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VELOCITY SPECTROMETER FOR HIGH ENERGY
PARTICLES

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FOR HIGH-ENERGY PARTICLES

Joseph J. Murray

May 7, 1957

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ABSTRACT

An apparatus is described in which crossed E and H fields will be used to separate high-energy charged particles of different masses. It is shown that use of coaxial geometry is expected to effect satisfactory separation within a considerably shorter system than would be possible in a parallel-plate system with a comparable separation criterion. A summary is given of the operational characteristics of a spectrometer now under construction.



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Introduction

In low-energy work, electrostatic analysis of particle beams is a very familiar technique, and one which, adapted to higher energies, will undoubtedly prove useful in work with rare particles such as K^- mesons and antiprotons. The purpose of this paper is to point out and analyze what seems to be a promising scheme for adapting the technique to high energy.

A charged particle with momentum p and velocity $\beta = v/c$ passing through a transverse static electric field E is deflected by an angle proportional to $EL/p\beta$, where L is the length of the deflector. This deflection, combined with momentum analysis, achieves an angular separation of particles of different masses. Since the mass sensitivity is dependent on the velocity ratio of the particles, the maximum momentum at which the technique is useful is limited roughly to 1, in units of the mass of the heavier of a pair of particles one hopes to separate. In a specified system, however, for a given momentum the maximum tolerable E sets a lower limit on the length of the system required to effectively separate particles of given mass ratio. At momenta corresponding to the K -meson or nucleon mass, the required length tends to become excessive.

A device is described here that is based on the foregoing principle (differing only in that crossed E and H fields rather than E alone are used), which as a result of somewhat unconventional design will be practical for separation of antiprotons from π^- mesons at $\lesssim 1$ Bev/c and--in a smaller version--for separation of K^- mesons from π^- mesons at $\lesssim 500$ Mev/c. As will appear, the use of coaxial geometry results in a considerably shorter system, for a given momentum, than can be realized, for example, with a parallel-plate deflector. This is of practical importance

- (a) because imperfections in beam optics restrict the useful length of a system,
- (b) in the case of K^- mesons, because of their short lifetime, and
- (c) (in any case) for the sake of economy.

General Description and Design Considerations

The coaxial spectrometer will consist, basically, of a cylindrical outer conductor at a high potential, and a concentric center conductor at approximately ground potential in which a current flows. Except for end effects, this produces inside the coax a crossed E-H field with both E and H inversely proportional to radius, so that E/H is a constant. The beam of particles to be analyzed is directed along the axis of the coax. The ratio E/H will be set approximately equal to the velocity of the desired particles, which then will pass through the coax essentially undeflected. Undesired particles of different velocity will be deflected inward or outward depending on their charge and the sign of E and H. In the specific applications to be considered here, the undesired particle will be π^- (or π^+) mesons with velocities greater than E/H, and the outer conductor will then be made negative (or positive) so as to sweep them outward.

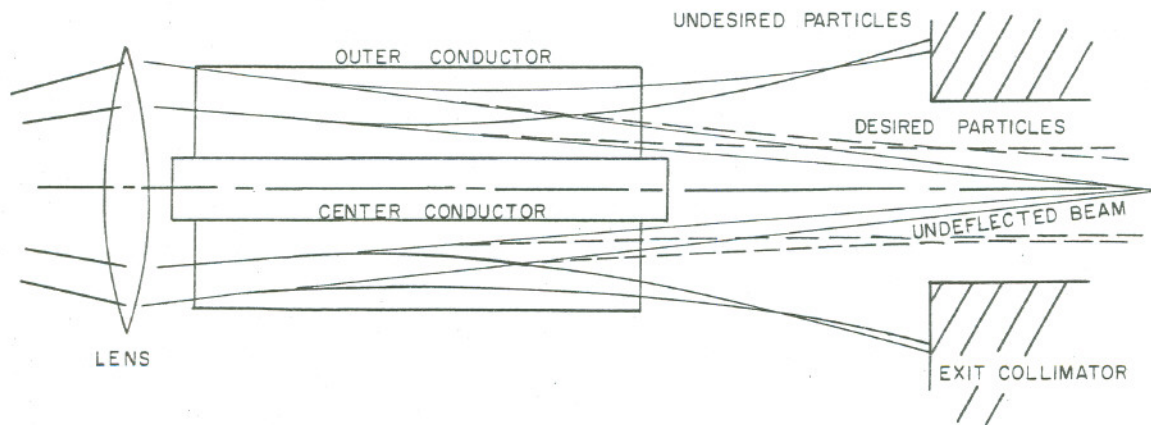
In simplest form a complete system will consist of a lens feeding the coaxial spectrometer with a field-free interval for divergence of the undesired particles between the end of the coax and the exit aperture, as shown in Fig. 1. The length of the system will be determined largely by the criterion that no trajectories of undesired particles shall exist which connect the lens and exit aperture directly. This criterion insures complete rejection of the undesired particles independent of source size, multiple scattering ahead of the lens, and optical aberrations.

Actually, of course, the rejection cannot be complete. It will be limited by contamination consisting chiefly of

- (a) products of decay in flight of the undesired particles, and
- (b) undesired particles themselves
 - (i) reflected by multiple Coulomb scattering from the outer conductor of the coax, and
 - (ii) scattered back into the system by an exit collimator if one is used.*

Contamination from decay can be controlled by exit collimation. Reflection by multiple scattering can be effectively controlled only by preventing the undesired particles from striking the outer conductor. This can be achieved to a high degree by feeding the coax with a converging beam having a diameter at the entrance somewhat smaller than that of the outer conductor. Collimator

* Contamination will also be caused by scattering of undesired particles in the center-conductor support and the window at the exit end of the coax. It appears to be practical, however, to use an aluminum diaphragm, as both center-conductor support and end window, that is sufficiently thin so as to cause a negligible amount of contamination from this source.



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Fig. 1. Schematic diagram of the coaxial velocity spectrometer system.

scattering can be controlled by restricting the exit aperture or by secondary collimation. With practical designs it appears to be possible to achieve rejection ratios exceeding 10^3 .

While maximizing the rejection of undesired particles, it is also necessary, of course, to maintain high efficiency for transmission of desired particles. The center-conductor diameter is determined by compromise between transmission efficiency and the maximum electric-field gradient. A ratio of incoming-beam diameter to center-conductor diameter of 4 is satisfactory and noncritical. Convergence of the beam results in greater than geometric interception by the center conductor. This effect can be reduced greatly, however, by setting the selected velocity, E/H , to a value different from that of the desired particles, so as to give them a slight outward deflection. As it turns out, a beam converging to a point focus near the exit aperture can thus be brought to a slightly fuzzy ring focus with a diameter somewhat larger than that of the center conductor. Analysis of the transmission of realistic beams (nondispersively analyzed, typically scattered beams with momentum spreads of $\pm 5\%$ to $\pm 10\%$) indicates that transmission efficiencies, neglecting decay, can be expected to be $\approx 50\%$.

Particle Dynamics

In a cylindrical coordinate system with Z along the axis of the coax, the fields \vec{E} and \vec{H} inside the coax and the potential V of the outer conductor are given by

$$\vec{E} = \vec{e}_\rho E \frac{\rho_1}{\rho} \quad \vec{H} = \vec{e}_\phi H \frac{\rho_1}{\rho}, \quad V = -\rho_1 E \ln \frac{\rho_2}{\rho_1}, \quad (1)$$

where E and H are the field magnitudes at the center conductor and ρ_1 and ρ_2 are the radii of the center and outer conductor, respectively. The radial equation of motion of a particle of charge e in these fields is

$$\ddot{\rho} = -\frac{e}{M} (\beta H - E) \frac{\rho_1}{\rho} + \frac{I^2}{M^2 \rho^3}, \quad (2)$$

where M is the relativistic mass, $I = M\rho^2 \dot{\phi}$ (a constant) is the angular momentum of the particle, and β is the particle velocity in units of c ; β is assumed to be constant because $V \ll Mc^2$, and the trajectories will be nearly normal to \vec{E} .

Values of $I \neq 0$ will result from finite size and dispersion of the source and optical aberration. Analog-computer solutions of Eq. (2) for $I \neq 0$ were used in detailed calculations of transmission efficiency and rejection ratios. For calculation of system length, however, it is sufficient to consider the case $I = 0$, since undesired particles with $I = 0$ will, in general, require more length to be swept from the system than will those with $I \neq 0$.

In terms of a dimensionless variable $r = \rho/\rho_1$, Eq. (2) becomes, for $I = 0$ and

$$\ddot{r} = \frac{K}{r}, \quad K = -\frac{eE}{M\rho_1} \left(\frac{\beta}{\beta_0} - 1 \right), \quad (3)$$

$$\beta_0 = E/H,$$

which can be integrated formally with the result

$$t_f - t_i = \int_{r_i}^{r_f} \frac{dr}{(2K \ln r + C)^{1/2}} = \frac{Z_f - Z_i}{\beta c}, \quad (4)$$

where i and f denote initial and final values and

$$C = \dot{r}^2 - 2K \ln r = \text{a constant.}$$

Formula for Estimating System Length

If the coax extends over the entire distance between the lens and exit aperture, then the length of the undesired-particle trajectory between the outer edges of the exit and lens apertures that is tangent to the center conductor would be the required system length,* according to the criterion discussed in an earlier section. It is assumed that undesired particles will be swept outward so that $K > 0$.

For a trajectory tangent to the center conductor we have $C = 0$, and the system length L , according to Eq. (4), is given by

$$L = \frac{\beta c}{(2K)^{1/2}} \int_{r_l}^{r_e} \frac{dr}{(\ln r)^{1/2}},$$

*This is strictly true only for exit aperture radii r_e less than about 2.5. For greater r_e , trajectories tangent at radii larger than that of the center conductor form the boundary of the "forbidden zone."

where r_ℓ and r_e are the lens and exit-aperture radii respectively, and the proper branch of the radical must be used. By change of variable to $y = (\ln r)^{1/2}$, this becomes

$$L = \beta c \left(\frac{2}{K} \right)^{1/2} \left[\int_0^{(\ln r_\ell)^{1/2}} e^{y^2} dy + \int_0^{(\ln r_e)^{1/2}} e^{y^2} dy \right] \quad (5)$$

$$\approx \left[-\frac{p\beta c}{eE} 2\rho_1 \frac{1}{\beta/\beta_0 - 1} \right]^{1/2} \left(\frac{r_\ell + r_e}{2} \right), \text{ for } r_\ell \text{ and } r_e > 2.$$

The integrals may be evaluated from the curve given in Fig. 2. In general it will be desirable to terminate the coax ahead of the exit aperture, so that the length of an actual system will exceed L somewhat, depending on the extent of the foreshortening of the coax.

Comparison with Parallel-Plate Deflector

A comparable parallel-plate deflector will be assumed to have plates separated by $2 r_\ell$ with a uniform gradient between them equal to that at the center conductor of the coax, and the outer conductor of the coax will be assumed to have a radius equal to r_ℓ . In analogy to the criterion used for the coax, let the required length L_{pp} of the parallel-plate system be the length of the trajectory, tangent to the opposite plate (as shown in Fig. 3), of an undesired particle between the edges of the lens and exit aperture. Then it is easily shown that we have

$$L_{pp} = \beta c \left(\frac{2}{K} \right)^{1/2} \left[(2r_\ell)^{1/2} + (r_\ell + r_e)^{1/2} \right].$$

Using the approximate form for L , one finds the ratio of lengths of the parallel-plate and coaxial systems to be

$$\frac{L_{pp}}{L} = 2 \frac{(2r_\ell)^{1/2} + (r_\ell + r_e)^{1/2}}{r_\ell + r_e};$$

r_ℓ is always greater than r_e , so that the ratio is greater than 1 for $r_\ell < 8$. For the typical values $r_\ell = 4$ and $r_e = 3$ the ratio is 1.55, that is, the parallel-plate deflector would have to be about 50% longer than the coax.

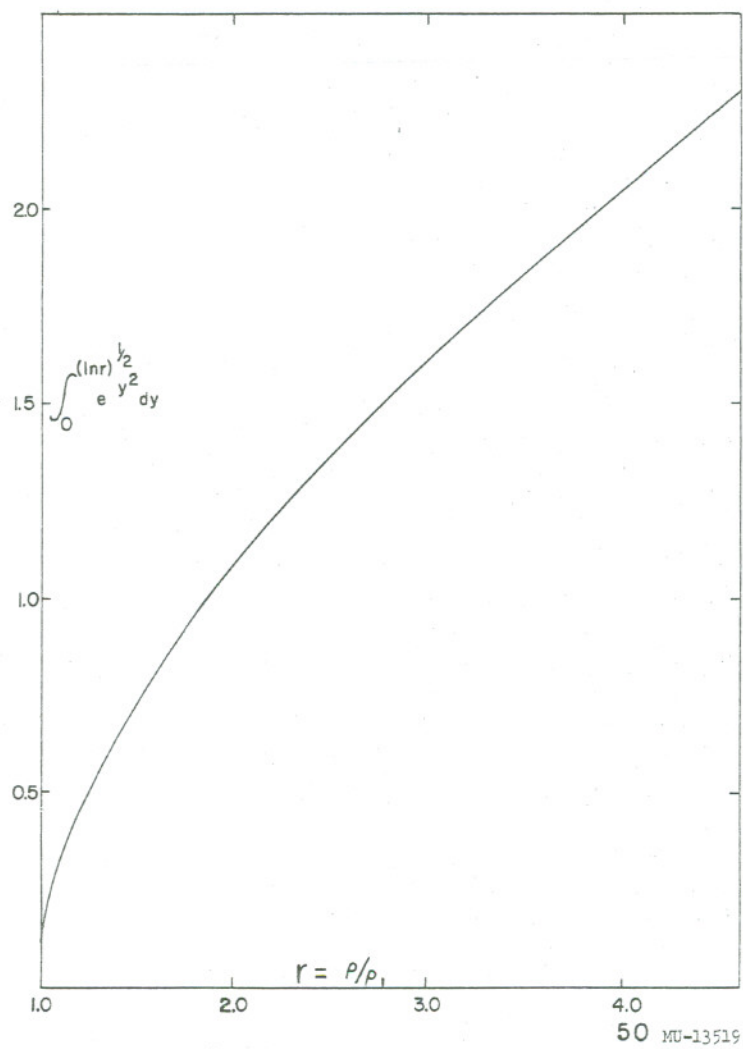


Fig. 2. Curve of evaluation of integrals appearing in Eq. (5).

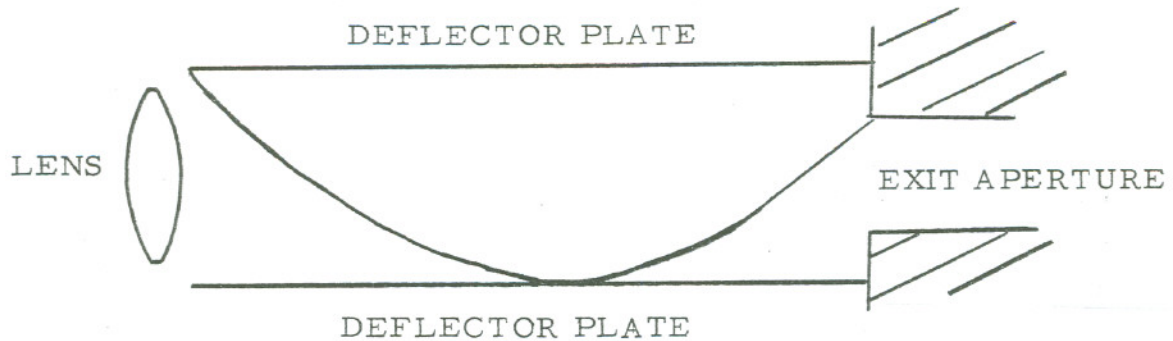


Fig. 3. Parallel-plate system used for comparison with the coaxial system.

In making the foregoing comparison, no account was taken of practical gradient and potential limitations. Using the rule-of-thumb "no spark" criterion that VE^2 is a constant, one concludes that the gradient E_{pp} in the parallel-plate deflector should not exceed 0.67 times the gradient E_{pp} at the center conductor of the coax if both are operated at the no-spark limit. This increases the L_{pp}/L ratio by the factor $(0.67)^{1/2}$. Still the comparison is not completely realistic, since the potential required for the comparable parallel-plate deflector may be very high. For example, for $r_l = 4$ and $E_{pp} = 0.67 E$, we have $V_{pp} = 7.5 V$. For $\rho_l = 5$ cm and $E = 10^5$ V/cm (a practical value for a coax), we have $V = 174$ kv, while V_{pp} would have to be 1300 kv.

It would be reasonable to relax the length criterion for the parallel-plate system, for example, to let the required length be that of the trajectory tangent to the centerline of the system of an undesired particle between the edges of the lens and exit apertures, if one could argue convincingly that only an insignificant number of undesired particles would have more extreme trajectories. In this case, for $E_{pp} = E$, L_{pp} and L would be about the same. Furthermore, the potential between the deflector plates could, of course, be divided symmetrically with respect to ground, thereby easing the high-voltage problem. Even so, practical high-voltage limitations would in general result in required lengths of parallel-plate systems considerably longer than those of comparable coaxial systems.

Brief Description of a Spectrometer Now Under Construction

Construction of a coaxial system intended primarily for use in K beams of 400 to 450 Mev/c is nearing completion. In its first application, however, it will be used in a 250-Mev/c beam as an aid in a search for particles of mass $560 m_e$. The system may also be useful in antiproton beams up to about 750 Mev/c.

Much painstaking work, including some pioneering mechanical and electrical design, has been carried out by Messrs. Robert A. Kilpatrick and Clarence A. Harris of this laboratory.

Rejection and transmission properties of the system have been analyzed in considerable detail with the aid of an electronic analog computer and much graphical work. This analysis was accomplished through the able efforts of Mr. Andre Dury, visiting science teacher from Lowell High School, San Francisco, Mr. Jonathon Young from the Mathematics Department, and Mr. Bill Boyd, an electrical engineering student at the University of California. Here only the results will be summarized.

The coax itself is 14 feet long with 3 feet between the coax and exit aperture. The system is designed for a maximum beam diameter of 4 inches and should be fed with a nondispersively analyzed beam delivered by a quadrupole lens immediately preceding the coax. For reasonable transmission efficiency, the optics should be sufficiently free of aberration and scattering to be able to form at the exit aperture an unobstructed image--that is, the image that would be formed in the absence of the coax--less than 2 inches in diameter.

The useful range of operation of the system is summarized by the characteristics given in Fig. 4. Contours labeled with a momentum parameter give the values of E and H (or potential and center-conductor current) that produce the minimum pion force constant, K_{π}/β_{π}^2 , required for effective pion rejection in accordance with the basic criterion discussed earlier. Heavy portions of these contours give the range of values of E and H for a particular selected particle (indicated by group labeling of the contours) that produce a ratio of force constants of selected particle and pions,

$$\frac{K}{\beta^2} \cdot \frac{\beta_{\pi}^2}{K_{\pi}} = F/F_{\pi}, \text{ having a value between 0 and 0.4, the useful}$$

operating range. The actual values of F/F_{π} , which varies linearly along the momentum contours, are given by the appended numbers.

Transmission of selected particles as a function of momentum is approximately gaussian, $e^{-\frac{(P/P_0 - 1)^2}{\delta^2}}$, where the width δ at the $1/e$

point depends largely on the focal length of the entrance lens, f_0 , which has been assumed to vary quadratically with momentum. The width δ is given approximately by the formula

$$\delta = \frac{0.6}{\frac{300}{f_0} + 1.3}, \quad f_0 \text{ in inches} \cdot$$

It should be understood that the width of this momentum transmission band does not imply a corresponding mass resolution. The system actually constitutes a very poor mass spectrometer, since by design it barely "resolves" pions and selected particles.

The unobstructed image distance at the central momentum p_0 required for maximum transmission efficiency is a function only of the ratio F/F_π at the central momentum and is given by the curve in Fig. 5. The average transmission efficiency for

- (a) an unobstructed image at the exit aperture about 2 inches in diameter,
- (b) a nondispersive beam with a gaussian momentum distribution having a $1/e$ width approximately equal to 2δ , and
- (c) optimum image distance at p_0

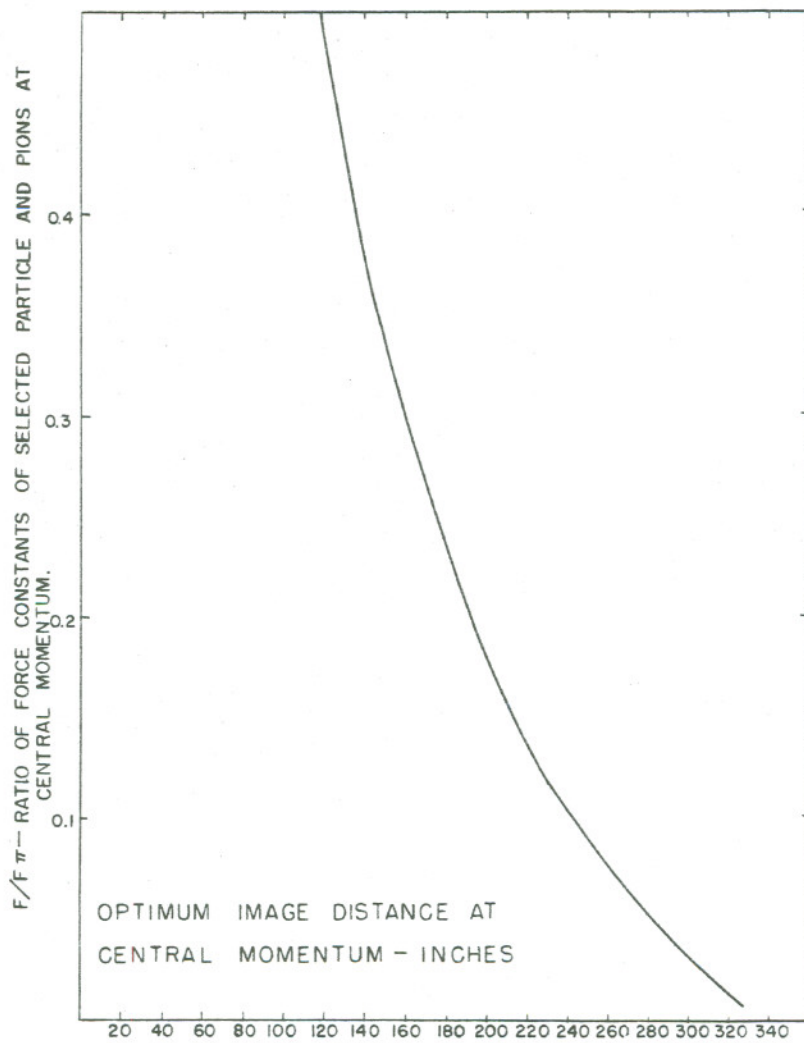
is maximum and $\approx 40\%$ for a value of F/F_π at p_0 equal to 0.2, and falls off to $\sim 20\%$ for F/F_π equal to 0 and 0.4 at p_0 .

Pion contamination resulting from multiple scattering in the exit diaphragm of the coax is expected to be negligible ($\ll 1$ per 10^4 pions into the system), and if necessary can be controlled by slight restriction of the exit aperture.

In order to minimize contamination from other sources, it appears to be desirable to employ a secondary collimator with an aperture 1.5 inches in diameter located about 8 feet behind the primary collimator (at the 3-inch exit aperture) and lying in the focal plane of a quadrupole lens following immediately behind the primary collimator. Such an arrangement causes essentially no loss, apart from decay, of transmission of selected particles, and gives at least an order-of-magnitude reduction of contamination by muons from π - μ decay and by pions multiply scattered from the primary collimator and from the outer conductor of the coax.

Muon contamination is expected to predominate. It should be roughly momentum-independent and, with secondary collimation, equal to about 1 muon per 3×10^3 pions entering the system. The contaminating muons will have momenta differing only slightly from the extremes of the muon momentum distribution characteristic in π - μ decay, and will come largely ($\sim 90\%$) from the high end of the distribution at or slightly above the primary pion momentum. (Hence postmomentum analysis would be of little value.)

With secondary collimation, the total pion contamination from all sources is expected to be < 1 per 10^4 pions entering.



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Fig. 5. Optimum image distance for the entrance lens as function of F/F_{π} .