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Test-Based Calibration of Safety Factors for Capacity Models

Giorgio Monti[1] and Floriana Petrone[2]

Abstract: A simple procedure to calibrate the safety factor of capacity models is presented. The calibration can be carried out based on any available database of experimental tests, even of limited size. The procedure aims to assess the model capability of predicting the test results and to calibrate the safety factor so that the capacity equation meets the target reliability level required by the code or sought by the calibrator. After predicting each test of the database with the capacity equation under consideration, the test-prediction pairs are checked for the property of linearity, and the relative error for the properties of homoscedasticity and normality. Once these properties are fulfilled—which may require a nonlinear transformation of the test values and/or the predictions—the closed-form equation proposed in this paper is employed to compute a target design value. The model safety factor is finally obtained by comparing such target design value with the design value obtained from the code. The paper also proposes two approximate analytical equations to compute the tolerance factor, used to attain any given fractile, as a function of the (even small) number of tests, with any assigned confidence level. A fundamental outcome of the procedure is that it yields an objective indicator of the model accuracy, measured by the standard deviation of its error, which may be regarded as a parameter useful for selecting the most reliable model among different competing ones. In the long run, the application of the proposed procedure will allow achieving a uniform reliability level throughout all capacity models used in codes and guidelines. A further advantage is that the partial safety factors so derived can be straightforwardly updated when more experiments become available. As an example, the proposed procedure is herein applied to the ACI 318 shear design capacity equation for concrete members unreinforced in shear. **DOI: 10.1061/(ASCE)ST.1943-541X.0001571.**

Author keywords: Test-based calibration; Safety factors; Structural safety and reliability; Calibration procedure.

Introduction

The development of any a priori analytical capacity model should go through a validation stage that implies an a posteriori testing of its predictive ability over a reasonably representative set of experimental results. For a meaningful comparison between predictions and experiments, the capacity model should be formulated in a fully probabilistic manner by including both the intrinsic uncertainties, concerning the underlying material and geometry variables, and the epistemic uncertainties, affecting the model itself (simplification, incompleteness, and approximation).

When designing a structural system to meet a certain failure probability, one should consider all uncertainties in the demand and all uncertainties in the capacity. The latter are expressed in terms of basic geometrical and mechanical properties, whose statistical description is usually available. In general, capacity models are developed either by using simplified relationships among basic variables or by neglecting others for concisely describing the main resisting mechanisms they aim to represent. This introduces a so-called modeling uncertainty, which should be adequately considered when assessing the reliability of a capacity model. Modeling uncertainty can be incorporated into a capacity model by introducing a random variable to represent the difference between measured and predicted response (Melchers 1987).

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Calibrated safety factors for construction materials and products are provided by codes [e.g., CEN (2005) for concrete] on the basis of probabilistic studies conducted on wide test databases, but no standard procedure is available, either in the literature or in codes, to define safety factors of capacity models although they are explicitly provided in modern codes (e.g., γ_{Rd} in Eurocodes and the ϕ in U.S. Codes). Even when some authors (Sedlacek and Kraus 2007; Burdekin 2007) refer to the statistical determination of capacity models, they raise the question about the modeling uncertainty but do not provide an explicit method for assessing it.

A complete procedure to calculate safety factors of any capacity model, regardless of its functional form, is here developed. It aims at assessing the model capability of predicting the outcomes of a given test database so as to meet a target exceedance probability.

Overall, this work underlines the importance of defining a coefficient able to fine-tune capacity models, which allows endowing a capacity model with an objective measure of its unavoidable error. In this sense, it would be desirable that all capacity models developed in the future be described in terms of (1) an unbiased expression predicting the experimental results in the average, (2) the corresponding design expression, (3) a measure of the modeling error, and (4) a safety factor calibrated on the currently available test database. In this manner, the reliability of each capacity model could be easily and conveniently updated as more tests become available, and a uniform reliability level among all capacity equations developed for codes and guidelines could be achieved.

Definitions

For introducing and explaining the meaning of the symbols used in the procedure, some basic definitions are provided in the following:

Basic Random Variables

The basic random variables (RVs) are those defining all parameters of geometric and mechanical nature involved in the model in hand. Following the notation of Montgomery and Runger (2011), a set of basic RVs can be conveniently collected in a vector $\boldsymbol{X}$, that is

$$\boldsymbol{X} = [X_1 \quad X_2 \quad \ldots \quad X_m]^T \tag{1}$$

where each $X_j (j = 1, \ldots, m)$ represents, e.g., a specimen size, a material strength, and so forth.

Test Database

A test database can be formally described as

$$Y_i = T_i(\boldsymbol{X}_i) + \in_{Y_i} \tag{2}$$

where Y_i = i-th test outcome; $T_i(\cdot)$ = functional application representing the i-th test, being $i = 1,2, \ldots, N_{\text{tot}}$ the number of tests and N_{tot} the total number of tests; and $\in_{Y_i}$ = error of the outcome, which includes the measurement error (on the outcome Y_i) and possible testing errors of the i-th test. Notice that each test $T_i(\cdot)$ has a different nature, being carried out by different authors in different years with different test setups and protocols.

Capacity Model

A capacity model is a function that predicts the outcome of experimental tests performed to check a specific resisting mechanism. It is expressed either in terms of force or displacement and is formally given as

$$C_i = C(\boldsymbol{X}_i + \in_{\boldsymbol{X}_i}) + \in_C \tag{3}$$

meaning that the i-th test outcome C_i is predicted through the function $C(\cdot)$, where the basic RVs $\boldsymbol{X}_i$ are those coming from the experimental tests and, as such, they include the measurement error $\in_{\boldsymbol{X}_i}$ (on the RVs). These measurement errors are hidden and per se partly responsible for the model prediction error. Then, the error of the model itself is represented by the term $\in_C$.

This amounts to saying that when calibrating a capacity model, the errors due to both the measurements and the model are included, where the latter accounts for inherent limitations that actually play the prevailing role.

Model Error

The validation of a capacity model is conducted by comparing each experimental result Y_i in the database with the corresponding prediction C_i, in a Y_i versus C_i scatter plot. Due to $\in_{\boldsymbol{Y}}$, $\in_{\boldsymbol{X}}$, and $\in_C$, test results and predictions are expected to be different, and each comparison shows an error given by

$$\in_i = Y_i - C_i \tag{4}$$

When the above error is deemed to be excessively high, the calibrator might consider removing the corresponding tests from the database (trimming), ending up with $N \le N_{\text{tot}}$ tests.

For the calibration to be efficiently carried out, the following fundamental assumptions have to be checked:

1. Linearity of test-prediction pairs: The points (C_i, Y_i) should be evenly and symmetrically distributed along a diagonal line. Violation of such property indicates the presence of a nonlinear relationship that reveals either systematic errors in the model or the inadequacy of the formulation throughout the entire range of the considered RVs. A viable solution is to apply an appropriate nonlinear transformation to Y_i and/or C_i. For example, if the data are strictly positive, as is always the case for capacity models, a log-transformation is typically of use.
2. Normality of the error: The error in Eq. (4) should have a normal distribution, which may be checked through the usual normal probability plot. The normality assumption is necessary when estimating fractiles corresponding to given target probabilities, which is the purpose of this work. Violation of normality often indicates that there is some conceptual problem with the model assumptions and/or the test results. If in the normal probability plot, few data points are detected to significantly deviate on one or both ends, they should be examined and removed from the database (trimming), if necessary. The error should also have zero mean. In general, this can be fixed through a linear regression.
3. Homoscedasticity of the error: Error should not increase as a function of the predicted value. The points $(C_i, \in_i)$ should be symmetrically distributed around the zero value, with an approximately constant variance. Violation of this property implies heteroscedasticity, which results in confidence intervals (used in the calibration) that do not have constant amplitude throughout the database. Homoscedasticity tests are available in the literature (e.g., Goldfeld and Quandt 1965). However, for our purposes, this property can be practically verified in the mean sense by checking that the trend of the squared error, which may be found through a linear regression of the squared error plot $(Y_i - C_i)^2$, is approximately constant throughout the database. Notice that heteroscedasticity may arise from a significant violation of the linearity assumption, and it is usually fixed as a byproduct of the nonlinear transformation mentioned above (Assumption 1). In some cases, it may be solved by removing from the database (trimming) those tests showing the highest squared errors.

If the (trimmed) database, counting $N \le N_{\text{tot}}$ experimental tests, does not meet the above three assumptions, it will undergo a series of modifications that will transform it as follows:

- Original database: (C_i, Y_i);
- Adjusted database: $(\tilde{C}_i, \tilde{Y}_i) \equiv f(C_i, Y_i)$ derived through nonlinear transformations. For example, $f(C_i, Y_i) \equiv (\log C_i, \log Y_i)$; and
- Fitted database $(\hat{\tilde{C}}_i, \tilde{Y}_i)$ obtained through linear regression on the adjusted database.

For the sake of notation simplicity, from now on (C_i, Y_i) is intended as $(\tilde{C}_i, \tilde{Y}_i)$ if the database has undergone a nonlinear transformation.

The relationship between the two variables, test and prediction, is now of linear nature and is described by the fitted capacity model

$$Y_i = \hat{C}_i + \hat{\in}_i \tag{5}$$

where

$$\hat{C}_i = \hat{\beta}_1 C_i + \hat{\beta}_0 \tag{6}$$

and

$$\hat{\beta}_1 = \frac{\sum_{i=1}^N C_i Y_i - \frac{1}{N}\sum_{i=1}^N C_i \sum_{i=1}^N Y_i}{\sum_{i=1}^N C_i^2 - \frac{1}{N}(\sum_{i=1}^N C_i)^2} = \frac{S_{CY}}{S_{CC}} \tag{7}$$

$$\hat{\beta}_0 = \frac{1}{N}\sum_{i=1}^N Y_i - \hat{\beta}_1 \frac{1}{N}\sum_{i=1}^N C_i = \bar{Y} - \hat{\beta}_1 \bar{C} \tag{8}$$

where $\bar{Y}$ and $\bar{C}$ = means of the two populations.

The error between the test outcome and the fitted model, called residual, is

$$\hat{\epsilon}_i = Y_i - \hat{C}_i \quad \text{with} \quad \hat{\epsilon} = N(0, \sigma^2) \tag{9}$$

which is a normal RV with mean zero and (unknown) variance σ^2, estimated by

$$\hat{\sigma}^2 = \frac{1}{N-2}\sum_{i=1}^{N}(Y_i - \hat{C}_i)^2 \tag{10}$$

Having verified the relaxed condition of homoscedasticity, as explained above, the variance can be considered as constant throughout the database. It represents a measure of the model accuracy relative to the database considered.

The calibration of the capacity model can be finally carried out.

Calibration of the Safety Factor

Having adjusted (by transformation and trimming) and fitted (by regression) the database and having checked normality and homoscedasticity of the error, the design value for the capacity is computed at the mean point of the database as follows:

$$\bar{C}_d = \bar{\hat{C}} - k_{\alpha,N-1,p}\hat{\sigma} \tag{11}$$

where

$$\bar{\hat{C}} = \frac{1}{N}\sum_{i=1}^{N}\hat{C} = \sum_{i=1}^{N}(\hat{\beta}_1 C_i + \hat{\beta}_0) \tag{12}$$

is the mean over the database $(\hat{C}_i, Y_i)$ of the fitted capacity model obtained from Eq. (5) and $\hat{\sigma}$ is given in Eq. (10).

In Eq. (11), $k_{\alpha,N-1,p}$ is the so-called tolerance factor, to be determined with confidence level $100(1-\alpha)\%$, referred to N tests (that is, $N-1$ degrees of freedom), and relative to a given fractile having exceedance probability p. As the tolerance factor is an essential component of the design capacity equation, especially when few tests are available, some literature formulas are presented in the next section, along with two simple closed-form equations proposed by the authors.

Finally, it is important to notice that the exceedance probability p is that relevant to the capacity design value, given by $p = \Phi(-\alpha_C\beta_{LS})$, being α_C the sensitivity factor associated to the capacity (0.8 in EN 1990) and β_{LS} the reliability index associated to the limit state under consideration [3.8 in EN 1990 (CEN 2002) and 3.5 in the U.S. code].

The design value obtained with Eq. (11), should be compared, at the mean point of the database, with that provided by the capacity equation being calibrated. In general, a design capacity equation can be formally expressed as

$$C_{d,\text{code}} = \eta C(\boldsymbol{X}_d) = \eta C\left(\frac{\boldsymbol{X}_k}{\gamma_X}\right) \tag{13}$$

where the capacity reduction factor η depends on the code considered: for example, it corresponds to $1/\gamma_{Rd}$ (with $\gamma_{Rd} \geq 1$) in EN 1990 (CEN 2002) and to $\phi \leq 1$ in the U.S. codes. The capacity is computed with reference to the design values $\boldsymbol{X}_d$ of the j-th basic variables, obtained by dividing the characteristic value X_k of each RV by the relevant partial safety factor γ_X. Both X_k and γ_X depend on the code format. For example, in EN 1990 (CEN 2002), the characteristic value is the 5% fractile of the RV distribution, obtained as $X_k = \bar{X} - k_{0.05}\sigma_X$ with $k_{0.05} = 1.645$, while in the U.S. codes, the characteristic value is the 9% fractile of the RV distribution, obtained as $X_k = \bar{X} - k_{0.09}\sigma_X$ with $k_{0.09} = 1.345$. In both equations, $\bar{X}$ is the mean of the RV and σ_X is its standard deviation. As for the partial safety factor, in EN 1990 (CEN 2002) each RV has its own γ_X, while in U.S. codes, it is always $\gamma_X = 1$. In both codes, for geometry properties, it is always $\sigma_X = 0$ and $\gamma_X = 1$, while for material properties the standard deviation σ_X is provided (e.g., in EN 1992 (CEN 2005), for concrete: $\sigma_X = 4.86$ MPa).

The sought safety factor γ_{cal} is finally obtained as the ratio between the mean of Eq. (13) over the database and Eq. (11) after applying the inverse transformation, f^{-1}

$$\gamma_{\text{cal}} = \frac{\bar{C}_{d,\text{code}}}{\bar{C}_d} = \frac{f^{-1}(\bar{\tilde{C}}_{d,\text{code}})}{f^{-1}(\bar{\tilde{C}}_d)} \tag{14}$$

where: $\bar{\tilde{C}}_{d,\text{code}} = \eta(1/N)\sum \tilde{C}(\boldsymbol{X}_{d_i})$, with the design values $\boldsymbol{X}_{d_i}$ of the RVs determined as explained above, with $\bar{\boldsymbol{X}}_i = \boldsymbol{X}_i$, i.e., the value given in each test.

If the safety factor is found to be lower than 1, then it may be concluded that the code equation is conservative with respect to the database examined. If it is larger than 1, then the code equation is nonconservative with respect to the database examined, and γ_{cal} should be applied as a divisor to $C_{d,\text{code}}$ in Eq. (13).

Tolerance Factors $k_{\alpha,N-1,p}$

Tolerance factors are used to estimate any fractile of a RV distribution sampled through a limited number N (in this section given as n) of experiments. Given an (assumed) normal population of results (in this case this is the population of the error between the test results and the model predictions already verified as normal), a tolerance bound is the value above which lies at least $100(1-p)\%$ of the population, with confidence level $100(1-\alpha)\%$. For example, a suggested value for the confidence level in EN 1990 (CEN 2002) is 75%, while 50% implies the median value of the sought fractile.

Finding a tolerance bound entails the use of the noncentral t-distribution to find the solution to a quite complex problem expressed by the following equation having the tolerance factor k as the unknown:

$$F_{Z_k}(z_p) = 1-\alpha; \quad z_p = \Phi^{-1}(p); \quad F_{Z_k}(z) = \int_{-\infty}^{z_p} f_{Z_k}(z)dz;$$

$$f_{Z_k}(z) = \begin{cases} \displaystyle\int_0^{\infty} \frac{2}{\Gamma(\frac{n-1}{2})}\sqrt{\frac{n}{2\pi}}\left(\frac{n-1}{2k^2}\right)^{(n-1)/2}(s)^{n-2}e^{-[(n-1)s^2]/2k^2}e^{(-n/2)(z-s)^2}ds & \text{for } k>0 \\ \displaystyle\int_{-\infty}^{0} \frac{2}{\Gamma(\frac{n-1}{2})}\sqrt{\frac{n}{2\pi}}\left(\frac{n-1}{2k^2}\right)^{(n-1)/2}(-s)^{n-2}e^{-[(n-1)s^2]/2k^2}e^{(-n/2)(z-s)^2}ds & \text{for } k<0 \end{cases} \tag{15}$$

with $\Phi^{-1}(p)$ representing the inverse cumulative standardized normal.

The solution to the above problem yields the inverse cumulative distribution function for the noncentral t-distribution. The expression for the so-called tolerance factor is in this case

$$k_{\alpha,n-1,p} = \frac{F^{-1}_{T,n-1,\delta}(\alpha)}{n-1} \quad (16)$$

where

$$\delta = \Phi^{-1}(p)\sqrt{n-1} = z_p\sqrt{n-1} \quad (17)$$

is the noncentrality parameter.

A disadvantage of this approach is that the function $F^{-1}_{T,n-1,\delta}(\alpha)$ does not have an easily treatable analytical form and is only available in tables. The following sections are therefore intended to identify the most suitable and reliable approximation to Eq. (16).

Wallis Approximation

An approximation for $k_{\alpha,n-1,p}$ comes from the following set of formulas (generally credited to Natrella 1963 but originally proposed by Wallis 1947):

$$k_{\alpha,n-1,p} \approx k_{W,\alpha,n-1,p} = \frac{z_p + \mathrm{sgn}(z_p)\sqrt{z_p^2 - ab}}{a}$$
$$\text{with } a = 1 - \frac{z_\alpha^2}{2(n-1)}; \qquad b = z_p^2 - \frac{z_\alpha^2}{n} \quad (18)$$

where $z_p = \Phi^{-1}(p)$ and $z_\alpha = -\Phi^{-1}(\alpha)$.

This approach is widely used in on-site estimate of concrete strength [see, for example, ACI 228.2R (ACI 2013)] or anchors strength [see, for example, ACI 355.3R (ACI 2011a)] or even concrete core strength [ACI 214R (ACI 201b)].

Welch Approximations

A better approximation, especially for low values of n, is that given by Jennett and Welch (1939) by means of the quantiles of the noncentral t-distribution

$$k_{\alpha,n-1,p} \approx k_{JW1,\alpha,n-1,p} = \frac{1}{\sqrt{n-1}}\frac{\delta b_n + z_\alpha\sqrt{b_n^2 + (1-b_n^2)(\delta^2 - z_\alpha^2)}}{b_n^2 - z_\alpha^2(1-b_n^2)} \quad (19)$$

where

$$b_n = \frac{\Gamma(\frac{n}{2})}{\Gamma(\frac{n-1}{2})}\sqrt{\frac{2}{n-1}} \quad (20)$$

A simpler expression was developed by Johnson and Welch (1940) by assuming $b_n \approx 1$ and $1 - b_n^2 \approx \frac{1}{2(n-1)}$, as follows:

$$k_{\alpha,n-1,p} \approx k_{JW2,\alpha,n-1,p} = \frac{1}{\sqrt{n-1}}\frac{\delta + z_\alpha\sqrt{1 + \frac{1}{2(n-1)}(\delta^2 - z_\alpha^2)}}{1 - \frac{z_\alpha^2}{2(n-1)}} \quad (21)$$

which yields slightly lower estimates, especially for $n < 30$.

However, the latter approximation does not give accurate results for $\alpha = 0.5$.

Two Proposed Approximations

By considering that Eq. (20) can be approximated as

$$b_n \approx 1 - \frac{1}{4n-5} \quad (22)$$

the following expression for calculating the tolerance factor is here proposed (where MP stands for Monti-Petrone):

$$k_{\alpha,n-1,p} \approx k_{MP1,\alpha,n-1,p} = \frac{z_p}{A}\frac{4n-6}{4n-5} + z_\alpha\sqrt{\frac{1}{n-1} + \frac{1}{2(n-1)}\left(\frac{z_p}{A}\right)^2}$$
$$\text{with } A = 1 - \frac{1}{2(n-1)}(1 + z_\alpha^2) \quad (23)$$

Eq. (23) represents an as accurate yet simpler version of Eq. (16) that allows avoiding the computation of the Gamma function.

The proposed equation can be further simplified to

$$k_{\alpha,n-1,p} \approx k_{MP2,\alpha,n-1,p} = \frac{z_p}{A} + \sqrt{\frac{z_\alpha^2}{n-1}\left(1 + \frac{1}{2}\frac{z_p^2}{A^2}\right)} \quad (24)$$

which yields slightly higher estimates, especially for $n < 10$.

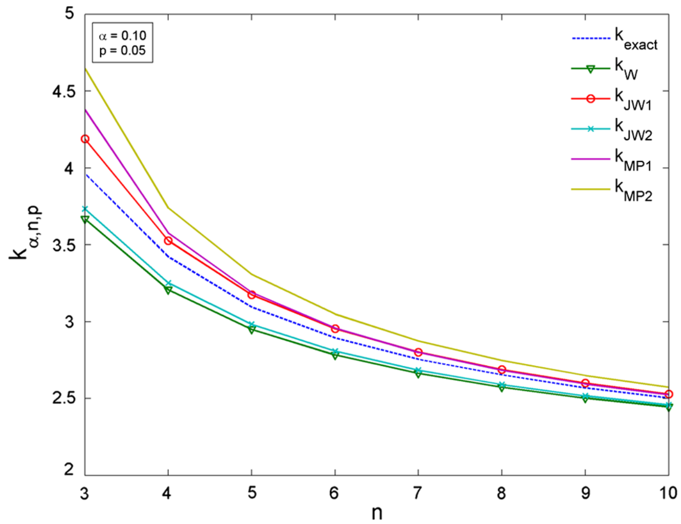

Fig. 1. Comparison of different approximations for the tolerance factor, for $p = 0.05$ and $\alpha = 0.10$ (90% confidence level) with $n = 3, \ldots, 10$

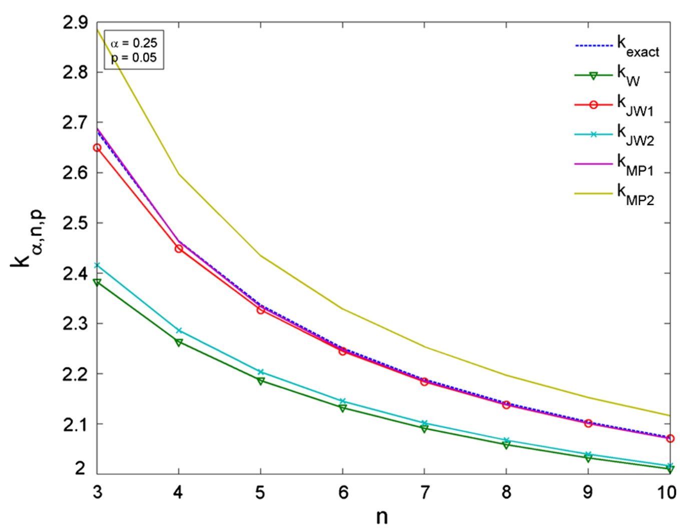

Fig. 2. Comparison of different approximations for the tolerance factor, for $p = 0.05$ and $\alpha = 0.25$ (75% confidence level) with $n = 3, \ldots, 10$

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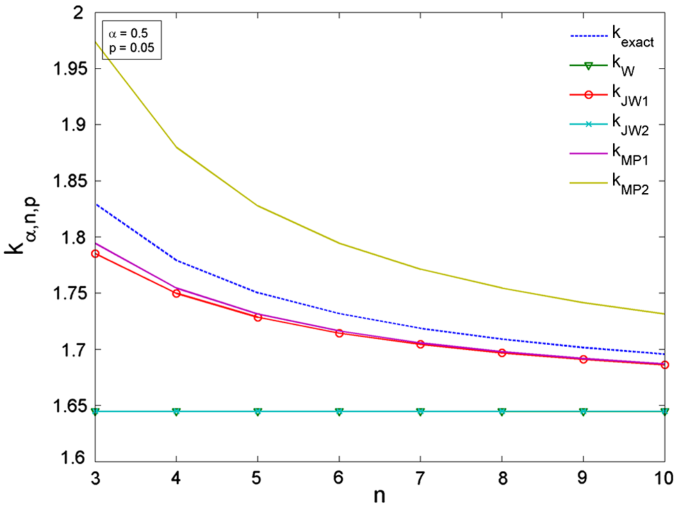

Fig. 3. Comparison of different approximations for the tolerance factor, for $p = 0.05$ and $\alpha = 0.50$ (median value) with $n = 3, \ldots, 10$

When n is large, say > 50, $A \approx 1$, and Eq. (24) simplifies to

$$k_{MP2,\alpha,n-1,p} = z_p + z_\alpha \sqrt{\frac{2 + z_p^2}{2(n-1)}} \tag{25}$$

Comparison among Approximations

The approximations proposed above are compared in Figs. 1–3, where, for the purpose of demonstration, the tolerance factor for $p = 0.05$ [the so-called characteristic value in EN 1990 (CEN 2002)] is sought. Since all approximations tend asymptotically to the exact solution, the comparison is only shown for $n = 3, \ldots, 10$, which refers to the condition of operating on a small-size database. The comparison is carried out between the exact solution found by solving Eq. (16) and the approximate solutions of Eqs. (18), (19), (21), (23), and (24).

Fig. 1 shows the comparison for $\alpha = 0.10$ (90% confidence level). It can be seen that Wallis (1947) considerably underestimates the tolerance bound by an amount comparable to Johnson and Welch (1940), while the second proposed approximation in Eq. (24), being simpler in its formulation, yields an overestimate. The approximation by Jennet and Welch (1939) is very satisfactory, such as the first proposed approximation in Eq. (23), though it slightly overestimates the tolerance factor for $N < 5$.

Fig. 2 shows the comparison for $\alpha = 0.25$ (75% confidence level). It can be seen that Wallis (1947) underestimates considerably the tolerance bound by an amount comparable to Johnson and Welch (1940), while the second proposed approximation in Eq. (24) again yields an overestimate. Both approximations by Jennet and Welch (1939) and the first proposed in Eq. (23) practically coincide with the exact solution, with the latter having the advantage of being much simpler.

Fig. 3 shows the comparison for a confidence level $\alpha = 0.50$ (median value). It can be seen that both Wallis (1947) and Johnson and Welch (1940) yield a constant value, thus failing to account for the variation with the number of data. The second proposed approximation in Eq. (24) overestimates the factor. Again, both the approximations by Jennet and Welch (1939) and the first proposed approximation in Eq. (23) are closer to the exact solution.

Overall, it can be concluded that both Eqs. (23) and (24) provide a better approximation than the equations available in the literature, and especially the former can be safely adopted in Eq. (14) for the calibration of the safety factor. Its simplified expression in Eq. (25) is recommended in the case of large databases.

Summary of the Calibration Procedure

The entire calibration procedure is here summarized in five steps, recalling the main equations to be used:

Step 1: Consider the entire database with N data. Predict each test result Y_i in the database through the capacity model, by plugging in the equation $C_i = C(\boldsymbol{X}_i)$ the values $\boldsymbol{X}_i$ given in the test, which already include the error $\in_{X_i}$. Therefore, N data pairs (C_i, Y_i) are obtained.

Step 2: Check the property of linearity of the test-prediction pairs, (C_i, Y_i). Compute the errors in Eq. (4) and check the properties of normality and homoscedasticity. If these properties (LNH) are not satisfied, treat the data by appropriate nonlinear transformations and/or by removing outliers and/or by performing linear regression to have $\hat{\in} = N(0, \sigma^2)$. The data pairs become $(\tilde{C}_i, \tilde{Y}_i)$. Repeat Step 2 until LNH are satisfied.

Step 3: Collect the coefficients $\hat{\beta}_1$ and $\hat{\beta}_0$ from Eqs. (7) and (8), and the error variance $\hat{\sigma}^2$ from Eq. (10). The latter represents the model accuracy with respect to the database considered.

Step 4: Compute the tolerance factor $k_{\alpha,N-1,p}$ for the desired values of α and p, with Eq. (23) or Eq. (24), [or Eq. (25) in the case of a large database].

Step 5: Calculate the design value of the capacity model $\bar{C}_d$ as per Eq. (11) and the design value as given by the relevant code $C_{d,\text{code}}$ as per Eq. (13), and finally compute the safety factor from Eq. (14).

Application of the Calibration Procedure

The calibration procedure is here applied to the ACI 318 (ACI 2014) design capacity equation, relative to the shear strength V_c of reinforced concrete (RC) members unreinforced in shear and subjected to shear and flexure only, given as

$$V_c = \phi \cdot 0.166\sqrt{f_c'}b_w d \tag{26}$$

where $\phi = 0.75$ = strength reduction factor; f_c' = specified compressive strength of concrete (in MPa); and $b_w \cdot d$ = cross sectional area (in mm^2). Therefore, the vector of the RVs takes on the form $\boldsymbol{X} = [f_c' \quad b_w \quad d]^T$.

A reference database of 216 experimental tests conducted by several authors and collected by Reineck et al. (2003) was considered. The appendix lists all the tests with the corresponding values of X and Y.

Step 1: The entire database with $N_{\text{tot}} = 216$ data is considered. Each test result Y_i in the database, as listed in the appendix, is predicted by means of the capacity model in Eq. (26), using the values of the RVs $\boldsymbol{X}_i$ given in the test (all safety factors are set equal to 1 as per U.S. provisions). Then, nine pairs (C_i, Y_i) showing an absolute error ratio ($|1 - C_i/Y_i|$) larger than 75% are detected, which might reveal some macroscopic error either in the test setup or in the measurement of input and/or output quantities. Keeping these values might alter the calibration procedure and, since in number statistically irrelevant, they are removed thus reducing the size of the database to $N = 207$. The resulting scatter plot (C_i, Y_i) is shown in Fig. 4 along with a logarithmic regression fitting the data.

Step 2: The errors are computed from Eq. (4). The error scatter plot $(C_i, \in_i)$ is shown in Fig. 5, while the error normality plot is shown in Fig. 6. Looking at the figures, it is clearly seen that the properties of linearity and normality are not satisfied. However, the relationship between tests and predictions appears to follow

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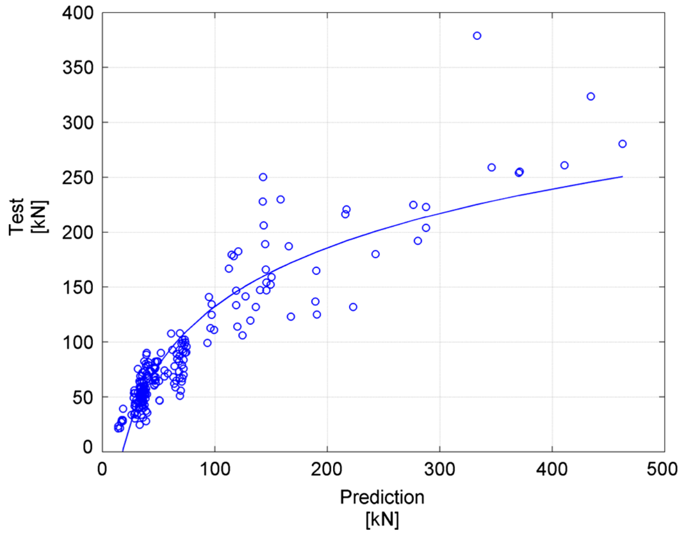

Fig. 4. Scatter plot of the pairs (C_i, Y_i) (original database)

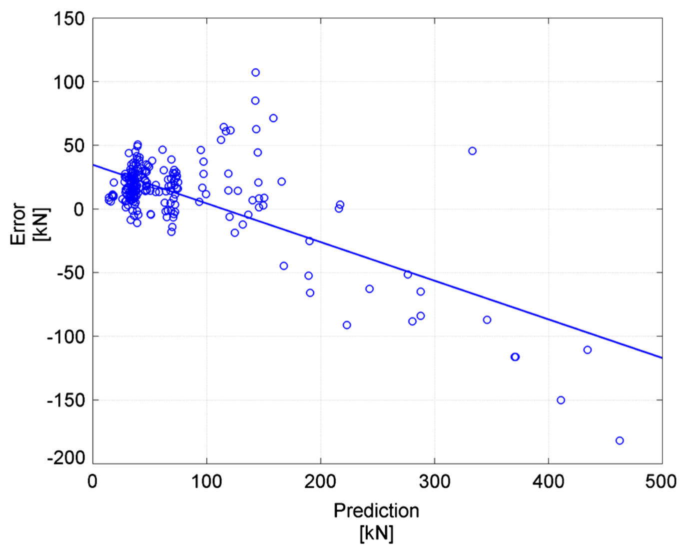

Fig. 5. Scatter plot of the error (original database)

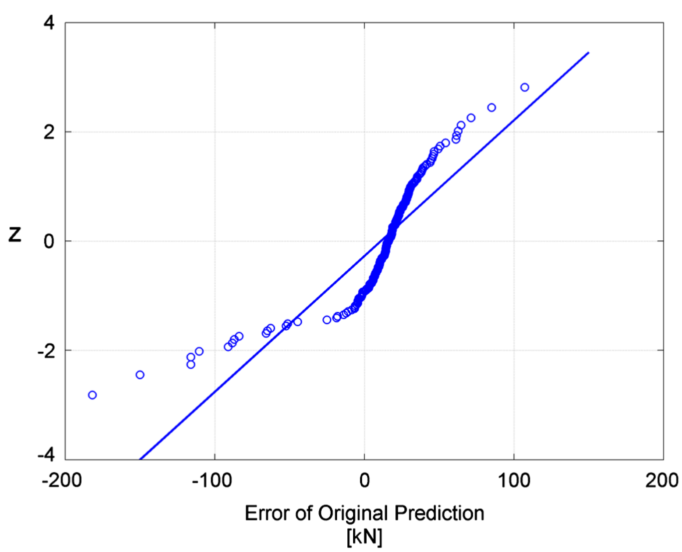

Fig. 6. Normality plot of the error (original database)

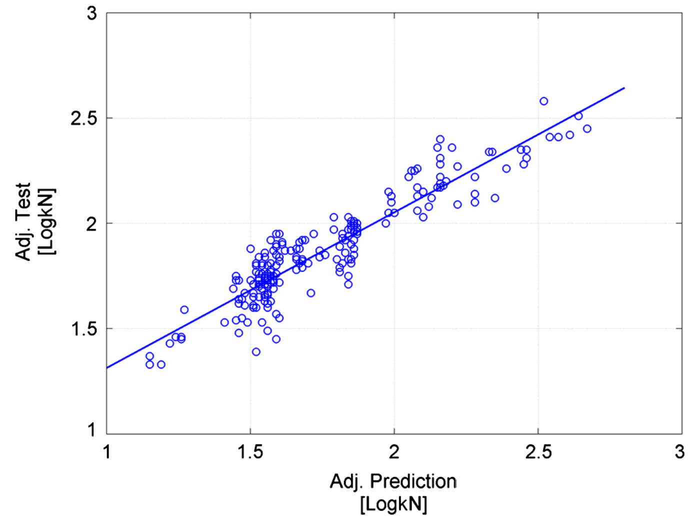

Fig. 7. Scatter plot of the pairs $(\tilde{C}_i, \tilde{Y}_i) \equiv (\log C_i, \log Y_i)$ (adjusted database)

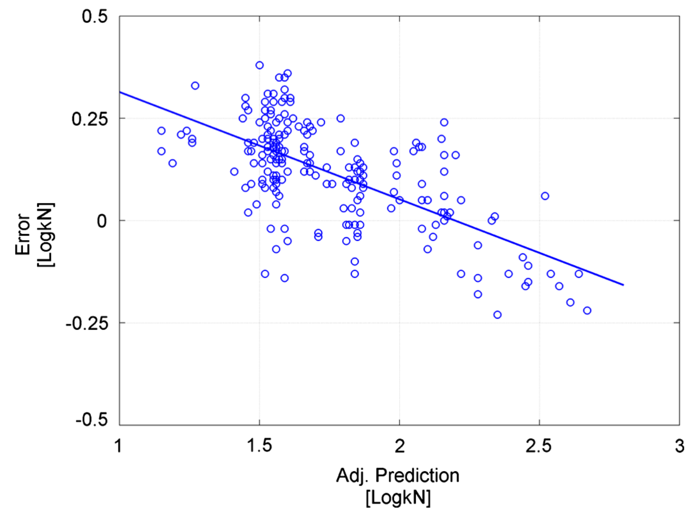

Fig. 8. Scatter plot of the error (adjusted database)

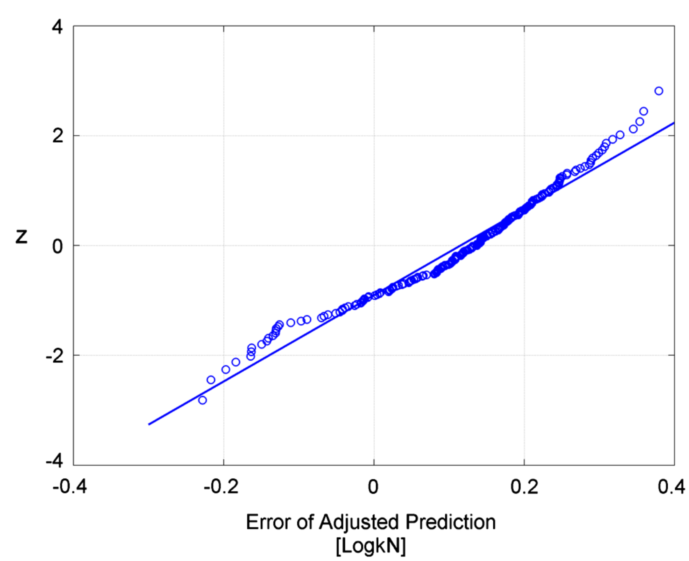

Fig. 9. Normality plot of the error (adjusted database)

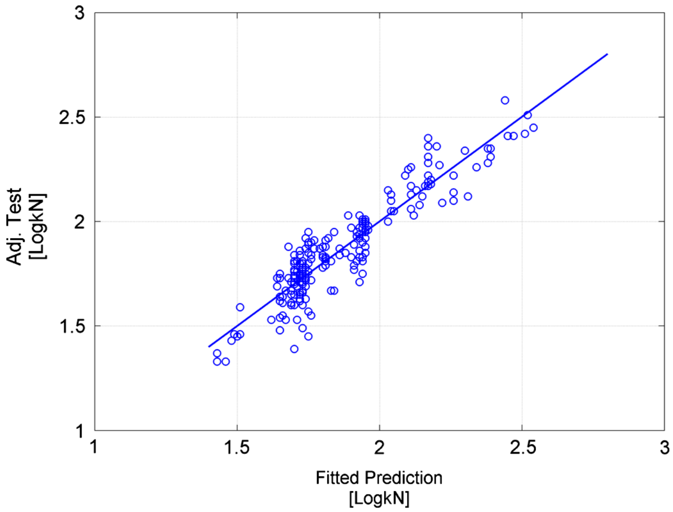

Fig. 10. Scatter plot of the pairs $(\hat{\tilde{C}}_i, \tilde{Y}_i)$ (fitted database)

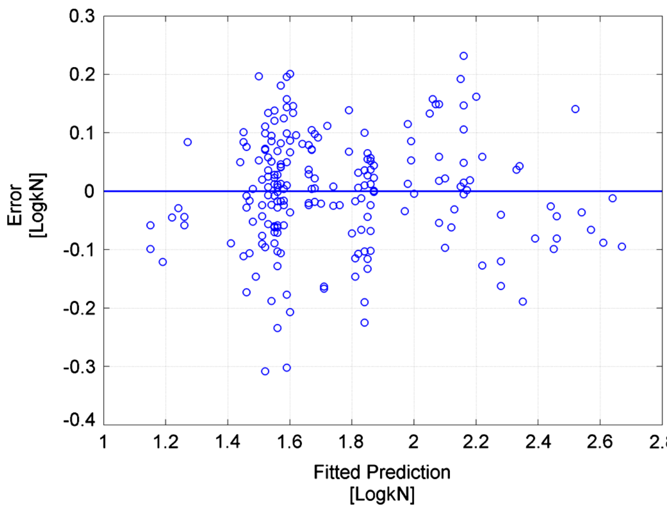

Fig. 11. Scatter plot of the error (fitted database)

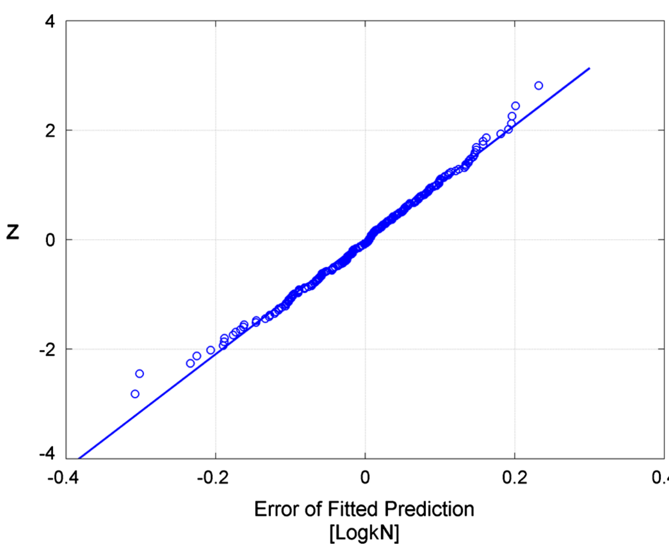

Fig. 12. Normality plot of the error (fitted database)

a logarithmic trend (solid line in Fig. 4). Then the data points are treated with a logarithmic transformation so that $(\tilde{C}_i, \tilde{Y}_i) = f(C_i, Y_i) = (\log C_i, \log Y_i)$, where log is the natural logarithm. The scatter plot of the $(\tilde{C}_i, \tilde{Y}_i)$ pairs, that of the error, and the normality plot of the adjusted database are shown in Figs. 7–9, respectively, where it can be seen that both the linearity and the normality properties of the scatter plot have significantly improved. Finally, since the error still shows a nonzero mean, a linear regression is performed.

Figs. 10–12 show the effects of this further transformation $(\hat{\tilde{C}}_i = \hat{\beta}_1 \tilde{C}_i + \hat{\beta}_0)$, as the error is now actually a normal RV with zero mean.

The only check left is to verify whether the error is homoscedastic. As mentioned above, this can be verified in the mean sense by observing that the mean of the squared error, see solid trend line in Fig. 13, is approximately horizontal throughout the database.

Step 3: The regression coefficients are $\hat{\beta}_1 = S_{\tilde{C}\tilde{Y}}/S_{\tilde{C}\tilde{C}} = 0.7384$ and $\hat{\beta}_0 = \bar{\tilde{Y}} - \hat{\beta}_1 \bar{\tilde{C}} = 0.5750$ obtained from Eqs. (7) and (8), respectively, and the variance $\hat{\sigma}^2$ is obtained from Eq. (10) as follows:

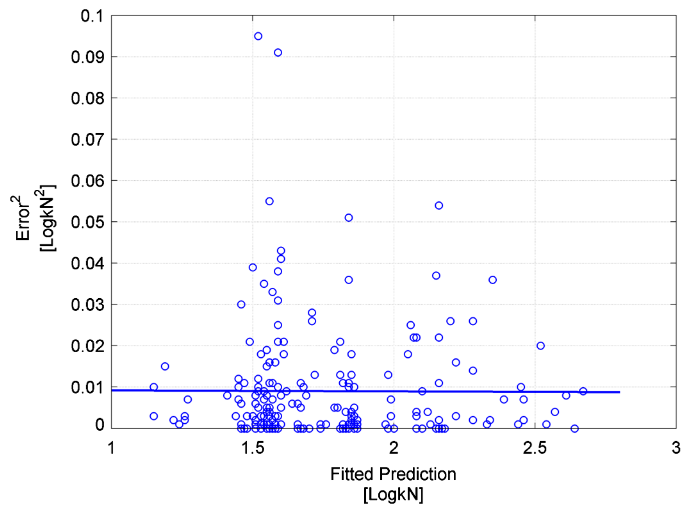

Fig. 13. Scatter plot of the squared error (fitted database)

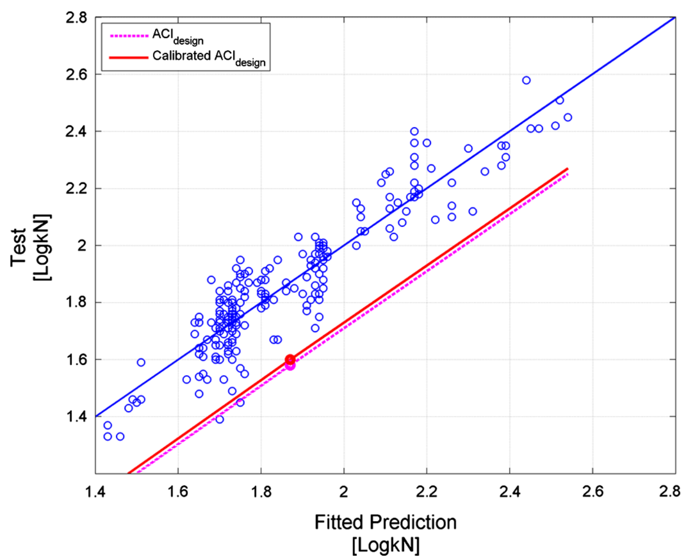

Fig. 14. Scatter plot of the pairs $(\hat{\tilde{C}}_i, \tilde{Y}_i)$ (fitted database) showing the design lines obtained with ACI 318 (ACI 2014) (solid line) and with the proposed procedure (dashed line)

$$\hat{\sigma}^2 = \frac{1}{N-2}\sum_{i=1}^{N}(\tilde{Y}_i - \hat{\tilde{C}}_i)^2 = 0.00911 \quad (27)$$

which is practically coincident with the value of the intercept of the regression line equation shown in Fig. 13. This can be regarded as a relaxed proof of homoscedasticity.

Step 4: The tolerance factor $k_{\alpha,N-1,p}$ is computed with Eq. (25) as the database is sufficiently large ($N = 207$). A confidence level of 75% ($\alpha = 0.25$) is assumed as suggested in EN 1990 (CEN 2002), from which $z_\alpha = 0.674$. For the exceedance probability $p = \Phi(-\alpha_C \beta_{LS})$ is considered, from which $z_p = \alpha_C \beta_{LS} = 0.8 \cdot 3.5 = 2.8$, so that

$$k_{MP2,0.25,207,p} = z_p + z_\alpha \sqrt{\frac{2 + z_p^2}{2(N-1)}} = 2.869 \quad (28)$$

Step 5: Finally, the safety factor is computed from Eq. (14) as follows:

$$\gamma_{\text{cal}} = \frac{\bar{C}_{d,\text{code}}}{\bar{C}_d} = \frac{\exp[\phi \frac{1}{207}\sum_{i=1}^{207}\tilde{C}(\boldsymbol{X}_i - k_{0.09}\sigma_{\boldsymbol{X}_i})]}{\exp(\frac{1}{207}\sum_{i=1}^{207}\hat{\tilde{C}} - k_{MP2,0.25,207,p}\,\hat{\tilde{\sigma}})} = 0.98 \quad (29)$$

where $\exp(\cdot) \equiv f^{-1}[\log(\cdot)]$, and where, again, $\tilde{C}(\cdot) \equiv \log C(\cdot)$. The standard deviation $\sigma_{\boldsymbol{X}}$ has been taken equal to zero for the geometry RVs, while for the concrete strength it has been assumed as $\sigma_{f_c} = 8/k_{0.05} = 8/1.645 = 4.86$ MPa, in agreement with EN 1992 (CEN 2005), so that $\sigma_{\boldsymbol{X}} = [4.86 \quad 0 \quad 0]^T$. The safety factor so obtained is very close to the unit and shows that the ACI 318 (ACI 2014) design equation for the shear capacity meets the reliability requirement of the U.S. code.

In the scatter plot of Fig. 14, relative to the pairs $(\hat{\tilde{C}}_i, \tilde{Y}_i)$, two lines are reported, which represent the design lines obtained with ACI 318 (ACI 2014) (dashed line) and with the proposed procedure (solid line). The two lines are very close, and it can also be observed that only two points fall in the unsafe zone below the design lines. These are the two outliers in Figs. 12 and 13 showing the highest $\hat{\tilde{\epsilon}}_i = \tilde{Y}_i - \hat{\tilde{C}}_i$.

Conclusions

This work presents a simple procedure to calibrate the partial safety factor of capacity models on the basis of experimental tests, even if they are available in limited number. The procedure aims to obtain a relationship between test results and predictions, which must fulfill the property of linearity with the error being normal and homoscedastic throughout the database. This relationship can be established through nonlinear transformation, trimming, and regression-fitting of the test-prediction database and is used to find the design value corresponding to the exceedance probability targeted in the code. The safety factor is then found by comparing such design value with the corresponding design value obtained with the capacity model provided by the code.

As a byproduct, the procedure yields an objective measure of the model accuracy with respect to the database considered, given by the error standard deviation. This should always be computed and made available by authors when validating models on experimental databases, so that it could be used as an objective measure of accuracy when comparing different models on the same database.

For the case of a limited number of tests, two approximate analytical equations for calculating the tolerance factors are proposed, which can be used to compute any fractile of a random variable with a given confidence level. The accuracy of the proposed tolerance factors is verified and proved by comparing them with the exact solution and with other formulas currently available in the literature.

The proposed methodology has been applied to the shear capacity equation of RC beams without shear reinforcement provided by ACI 318 (ACI 2014). The calibration, carried out on 216 experimental tests collected from the literature, demonstrated that that equation fulfills the reliability requirements of ACI and, above all, that the proposed procedure is of straightforward applicability.

Appendix. Database of Experimental Tests Considered for the Application of the Calibration Procedure

Reference authors	Beam name	b_w (mm)	d (mm)	f_{1c} (MPa)	V_{exp} (kN)
Ahmad and Kahloo (1986)	C2	127	184	62.9	75.6
Ahmad and Kahloo (1986)	A2	127	203	59.3	68.9
Ahmad and Kahloo (1986)	B2	127	202	65.5	69.1
Ahmad and Kahloo (1986)	A1	127	203	59.3	57.8
Ahmad and Kahloo (1986)	C1	127	184	62.9	54.3
Ahmad and Kahloo (1986)	B1	127	202	65.5	51.3
Ahmad and Kahloo (1986)	A8	127	208	59.3	48.9
Ahmad and Kahloo (1986)	B8	127	208	65.5	46.7
Ahmad and Kahloo (1986)	C7	127	207	62.9	45.5
Ahmad and Kahloo (1986)	B7	127	208	65.5	46.3
Ahmad and Kahloo (1986)	C8	127	207	62.9	44.6
Aster and Koch (1974)	3	1,000	250	26	220.7
Aster and Koch (1974)	2	1,000	250	25.7	216.3
Aster and Koch (1974)	12	1,000	500	26	323.7
Aster and Koch (1974)	10	1,000	500	19	255.1
Aster and Koch (1974)	9	1,000	500	18.9	254.1
Aster and Koch (1974)	16	1,000	750	28.8	392.4
Aster and Koch (1974)	11	1,000	500	23.3	261
Aster and Koch (1974)	8	1,000	500	29.5	280.6
Aster and Koch (1974)	17	1,000	750	27.3	349.2

Appendix (*Continued.*)

Reference authors	Beam name	b_w (mm)	d (mm)	f_{1c} (MPa)	V_{exp} (kN)
Bhal (1968)	B1	240	300	22.5	71.5
Bhal (1968)	B3	240	900	26.7	165
Bhal (1968)	B6	240	600	24	114
Bhal (1968)	B2	240	600	28.8	119.5
Bhal (1968)	B4	240	1,200	24.5	180
Bhal (1968)	B5	240	600	25.8	106
Bhal (1968)	B7	240	900	26.5	137
Bhal (1968)	B8	240	900	26.9	125
Bresler and Scordelis (1963)	0A-2	305	466	23.3	178
Bresler and Scordelis (1963)	0A-1	310	461	21.4	166.9
Bresler and Scordelis (1963)	0A-3	307	462	35.9	189.1
Chana (1981)	2.3	203	356	33.9	99.4
Chana (1981)	2.1	203	356	37	96
Chana (1981)	2.2	203	356	31.2	87.4
Collins and Kuchma (1997)	B100	300	925	34.2	225
Collins and Kuchma (1997)	B100L	300	925	37.1	223
Collins and Kuchma (1997)	B100B	300	925	37.1	204
Collins and Kuchma (1997)	B100H	300	925	93.2	193
Cossio and Siess (1960)	A-12	152	254	25.4	59
Cossio and Siess (1960)	A-14	152	254	26.1	54.7
Cossio and Siess (1960)	A-13	152	254	21	46.9
Cossio and Siess (1960)	A2	152	254	29.9	41.8
Cossio and Siess (1960)	A3	152	254	18.5	34.3
Elzanaty et al. (1986)	F12	178	269	19.7	54.6
Elzanaty et al. (1986)	F10	178	268	62.2	78
Elzanaty et al. (1986)	F14	178	269	38	64.6
Elzanaty et al. (1986)	F15	178	268	60.3	68
Elzanaty et al. (1986)	F11	178	272	19.7	45.5
Elzanaty et al. (1986)	F2	178	270	75.3	67
Elzanaty et al. (1986)	F6	178	267	62.2	62
Elzanaty et al. (1986)	F9	178	268	75.3	64
Elzanaty et al. (1986)	F1	178	272	62.2	58.6
Elzanaty et al. (1986)	F13	178	272	38	47
Elzanaty et al. (1986)	F8	178	273	38	46.7
Feldman and Siess (1955)	L-2A	152	252	34.9	80.1
Feldman and Siess (1955)	L-3	152	252	26.6	53.4
Feldman and Siess (1955)	L-4	152	252	24.5	51.2
Feldman and Siess (1955)	L-5	152	252	26.5	51.2
Grimm (1997)	s2.4	300	328	89.4	229.8
Grimm (1997)	s3.4	300	690	89.4	379
Grimm (1997)	s2.2	300	348	86.7	187.1
Grimm (1997)	s1.3	300	146	89	98.6
Grimm (1997)	s3.2	300	718	89	259.1
Grimm (1997)	s1.2	300	152	86.6	75.8
Grimm (1997)	s1.1	300	153	85.6	70.1
Grimm (1997)	s2.3	300	348	89	123.1
Grimm (1997)	s3.3	300	746	89.7	192.8
Hallgren (1994)	B91SD2-4-61	156	195	57.8	90
Hallgren (1994)	B91SD1-4-61	156	194	57.8	88.5
Hallgren (1994)	B91SD6-4-58	150	196	55.4	82.5
Hallgren (1994)	B91SD3-4-66	156	195	62.4	81.5
Hallgren (1994)	B91SD5-4-58	156	196	55.4	78
Hallgren (1994)	B91SD4-4-66	155	195	62.4	79
Hallgren (1994)	B90SB13-2-86	163	192	81.9	82.5
Hallgren (1994)	B91SC1-2-62	156	193	58.7	71
Hallgren (1994)	B90SB5-2-33	156	191	31.2	56
Hallgren (1994)	B90SB18-2-45	155	194	42.7	63
Hallgren (1994)	B91SC4-2-69	156	195	65.6	74
Hallgren (1994)	B91SC2-2-62	155	196	58.7	69.5
Hallgren (1994)	B90SB10-2-31	157	193	29.5	53.5
Hallgren (1994)	B90SB14-2-86	158	194	81.9	76.5
Hallgren (1994)	B90SB6-2-33	156	194	31.2	53.5
Hallgren (1994)	B90SB22-2-85	158	193	80.4	75.5
Hallgren (1994)	B90SB17-2-45	157	191	42.7	59
Hallgren (1994)	B90SB9-2-31	156	192	29.5	49
Hallgren (1994)	B90SB21-2-85	155	194	80.4	69
Hamadi and Regan (1980)	G2	100	372	22.3	41

Appendix (*Continued.*)

Reference authors	Beam name	b_w (mm)	d (mm)	f_{1c} (MPa)	V_{exp} (kN)
Hamadi and Regan (1980)	G1	100	370	28.8	44.5
Hamadi and Regan (1980)	G4	100	372	20.9	30.3
Hanson (1961)	B2	152	267	29.3	52.4
Hanson (1961)	B4	152	267	29.4	42.8
Hanson (1961)	BW4	152	267	28.2	40
Hanson (1961)	A4	152	267	19.9	33.8
Kani (1967)	274	612	270	25.8	250.2
Kani (1967)	272	611	271	25.6	227.8
Kani (1967)	273	612	271	25.8	206.2
Kani (1967)	74/75	152	524	25.9	107.9
Kani (1967)	83	156	271	26.1	65
Kani (1967)	60	155	139	25.4	39.3
Kani (1967)	97	152	276	25.9	62.5
Kani (1967)	71	155	544	26	102.1
Kani (1967)	3,043	154	1,092	25.7	166
Kani (1967)	63	154	543	24.9	93.2
Kani (1967)	3,044	152	1,097	28	159.1
Kani (1967)	96	156	275	24	56.3
Kani (1967)	3,046	155	1,097	25.4	154.2
Kani (1967)	84	151	271	26.1	55.4
Kani (1967)	66	156	541	25.1	90.8
Kani (1967)	3,045	155	1,092	26.9	152.4
Kani (1967)	3,047	155	1,095	25.4	147.1
Kani (1967)	81	153	272	26.1	51.2
Kani (1967)	91	154	269	26.1	51
Kani (1967)	79	153	556	24.8	83.7
Kani (1967)	52	152	138	23.6	28.9
Kani (1967)	64	156	540	24.4	79
Kani (1967)	58	152	138	25.9	28.9
Kani (1967)	48	152	133	23.5	27.1
Kani (1967)	92	152	270	26.1	45.9
Kani (1967)	56	153	137	25.9	28
Krefeld and Thurston (1966)	18B2	152	316	18.9	72.1
Krefeld and Thurston (1966)	18C2	152	316	21.5	73.4
Krefeld and Thurston (1966)	12A2	152	238	28.6	64.1
Krefeld and Thurston (1966)	18A2	152	316	18.3	63.2
Krefeld and Thurston (1966)	11A2	152	314	28.7	73.4
Krefeld and Thurston (1966)	OCA	254	456	36.4	146.9
Krefeld and Thurston (1966)	6CC	152	250	36.5	63.2
Krefeld and Thurston (1966)	4AAC	152	254	27.7	57.9
Krefeld, andThurston (1966)	6AAC	152	250	32.7	60.1
Krefeld and Thurston (1966)	18D2	152	316	21	60.1
Krefeld and Thurston (1966)	6AC	152	250	32.4	59.2
Krefeld and Thurston (1966)	OCB	254	456	36.4	133.5
Krefeld and Thurston (1966)	5AAC	152	252	31.2	57
Krefeld and Thurston (1966)	4AC	152	254	29	53.8
Krefeld and Thurston (1966)	C	203	483	15.9	84.6
Krefeld and Thurston (1966)	5CC	152	252	35.6	57.4
Krefeld and Thurston (1966)	5AC	152	252	31.2	54.3
Krefeld and Thurston (1966)	3AAC	152	256	32.8	55.6
Krefeld and Thurston (1966)	3AC	152	256	30.3	53.4
Krefeld and Thurston (1966)	17A2	152	243	20.9	44.1
Krefeld and Thurston (1966)	3AC	152	256	19.8	44.1
Krefeld and Thurston (1966)	OCB	152	254	33.9	52.5
Krefeld and Thurston (1966)	16A2	152	240	21.1	41.8
Krefeld and Thurston (1966)	4CC	152	254	36.5	52.5
Krefeld and Thurston (1966)	OCA	152	254	33.9	48.5
Krefeld and Thurston (1966)	3CC	152	256	19.5	35.6
Laupa and Siess (1953)	S4	152	263	29.3	55.6
Laupa and Siess (1953)	S13	152	262	24.9	49.8
Laupa and Siess (1953)	S3	152	265	30.7	53.1
Laupa and Siess (1953)	S5	152	262	28.4	49.8
Laupa and Siess (1953)	S2	152	269	25.6	42.5
Laupa and Siess (1953)	S11	152	267	14	33.8
Leonhardt and Walther (1962)	P9	500	146	24.2	107.8
Leonhardt and Walther (1962)	5r	190	270	28	76.5
Leonhardt and Walther (1962)	P8	502	148	24.2	92.8

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Appendix (*Continued.*)

Reference authors	Beam name	b_w (mm)	d (mm)	f_{1c} (MPa)	V_{exp} (kN)
Leonhardt and Walther (1962)	6r	190	270	28	68.2
Leonhardt and Walther (1962)	7–2	190	270	29.4	68.2
Leonhardt and Walther (1962)	C4	225	600	37.2	147.2
Leonhardt and Walther (1962)	D4/l	200	280	33.6	74.1
Leonhardt and Walther (1962)	D4/2l	200	280	33.6	74.1
Leonhardt and Walther (1962)	8–1	190	270	29.4	65.7
Leonhardt and Walther (1962)	8–2	190	270	29.4	65.7
Leonhardt and Walther (1962)	C2	150	300	37.2	64.8
Leonhardt and Walther (1962)	7–1	190	270	29.4	62.3
Leonhardt and Walther (1962)	61	190	270	28	60.8
Leonhardt and Walther (1962)	D4/2r	200	280	33.6	68.7
Leonhardt and Walther (1962)	51	190	270	28	60.3
Leonhardt and Walther (1962)	D3/l	150	210	36.6	46.4
Leonhardt and Walther (1962)	C3	200	450	37.2	99.1
Leonhardt and Walther (1962)	D3/2r	150	210	36.6	44.5
Leonhardt and Walther (1962)	D2/2	100	140	35.4	23.3
Leonhardt and Walther (1962)	D3/2l	150	210	36.6	41.2
Leonhardt and Walther (1962)	D2/l	100	140	35.4	21.2
Leonhardt and Walther (1962)	C1	100	150	37.2	21.6
Marti and Pralong Th limann (1977)	PS11	400	144	28.1	90
Mathey and Watstein (1963)	IIIa–17	203	403	27.8	90
Mathey and Watstein (1963)	IIIa–18	203	403	23.9	82.5
Mathey and Watstein (1963)	Va–20	203	403	24.3	67.4
Mathey and Watstein (1963)	Va–19	203	403	22.3	64.7
Mathey and Watstein (1963)	VIa–24	203	403	25	55.7
Mathey and Watstein (1963)	VIa–25	203	403	24.5	51
Morrow and Viest (1957)	B65A6	308	356	37.9	179.5
Morrow and Viest (1957)	B70A6	305	356	42.7	182.5
Morrow and Viest (1957)	B56A4	305	375	23.7	141.1
Morrow and Viest (1957)	B70A4	305	368	25.9	134.5
Morrow and Viest (1957)	B56B2	305	368	14	102.2
Morrow and Viest (1957)	B56B4	305	368	25.9	124.8
Morrow and Viest (1957)	B56B6	305	372	43.4	141.6
Morrow and Viest (1957)	B84B4	305	363	25.9	112.7
Morrow and Viest (1957)	B70B2	305	365	15.5	90.6
Morrow and Viest (1957)	B56E4	305	368	27	111
Mphonde and Frantz (1984)	AO–7–3a	152	298	36.9	82
Mphonde and Frantz (1984)	AO–3–3b	152	298	20.3	64.3
Mphonde and Frantz (1984)	AO–7–3b	152	298	40.7	82.4
Mphonde and Frantz (1984)	AO–3–3c	152	298	26.6	66.6
Mphonde and Frantz (1984)	AO–15–3b	152	298	91.7	99.7
Mphonde and Frantz (1984)	AO–15–3c	152	298	89.9	97.4
Mphonde and Frantz (1984)	AO–15–3a	152	298	79.6	92.9
Mphonde and Frantz (1984)	AO–11–3b	152	298	73	89.2
Mphonde and Frantz (1984)	AO–11–3a	152	298	73.3	89.2
Niwa and Yamada (1987)	2	600	2,000	24.9	382
Niwa and Yamada (1987)	1	600	2,000	25.8	402
Niwa and Yamada (1987)	3	300	1,000	23.4	102
Podgorniak-Stanik (1998)	BN50	300	450	35.2	131.9
Podgorniak-Stanik (1998)	BN25	300	225	35.2	73
Podgorniak-Stanik (1998)	BN100	300	925	35.2	192.1
Podgorniak-Stanik (1998)	BN12	300	110	35.2	40.1
Podgorniak-Stanik (1998)	BH50	300	450	94.1	131.9
Podgorniak-Stanik (1998)	BRL100	300	925	89.3	164.1
Podgorniak-Stanik (1998)	BH100	300	925	94.1	193.1
Rajagopalan and Ferguson (1968)	S–13	152	265	22.5	40
Rajagopalan and Ferguson (1968)	S–2	154	265	31.4	37.4
Rajagopalan and Ferguson (1968)	S–5	152	262	26.5	33.6
Rajagopalan and Ferguson (1968)	S–1	154	259	34.7	35.6
Rajagopalan and Ferguson (1968)	S–3	152	267	27.5	31.1
Rajagopalan and Ferguson (1968)	S–9	152	262	23.8	24.5
Rajagopalan and Ferguson (1968)	S–4	152	268	31.4	28

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