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**CALIFORNIA PATH PROGRAM
INSTITUTE OF TRANSPORTATION STUDIES
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Transitional Platoon Maneuvers in an Automated Highway System

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Abstract

The report deals with an introduction to the control aspect of platoon maneuvers in Automated Highway Systems. The different platoon maneuvers have been introduced. The first part of the report consists of a review of the existing literature in this area. The survey is split up into the quasi-synchronous and vehicle follower controller methods of approach to the problem. The subsequent sections deal with the mathematical description of the problem with two representations of the system under study, varying only in level of model complexity. The simplified model has been studied with a linear based control as well as the nonlinear sliding mode approach. The results only reinforce the need for nonlinear systems analysis and control. The sliding control technique has also been applied to study the detailed version of the plant.

1 Introduction

The concept of automated highway systems was first introduced to the public in the 1939 World Fair at the GM Pavillion. The first research efforts can be traced to the Radio Corporation of America then working in association with GM. Since then a considerable amount of time and money has been invested in this concept. The PRT and AGT studies of the 60's and 70's have addressed the feasibility, management and operational concepts of the system. Similar studies have been carried out in Europe.

The automated highway system is primarily aimed at reducing congestion on highways through closer packing of vehicles per unit mile of the highway. The system will also result in improved safety on the highway and easier and more comfortable ride for the individual. The longitudinal and lateral control studies of automobiles on the highway have established the theoretical feasibility of the concept. Other considerations include communications, actuator and sensor requirements and system management issues.

The control of the automobile on the highway not only includes longitudinal and lateral control but must also take into consideration issues such as platoon formation and splitting, lane changing, entry and exit of vehicles to and from the automated lanes.

2 Platoon Maneuvers

This project is aimed at investigating the intermediate maneuvers in automated highway systems. This includes the control of vehicles involved in changing lanes, joining platoons and splitting from platoons. The control of vehicles during these maneuvers constitutes the platoon maneuver control problem in Automated Highway Systems.

2.1 Platoon Maneuver Scenarios

Figure 1. shows the lane change operation. Some of the factors that determine the type of lane change operation are the velocities of the maneuvering and platoon vehicles, the time and distance constraints, and the types of vehicles involved.

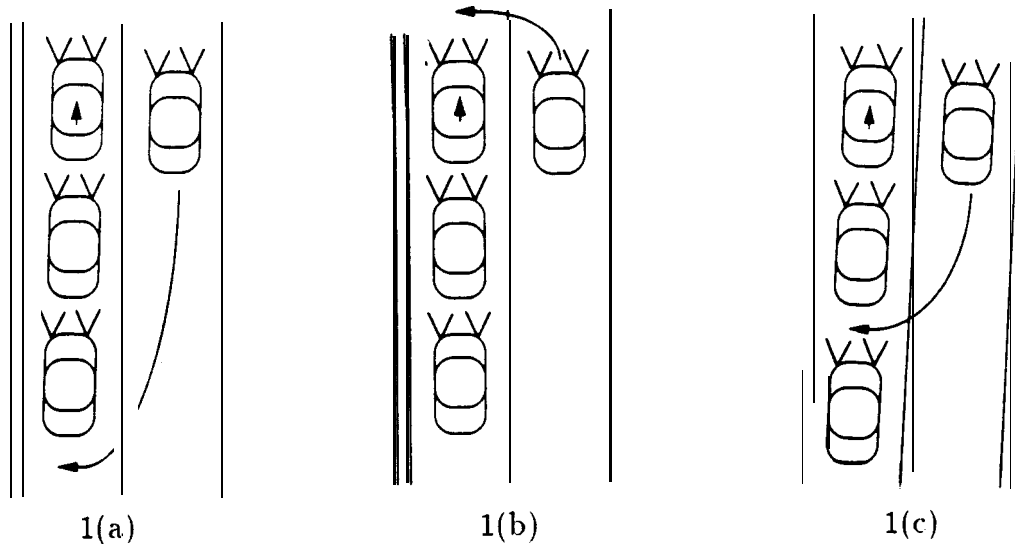


Figure 1: Lane change scenarios (a) type 1 (b) type 2 (c) type 3

Based on the longitudinal position and speed of the maneuvering vehicle relative to the position of the platoon on the highway the vehicle can essentially change its lane such that it joins the nearest platoon in the front of (type 3. Figure 1 (c)) or at the rear of the platoon (type 1, Figure 1(a)).

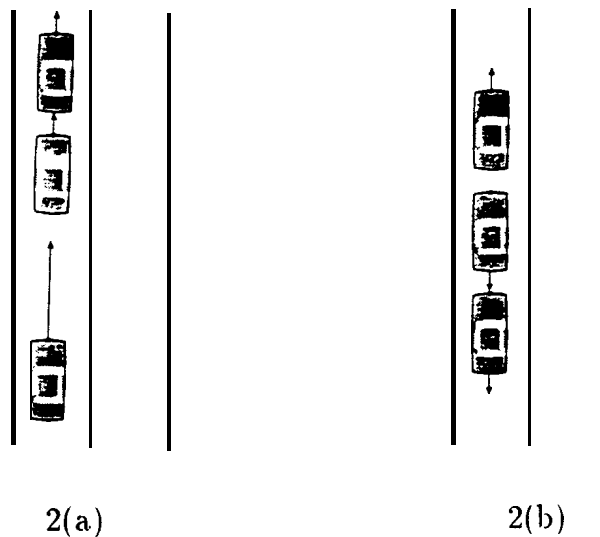


Figure 2: (a) Merge scenario (b) Split scenario

A lane change of type 1 is preferred over the mid-platoon lane change (type 2, Figure 1(b)) from a safety point of view. Sometimes when the platoon size is big and we have a restriction on the maneuvering distance for the maneuvering vehicle the mid-platoon lane change will be favored. In this the vehicle changes lanes in such a manner that it enters the automated lane between two vehicles of the platoon. Figure 2(a) shows the merging procedure wherein a vehicle in a particular lane joins either at the front(rear) of the platoon behind(ahead) it. The split procedure (Figure 2(b)) involves vehicles that wish to leave a particular platoon. The merge and split procedures differ from the lane change procedure mainly in that the former involve vehicles all in the same lane of the highway which is not the case for the lane change operation in which a vehicle either enters or leaves the automated lane.

2.2 Maneuver Control Tasks

The process of the maneuver would require a higher/system level of control for making decisions regarding the type and time of initiation of the maneuver. The decisions could then be transmitted through communication links to the maneuvering vehicle and the vehicles of the platoon. The maneuver control task can be split up into two parts.

1. Maneuver logic – The logic used to decide the order of maneuvering of the vehicles in the maneuver area. A maneuver logic is to be defined which will dictate the actual position and velocity of the maneuvering vehicle on the maneuver lane besides defining which vehicles from the platoon should be involved in the maneuver and in what order. Maneuver logic deals more with the method of assigning “which” vehicle will go “where” rather than “how” will it get there. Several algorithms have been proposed for the maneuver process but they deal with vehicles following the point or slot follower technique.
2. Vehicle control – control of the vehicles to accomplish the order desired. The design must take into the account the constraints imposed on the maneuver which include space considerations, vehicle capabilities and time constraints.

It must be noted that the maneuver control problem we have defined is referred to broadly as the **Merge Problem** in the existing literature and this is how it is referred to in the following review. The rest of the document uses the terminology as defined at the beginning of the paper.

3 Literature Review

The automated highway concept is receiving renewed interest. The AGT and PRT studies of the 60s and 70s, in this country, yielded a number of articles devoted to the

management and operational aspects of the automated highway. Hence a considerable amount of work was also done in the area of merge control.

Prior studies in the area are broadly concentrated on the following three methods of approach.

1. Synchronous control scheme : vehicles are allotted to slots that move along the guideway at predetermined speeds. Slot positions are assigned at the beginning of the trip and slot positions are to be ensured for each vehicle throughout the trip. So the maneuver problem is solved by a centralized control scheduler. Once assignments are made, no possibilities of conflict exist.
2. Quasi-synchronous control : vehicles assigned to slots on the guideway. Conflicts at merge junctions are resolved by a local controller. At the initiation of the maneuver, vehicle movement is dependent on relative positions and velocities of other vehicles at the junction.
The above method is also termed as moving cell/slot control technique.
3. Asynchronous control : vehicle control at all times is dependent on the relative positions and velocities of the preceding vehicles of the platoon.

The review is not chronological but is based on the approach. The merge problem is closely related to the longitudinal control problem and hence a number of papers related to the longitudinal control problem also discuss the merge control aspect.

The fully synchronous control technique, though safe as far as maneuver control is concerned, has not been given much attention. This is because of the need to store a great deal of information. The method is not decentralized and a central scheduler is needed to oversee the movement of all the vehicles in the system. Prior studies have been focussed in the areas of the quasi-synchronous and vehicle follower control methodologies. Current research in the area of longitudinal control of vehicles in the PATH program at UC Berkeley, is of the vehicle follower type. This approach is therefore of particular interest to the study of the maneuver control problem.

3.1 Quasi-Synchronous Control Techniques

The quasi-synchronous control technique has been based on the moving cell technique. Vehicles are assigned to slots and vehicle control is restricted to tracking the center of the slot while each slot moves at a preassigned speed. Control at the intersections and during merge is effected by a local computer that assigns the movements of each vehicle based on relative positions of other vehicles at the merge area.

One of the earliest studies of the merge control problem can be attributed to Godfrey who in his thesis [13] evaluated the application of the quasi-synchronous control methodology to the merge control problem. He used the mean square delay time as the criterion for evaluation of six merging strategies. The merging strategies

studied by him include 1. One lane with priority. 2. Switch priority if vehicle on other lane and all delayed vehicles on current priority lane have been served. 3. Switch priority of vehicle on other lane and empty slot on current priority lane. 4. First in first out (FIFO). 5. random choice. 6. Alternate lane merge.

The study revealed that the mean square delay time was independent of the strategy adopted. He used the Monte Carlo simulation technique to generate arrivals to the merge junction due to the time dependent nature of vehicle flows in the AHS. The FIFO strategy was found to be the best one. But, Godfrey did not explicitly consider the actual vehicle control aspects of the problem.

Sakasita [19] extended Godfrey's work to evaluate queueing policies of the vehicles requiring a merge. He considered three queueing policies. 1. Stop queue policy characterized by sequential occupancy of slots and creep of vehicles while in queue is permissible. 2. Shockwave queue policy. 3. Slot queue policy - distance between vehicles is the same whether being merged or not. The author recommends an optimal merge control based on the proper selection of a merging strategy and a queueing policy. The performance was evaluated based on four criteria:

1. to minimize the number of vehicles delayed.
2. to minimize the scatter of the delay time.
3. to minimize maximum delay.
4. to minimize maximum queue.

The study confirms Godfrey's findings on the effectiveness of the FIFO strategy. The stop queue policy turned out to be most effective when used with the FIFO strategy. The author used this study to predict the lengths of maneuvering sections needed for various flow densities.

Caudill and Youngblood [6],[7] have proposed several algorithms for merging under a quasi-synchronous control scheme. Their approach to the problem is analytical. The criterion for comparison of the various algorithms is expressed in probabilistic terms as the miss-turn rate. These algorithms define specifically an aborted merge. The algorithms proposed include

1. Drop-back-one-slot algorithm.
2. Advance-one-slot algorithm.
3. Drop-back-two-slots algorithm.
4. Drop-back-one-slot-or-advance-one-slot algorithm.
5. Cycle-of-five-slots algorithm.

The last algorithm can be extended to make it a cycle -of-n slots algorithm. The study though reveals that the cycle-of-five-slots algorithm is good enough for most maneuvers except those of very high traffic densities.

Stefanek discusses the issues involved in the design of a dual mode transportation systems. The effectiveness of the dual mode vehicle (DMV) was demonstrated in an earlier paper [27]. The use of the moving cell technique should solve the problems associated with the DMV problem. This paper [26] describes the need for preprogramming for the DMV methodology. The authors present a method for time synchronization and a method to improve entrance to the DMV network.

Wilkie's [28] optimal control approach to the merge problem is a variation of the Levine and Athans[14] approach applied to the moving cell control technique. The performance index to be minimized is a sum of the squares of the tracking error, the vehicle velocity deviations and the allowable accelerations of the controlled vehicle only. The problem is set up as a linear quadratic regulator problem with the aim of minimizing the performance index subject to the system constraints governed by the state equations.

Various quasi-synchronous merge algorithms have been proposed by McGinley[15]. The survey introduces the reader to the interchange management problem. The performance indicators for the evaluation of the different algorithms are the abort rate and the inclusion and effectiveness of priority measures. The paper deals with the description of the main-line-man, ramp-man, and the loop-former algorithms. A comparison of the algorithms reveals that the loop former algorithms - constant capacity and constant spacing result in the lowest abort rates at all levels of vehicle flow. The loop former-constant spacing algorithm requires larger queueing area but handles priority measures best. The author suggests the use of a combination of the two algorithms depending on the type of vehicle flow through the merge area. The loop-former algorithms have been dealt with in detail in an earlier paper by the same author [16] which deals with the comparison of the loop former algorithms with the chain former algorithm presented by Brown. The loop former algorithms do not have the restrictions of the chain-former algorithm and hence are seen to perform better over the entire range of vehicle flows and densities.

The chain former algorithm was first introduced by Brown [4]. The algorithm accomplishes the intersection control by formation of a virtual chain of directional links with each link representing a maneuver. The maneuver is a slipping maneuver required to bring the vehicle in the cell at the link's tail to the the slot currently positioned at the head of the link. The drawback of the algorithm lies in the restrictions of the algorithm. The maneuvering is restricted only to vehicles on the interchange ramp. Moreover the quantity of information available to the interchange control computer is restricted to the states of the cells occupying the known status region (KSR) that has been described in the paper. This could lead to several avoidable aborts.

The first complete strategy for the merging of two high speed streams of traffic is due to Sarachik and Chu[21]. The control task is split into an assignment part which is the region where future assignments are made to the slots in the forward

blocks. A constrained optimization program is used to minimize a cost function to assign vehicles to cells during a merge subject to the constraint that a cell cannot be occupied by more than one vehicle. The control of the vehicle dictated by the maneuvers suggested by the assignment part is effected by an optimal controller. The control is calculated by minimizing a cost function that includes penalties on errors of all vehicles in the string. The control law is simply a linear feedback law.

3.2 Asynchronous Control Techniques

The literature on vehicle follower control defines two modes of operation – the velocity command mode and the regulation mode. The velocity command mode is the mode the system is in when the vehicle spacing is large i.e when the vehicle is not a part of the platoon. The regulation mode is when the vehicle is a part of a platoon with spacings between vehicles of the order of less than 5-10 metres. Most articles devoted to this problem have dynamics similar to that described below.

$$\begin{aligned}\dot{x}_i &= v_i \\ \dot{v}_i &= a_i \\ m\dot{a}_i &= \dot{f}_i - \alpha v_i\end{aligned}$$

where,

x_i, v_i and a_i are the position, velocity and acceleration of the i th vehicle respectively. f_i is the input to the i th vehicle and α is the linearized drag coefficient about the platoon velocity.

Levine and Athans[14] used the optimal control approach to address the longitudinal control problem. Simplified dynamics were assumed for the plant which included the nonlinear drag term linearized about the steady state velocity of the vehicles in the string. The paper does not explicitly deal with merge control. The authors have defined the corresponding error variables for the position, and velocity of the controlled vehicle. This is to enable the formation of a performance index that is minimized subject to the system constraints. The approach though computationally very intensive yielded good results.

This method has been extended to address the merge problem as well [1]. A certain length upstream and downstream of the junction point (of two feeder guideways merging into one) is specified. This is the decision area. For a given set of vehicles in the decision area there is a finite number of orderings possible for the vehicles to effect a merge. The problem has been set up as an optimal control problem that seeks to minimize the sum of the squares of the position and velocity errors (of all the vehicles in the string) and the inputs to the system. The gains are calculated for the linear feedback law and the performance index is calculated for each of the orderings possible. The index corresponding to the ordering that yields the lowest value of the performance index is chosen as the merging order. The drawback of the method lies in that it does not account for merge performance measures such as delay time

of vehicles, length of maneuvering region required, etc. Moreover the computation and storage requirements increase with the increase of the number of vehicles in the system.

Brown's [3] approach to merge control is based on vehicle follower control. The paper does not lay much stress on plant dynamics (represented as a unity transfer function) but seeks to demonstrate the application of longitudinal control techniques to the merge problem. The system operates in either the regulation or the velocity command mode. The control consists of two components, the control component and the assignment component. The assignment component requires the data on the vehicles involved in the merge. It effects the execution of the merge logic based on the relative positions and velocities of vehicles and causes changes in the lead vehicle assignments. The control component on the other hand issues the necessary acceleration commands depending on the assignment commands and which mode the vehicle is in. The adaptive merge control law [4] is used here. The study revealed that the constant headway policy was effective for low maneuvering distance merging but produced rather high velocity transients. The constant K-factor method can be used to avoid this.

The longitudinal control problem has been treated in great detail by Shladover [22]. The accuracy requirements were stringent with spacings between vehicles of the platoon of the order of 30-60 cms. A third order model has been assumed for the plant with a first order lag on the propulsion system. This is to study the bandwidth requirements of the system. The author suggests a control that takes care of the regulation mode as well as the low gain velocity command mode. The gains vary as a function of the spacing between the controlled and the preceding vehicle. There is a detailed treatment of the destabilizing effect of the jerk limiter which dominates the performance of the system more than the effects of the acceleration limiter. A describing function analysis was used to study the destabilizing effects of the jerk limiter. The analysis is used to come up with constraints on the states which when not violated ensure system stability. If the system tends to violate the constraints the author suggests switching to a lower gain mode. A study of the effect of random input disturbances has also been done. The phenomenon of jump resonance has been shown to exist because of the nonlinearity due to the jerk limiter.

Shladover extended the concept presented in the longitudinal control problem [23] to the problem of dynamic entrainment and extrainment of vehicles to and from platoons [23]. The problem deals with the control of an individual vehicle that either wishes to join a platoon (at the back of the existing platoon) or wishes to leave a platoon (from the front of the platoon). The control law implemented for this is a nonlinear function of the spacing and velocity error between the vehicle and the preceding one. The controller gains for nominal small spacing(15 cms) and those for nominal large spacing(45m) are determined based on fixing the dynamics at these two extreme cases through pole placement. Intermediate controller gains have been obtained through a curve fit. Good performance has been shown with the use of this controller. The jerk limiter was not introduced though. Responses to external

force loadings reveal system destabilization with integral compensation. The author suggests the use of a disturbance estimator.

Y and X merge and diverge junctions have been considered by several researchers before. Shladover [24] examined the merge performance at a Y junction under two different types of merge philosophies - isocapacity (vehicles leave the merge junctions as they arrived as individual entities or trains) and concatenating (vehicles approaching the junction bunch up into a single train as they leave the merge junction). Extensive simulation results show that the latter philosophy is better both in terms of safety and reduction in the maneuvering distance required for merge (under equivalent traffic densities). The merge performance was best only for intermediate train lengths.

Another vehicle follower controller with variable gains is due to Chiu, Stupp and Brown [8]. A standard second order linearized plant dynamics is assumed. The damping and bandwidth specifications required can be used to design a controller that is a linear function of the spacing and velocity error between a vehicle and its preceding vehicle. In order to account for maneuvers such as merging and overtaking a gain varying method was proposed. The gain varies as a function of the headway of the vehicle. The gains of the controller are expressed as a function of the headway. So by specifying the way the headway should change the gains of the controller can be calculated. The linearization of the plant could lead to potential problems but the controller was shown to perform well over a wide range of initial conditions and maneuvers.

The constraints on the allowable acceleration and jerk levels of vehicles has led many investigators to look into the area of state constrained controllers. Pue designed a vehicle follower controller for longitudinal control. The controller has been designed by explicitly taking into account the constraints imposed on the states of the vehicle such as acceleration and jerk. A kinematic constraint was formed and the control law is such that it is linear so long as the constraint is satisfied but the instant it is not a nonlinear controller is introduced. The solution is based on the solutions to the optimal control law subject to such constraints [20]. Several sub-optimal controllers have been designed by the author through small modifications in the constraint function. The control is optimal in the sense that desired spacing is achieved but the control is not necessarily a minimum time control. The purpose of suboptimal controllers is to reduce the necessary information for control. The stability of the system was checked by using the describing function analysis.

Chiu applied the state constrained controller technique of Pue [18] to the station egress problem [9]. This is very similar to the merge problem with the difference that the egressing station vehicle (ESV) starts from rest and accelerates up to line speed to merge between the preceding guideway vehicle (PGV) and trailing guideway vehicle (TGV). The main part of the problem lies in developing an appropriate constraint function. The constraint function in this case is the difference between the spacings (between ESV and PGV) required for the ESV to merge safely when the PGV is performing a normal maneuver as against a maneuver on the service limit. The problem

is complicated more by the presence of finite egress lane lengths. The egress lane lengths required at short headway were found to be too long. Hence an egress controller was designed for short headways. The controller consists of the egress initiator which controls the motion of the ESV at the initial part of the egress and can be superseded by the egress assurance that ensures against any collisions. The structure of both controllers are similar to the regulation controller suggested by Pue[18].

4 Control Objective

The maneuver and longitudinal control problems differ primarily in the range of maneuvering distances encountered. The control in the longitudinal problem involves maintaining separation distances (between succeeding members of the platoon) about a certain nominal distance. This results in small changes in the relative control effort between the lead and the controlled vehicle. This is not true for the platoon maneuver case.

The control objective is to maneuver a vehicle (laterally and longitudinally) so as to facilitate its entry to or departure from a platoon of vehicles travelling in an automated lane of the highway.

The initial focus has been on the longitudinal control aspect of the vehicle during the maneuver. Keeping this in mind the following objectives have been defined for the different maneuvers.

- **Lane change** : The objective during the lane change operation is to maneuver a vehicle (called the maneuvering vehicle) from its position at the initiation of the maneuver to a longitudinal position behind the last vehicle of a platoon. At the completion of the lane change operation the maneuvering vehicle must be travelling at the platoon velocity maintaining the specified separation distance for platoon operation. Similar objectives can be defined for type 2 and type 3 lane change procedures.
- **Merge Procedure** : The merge procedure involves maneuver of a vehicle to a position behind or ahead of the platoon such that it is travelling at the platoon velocity and maintaining the required distance for platoon operation i.e. it is a member of the platoon.
- **Split Procedure** : The split procedure involves maneuver of a vehicle to a position behind or ahead of the platoon it was a member of. At the end of the split operation the distance between the maneuvered vehicle and the platoon must be at least the minimum specified inter-platoon distance i.e. the maneuvered vehicle is a free agent at the end of the maneuver.

The lane change procedure as seen from the above definitions not only requires steering control for a change of lane but also includes the merge procedure by which the maneuvering vehicle becomes a member of the nearest platoon. The general case

of the lane change thus assumes that any vehicle entering the automated lane does not remain as a free agent in that lane i.e. the procedure includes a further maneuver into the nearest platoon. **The merge and split procedures for this study from a analysis point of view are therefore specialized cases of the lane change maneuver.** The study also assumes that a separate dedicated lane exists for vehicles that are in the process of changing lanes.

5 System Description

A preliminary study of the maneuver control problem involved a simplified representation of the system. Subsequently a higher order system depicting engine dynamics, [17],[10], was chosen to study the system further to account for the nonlinearities present in the system.

5.1 Simplified System Representation

The simplified system consists of a two state model for each vehicle. Several assumptions have been made for characterizing the system which have been listed below.

1. Vehicles are treated as point masses.
2. Vehicle dynamics are simplified and of the form:

$$\ddot{x}_i = u_i - C_{di}\dot{x}_i^2 \quad \forall i$$
 where, x_i is the position, u_i is the control input and C_{di} is the drag coefficient of the i th vehicle.
3. The vehicles are under longitudinal control only. No tire force-slip characteristics are considered.
4. Steering dynamics are not considered.

5.2 Detailed System Represent at ion

The detailed model is a four state model. The model is similar to that used for longitudinal control studies, [12] and captures the engine and transmission dynamics.

A rigid coupling between the engine shaft and the vehicle wheels has been assumed. The model includes torque converter dynamics but has no shift scheduler. The inputs to the model are the throttle position and the brake torque.

The mathematical formulation of the detailed system is as below.

$$\begin{aligned} \dot{m}a_i &= \dot{m}a_i - \dot{m}a_o \\ \omega_{e,i} &= (t_{net_i} - r_i^* * t_{load_i}) / j_i^* \\ \dot{t}_{br,i} &= (t_{bc_i} - t_{br,i}) / \tau_b \\ \dot{v}_i &= (f_{tr_i} - f_{rtot_i} - f_{a_i}) / mass \end{aligned}$$

where,

The subscript i corresponds to the i th vehicle.

m_{a_i} is the mass of air in the intake manifold.

$\omega_{e,i}$ is the engine speed.

$t_{br,i}$ is the brake torque.

v_i is the vehicle speed.

m_{a_i} is mass flow of air into intake manifold.

m_{a_o} is mass flow of air out of the intake manifold.

$t_{net_i}, t_{bc_i}, t_{load_i}$ are the net, commanded brake and load torques respectively.

r_i^*, j_i^* are the effective reductions and inertias respectively

$f_{tr_i}, f_{rtot_i}, f_{a_i}$ are the tractive force, total friction force and drag force respectively.

m_{a_o} , **t_{net}** ; are calculated through table look-up procedures. The tables were obtained from steady state engine maps generated for various engine speeds and manifold pressures. The mass flow rate of air into the intake manifold, m_{a_i} is calculated from the relation

$$m_{a_i} = m_{a_o} \cdot p_{ri} \cdot TC(\alpha) \quad (1)$$

where,

m_{a_o} : is an engine constant.

p_{ri} is the pressure influence function that is a function of the ratio of pressure in the manifold, p_m to the atmospheric pressure.

TC is the throttle characteristic, dependent on the throttle angle, α .

6 Assumptions and Initial Conditions

This study concentrates on the longitudinal positioning of vehicles to facilitate the maneuver. Some of the assumptions made are listed below.

6.1 Lane Change Operation

1. The platoon has been replaced by 1 vehicle (the last vehicle of the platoon for type 1 lane change) called the lead vehicle. The vehicle entering (leaving) the automated lane is called the maneuver vehicle.
2. The lead vehicle moves at a constant velocity i.e. the maneuver vehicle is the only one whose speed is adjusted.
3. The initial conditions are such that the maneuver vehicle is longitudinally ahead or parallel to the lead vehicle and not necessarily travelling at the same velocity as the lead vehicle.
4. The maneuver is considered complete when the maneuver vehicle is at the correct longitudinal position relative to the vehicles of the platoon i.e. it is behind the lead vehicle at a distance equal to the specified desired in-platoon spacing.

6.2 Merge/Split Operation

The philosophy behind the control of vehicles for the merging and splitting procedures is similar to the lane change operation. The maneuvering distance is larger in this case also when compared to pure longitudinal control.

1. Only 2 vehicles are considered. During the merge operation the lead vehicle represents the vehicle of the platoon ahead or behind which the maneuver vehicle is to join the platoon.
2. The lead vehicle moves at constant velocity. The maneuver vehicle is the only one whose speed is being adjusted.

Studies with the simplified model have been restricted to the more general case of lane change.

7 Designing a Desired Trajectory

When the maneuver is initiated a step change occurs in the desired position of the maneuver vehicle and this results in a sudden surge in the control effort which is exhibited in the sudden deceleration/acceleration of the maneuvering vehicle.

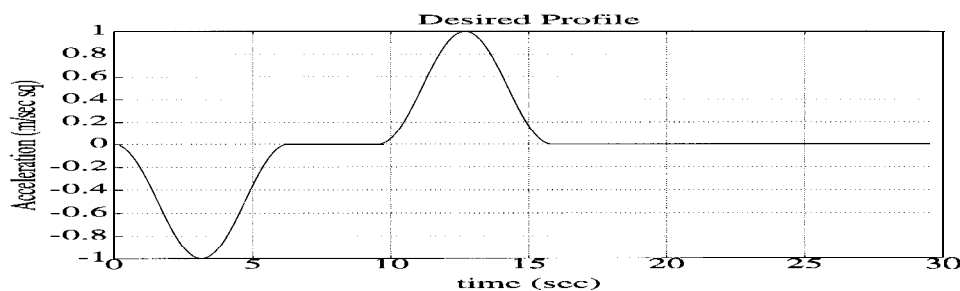


Figure 3: Desired Maneuver Vehicle Acceleration (m/sec sq)

Prior studies in maneuver control have indicated that a limit must be placed on the allowable jerk and acceleration changes required by the controller during the process of the maneuver. The limits have been obtained from literature and are based on vehicle capabilities and passenger ride comfort requirements. The allowable limits have been specified as .2g for acceleration, .3g for deceleration and .1g/sec for maximum allowable jerk. These limits have been used to design the desired trajectory. An additional constraint in designing the trajectory is that the maneuver must be accomplished in minimum time.

The spacing trajectory sp_d was obtained from a desired acceleration trajectory $\ddot{s}p_d$. A maneuver that would require the maneuver vehicle to fall back a certain distance would then involve three phases: a smooth deceleration to the possible limit and then back to zero acceleration again, a period of no acceleration and then a smooth

acceleration phase (a mirror image of the first phase) is shown in Figure 3 and Figure 4. The times of deceleration and acceleration are found from the constraints on maximum acceleration and desired spacing change. The above plots are for the case

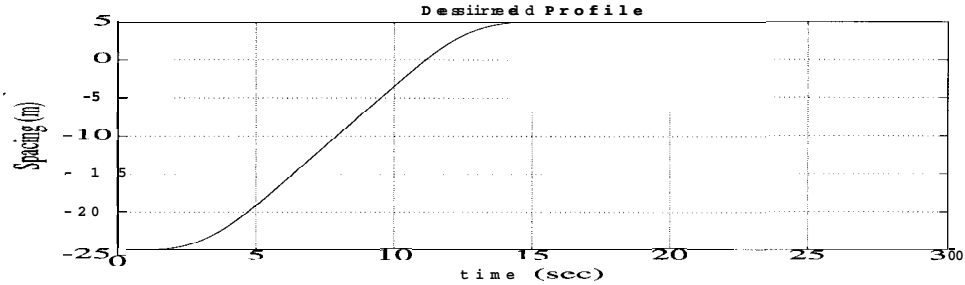


Figure 4: Desired Spacing Change (m)

where the net velocity change is zero and is for a case where the initial velocities of the lead and merge vehicle coincide at the initiation of the merge. Trajectory design for the case of differing initial velocities of the maneuver and lead vehicles is more complicated but is just an extension of the above principle.

It must be noted that the above design is not a minimum time desired trajectory because the acceleration limit has been chosen less than the maximum allowable. This is a milder case than would be possible if the allowable limits were used.

8 A Linear Approach to Platoon Maneuver Control

A linear approach is first investigated to study the performance of the nonlinear system operating under a control law based on the linearized plant. The system is therefore first linearized about an equilibrium point. The simplified representation of the plant has been used to simulate the response of the system under the linear approach.

8.1 System Linearization

Considering two vehicles only - a lead and a maneuver vehicle the system can be described by the following equations.

$$\begin{aligned}\dot{x}_1 &= x_2 \\ \dot{x}_2 &= u_1 - Cd_1x_2^2 \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= u_2 - Cd_2x_4^2\end{aligned}$$

where,

x_1, x_3 are positions of the lead and maneuver vehicles respectively.

x_2, x_4 are velocities of the lead and maneuver vehicles respectively.

u_1, u_2 are the control efforts of the lead and maneuver vehicles respectively.

Cd_1, Cd_2 are the drag coefficients of the lead and maneuver vehicles respectively.

The equilibrium point can be found by setting the derivatives of the states to zero.

This yields 'an equilibrium point that corresponds to the system in a state of rest. We

wish to linearize the system about the steady state velocity of the lead vehicle of the platoon (x_{2e}). Hence we redefine the system.

$$\begin{aligned} x_i &= \delta x_i + x_{ie} \\ x_{ie} &= x_{(i+1)e}t + x_{ie0} \quad \forall i = 1, 3 \\ x_j &= x_{je} + \delta x_j \quad \forall j = 2, 4 \\ \mathbf{u}_k &= \mathbf{u}_{k e} + \delta u_k \quad \forall k = 1, 2 \end{aligned}$$

where, $u_{ke} = Cd_k x_{je}^2$

x_{1e} and x_{3e} are the positions at time t if the vehicles were travelling at their initial velocities, x_{2e}, x_{4e} . Here both vehicles are initially assumed to be travelling at the platoon velocity x_{2e}

x_{1e0} and x_{3e0} are positions of lead and maneuver vehicles at time, t equal to 0.

$\delta x_i (i = 1, 3), \delta x_j (j = 2, 4), \delta u_k (k = 1, 2)$ are variations from the equilibrium values.

The redefined system is now reduced to the form below.

$$\begin{aligned} \delta \dot{x}_1 &= \delta x_2 \\ \delta \dot{x}_2 &= \mathbf{0.0} \\ \delta \dot{x}_3 &= \delta x_4 \\ \delta \dot{x}_4 &= \delta u_2 - 2Cd_2 x_{4e} \delta x_4 + \delta x_4^2 \end{aligned}$$

Using Taylor series expansion we linearize the system. Neglecting the higher order terms we obtain the linearized system:

$$\begin{aligned} \delta \dot{x}_1 &= \delta x_2 \\ \delta \dot{x}_2 &= \mathbf{0.0} \\ \delta \dot{x}_3 &= \delta x_4 \\ \delta \dot{x}_4 &= \delta u_2 - 2Cd_2 x_{4e} \delta x_4 \end{aligned}$$

8.2 Controller Design

The output of the system y_1 is defined as the spacing between the lead and maneuver vehicles. The corresponding error in spacing, e_1 , is the difference between the actual and the desired output. The output and error have been defined with respect to the original system and not the linearized one.

$$y_1 = x_1 - 23$$

$$e_1 = y_1 - y_{1d}$$

Since we have to differentiate the error equations twice before we hit the control, the control input to the merge vehicle is calculated from the following error dynamics equation

$$\ddot{e}_1 + \alpha_1 \dot{e}_1 + \beta_1 e_1 = 0.0$$

Proper selection of α_i and β_i ensures that the controller will drive the error to zero in finite time. The control law is then chosen as:

$$\delta u_2 = 2C d_2 x_{4e0} \delta x_4 + \alpha_1 \dot{e}_1 + \beta_1 e_1 - \ddot{y}_{1d}$$

The control input is obtained from the linearized version of the plant. This control is then used in the nonlinear system for the maneuver.

The plant does not possess any inherent damping and hence the high rate of change of the control effort. Selection of poles closer to the origin will reduce the rate of change of the control effort but at the same time will result in more sluggish response of the system to changes in the input.

8.3 Simulation Results and Analysis

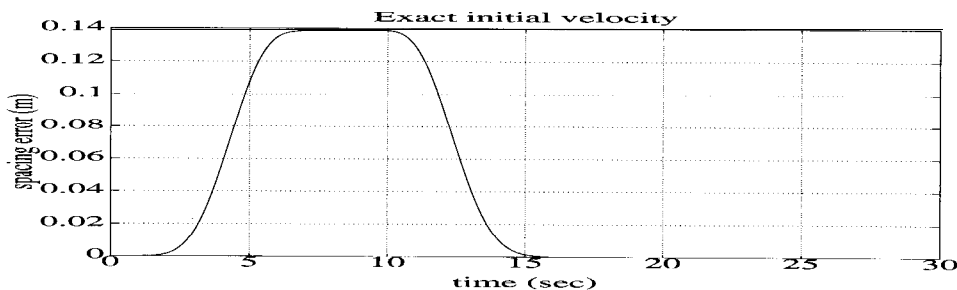


Figure 5: Spacing Error

The maneuver is initiated at $t=0.20$ sec. The maneuver vehicle at this time is 25m ahead of the lead vehicle and not necessarily travelling at the same velocity. The velocity of the lead vehicle remains constant at 16 m/sec. The spacing error and vehicle acceleration (maneuver vehicle) plots are shown in Figure 5 and Figure 6. The results correspond to the case where the maneuver vehicle initial velocity is the same as the lead vehicle velocity. The results indicate that the controller is able to drive the spacing error to zero. The maneuver is accomplished in 15 seconds. The deceleration is within limits. Simulations requiring the maneuver vehicle to fall back or advance more than 5m under the same initial velocity indicate comparable performance.

The performance of the controller is poor in the case of maneuver vehicle initial velocity differing from the lead vehicle velocity with performance degradation for greater initial velocity variations (due to the limitations of the linearized model away from the equilibrium velocity). The performance of the system is good only if the maneuver vehicle has initial conditions around the equilibrium velocity of **16** m/sec.

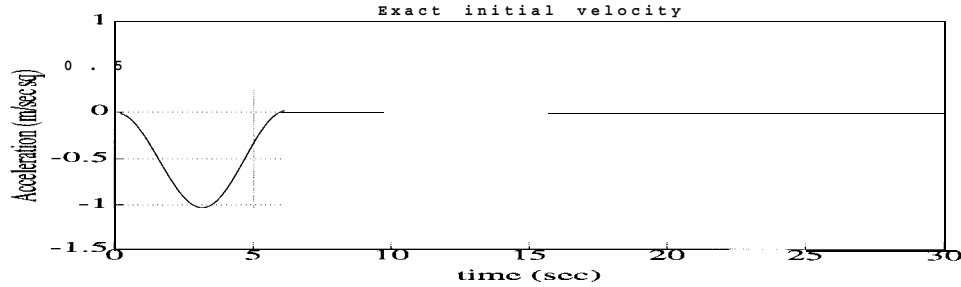


Figure 6: Maneuver Car Acceleration

Transitional platoon maneuver control involves greater variations in vehicle velocity and maneuvering distance than in longitudinal control. There is a marked degeneration in system performance away from the equilibrium platoon velocity. The linear controller is limited by its range of operation. Hence we require a control method that not only has a wide range of operation but also performs well in the face of modelling errors and uncertainties in the system. The sliding control methodology besides guaranteeing stability also ensures system performance in the face of modeling errors and system uncertainties.

9 Sliding Control

Given a system of the form:

$$\dot{x} = f(x) + g(u)$$

where,

x is a n dimensional state vector.

$f(x)$ is a n dimensional vector of nonlinearities.

$g(u)$ is a m dimensional input vector.

We also define a n dimensional error vector \tilde{x} which is the difference between the state and the desired state. In sliding control we define a surface S by :

$$S = \left(\frac{d}{dt} + \lambda\right)^{n-1} \tilde{x}$$

where λ is a positive constant. The tracking problem is then reduced to remaining on S for all time. If we can ensure the *sliding condition*

$$\frac{1}{2} \frac{d}{dt} S^2 \leq -\eta S^2 \text{ for } \|S\| > \phi$$

where, η is a positive constant and ϕ is the desired tracking accuracy, then the surface S becomes an invariant space and trajectories reaching the surface will remain there for all time. Once the surface is reached, the tracking error exponentially converges to the boundary layer of width ϕ [25].

10 Application of Sliding Control Methodology to Transitional Platoon Maneuver Control

The sliding control methodology has been applied to both the simplified and complex models. The basic control algorithm involves the definition of error variables based on a comparison between the actual and desired vehicle spacing profiles. The definition of the surface varies with the representation of the plant that has been adopted for study and with the control objective.

10.1 Control Law Development Based on the Simplified Model

The surface is based on the spacing error of the maneuver vehicle. The surface for the maneuver vehicle is as described below.

$$S = \dot{e}_1 + \lambda_1 e_1 + \lambda_2 \int e_1 dt$$

where the errors are as described in the linear case.

The control is obtained by differentiating the surface once. To achieve convergence of the trajectories to the surface we satisfy the sliding condition by requiring that $\dot{S} = -kS$ where, k is the sliding gain of the maneuver vehicle. The model chosen is of relative order 2 i.e. the output must be differentiated twice before we obtain the control input.

The control law obtained using this method is shown below. By proper selection of k and λ , positive constants, we can guarantee convergence of trajectories to the surface.

$$u_2 = C d_2 x_4^2 + \lambda_1 \dot{e}_1 - \ddot{s} p_d + kS$$

The designed desired trajectory was adopted in this study as well. The performance of the system was evaluated for several initial conditions. The initial runs assume a perfect model.

10.1.1 Simulation Results and Analysis

When the maneuver is initiated at $t=0.2$ sec, the maneuver vehicle is 25m ahead of the lead vehicle. The control parameters in the sliding definition, k, λ are chosen to be 1. The designed trajectory for the linear approach was adopted in this case too.

With exact initial velocity of the maneuver and lead vehicles the system performance in the nonlinear case is marginally better than that obtained in the linear case. The spacing error (Figure 7) is lower in this case. The acceleration profile (Figure 8) follows that of the desired profile and has been limited to a maximum of .1g.

The system does perform markedly better than the linear controller for variations in the initial velocity of the maneuver vehicle from that of the lead vehicle. Even for large variations in initial error the controller performs well.

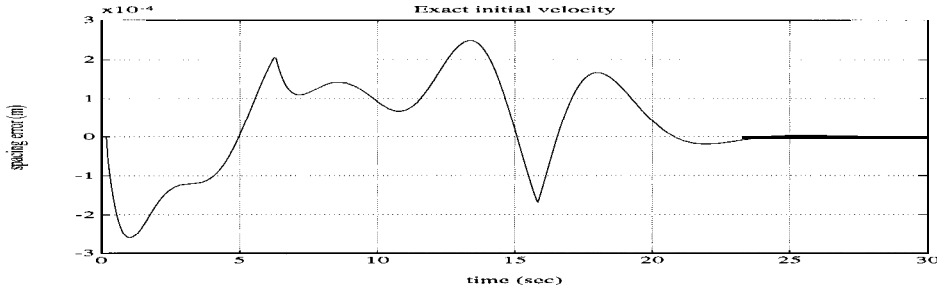


Figure 7: Spacing Error (m)

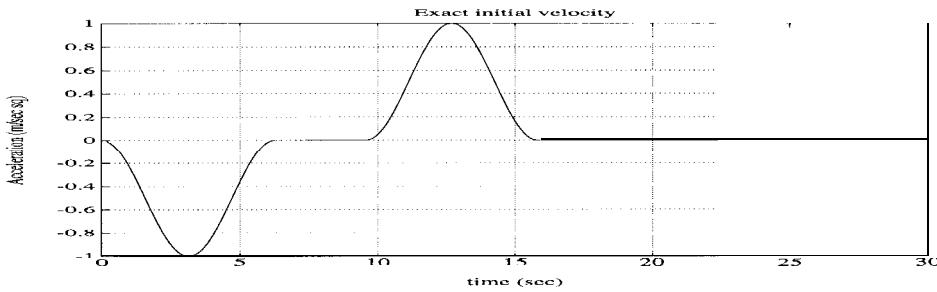


Figure 8: Maneuver Vehicle Acceleration (m/sec^2)

10.2 Control Law Development Based on the Detailed Model

The sliding surface as seen in the previous definitions is a function of the spacing and/or the velocity error between the vehicles. When using this definition with the detailed model, more than one differentiation of the surface is required before the control input appears. Moreover, successive differentiations still do not give the control input since a closed form expression does not exist for several intermediate variables. This is due to the fact that several of these system variables are calculated from table look-up procedures.

This leads to the 2 surface approach used successfully before by Green and Hedrick [11] and McMahon and Hedrick [17]. The first surface is similar to the one defined for the simplified model. The first surface for each vehicle is defined as below.

$$S_1 = \dot{e}_1 + \lambda_1 e_1 + \lambda_2 \int e_1 dt$$

Differentiating the first surface and requiring that the sliding condition is satisfied ($\dot{S}_1 = -k_1 S_1$) we obtain a synthetic control input which is $ma_{des,2}$ if the throttle control algorithm is in effect or is $tbr_{des,2}$ if the braking control algorithm is being used. The second surface is defined based on this calculated value. The second surface is as defined below.

$$S_2 = ma_2 - ma_{des,2}$$

($S_2 = tbr - tbr_{des,2}$ for braking) Requiring that the sliding condition be satisfied ($\dot{S}_2 = -k_2 S_2$) we obtain the control law :

$$tc_{des,2} = (m\dot{a}_{o2} + ma_{des,2} - k_2 s_2)/(max \cdot pripri)$$

The desired brake torque $tbr_{des,2}$ can be calculated in a similar fashion.

10.2.1 Simulation Results and Analysis

The different types of paltoon maneuver scenarios have been simulated to evaluate the performance of the controller. The structure of the control law remains essentially the same but the desired trajectory differs depending on the type of maneuver that is being considered.

1. **Lane Change operations** When the maneuver is initiated $t=0.2$ sec the ma-

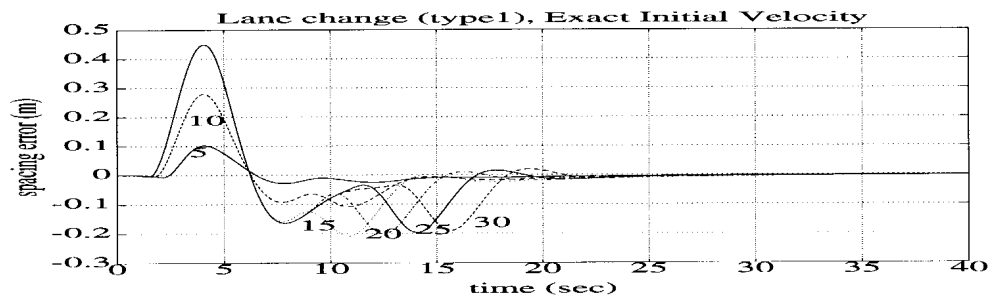


Figure 9: Spacing error(m)

neuver vehicle is assumed to be ahead of the lead vehicle. The initial velocity of the maneuver vehicle is assumed to be the same as that of the lead vehicle. The performance of the system was simulated for several initial conditions corresponding to the position of the maneuver vehicle at the initiation of the lane change operation. Figure 9. shows the variation of the spacing error during the process of lane change. The spacing error decreases with the maneuvering distance. As expected, with lower required maneuvering distance the maximum acceleration/deceleration (Figure 10) of the vehicle during the lane change process is also less. The acceleration plots indicate a poor performance during the deceleration phase. This is due to the fact that the brake dynamics have not been characterized completely and hence during braking we often get greater

decelerations than required which necessitates some throttle action to keep the spacing error low. So for a brief period of time we have a high frequency switching between the braking and throttle control routines. Figure 11 shows the variation of the actual spacing of the vehicles during the maneuver.

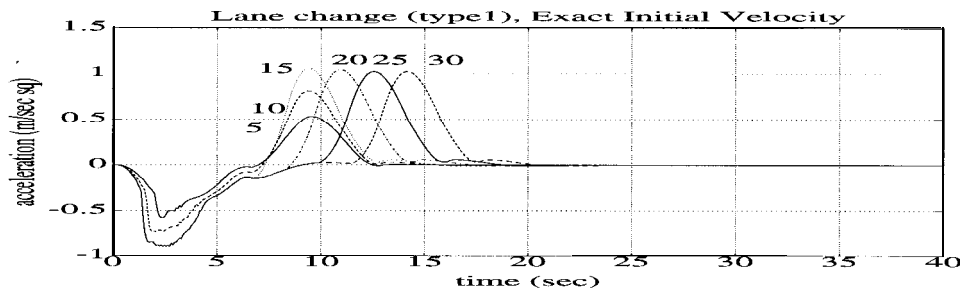


Figure 10: Maneuver Vehicle Acceleration (m/sec sq)

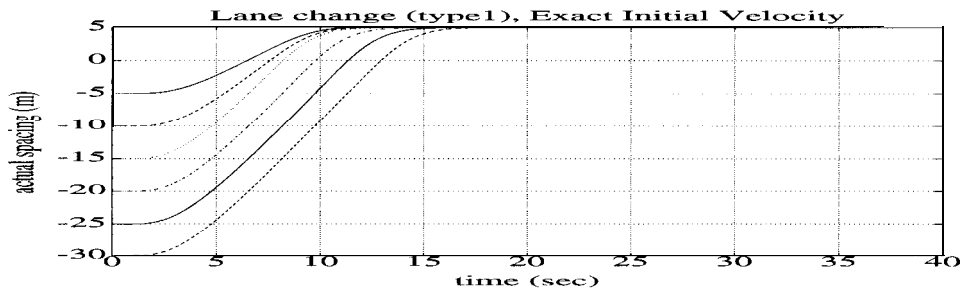


Figure 11: Actual Spacing (m)

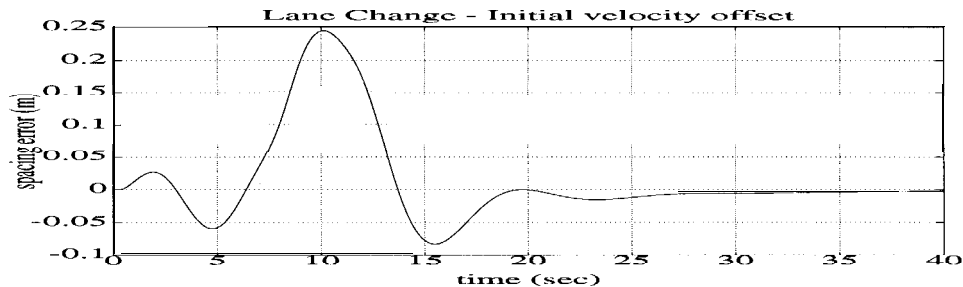


Figure 12: Spacing error(m)

For differing initial velocity of the maneuver vehicle from the lead vehicle, we can follow two approaches : first apply velocity control and when velocities match, we can apply the above strategy to effect a lane change or second, design a desired trajectory such that the required spacing is achieved simultaneously with the required velocity change. We have adopted the latter strategy because

it results in relatively shorter times required for lane change and used the desired trajectory specification to effect adequate control. Figure 12 shows the variation of the spacing error for this case while Figure 13 shows the variation of the maneuver vehicle velocity during the process of the lane change operation. The controller is seen to perform well over a large range of maneuvering distances and initial maneuver vehicle velocities.

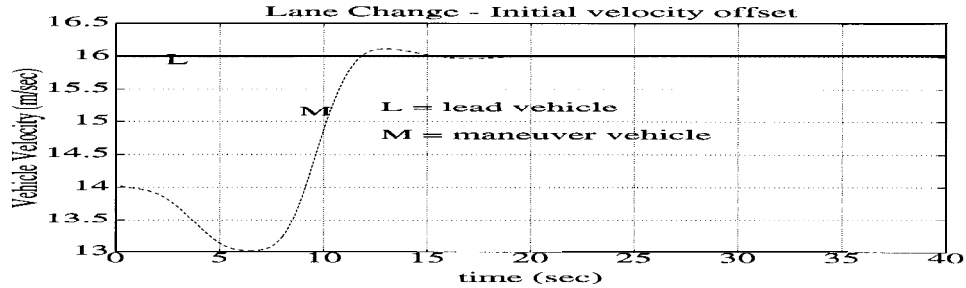


Figure 13: Maneuver Vehicle Velocity (m/sec)

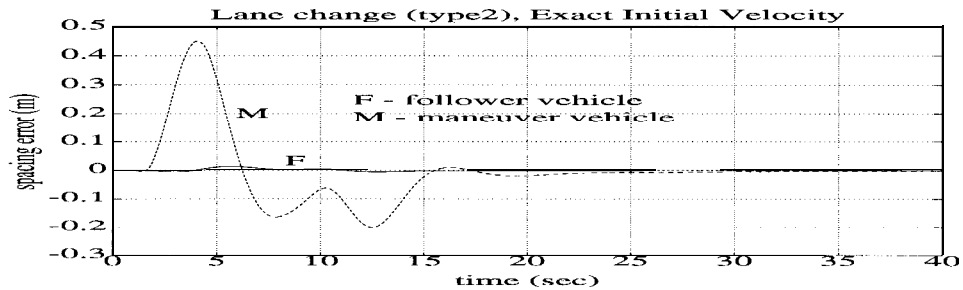


Figure 14: Spacing error (m)

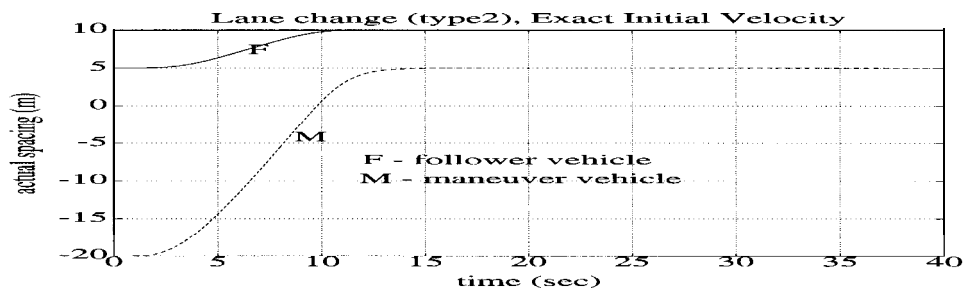


Figure 15: Actual Spacing (m)

A lane change operation resulting in the maneuver vehicle joining in between two vehicles of the platoon was also simulated. In this case we represent the platoon by 2 vehicles, the lead and follower vehicles, between which the maneuver vehicle joins the platoon. The lane change procedure then requires that at the end of

the lane change operation the second vehicle of the platoon must be dropped back a sufficient distance to allow for the entry of the maneuver vehicle in the platoon. At this time all vehicles must be travelling at the common platoon velocity maintaining the required spacing distance between succeeding vehicles. Figure 14 shows the variation of the spacing error during the lane change. Figure 15 shows the variation of the actual spacing and indicates that the lane change process occurs smoothly with no chances of any collisions.

2. Merge/Split operations

We have assumed a distance of 35 m for the normal

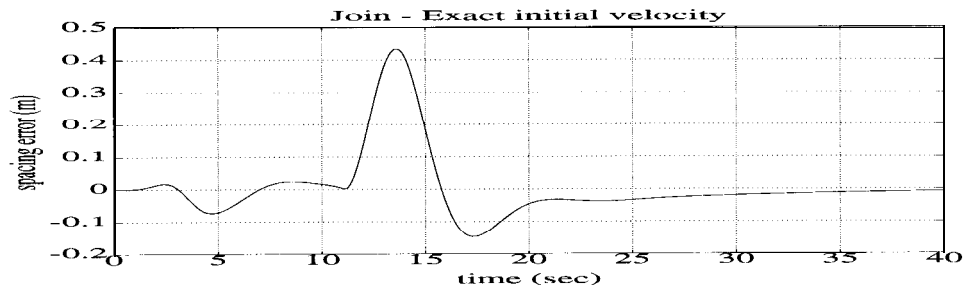


Figure 16: Spacing error (m)- Merge operation

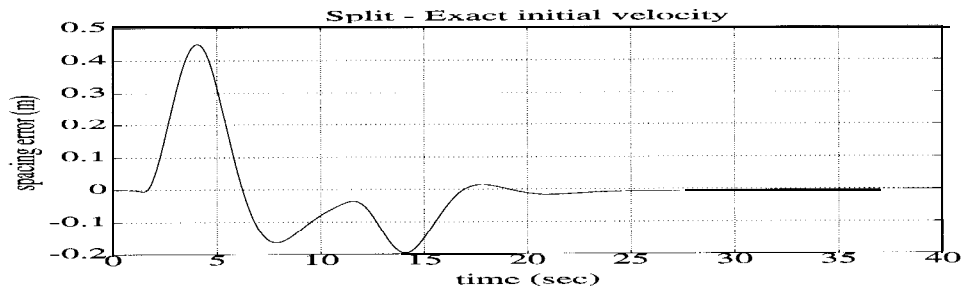


Figure 17: Spacing error (m)- Split operation

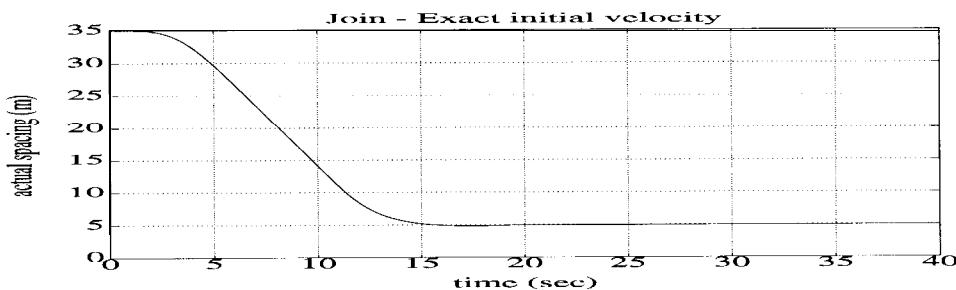


Figure 18: Actual Spacing (m)- Merge operation

separation distance *between* platoons. So in order for a vehicle to leave the

platoon the vehicle must be moved such that it is at least 35 m away from the lead (last) vehicle of the platoon and must be travelling at a velocity greater than or equal to the platoon velocity. Figure 16 and Figure 17 show the spacing errors for the merge and split cases and it is observed that the magnitude of spacing errors are comparable to those of the standard lane change cases. Figure 18 and Figure 19 show the variation of the actual spacing during the join and split processes respectively.

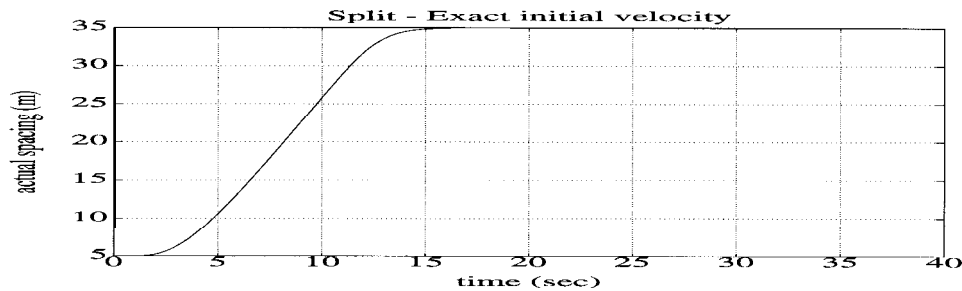


Figure 19: Actual Spacing (m)- Split operation

11 Sensor Requirements

Designing control algorithms for automated highway system technology must go hand in hand with implementation issues. So the control law developed must be based on the measurements that can be obtained from the different sensors available on-board the vehicle.

The methodology followed in this primary approach relies heavily on obtaining the desired trajectory. Hence we must have a good estimate for the maneuver vehicle position and velocity. For the merge and split operations the initial position estimates are only limited by the radar range. So long as the vehicle is in radar range the desired trajectory can be generated through radar measurements and communication transfer of maneuver vehicle velocity and acceleration.

The distance estimate is more difficult to obtain when a lane change process is required. Currently the radar is fixed and is not allowed to rotate. Therefore a method must be developed to detect vehicles in adjacent lanes.

In addition to the above, accelerometers, engine speed sensor, and mass of air flow sensors are required for the control of the individual vehicles.

The platoon maneuver logic decisions must be made in one level higher than the vehicle control. It is therefore expected that wayside processors/sensors might be required in areas designated for maneuvering to detect the relative positions

of the maneuver vehicle and the vehicles of the nearest platoon which could then be used to decide the type and time of maneuver required. This information could then be transmitted via communication links to the maneuvering as well as platoon vehicles.

12 Conclusions

The study of the platoon maneuver control problem was restricted to longitudinal vehicle control. The vehicles have only one degree of freedom with no lateral dynamics taken into consideration. The simplified system representation neglects some of the nonlinearities in the vehicle dynamics to rectify which a detailed model has been proposed. Nevertheless this system gives us valuable insights into the application of two types of control methodologies to the problem of maneuver control. The sliding control method performs better than the linear controller.

With the detailed model we are able to obtain a direct parametrization of the control inputs to the model.

The control law used with the detailed model is similar to the one used for longitudinal control. This clearly indicates that it is possible to use the same control law for short separation distance regulation control and larger separation distance maneuver control. This also ensures that switching between these two modes is guaranteed to be smooth.

13 Proposed Future Research

The first phase of the platoon maneuver control studies yielded valuable insights into several aspects of the problem. This leads us to several areas where further research can be focussed.

- **Model Development :**

- (a) Introduction of lateral dynamics for steering control required during lane change operation.
- (b) Introduction of shift scheduler into current model.

- **Trajectory Design :**

- (a) Study and introduction of different types of trajectories for optimal operation under given system and vehicle constraints.
- (b) Simulation studies using different trajectories

- **Control Design :**

- (a) Testing robustness of current control scheme under modelling uncertainties, disturbances and sensor noise.
 - (b) Investigation of alternate control strategies using MIMO
 - (c) Investigation of control strategies for effecting steering control.
 - (d) Investigation of sensor requirements for effective control.
- **Experimental Implementation :**
 - (a) Implementation of longitudinal maneuvering control.
 - (b) Implementation of lane change operation.

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