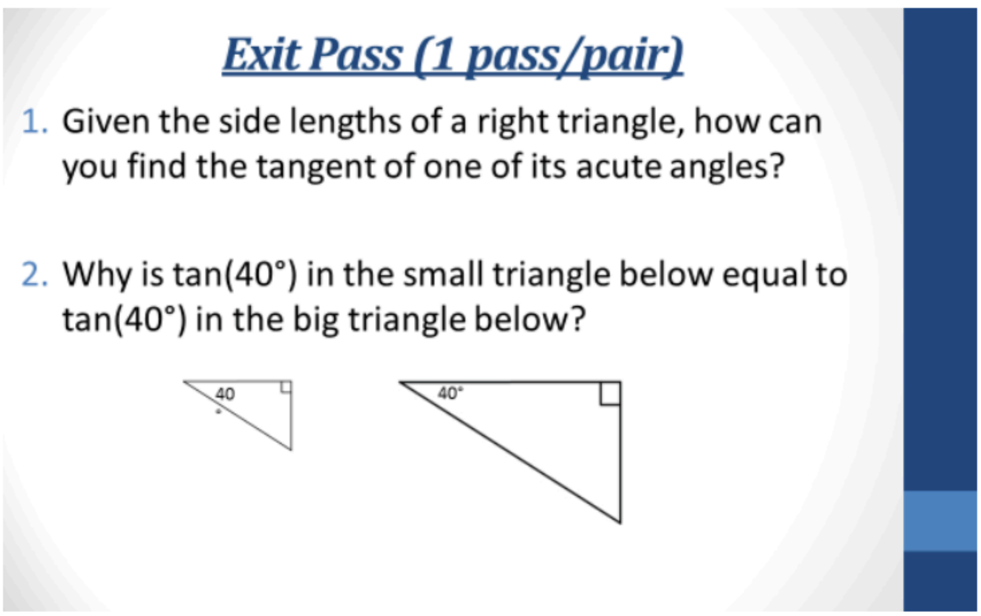


$a)\ r = 1$
$b) r = 0.86$
$c)\ r = -0.75$
$d) r = 0.40$

Tasks at the second highest level of cognitive demand, *procedures with connections,* are tasks that ask students to focus on the procedure while making meaningful connections to other contexts, such as real-world concepts or other mathematical topics. While all six preservice mathematics teachers incorporated tasks that I coded as *procedures with connections*, the three high cases incorporated more of these tasks than the three low cases. As discussed above, both Nellie and Emma had the most tasks that were coded as *doing mathematics*, followed by *procedures with connections*. Tierney, the third high case, had the opposite situation in that she had more tasks that were coded as *procedures with* *connections*, and then followed by *doing mathematics*. Janet, Leanette, and Lea, the low scoring cases, only incorporated a few tasks that were of this demand level into their planning commentaries. Figure 5 below is an example of a task Tierney gave to her students, which I coded as *procedures with connections.* While the focus of the first part of the task asks students to provide procedural information, which they must complete to find the tangent of one of the right triangle's acute angles, the second half of the task asks students to make a connection between concepts, developing a deeper level of their mathematical understanding.

Figure 5

Procedures With Connections Task From Tierney's edTPA



All six participants also had tasks that were coded as the first of the two low cognitively demanding tasks, *procedures without connections.* While the three high cases did not have nearly as many tasks coded at this low level of demand, Tierney had 20% of her tasks coded as procedures without connections, almost 10% more than both Nellie and Emma. However, Janet, Leannette, and Lea, the three low cases, all had a much higher percentage of tasks coded at this level when compared to the three high cases. Tasks that are considered to be *procedures without connections* require students to use minimal cognitive demand and have no meaning behind the procedure, whereas memorization tasks are simply based on students relying on facts, formulas, or definitions from memory. All three had over 20% of their tasks that were coded at this level, with Lea having the greatest amount with 40% of her tasks coded as *procedures without connections*, equal to the number of tasks she had coded as *memorization*. Figure 6 below is a task used by Lea in her planning section, which I coded as *procedures without connections*. While this task requires students to display procedural understanding, it makes no connections to the processes or meaning that underlie the procedure being used. In addition, it requires no explanations from students.

Figure 6

Procedures Without Connections Task From Lea's edTPA

Is Ms. Lien correct?

Ms. Lien says the expression $\frac{(m+3)^2}{2n}$ is the sum of m and 3, divided by twice of n, then raised altogether to the power of 2. Is Ms. Lien correct?

Finally, we again see that tasks coded as the lowest level of cognitive demand, *memorization*, were mostly present in the three low cases. Both Nellie and Emma had very few tasks that were coded as this, and Tierney had none. Janet, Leannete, and Lea, however, all had tasks coded as *memorization* more than or equal to any other level. All three low cases had at least 40% of their tasks coded as *memorization*, which was the highest for both Janet and Leannette. *Memorization* and *procedures without connections* were tied at a little over 40% for Lea. Figure 7 below shows a task Leanette used in her planning commentary that was coded as *memorization.* The task requires students to show a very limited amount of cognitive demand and asked for them to display their mathematical understanding through previously memorized rules.

Figure 7

Memorization Task From Leanette's edTPA

Use exponents to write each of the following expressions as simply as possible.

$\left(x^2\right)\left(x^5\right)$

e) $\frac{x^3}{x^1}$

$x^7 \cdot x^5$

f) $x^3 \cdot x^4$

$y^8 \cdot y^6$

g) $m^{13} \cdot m^{14}$

Standards for Mathematical Practice Present in Preservice Mathematics Teachers' Various Levels of Cognitively Demanding Tasks

For my second research question, I wanted to investigate which Standards for Mathematical Practice were present in each of the six participants' cognitively demanding tasks. In addition, I sought to investigate whether there were any similarities or differences in the practices present in the various levels of tasks, and how this compared within and across both high and low cases. Once I coded each cognitively demanding task for the presence of any of the eight Standards for Mathematical Practice, I created a table for each of my six participants that displays the practices that were present in the participants' various tasks. By doing this, I was able to see if certain levels of cognitively demanding tasks had a certain type of practice or practices present and to compare this across all cases.

Table 9 below shows the practices that were present in the different levels of cognitively demanding tasks from the three high case participants. In terms of similarities, all three participants showed evidence of Practice 1 and Practice 4 in tasks coded as *doing mathematics* and *procedures with connections*, the two high-level tasks. In addition, there were multiple practices present in each of these two high-level tasks for all three participants. However, only Nellie and Emma also had Practice 3 and Practice 2 present in tasks coded as *doing mathematics* and *procedures with connections*. Figure 8 below is an example of a task Emma used where Practices 1, 2, and 3 are present and the task itself was coded as *doing mathematics*. This task required her students to make sense of and quantitatively explore the nature of the mathematical concepts, while constructing arguments to support their work. Tierney did not have either Practice 2 or 3 in any of her tasks. While Practice 8 is not present in any of these three participants' tasks, Tierney did have both Practices 6 and 7 in her tasks labeled as *doing mathematics* and *procedures with connections*. So, while all three high cases had multiple practices present in each level of task, as well as the presence of both Practices 1 and 4, Nellie and Emma had more similarities with Tierney being slightly different.

Table 9

Practices Present in Four Levels of Cognitively Demanding Tasks: High Cases

	Doing Mathematics	Procedures With Connections	Procedures Without Connections	Memorization
SMP 1	● ■ ◆	● ■ ◆		
SMP 2	● ■	● ■		
SMP 3	● ■	● ■		
SMP 4	● ■ ◆	● ■ ◆	● ■ ◆	
SMP 5	● ◆	● ◆	● ◆	●
SMP 6	◆	■ ◆	◆	■
SMP 7	◆	◆	■ ◆	■
SMP 8				■

**Note: Nellie= blue circle, Emma=green square, Tierney=orange diamond*

Figure 8

Doing Mathematics Task With Practices From Emma's edTPA

1. Carly is asked to use the table to solve the equation $64 = 5^t$. She says she can't use the table because there are no powers of 10 in her equation.

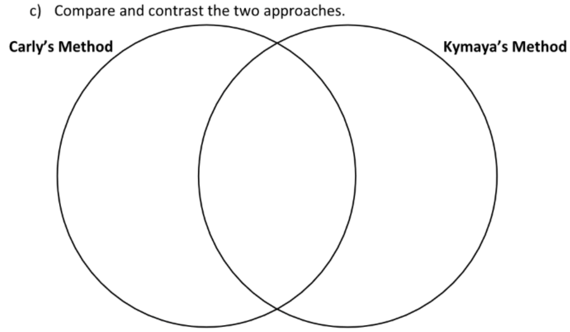
   a) Show Carly how to rewrite her equation so that there are powers of 10, and then use the table to approximate a solution.

   b) Kymaya has a different idea to solve $64 = 5^t$ and wrote the first two steps below.

   $64 = 5^t$
   $\log(64) = \log(5^t)$

   What did Kymaya do? Continue her method to finish the problem.

   c) Compare and contrast the two approaches.



For the three low case participants, Table 10 below show the breakdown of what practices were present in the four levels of tasks. To start, we can see that there are no practices present in the highest level of cognitively demanding tasks, *doing mathematics*, because they did not implement these kinds of tasks. Moving to the second highest level, *procedures with connections*, all three participants displayed evidence of Practice 1 in their tasks. All three participants also had Practice 6 present in both the two low level tasks, *procedures without connections* and *memorization*. Figure 9 below shows a task used by Janet, which I coded as procedures without connections, as there is little connection to underlying concepts, and the focus is more procedural than anything else. While there is no presence of Practice 1 or 3 in this task, there is an emphasis on attending to precision with correctly labeling and scaling, or Practice 6. In terms of differences, Janet was the only low case who had Practices 2, 3, and 5 evident in her mathematical tasks, which had a presence in *procedures with connections*. She was also the only participant to have at least three practices present throughout any level of task. Both Leannette and Lea only had one to two practices present, and neither of these participants had Practice 3 present in any of their tasks. Janet showed evidence of six of the eight practices in her tasks, excluding Practices 7 and 8.

Table 10

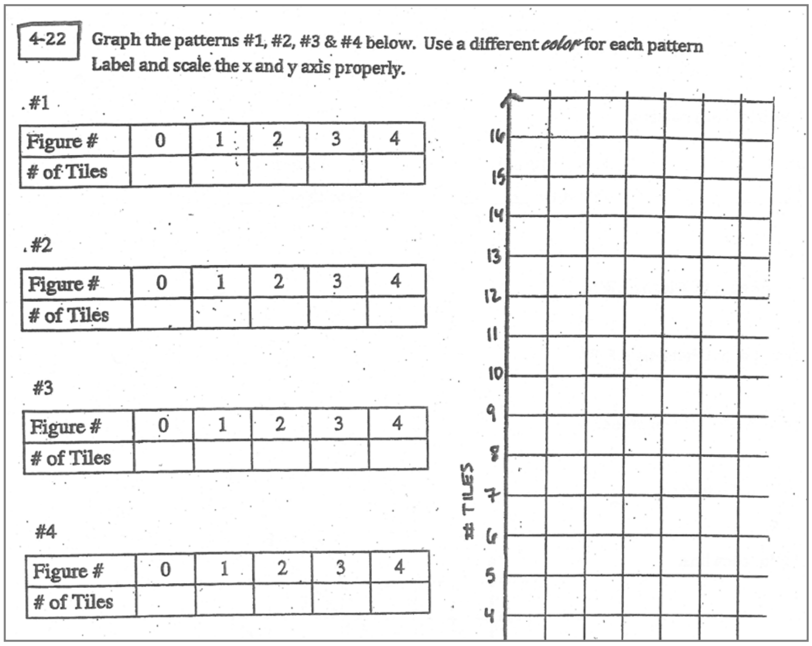
Practices Present in Four Levels of Cognitively Demanding Tasks: Low Cases

	Doing Mathematics	Procedures With Connections	Procedures Without Connections	Memorization
SMP 1		● ■ ♦		
SMP 2		●	●	●
SMP 3		●		
SMP 4		■	●	
SMP 5			●	●
SMP 6		♦	● ■ ♦	● ■ ♦
SMP 7				
SMP 8			■	■

******Note: Janet=black circle, Leanette=gray square, Lea=purple diamond*

Figure 9

Procedures Without Connections Task With Practice 6 in Janet's edTPA



4-22 Graph the patterns #1, #2, #3 & #4 below. Use a different color for each pattern
Label and scale the x and y axis properly.

#1

Figure #	0	1	2	3	4
# of Tiles					

#2

Figure #	0	1	2	3	4
# of Tiles					

#3

Figure #	0	1	2	3	4
# of Tiles					

#4

Figure #	0	1	2	3	4
# of Tiles					

Overall, I found several major differences between the high and low case groups. While all six participants had Practice 1 present in tasks that were coded as *procedures with connections*, only the three high cases also had Practice 3 and Practice 4 present in these tasks as well. In addition, the three high cases also had Practice 1 and Practice 4 present in the highest level of cognitively demanding tasks, *doing mathematics.* As previously stated, none of the low cases had any tasks coded at this level. In addition, among all participants who had tasks coded at the lowest level, *memorization*, there was the absence of Practices 1, 3, and 4, with a mix of Practices 5, 6, 7, and 8 visible throughout. Finally, when looking at the sheer number of practices visible in each task, the three high case participants had a greater number of practices as compared to the three low cases.

Discussions and Implications

With in-service mathematics teachers in the United States faced with the challenge of providing curriculum and instruction to their students that supports them in learning mathematical concepts in new and demanding ways, teacher education programs must focus on preparing their preservice mathematics teachers to meet this demand as well. By incorporating high-level cognitively demanding tasks that draw on the Standards for Mathematical Practice, teachers can provide their students with the authentic instruction needed to be successful in mathematics (Boston & Wilhelm, 2015). However, research focusing on how preservice mathematics teachers incorporate cognitively demanding tasks and the Standards for Mathematical Practice is limited, especially when looking for evidence in their edTPAs. This study provides needed insight by investigating not only what levels of cognitively demanding tasks these six preservice teachers incorporated into their edTPA planning section, but also what Standards for Mathematical Practice were visible in these different tasks.

I organized this section based on discussion around the findings for my high and low cases, and not by research question. To start, most of the tasks that the three high cases incorporated into their planning section were coded as the two highest levels of demand, *doing mathematics* and *procedures with connections*. All three high cases had 75% of their tasks coded at these two high levels of demand. It is interesting to note that one of the three high cases, Tierney, had more tasks coded as *procedures with connections* as compared to tasks coded as *doing mathematics*. The other two high cases, Nellie and Emma, had the opposite situation, with most tasks coded as *doing mathematics*, followed by *procedures with connections.* In addition, Tierney was the only participant to have no tasks coded as the lowest level of cognitive demand, *memorization*.

As we look to the findings from the second research question, one possible explanation for why Tierney had more tasks coded as *procedures with connections* and not *doing mathematics* may be because she did not show evidence of Practice 3 in either of these kinds of tasks, suggesting that this practice may contribute to the level of demand of the tasks. However, we do see evidence of both Practice 1 and Practice 4 throughout these three cases. Tasks that are considered to be high cognitively demanding tasks involve students paying attention to the purpose behind a procedure and require students to both analyze the task and support their mathematical reasoning through detailed explanations (Smith & Stein, 2011). These are all skills we see present in Practices 1, 3, and 4, suggesting that high-cognitively demand tasks naturally support the presence of these practices. While Practice 4 is one that teachers have reported having difficulty understanding in the past, its visibility in high cognitively demanding tasks suggests that preservice mathematics teachers may be drawing on it without realizing it (Jung & Newton, 2018). A final reason for the difference among the three high cases may be due to the fact that Nellie and Emma were both from the same teacher education program at University 4, while Tierney was from University 1.

None of the three low cases incorporated tasks at the highest level of demand, *doing mathematics.* While we know that mathematics teachers tend to strip these tasks of their high demand, this is usually when they are put into practice, and not at the planning stage (Cohen, 1990; Minor et al., 2016; Spillane, 2000; Spillane & Zeuli, 1999). Therefore, it is concerning that the three low scoring cases incorporated no tasks that were considered *doing mathematics*, and there were only a few tasks that they incorporated at the second highest demand level, *procedures with connections.* All three high cases had at least Practice 1 visible in tasks that were of highest demand, something missing from all three low cases. As problem-solving in mathematics has been a basic principle and skill required of students for decades (Yuristia & Musdi, 2019), it is not surprising that I found evidence of this practice in tasks coded as *doing mathematics* and *procedures with connections*, suggesting that the presence of this practice is essential to these high demand tasks. Equally as vital, and referenced in mathematics education throughout the years, is a student's ability to support their problem-solving with both mathematical arguments and questioning the reasoning of their peers, all of which are represented by Practice 3, *construct viable arguments and critique the reasoning of others.* I only found evidence of the presence of this practice once in a task Janet selected that was coded as *procedures with connections*. In addition, all three cases showed evidence of Practice 6 in their low demand tasks. While this is slightly surprising as this practice has been reported as one that teachers struggle with understanding (Jung & Newton, 2018; Otten et al., 2017), it may suggest that the presence of this practice with the absence of Practices 1 and 3 lead to lowering the cognitive demand of mathematical tasks.

Further, there were distinct differences among the three low cases. First, while none of the three cases had tasks coded as *doing mathematics*, Janet did have more tasks coded as *procedures with connections* than both Leannette and Lea and she also had more practices present in her tasks. Janet was also the only low case to show the presence of Practice 3 in any of her tasks, which is something we see from our high cases. One major reason for these differences might be that Janet was from University 2, whereas both Leannette and Lea were from University 3 and were in the same cohort. In addition, University 3 was the undergraduate university, which may further be a reason behind these differences.

The findings of this paper have implications for preparing preservice mathematics teachers to incorporate high-demand tasks that show the presence of certain Standards for Mathematical Practice. Across all six cases, I found that the three preservice mathematics teachers who scored high on their edTPA incorporated more tasks that were in the two high cognitive demand levels. In addition, these tasks showed evidence for not only multiple practices, but specifically Practices 1, 3, and 4. The three preservice mathematics teachers who scored low on their edTPA incorporated mostly low cognitively demanding tasks in their edTPA planning section. The tasks they did incorporate drew on very few practices, with Practices 1, 3, and 4 much less visible when compared to the three high cases. This work highlights the importance of preparing our preservice mathematics teachers not only in understanding how to draw on high cognitively demanding tasks, but how these tasks increase the number of practices students will engage in, including practices that are essential to their development as successful mathematics students. In addition, these findings suggest that the program a preservice mathematics teacher attends may influence how they understand cognitively demanding tasks and the Standards for Mathematical Practice.

Conclusion and Limitations

I recognize that this study had several limitations. First, the preservice mathematics teachers in this study did not all incorporate the same number of tasks into their planning sections, impacting the opportunities available to see which Standards for Mathematical Practice were visible in their tasks. In addition, I acknowledge that the preservice mathematics teachers' edTPA scores may have little relationship to how successful they will be as in-service teachers and influence student outcomes. However, I see this as an opportunity for future research to look at how to best prepare preservice mathematics teachers to fully understand and incorporate cognitively demanding tasks and the Standards for Mathematical Practice, and the relationship that may exist between these two constructs.

In this study, I sought to understand what level of cognitively demanding tasks preservice mathematics teachers who scored high and low on their edTPA incorporated into their edTPA planning section. In addition, I investigated which of the eight Standards for Mathematical Practice were visible in these levels of cognitively demanding tasks, and what differences, if any, existed. I found evidence that not only did preservice mathematics teachers who scored high on their edTPA incorporate mostly high cognitively demanding tasks rather than the low cases, but they also incorporated more of the eight practices, specifically Practices 1, 3, and 4. These findings suggest that high-demand tasks provide students with the opportunities to engage in more practices, specifically practices that are considered to be the benchmark of mathematics education. Future research should continue to look at the connections between the Standards for Mathematical Practice and cognitively demanding tasks, as well as providing opportunities for preservice mathematics teachers to learn how to implement and engage their students in both, and how they possibly work together to provide the best mathematical learning experience for students.

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Chapter 5: ***Conclusions***

The goal of this dissertation was to take a deeper look at how preservice mathematics teachers conceptualized and implemented the Standards for Mathematical Practice. While the Standards for Mathematical Practice have been a major focus for mathematics education research in the last 10 years, research specifically focused on preservice mathematics teachers remains sparse. Furthermore, there is a clear gap in the literature when looking at how preservice mathematics teachers understand and implement all eight of the practices, and how this is demonstrated in the planning and instructional sections of their edTPA. This research draws attention to how preservice mathematics teachers learned, understood, and engaged their students in the Standards for Mathematical Practice after a year of learning about them in their mathematics methods courses and field placements.

In this study, I began by first looking at initial and follow-up interviews from 47 preservice mathematics teachers from four separate teacher education programs and their responses to two questions regarding which practices they thought were the most important to teach and which they needed more help with understanding. Preservice mathematics teachers overwhelmingly reported Practice 1, *make sense of problems and persevere in solving them,* and Practice 3, *construct viable arguments and critique the reasoning of others*, as the two most important practices to teach, with trends holding regardless of university or initial or follow-up interview. Most preservice mathematics teachers reported one of these two practices because they felt that they supported students in building skills that they could apply beyond the mathematics classroom and were the foundation for which other skills build off of.

Similar results were found when looking at which practices the participants reported as needing further help to understand, with Practice 4, *model with mathematics*, and Practice 8, *look for and express regularity in repeated reasoning,* as the two most reported. Here, preservice mathematics teachers were confused as to what these practices actually meant and what implementation looked like in the classroom. They also said that they felt there was a lack of resources available to help them fully understand these practices and engage their students in them. From this study, I was able to understand which practices these 47 preservice mathematics teachers valued the most and which they struggled with. This has implications for how teacher education programs are preparing and supporting their preservice mathematics teachers with the Standards for Mathematical Practice.

In my second paper, I looked at the extent to which preservice mathematics teachers showed evidence of the practices in their edTPA video clips. In order to accomplish this, I watched all of the participants' video clips and scored them using the $MCOP^2$, which is a validated tool designed to assess how preservice mathematics teachers implement the Standards for Mathematical Practice through classroom observations. After watching and scoring all the video clips, I created a correlation coefficient with participants' $MCOP^2$ scores and both their instructional commentary and full edTPA scores. I found there to be a high, positive correlation between the scores, implying that preservice mathematics teachers who incorporated the Standards of Mathematical Practice successfully into their edTPA instructional videos received high scores, and vice versa for low scores. This research allows us to look closely at both high and low cases and see how they implemented and engaged their students in the Standards for Mathematical Practice. As the edTPA is supposed to be an important opportunity for preservice teachers to display their readiness for teaching full-time, I was able to better understand how prepared our preservice mathematics teachers were to bring these practices to their future classrooms.

Finally, for my third paper, I took a closer look at six of these high and low cases from my second paper to see not only what levels of cognitively demanding tasks preservice mathematics teachers planned on implementing in their edTPA lessons, but which of the eight practices were visible in these different tasks. I first examined all of the materials in the edTPA planning section of my six focal cases to see what tasks they planned on implementing in their lessons. I placed these tasks on the "Levels of Cognitive Demand" framework created by Smith and Stein (2011) to see if they were one of the two high cognitively demanding tasks, *doing mathematics* and *procedures with connections*, or one of the two low cognitively demanding tasks, *procedures without connections* and *memorization*. Overall, I found that preservice mathematics teachers who scored high on the $MCOP^2$ and their edTPA included higher cognitively demanding tasks, whereas those who scored low included mostly low cognitively demanding tasks. I then reviewed all of these tasks again to look for the presence of the eight Standards for Mathematical Practice in order to see if certain levels of tasks supported certain practices than others. I found all high cognitively demanding tasks displayed evidence of Practice 1 and Practice 4. In addition, there were more practices visible overall in these tasks, with at least four visible in any one task. The low cognitively demanding tasks lacked the presence of most of the practices, specifically Practices 1, 3 and 4. While there is a plethora of research on how vital high cognitively demanding tasks are to implement in the classroom, the findings in this study draw attention to how these tasks also incorporate more of the eight Standards for

Mathematical practice. In addition, these tasks draw on the practices that participants reported as the most important to teach from my first study.

This dissertation contains several limitations that should be considered when evaluating the findings from each of the three papers. First, while the sample size of 47 participants for paper 1 and paper 2 was substantial, there were not equal participants across each university and cohort. Overall, most of the participants were from the teacher education programs at University 1 and 3, with much fewer from University 2 and 4. In addition, participants from University 3 were undergraduates, and this may have played a role in their level of experience with classroom teaching. Second, the $MCOP^2$ was designed as a tool to observe an entire classroom lesson. In my second paper, I used it to score lessons that were between 5 and 20 minutes in length, and not an entire mathematics lesson. However, these were the time restrictions put on the edTPA video submissions, and as the goal was to score these specific videos. Finally, the preservice mathematics teachers in my third and final paper all incorporated different numbers of mathematical tasks into their planning section. This may have possibly impacted the opportunities available to see which practices were visible in their tasks.

This research has clear implications for the field of preservice mathematics teacher education. We saw a discrepancy in not only the number of practices that preservice teachers implemented in their edTPA, but the quality of these practices as well. These findings suggest that preservice mathematics teachers need to learn to value, understand, and implement all eight practices equally. They also suggest that the practices themselves might need to be reevaluated. Either way, these findings indicate that future research needs to be conducted on the value of the practices themselves and how we prepare and support preservice mathematics teachers to understand and implement them.

We have made mathematics education a priority in the United States in order to make sure that our students acquire the skills and knowledge to be successful in mathematics. While standards, practices, curriculum, and content all play a major role in this endeavor, teachers are the foundation that exist in order for this to happen. As vital as the Standards for Mathematical Practice are to our students' mathematics education, preparing our preservice mathematics teachers to fully understand and implement the practices is just as important. In order for our students to be successful, our teachers must be successful. This starts with supporting our preservice mathematics teachers to develop a rich understanding of the Standards for Mathematical Practice in their teacher education programs.

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Appendix A: The CCSSM Eight Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them: Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. Reason abstractly and quantitatively: Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; knowing and flexibly using different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others: Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is.

4. Model with mathematics: Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically: Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision: Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure: Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well- remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8. Look for and express regularity in repeated reasoning: Mathematically proficient students notice if calculations are repeated and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Appendix B: Mathematical Classroom Observation Protocol for Practices Descriptors

Item #1: Students engaged in exploration/investigation/problem solving.

	The role of exploration, investigation, and problem solving is central in teaching mathematics as a process. In order for students to develop a flexible use of mathematics, they must be allowed to engage in exploration, investigation, and/or problem-solving activities which go beyond following procedures presented by the teacher. Furthermore, problem solving can be developed as a valuable skill in itself (Barker, et al., 2004) and a way of thinking (NCTM, 1989), rather than just as the means to an end of finding the correct answer. Student exploration may also promote a stance of mathematics as a discipline that can be explored, reasoned about, connected to other subjects, and one that 'makes sense' (Barker, et al., 2004). Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010). If students are following a procedure established by the teacher, then it does not count as exploration/investigation/problem solving. Instead, students should be determining their own solution pathway without necessarily knowing that the path will lead to the desired result.
Score	
3	Students regularly engaged in exploration, investigation, or problem solving. Over the course of the lesson, the majority of the students engaged in exploration/investigation/problem solving
2	Students sometimes engaged in exploration, investigation, or problem solving. Several students engaged in problem solving, but not the majority of the class.
1	Students seldom engaged in exploration, investigation, or problem solving. This tended to be limited to one or a few students engaged in problem solving while other students watched but did not actively participate
0	Students did not engage in exploration, investigation, or problem solving. There were either no instances of investigation or problem solving, or the instances were carried out by the teacher without active participation by any students.

Item #2: Students used a variety of means (models, drawings, graphs, concrete materials, manipulatives, etc.) to represent concepts

	In mathematics instruction it is common for the teacher to use various representations (models, drawings, graphs, concrete materials, manipulatives, graphing calculators, compass & protractor, i.e. tools for the mathematics classroom) to focus students' thinking on and develop their conceptions of a mathematical concept. It is also important for students to interact with and develop representations of mathematical concepts and not merely observe the teacher presenting such representations. Thus, this item is concerned with whether the students use representations to represent mathematical concepts. The representations can be student generated (a drawing or a graph) or provided by the teacher (manipulatives or a table), but it is the students that must then use the representation. Just because there is a representation in a lesson, if it is only used by the teacher while students watch (such as a graph on a PowerPoint slide), it is not considered to be used by students unless the students manipulate and interact with the representation. Students' notes can count as a type of representation if the students themselves offer some sort of input. For instance, if a student corrects a teacher's mistake in a problem he or she is copying down then the notes are actually being manipulated by a student and should therefore count as a type of representation.
Score	
3	The students manipulated or generated two or more representations to represent the same concept, and the connections across the various representations, relationships of the representations to the underlying concept, and applicability or the efficiency of the representations were explicitly discussed by the teacher or students, as appropriate
2	The students manipulated or generated two or more representations to represent the same concept, but the connections across the various representations, relationships of the representations to the underlying concept, and applicability or the efficiency of the representations were not explicitly discussed by the teacher or students
1	The students manipulated or generated one representation of a concept.
0	There were either no representations included in the lesson, or representations were included but were exclusively manipulated and used by the teacher. If the students only watched the teacher manipulate the representation and did not interact with a representation themselves, it should be scored a 0.

Item #3: Students were engaged in mathematical activities

	This item is concerned with the extent of student engagement in activities that are mathematical. Students are considered to be engaged in a mathematical activity when they are investigating, problem solving, reasoning, modeling, calculating, or justifying (each of these could be written or verbal). Note "most of the students"

	in an undergraduate mathematics classroom is accepted here to mean more than one-third of the students in the classroom were engaged in mathematical activity, while in a K-12 mathematics classroom it means more than one-half. It is important to note that one should only focus on what actually happens—not what the teacher assigns watching for students who are off-task
Score	
3	Most of the students spend two-thirds or more of the lesson engaged in mathematical activity at the appropriate level for the class. It does not matter if it is one prolonged activity or several shorter activities. (Note that listening and taking notes does not qualify as a mathematical activity unless the students are filling in the notes and interacting with the lesson mathematically.)
2	Most of the students spend more than one-quarter but less than two-thirds of the lesson engaged in appropriate level mathematical activity. It does not matter if it is one prolonged activity or several shorter activities.
1	Most of the students spend less than one-quarter of the lesson engaged in appropriate level mathematical activity. There is at least one instance of students' mathematical engagement.
0	Most of the students are not engaged in appropriate level mathematical activity. This could be because they are never asked to engage in any activity and spend the lesson listening to the teacher and/or copying notes, or it could be because the activity they are engaged in is not mathematical – such as a coloring activity.

Item #4: Students critically assessed mathematical strategies

	In order for students to flexibly use mathematical strategies, they must develop ways to consider the appropriateness of a strategy for a given problem, task, or situation. This is because not all strategies will work on all problems, and furthermore the efficiency of the strategy for the given context needs to be considered. For students to make such distinctions it is important that they have opportunities to assess mathematical strategies so that they learn to reason not only about content but also about process. This item is concerned with students critically assessing strategies, which is more than listening to the teacher critically assessing strategies or asking peers how they solved a task. Examples of critical assessment include students offering a more efficient strategy, asking "why" a strategy was used, comparing/contrasting multiple strategies, discussing the generalizability of a strategy, or discussing the efficiency of different ways of solving a problem (e.g. the selection appropriate tools if needed).
Score	

3	More than half of the students critically assessed mathematical strategies. This could have happened in a variety of scenarios, including in the context of partner work, small group work, or a student making a comment during direct instruction or individually to the teacher.
2	At least two but less than half of the students critically assessed mathematical strategies. This could have happened in a variety of scenarios, including in the context of partner work, small group work, or a student making a comment during direct instruction or individually to the teacher.
1	An individual student critically assessed mathematical strategies. This could have happened in a variety of scenarios, including in the context of partner work, small group work, or a student making a comment during direct instruction or individually to the teacher. The critical assessment was limited to one student.
0	Students did not critically assess mathematical strategies. This could happen for one of three reasons: 1) No strategies were used during the lesson; 2) Strategies were used but were not discussed critically. For example, the strategy may have been discussed in terms of how it was used on the specific problem, but its use was not discussed more generally; 3) Strategies were discussed critically by the teacher but this amounted to the teacher telling the students about the strategy(ies), and students did not actively participate.

Item #5: Students persevered in problem solving

	One of the Standards for Mathematical Practice (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010) is that students will persevere in problem solving. Student perseverance in problem solving is also addressed in the Mathematical Association of America's Committee on the Undergraduate Program in Mathematics Curriculum Guide (Barker, et al., 2004): Every course should incorporate activities that will help all students approach problem solving with a willingness to try multiple approaches, persist in the face of difficulties, assess the correctness of solutions, explore examples, pose questions, and devise and test conjectures. Perseverance is more than just completion or compliance for an assignment. It should involve students overcoming a roadblock in the problem-solving process.
Score	
3	Students exhibited a strong amount of perseverance in problem solving. The majority of students looked for entry points and solution paths, monitored and evaluated progress, and changed course if necessary. When confronted with an obstacle (such as how to begin or what to do next), the majority of students continued to use resources (physical tools as well as mental reasoning) to continue to work on the problem.

2	Students exhibited some perseverance in problem solving. Half of students looked for entry points and solution paths, monitored and evaluated progress, and changed course if necessary. When confronted with an obstacle (such as how to begin or what to do next), half of students continued to use resources (physical tools as well as mental reasoning) to continue to work on the problem.
1	Students exhibited minimal perseverance in problem solving. At least one student but less than half of students looked for entry points and solution paths, monitored and evaluated progress, and changed course if necessary. When confronted with an obstacle (such as how to begin or what to do next), at least one student but less than half of students continued to use resources (physical tools as well as mental reasoning) to continue to work on the problem. There must be a roadblock to score above a 0.
0	Students did not persevere in problem solving. This could be because there was no student problem solving in the lesson, or because when presented with a problem-solving situation no students persevered. That is to say, all students either could not figure out how to get started on a problem, or when they confronted an obstacle in their strategy, they stopped working.

Item #6: The lesson involved fundamental concepts of the subject to promote relational/conceptual understanding.

	Relational/conceptual understanding is “knowing both what to do and why” (Skemp, 1976). This is in contrast to a procedural understanding as being able to compute certain mathematical activities, but not understanding how the computation works or when one would need to use such a computation and what the answer would mean. According to the NCTM (2006), certain topics are core to the mathematics learned at each grade level and can form the backbone of the K-8 curriculum. The NCTM extended this concept to the high school level with an emphasis on using these fundamental concepts to make sense of mathematics and deepen students’ relational and conceptual understanding (Martin, et al., 2009). Similar to the NCTM’s guidelines for middle school and high school mathematics lessons, at the undergraduate level the Mathematical Association of America has recommendations in the Committee on the Undergraduate Program in Mathematics Curriculum Guide (Barker, et al., 2004) for departments, programs, and all courses to promote relational/conceptual understanding for both mathematics majors and non-mathematics majors.
Score	
3	The lesson includes fundamental concepts or critical areas of the course, as described by the appropriate standards, and the teacher/lesson uses these concepts to build relational/conceptual understanding of the students with a focus on the "why" behind any procedures included.

2	The lesson includes fundamental concepts or critical areas of the course, as described by the appropriate standards, but the teacher/lesson misses several opportunities to use these concepts to build relational/conceptual understanding of the students with a focus on the "why" behind any procedures included.
1	The lesson mentions some fundamental concepts of mathematics, but does not use these concepts to develop the relational/conceptual understanding of the students. For example, in a lesson on the slope of the line, the teacher mentions that it is related to ratios, but does not help the students to understand how it is related and how that can help them to better understand the concept of slope.
0	The lesson consists of several mathematical problems with no guidance to make connections with any of the fundamental mathematical concepts. This usually occurs with a teacher focusing on procedure of solving certain types of problems without the students understanding the “why” behind the procedures.

Item #7: The lesson promoted modeling with mathematics

	Following the “Standards for Mathematical Practice” from the Common Core State Standards (2010) and the recommendations from the MAA’s CUPM Curriculum Guide (Barker, et al., 2004), this item describes lessons that help students to “apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another” (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010). In an undergraduate classroom, a lesson that promotes modeling might use “radiocarbon dating to illustrate how an initial value problem (IVP) can model a real world situation, and the solution of the IVP then yields obviously useful and interesting results” or “a simple system of differential equations to predict the cyclical population swings in a predator-prey relationship” or even “how modular arithmetic is used in cryptography and the transmission of encoded information” (Barker, et al., 2004).
Score	
3	Modeling (using a mathematical model to describe a real-world situation) is an integral component of the lesson with students engaged in the modeling cycle (as described in the Common Core State Standards).
2	Modeling is a major component, but the modeling has been turned into a procedure (i.e. a group of word problems that all follow the same form and the teacher has guided the students to find the key pieces of information and how to plug them into a procedure.); or modeling is not a major component, but the

	students engage in a modeling activity that fits within the corresponding standard of mathematical practice.
1	The teacher describes some type of mathematical model to describe real-world situations, but the students do not engage in activities related to using mathematical models.
0	The lesson does not include any modeling with mathematics.

Item #8: The lesson provided opportunities to examine mathematical structure. (Symbolic notation, patterns, generalizations, conjectures, etc.)

	Following some of the "Standards for Mathematical Practice" (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010) and the recommendations in the MAA's CUPM Curriculum Guide (Barker, et al., 2004), lessons should include opportunities for students to contextualize and/or decontextualize in the process of solving quantitative problems, explore and make use of mathematical structure, or to use repeated reasoning to generalize certain categories of problems and their solutions.
Score	
3	The students have a sufficient amount of time and opportunity to look for and make use of mathematical structure or patterns
2	Students are given some time to examine mathematical structure but are not allowed adequate time or are given too much scaffolding so that they cannot fully understand the generalization.
1	Students are shown generalizations involving mathematical structure but have little opportunity to discover these generalizations themselves or adequate time to understand the generalization.
0	Students are given no opportunities to explore or understand the mathematical structure of a situation.

Item #9: The lesson included tasks that have multiple paths to a solution or multiple solutions.

	As part of having students "make sense of problems and persevere in solving them" (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010), students must be encouraged to look for multiple methods of solving a problem and to deal with problems that have multiple solutions based upon various assumptions. Additionally, selected tasks with multiple paths to a solution or multiple solutions can increase the cognitive

	demand of the task for all students through the interaction of the teacher to ask questions of each student at their ability level (Stein & Smith, 1998). This flexibility, “switching (smoothly) between different strategies,” and adaptivity, “selecting the most appropriate strategy” (Verschaffel, Luwel, Torbeyns, & Van Dooren, 2009) enables students to solve problems for which a solution path is not obvious.
Score	
3	A lesson which includes several tasks throughout; or a single task that takes up a large portion of the lesson; with multiple solutions and/or multiple paths to a solution and which increases the cognitive level of the task for different students.
2	Multiple solutions and/or multiple paths to a solution are a significant part of the lesson, but are not the primary focus, or are not explicitly encouraged; or more than one task has multiple solutions and/or multiple paths to a solution that are explicitly encouraged.
1	Multiple solutions and/or multiple paths minimally occur and are not explicitly encouraged; or a single task has multiple solutions and/or multiple paths to a solution that are explicitly encouraged.
0	A lesson which focuses on a single procedure to solve certain types of problems and/or strongly discourages students from trying different techniques.

Item #10: The lesson promoted precision of mathematical language.

	This item follows the Standard of Mathematical Practice to “attend to precision”. As such, “Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem” (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010). This item also follows the MAA’s CUPM Curriculum Guide recommendation to “develop mathematical thinking and communication skills” which states: “Students should read mathematics with understanding and communicate mathematical ideas with clarity and coherence through writing and speaking” (Barker, et al., 2004)
Score	
3	The teacher “attends to precision” in regard to communication during the lesson. The students also “attend to precision” in communication, or the teacher guides students to modify or adapt non-precise communication to improve precision.

2	The teachers “attends to precision” in all communication during the lesson, but the students are not always required to also do so.
1	The teacher makes a few incorrect statements or is sloppy about mathematical language, but generally uses correct mathematical terms.
0	The teacher makes repeated incorrect statements or incorrect names for mathematical objects instead of their accepted mathematical names.

Item #11: The teacher’s talk encouraged student thinking.

	This item assesses how well the teacher’s talk promotes a number of the mathematical practices. Specifically, the practices requiring students to be able to think, reason, argue, and critique during the study of mathematical concepts (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010). Teachers can greatly impact the level of student thinking and discussion simply by what questions are asked of students. In line with Stein, et al. (2009), the cognitive task level should be maintained at a high level, i.e. procedures with connections and doing mathematics, while questions which are over scaffolded, rhetorical, or cursory to the level of the students, would score a 1 or a 0. Specifically, about the teacher’s talk, this item is referring to the content of the question or statements put forth in the classroom for students to reason and/or discuss. A well-planned lesson may contain rich tasks for students to explore or problems to solve, but if the teacher’s talk drops or removes student reasoning and problem solving, it has removed or reduced student thinking.
Score	
3	The teacher’s talk focused on high levels of mathematical thinking. The teacher may ask lower-level questions within the lesson, but this is not the focus of the practice. There are three possibilities for high levels of thinking: analysis, synthesis, and evaluation. Analysis: examines/ interprets the pattern, order or relationship of the mathematics; parts of the form of thinking. Synthesis: requires original, creative thinking. Evaluation: makes a judgment of good or bad, right or wrong, according to the standards he/she values.
2	The teacher’s talk focused on mid-levels of mathematical thinking. Interpretation: discovers relationships among facts, generalizations, definitions, values and skills. Application: requires identification and selection and use of appropriate generalizations and skills
1	Teacher talk consists of "lower order" knowledge-based questions and responses focusing on recall of facts. Memory: recalls or memorizes information. Translation: changes information into a different symbolic form or situation.
0	Any questions/ responses of the teacher related to mathematical ideas were rhetorical in that there was no expectation of a response from the students.

Item #12: There were a high proportion of students talking related to mathematics.

	The focus of this descriptor is on the proportion of students talking (frequency). The Standards for Mathematical Practice (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010) encourages students to be active in making conjectures, exploring the truth of those conjectures, and responding to the conjectures and reasoning of others. In a classroom dominated by only a few students, classroom discourse may appear to be high, but all students must be engaged.
Score	
3	More than three quarters of the students were talking related to the mathematics of the lesson at some point during the lesson.
2	More than half, but less than three quarters of the students were talking related to the mathematics of the lesson at some point during the lesson.
1	Less than half of the students were talking related to the mathematics of the lesson
0	No students talked related to the mathematics of the lesson.

Item #13: There was a climate of respect for what others had to say

	This item adheres to the expectation provided in the third Standard for Mathematical Practice, "Construct viable arguments and critique the reasoning of others." Given that practice, students are expected to communicate with each other as part of an effective classroom community. Effective communication means that students will listen, question, and critique; this is part of the discourse expected in a mathematics classroom (Sherin, Mendez, & Louis, 2004). This item also encompasses the literature on equity and mathematics in that all students have valuable ideas, strategies, and thinking to share within the mathematics classroom (Boaler, 2006). Equitable spaces include the interactions of students within a mathematical community that increase participation and engagement of all students and work to remove potential barriers (Diversity in Mathematics Education Center for Learning and Teaching, 2007; Gutierrez, 2007; Hiebert & Grouws, 2007; NCTM, 2000; Sherin, Mendez, & Louis, 2004; Yackel & Cobb, 1996). This means creating a climate of respect.
Score	
3	Many students are sharing, questioning, and commenting during the lesson, including their struggles. Students are also listening (active), clarifying, and recognizing the ideas of others.
2	The environment is such that some students are sharing, questioning, and commenting during the lesson, including their struggles. Most students listen.

1	Only a few share as called on by the teacher. The climate supports those who understand or who behave appropriately. Or Some students are sharing, questioning, or commenting during the lesson, but most students are actively listening to the communication
0	No students shared ideas.

Item #14: In general, the teacher provided wait-time.

	The appropriate wait time must align with the question/task. In the elementary grades, a teacher may ask students to explain a situation that represents the expression 24*(1/2)*3. In middle school, the teacher may ask students to describe why the slope is positive. High school teachers may ask students to explain how linear and exponential functions are similar and different. In each instance, these questions/tasks are not simple yes/no answer and require wait time to provide an answer with meaning and understanding. Simple Yes/No questions could be asked, but must be accompanied by an explanation. Simple skills or procedural problems should require explanations with the computation and/or procedures. If the class is dominated by rhetorical questions, a score of 0 or 1 is warranted. Even if rhetorical questions are asked, it is possible to score a 2 or 3 if there are questions asked sometimes or frequently that require students to reason, make sense, and articulate thoughtful responses.
Score	
3	The teacher frequently provided an ample amount of "think time" for the depth and complexity of a task or question posed by either the teacher or a student.
2	The teacher sometimes provided an ample amount of "think time" for the depth and complexity of a task or question posed by either the teacher or a student.
1	The teacher rarely provided an ample amount of "think time" for the depth and complexity of a task or question posed by either the teacher or a student.
0	The teacher never provided an ample amount of "think time" for the depth and complexity of a task or question posed by either the teacher or a student.

Item #15: Students were involved in the communication of their ideas to others (peer-to-peer)

	Both the National Council of Teachers of Mathematics and The Eight Standards for Mathematical Practices expect teachers to create a mathematical community that includes dialogue around the mathematics content and learning. Students are expected to talk and participate in the discourse of the classroom (Manouchehri & St John, 2006). This item highlights the need for all students to be active

	participants in the classroom dialogue. Without teacher support and expectations, the classroom discourse can be monopolized or biased against certain populations (Mercer & Wegerif, 1999; Mercer, Wegerif, & Dawes, 1999; Rojas-Drummond & Mercer, 2003; Rojas-Drummond & Zapata, 2004). This descriptor focuses on the amount of time students spend in communication with their peers at any level, including pairs, groups, informal settings, or whole class settings.
Score	
3	Considerable time (more than half) was spent with peer to peer dialog (pairs, groups, whole class) related to the communication of ideas, strategies and solution.
2	Some class time (less than half, but more than just a few minutes) was devoted to peer to peer (pairs, groups, whole class) conversations related to the mathematics
1	The lesson was primarily teacher directed and little opportunities were available for peer to peer (pairs, groups, whole class) conversations. A few instances developed where this occurred during the lesson but only lasted less than 5 minutes
0	No peer to peer (pairs, groups, whole class) conversations occurred during the lesson

Item #16: The teacher uses student questions/comments to enhance conceptual mathematical understanding

	Driscoll (1999; 2007) and Reys, et al. (2009) discuss how teacher questioning can build on student thinking to foster deeper mathematical thinking. In the elementary grades, students can make "over generalized" statements that have a correct nature about them. This is a teachable moment to use. A teacher can ask a question that has the student(s) reexamine their thoughts that would help simplify the over generalizing statement into precise understanding. Reys, et al. (2009) present a simple example, "Student: So every even number is composite. Teacher: Every even number? <Pause with wait time> What about 2?" The teacher's question stimulates further thought by the student. In secondary grades, Driscoll (1999) indicates that well-timed questions to students should help them shift or expand their thinking, or at least have students thinking about what is important to pay attention to during a lesson. When students are examining expressions, a teacher can ask questions to facilitate mathematical flexibility (Heinze, Star, & Verschaffel, 2009). For example, "What other ways can you write that expression to bring out the hidden meaning? How can you write the expression in terms of the important things you care about?"
Score	

3	The teacher frequently uses student questions/ comments to coach students, to facilitate conceptual understanding, and boost the conversation. The teacher sequences the student responses that will be displayed in an intentional order, and/or connects different students' responses to key mathematical ideas.
2	The teacher sometimes uses student questions/ comments to enhance conceptual understanding
1	The teacher rarely uses student questions/ comments to enhance conceptual mathematical understanding. The focus is more on procedural knowledge of the task versus conceptual knowledge of the content.
0	The teacher never uses student questions/ comments to enhance conceptual mathematical understanding