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Small-Strain Shear Modulus Model for Saturated and Unsaturated Soil

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ABSTRACT: The small-strain shear modulus depends on soil type, void ratio, stress level, and degree of saturation. Models have been proposed to describe G_0 of either saturated or unsaturated soils, yet none of them can predict G_0 for soils under both conditions. In this paper, a unified model for G_0 was proposed for soils based upon the consideration of effects of both material hardening/softening and inter-particle contact forces. These two mechanisms are characterized by the degree of saturation and effective stress, respectively, as the primary variables. Suction stress theory was used to determine the effective stress to for soil under both saturated and unsaturated conditions. It is shown that G_0 is correlated with the soil-water retention curve for unsaturated soils by using suction stress as the effective stress framework. The proposed model fit well for 17 soils, including those from the literature and new experiments on soils under different saturation conditions and various loading conditions. A correlation between the fitting parameters of the proposed model and the inverse of the air-entry suction was found to provide good predictions of the small-strain shear modulus for unsaturated soils under various loading conditions.

1. INTRODUCTION

The small-strain shear modulus as a fundamental property of soils in context of liquefaction evaluation, earthquake ground response analysis, and static and dynamic soil-structure interactions has been the subject of continued research for decades (Clayton 2011). The shear strain is usually less than 0.001%, which is within the range of particle vibration for elastic wave propagation. Below this strain limit, G_0 represents the stiffness of the soil skeleton where slippage at particle contacts is negligible and the soil fabric is unchanged. In this case, the soil exhibits an elastic

response where the deformation is governed by contact force and is reversible without loading and unloading hysteresis (Santamarina et al. 2001).

There are various of affecting factors of G_0 , and plenteous of studies have been conducted on those for dry or saturated soils, such as void ratio, over-consolidation ratio (OCR), stress-strain history, and strain rate on G_0 (Hardin and Richart 1963; Rampello et al. 1997; Viggiani and Atkinson 1995). For unsaturated soils, several efforts have also been spent to understand the effect of particle size, compaction energy, degree of saturation, matric suction, and hysteresis on G_0 for unsaturated soils (Cho and Santamarina 2001; Khosravi and McCartney 2012; Heitor et al. 2013; Oh et al. 2014). However, we still have not seen a universal model that generalizes the G_0 for soils at different stress levels under both saturated and unsaturated conditions.

Many measurement techniques for G_0 have been investigated both in the field and in the laboratory (Kurtulus and Stokoe 2008). Among them, the bender element method is a versatile and simple approach to determine G_0 for all soils (Lee and Santamarina 2005; Leong et al. 2005). Nevertheless, the current testing techniques of bringing soil samples to equilibrium under a specified stress state and matric suction for unsaturated soils are time-consuming, and the data in the literature are not complete due to the limited range of applied suction. With this consideration, the Drying Cake (DC) method of Lu and Kaya (2013) was employed in this study to dry soil specimens over a wide range of suction. The bender element method was incorporated to characterize the G_0 evolution with respect to degree of saturation of unsaturated soils. This paper presents results and data from the literature to verify a proposed unified model of G_0 for all saturated and unsaturated soils.

2. STRESS DEPENDENCY AND HARDENING/SOFTENING OF G_0

The pioneering micromechanical analyses of soil by Hardin and Richart (1963) suggest that the small-strain stiffness is stress dependent and can be fitted well by the general power-form equation in terms of effective stress. However, this model only applies for dry and fully saturated soils. To extend this principle to unsaturated soils, several investigators used the effective stress concept of Bishop (1959), to consider the effect of external loading stress and matric suction on the G_0 (Ng et al. 2009; Sawangsuriya et al. 2009; Khosravi and McCartney 2012; Heitor et al. 2013; Oh et al. 2014). For instance, Sawangsuriya et al. (2009) proposed a G_0 model to study the effect of saturation and matric suction on G_0 for limited types of soil. To avoid the difficulty of experimentally deriving Bishop's effective stress parameter χ , they adopted an empirical fitting equation based on the degree of saturation $\chi = S^k$ to evaluate the effective stress. However, this model only works well for a limited range of net confining stress. Oh et al. (2014) proposed a G_0 model considering the non-linear dependency of G_0 by using matric suction as the main variable multiplying another exponential term of saturation with a fitting parameter to capture the nonlinear behavior of G_0 . However, the two fitting parameters are competing against with each other and do not have a solid physical meaning. Further, this model only applies for non-plastic sandy soils under a limited confining stress.

Despite the fact that matric suction is an important state variable, and is inter-correlated with the net normal stress in describing the state of stress in unsaturated soils, matric suction itself does not directly generate the inter-particle stress acting on the contacts. External confining and matric suction are intrinsically coupled when both applied to unsaturated soils. Therefore, the correct interpretation of effective stress framework becomes the key to the stress-dependency analysis. For this reason, all previous models cannot uncouple the effect of water content or matric suction with stress level for unsaturated soils. Lu and Likos (2006) and Lu et al. (2010) proposed suction stress, which conceptualize different types of inter-particle forces at different soil water energy level. Through the inherent correlation between suction stress and soil water retention curves (SWRC) and the model of van Genuchten (1980), the mean effective stress can be calculated using the following equations:

$$\sigma' = (\sigma - u_a) - \sigma^s \quad (1)$$

$$\sigma^s = -S_e(u_a - u_w) = -\left(\frac{S_e}{\alpha}\right) \left[S_e^{n/(1-n)} - 1\right]^{1/n} \quad (2)$$

where σ is the mean total stress, u_a and u_w are the pore air and water pressures, respectively, σ^s is the suction stress, and S_e is effective degree of saturation. α and n are empirical SWRC fitting parameters, where α is the inverse of air-entry pressure [kPa⁻¹], and n is a dimensionless soil pore-size distribution parameter. S_e is used instead of S as S_e is directly used in the SWRC model of van Genuchten (1980).

Beside the stress dependency, shear modulus has been found to be related to material hardening/softening effect for unsaturated soils. Lu and Kaya (2014) developed a power law of elastic moduli (both Young's modulus and shear modulus) at finite-strain level (~1%) for unsaturated soils. In a multiphase soil with air, water and solids, each constituent of the mixture contributes to the modulus. At different water content, the stiffness is different. An empirical relationship was developed that assumes G_0 is proportional to a power of the degree of saturation as follows:

$$G'(S) \propto (1/S)^\beta \quad (3)$$

where β is an empirical fitting parameter. The proportionality to the inverse of the degree of saturation $1/S$ reflects a material softening mechanism as the degree of saturation increases. The relationship defined by Lu and Kaya (2014) was based on values of shear modulus that were lower than G_0 due to the finite-strain level in their tests (e.g., from <0.0001% to 1%). At larger strains there will be degradation associated with the redistribution of contact forces in the soil skeleton due to changes in soil texture and fabric during the loading to high strains. However, the degree of saturation is still expected to play the same role in G_0 at small strains.

Based on the above discussion and analysis of G_0 for unsaturated soils, we propose a new model for G_0 extended from the finite-strain power law and covering all soils under both saturated and unsaturated conditions, by using the suction stress framework for effective stress. For G_0 , the role of effective stress acting on soil skeleton was considered as a separate mechanism from the role of material softening/hardening due to changes in saturation. Figure 1(a) shows the schematic demonstration of inter particle forces in a representative elementary volume (REV).

Figure 1(b) shows the conceptual model consisting of two mechanisms: the material softening/hardening effect, demonstrated by the springs inside of the particles; and the contact force contribution represented by the springs at particle contacts. The stress-dependency will follow the form of Hardin's equation, but using suction stress to incorporate the effect of transferable external total stress and non-transferable inter-particle stress for unsaturated soils. This can be quantified as a similar proportionality of power law of effective stress normalized by atmospheric pressure:

$$G''(\sigma') \propto (\sigma'/P_{\text{atm}})^{\gamma_0} \quad (4)$$

where the mean effective stress σ' is normalized by the atmospheric pressure P_{atm} , and γ_0 is an empirical fitting parameter.

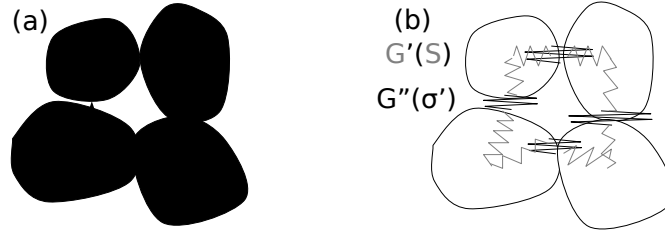


FIG 1. Schematic illustration of (a) soil matrix and inter-particle forces in unsaturated condition with external loading, and (b) conceptual model of small-strain shear modulus consisting of material softening and stress level.

Based on the conceptualized principles illustrated in Figure 1, G_0 can be determined by a combination of soil matrix component due to saturation and the contact behavior component due to the effective stress. In such a REV, the two components can be linked together in a multiplicative form:

$$G_0 = G'(S) \cdot G''(\sigma') = A_0 \left(\frac{1}{S} \right)^{\beta} \cdot \left(\frac{\sigma'}{P_{\text{atm}}} \right)^{\gamma_0} = G_0^{\text{sat}} \left(\frac{1}{S} \right)^{\beta} \cdot \left(\frac{\sigma'}{P_{\text{atm}}} \right)^{\gamma_0} \quad (5)$$

where A_0 is an empirical fitting parameter having the same units as the small-strain shear modulus (typically MPa). For saturated soil ($S = 1$) under a reference confining stress ($\sigma' = P_{\text{atm}}$), A_0 can be defined as the G_0 of saturated soil under a reference mean effective stress G_0^{sat} . The value of G_0^{sat} also reflects the effect of the initial void ratio. Thus for a saturated soil this value may also vary, depending on the initial conditions and the stress history. For saturated soils ($S = 1$) and under a constant mean effective stress σ' , the proposed equation reduces to the same form as Hardin's equation:

$$G_0 = G_0^{\text{sat}} \left(\frac{\sigma'}{P_{\text{atm}}} \right)^{\gamma_0} \quad (6)$$

where the mean effective stress reduces to Terzaghi's effective stress in saturated soil, (pore water pressure subtracted from total stress). Therefore, the proposed equation unifies the G_0 by extending soils from saturated condition to unsaturated conditions.

3. VALIDATION OF THE PROPOSED MODEL

Six types of soil were tested in this study. The sample preparation and drying technique are followed by Lu and Kaya (2013). Samples with 3-inch diameter and 0.75-inch thickness were compacted, and saturated by applying vacuum. The sensors and signal system follow the set up developed by Truong et al. (2011). All types of soils studies in this paper according to their sources is summarized in Table 1 based on their testing conditions, which includes 17 soils in total: 6 soils tested in this study using bender elements by air drying (soils No. 1-6); 5 soils from Sawangsurriya et al. (2009) (soil No. 7-11) by suction control; 2 soils from Ng et al. (2009) (soil No. 12-13) by suction control; 4 soils from Yoon et al. (2008) (soil No. 14-17) by triaxial. Table 1 lists the soil classifications and basic properties studied in the paper (porosity, void ratio), loading condition and applied net confining stress, the van Genuchten (1980) SWRC model parameters, and the fitting parameters for G_0 in the proposed unified model. The SWRCs of soils 1-17 were experimental obtained or adopted from the literature. The data sets from Ng et al. (2009) represent the behavior of the same soil, but following wetting and drying SWRC paths under different net normal stresses. The examined soils cover unsaturated soils (No. 1-13) and saturated soils (No. 14-17). Soil types include sandy soil, silty soil, and clayey soil. The loading conditions cover the net confining stress range of 0 to 300 kPa for unsaturated soils.

Table 1. Physical properties, SWRC parameters, and test conditions of soils

Soil No	Soil name and reference *	USCS	Porosity ϕ	Void ratio e	van Genuchten α n [kPa ⁻¹]		Test conditions	Net confining stress $\sigma_0 - u_a$ [kPa]
1	Bonny silt ^a	ML	0.51	1.0	0.06	1.54	Unsaturation	0
2	Hopi silt ^a	SC	0.36	0.5	0.03	1.77	Unsaturation	0
3	BALT silt ^a	ML	0.41	0.7	0.08	1.63	Unsaturation	0
4	Iowa silt ^a	ML	0.49	0.9	0.05	1.76	Unsaturation	0
5	Denver	CL	0.51	1.0	0.02	1.50	Unsaturation	0
6	Missouri clay ^a	-	0.41	0.6	0.05	1.37	Unsaturation	0
7	SC-Std-Opt ^b	SC	0.27	0.3	0.007	1.33	Unsaturation	35
8	ML-Std-Opt ^b	ML	0.39	0.5	0.028	1.32	Unsaturation	35
9	CL1-Std-Opt ^b	CL	0.38	0.6	0.004	1.30	Unsaturation	35
10	CL2-Std-Opt ^b	CL	0.33	0.4	0.005	1.27	Unsaturation	35
11	CH-Std-Opt ^b	CH	0.54	1.1	0.002	1.37	Unsaturation	35
12	dw-p110 ^c	ML	0.35	0.5	0.029	1.71	Unsaturation	110
13	dw-p300 ^c	ML	0.35	0.5	0.031	1.33	Unsaturation	300
14	TS-d ^d	SP	0.41	0.7	-	-	Saturated	50, 100, 200,
15	TS-l ^d	SP	0.44	0.8	-	-	Saturated	50, 100, 200,
16	SS-d ^d	SP	0.41	0.7	-	-	Saturated	50, 100, 200,
17	SS-l ^d	SP	0.43	0.7	-	-	Saturated	50, 100, 200,

Table 2. Fitting parameters and coefficient of correlation for unsaturated soils

Soil #	1	2	3	4	5	6	7	8	9	10	11	12	13
G_0^{sat}	45	40	159	43	51	119	117	40	66	125	21	46	46
β	0.99	1.13	0.47	0.73	1.01	1.25	3.08	1.58	3.58	4.03	4.32	0.10	0.31
γ_0	0.31	0.85	0.41	0.45	0.70	0.32	0.10	0.09	0.09	0.08	0.46	0.29	0.35
R^2	0.98	0.97	0.86	0.99	0.95	0.91	0.98	0.94	0.99	0.92	0.94	0.89	0.89

Table 3. Fitting parameters and coefficient of correlation for saturated soils

Soil #	14	15	16	17
G_0^{sat}	17	13	16	15
β	-	-	-	-

γ_0	0.41	0.44	0.42	0.42
R^2	0.99	0.99	0.99	0.99

The SWRCs of selected soils types with available experimental measurements are shown in Figure 2. In Figures 2(a-b), all data are used from the papers and the smooth curves are identified by fitting the parameters of the SWRC model of van Genuchten (1980). Figures 2(c-d) present the related suction stress characteristic curves (SSCC) calculated from the fitted SWRC parameters using Eq. 2. From the figures, suction stress increases significantly over the tested saturation range with 2~4 orders of magnitude as the soil dries. This suggests that even without external loading, there is a dramatic variation in effective stress for different unsaturated soils.

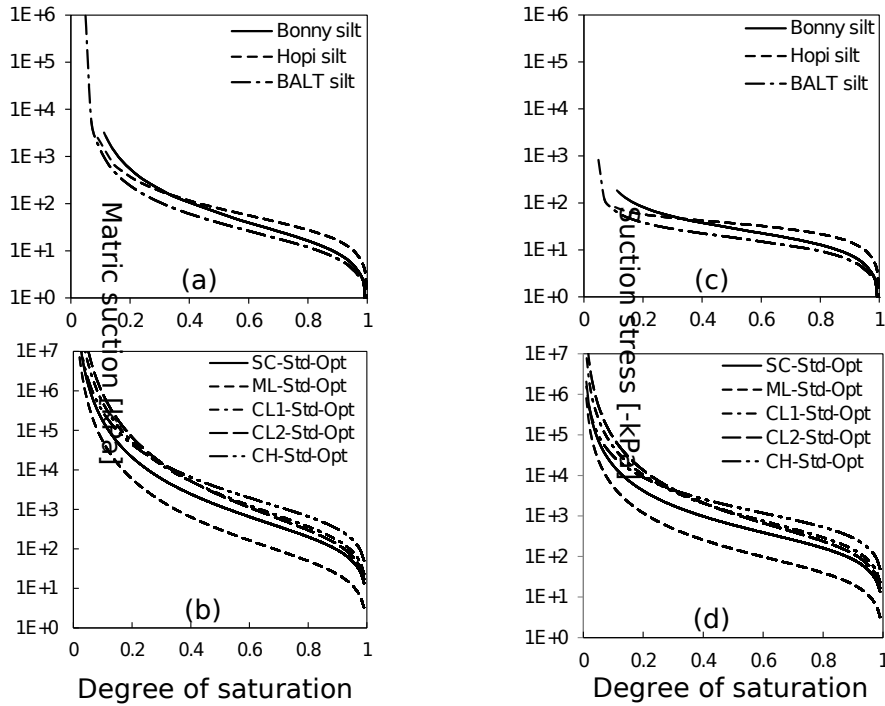


FIG 2. Soil water retention curves of all unsaturated soils tested (a-d), and their corresponding suction stress characteristic curves (e-h).

The least squares fitting technique was used for determination of the fitting parameters. The fitting results for different soils using the new model are presented in Figures 3 to 5. The G_0 measurements and fitting result for silts under unsaturated conditions with no external loading are shown in Figure 3. The G_0 evolution with degree of saturation, and the comparison between G_0 and mean effective stress for unsaturated soils with mean net normal confining stresses ranging from 35 to 300 kPa are shown in Figure 4. The fitting results of the new model and the experimental data for saturated sands under different external stress levels are shown in Figure 5. For all soils, G_0 increases with decreasing degree of saturation due to the material hardening effect. On the other hand, the shear modulus increases with the effective stress, due to the stress dependency effect. All the trends comply with the two mechanisms

described in the conceptual model. The general behavior of proposed unified model fits well with the experimental results. Through the comparison of categorized groups of soils under different loading conditions, the proposed model captures soil behavior well for both saturated and unsaturated states and under various loading conditions.

The fitting parameters and coefficient of correlation are listed in Tables 2 and 3. The value of G_0^{sat} varies from 21 to 160 MPa, depending on soil type and initial void ratio. Parameter β varies from 0.1 to 4.3. Parameter γ_0 varies from 0.09 to 0.85. All soils have R^2 value between 0.86 and 0.99, indicating that the proposed Eq. 5 can reasonably represent G_0 .

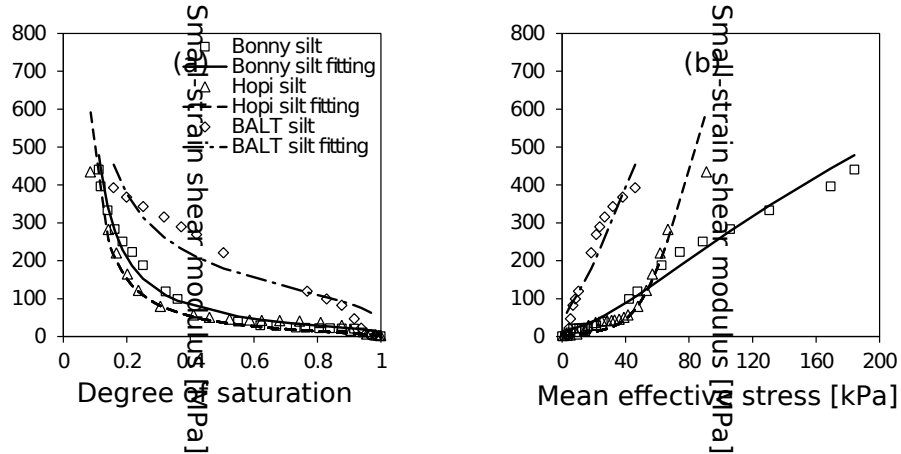


FIG 3. Small-strain shear modulus evolution with degree of saturation (a), and comparison between small-strain shear modulus with mean effective stress (b), for selected types of soil with no external loading.

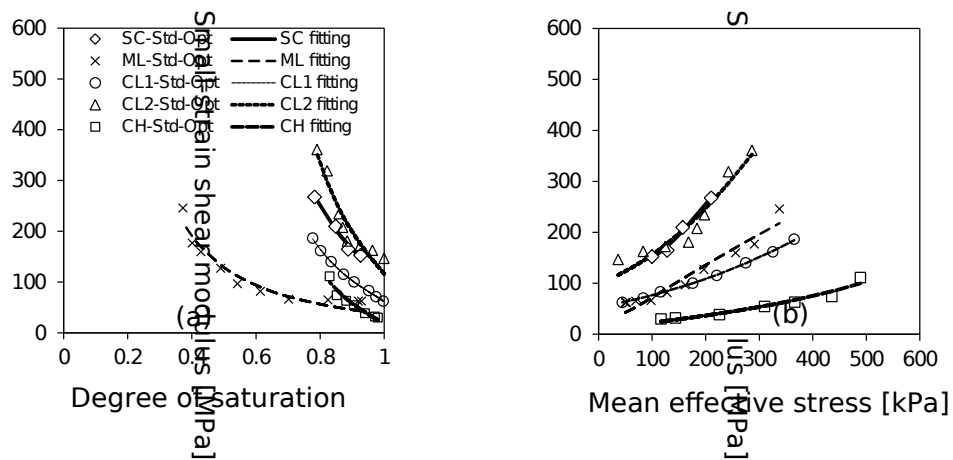


FIG 4. Small-strain shear modulus evolution with degree of saturation (a), and comparison between small-strain shear modulus with mean effective stress (b), for selected types of soil with external loading.

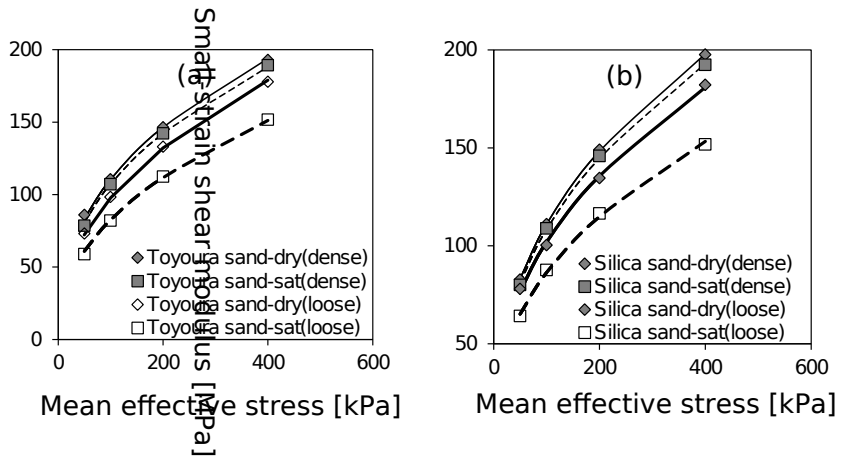


FIG 5. Comparison between small-strain shear modulus with mean effective stress (a-b), for saturated sands.

4. CORRELATIONS WITH SWRC PARAMETERS

The SSCC is intrinsically related to the SWRC. This suggests that the proposed G_0 model interpreted by using suction stress also has some correlation with soil water retention. In van Genuchten's SWRC model, the α parameter represents the air entry of the desaturation process, indicating the largest pore size and also reflect the effect of soil density. Thus, the correlation between fitting parameters of the proposed unified model and α was not clearly observed. Besides, the G_0^{sat} parameter already incorporates the effect of initial porosity or void ratio. At the same time, the stress power parameter γ_0 , reflects how strongly G_0 depends on the effective stress, which is essentially determined by the stiffness of the soil grains or minerals. The average value of γ_0 for the soils investigated in this study is around 0.5, which corresponds with the theoretical value expected from the Hertzian contact theory, and the range of parameters is consistent with those observed by Stokoe et al. (1999) for different soils. Hence, G_0^{sat} and γ_0 were not clearly related with the SWRC parameters.

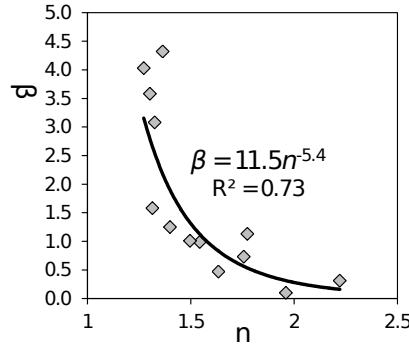


FIG 6. Correlation between the proposed model parameter and soil water retention curve of unsaturated soils: β vs n .

One the other hand, the soil matrix stiffness term, which expressed in term of saturation with a power of β , is obviously related to how much water that soil matrix retained. A strong empirical correlation between the β parameter and soil water retention parameter n was observed in Fig. 6. Higher β values were found with low n values (clayey soils), while the β value decreases as the n value increases (sandy soils). An empirical fitting equation was identified as $\beta = 11.5 \cdot n^{-5.4}$, which can be used for estimation of G_0 of unsaturated soil using the SWRC model of van Genuchten (1980) for different soil types. Another implication of this inter-correlation between material hardening/softening parameter β and soil pore size distribution parameter n is that the material softening/hardening mechanism mainly comes from the cementation effect of clay content of the soil. The more fine particles soil has, the stronger cementation force and larger the stiffness as the soil dries. Clayey soils with lower n values have higher parameter β numbers, reflecting higher sensitivity of G_0 to the degree of saturation for fine soils, leading to higher material softening/hardening effects during wetting/drying.

5. CONCLUSIONS

This study discussed about the effect of degree of saturation and particle contact force on the small-strain shear modulus G_0 by using suction stress theory for unsaturated soils. G_0 data were obtained for a wide range of soil types under various degrees of saturation and external loading conditions. Two major mechanisms were quantified using two exponential terms with empirical fitting parameters to implement the effects of softening/hardening of the soil matrix and the contact behavior of the soil skeleton, respectively. The suction stress framework provides a powerful interpretation of particle contact behavior, as it conceptualizes all kinds of particle contact forces and upscales the impact of matrix suction to an REV manifestation. A unified model was proposed based on the schematic spring networks representing impacts of degree of saturation on the soil matrix and the impact of effective stress on contact behavior. Compared to previous models for G_0 for unsaturated soils, the new model incorporates parameters that have physical meaning related with the soil water retention behavior. Specifically, the change of material stiffness of the soil matrix with the changing degree of saturation was observed to be empirically related to the pore-size distribution parameter of the van Genuchten SWRC model. The evolution of G_0 with degree of saturation was found to be more sensitive for silty and clayey soils than sandy soils. Further, the mathematical formulation of the new model can be easily extended from classic Hardin's equation for saturated soils to unsaturated soils, and applied for different cases of loading conditions with or without external confining, and with various confining stress conditions.

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