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Publication Date
2003-03-26

LBNL-52402
March 2003

On the Speed of Gravity and the v/c Corrections to the Shapiro Time Delay

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Abstract

Using a relatively simple method, I compute the v/c correction to the gravitational time delay for light passing by a massive object moving with speed v, and I find disagreement with previously published results. It is also argued that the speed of gravity formula that was recently used in the conjunction of Jupiter and quasar J0842+1845 is frame dependent.

Subject headings: gravitation - relativity - quasars: individual (QSO J0842+1835)

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I. Introduction

On September 8, 2002, a conjunction of quasar J0842+1835 and Jupiter took place. This event was used to measure the Shapiro time delay of the quasar signal due to the gravity of Jupiter (Fomalont & Kopeikin 2002).

Many years ago, I. I. Shapiro (1964) proposed one of the classic tests of general relativity in which radio signals are bounced off an inner planet during a superior conjunction with the Sun. The effect of the Sun's gravity is to create a delay in the time required for the radio waves to return to Earth. In subsequent years, measurements performed using Mercury confirmed Einstein's theory (Shapiro et al. 1968; Shapiro et al. 1971), and the PPN parameter γ was measured to be its expected value of 1 to within 10%.

Because Jupiter's gravity is weaker than the Sun's, the QSO J0842+1835 measurement required remarkable accuracy: 10^{-12} seconds. This was achieved using very long baseline interferometry. Motivation for undertaking this experiment stems from a proposal (Kopeikin 2001) that it can be used to measure the speed of gravity c_g. The idea of testing whether c_g equals the speed of light c, as should be the case in general relativity, has attracted considerable attention both in the astrophysics community and in the media. The measurement yielded $c_g/c = 1.06 \pm 0.21$ (Fomalont & Kopeikin 2002) and was hailed as a confirmation of Einstein's general theory of relativity.

The purpose of this *Letter* is to point out an error in the theoretical formula used to analysis the Jupiter/quasar experiment. In the proposed theoretical formula, a v/c_g correction to the Shapiro time delay is proportional to $1/\theta^2$, where θ is the angle between the quasar and Jupiter. Since θ is small, an enhancement occurs thereby making the measurement feasible. However, using a simple method, this *Letter* computes the v/c corrections and finds no such term. The discrepancy between formula of the current work and the one used in the experiment is understood: The angle θ in latter was actually not the observable one but an artificially defined angle.

The last part of this *Letter* comments on whether the speed of gravity measurement has meaning for the Jupiter/quasar situation. The issue is whether the experiment is sensitive to the speed of gravity or the speed of light.

Our notation conforms to that of Dr. S. Kopeikin (Kopeikin 2001; Kopeikin 2003; (henceforth, we refer to these two references as A and B)): Quasar J0842+1835 is located in the direction of the unit vector $\vec{K}$. See Figure 1. Radiation for the quasar arrives at two observational points 1 and 2 on Earth, which are separated from one another by the distance $\vec{B}$. The impact parameters for each of these two points is respectively denoted by $\vec{\xi_1} = \xi_1 \vec{n}$ and $\vec{\xi_2} = \xi_2 \vec{n}$. Here, $\vec{n}$ is a unit vector perpendicular to $\vec{K}$ going from Jupiter to the closest approach of the electromagnetic radiation of the quasar. Since the difference of the impact parameters is small compared to either impact parameter, we use ξ to denote the value of either when a distinction is not important. The velocity of Jupiter is indicated as $\vec{v}_J$, and the Earth-Jupiter distance is denoted by R_{EJ}.

We are interested in the most significant corrections to the Shapiro time delay. Therefore, we neglect terms proportional to the product of, two of or the square of, any of the following quantities: $\frac{G_N M_J}{\xi c^2} \approx 6 \times 10^{-9}$, $\frac{v_J}{c} \approx 4.5 \times 10^{-5}$, $\frac{B}{\xi} \leq 0.006$ and $\theta_{obs} = \frac{\xi}{R_{EJ}} \sim 0.001$ (which is angle that an astronomer observes between Jupiter and the quasar). Here, G_N is Newton's constant and M_J is the mass of Jupiter. As the above numbers show, the dimensionless parameters in the list are all small for the Jupiter/quasar experiment. Since $\frac{v_J^2}{c^2}$ effects are dropped, we may use non-relativistic formulas in relating measurements made by two different inertial reference frames.

II. The v_J/c Corrections

If Δt_1 and Δt_2 denote the Shapiro time delays at the points 1 and 2, then the quantity of interest is the difference $\Delta(t_1, t_2) = \Delta t_2 - \Delta t_1$ of these two time delays:

$$t_2 - t_1 = |\vec{x}_2(t_2) - \vec{x}_0|/c - |\vec{x}_1(t_1) - \vec{x}_0|/c + \Delta(t_1, t_2) \quad . \tag{1}$$

Here, t_1 and t_2 are respectively the times at which the signals are measured at the two points $\vec{x}_1(t_1)$ and $\vec{x}_2(t_2)$, $\vec{x}_0$ is the position of the quasar, and $|\vec{x}_2(t_2) - \vec{x}_0|/c - |\vec{x}_1(t_1) - \vec{x}_0|/c$ is the time difference that occurs when gravitational effects are absent.

If $\vec{B} = \vec{x}_2 - \vec{x}_1$ and $\vec{n}$ are oppositely oriented, or more precisely $\vec{B} \cdot \vec{n} < 0$, then $\xi_1 > \xi_2$ and $\Delta(t_1, t_2)$ is positive because the electromagnetic radiation that arrives at 2 undergoes more time delay because it passes closer to Jupiter. This is the case illustrated in Figure 1.

If Jupiter were not moving, which is the static situation, then the Shapiro time delay for a single wave is (Weinberg 1972)

$$\Delta t = \frac{2G_N M_J}{c^3}\left(1+\ln\left(\frac{4R_{JQ}R_{EJ}}{\xi^2}\right)\right) \quad , \tag{2}$$

where R_{JQ} is the distance from Jupiter to the quasar. The leading contribution to $\Delta\left(t_1,t_2\right)$ is therefore

$$\Delta\left(t_1,t_2\right) = \Delta t_2 - \Delta t_1 = \frac{4G_N M_J}{c^3}\ln\left(\frac{\xi_1}{\xi_2}\right) = \frac{4G_N M_J \Delta\xi}{\xi c^3} \quad . \tag{3}$$

Let us determine $\Delta\xi = \xi_1 - \xi_2$ in terms on $\vec{B}$. The electromagnetic rays that originate from the quasar are bent slightly as they pass by Jupiter by an amount $\Delta\varphi$ given by (Weinberg 1972)

$$\Delta\varphi = \frac{4G_N M_J}{\xi c^2} \quad .$$

The angle that eventually arises between the two rays is

$$\delta\Delta\varphi = \Delta\varphi_2 - \Delta\varphi_1 = \frac{4G_N M_J R_{EJ}\Delta\xi}{\xi^2 c^2} \quad .$$

Since the separation between the rays starts as $\Delta\xi$ and increases as the distance times $\delta\Delta\varphi$,

$$-\vec{n}\cdot\vec{B} = \Delta\xi + R_{EJ}\delta\Delta\varphi = \Delta\xi\left(1+\frac{4G_N M_J R_{EJ}}{\xi^2 c^2}\right) \approx \Delta\xi \quad ,$$

because

$$\frac{4G_N M_J R_{EJ}}{\xi^2 c^2} \leq \frac{4G_N M_J R_{EJ}}{R_J^2 c^2} \sim 0.001 \quad ,$$

where R_J is the radius of Jupiter. In other words, within the solar system the angular deflection created by Jupiter can be neglected, and the separation between the rays remains essentially constant. Furthermore, we are interested in corrections proportional to v_J. The final result is

$$-\vec{n}\cdot\vec{B} = \xi_1 - \xi_2 \quad . \tag{4}$$

By substituting Eq.(4) into (3), one obtains the result for a static Jupiter

$$\Delta\left(t_1, t_2\right) = -\frac{4 G_N M_J \vec{n} \cdot \vec{B}}{\xi c^3} \quad . \tag{5}$$

When $\xi = \theta_{obs} R_{EJ}$ is used in Eq.(5), it gives rise to the leading term of references A and B.

Let us now compute the v_J/c corrections. This is simple to do by selecting an appropriate reference frame.

During the time in which the rays propagate from Jupiter to the Earth, Jupiter moves almost in a straight line with constant speed. In other words, the orbital motion of Jupiter around the Sun is not important. The same is true for the Earth. Therefore, observers on both planets can be considered as being inertial. Let us select an observational frame for which Jupiter is motionless. In this frame, the Earth appears to be moving with a velocity $\vec{v}_E$ equal to $-\vec{v}_J$. Since Jupiter is not moving, Eq.(5) applies. However, the distance $\vec{B}_{sf}$ between points 1 and 2 as measured in the static frame is not equal to $\vec{B}$ as measured on Earth. Place a static observer at the point 1 at time t_1 and another static observer at the point 2 at time t_2. Have these observers make the time measurements. Then the situation is completely static and the formulas for the static case may be used.

During the time $t_2 - t_1$, the Earth moves a distance $\vec{v}_E\left(t_2 - t_1\right)$. Next, note that the leading contributions to $t_2 - t_1$ are

$$t_2 - t_1 \approx -\frac{\vec{K} \cdot \vec{B}}{c} + \frac{\vec{n} \cdot \vec{B} \theta_{obs}}{c} + \Delta\left(t_1, t_2\right), \quad , \tag{6}$$

of which the first is the largest. Therefore,

$$\vec{B}_{sf} = \vec{B} - \frac{\vec{K} \cdot \vec{B}}{c} \vec{v}_E + \frac{\vec{n} \cdot \vec{B} \theta_{obs}}{c} \vec{v}_E + \Delta\left(t_1, t_2\right) \vec{v}_E \quad . \tag{7}$$

The motion of Earth leads to two corrections to the static time delay difference in Eq.(5). Using $\vec{n} \cdot \vec{B}_{sf}$ in Eq.(5) leads to an additional term $\frac{4 G_N M_J \vec{n} \cdot \vec{v}_E \vec{K} \cdot \vec{B}}{\xi c^4}$. The other correction arises if the Earth moves toward Jupiter. In this case, the time delay is reduced (or increased) by the time $\Delta\left(t_1, t_2\right)$ it takes light to travel the distance determined by the difference between $\vec{B}_{sf}$ and $\vec{B}$. The corresponding correction due to the second and third terms of Eq.(7) is independent of G_N and is a contribution to the first part of Eq.(1) that involves the difference in distances between the positions of the quasar and the observation points 1 and 2. The fourth term in Eq.(7) leads to

$$\delta\Delta\left(t_1,t_2\right) = -\frac{\vec{K}\cdot\vec{v}_E}{c}\Delta\left(t_1,t_2\right)$$

One switches to the Earth frame using $\vec{v}_E = -\vec{v}_J$. The final result is

$$\Delta\left(t_1,t_2\right) = -\frac{4G_N M_J}{\xi c^3}\left(\vec{n}\cdot\vec{B}\left(1+\frac{\vec{K}\cdot\vec{v}_J}{c}\right)+\frac{\vec{K}\cdot\vec{B}\vec{n}\cdot\vec{v}_J}{c}\right) \quad . \tag{8}$$

The correction factor $\frac{\vec{K}\cdot\vec{v}_J}{c}$ is present in references A and B. However, we find no $1/\theta^2$ terms. In its place is the $\vec{K}\cdot\vec{B}\vec{n}\cdot\vec{v}_J/c$ term of Eq.(8).

Although the Shapiro time delay has effects created by the long-ranged gravitational force (e.g. see Eq.(2)), these effects cancel in the time difference of Eq.(1). In the static case, this is illustrated by Eq.(5), in which $\Delta\left(t_1,t_2\right)$ is expressed in terms of the impact parameters of the electromagnetic waves, that is, quantities measurable in the vicinity of Jupiter. One therefore expects that long-ranged effects should not be present in $\Delta\left(t_1,t_2\right)$ even in the non-static case. The $1/\theta_{obs}^2$ terms of reference A and B, however, grow with the Earth-Jupiter distance. On physical grounds, it seems unlikely that such terms are present, and our computation confirms this.

III. Comparison to References A and B

It is easy to find the source of the $1/\theta^2$ effects in references A and B. In those works, the times $s_1 = t_1 - \left|\vec{x}_1\left(t_1\right) - \vec{x}_J\left(s_1\right)\right|/c$ and $s_2 = t_2 - \left|\vec{x}_2\left(t_2\right) - \vec{x}_J\left(s_2\right)\right|/c$ at which rays 1 and 2 pass by Jupiter are expanded in terms of the times t_1 and t_2 when the rays are observed at the points $\vec{x}_1$ and $\vec{x}_2$ on Earth. The differences between the s_i and t_i are sizeable, of order of R_{EJ}/c. During this time, Jupiter moves a significant distance. See the dotted circle in Figure 1. References A and B define the angle θ in terms of the position of Jupiter at t_1. This is not the physically observed angle θ_{obs}. For clarity, denote the angle of references A and B by θ_{AB}.

When the electromagnetic waves from the quasar pass by Jupiter, sunlight that has been reflected off of Jupiter also heads toward Earth. Eventually, the various waves arrive on Earth. See Figure 1. It is evident that the angle θ_{obs} between the quasar and Jupiter observed by an astronomer on Earth is determined by Jupiter's position at time s_1 and not t_1.

The reason for the $1/\theta^2$ term in references A and B is due to the use of the artificial angle θ_{AB}. The relation between θ_{obs} and θ_{AB} is

$$\theta_{obs} \approx \theta_{AB} + \frac{\vec{n} \cdot \vec{v}_J}{c} \quad . \tag{9}$$

When this result is substituted into the leading term of Eq.(5),

$$\Delta(t_1, t_2) = -\frac{4G_N M_J \vec{n} \cdot \vec{B}}{R_{EJ} \theta_{obs} c^3} = -\frac{4G_N M_J \vec{n} \cdot \vec{B}}{R_{EJ} \theta_{AB} c^3} \left(1 - \frac{\vec{n} \cdot \vec{v}_J}{c \theta_{AB}^2}\right) \quad , \tag{10}$$

and the $1/\theta^2$ effect emerges.

References A and B express $\Delta(t_1, t_2)$ as

$$\Delta(t_1, t_2) = \frac{4G_N M_J}{c^3} \ln \left[\frac{r_{1J}(s_1) + \vec{K} \cdot \vec{r}_{1J}(s_1)}{r_{2J}(s_2) + \vec{K} \cdot \vec{r}_{2J}(s_2)}\right] \quad , \tag{11}$$

and then expands unwisely about t_1. The expansion is somewhat subtle since factors such as $r_{1J}(s_1) + \vec{K} \cdot \vec{r}_{1J}(s_1)$ are proportional to the small quantity θ_1^2. A careful analysis reveals that Eqs.(8) and (29) of references A and B should have used

$$\ln \left[\frac{r_{1J}(s_1) + \vec{K} \cdot \vec{r}_{1J}(s_1)}{r_{2J}(s_2) + \vec{K} \cdot \vec{r}_{2J}(s_2)}\right] = \left[\frac{r_{1J}(t_1) + \vec{K} \cdot \vec{r}_{1J}(t_1)}{r_{2J}(t_1) + \vec{K} \cdot \vec{r}_{2J}(t_1)}\right] + \frac{2\vec{n} \cdot \vec{v}_J \vec{n} \cdot \vec{B}}{c r_{1J} \theta_{AB}^2} \quad . \tag{12}$$

When Eq.(12) is substituted into Eq.(11), the $1/\theta_{AB}^2$ term of Eq.(10) due to the expansion in Eq.(9) is reproduced. This shows that our equations are consistent with the method used in references A and B. By the way, the first term in Eq.(12) is approximately $-2\vec{n} \cdot \vec{B}/\xi(s_1) - \vec{K} \cdot \vec{B}/R_{EJ}$, the latter being a small non-v/c term down by a factor of order ξ/R_{EJ}.

Summarizing, (1) an analysis using a static frame allows one to easily compute the v_J/c corrections from the static result and one finds no $1/\theta_{obs}^2$ terms, (2) physical considerations suggest that such terms are absent, (3) the terms arise from an ill-advised expansion about the arrival time t_1 in references A and B, (4) using this expansion produces artifical $1/\theta_{AB}^2$ effects, and (5) finally, these terms match those of references A and B after the correct expansion in Eq.(12) is used.

The v_J/c corrections in Eq.(8) for the Jupiter/quasar experiment are at least 10,000 times smaller than the leading term thereby rending them beyond detection for current radio telescopes. They are also masked by larger corrections such as terms down by B/ξ and θ_{obs}, which are present but not shown in this *Letter*.

IV. On the Notion of the Speed of Gravity

It is clear from the derivation using the static frame in Section II that the leading v_J/c corrections involve the speed of light and not the speed of gravity, and there is a recent analysis (Will 2003) that supports this claim. However, references A and B argue that v_J/c_g should appear. The issue here is how does one extend Einstein's general theory of relativity to allow the possibility that the speed of gravity c_g is not equal to c. A reasonable approach is to assume that the effect of gravity propagates at c_g instead of c. For example, in the retarded times and positions of Jupiter in formulas, one replaces v_J/c by v_J/c_g. Hence, in the frame in which Jupiter is moving and the Earth is at rest, the v_J/c effects are generated in the vicinity of Jupiter, and v_J/c_g should appear in lieu of v_J/c in $\Delta(t_1, t_2)$. But consistancy demands that the computation of $\Delta(t_1, t_2)$ be frame independent up to v_J^2/c^2 relativistic effects. Thus, there does not seem to be a consistant way to define the speed of gravity concept for the Jupiter/quasar experiment. In the static frame, the corrections are due to the speed of light, while in the Jupiter-moving frame they are due to the speed of gravity.

How then might one try to test $c_g \neq c$ in Einstein's theory of relativity? The static and Jupiter-moving frames are both inertial. If Jupiter happened to be accelerating toward (or away from) the quasar's electromagnetic waves as they passed by the planet, then one would not be able to go back and forth between the two frames. Therefore, there is a reasonable chance that the speed of gravity concept could be defined for such a situation. The parameter c_g would not be attached to velocity-dependent terms but to acceleration effects. Although it is worth exploring this possibility theoretically, it is unlikely that a sytem within or beyond our Solar System exists that generates an effect sufficiently large to be measurable with current instruments.

Acknowledgments

This work was supported in part by the Director, Office of Science, Office of High Energy and Nuclear Physics, of the Department of Energy under contract number DE-AC03-76SF00098.

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Figure Captions

Figure 1. The Motion of Electromagnetic Waves Relevant for the Jupiter/Quasar Experiment.

For clarity, the diagram is not drawn to scale.