Title
Learning Through the Use of Instructional Materials: Secondary Mathematics Teachers' Enactment of Formative Assessment Lessons

Author
Seashore, Kimberly Holmes

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Learning Through the Use of Instructional Materials: Secondary Mathematics Teachers' Enactment of Formative Assessment Lessons

By

Kimberly Holmes Seashore

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Committee in charge:

Professor Alan H. Schoenfeld, Chair
Professor Na'ilah Suad Nasir
Professor Deborah A. Nolan

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Abstract

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This dissertation is an exploratory study that investigates what and how teachers learn from use of instructional materials. Over the course of one school year, eight secondary mathematics teachers in two urban schools interspersed their usual instruction with "formative assessment lessons" (FALs), based on lessons and lesson guides designed to allow teachers to use student thinking to inform instructional decision-making. The focus of the study was on understanding how the teachers interpreted the goals and uses of these materials for their own and their students' learning and how the materials supported teachers' learning about formative assessment.

Teachers were expected to enact five FALs, which they selected from the Mathematics Assessment Project web site (map.mathshell.org) to place in curriculum units in accordance with their instructional goals. The lessons were planned and enacted without formal professional development. However, five collaboration meetings allowed teachers time to plan with colleagues and reflect on their experiences. Teacher surveys, classroom observations, and classroom artifacts were analyzed in order to examine the placement and enactment of the lessons as well as the influence of enacting the lessons on teachers' classroom practices and use of formative assessment. The teachers' choices in selecting, placing, planning, and implementing the lessons significantly limited their potential to support teacher learning. However, analyses of student explanations and the teachers' interactions with small groups of students provide evidence of changes in classroom practices that allowed instruction to be better informed by student thinking.

Two case studies detail how teachers' knowledge, goals, and beliefs, their perceptions of the lesson designers' intentions, and the school environment influenced their use of the FALs and other materials, e.g., a monitoring sheet. Although the materials provided affordances that could support a range of teacher learning, the teachers' choices played a fundamental role in determining what they learned from teaching the FALs and using the materials. In one case, the teacher developed and refined new classroom routines that reflected his goals for his students about communication and accountability. In the other case, the teacher began to change her teaching practices when using the FALs and monitoring sheet, but later adapted her use of the FALs and altered the monitoring sheet to be more consistent with her routine instruction, limiting her opportunities for exploration and learning. The implications of these findings for research on teacher learning, for revision of the FALs, and for professional development are discussed.

In the memory of Randi Engle and
Charles N. Seashore and Edith Whitfield Seashore

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Chapter 1

An Investigation of Teachers' Use of Instructional Materials

Introduction

Beginning in 2010, state adoptions of the Common Core State Standards for Mathematics (CCSSM) renewed focus on the efforts of the past 20 years to reform teaching and learning of school mathematics (NCTM, 1989, 2000; Kilpatrick, Swafford, & Findell, 2001). Mathematics educators have worked to promote teaching of mathematics that results in equitable learning outcomes for students, and at the same time is more rigorous, relevant, and internationally competitive than in the past. This movement has been driven by the recognition that all students need to make sense of mathematical concepts, to reason with mathematics, and to interpret and solve complex problems using mathematics. These ways of engaging in mathematical activity are described in the CCSSM's Standards for Mathematical Practice. Together with the Standards for Mathematical Content, which describe trajectories of concepts and skills that students should develop via schooling, the CCSSM frame mathematics as a coherent collection of powerful tools that enable students to describe, understand, and solve problems in the world they live in. In addition to the shift in what needs to be learned, the reform movement emphasizes that learning is an active process of constructing knowledge within a social context.

Success in enacting reforms depends on teachers, the front line of reform efforts. Their ability to create classroom interactions that foster learning is critical. Reform asks teachers to build on student understandings. This differs from telling or showing students what to do without consideration of their ideas and perceptions. It also differs from unguided "discovery" of mathematics. Instead, reform recognizes that students come to class with their own conceptions and make their own meanings of classroom activities. The teacher's job is to guide and monitor their progress, designing instruction in response to student understandings (NCTM, 2000).

For teachers, learning to enact lessons that allow them to assess and build on student thinking, creating an environment that provides students with opportunities to share and refine their understanding, is not only challenging, but frequently runs counter to teachers' own experiences of learning mathematics and to school culture (Lortie, 1975). Moreover, reform is frequently seen as conflicting with the national focus on high-stakes testing and emphasis on mastery of solutions for test questions rather than on building students' reasoning, analytic skills, and explanations. Unfortunately, procedural knowledge that is not supported by conceptual understanding is fragile. This is evidenced by students' struggles to retain what they learn in mathematics classes or to build on it in future classes.

Instructional materials that are aligned to the standards and provide teachers with models for student centered classroom activity are often considered essential tools for supporting reform in mathematics teaching. Many studies have shown the promise of such materials to support teachers in changing their classroom practices. However, studies also show how teachers can use these materials in the classroom in ways that reduce their cognitive demand (Stein, Grover, & Henningsen, 1996), constraining students' access to mathematical concepts and practices. Although instructional materials are important and perhaps essential in supporting teachers in changing their classroom practices, there is little or no evidence that materials alone suffice. Additional support from peers and professional development appears to be necessary.

One set of instructional materials designed to support secondary teachers in implementing the CCSSM is the Formative Assessment Lessons (FALs) developed as part of the Mathematics Assessment Project (MAP),[1] a collaboration that includes researchers at the University of California, Berkeley. The FALs are a collection of 100 lessons, freely available for download via the MAP website (www.mathshell.org). To date, there have been more than 4 million downloads of these lessons, and many schools and districts have embedded the FALs into their curricula for the Common Core. The lessons focus on core concepts at each grade level from 6–11. They are designed to surface students' prior understanding of concepts, promote students' use of the mathematical practices described in the CCSSM, and to support resolution of difficulties and misconceptions through structured small group and whole class discussion.

The FALs provide many affordances for supporting student thinking, but they act only as exemplars. Use of five or ten FALs in a course each year could provide a rich supplement to the course curriculum. However, by themselves, five or ten innovative lessons are not likely to change students' experiences in mathematics classes. Their real power of the FALs is their potential to support teachers in learning to use formative assessment in *all* of their lessons. The FALs are rich in mathematical concepts, tasks with high cognitive demand and multiple access points, and structured opportunities for students to develop and explain their own understanding. They provide opportunities for teachers to elicit student understanding and lesson guides to help teachers interpret the student responses that are likely to occur. Currently, however, there is very little coordinated professional development provided in the use of these lessons, and even less in how to learn from them. Thus, the questions for this study were:

- How do teachers make use of these lessons in the absence of professional development?
- In what ways can teaching these lessons without structured professional development support teachers in learning to enact formative assessment?

[1] The development and distribution of these lessons was funded by the Bill and Melinda Gates Foundation, which also funded much of the research conducted by the MAP. During their development, the lessons were renamed Classroom Challenges. However, because they were called Formative Assessment Lessons when the study began, they are referred to as FALs throughout this dissertation.

To address these questions, eight teachers from two urban public schools were recruited to teach at least five FALs of their choosing during one academic year. The teachers were given a brief overview of the FALs and access to the research team, who had worked closely with the lesson designers and were very familiar with the lessons. The research team convened five collaborative planning meetings during the school year, which included providing the teachers with dinner. During these meetings, the teachers had dedicated time to plan the lessons and the opportunity to talk with each other or members of the team while they planned. But, there was no structured professional development and the teachers were ultimately responsible for selecting which lessons to teach, where to place them in their curriculum, and how to enact them in their classrooms. Along the way, new questions emerged: Given the wide range of affordances of these lessons, how do teachers choose to use the lessons to pursue their own goals for supporting students' learning? How can use of these lessons serve to reveal teachers' goals and beliefs, and in that way, serve to inform professional development?

This dissertation takes up the first questions in Chapter 3, using a cross-case analysis of the eight teachers in this study. The second question is addressed in Chapters 4 and 5 using case studies of a middle school and a high school teacher. Chapter 6, the final chapter, discusses the implications of this study for designing effective professional development on formative assessment and for using well designed, student centered lessons as a site for teacher learning.

Literature review

This dissertation is informed by research in the following areas:

- Changes in our understanding about how people learn, and their implications for teaching.
- Formative assessment as a pedagogical approach.
- Teacher knowledge, beliefs, decision-making, and learning.
- Teacher learning in the context of teaching and through the use of instructional materials.

How people learn and implications for teaching

Key findings about student knowledge and learning (that apply to all learners, including teachers) are summarized in the National Research Council report *How People Learn* (Bransford, Brown, & Cocking, 1999). Students come to class with their own conceptions. In order for learning to occur and be sustained, students' understandings must be engaged, integrated, and reconciled with the conceptions that they bring to the classroom. Competence is a combination of factual knowledge organized within a conceptual framework that facilitates retrieval, application, and extension. Metacognitive processes, such as defining learning goals and monitoring one's own progress toward achieving these goals, support learning.

In consequence, *How People Learn* argues that learning environments must be:

- Learner centered – teachers must have and act on their knowledge of individual learners' backgrounds, culture, knowledge, skills and interests.

- Knowledge centered – attention is paid to conceptual depth in content, assessing understanding and application of ideas, beyond skills and teaching of meta-cognition.

- Assessment centered – where assessment is used to provide opportunities for students to revise and improve their work, monitor their own progress, and help teachers become aware of the ideas (both correct and incorrect) that students bring to class or develop in response to instruction.

- Community centered – establishing a supportive community of learners within the classroom and bridging connections to students' home communities.

The lessons examined in this dissertation are intended to foster these features of learning environments by providing information about students' knowledge, focusing on conceptual understanding and processes such as generalization and categorization, and engaging students in collaborative learning activities. However, most central is supporting teachers' understanding of and ability to employ assessment to support student learning.

Using formative assessment to support student learning

Formative assessment is a pedagogical approach that emphasizes the importance of both teachers and students using students' understanding to inform instruction in the service of desired learning goals (Black & Wiliam, 2009).

Black and Wiliam describe formative assessment as:

> Practice in a classroom is formative to the extent that evidence about student achievement is elicited, interpreted, and used by teachers, learners, or their peers, to make decisions about the next steps in instruction that are likely to be better, or better founded, than the decisions they would have taken in the absence of the evidence that was elicited.

This description includes three stages: (A) eliciting evidence about student thinking to make it accessible to both students and teacher; (B) interpreting the elicited evidence; and (C) instructional responses that move students' learning forward. In practice there may be some fluidity between these stages, however, I consider it helpful to consider the stages as distinct parts of a formative assessment cycle (Figure 1.1), where the instructional response in Stage C feeds into collection of further information about students' developing understanding.

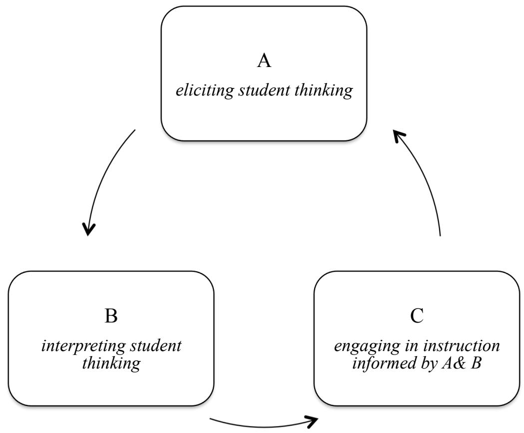

Figure 1.1. Formative assessment cycle

This description of formative assessment was distilled from observation of practices that support use of assessment to promote learning rather than as a summary of student knowledge. These practices have been distilled as a set of five key strategies that support or promote learning:

1. Clarifying learning intensions and criteria for success.
2. Engineering effective classroom discussions and learning tasks that elicit evidence of student understanding.
3. Providing feedback that moves learning forward.
4. Activating students as instructional resources for one another.
5. Activating students as the owners of their own learning.

The purpose of articulating these strategies is to make formative assessment more tangible and actionable for teachers (William & Thompson, 2007). There is a strong need to better understand how and under what circumstances these strategies are effective. Moreover, there is scant research on how teachers actually learn to implement the five strategies in ways that, when put together, effectively create cycles of formative assessment.

Implementation of any one stage of formative assessment poses substantial challenges for teachers. Eliciting evidence of student thinking requires good tasks and the creation of an environment where students are willing and able to share possibly incorrect ideas. Interpreting student thinking takes time and pedagogical content knowledge (Shulman, 1986) that allows teachers to make sense of students' responses. Finally, even if teachers have been able to elicit evidence of student thinking and interpret this evidence to make sense of the students' understandings, designing an instructional response that empowers students as "owners" of their learning, providing feedback that moves learners forward, and activating students as resources for each other's learning is a very tall order.

The Formative Assessment Lessons from the Shell Center were developed to provide support for teachers in implementing the process shown in Figure 1.1. Each lesson has a pre-assessment task intended to elicit evidence of student thinking about the central concept or problem of the lesson, a common issues table to help teachers interpret this evidence, and an instructional response – tasks connected with the most common student conceptions and challenges addressed in the lesson. These tasks are based on extensive research and experience in the development of the Mathematics Assessment Resource Service assessments and lessons developed as part of a wide variety of projects to support shifts in classroom practices and teachers' beliefs (Foster, Noyce, & Spiegel, 2007; Swan 2007). The lesson activities and guidance for class discussions have been developed and refined using a design-based research approach to support students' engagement with existing conceptions and their building of new conceptions through resolution of cognitive dissonance. The post-assessments that follow the lessons are intended to support students in responding to feedback from their work on the pre-assessments, consolidating their understanding based on their work on the lesson activities, and assessing their new understanding.

Teacher knowledge, beliefs, decision-making, and learning

The central focus of this study is on teachers learning to enact new classroom practices. Teacher learning, like all learning, is informed by the learner's experiences. In particular, a teacher's learning is informed by his or her experience in the classroom. These experiences are, in turn, the result of the teacher's decision-making which, as Schoenfeld (2010) puts it, is a function of their resources, orientations, and goals. Learning is also situated (Lave & Wenger, 1991). In particular, teacher learning is situated in the context of schools. This context includes the expectations of students, parents and administrators, in particular, expectations for curriculum coverage and test performance.

Clarke and Hollingsworth (2002) construe teacher learning as an interaction of four analytic domains (personal, external, practice, and consequence) that influence one another in iterations of enactment and reflection. Their model emphasizes the mutual dependence of teacher knowledge, goals, and beliefs; external source of information or stimulus; professional experimentation; salient outcomes. An important feature of this model is that each of these domains may influence the other. For example, goals and beliefs may affect choices in professional experimentation, e.g., choices made in using instructional materials. Knowledge or beliefs may affect the outcomes of experimentation that are salient to the teacher.

In their review of research on mathematics teacher learning, Goldsmith, Doerr, and Lewis (2014) find that studies of teacher learning focus on the efficacy of particular professional development programs or approaches (e.g. professional learning communities or lesson study) but the broader question of how teacher learning is conceptualized remains undertheorized. They advocate a more coherent approach to teacher learning based on the model of Clarke and Hollingsworth.

Teacher learning through the use of instructional materials

Throughout the mathematics reform movement, instructional materials have been considered crucial tools for supporting instructional change. Several of the curricula developed in response to the National Council of Teachers of Mathematics standards (NCTM, 1989) have been important resources for teachers in implementing reform mathematics instruction and have had positive effects on student learning (Senk & Thompson, 2003). However, questions remain about how to support teachers in implementing these curricula effectively. Studies of curricular implementation often focus on “fidelity” – curricular implementation in ways consistent with the designers’ intentions (Ball & Cohen, 1996, Collopy 2003; Stein, Grover, & Henningsen, 1996). Remillard (2005) notes the distinction between intended and enacted instructional materials. She makes the point that although some researchers focus on fidelity in implementation, others do not assume that fidelity between text and teaching is possible.

Much of the research on teacher learning from instructional materials has teachers use an instructional program for a sustained period of time, accompanied by professional development designed to support teachers in using the program (Goldsmith, Doerr, & Lewis, 2014). In these studies, the materials are assumed to be appropriate for supporting student learning. Most of these studies examine the question of how teachers adapt and interpret the materials in order to meet their learning goals for students.

Collopy (2003) studied how two upper elementary school teachers learned from the use of TERC’s *Investigations in Number, Data, and Space* in the absence of additional professional development after a two-day workshop. She found that one teacher’s instructional practices changed dramatically while the other’s remained stable, and concluded that “curricular materials can be an effective professional development tool, but perhaps not for all teachers.” Remillard and Bryans (2004) studied eight elementary teachers at the same school who used *Investigations in Number, Data, and Space*. Both pairs of researchers found that teachers’ orientations influenced their use of the curriculum materials, leading to different opportunities for student and teacher learning.

Swan (2006) explored how teachers’ beliefs and practices changed as a result of using replacement units with tasks that were designed to engage students in collaborative work and provoke cognitive conflict to be resolved through exploration and discussion. This study, conducted with 36 teachers (who were the UK equivalent of community college instructors), provided four full days of professional development that included work on tasks. The teachers reflected on their beliefs and practices, and how the use of these tasks with students challenged those beliefs. At the beginning of the study, the teachers’ orientations toward mathematical knowledge, student learning, and teaching were categorized as transmission, discovery, or connectionist. Swan found that the use of the tasks did not did not change the beliefs of teachers with transmission or connectionist orientations, but that the discovery-oriented teachers became more connectionist in their orientations.

Few studies take up the question of how intermittent use of supplementary instructional materials influences teacher learning about pedagogical strategies.

Teacher learning about aspects of formative assessment

The instructional materials studied in this dissertation are based on the five strategies and three stages of formative assessment described earlier. In the FALs, learning mathematics happens as students engage with each other in activities that challenge them to explain connections between different concepts, build new conceptions from existing ideas, and test their new conceptions. In facilitating these lessons, teachers have several potentially new roles. They need to guide students to give explanations and justifications to each other; to help students articulate, clarify, and evaluate their justifications; to support students in holding each other accountable for their justifications; and to facilitate whole class discussions.

The three stages illustrated in Figure 1.1 indicate other potentially new roles for teachers. Teachers need to elicit student thinking, interpret it, and use it as a basis for making decisions about instructional activities. Researchers have documented efforts to support teachers in learning these roles: eliciting student thinking (Borko et al., 2000; Doerr & English, 2006); making sense of student thinking (e.g., Borko et al., 2008; Carpenter et al., 1989; Collopy 2003; Kazemi & Franke, 2004; Kersaint & Chappell, 2001; Sherin & van Es, 2009); and using student thinking as the basis for instructional decision-making (e.g., Kazemi & Franke, 2004; Remillard & Bryans 2004). Most of this research is based in the context of formal professional development programs and the majority of this research concerns elementary school teaching. Although the question of how to support teachers in developing the knowledge and skills to engage in all three stages of formative assessment in secondary mathematics is informed by this work, it is not directly addressed by it.

Reform-oriented teaching often uses a three-part lesson structure: a launch; collaborative activity in small groups; and whole class discussion that builds on the results of the activity. The most difficult of these for teachers is structuring the class discussion, making it lead to greater mathematical understanding rather than a “show and tell” of student work (Baxter & Williams, 2010). Stein, Engle, Smith, and Hughes (2008) suggest a set of five practices to use in orchestrating more productive class discussions. Before the lesson, the teacher *anticipates* student thinking. During the activity, the teacher *monitors* student work and *selects* work to share later in the class discussion. Before the discussion, the teacher *sequences* the work that students will share, making it easier during the discussion for the teacher to *connect* student ideas and create a coherent discussion. To support this process, Stein et al. created a “monitoring sheet” for teachers to record student thinking and use in selecting, sequencing, and connecting student presentations. In addition, the monitoring sheet can help the teacher to stay focused on observing and making sense of student thinking, rather than intervening. Figure 1.2 shows how the five practices are connected with the stages of formative assessment.

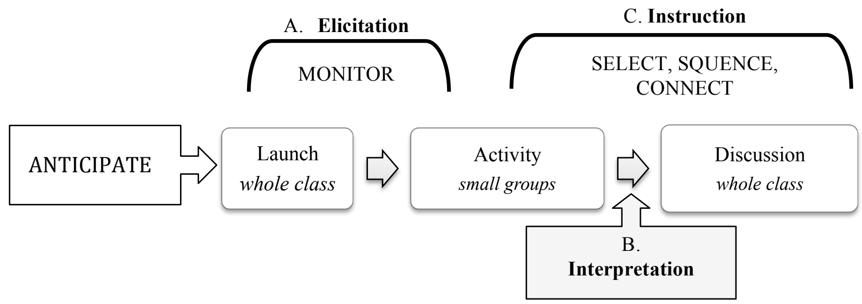

Figure 1.2. Connection of Stein et al.'s five practices (in capitals) and the three stages of the formative assessment cycle (in bold)

Teacher learning from the FALs

Research with earlier versions of the FALs in remedial college courses in the UK (Swan, 2008) shows that working with carefully designed lessons that supported eliciting and building on student thinking can change teachers' beliefs about collaborative learning and change their teaching practices. Building on these and other findings on the use of lessons to engage students in cognitive conflict and promote student sense-making through mathematical discourse, Swan and his colleagues created the FALs to promote teacher learning about productive practices for ongoing formative assessment as they advanced student learning.

Closer to home, a report on a three-year project involving extensive professional development for Kentucky teachers on the use of the FALs found some positive results for both teachers and students (Herman, Epstein, Leon, La Torre Matrundola, Reber, & Choi, 2015). In survey responses, the teachers reported finding the FALs helpful in meeting a variety of goals, including use of formative assessment and incorporating more complex thinking and problem solving in curriculum and instruction. They also found moderate learning gains for students in classes using the FALs when compared with a similar population. Their findings provide support for continuing to study how to use these lessons most effectively. At the same time, they conclude that "mathematics teachers who implement MDC [the FALs] likely will be more successful to the extent they possess sufficient pedagogical-content knowledge to effectively scaffold students' productive struggle with complex mathematical concepts and problems." Essentially, they find that the lessons worked best when the teachers already had the knowledge and skills to use them well. However, the question of how the FALs themselves support teachers in acquiring this knowledge is left unaddressed.

Concluding remarks

To understand how and what teachers may learn about their practice from instructional materials, in contrast to how they learn to implement these material, we need to consider them as learners, asking: What understandings do they bring to their work? What constraints shape their opportunities to learn? And, what understandings do they build in consequence? We must study what they do, and try to understand what they consider to be important, and how that shapes what they focus on.

Overview of the dissertation

The data analyzed in this dissertation are part of a larger corpus of data collected to support the ongoing development of the FALs. These were collected as part of the Mathematics Assessment Project (MAP), a collaboration between the University of California, Berkeley, the Shell Centre at the University of Nottingham, and the Silicon Valley Mathematics Initiative that includes designing and evaluating resources for teaching and professional development. Throughout this dissertation, "we" refers to members of the UC Berkeley research group working with MAP and "I" refers to the author.

Chapter 2 describes the study design and methods of analysis. For this study, eight secondary mathematics teachers from two schools selected, planned and taught between three and five FALs during the 2013–14 school year. The schools, one middle school and one high school, were in urban school districts serving diverse student populations. The lessons were primarily taught in non-tracked, heterogeneous classes.

Chapter 3 takes up questions about how the eight teachers in this study selected, planned, and enacted lessons based on the FALs, and how these choices afforded or constrained the teachers' opportunities to learn from engaging in new instructional practices. Specific findings of Chapter 3 include the different ways that teachers interpreted the FAL designers' intentions, creating goals for their and their students' learning. Some teachers focused primarily on the content of a given FAL, opting to use portions of the FAL that they thought most clearly illustrated or moved students toward those goals, while other teachers interpreted the FALs as vehicles for changing the type of interactions that they and their students had while working toward mathematical understanding. The decisions that teachers made affected what and how they learned from the lessons while also revealing the teachers' understanding of formative assessment and student learning. In this chapter, the unit of analysis is a FAL as enacted by a teacher with a particular class of students. Each enactment of a FAL by a teacher with a group of students is seen as an opportunity for that teacher to learn and we are interested in seeing how and why those get taken up differently. This chapter does not attempt to address changes in a teacher's use of the FALs over time, nor does it attempt to evaluate the efficacy of these lessons on student learning.

Chapters 4 and 5 present case studies of the evolving goals and practices of two teachers, Mr. Davidson and Ms. Elmore, as they enact a series of FALs with their students. These teachers were selected in part because of the teachers' interest using their experiences with the FALs to change their own and their students' classroom practices throughout the school year. In these chapters, the unit of analysis is a teacher and class of students.

Chapter 4 is a case study of Mr. Davidson, a middle school teacher. All of the FALs have the same "lesson structure," which is described in the lesson guide. This structure consists of five phases: a pre-lesson assessment designed to elicit students' understandings of the lesson topic; a launch in which the teacher introduces the collaborative activity; the collaborative activity (including work in small groups); a whole-class discussion; and a post-lesson assessment.

Mr. Davidson used his work with the FALs to influence interactions in his classroom in two different ways. First, interactions between Mr. Davidson and his students during collaborative group activities focused increasingly on making student thinking available to both the students and the teacher. Second, Mr. Davidson leveraged lessons as settings to develop and refine classroom routines to support students' presentations of their mathematical ideas. These two changes represent two different aspects of learning for Mr. Davidson, learning through the structures built into the FALs and learning through innovations with support provided by the FALs. Over the course of several lessons (taught over the course of the school year), we see changes in learning processes, for both Mr. Davidson and his students. These changes seem to stabilize in new classroom practices more aligned with Mr. Davidson's goals and the intentions of the FAL designers.

Chapter 5 is a case study of Ms. Elmore, a high school teacher. Ms. Elmore's case adds a different perspective on the interaction between teachers' goals and their interpretations and implementation of the FALs. The collaborative aspects of the FALs were a significant departure from her regular teaching. Ms. Elmore's use of the FALs also showed substantial increases in the level of interaction with student thinking during the classroom activity. However, Ms. Elmore struggled with how to conduct a whole class discussion that connected and built on the collaborative activity. Her use and modification of a monitoring tool designed to connect student thinking with the whole class discussion reflected her modifications of the FALs to align with her normal teaching rather than support her learning of different instructional practices.

Chapter 6 discusses the implications of these findings from three perspectives: design of instructional resources, specific recommendations for professional development, particularly with respect to formative assessment; and ultimately teacher decision-making and learning.

The findings presented in Chapter 3 indicate that the teachers had some difficulty interpreting key aspects of the FALs from the lesson guides about the type of prior experience that would support students' engagement with the lesson. They also had difficulty in deciding "how to land" these lessons during the whole-class discussion phase and all struggled to determine the main messages of the lesson. These findings suggest how the FALs might be modified to include planning materials that could be available on the website to help teachers make decisions before and during the lesson.

The two case studies provide more specific insights into what the teachers were able to learn on their own from working with the lessons and where professional development could have provided more support. In particular, teachers struggled to connect the elicited student thinking from the pre-lesson assessment and the activity phase with the discussion phase and the post-lesson assessment. The FALs and the five practices framework, when coordinated offer promising resources for supporting teachers in making these connections, but only if careful attention is paid to when and how the lessons are used and time for reflection during the lessons is provided for teachers to interpret and make instructional decisions based in student thinking.

Finally, at a broader level, the two case studies show how teacher learning, like all learning, is situated, and strongly influenced by the teacher's goals and values, as well as by perceived external forces. Both teachers acted in what they perceived to be the best interests of their students in making use of the formative assessment lessons. Their decisions and actions were guided by their perceptions of the affordances of the lessons and their understanding of the role that student presentations should play in a Common Core classroom.

Chapter 2

Research Design and Analytical Methods

Study design

This dissertation is part of a larger study intended to better understand the choices that teachers make in using the formative assessment lessons developed by the Mathematics Assessment Project (MAP) and, given those choices, the opportunities made available for teachers to learn from their teaching of these lessons.

In designing the MAP study, several key features stood out as important for selecting teachers based on earlier pilot studies and professional development workshops with teachers interested in using the Formative Assessment Lessons (FALs) described later in this chapter. Teachers needed to commit to using three or more FALs with the same group of students in a single course in order to allow them and their students the opportunity to become familiar with the new roles and expectations in these lessons. Second, teachers needed to select which FALs to teach and where to place them in their course curriculum units, so that they would see the lessons as relevant to their students' learning in that course. Researchers could help by suggesting lessons that might fit with the teachers' syllabi, but teachers needed to make the decisions, including the decision not to teach a previously selected lesson.

School contexts

The teachers in this study were drawn from two urban schools in the San Francisco Bay Area serving diverse student populations which I call Adams Middle School and Brookfield High School. Table 2.1 shows the percent of students who identify with each of the racial categories, percent of students considered socioeconomically disadvantaged, and the percent of students classified as English language learners on the California Department of Education School Report Card for the 2013–14 school year. This demographic information shows that Adams has a substantially higher percentage of students considered "at-risk" than Brookfield.

Table 2.1. School demographics

	Adams Middle School % of total enrollment	Brookfield High School % of total enrollment
Racial identification		
Black or African American	21.6	21.1
Asian	0.8	8.6
Hispanic or Latino	76.1	20.9
White	0.2	37.8
Other categories	2.3	22.6
Socioeconomically disadvantaged	91.8	34.0
English learners	33.7	6.2

The 6th and 7th grade mathematics classes at Adams were heterogeneous and used a curriculum called Springboard[2] aligned with the Common Core State Standards for Mathematics, supplemented with substantial individual work on computers. Adams has three different options for 8th grade mathematics. Most students took Math 8, which used a Springboard curriculum and individual skill practice on computer programs such as those from iXL and Khan Academy. About 25 8th grade students were placed in an accelerated Algebra 1 class whose curriculum was aligned with the 2001 California State Standards for Mathematics. The remaining 25 8th graders took mathematics during the first two periods of the school day, first Geometry with Mr. Davidson, then Algebra 1 with Ms. Castle. The Geometry class was also based on the 2001 California State Standards for Mathematics, but is guided by *Discovering Geometry*, a reform text.

The three courses observed at Brookfield High School were: a regular (non-honors) Geometry – which followed the California State Standards for Mathematics (2001) and used *Discovering Geometry*; the second course of a three-year untracked integrated mathematics sequence that used the Interactive Mathematics Program (IMP) curriculum; and a senior level course focused on advanced topics that included elementary calculus and statistics that was part of the International Baccalaureate Program (IB).

Participating teachers

Two teachers, Ms. Belle[3] and Mr. Golden, participated in a summer internship with our research group during the summer of 2013 as part of a teacher leadership fellowship. They served as participants and grade-level leaders during a week-long summer professional development workshop on the FALs facilitated by the author. They transcribed and coded classroom observations from pilot studies of the FALs in local classrooms and analyzed mathematical affordances in several of the lessons. Based on their interest in the FALs, these teachers were asked to help design this study so that it would serve teachers' as well as researchers' goals.

Both Ms. Belle and Mr. Golden contributed to the design decisions above and offered to recruit teachers from their schools – Adams Middle School and Brookfield High School, respectively – to participate in the study. Five teachers from Adams and six teachers from Brookfield expressed interest, and four teachers were selected from each site for the study based on their expressed desire to use the FALs to improve their classroom instruction. We also established, either through prior classroom observations or recommendations from department chairs or administrators, that each of the teachers had clearly established classroom norms for behavior and consistent routines for interactions.

The teachers each received a stipend of $250 for participating in the study and attending collaboration meetings and $175 per lesson for planning and teaching up to five

[2] http://springboardprogram.collegeboard.org/mathematics/

[3] All names of teachers, students, schools and district have been changed to pseudonyms.

FALs. All mathematics teachers at both schools were invited to participate in collaboration meetings, to teach the FALs, and to ask the research team for support in selecting and planning lessons.

Most of the teachers were in their first ten years of teaching. Table 2.2 shows the grade level, teaching experience, and prior professional development and teaching experience with FALs and other similar resources such as Mathematics Assessment Resource Service tasks (also developed by the FAL designers). Prior professional development and teaching experience were each ranked with a score of 0, 1, 2 with: 0 = no prior experience with the lessons, 1 = some exposure, short, one time workshop, or tried a lesson once in the classroom, and 2 = multi-day professional development experience or used more than one lesson or task in the prior year of classroom teaching. Professional development and teaching experience scores were added to create a score between 0 and 4.

Table 2.2. Teacher demographics

<table>
<tr><th></th><th>Years teaching</th><th>2013–14 Grade & Course</th><th>Prior experience with FALs 0–4</th><th>Prior experience with similar resources 0–4</th></tr>
<tr><td colspan="5">Adams Middle School</td></tr>
<tr><td>Ms. Amador</td><td>3</td><td>Math 6</td><td>0</td><td>4</td></tr>
<tr><td>Ms. Belle</td><td>6</td><td>Math 7</td><td>2</td><td>4</td></tr>
<tr><td>Ms. Castle</td><td>3</td><td>Math 7
Algebra 8</td><td>2</td><td>0</td></tr>
<tr><td>Mr. Davidson</td><td>5</td><td>Math 8
Geometry 8</td><td>1</td><td>2</td></tr>
<tr><td colspan="5">Brookfield High School</td></tr>
<tr><td>Ms. Elmore</td><td>8</td><td rowspan="2">Geometry
Grades 9–11

IB Grade 12</td><td>1</td><td>3</td></tr>
<tr><td>Ms. Feldman</td><td>23</td><td>3</td><td>4</td></tr>
<tr><td>Ms. Golden</td><td>7</td><td rowspan="2">IMP 3 (Geometry and Algebra II)
Grades 9–11</td><td>2</td><td>2</td></tr>
<tr><td>Ms. Heather</td><td>4</td><td>2</td><td>4</td></tr>
</table>

Six of the eight teachers in the study identify as white females; Mr. Davidson is a white male and Ms. Amador is an African-American female. All eight teachers are native English speakers. As is frequently the case in urban schools, teacher demographics differ significantly from student demographics.

Resources

The primary material resources in this study were the Formative Assessment Lessons (FALs). Teachers had access to all the lessons that were publicly available on the Mathematics Assessment Project website[4] as well as additional FALs that were still under development. The MAP website also provides background research, overview and justification for the design, and suggested implementation guides for each of these lessons. Participants were strongly encouraged to download and read *A Brief Guide for Teachers and Administrators* from the MAP website.

A central focus of the MAP study was how teachers make sense of these lessons in their regular teaching contexts. Thus, there was no professional development specific to the FALs during the academic year. However, as part of the study design, support for teacher planning was provided in the form of five collaborative planning meetings at each site, approximately every six weeks. At these meetings, teachers in the study gathered with members of the research team to reflect on the lessons that they had taught and to prepare for upcoming lessons. These meetings were very loosely structured, with most of the time dedicated to teachers working in pairs or teams to prepare future lessons.

Overview of Formative Assessment Lessons

The Formative Assessment Lessons used in this study were developed by the Shell Centre in Nottingham, UK, with funding from the Bill and Melinda Gates Foundation. These lessons were developed through an iterative design process, including extensive piloting in the United States. The design of the lessons and in many instances the actual tasks are based on those used in other reform-oriented mathematics professional development projects (Swan, 2006). The lessons aligned to the content standards and incorporate and exemplify the standards for mathematical practice of the Common Core State Standards for Mathematics (CCSSM). They are intended to be versatile enough to be inserted into any Common Core-aligned mathematics course, regardless of the specific curriculum, to help teachers assess and support their students' emerging conceptual understanding and engagement with mathematical practices. These lessons are intended to support students' mathematical development and to provide teachers with models of instruction that can support their own professional growth with respect to effective formative assessment practices.

The corpus of lessons consists of approximately 20 lessons per grade level for grades 6–11. They are publicly available for free download at map.mathshell.org., As of August 2015, there have been approximately 3,000,000 downloads of the lessons. Although they were initially called Formative Assessment Lessons (FALs), their name has been changed to Classroom Challenges. For the remainder of this study, I refer to them as FALs because I believe that the emphasis should be on the affordances in the lessons for formative assessment. For teachers in the United States, "Classroom Challenges" seems to carry the implication that the lessons are only for "advanced" students needing additional challenge

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[4] All of the lessons in this study can be accessed at *map.mathshell.org.*

beyond the regular course. The view of the researchers is that these lessons have the potential to support teacher and student learning in all secondary classroom settings, and we are particularly interested in how these lessons can support teachers and students in engaging in formative assessment processes.

Each FAL comes with an overview, a lesson guide that include solutions for the primary activities in the lesson, blackline masters, and slides for teacher use. The lessons guides give brief descriptions of the lessons, the CCSSM content and practice standards that are most relevant to the lesson, and suggestions about structuring and facilitating the lesson.

There are two types of FALs, *concept development* and *problem solving* lessons, each with somewhat different goals and structures. Concept development lessons focus on key content standards for the grade level for which the lessons are designed. Problem solving lessons are designed to support students in applying mathematical understanding to unstructured, non-routine problems. Their goal is to support students in developing and refining solution strategies. At each grade level, approximately 65% of the available lessons are concept development and 35% are problem solving. Because the goals and formats of these two types of lessons differ in significant ways, I describe the two types separately below.

Concept development lessons

Concept development (CD) lessons are intended to support students in building deeper connected understandings. Each of the concept development lessons is structured around one or two collaborative activities, followed by a whole class discussion. The collaborative activities are designed to continue to surface student thinking and encourage students to makes sense of inconsistencies and overgeneralizations in their understandings. The activities are also meant to provoke cognitive conflict that is negotiated and resolved through the small group discussion with peers and during a whole class discussion at the end of the lesson. Prior to the lesson, students individually complete a pre-assessment designed to elicit their understandings of central concepts in the lesson. After the lesson, students individually complete a post-assessment that is parallel to the pre-assessment.

I use the lesson Lines and Linear Equations to exemplify, in detail, the different phases of a concept development lesson. This lesson addresses two particular 8th grade CCSSM content standards: 8.EE Understand the connections between proportional relationships, lines, and linear equations; and 8.F Define, evaluate, and compare functions.

Pre-assessment phase

The first phase of the lesson is the pre-assessment. For this lesson, the pre-assessment (shown in Figure 2.1) poses questions about a race between two women running at constant speeds. A graph shows their distances from the starting line as functions of time. The second page shows the same race, but the functions now indicate distances from the finish line. The slopes of the two functions for each runner have opposite signs and the sum of their *y*-intercepts is the length of the race.

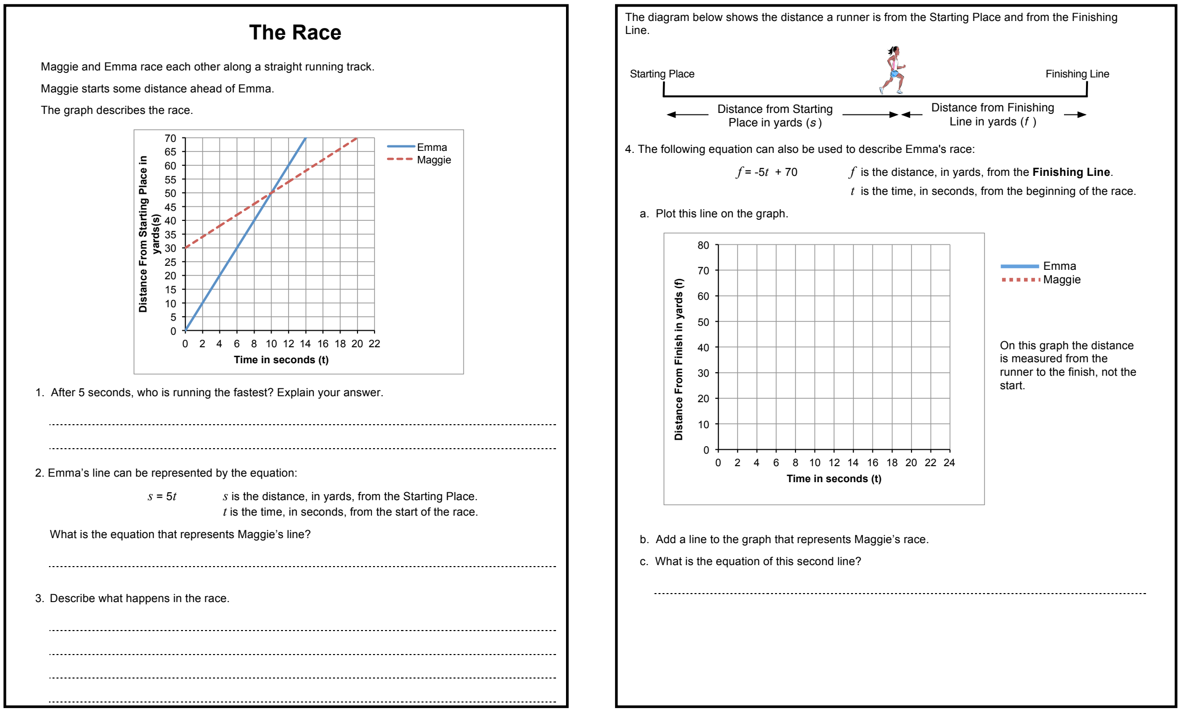


Figure 2.1. Pre-assessment for Lines and Linear Equations

Students complete the pre-assessment several days before the lesson, in order to give their teachers time to interpret their work and provide written feedback in the form of questions. The lesson guide includes a list of common issues that students encounter on the pre-assessment and suggested questions to ask in order to support students in revising their thinking.

Launch phase

The classroom lesson begins with a *launch*, where the students are introduced to a similar situation: six units of water in a closed, hourglass shaped container where water flows from a top container to the bottom container (Figure 2.2).

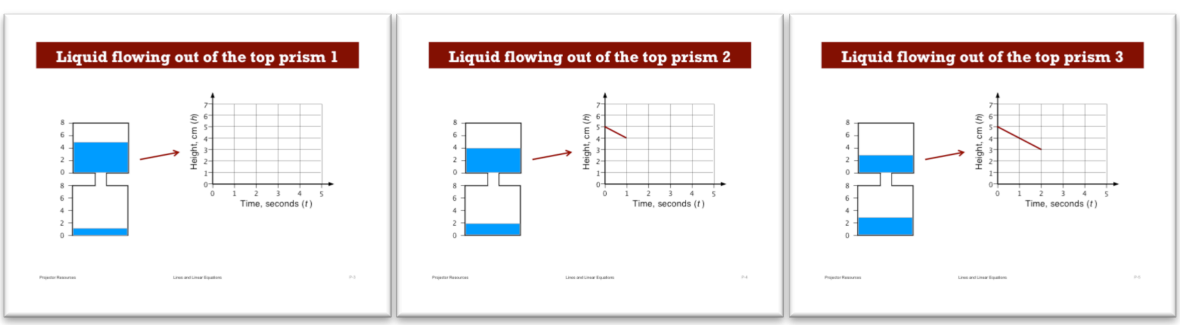


Figure 2.2. Launch – similar situation

In this scenario, as the top container empties, the bottom container fills. The teacher shows the students four graphs (Figure 2.3) and asks the students to share strategies for deciding which graphs could represent the top and bottom of the same container. Some strategies are: the height of water for the top container must be decreasing, so it must be represented by G2 or G5; the height of water in the bottom container is increasing, so it must be represented by G3 or G6; the height of water in the top container must be at zero at the same time as the height of water in the bottom container is at 6, as is the case for G2 and G6.

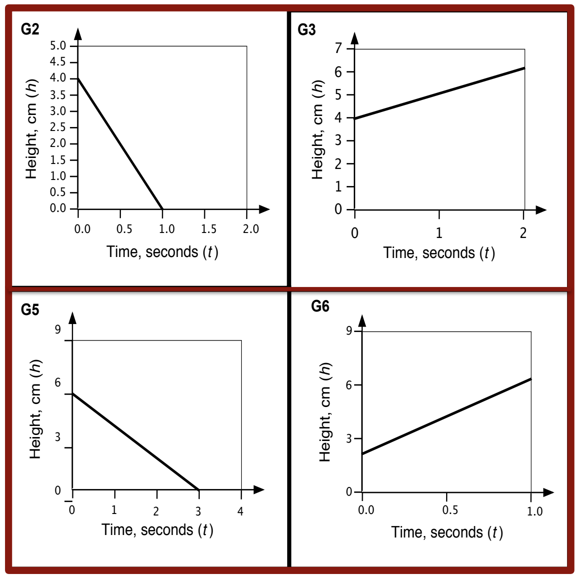


Figure 2.3. Launch – sample graphs

The goal of the launch is for the students to understand the context – what the graphs represent and that they are to look for ways to justify matches between graphs of the top and bottom prisms of the same container.

Activity phase

Each group of students then gets a set of cards (Figure 2.4) showing graphs to match. The lesson guide provides an overhead slide with directions for how the students should work together to find the matches (Figure 2.5).

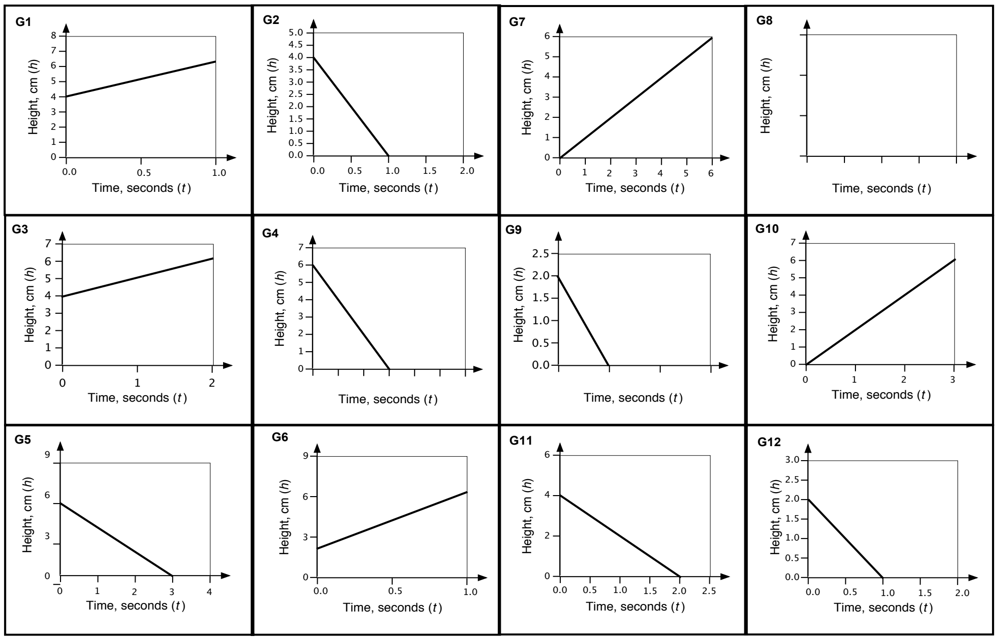


Figure 2.4. First activity – cards to match

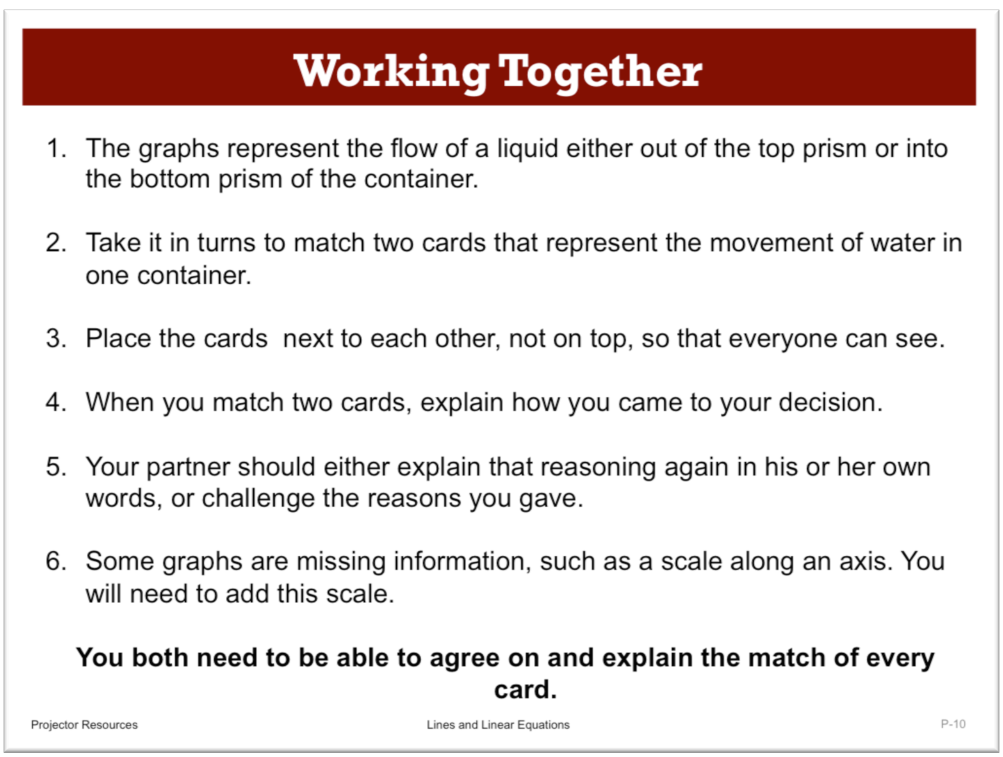


Figure 2.5. First activity – instructions

This task requires students to look closely at key features of the graphs, particularly because the scales on the graphs are different. In several cases, students need to fill in the scale on the graph once they have decided on a match. There is also one blank graph to use for the final match. Students are to glue the matched cards to a poster.

After matching the cards, one member of each group is to write the numbers of each of the matched pairs of graphs, e.g., G2 and G6, then move to another group to discuss their answers. In this way, students practice explaining their groups' ideas to other students and are also able to learn from someone who was working with a different set of students. The lesson guide provides a slide with directions for this jigsaw activity (Figure 2.6).

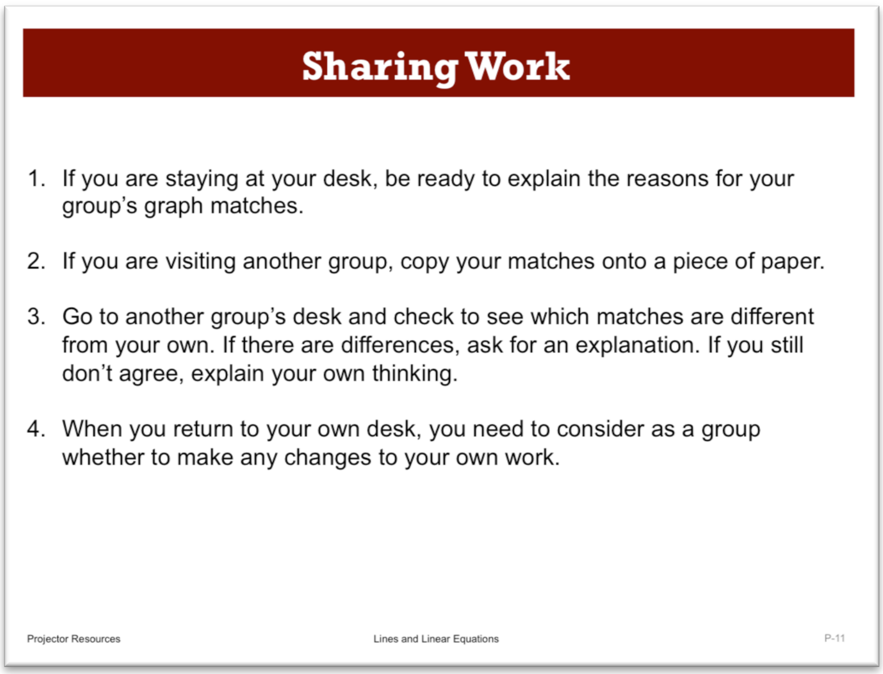
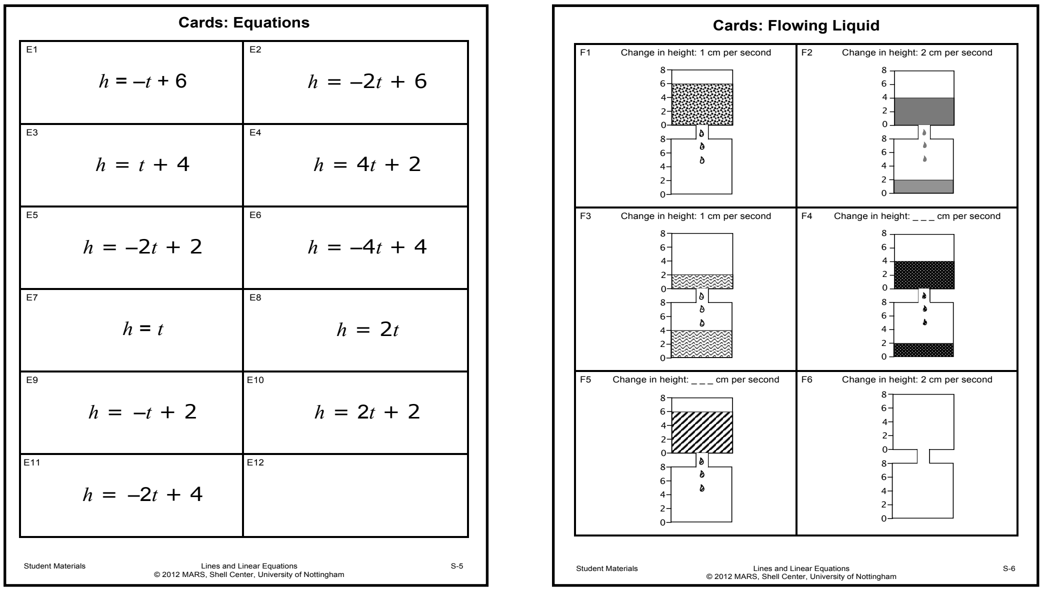


Figure 2.6. First activity – instructions for sharing work

After all of the students return to their groups, they are given cards with equations and cards showing the starting position of the containers with the rate that the water flows between the containers written in a sentence to add to their posters (Figure 2.7). This allows the students to connect the three representations of the same situations.



2.7. First activity – Lines and Linear Equations card sets with equations and containers

Completed matches have five cards: two graphs, two equations, and one diagram of a container (see Figure 2.8). As with the graphs, some of the container diagrams are missing information and one of the equations is blank. This allows students to generate information rather than using the process of elimination to create the matches. Students are encouraged to annotate their posters with words and symbols that explain the matches and any connections they find between the representations.

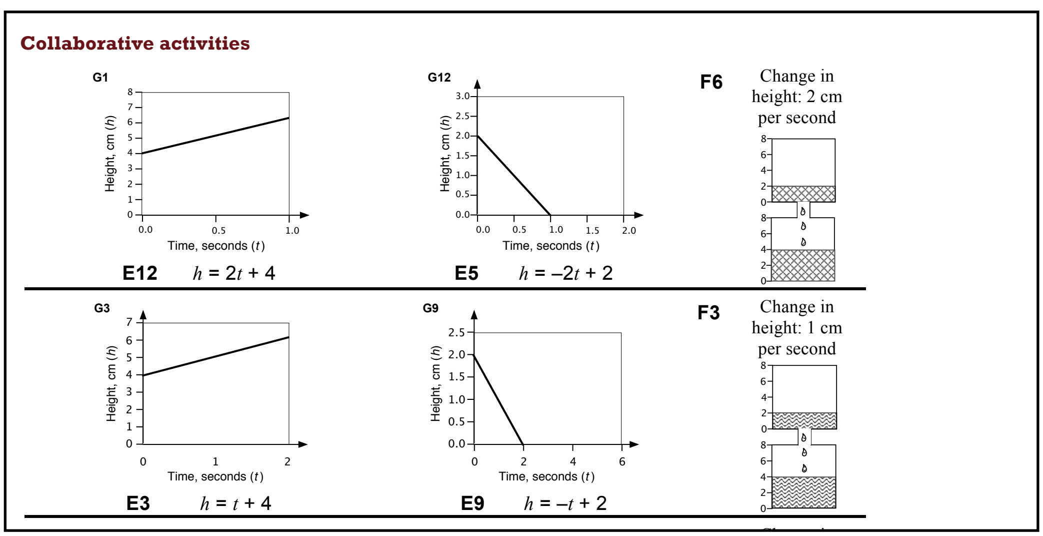


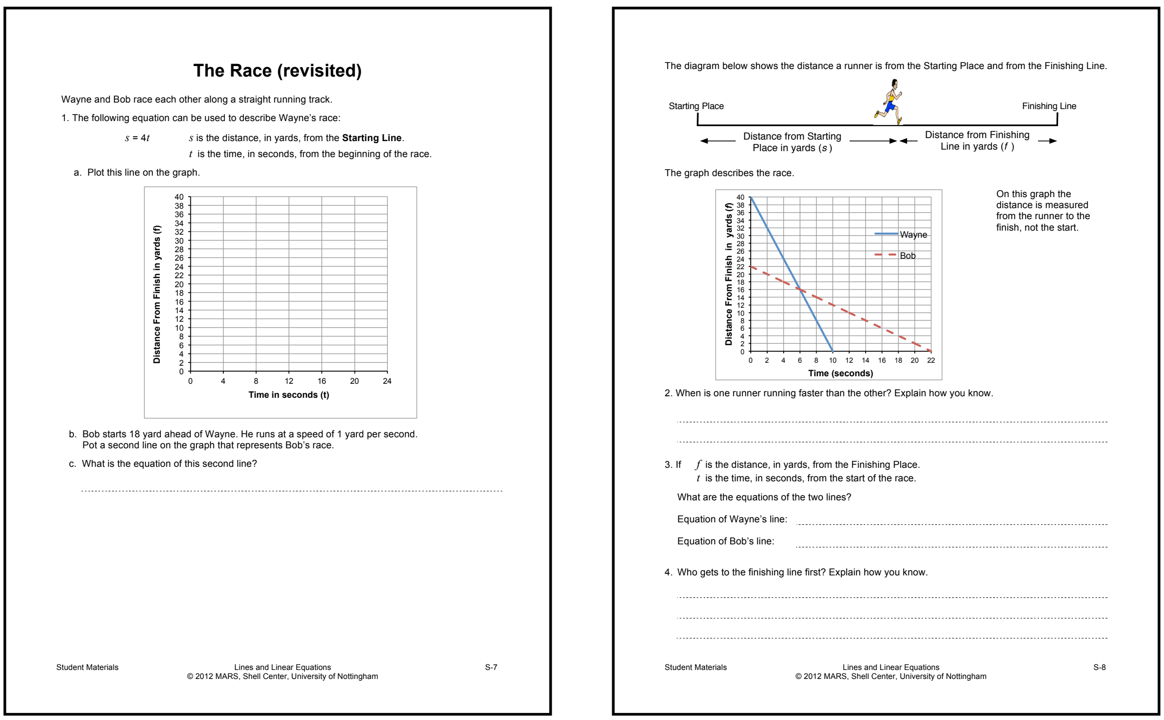
Figure 2.8. First activity – sample matches

Discussion phase

The next phase of the lesson is the class discussion. The teacher asks groups to share how they found their matches and to share patterns that they noticed. Other groups are encouraged to look for those same patterns on their posters. The students might discuss how they filled in missing information on the cards, using particular examples. The lesson guide suggests that the teacher provide the equation $h = 5t + 1$, then ask students if this represents the bottom or top of a container and to find the matching equation.

Post-assessment

The final phase of the lesson, listed in the guide as "follow-up lesson," is the post-assessment (Figure 2.9). The post-assessment has a context and questions similar, but not identical to, those of the pre-assessment. During the post-assessment, students are given back their pre-assessments with feedback in the form of questions to help guide the revision of their thinking. Neither the pre-assessments nor the post-assessments are scored for an evaluative grade. The primary purpose of the post-assessment is for students to revisit and improve upon their earlier work based on what they learned during the activity and class discussion.



The Race (revisited)

Wayne and Bob race each other along a straight running track.

1. The following equation can be used to describe Wayne's race:

$s = 4t$ — s is the distance, in yards, from the **Starting Line**. t is the time, in seconds, from the beginning of the race.

a. Plot this line on the graph.

b. Bob starts 18 yard ahead of Wayne. He runs at a speed of 1 yard per second. Pot a second line on the graph that represents Bob's race.

c. What is the equation of this second line?

Student Materials — Lines and Linear Equations — S-7
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The diagram below shows the distance a runner is from the Starting Place and from the Finishing Line.

The graph describes the race.

2. When is one runner running faster than the other? Explain how you know.

3. If f is the distance, in yards, from the Finishing Place. t is the time, in seconds, from the start of the race.

What are the equations of the two lines?

Equation of Wayne's line:

Equation of Bob's line:

4. Who gets to the finishing line first? Explain how you know.

Student Materials — Lines and Linear Equations — S-8
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Figure 2.9. Post-assessment

Flow of concept development lessons

As in the example above, in describing the flow of the concept development lessons for teachers, the researchers have found it useful to separate and emphasize the launch phase. The lesson guide emphasizes that the focus of the activity phase is on students explaining and justifying the reasoning that supports their solutions rather than completing the activity correctly or completely. Thus, the launch should orient students both to structure and (if necessary) the materials in the activity and to the expectations for interaction with each other in justifying and explaining their thinking during the activity, without specifying the mathematical processes to be used.

For teachers and students who are new to this lesson format, particularly those operating in a gradual release of responsibility model, there is a risk of using the launch as an example of the mathematical approach to be used, constraining the students from revealing misconceptions or finding different productive approaches. Alternatively, some teachers, recognizing the emphasis on student-centered work, provide so little guidance before the activity that students do not know what they are supposed to be doing. This lack of structure can lead to students getting unnecessarily tangled in the specifics of the activity. Our framing of the introductions in the lesson guide as *launching* the work of the collaborative activity is intended to focus teachers on clarifying the task and the terms without guiding the mathematical processes that students should use.

Making the formative assessment in the concept development FALs explicit

As discussed in Chapter 1, each concept development lesson can be viewed as a single complete formative assessment cycle, while also containing one or more embedded formative assessment cycles (shown in Figure 2.10). At the lesson level, information about student thinking is elicited in the pre-assessment and analyzed by teachers before launching the activity and class discussion. The information about student thinking from the pre-assessments is used to inform the teacher's interactions with students during the lesson phases. The launch, collaborative activity, and whole class discussion provide the basis for instructional activities that can be used to move student thinking forward. In addition, the teacher's written comments on the pre-assessments provide individual feedback that student can use to guide the revision of their thinking after they engage in the learning activities. The lesson guide provides a table of common issues that may emerge on the pre-assessments to help teachers interpret the thinking that may be behind students' responses. The common issues table also includes suggested questions for teachers to use for feedback on the pre-assessments. The students are supported in reflecting on their own progress toward the learning goals when they review their work and the teacher's questions on the pre-assessment during the post-assessment. The post-assessment is an opportunity for students to reflect on their learning and can serve as the start of another formative assessment cycle.

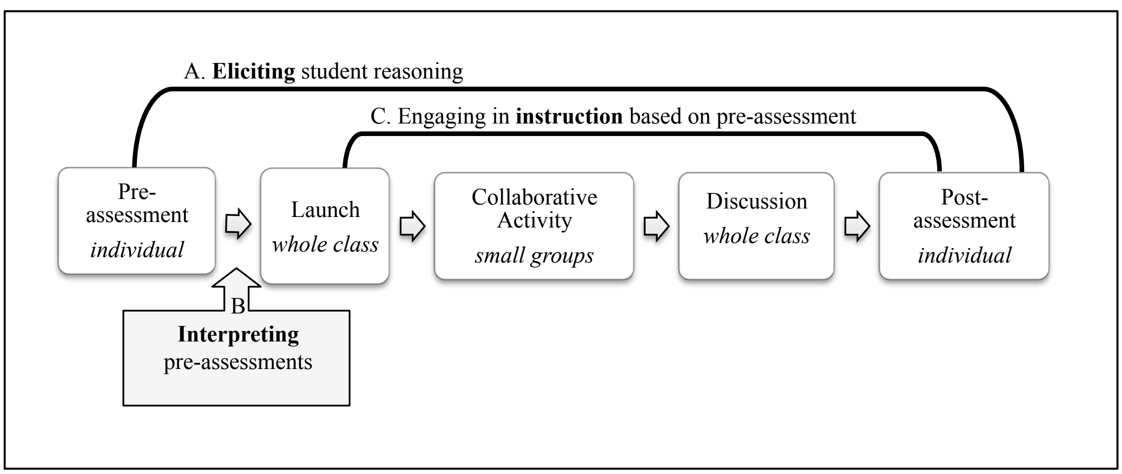


Figure 2.10. Connecting formative assessment cycle with phases of the FALs

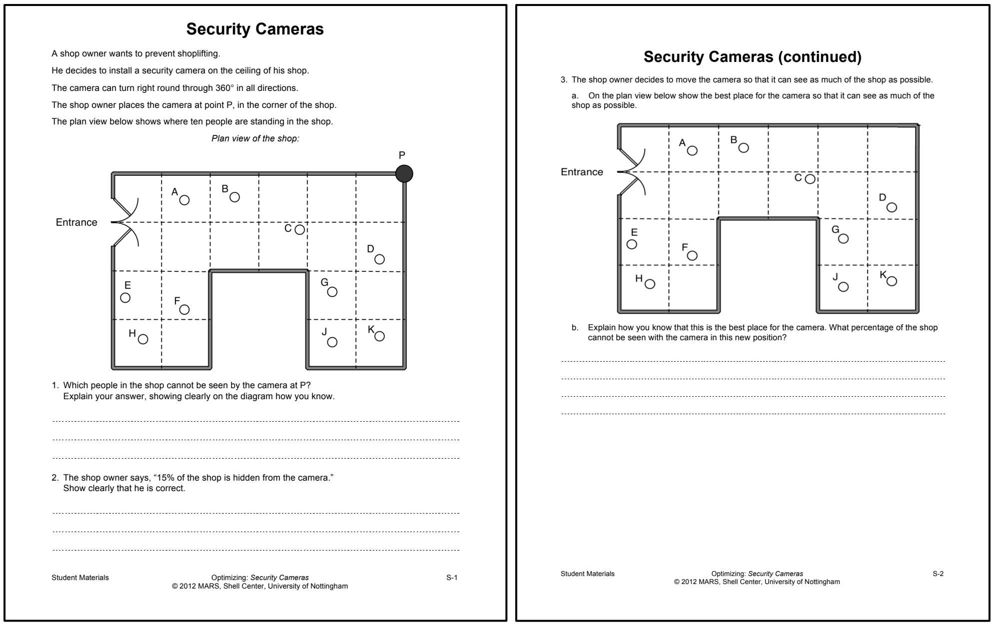
At a finer grain size, the lesson launch and collaborative activity can be used as opportunities for the teachers to elicit information about the students' thinking, and the whole class discussion can be considered as the instructional response to the student thinking. Thus, the activity and whole class discussion comprise a smaller assessment cycle within the larger formative assessment cycle.

Designers' suggested lesson placement for concept development lessons

The FAL design team suggests that concept development lessons are most useful about two-thirds of the way through a unit. At this point in the unit, students have learned enough to be able to discuss and refine their ideas about the concept, but still have room to build on the thinking that emerges from the lesson. The lesson designers also suggest that teachers may use them at the beginning of a unit to review knowledge from a prior course or unit that will be built on in the next unit. They do not encourage using concept development lessons to introduce a concept that students have not yet studied, because the students may not have enough background to effectively engage with the materials.

Problem solving lessons

Problem solving lessons are designed to support students in applying mathematical understanding to non-routine problems. Problem solving lessons differ from concept development lessons in that all of the work in a problem solving lesson is focused on a single complex problem situation. For example, students might use what they know about lines, areas, ratios and percentages to develop strategies for approaching a problem in a real world context, as in Security Cameras (Figure 2.11); or they might be asked to generate strategies for systematically investigating and justifying patterns generated in a mathematical context, as in The Difference of Two Squares (Figure 2.12). In problem solving lessons, the focus is on the construction, evaluation, comparison, and refinement of the strategies used to approach the problem. These problems also frequently involve mathematical modeling where students need to determine what assumptions they are making in order to be able to model the problems. The developers caution teachers to avoid posing specific suggestions or questions to guide students to use a particular approach, but suggest that teachers instead ask questions about the students' approaches and strategies such as observing patterns.


Security Cameras

A shop owner wants to prevent shoplifting.

He decides to install a security camera on the ceiling of his shop.

The camera can turn right round through 360° in all directions.

The shop owner places the camera at point P, in the corner of the shop.

The plan view below shows where ten people are standing in the shop.

Plan view of the shop:

1. Which people in the shop cannot be seen by the camera at P?
   Explain your answer, showing clearly on the diagram how you know.

2. The shop owner says, "15% of the shop is hidden from the camera."
   Show clearly that he is correct.

Student Materials Optimizing: *Security Cameras* S-1
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Security Cameras (continued)

3. The shop owner decides to move the camera so that it can see as much of the shop as possible.
   a. On the plan view below show the best place for the camera so that it can see as much of the shop as possible.

   b. Explain how you know that this is the best place for the camera. What percentage of the shop cannot be seen with the camera in this new position?

Student Materials Optimizing: *Security Cameras* S-2
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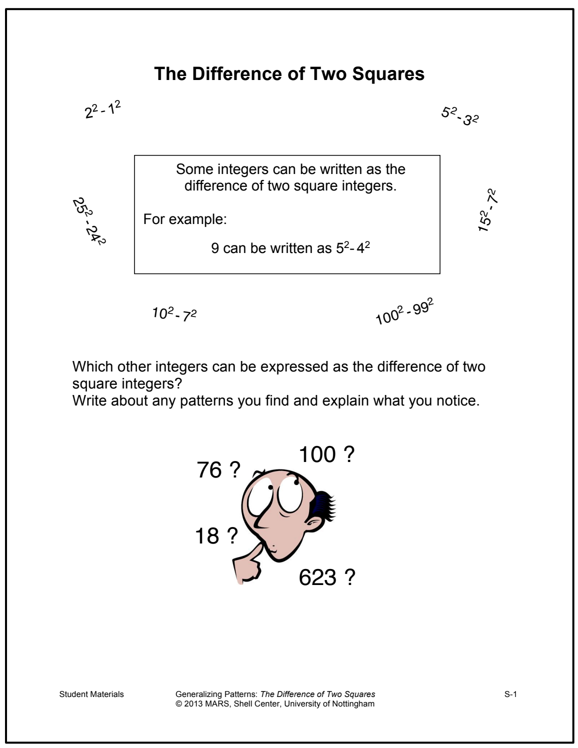
Figure 2.11. Pre-assessment for Security Cameras


The Difference of Two Squares

Some integers can be written as the difference of two square integers.

For example:

9 can be written as $5^2 - 4^2$

Which other integers can be expressed as the difference of two square integers?
Write about any patterns you find and explain what you notice.

Student Materials Generalizing Patterns: *The Difference of Two Squares* S-1
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Figure 2.12. Pre-assessment for The Difference of Two Squares

The lesson structure for problem solving lessons is slightly different than for concept development lessons, because these lessons focus on the same problem from the pre-assessment, through the activities and whole class discussion, to the post-assessment.

Pre-assessment phase

For the pre-assessment in problem solving lessons, students work individually to develop an initial approach or solution. The teacher collects these first attempts and writes comments to encourage the students to elaborate their reasoning or consider alternative approaches.

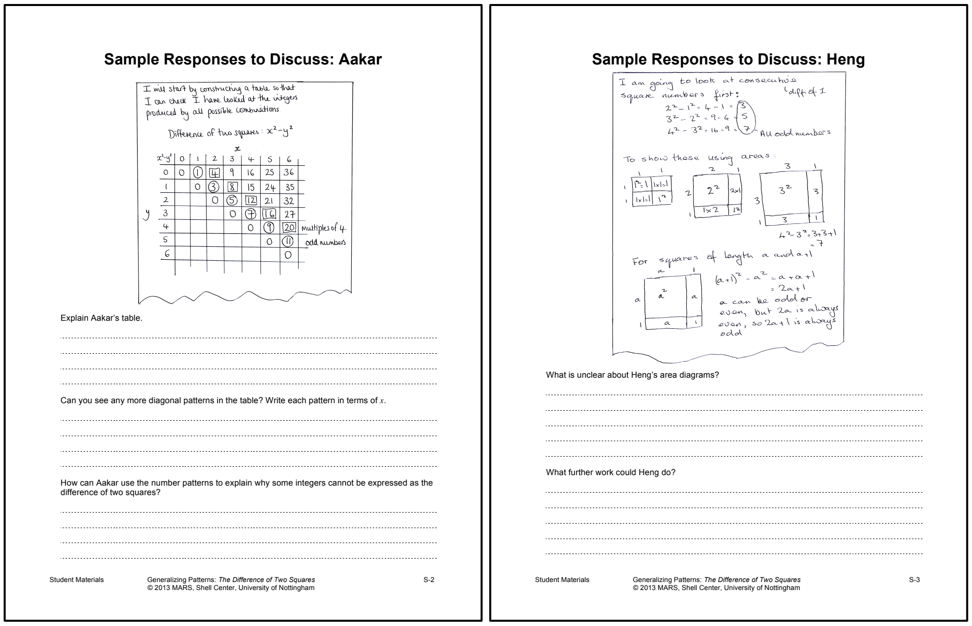
Activity phase

At the beginning of the lesson, students are given back their initial work with comments, and given an opportunity to read and respond to some of the comments. They then engage in three different collaborative activities. First, they work with partners or in small groups to share strategies and together decide to pursue and improve upon one of the strategies. Next, students visit other groups to share different approaches to the problem. During this activity, students are encouraged to ask questions about the other groups' approaches that will help their own group refine their work. They are able to bring ideas back to use with their own group.

The third activity in the problem solving lessons is looking at sample student work. Figure 2.13 shows the sample student work for the Difference of Two Squares. These pieces of incomplete sample work allow students to consider approaches that may not have been mentioned in their group or in their visits to other groups. The sample work also allows students to critique the presentation of the strategies and suggest ways to make the strategy more accurate or effective as well as making the explanation of the reasoning clearer.

Discussion and post-assessment phases

This is followed by a class discussion about the different strategies in the sample student work and a comparison with the work of the students in the class. For the post-assessment, students revisit their own work on the problem, either individually or in groups, and apply what they have been learning in order to revise their strategies and clarify their explanations to create an improved final solution.



Sample Responses to Discuss: Aakar

I will start by constructing a table so that I can check I have looked at the integers produced by all possible combinations.

Difference of two squares: $x^2 - y^2$

x^2-y^2	0	1	2	3	4	5	6
0	0	1	4	9	16	25	36
1		0	3	8	15	24	35
2			0	5	12	21	32
3				0	7	16	27
4					0	9	20
5						0	11
6							0

multiples of 4
odd numbers

Explain Aakar's table.

Can you see any more diagonal patterns in the table? Write each pattern in terms of x.

How can Aakar use the number patterns to explain why some integers cannot be expressed as the difference of two squares?

Student Materials Generalizing Patterns: *The Difference of Two Squares* S-2
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Sample Responses to Discuss: Heng

I am going to look at consecutive square numbers first: (diff. of 1)
$2^2 - 1^2 = 4 - 1 = 3$
$3^2 - 2^2 = 9 - 4 = 5$
$4^2 - 3^2 = 16 - 9 = 7$
All odd numbers

To show these using areas:

$4^2 - 3^2 = 3 + 3 + 1 = 7$

For squares of length a and a+1

$(a+1)^2 - a^2 = a + a + 1 = 2a + 1$

a can be odd or even, but 2a is always even, so 2a+1 is always odd

What is unclear about Heng's area diagrams?

What further work could Heng do?

Student Materials Generalizing Patterns: *The Difference of Two Squares* S-3
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Figure 2.13. Sample student work for The Difference of Two Squares

Problem solving lesson placement

The lesson designers recommend that problem solving lessons be used every four to eight weeks. These lessons focus on problem solving – monitoring, selecting, and evaluating solution strategies, explaining and justifying assumptions and conclusions. In order for students to be able to engage in these practices, the mathematical skills and concepts needed to approach the problem must be accessible to the students, and it should be possible to make significant headway on the problems using more than one strategy. It is also important that the students have not recently solved a problem so similar to this problem that they mostly recreate the same solution strategy previously used.

FAL lesson structure

Problem solving and concept development FALs have a similar format: students work alone on the pre-assessment; work together on one or more collaborative activities; and participate together in the whole class discussions. Although there are a range of activity types, e.g., matching cards with different representations of the same information, classifying statements as sometimes, always or never, and analyzing sample student work, the purpose of the activities is for the students to make their thinking public and to make sense of the problems collaboratively.

Potential opportunities for teacher learning

Beyond the opportunities that lessons provide for supporting student learning, these lessons are also intended to support teacher learning. Teachers can learn about student thinking by listening, observing, and recording different students explaining their reasoning, and by asking students to elaborate on those explanations. Teachers might learn how to facilitate group interactions better, allowing them to elicit ideas from more students. They may also learn how to have students present and share their thinking in ways that allow learning to be shared by the class. As this dissertation will show, what teachers actually learn from teaching these lessons depends very much on their goals and their choice of focus.

The lesson designers have specific suggestions about the teacher's role during these lessons. Teachers are to prompt students to reflect and reason, and to ask questions that support students in revisiting and revising their own thinking. Teachers are not supposed to provide answers or validate solutions.

Pilot use of the FALs identified two specific areas where teachers sought additional tools: Analyzing and providing feedback on the pre-assessments; and tracking and structuring students' ideas to be shared during the group discussion. As a result, teachers in this study were provided with two optional resources developed by the research team that supplemented the materials in the lesson guides.

Pre-assessment tally

First, a pre-assessment tally form was developed as a tool to help teachers keep track of issues that emerged on the students' pre-assessments and count the number of students with similar conceptions. Figure 2.14 shows a blank version of this tool and a completed version based on the lesson Lines and Linear Equations. Each lesson guide includes a table of common issues to be used for a similar purpose, but during the pilot study, teachers mentioned difficulties in matching their students' work with the entries in the common issues table. This tally form was intended to both support the teachers in providing meaningful written feedback on their students' pre-assessments and to guide them to look for commonalities and outliers in types of thinking.

Pre/Post-assessment Analysis
Lesson: ____________________
(to inform teaching the lesson and giving feedback)

Look through the pre-assessments and make note of issues that you see coming up for students in different parts of the assessment. After you have complete your own analysis, look at the analysis in the lesson guide.

Common Issues	Question #	Tally (PRE)	Tally (POST)	Questions to help students reflect on this issue
Did not answer question _____ on the pre-assessment				
Did not answer question _____ on the pre-assessment				
Did not answer question _____ on the pre-assessment				

Pre/Post-assessment Analysis
Lesson: Lines & Linear Equations
(to inform teaching the lesson and giving feedback)

Look through the pre-assessments and make note of issues that you see coming up for students in different parts of the assessment. After you have completed your own analysis, look at the analysis in the lesson guide.

Common Issues	Question #	Tally (PRE)	Tally (POST)	Questions to help students reflect on this issue
Maggie is faster	1	||		
no explanation or limited	1	|||		what more can you tell me about the race?
Incorrect slope for Maggie	2	𝍸 |		How fast is Maggie going?
Incorrect y-int for Maggie	2	𝍸 𝍸 |		where did Maggie start?
limited description	3	𝍸 |		who wins the race? How long does it take for each runner to reach the finish line?
Incorrect graph(s)	4a	𝍸		How long does it take ___ to reach the finish line?
Did not answer # 2		||		
Incorrect equation	4c	𝍸		
Did not answer question 4a on the pre-assessment		𝍸 |		
Did not answer question 4b on the pre-assessment		𝍸 𝍸 ||		
Did not answer question 4c on the pre-assessment		𝍸 𝍸 |||		

Figure 2.14. A blank and completed pre-assessment tally form

Monitoring sheet

Second, to provide more support for planning and orchestrating the discussion phase of the lesson, teachers were provided the article "Orchestrating Discussion" (Stein et al., 2008a) which describes five practices – anticipating, monitoring, selecting, sequencing, and connecting – that can support teachers' facilitation of whole group discussions of complex mathematical problems. The article contains a tool for monitoring the strategies that emerge during the exploratory phase of a lesson – or in the FALs, the activity phase – so that teachers can make decisions about which student should present a given strategy and about how to sequence the presentations in ways that support connections between different methods. The teachers in this study were provided with a blank copy of that monitoring sheet to adapt for their lessons if they wanted to try the five practices (Figure 2.15).

Monitoring Sheet

Lesson/Problem: _________________

		Who/notes	Order
Other:			

Figure 2.15. Blank monitoring sheet adapted from Stein et al. (2008)

Data collection

Six of the eight teachers' focal classes were observed during one or two routine lessons. Ms. Castle initially rescheduled and then cancelled the observation of her routine teaching but agreed to be observed teaching the FALs. Ms. Heather was observed teaching a routine lesson, but the observation was not videotaped. The launch, collaborative activity, and discussion phases of three to five FALs during the 2013–14 school year were observed and videotaped for all eight teachers. During whole class discussions, the camera was positioned to capture as much of the class as possible. During the small group interaction, the camera followed the teachers' movement between groups with a lapel microphone recording the interactions between the teacher and small groups of students.

All student work, including the pre-assessments, post-assessments, and student created posters, was collected after the teachers had finished using them for instructional purposes. In many cases, the student work was scanned and left at the school so that the teacher or students could continue to refer to them. Any additional materials that teachers developed for use during the FALs, including pre-assessment tally forms, monitoring sheets, graphic organizers, or overhead slides were also collected.

Prior to the study, each teacher completed an application indicating his or her goals for participating in the study. Before each lesson, teachers completed a written pre-lesson reflection, and following each lesson, the teachers completed a post-lesson reflection or interview. In early March, after all of the teachers had taught two or three lessons, they completed a mid-project survey.

The most challenging part of the data collection for both the teachers and the research team was the teachers' completion of the pre- and post-lesson reflections. Several of the teachers consistently did not turn in a pre-lesson reflection or did not submit a post-lesson reflection. In other cases, the teachers' responses in the written reflections were cursory, superficial, or incomplete. This seemed to be an instance where having teachers provide us with their thinking in this format was an imposition on their time and did not serve a useful purpose for the teachers. Those who completed the reflections did so in a perfunctory manner while others simply did not complete them. In several cases, we were able to record interviews with the teachers before, during or after the lessons. In these cases, I have used these interviews for analysis in place of the reflections.

All teachers participated in four or five collaborative planning meetings. Two of these meetings were recorded to provided evidence of the types of activities that teachers engaged in when planning lessons.

Table 2.3 gives the number and type of FALs taught by each teacher. Ms. Golden and Ms. Heather co-planned all five of their lessons. Ms. Elmore had a student teacher, Ms. Iris, for most of the school year, with whom she collaborated in planning the lessons. Other teachers taught the same lessons, but either did not collaborate during the lesson or one teacher taught the lesson only after learning about the other teacher's experience in teaching the lesson, so the collaboration only supported the second teacher. In all, 33 FALs were observed: 21 concept development lessons and 12 problem solving lessons. A complete list of the lessons taught appears in the Appendix.

Table 2.3. Summary of FALs observed

Teacher	observed	CD	PS
Ms. Amador	4	3	1
Ms. Belle	3	3	0
Ms. Castle	2	2	0
Mr. Davidson	5	4	1
Ms. Elmore	5	4	1
Ms. Feldman	4	3	1
Ms. Golden	5	1	4
Ms. Heather	5	1	4
Total lessons observed	33	21	12

Data analysis

This section gives an overview of the analytic techniques used in this dissertation. The approaches for analysis in this dissertation differ by chapter. Chapter 3 addresses the question of how the choices that teachers made in selecting, planning, and implementing the lessons influenced the opportunities that they had to learn to enact formative assessment processes in their teaching. Chapters 4 and 5 present two case studies (Yin, 2013) of teacher learning specifically related to the activity and class discussion phases of the lesson. These cases provide evidence of the different ways that teachers' goals and orientations influence change in their teaching practices.

Analysis of lesson selection, placement, and planning

The phases of the enacted FALs were compared with the lesson guides. Each phase was coded as completed *consistent* with the lesson guide, significantly *altered*, *added or omitted*. The placement of each FAL was compared with the course syllabus, surrounding lessons, and the objectives on the teacher's pre-lesson reflection to determine how the lesson had been placed relative to the development of mathematical concepts and skills in the course.

The placement of enacted concept development lessons was coded as review from a prior course, review from earlier in the current course, partway through a unit of study on the relevant concepts, or introductory to a topic.

The placement of problem solving lessons was coded differently, based on the goal of examining and comparing multiple strategies for approaching the central problem situation. Problem solving lesson placements were coded *appropriate* if students' prior knowledge and experience allowed them access to multiple solution strategies as evidenced by the student work during the lesson. A lesson was coded as *too soon* if the students did not know enough of the mathematics in the problem to have access to more than one strategy for work on the problem or when the problem was placed so close to a similar problem for which there is a single known solution strategy that students do not consider alternative approaches.

Teachers' comments on the pre- and post-lesson reflections and the mid-project survey were compared and coded for common themes, particularly in response to their and their students' learning from the lessons and to their use of particular tools.

Analysis of student explanations

The analysis for Chapter 3 centers on the activity phase of the FALs. We chose this phase for analysis because it was used by all the teachers in every observed FAL enactment. Also, the activity phase was the phase most frequently taught in alignment with the lesson guide. Because one of the primary formative assessment goals of the activity phase is the eliciting of student explanations, we analyzed data from this phase for instances of student explanations in the presence of the teacher.

Coding of student explanations

The small group activity phase of each lesson was parsed into episodes, called *group visits.* A group visit is an interaction of more than 20 seconds between the teacher and a small group of students working on the problem. The number of turns of student dialog that contained a *student explanation,* as described below, was recorded as a single *explanation*. A new instance was not counted if the same student simply repeated the same explanation in a future turn of dialog without further elaboration or justification.

Rules for determining what counted as an explanation were initially established using transcripts of portions of lessons from several different classrooms in the study. Once the coding rules were established, two coders coded several episodes directly from video and reconciled any discrepancies. Finally, the remaining episodes were coded directly from video, with coders noting any ambiguous cases. These cases were resolved by discussion with the lead researcher.

Instances of elicited student reasoning were identified and, using a classroom layout diagram for each lesson, these explanations were coded by student and group. This allowed for further analysis of the distribution of the teacher's group visits throughout the class and the distribution of the student reasoning being elicited.

The data were then aggregated to show the total time the teacher spent in group visits, the rate of explanations per hour, and the number of group visits per hour. This analysis was also used to show the number of group visits per hour, the number of group visits per group, and which students ideas the teacher had been exposed to.

Counting student explanations

Each time that a student verbalized information about how they were thinking about the mathematics in the question, problem, or activity was coded as a potential student explanation. In order to count as an explanation, the thinking expressed must be coherent and extend beyond using procedures or repeating definitions, but it need not be correct. For example, a teacher asked a student "What is the definition of 'mean'?" and the student responded by reading a definition from her notes. Although this was an extended turn of student dialog, it was not considered an explanation. Similarly, a student telling which frequency graph and table of measures of central tendency her group had matched together without justifying the match, even if the match was correct, did not count as an explanation.

On the other hand, consider a student who explains how she found the mean of data shown in frequency graph using the number of possible outcomes rather than the number of data points as the divisor. Although her process and answer are incorrect, her explanation reveals how she interprets the frequency graph as a representation of data, and what she understands about the mean. Therefore, this would be counted as an explanation.

If a student shares multiple explanations in the same turn of dialog, it was still coded as one explanation. If a second student or the teacher participated in the discussion between two turns of explanation from the same student, these were counted as two explanations, as long as the student added some new idea to the first explanation. A simple reiteration of the same idea was not coded as a new explanation.

Chapter 3

How Lesson Enactments Influence Opportunities for Teachers to Engage In and Learn About Formative Assessment

Introduction

The Formative Assessment Lessons (FALs) were designed with ambitious learning goals for both students and teachers. As curricula, the FALs were designed to enable students to develop rich understandings of mathematical content and practices. But, equally important, they were designed to provide opportunities for teachers to develop deeper understandings of formative assessment and to become more effective at implementing it – not only while teaching the FALs, but in their ordinary practice as well.

This chapter focuses on the affordances of the FALS for teacher learning, and the use that teachers did or did not make of those affordances. I begin by discussing how the FALs were designed to scaffold teacher learning, describing one lesson implementation that shows that it was possible for teachers to implement the FALs with some fidelity to the designers' intentions. I proceed to discuss various ways in which teachers, who had their own goals for lesson implementation and were of course subject to the standard pressures of time, coverage, and testing, did or did not make use of the potential learning opportunities in the FALs. I then summarize FAL usage across the teachers, as a way of contextualizing my selection of two teachers for the more detailed case studies that appear in chapters 4 and 5.

It is important to stress that the FALs provide opportunities that may or may not be used, depending on the teacher's knowledge, beliefs, and goals. By way of analogy, consider how various people might use an elegant cookbook. Some might try to learn from it, starting by trying to follow recipes closely (if imperfectly) and then, as they become more practiced, taking some liberties. Presumably there would be some transfer, as the techniques learned with the cookbook's support become part of their repertoire. Other cooks might take shortcuts at the beginning, either in technique or by combining or skipping steps. Not only would the meals they produced not be as good as they might have been, but those cooks would have learned less as well – possibly emerging with a few more dishes in their repertoire, but being little improved as cooks. Still others might possess the cookbook for the pleasure of reading about food and not improve at all as cooks. Similarly, some teachers might try to learn very explicitly from the FALs; some might learn aspects of formative assessment from using them; and some might teach the FALs in ways that eliminate opportunities to learn what the designers intended.

In Chapters 1 and 2, I described how the FALs could be envisioned as scaffolding formative assessment cycles. Here I briefly review the FALs' lesson structure and the affordances it might offer, and then discuss the reality of implementation. My purpose is to understand the obstacles that teachers face in engaging with the formative assessment lessons as designed, in order to inform the creation of materials for teacher support and professional development.

Intended FAL usage

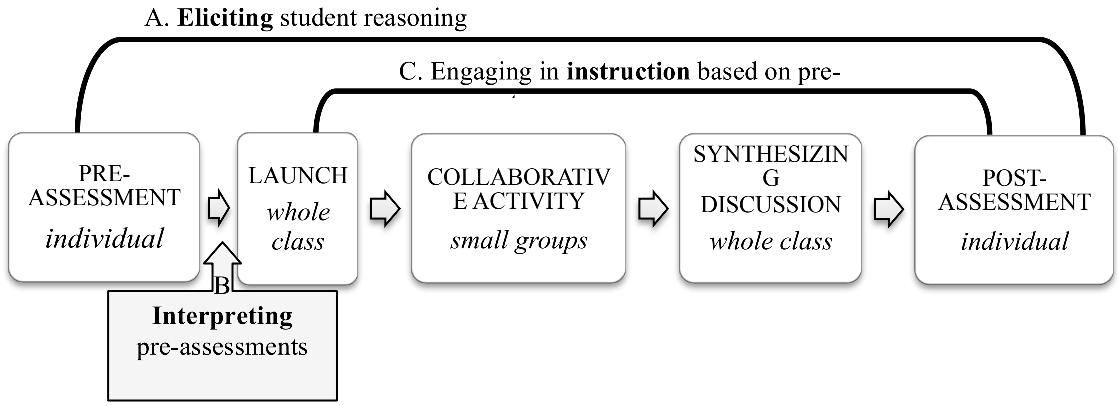

Figure 3.1. The designers' intentions

As depicted in Figure 3.1 and described in Chapters 1 and 2, the designers intended the FALs to provide multiple opportunities for students and teacher to get student reasoning "out in the open" and to work with it. The pre-assessment helps the teacher to elicit and understand the most common issues that students face in understanding a core concept, thus preparing the teacher for some (mis)understandings that might otherwise seem to come "out of the blue" while teaching. In addition, the common issues tables provided in the lesson guides supports teachers in interpreting the student work on the pre-assessment and in preparing responses to issues that are likely to come up. With this support, the teachers have to do less in-the-moment invention, making it easier to respond to student comments. The four subsequent phases – the launch, the cooperative activity, the class discussion, and the post-assessment, are all parts of the instructional response to the information obtained from the pre-assessment. The tasks in the activity are intended to provoke students, with the support of peers and teacher, to confront and resolve conflict in their current conceptual understandings. In the synthesizing discussion, students share their new understanding of the central concepts or problem solving strategies used in the lesson. During this phase, the teacher uses her or his observations of the students' work during the activity phase to decide which ideas should be shared and how the ideas should be connected. In the last phase, the post-assessment, students re-engage with their initial understandings. The feedback on the pre-assessment should guide them to reference their experiences with the activity and discussion, and to revisit their initial responses. It is an opportunity for students to assess the progress they are making toward the learning goals and determine where they are still uncertain.

The FALs are designed to provide scaffolding for the teachers in the instructional segment of the formative assessment cycle (Part C of Figure 3.1). The cooperative activity in a FAL is designed to support students in confronting and resolving the frequently occurring conceptual issues found on the pre-assessments. Thus, in teaching a FAL, teachers are not expected to design the instructional response to the pre-assessment, rather they can observe how students respond to the activity and discussion while facilitating these phases of the lesson. The teacher's role throughout this process is to use knowledge of individual students' misconceptions as well as the whole class's experiences to guide the students' engagement with the activity, highlight and connect learning during the whole class discussion, and provide feedback on the pre-assessment that will allow students to reflect, self-assess, and further revise their thinking during the post-assessment. However, in order for this to happen, the teachers must select and place lessons where the students have sufficient prior knowledge of the concepts to engage in the activity and move their thinking forward.

Note that there are various ways in which the potential impact of the lesson, on both student learning and teacher learning, can be affected negatively. It is possible, for example, to omit the pre-assessment and begin with the launch (i.e., implementing just Part C of the lesson cycle). However, the successful implementation of Part C by itself requires gleaning information about student understandings "on the fly" and reacting to it in the moment. Doing so is not easy.

Similarly, lesson placement affects lesson usage. The choice of where to place a lesson reflects the teacher's goals for the lessons as well as the possibilities for what prior knowledge students will be able to bring to the lesson. Teaching a lesson that *introduces* a substantial amount of new content may engage the students in interesting activities, for example – but the lesson no longer serves as a mechanism for teacher or student to reflect upon and deepen understandings of content that has already been studied.

In the following section, we look at an episode in which a teacher works through an entire lesson with reasonable fidelity to the designers' intentions. This shows that the lessons *can* be taught as intended. Later in the chapter, we examine more typical enactments of the FALs.

An enactment of a FAL with a formative assessment cycle

Ms. Heather's enactment of the concept development FAL Comparing Lines and Linear Equations provides a solid example of a complete formative assessment cycle supported by the FAL materials. In this lesson, there are two water containers connected at the necks to form a closed system with 6 units of water. Water flows out of the top container into the bottom container. The students match graphs of the flow with equations and diagrams. (A detailed description of this lesson appears in Chapter 2 as an illustration of a concept development FAL.)

Ms. Heather and her colleague, Ms. Golden, selected this lesson to teach just before they began a unit on systems of linear inequalities and linear programming in a second year integrated math class. Their goals in using this lesson were to find out what the students knew about linear equations from the prior course and to support their students in connecting meanings of slope and y-intercept in graphs and equations and interpreting them in a context. In her pre-lesson reflection, Ms. Heather wrote about what she hoped to find out about her students' thinking in teaching this FAL:

I anticipate learning how much my students can apply their knowledge of linear functions to a real-life context and use mathematics to make sense of a real situation, especially regarding rate of change (slope) and the relationship between filling and emptying/distance from start vs. distance from finish.

She also wrote about her concerns for the lesson and questions that she was wrestling with about the implementation of the lesson:

I am concerned about how to manage the class if some groups finish their matching before others. How much time should I give for the matching activity?
How am I going to have groups share the work they've done with the rest of the class?

Several days before the lesson, Ms. Heather's students worked on the pre-assessment in class. Ms. Heather used a pre-assessment tally along with the common issues table in the lesson guide, shown in Figure 3.2, to categorize student responses and generate written questions for the students about their work.

Common Issues	Question #	Tally (PRE)	Tally (POST)	Questions to help students reflect on this issue
Maggie is faster	1	II		
no explanation or limited	1	III		what more can you tell me about the race?
Incorrect slope for Maggie	2	卌I		How fast is Maggie going?
Incorrect y-int for Maggie	2	卌卌I		where did Maggie start?
limited description	3	卌I		who wins the race? How long does it take for each runner to reach the finish line
Incorrect graph(s)	4a	卌		How long does it take ___ to reach the finish line?
Did not answer #2		II		
Incorrect equation	4c	卌		
Did not answer question 4a on the pre-assessment		卌I		
Did not answer question 4b on the pre-assessment		卌卌II		
Did not answer question 4c on the pre-assessment		卌卌III		

Common issues:	Suggested questions and prompts:
Student assumes Maggie is running the fastest because at five seconds her line is above Emma's line (Q1.)	• During the race, does Emma's or Maggie's speed change? • How can you figure out the speed of each runner?
Students description of the race is limited For example: The student does not mention speed, or the time it took for each person to complete the race.	• What more can you tell me about the race? • Does one runner overtake the other one? If so, at what point does this happen? • Who wins the race? How far ahead are they when they cross the finishing line? • What are the race times for each runner?
Student misinterprets the scale For example: The student fails to notice the distance goes up in 5s not 1s (Q1.) Or: The student does not notice the scales for the axes on the two graphs are different.	• What is the scale on the vertical/horizontal axis for each graph?
Student draws an incorrect graph (Q4) For example: The student draws a graph with a positive slope. Or: The student draws a slope with an incorrect y-intercept, e.g. $f = 30$. Or: The student draws a non-linear graph. Or: The student draws an incomplete graph.	• As the race progresses will the distance, f, increase or decrease? How can you show this on your graph? • At the beginning of the race, how far are the runners from the finishing line? How can you show this on your graph? • Does Maggie run at a constant speed? How have you shown this speed on your graph? • Your graph should represent all of the race. When will Emma/Maggie have completed the race? How can you show these points on the graph?
Student's equations are incorrect For example: The student writes an equation without a variable for the time.	• Explain your equation in words. • Does your equation describe how the distance changes as the race progresses?

Figure 3.2. Ms. Heather's identification and tally of common issues on pre-assessments (left) and the common issues table from the lesson guide for Comparing Lines and Linear Equations

From the tally marks, it appears many of the students did not answer substantial portions of the fourth question. This makes using formative assessment really challenging for the teacher because the students did not provide information. Table 3.1 shows the issues that Ms. Heather identified on the students' pre-assessments (given in bold) with similar or corresponding issues from the common issues table in the lesson guide. Several of the issues that Ms. Heather identified were precisely what was anticipated by the lesson designers. In some cases, her instructional response used the questions from the lesson guide, and in others cases she wrote her own questions.

Table 3.1. Common issues and questions for students: Ms. Heather and lesson guide

<table>
<tr><th>Teacher: Pre-assessment Tally Common Issues</th><th>Lesson Guide: Common Issues Table</th><th>Teacher: Questions for Students</th><th>Lesson Guide: Questions for Students</th></tr>
<tr><td>**Maggie is faster (Q1)**</td><td>• Student assumes Maggie is running the fastest because at five seconds her line is above Emma’s line (Q1)</td><td>**(None)**</td><td>• During the race, does Emma’s or Maggie’s speed change?
• How can you figure out the speed of each runner?</td></tr>
<tr><td>**No or limited explanation (Q1)**</td><td rowspan="2">• Students description of the race is limited
• For example: The student does not mention speed, or the time it took for each person to complete the race.</td><td>**What more can you tell me about the race?**</td><td rowspan="2">• What more can you tell me about the race?
• Does one runner overtake the other one? If so, at what point does this happen?
• Who wins the race? How far ahead are they when they cross the finishing line?
• What are the race times for each runner?</td></tr>
<tr><td>**Limited description (Q3)**</td><td>**Who wins the race? How long does it take for each runner to reach the finish line?**</td></tr>
<tr><td>**Incorrect slope for Maggie (Q2)**</td><td rowspan="3">• Student’s equations are incorrect
• For example: The student writes an equation without a variable for the time.</td><td>**How fast is Maggie going?**</td><td rowspan="3">• Explain your equation in words.
• Does your equation describe how the distance changes as the race progresses?</td></tr>
<tr><td>**Incorrect y-int for Maggie (Q2)**</td><td>**Where did Maggie start?**</td></tr>
<tr><td>**Incorrect equation (Q4c)**</td><td>**(None)**</td></tr>
<tr><td>**Incorrect graph(s) (Q4a)**</td><td>• Student draws an incorrect graph (Q4)
• For example: positive slope, incorrect y-intercept, non-linear graph, incomplete graph
--
• Student misinterprets the scale
• For example: The student fails to notice the distance goes up in 5s not 1s (Q1.)
• Or: The student does not notice the scales for the axes on the two graphs are different.</td><td>**How long does it take ___ to reach the finish line?**</td><td>• As the race progresses will the distance, **f**, increase or decrease? How can you show this on your graph?
• At the beginning of the race, how far are the runners from the finishing line? How can you show this on your graph?
• Does Maggie run at a constant speed? How have you shown this speed on your graph?
• Your graph should represent all of the race. When will Emma/Maggie have completed the race? How can you show these points on the graph?
--
• What is the scale on the vertical/ horizontal axis for each graph?</td></tr>
</table>

On the pre-lesson reflection, Ms. Heather summarized what she had learned from the pre-assessment and how this influenced her thinking as she was starting to teach the lesson:

> Students had trouble with the notation of the equations, which used the variables *s* and *t* instead of *x* and *y*. I anticipated that more students would misinterpret the graph to say that Maggie was going faster since her line was above Emma's, but the majority of students realized that just because Maggie was AHEAD in the race did not mean that she is running faster. Many students made the connection that the steepness of the line, or slope, is the indicator of speed.

These comments indicate that formative assessment cuts two ways: it reveals student difficulties but also shows unanticipated student strengths. Ms. Heather had focused on the errors that she was anticipating that students would make, and the students surprised her with what they knew. This is an important discovery for a teacher to make before a lesson and could have informed her response on the pre-lesson reflection about how quickly students would be able to move through the activity. Absent from these observations about the pre-assessments is how Ms. Heather used them to anticipate what she might do with the lesson activity and how the activity might help them to move forward on the issues that she identified in the table.

Ms. Heather allotted two days for launch, activity, discussion. and the post-assessment phases of the lesson. She launched the lesson using slides provided in the lesson materials and showed the students a model of the system of containers constructed from Fiji water bottles. On the first day, she only had students work on matching the graphs of the top and bottom containers. As she had been concerned might happen, some groups finished this activity much more quickly than others and the students were more concerned with making the matches than justifying their matches. She began the second day with students in different places, and gave out the cards with the equation and containers for students to add to their graphs.

On the pre-lesson reflection, she had indicated her intentions for how she would interact with students during the activity:

> I will ask them what patterns they notice in the pairs of corresponding graphs and equations. I will not answer the question "Is this right?" and instead challenge groups to justify to me WHY their matches are correct.

During the activity phase of the lesson, Ms. Heather began her interactions with groups with questions about their work such as:

> Are you ready for the pictures? I want to know if you notice any patterns in your data?
>
> All right, are you done with equations? There's one blank equation. Have you identified the equations?
>
> So what do you notice about the equations? Is there any pattern that you notice?
>
> So every time you ask me "Is this right?" have you learned what I always respond with? "How can you justify why it's right or wrong?"

As the interactions with the groups proceeded, Ms. Heather frequently ended up explaining to the students why their matches were correct or leading individual students in the group to particular correct patterns or matches.[5] As a result, by the middle of the second day, all of the groups had created posters with mostly correct matches, but not all of the students necessarily understood how to justify the matches on their posters.

Ms. Heather brought the class together for a whole class discussion. She asked the students to consider the equation $h = 5t + 1$, decide if it would represent the height of water in a top or bottom container, and write the equation for the other container. A student volunteered the equation $h = -5t + 1$, which led to a short discussion about the reason for the opposite slopes and the need for the y-intercepts of the two equations to have the sum 6. This enabled a few students to verbalize their conclusions about the connections between the containers and the relationship between the coefficients in the equations and the task context. After this discussion, Ms. Heather handed back the students' pre-assessments with the following directions:

> So I'll pass these back and I asked questions. I didn't grade you, but I did ask you questions on here that should really push your thinking. So your post-assessment is your exit ticket today. Um, hopefully you'll be able to answer with confidence every single question on the front and the back . . . Take a look at your questions. Make sure you return this to me with your post-assessment.

Here, she presented the purpose of the questions on the pre-assessment as pushing the students to reflect on their initial answers. She then handed out the post-assessment with the following explanation:

> On this Race Revisited, hopefully you looked over what I wrote on your original Race. This time we are looking at Lane and Bob . . . I'm really looking forward to comparing your original pre-assessment to The Race Revisited to see how much you learned, or how much this activity helped you, or shaped the way that you think about these linear equations.

In both of these directions to students, she positioned the role of the post assessment as helping both her and the students reflect on how much the students' thinking progressed after their work in this lesson. Figure 3.3 shows a student's pre-assessment with the Ms. Heather's comments along with the same student's post-assessment. On both of these assessments, the student creates the correct equations. However, on the post-assessment, he also correctly

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[5] This, in my experience with the Formative Assessment Lessons, is typical of their use early in a teacher's learning trajectory – it takes some time to be comfortable with the idea that students will sort out many of the content-related issues by themselves if allowed to engage with the lesson as designed. The claim here is not that the lesson was implemented as well as possible; rather, that Ms. Heather made a good faith attempt to implement all parts of the lesson as designed.

identifies which runner is faster, using his slope as justification and on the last question, he draws a line to the graph to indicate how he knows that when that runner as finished the race.

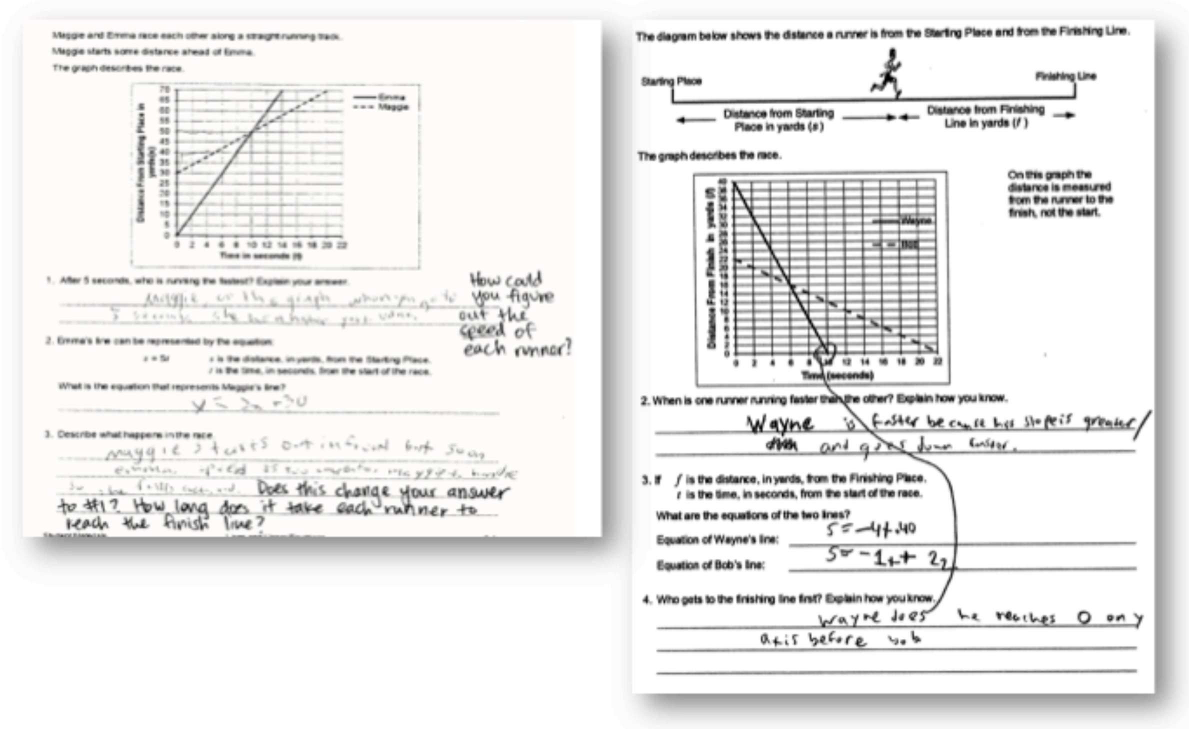

Figure 3.3. Sample of a matched student pre-assessment (left) and post-assessment (right) for Comparing Lines and Linear Equations taught by Ms. Heather. The questions posed by Ms. Heather to the students during her class are shown in dark writing on the pre-assessment.

This was the first concept development FAL that Ms. Heather taught in her class. Although many aspects of this lesson could have been better, e.g., more complete responses on the pre- and post-assessments, more student talk during group work, ways to engage more of the students in the discussion, there were many things that went very well. Ms. Heather attempted to use the pre-assessment to gauge her students' understandings prior to the lesson and to allow herself to be more responsive to the types of issues, such as reading a starting point from a graph, that they would encounter during the post-assessment. She had framed the three phases of the lesson (launch, activity, discussion) in terms of moving students forward from their initial thinking and frequently called on students to explain their reasoning to one another. From the perspective of a teacher learning about formative assessment, this was a promising start. If this were typical of the enactments of FALs in this study, then I could have investigated questions about how frequently use of FALs was needed for teachers to begin to use similar cycles of formative assessment in their routine lessons.

Unfortunately, however, this enactment, was atypical of the FAL enactments observed in this study. As a result, many uses of the FALs provided few opportunities for the teachers to experience and reflect on what it means to teach a complete formative assessment lesson.

FAL selection, placement, and implementation

In this section, I describe how the teachers selected and placed FALs in their curriculum and which phases of the FALs they used in their enactment of the lessons. Each of these decisions affected their opportunities to engage in the type of formative assessment shown in Ms. Heather's class. Although the primary unit of analysis here is individual lessons, it should be clear that for any particular teacher, the affordances that the teacher took advantage of in the enactment of the collection of FALs that he or she taught provides the major set of opportunities for his or her learning.

As indicated by the teacher applications and conversations in the first collaborative meetings, all of the teachers had seen the FALs and the MAP website before and they had a range of ideas about these lessons. All of the teachers expressed the opinion that the FALs contained interesting activities, but they had a variety of ideas about where the formative assessment was in the lesson. Three of the teachers, Ms. Belle, Ms. Castle, and Ms. Golden, had participated in a summer workshop on formative assessment in which these FALs were introduced, each phase was examined in relation to the formative assessment cycle, and the teachers collaboratively planned for teaching a FAL. The other five teachers had encountered the student materials from one or more of the FALs at a department meeting or district workshop, but had not seen the lesson guides.

Teachers' reasons for using the lessons were all focused on either learning to teach these particular lessons to their students or using lessons to further a student learning goal. None of the teachers mentioned wanting to understand more about the use of formative assessment in general as one of their goals, despite the fact that we asked what they hoped to learn about teaching from the study.

In most cases, teachers selected lessons based on the activity and the corresponding mathematical content standards. If the activity looked fun and engaging and the related standards were relevant to what they were teaching, they were willing to give the lessons a try. Ms. Castle's application contained a clear expression of this sentiment:

> I would incorporate Formative Assessment Lessons into my curriculum where the content was aligned with the activity. I would use these activities as a way for students to richly explore concepts introduced during math class and to build their critical thinking and problem solving skills. I believe these activities will push my students to understand the math content at a deep level and will be an engaging way for them to explore math content. (Ms. Castle's FAL study application, 9/2013)

Two of the other teachers, Ms. Elmore and Ms. Feldman, expressed both explicitly and implicitly that they thought a primary purpose of this study was for them to test the FALs and provide feedback to the developers. This was despite explicit explanations that the object of study how teachers were learning from their teaching of these lessons, and that this was separate from the lesson piloting for development and revision.

FAL selection and placement

Of the 33 lessons observed, 23 of the FALs taught were concept development lessons (CD) and 10 were problem solving lessons (PS). Table 3.2 shows the types of lessons, in the order in which they were taught by each teacher. Six of the teachers taught one or no problem solving lessons; two of the teachers, Ms. Golden and Ms. Heather, each taught three problem solving lessons. The other six teachers all expressed a preference for concept development lessons, because they would be more helpful in meeting the expectations of the grade level content standards.

Two of the teachers, Mr. Davison and Ms. Feldman chose to teach lessons in an advanced course in addition to the non-tracked, grade level class where they were initially observed. Mr. Davidson was observed teaching one concept development lesson in an 8th grade geometry class and Ms. Feldman was observed teaching two concept development lessons in a capstone class for students preparing for the International Baccalaureate exam. These lessons are indicated with an asterisk in Table 3.2.

Table 3.2. FAL type and order taught, by teacher

Teacher	Lesson Type and Order					Totals	
						CD	PS
Ms. Amador	CD	CD	PS	CD	–	3	1
Ms. Belle	CD	CD	CD	–	–	2	0
Ms. Castle	CD	CD	–	–	–	2	0
Mr. Davidson	PS	CD	CD	CD*	CD	4	1
Ms. Elmore	PS	CD	CD	CD	CD	4	1
Ms. Feldman	PS	CD	CD*	CD*	–	3	1
Ms. Golden	PS	CD	PS	CD	PS	2	3
Ms. Heather	PS	CD	PS	CD	PS	2	3
Totals						23	10
* lesson taught in a more advanced course.							

To produce Table 3.3 below, each of the lessons in Table 3.2 was coded for its placement relative to the course curriculum and students' prior knowledge. Coding was determined from the teacher's pre-lesson reflection description of why they chose the lesson and how it fit with what they were teaching, their students' work on pre-assessments, observations from the lesson enactment, and teacher post-lesson reflections (when available).

Based on the developers' recommendations in the *Brief Guide for Teacher and Administrators* (Mathematics Assessment Resource Service, 2013 April 9), lesson placement was coded as target (T) if it was taught part of the way through a relevant unit for a concept development lessons. Target (T) placement for a problem solving lesson was any time when the students had sufficient prior knowledge to engage with multiple strategies for approaching the central problem.

Codings of off-target concept development FAL placements

The three other codings of concept development FAL placements are: activating Prior Knowledge before starting a new unit that would build on that knowledge (coded PK); Introducing a new concept or skill (coded I); or Reviewing a concept taught earlier in the course that the teacher did not intend to build on in ensuing lessons (coded R). The lesson designers specifically mention the use of concept development lessons for accessing knowledge from a prior course to be used in ensuing lessons. For this reason, lessons coded as PK were counted with target lessons as ones that were well placed to foster teachers' use of formative assessment. Table 3.3 shows that approximately 40% (9 of the 23 lessons) of the concept development lessons were used at times other than the target.

Table 3.3. Lesson type and curriculum placement, by teacher

Teacher	Lesson Type and Placement					Totals				
						CD		PS		T or PK
						T or PK	I or R	T	A or N	
Ms. Amador	CD *T*	CD *I*	PS *T*	CD *T*	–	2	1	1	0	3
Ms. Belle	CD *T*	CD *T*	CD *I*	–	–	2	1	0	0	2
Ms. Castle	CD *T*	CD *T*	–	–	–	2	0	0	0	2
Mr. Davidson	PS *A*	CD *I*	CD *T*	CD* *I*	CD *T*	2	2	0	1	3
Ms. Elmore	PS *T*	CD *I*	CD *T*	CD *I*	CD *T*	2	2	1	0	4
Ms. Feldman	PS *A/N*	CD *I*	CD* *PK*	CD* *PK*	–	2	1	0	1	2
Ms. Golden	PS *N*	CD *PK*	PS *T*	CD *R*	PS *T*	1	1	2	1	3
Ms. Heather	PS *N*	CD *PK*	PS *T*	CD *R*	PS *T*	1	1	2	1	3
Totals						14	9	6	4	20
						23		10		

* lesson taught in a more advanced course.

A main focus of this study is to examine the potential for teacher learning when teachers make use of the FALs in ways that at least resemble the intentions of the designers, both with regard to lesson placement and with regard to implementation. Hence my focus, both in my general analyses and in my selection of teachers for more detailed case studies, will be on Target and Prior Knowledge lessons. However, it is worth noting how the teachers made use of the lessons. That discussion follows.

Prior knowledge (PK). Four of the 23 concept development lesson implementations, all in high school, were coded PK. Two were taught by Ms. Feldman at the beginning of a unit on statistics in a capstone class for seniors preparing for the International Baccalaureate test. Because she had not taught the students in the prior course, she was very uncertain about what their knowledge of representations of large sets of statistical data, in particularly how to read and make sense of frequency graphs and box-plots. She used the pre-assessments from the lessons Representing Data I and II, to surface prior knowledge and understanding. Students struggled to connect representations to descriptions of data. Based on the pre-assessment, she had the students engage in the lesson activities. These gave opportunities for some students to refresh their thinking, others to sort out confusion, and still others to learn about the representations from their peers. This provided a more solid foundation as they went forward with techniques for calculating measures of central tendencies for continuous data sets. Although these FALs did not occur within a unit, Ms. Golden used the data about student thinking that she gathered from the pre-assessments to inform her choices in the discussion of this lesson and to reference in subsequent lessons. The other two lessons coded PK were Ms. Heather's and Ms. Golden's uses of Comparing Lines and Linear Equations discussed at the beginning of this chapter.

Introductory (I). Seven of the concept development lessons were used to introduce new concepts to students. In these cases, the activity was really the only useful phase of the lesson. The students were not able to provide the teachers much information on the pre-assessments so there was little opportunity to see how the lesson activity helped students to re-engage their misconceptions. These enactments of the FALs often required more introduction during the launch, and in each case, the teacher provided direct instruction just before launching the activity or during the activity. The teachers' reflections on these lessons included appreciation of the lessons' opportunities for the students to learn in a hands-on way, and their hopes to build on the experiences in these lessons during the subsequent unit. But, as Ms. Elmore wrote on the post-lesson reflection for a concept development lesson that she used as an introduction to equations of circles:

> From the post-assessment, there was general improvement with the ability to write the equation of the circle, but very few students were able to explain it – were they not able, or did they not understand what it means to explain? A large number of students still had a very hard time with the (2,*m*) question, but there was general improvement across the board (easy to improve from zero). (Ms. Elmore, post-lesson reflection, 3/17/2014)

Another example of a FAL coded as an introductory lesson was the second FAL that Mr. Davidson taught in Math 8, Translating Between Repeating Decimals and Fractions. The students had learned in 6^{th} and 7^{th} grade how to find a decimal equivalent for a fraction by dividing the numerator by the denominator. However, most students had no prior experience of converting a repeating decimal to an equivalent fraction except in cases such as .333 . . . = 1/3 where they already knew the answer. During preparation for this FAL, Mr. Davidson revealed that he himself did not have a method for conversion. After I showed him the method that is usually taught in algebra, he then taught exactly this method to his students in the lesson. On their pre-assessments, almost all students left the problem with repeating decimals blank. As a result, during the FAL, Mr. Davidson spent a day teaching the students the technique for converting repeating fractions to decimals. This meant that rather than his seeing how the activity built on students' prior knowledge, the lesson became an exercise in students applying, with varying levels of success, the technique that they had just learned. As such, it was an interactive and motivating activity for students to use what they had just been taught, but it did not result in much discussion of why the technique worked, nor how the decimal and fraction representations were connected, except via the conversion method. There were few opportunities for Mr. Davidson to use students' existing ideas to move their learning forward.

Review (R). Two enactments of the 23 concept development FALs were coded R. The teachers explained that their reason for using these FALs was to prepare students for a semester exam and that they did not intend to use what they learned about student thinking to follow up on the lesson. Not surprisingly, the teachers found that the students were mostly successful in completing the activity and there was not much purpose for the discussion because the students had resolved all of the issues in their groups. These lessons appeared to be satisfying for the students, in part because they succeeded easily in the lessons. They provided the teachers with good opportunities to work with students on good norms for working in groups, but they did not provide much opportunity for teachers to see how to leverage students' prior knowledge to move learning forward.

Codings of off-target problem solving FAL placements

Although the suggested target placement for problem solving FALs is much broader than for concept development FALs, among the observed FAL enactments, problem solving FALs were also placed in the curriculum at times other than the target. Off-target placements were coded in two categories: Advanced (A) and Near (N). Enactments were coded as Advanced, if the problems in the FAL were too advanced for the students to be able to use multiple strategies. They were coded as Near if students had learned to solve a similar problem and approached the FAL problem as an application of one particular method without use of multiple strategies.

All four of the teachers at Brookfield High School selected problem solving FALs as the first FAL taught. Of these lessons, only one was coded as target (see Table 3.3), despite the fact that it was listed on the MAP website as a 6th grade lesson. This lesson, taught by Ms. Elmore, allowed students to engage with multiple strategies, share their approaches, critique sample student work, and present their revised group solutions to the rest of the class. Ms. Golden's and Ms. Heather's first problem solving FAL, Floodlights, was taught during a unit with very similar content. In fact, they selected the lesson because of how well the problem was aligned with the unit. However, this alignment turned out to be somewhat problematic for the goals of a problem solving lesson. As Ms. Heather wrote in her post-lesson summary:

> My students seemed very engaged by this problem and lesson . . . My students have already been exposed to the concepts of similar triangles and proportions in our current unit [called "Shadows"], and just before I taught the FAL, students had derived a formula to find the length of a shadow given the height of the object casting the shadow, the height of the light source, and the distance between object and light source. I found that having this tool limited students' creativity in their approach to #2 of the Floodlights FAL, which asked students to solve for the total combined length of both of the football player's shadows. Every group used this shadow formula but did not justify why it could be applied (by showing that the triangles in the diagram are similar.) I would be curious to see how students approached and persisted with this problem if they didn't have this formula and hadn't been previously exposed to problems of almost the exact same type.

Ms. Golden described a similar experience with her class, where students interpreted this problem as an application exercise based on the mathematics that they had just been doing in their class:

> Many students used the shadow "formula" we derived last week [$H/L = S/(S + D)$]. When I asked them about it in their groups, a lot of students said that they just plugged the numbers into the formula and when I probed them about where the formula came from or how you could solve it if you forgot the formula, they were stuck.

Although both teachers indicated that the students enjoyed working on the problem, they also realized that the students did not really engage in finding different strategies for solving it. The problem has three questions. Because of the FAL placement in the unit, the only real challenge came in the third question. Both the teachers' reflections and the students' work show that most of the students did not work on the third question.

Mr. Davidson and Ms. Feldman also selected the same problem solving lesson, The Difference of Squares, for their first FAL but taught it in courses at two different levels. Its central question is which numbers can (and can not) be written as the difference of two square integers. Mr. Davidson chose this lesson because his Math 8 students were studying patterns in arithmetic and geometry sequences. He saw the problem as building on the strategies that they were using for making systematic lists and looking for patterns. For Mr. Davidson's students, this problem was much larger than any that they had approached before, and they were very confused by the exponents. They spent much of the lesson simply trying to understand that they were not solving individual problems, but looking for a generalization. This enactment of the FAL was coded as advanced (A).

Ms. Feldman's students initially had trouble making sense of square numbers and finding their differences, however, that was quickly identified and cleared up. In this case, students began looking for patterns and defined their job as describing the individual patterns that they found using algebra. The students' prior experience with area models for multiplying binomials from algebra, and with linear and quadratic functions provided some of them with more access to the sample student solutions.

Ms. Feldman's approach to the lesson was that it was exploratory and that her job was to allow students to figure something out. The result was an undirected exploration. Students used symbols to write patterns that they had identified, but without any justification. The sample student work was used as examples of ways that students might want to think about solving the problem, and the lesson ended without any comparisons or critiques of strategies. The students worked on a big problem that some of them found interesting and engaging, but as a class, they did not work on problem solving. After the lesson, Ms. Feldman recognized that she did not have clear goals prior to teaching it. As she wrote in her post-lesson reflection, she was not sure where she wanted the lesson to go for any of her students.

> It would be better to have worked through the math more completely and decided what my academic goals were before starting the problem [i.e. just identifying patterns, going from pattern to algebraic representation, beginning to see number theory ideas]. It would also have been better to have figured out what the weakest students should accomplish, and the purpose of the task for them. I think it was a great experience for my gifted students, but I think it would have been better for the weaker students if I'd come up with a clearer way for them to feel like they had accomplished something.

This also shows that, as used, this lesson reinforced differences in status among students in the class for Ms. Feldman. Contrasts in the roles of gifted and weaker students were highlighted in the enactment of the FAL. Students who did not initially have access to the problem spent much of the class doing very little.

Six of the ten problem solving lessons observed in this study were taught at target times. For this reason, I leave further analysis of the problem solving FAL enactments for the future. The remainder of this chapter will be on the concept development FALs.

Enactment and elimination of phases in concept development FALs

In the analysis of FAL placement in the previous sections, the central concern was whether or not the FAL could be used to move student thinking forward. The assumption was that the teachers enacted all of the phases of the lesson more or less as they were presented in the lesson guide. However, that was also not always the case.

Pre-assessments were administered for twelve of the fourteen concept development FALs coded as T or PK. However, evidence indicates that the teachers looked at the pre-assessments prior to the lesson for only eight. Table 3.4 shows the concept development lessons by teacher with lessons where the pre-assessment was not administered or not interpreted by the teacher prior to the lesson shaded diagonally. Table 3.4 shows that for five teachers the first FALs were the only FALs that were well timed and for which they had access to student thinking before starting the launch and activity phases.

On the mid-project survey, several teachers expressed concern about how much time it took to interpret and provide feedback on the pre-assessments. One of the teachers felt strongly that information gained was not worth the time and effort invested. Teachers who administered the pre-assessments and attempted to look at them for multiple lessons commented:

> I hate the pre & post assessments. Too much time-intensive grading and I haven't found the pre-assessments informative. I honestly haven't more than scanned the most recent post-assessments. (Ms. Elmore, mid-project survey, 3/13/2014)

> Pre-assessment tally form [is] very time consuming, but helpful to see the overall picture and places where student[s] had the most misconceptions and needed the most support. (Ms. Golden, mid-project survey, 3/13/2014)

> I have used [the pre-assessment tallies] and find them extremely time-consuming AND extremely useful. (Ms. Heather, mid-project survey, 3/13/2014)

Ms. Belle and Ms. Amador administered the pre-assessment in their first concept development FAL, but not for subsequent FALs. Ms. Feldman and Ms. Castle administered the pre-assessments for the second concept development FALs that they taught, but did not provide feedback to the students. Ms. Belle wrote: "not enough time to give feedback on pre-assessments."

There were three reasons that the teachers were so frustrated by the pre-assessments. First, as is evident in their comments above, they found interpreting and providing feedback on the assessments very time-consuming. The second issue was more to the point – the FALs were structured to be effective for student learning independent of the results that the teacher found on the pre-assessment. As a result, the teachers did not experience a substantial value added from the time invested in the pre-assessments. Finally, examination of a sample of the student responses for pre-assessments from the eight lessons shows that students frequently left large portions unanswered. As a result, the teachers found that administering the pre-assessments only provided information from students who were likely to volunteer that information in class anyway, and was a burdensome and ineffective way to get information about the students who were most at risk. However, without the pre-assessments, the teachers began the activity phase with little information about individual students or of the class as a whole. This reduced or eliminated their opportunities to learn about formative assessment through participation in the formative assessment cycle.

Table 3.4. Concept development FAL placement and use of pre-assessment

Teacher	Placement, Use of Pre-assessment				
Ms. Amador	CD *T*	CD *I*	–	CD *T*	–
Ms. Belle	CD *T*	CD *T*	CD *I*	–	–
Ms. Castle	CD *T*	CD *T*	–	–	–
Mr. Davidson	–	CD *I*	CD *T*	CD* *I*	CD *T*
Ms. Elmore	–	CD *I*	CD *T*	CD *I*	CD *T*
Ms. Feldman	–	CD *I*	CD* *PK*	CD* *PK*	–
Ms. Golden	–	CD *PK*	–	CD *R*	–
Ms. Heather	–	CD *PK*	–	CD *R*	–

* lesson taught in a more advanced course.

Formative assessment during the activity phase and changes in elicited student thinking

In the previous section, I showed that the FALs as used by the teachers in this study provided few opportunities to engage in a full cycle of formative assessment. However, this is only part of the story of how teachers can learn from teaching these lessons. When asked what, if anything, they found most beneficial for them and their students about using the FALs, the teachers had a lot to say:

> Engaging group work activities that get students into good conversations. (Ms. Golden, mid-project survey, 3/13/2014)
>
> They get students talking to each other ABOUT MATH! (Ms. Heather, mid-project survey, 3/13/2014)
>
> Very helpful! The biggest impact has been the willingness and completeness of student explanations. (Mr. Davidson, mid-project survey, 3/13/2014)
>
> Listening to student conversations unearths misconceptions I did not realize existed. For instance, I thought my Algebra students would have an easy time completing the 7th grade "steps to solving equations" FAL, but they struggled a lot more than I

anticipated, especially on the problems where it was unclear what *x* represented. (Ms. Castle, mid-project survey, 3/13/2014)

What they found useful and what we observed in watching the lessons unfold is that the activity phases of the lessons provided rich problems for students to discuss and provided teachers lots of opportunities to interact with their students' thinking. This suggested that perhaps, although the teachers were not engaging in a large cycle of formative assessment using the entire FAL as depicted in Figure 3.1, they might be learning to engage in a smaller formative assessment cycle comprised of the launch, activity and discussion phases of the lessons (Figure 3.3).

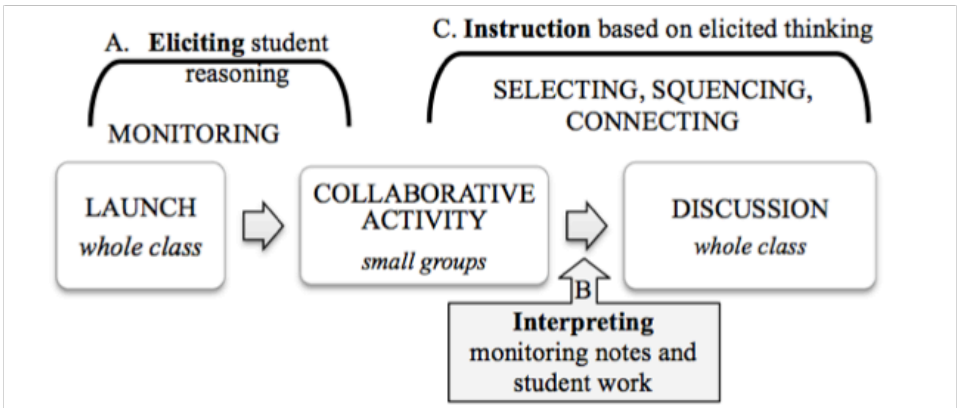

Figure 3.3. Small formative assessment cycle

Further investigation showed that although the teachers found the activity phase of the lesson exciting and productive for both them and their students, they struggled with how to manage the class discussion. Of the 23 observed FALs, only 13 had any substantial whole class discussion that built on the student work during the activity phase. Consistent with these observations, the teachers' comments on the mid-project assessment reflected their struggles with how to productively use the student work from the activity phase in the whole class discussion phase. Many of the challenges they expressed on the mid-project survey focused on the discussion:

> It's a little frustrating that the lessons are not straightforward. This leads to multiple entries and the richness, but the lesson can go on and on. A teacher has to create a learning goal before going on. Also, it feels as though it's all discovery, however there are times I want to tell them something so we can move to another concept or go deeper into a concept. (Ms. Amador)

> I have been frustrated with how to provide closure and a sense of "oh, now I get these problems more" at the end of a FAL. (Ms. Castle)

> I'm still struggling on a solid "wrap-up". It feels like kids should finish with a big "ta-da" but that clearly isn't always possible. (Mr. Davidson)

> I've had difficulty in the whole class discussions after student have spent time working in small groups and making discoveries. I struggle with how to get students to share out what they have learned with the whole class, how to keep the rest of the class engaged, and how to keep it snappy, deep, meaningful and not redundant. My biggest successes have been in small groups, but bringing it back is difficult. (Ms. Heather)

Similarly, when asked "What goals do you have for yourself as a teacher in the next several FALs you teach?," the teachers' responses focused on the class discussion and wrapping up the lessons.

> My goal is to clearly define the goal for the FALs and to recognize when I accomplish the goal. In addition, have a suitable end product. (Ms. Amador)
>
> To let as much as possible be stated by the students; to come up with good **discussion** questions. (Ms. Castle, emphasis in the original)
>
> Orchestrate better class discussions (more participation and engagement). (Ms. Golden)
>
> I would like to feel as strong about the end of the FAL as I do about the beginning, particularly the whole-class discussion part. Often I don't feel like students do justice to the amazing discoveries they made in small groups when it comes to full class discussion. (Ms. Amador)

The teachers' concerns about the final discussion, along with the observation data which shows that several of the teachers omitted or seriously abbreviated the discussion phase of the lesson, indicate that the teachers were not yet engaging in complete cycles of formative assessment. Whereas the primary obstacle to the formative assessment for the whole lesson was the teachers' eliciting and interpreting of student thinking, within the lesson, the greater obstacle seems to be teachers knowing what to do with the student thinking that is elicited during the activity phase of the lesson. Making effective use of formative assessment means focusing on student thinking – building on it, and providing students opportunities to refine and correct their own ideas. To the degree that teachers fell back on well-established habits and the beliefs that underpin them – that it was their responsibility to lay out the mathematics and to correct students by showing how to do things right – the teachers' "default" pedagogical beliefs and practices stood in the way of their implementing the FALs in ways consistent with the designers' intentions.

Implications for FAL lesson guides and professional development

It is evident from a few instances such as Ms. Heather's implementation (described at the beginning of this chapter) that the FALs can support teachers' use of formative assessment. However, it is also clear that to be able to take full advantage of the FALs, teachers need substantially more support and guidance than were provided in the lesson guides and the *Brief Guide for Teachers and Administrators*.

Thus, professional development should focus on teachers more effectively supporting students in revealing their thinking on the pre-assessments, tools and efficient methods for teachers to interpret the results of the pre-assessments, and ways for teachers to connect the remainder of the lesson to the pre-assessment. A second focus of professional development should be on supporting teachers in interpreting the work that students have done during the activity phase of the lesson and structuring class discussions that leverage and build on this work. Although the FALs provide rich affordances for these connections, the lesson guides did not provide enough support for the teachers in transitioning from the activities to productive classroom discussions.

How did the activity phase of the FALs support changes in teacher and student practices?

So far, this chapter has documented the fact that (with some exceptions such as Ms. Heather's implementation of the Comparing Lines and Linear Equations FAL discussed above) there were significant challenges for the teachers in engaging in a complete cycle of formative assessment using the FALs. However, it is still possible that these lessons could support teachers and students in learning to engage in practices that could support formative assessment. In particular, because all of the observed FAL enactments included the activity phase and teachers' reflections on this phase were mostly positive, it is worth knowing if this phase of the lessons, as enacted, provided access to student thinking for teachers and students. The remainder of this chapter focuses on this issue.

To address this issue, I analyzed videos of two lessons for each of six teachers in this study[6]: for a routine lesson taught before the second or third FAL. The question is: Do students (with the help of the teacher) become more proficient at one of the key practices aimed at in the FALs, providing explanations of the mathematical content being studied? To answer this, I counted the number of instances in which students provided explanations or justifications during small group interactions with the teacher and during whole class discussions. Student explanations as a percentage of total student contributions to the class discussion is used to demonstrate the range of experiences available to students in this study during the teaching of the FALs. Portions of these videos were double coded to ensure reliability and validity.

Six out of the eight teachers in the study were observed teaching a routine (non-FAL) classroom lesson near the beginning of the school year. We asked the teachers to invite us to a lesson that they thought was representative of routine instructional activity where there would be significant interaction between the students and the teacher. We asked that the primary focus of the lesson on these days be on learning something new, rather than reviewing or

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[6] Two of the teachers, Ms. Castle and Ms. Heather, were not observed during routine lessons.

assessing previously taught material. The purpose of these observations was to provide a baseline indication of student explanations, prior to the enactment of the FALs.

In all of the FAL enactments observed, the teacher and students engaged in the collaborative activity phase of the lesson using the materials that were mostly consistent with their presentation in the lesson guides. Although there was some modification of the activity materials, it was relatively minor in most cases. More discussion of the consequential modification to collaborative activities is given in the case study in Chapter 5.

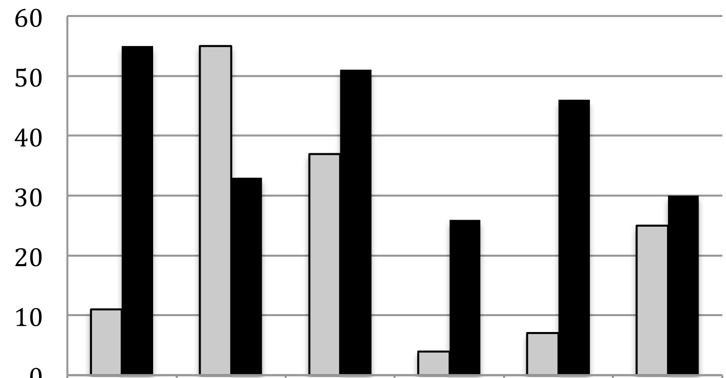
From the lesson designers' perspective, the activity is crucial in allowing the students to confront their existing conceptions and integrate them into their emerging understandings through justification, critique, and negotiation with one another. For the teachers, this phase of the lesson provides another potential source of information about the students' thinking. However, this potential can only be realized if the students have the opportunity to share their understandings of how they are making sense of the tasks in the activity. The question is: How did the teachers' opportunities to learn about student thinking through student explanations during enactment of the activity phase of the FAL compare to their interactions with individual or small groups of students during their routine lessons?

To measure this, I coded the interactions between students and teachers during the times that the students worked in groups. Each time that a teacher interacted with a group (or any members of that group) was coded as a "group visit." The length of each group visit and number of group members were recorded. Turns of student talk that included an explanation by a student of thinking related to the task were coded as student explanations. The explanations did not need to be accurate, or even complete thoughts, but needed to be long enough to provide some additional insight about the students' thinking – not simply an answer to a factual or definitional question. Reiterations of the same thinking during the same group visit were not counted. However, if in a later turn of dialog, the student shared a different idea, that was coded as a new explanation. Only student explanations that were recorded with the teacher present and engaging with the group were counted.

To compare group visit interactions during the routine lesson to FALs, for each teacher we selected one FAL to code in the manner described above. The FALs were selected based on the criteria below. When no lesson could be found that met all four criteria, I selected a lesson that met the first three.

1. The comparison lesson must be a concept development FAL.
2. The comparison lesson was in the same course as the routine lesson and where possible in the same class period.
3. The comparison lesson placement was coded T or PK.
4. The comparison lesson was not the first or fifth FAL taught by the teacher. When teaching the first, teachers were becoming familiar with the FALs. For many teachers, the fifth FAL occurred at the end of the year when they were under pressure to complete the curriculum.

Figure 3.4 shows the number of student explanations during teacher group visits in the routine lesson and the FAL by teacher. In different lessons, different lengths of time were spent in groups. To account for this, the number of student explanations was normed to one hour: the observed number of explanations was divided by the total time for teacher group visits in minutes and multiplied by 60.



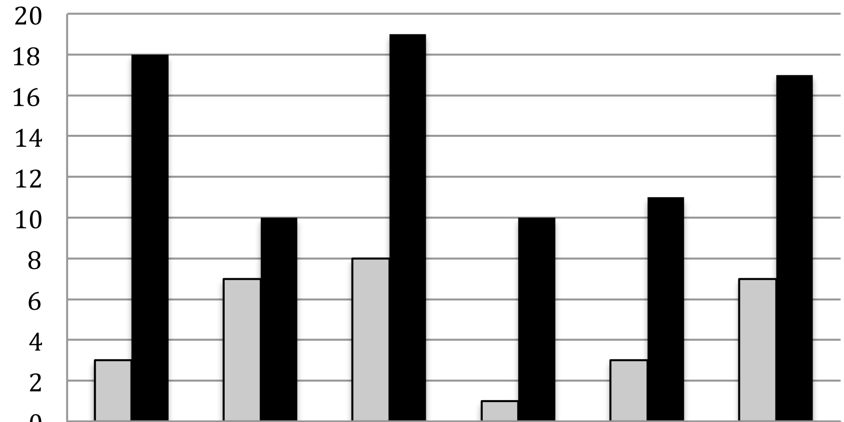
	Ms A	Ms. B	Mr. D	Ms. E	Ms. F	Ms. G
Routine Lesson	11	55	37	4	7	25
FAL	55	33	51	26	46	30

Figure 3.4. Average rate of student explanations (number of student explanations per 60 minutes of teacher group visits) observed during routine lesson and FAL

In five of the six cases, the number of student explanations increased between the routine lesson and the FAL. Several were dramatic increases, e.g., 7 to 46. The one case of non-increase is Ms. Belle who already elicited many explanations from her students during her routine lesson. She was particularly concerned about the amount of time needed for the FALs and a lack of clear direction toward specific content goals. As a result, her implementation of the FALs created fewer opportunities for students to explain their own thinking than her routine lesson. In all other cases, the FALs offered the teachers at least the same level of access to student thinking as their routine lessons, and perhaps substantially more.

This comparison supports the claim that the FALs contribute to increase in student agency with respect to mathematical ideas discussed. Agency is generally associated with students' opportunities to explain their thinking and to promote their ideas about mathematical content. Increases in the number of student explanations in the presence of the teacher (rather than only during small group discussions) indicates that the students expressed as much if not more mathematical thinking than in during group work in the routine lesson.

Was this increase due to a few vocal students becoming more vocal or were the teachers gaining access to the mathematical thinking of more students? Figure 3.5 shows the number of distinct students in each lesson who gave at least one explanation during group visits.



	Ms A	Ms. B	Mr. D	Ms. E	Ms. F	Ms. G
Routine Lesson	3	7	8	1	3	7
FAL	18	10	19	10	11	17

Figure 3.5. Number of distinct students offering explanations during teacher group visits during observed routine lesson and FAL

There were between 20 and 28 students in all of the lessons observed. These numbers varied by teacher and by lesson. For this reason, it did not make sense to look at the percent of students participating. Figure 3.6 shows that for all of the teachers, more students offered an explanation during the FALs than during the routine lessons. Although the differences were only a few students for Ms. Amador and Ms. Belle, for the other four teachers, Mr. Davidson, Ms. Elmore, Ms. Feldman, and Ms. Golden, more than twice as many students offered explanations. It also shows that during the FALs taught by Ms. Amador, Mr. Davidson, and Ms. Golden, more than half of the students provide at least one explanation during group visits, something that did not occur during any of the routine lessons.

The final question that I want to address with these data is: Does teacher behavior in eliciting student thinking during group visits change? For example, do all of these explanations come during a few visits where the teacher asked the students to explain with the remaining group visits more teacher directed? Or, were the teachers more frequently listening to student thinking when interacting with groups? Figure 3.6, Table 3.5, and Figure 3.7 are intended to address these questions. Figure 3.6 and Table 3.5 display percents of group visits with one or more student explanations during the routine lessons and the FALs. These show that the percents of such group visits increased for all teachers between routine lesson and FAL, and that for at least three of the teachers, Ms. Amador, Ms. Elmore, and Ms. Feldman, there were substantial gains.

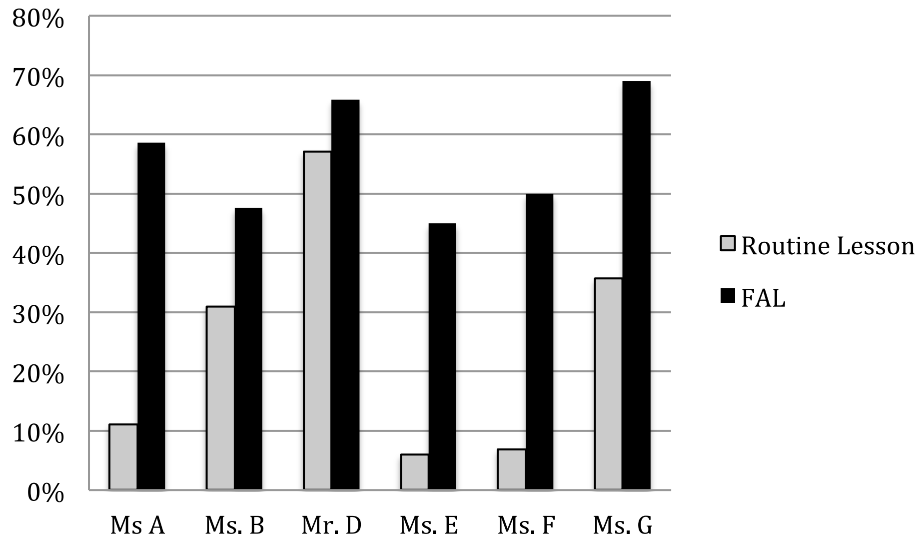


Figure 3.6. Percent of group visits with at least one student explanation

Table 3.5. Percent of group visits with at least one student explanation

Teacher	Ms A	Ms. B	Mr. D	Ms. E	Ms. F	Ms. G
Routine lesson	11% (1/9)	31% (5/12)	57% (8/14)	6% (1/16)	7% (2/29)	36% (5/14)
FAL	59% (17/27)	48% (10/21)	66% (27/41)	45% (9/20)	50% (12/24)	69% (20/29)

Note. These statistics are shown in Figure 3.6. The fractions in parentheses show the number of group visits with at least one student explanation over the total number of group visits.

Table 3.5 shows that, in contrast to the routine lessons, during the FALs all of the teachers were offered student explanations in a large percent of their group visits, more than 45% in all cases. The teacher with the smallest increase, not surprisingly, had by far the highest percent of group visits with explanations during the routine lesson.

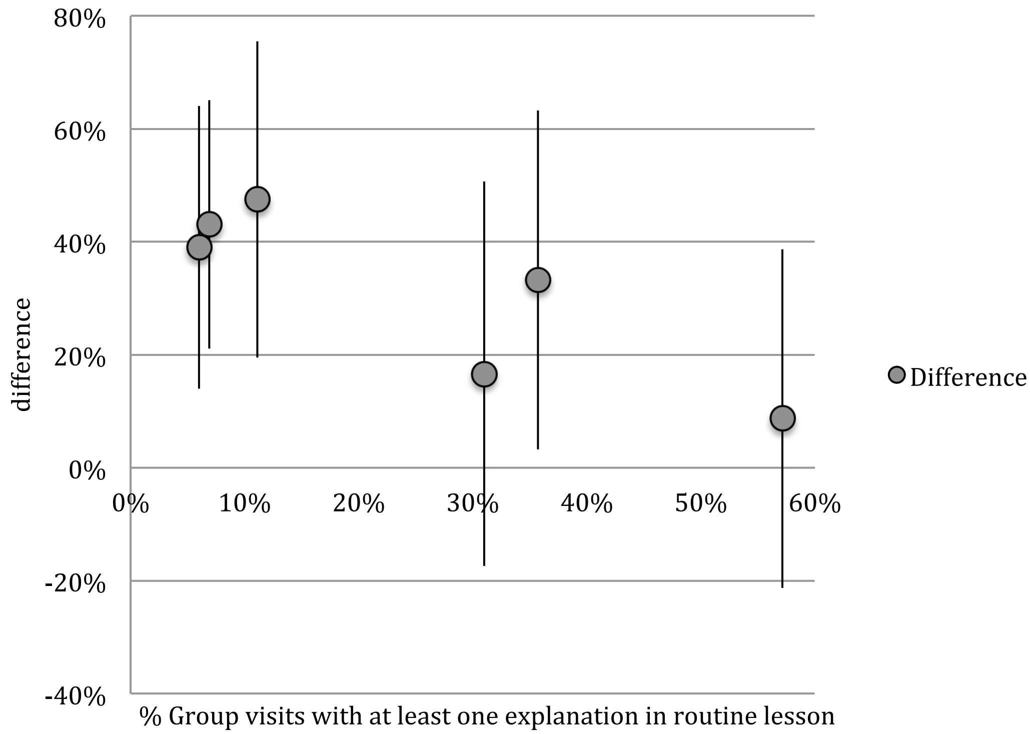


Figure 3.7. Differences in percent of group visits with at least one student explanation between FAL and routine lesson as a function of percent of group visits in routine lesson with at least one student explanation. Error bars show two standard deviations.

Figure 3.7 shows the *difference* in the percent of group visits between FAL and routine lesson with at least one student explanation as a function of percent of group visit with at least one explanation during the routine lesson. This graph indicates that the teachers who elicited student explanations most frequently during the routine lesson had the smallest change, while those who elicited student explanations in few of their group visits changed substantially when teaching the FAL. The error bars[7] show two standard deviations from the observed difference, indicating that for four of the six teachers there is a 95% likelihood that the change from the routine lesson to the FAL was positive.

In Chapters 4 and 5, I illustrate learning trajectories for two of these six teachers. These teachers were initially chosen for case studies because they taught the largest numbers of concept development FALs (see Table 3.3). They were also chosen because each took their participation in the study very seriously, completed pre- and post-lesson reflections,

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[7] The error bars in this figure were calculated by treating teacher group visits as binomial distributions (X = number of group visits with at least one explanation during routine lesson and Y = number of group visits with at least one explanation during FAL, n (or m) = number of group visits, p (or q) = changes of at least one student explanations). The error bars show two standard deviations away from the observed percentage of successes.

administered pre- and post-assessments for almost every lesson, and were eager to share their experiences in learning to teach the FALs. Most important, however, was that in using the FALs each of these teachers felt that they were developing some aspect of their practice.

The case studies in Chapters 4 and 5 serve two purposes. First, they allow me to look more closely how measurements of student explanations during group visits changed across a set of FALs for each of the case study teachers. This provides a more detailed understanding of the variation between enactments of FALs by the same teacher, and how the teachers' interactions with their students while teaching FALs changed over the course of the year.

The second purpose is to examine how the teachers chose to use and adapt parts of the FALs, in particular how they approached the discussion phase of these lessons, which most of the teachers found challenging. In these analyses, I set aside the focus on the goal of teachers learning about formative assessment and instead try to analyze the teachers' use of the lesson materials to move toward their own goals.

Chapter 4

Case Study of Mr. Davidson's Use and Adaptation of FALs

Introduction

This chapter presents a case study of Mr. Davidson, one of the Adams Middle School teachers. Here, I examine how Mr. Davidson engaged in professional learning. First, Mr. Davidson's interactions with students working in small groups shifted to focus more on student thinking and less on guiding students to correct answers. Second, Mr. Davidson used the FALs as a context for innovating his teaching practice by developing an alternative approach to the discussion phase of the lessons. Mr. Davidson's modifications to the lessons were new to him and directed toward his goals in using the FALs.

Mr. Davidson's case illustrates two ways in which curricular materials can support teacher learning: through the use of the lessons as written, and through the affordances of the materials for experimentation.

Background and context

Mr. Davidson was an 8^{th} grade math teacher at Adams Middle School, where, together with Ms. Belle, he co-chaired the department. He taught three sections of Math 8, the standard heterogeneous class for 8^{th} grade students; one section of Strategic Support for students in his Math 8 class with a history of low achievement in math class; and one section of Geometry for 8^{th} grade accelerated students.

Mr. Davidson, like all of the teachers at Adams, integrated computer technology into his classroom teaching using an instructional model called blended learning. Classes met for one hour on Monday, Tuesday, Thursday and Friday. On these days, the 30 students were divided into two groups. Fifteen students worked in small groups of three or four of the new lessons at a table in the center of the classroom. Meanwhile, the other fifteen students worked individually on computers practicing skills using programs like IXL or Khan Academy on stools at a counter along the back and side of the classroom. After 30 minutes, the two groups would switch. On Wednesdays, classes met for only 40 minutes. Mr. Davidson planned to use Wednesdays for the whole class to work in groups on problem solving activities.

He was observed teaching one routine lesson, one problem solving FAL (PS1) and three concept development FALs (CD1, CD2, and CD3) in his Math 8 classes, and one concept development FAL (CD4) in the Geometry class. In order to be able to track changes in the interactions between Mr. Davidson and the same group of students, the analysis in this chapter is limited to data from the routine lesson and the three concept development FALs (CD1, CD2, and CD3), all of which took place in his third period Math 8 class.

Mr. Davidson's goals and routine classroom instruction

In response to the application question: "Why are you interested in using the Formative Assessment Lessons in your teaching?" Mr. Davidson wrote:

> I would like to use FALs at least once every unit this year because I believe they are a fantastic bridge for our students as they move from "answer-based" thinking to more "process-based" thinking. Additionally, I am excited to take advantage of Common Core-aligned tasks and push my students to reflect on their learning process and written/verbal communication.

At the first meeting with the teachers at his school, he reiterated the goal of pushing his students to explain their thinking and make sense of the mathematics that they were learning. Mr. Davidson's statement above has three goals that became themes in my observations of Mr. Davidson and served as lenses for thinking about his teaching. First, he wanted his students thinking about mathematics, and presumably his own thinking, to shift from "answer-based" to "process-based." This goal seems to be very closely related to the second goal of communication. Together, these indicate a goal for students to recognize how they are solving problems rather than whether or not they are getting the right answer, and then to communicate how they reached their answer. A third goal is personal accountability. Underlying the goal of pushing students to reflect on their learning processes and communication is the goal of accountability to each other and to themselves for the mathematics they are learning and how they are understanding it.

This interpretation of Mr. Davidson's goals might seem like reading a lot into his short response on an application; however, the argument in this chapter is that Mr. Davidson used the FALs to move towards these goals and in the process, what he meant by these goals became clearer to him and to his students. I claim that the evolution of Mr. Davidson's classroom routines and practices constitutes teacher learning, and that the FALs served to create a productive context for this learning. Finally, I argue that these changes were consistent with developing practices of formative assessment, even when the enacted lesson deviated significantly from the lesson guides for the FALs. This last point is significant because when instructional materials are used to support teacher learning, much emphasis is frequently placed on fidelity. As the analysis in Chapter 3 showed, teachers' lack of fidelity in their use or interpretation of the FALs can constrain their opportunities to enact formative assessment cycles. This chapter shows another side of lack of fidelity – teacher learning through innovation.

Overview of chapter

This chapter has three sections. The first section is a close analysis of teacher–student interactions during group visits (see Chapter 3) in two different lessons. The first group visit occurred during a routine lesson about three weeks into the school year. The second occurred during the second concept development FAL (CD2) that Mr. Davidson taught in the same class. The purpose of this analysis is to illustrate how the interactions between Mr. Davidson and students change.

The second section presents evidence that the differences between these two episodes are representative of a trend in changing interactions over the course of the three concept development FALs Mr. Davidson taught. Using quantitative analysis of Mr. Davidson's interactions with groups during the activity phase of the FALs and the distribution and frequency of student explanations, I show that the interactions during the FALs afforded more opportunities for students and teacher to achieve Mr. Davidson's goals.

The final section of this chapter traces how Mr. Davidson used the FALs as an opportunity to create and refine his own routines for supporting students' mathematical communication and presentation. This section provides a detailed example of teacher learning that is supported by the affordances of the formative assessment lessons, but is directed by the teacher's goals rather than the intentions of the FAL designers.

Two vignettes from group visits

Vignette 1: Teacher–student interactions during a routine lesson

The routine lesson observation in Mr. Davidson's class was on September 18, 2013. Mr. Davison directed the students who were working in small groups to first discuss their answers to homework questions and then complete Problem 15 from their workbook. Problem 15 consisted of completing a table with information about sequences of rational numbers (Figure 4.1). Mr. Davidson circulated through the class, alternating between visits to the small groups to ask about their answers to the homework and classwork questions, and monitoring the behavior and troubleshooting technology questions for the students working on the computers.

About 15 minutes after the beginning of the class, Mr. Davidson visited a group of three students – Kayla, Hector, and Leo – who were working on Problem 15. Figure 4.1 shows the problem as it appears in the book (in bold) along with possible correct responses based on Mr. Davidson's interactions with the students (in italics). The photo beside the problem shows Hector's work at the time Mr. Davidson visited the group.

Problem 15 is mathematically problematic because it makes the (unstated) assumption that each of the sequences will be arithmetic or geometric. However, the students and Mr. Davidson make the same assumption, so that is not part of the difficulty for the students. Mr. Davidson and the textbook writers expect the following for the first two rows of the table: The first sequence is increasing by 5. This pattern is likely to be so familiar to 8^{th} grade students that they may not be able to explain how they know it. The second sequence is more challenging. Each term is the prior term divided by 2, or half of the prior term, so the next term would be half of minus 1, which is –1/2. This problem provides students with a fair amount to talk about. Because the terms are negative, they are being divided by 2, while the sequence is increasing, a likely source of some confusion. This provides the potential for rich conversations and access for the teacher to student thinking.

15. Complete the table by investigating each sequence.

Sequence	Increasing or Decreasing?	Next Term in the Sequence	Description of Patterns
0,5,10,15,…	*increasing*	*20*	*add 5*
-8,-4,-2,-1,…	*increasing*	*-½ or -0.5*	*divide by 2 (or multiply by ½)*
1.5,2.75,4…	*increasing*	*5.25*	*add 1.25*
1/8,1/4,1/2,…	*increasing*	*1*	*multiply by 2*
2, 5/4, 1/2, …	*decreasing*	*-1/4*	*subtract ¾*

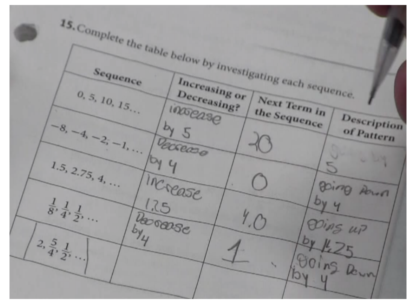


Figure 4.1 Problem 15 with target solutions and student work

The transcript below shows the conversation between Mr. Davidson and the students during the group visit. The students began by indicating that the last problem was a "tricky one." Mr. Davidson agrees that it is "tricky," but they do not discuss why it is tricky or how it might or might not be related to the previous problems. Instead, Mr. Davidson leads them back to their work on the earlier problems.

Turn	Time	Speaker	Transcript
1	0:00	Hector:	You gave us a tricky one [points to the last pattern in the table].
2	0:02	Mr. D:	I gave you a tricky one? I gave you the same one as everyone else is doing. They're all the same.
3	0:06	Hector:	No, like this one is hard.
4	0:07	Mr. D:	Oh, that one *is* tricky. Well let's check the other ones first – see how we're doing with them, ok? The first one, how hard was it?
5	0:14	Kayla:	Easy.
6	0:15	Mr. D:	Pretty easy? Ok. What about the second one?
7	0:17	Students:	Easy.
8	0:18	Mr. D:	Tell me about the second one.
9	0:20	Kayla:	What you want to know about it?
10	0:21	Mr. D:	I want to know is it increasing or decreasing, I want to know what the next term is, I want to know what the description is…
11	0:26	Leo:	It's decreasing and the next term is 0 and it goes by…and it's decreasing

Turn	Time	Speaker	Transcript
			by 4
12	0:34	Mr. D:	Interesting. Did anyone think anything differently? Was everyone like "This is it! Done and done and done!"?
13	0:40	Hector:	We, at first we thought it was decreasing by 2, because like…
14	0:44	Mr. D:	Oh, first you thought it was decreasing by 2?
15	0:45	Hector:	Because it went from 2 to 4 and then…
16	0:49	Mr. D:	Now what I am noticing is that some of you are using words like *going* and *increasing*, [student starts erasing the words *going by*], I want to help you with something [pause] Operations. We've got four of them we use all the time
17	1:07	Kayla:	Addition, subtraction, multiplication and division.
18	1:10	Mr. D:	Got 'em. [Hector now erases everything he has written in the table cells]. Now in the first one you said *going by 5* or *by 5* or *increasing by 5.* Is there a more specific way that you could write that?
19	1:23	Leo:	Increasing?
20	1:26	Kayla:	Adding
21	1:27	Mr. D:	What's it doing?
22	1:28	Kayla:	Adding
23	1:29	Mr. D:	Uh – so does that sound ok? Adding by 5.
24	1:35	Kayla:	Is this going to be on our test? This part right here.
25	1:40	Mr. D:	You're talking about when you go over to the computers. Just this right here, just the three questions. So that's adding by 5. What's the second one doing? Use an operation.
26	1:49	Hector:	Subtracting.
27	1:51	Mr. D:	Subtracting. Anyone think anything else? Or anyone agree? Remember you want to use an operation: addition, subtraction, multiplication and division.
28	2:01	Kayla:	It's subtraction
29	2:03	Mr. D:	Kayla is like, *"it's subtraction."* What are you subtracting by, if you are so sure that you are subtracting?
30	2:08	Hector:	By 4
31	2:09	Mr. D:	So, let's go through it…
32	2:10	Kayla:	No, dividing by 4
33	2:12	Mr. D:	Oh. She just changed her answer by a huge thing. Subtracting and dividing are very different. So now it sounds like there's something to talk about. Let's figure out which operation it is and what it is going by.

On the surface, Mr. Davidson's classroom routines appear well aligned with the activity phase of the FALs. The students were engaged in talking together about their answers and when Mr. Davidson visits the group, he addresses and engages with all members of the group, not just one or two students. He and his students appear to have norms about group work that include talking about mathematics, working together, and persisting with challenging tasks. Mr. Davidson interacts with students by asking questions rather than directly pointing out errors or just providing correct solutions. These norms and questioning are consistent with productive group work and student-centered instruction. At the end of this exchange, the students reach a disagreement and Mr. Davidson walks away from the group leaving them "with something to talk about." It seems that these teaching practices should provide him with many opportunities to formatively assess the students' thinking.

However, closer analysis of the transcript shows that these norms are not sufficient to support students' development of the type of mathematical reasoning and communication that Mr. Davidson described in his application. In the transcript excerpt above, reaching correct answers dominates the discourse of both Mr. Davidson and his students. Students do not attempt to explain the thinking that supports their answers, especially incorrect answers, nor are they encouraged to attempt these explanations. There was little evidence that Mr. Davidson was viewing explanations for incorrect solutions as valuable in understanding student thinking.

Mr. Davidson began by agreeing with Hector that there is something "tricky" about the last problem, rather than asking what about it is "hard." Then, in line 4, he says, "let's check the other ones first" shifting the focus to checking answers. When the students respond that the first sequence, for which they have mostly correct answers, was easy, Mr. Davidson accepts that without question. However, on the second question, where he sees that their answer is incorrect, he does not accept their response that this was "easy" (line 7), and instead asks them to tell him about it. Kayla's response, "What do you want to know about it?" suggests that explaining an answer is not an established practice. Mr. Davidson's response (line 10) maintains the focus on answers by reading the column headers, rather than asking the students to explain how they decided on those answers.

Turn	Time	Speaker	Transcript
1	0:00	Hector:	You gave us a tricky one [points to the last pattern in the table].
2	0:02	Mr. D:	I gave you a tricky one? I gave you the same one as everyone else is doing. They're all the same.
3	0:06	Hector:	No, like this one is hard.
4	0:07	Mr. D:	Oh, that one *is* tricky. Well let's check the other ones first – see how we're doing with them, ok? The first one, how hard was it?
5	0:14	Kayla:	Easy.
6	0:15	Mr. D:	Pretty easy? Ok. What about the second one?
7	0:17	Students:	Easy.
8	0:18	Mr. D:	Tell me about the second one.
9	0:20	Kayla:	What you want to know about it?

Turn	Time	Speaker	Transcript
10	0:21	Mr. D:	I want to know is it increasing or decreasing, I want to know what the next term is, I want to know what the description is …
11	0:26	Leo:	It's decreasing and the next term is 0 and it goes by … and it's decreasing by 4.

In the next part of the exchange, Mr. Davidson starts to open up the discussion by asking if anyone in the group had a different idea. But when Hector tries to share what he was thinking that was different, Mr. Davidson interrupts him and switches the conversation to the words that they are using to describe the patterns. This intervention removes the possibility for Mr. Davidson to learn why the students are interpreting the sequence as decreasing by either 4 or 2, or how they are making sense of the terms "increasing" and "decreasing." The students immediately start erasing the words "going by," a strong indication of their understanding that it is more important to get the correct answer than to explain what the meaning of the description that they had written. It is not clear what connection Hector makes between the way that he sees these sequences as "going by" some number and the operations that Mr. Davidson asks them to write.

Turn	Time	Speaker	Transcript
12	0:34	Mr. D:	Interesting. Did anyone think anything differently? Was everyone like "This is it! Done and done and done!"?
13	0:40	Hector:	We, at first we thought it was decreasing by 2, because like…
14	0:44	Mr. D:	Oh, first you thought it was decreasing by 2?
15	0:45	Hector:	Because it went from 2 to 4 and then…
16	0:49	Mr. D:	Now what I am noticing is that some of you are using words like *going* and *increasing*, [student starts erasing the words *going by*], I want to help you with something [pause] Operations. We've got four of them we use all the time
17	1:07	Kayla:	Addition, subtraction, multiplication and division.
18	1:10	Mr. D:	Got 'em. [Hector now erases everything he has written in the pattern boxes]. Now in the first one you said *going by 5* or *by 5* or *increasing by 5.* Is there a more specific way that you could write that?
19	1:23	Leo:	Increasing?
20	1:26	Kayla:	Adding
21	1:27	Mr. D:	What's it doing?
22	1:28	Kayla:	Adding.
23	1:29	Mr. D:	Uh – so does that sound ok? Adding by 5.
24	1:35	Kayla:	Is this going to be on our test? This part right here.
25	1:40	Mr. D:	You're talking about when you go over to the computers. Just this right here, just the three questions. So that's adding by 5. What's the second one doing? Use an operation.

Toward the end of this group visit, Mr. Davidson returns to the second sequence, which has incorrect answers. When Hector ventures substituting the operation "subtracting" for the phrase "going by," Mr. Davidson does not ask him to justify it, but again asks if there is a different idea. The suggestion of "a different idea," the prompt "remember, you have four operations?," and the challenge (line 29), "What are you subtracting by if you are so sure that you are subtracting?" all signal to the group that they should look for a different answer. In line 31, it seems as if Mr. Davidson is about to abandon the strategy of leading students to answers by questioning, when Kayla offers another guess – "dividing by 4." Having finally extracted a different response, close to the answer he was looking for, Mr. Davidson challenges the students to discuss and decide on which operation and number they should use, and walks away from the group.

Turn	Time	Speaker	Transcript
26	1:49	Hector:	Subtracting.
27	1:51	Mr. D:	Subtracting. Anyone think anything else? Or anyone agree? Remember you want to use an operation: addition, subtraction, multiplication and division.
28	2:01	Kayla:	It's subtraction.
29	2:03	Mr. D:	Kayla is like, *"it's subtraction."* What are you subtracting by, if you are so sure that you are subtracting?
30	2:08	Hector:	By 4.
31	2:09	Mr. D:	So, let's go through it…
32	2:10	Kayla:	No, dividing by 4.
33	2:12	Mr. D:	Oh. She just changed her answer by a huge thing. Subtracting and dividing are very different. So now it sounds like there's something to talk about. Let's figure out which operation it is and what it is going by.

Throughout this group visit, it was obvious that student voice was very important to Mr. Davidson and that he wanted the students to reach the correct answers by talking with each other rather than asking him for validation. Despite that, he managed to validate correct answers, indicate incorrect answers, and place the focus squarely on reaching answers that were correct and used specific mathematical language. The authority for validating the correctness of answers resided with the teacher, rather than being based in mathematical justification. Correct responses for the first sequence were accepted without justification, so they did not provide a model for how to resolve the confusion about the second sequence. After Mr. Davidson left the group, the students decided that the operation for this second sequence was probably division. This decision was based on their idea that the sequence was decreasing and subtraction was clearly not the answer that Mr. Davidson was looking for.

In the whole class discussion that followed, Mr. Davidson told the class that there was lots of discussion about the second sequence and that among the groups, he heard talk about all four operations. He told the students that he wanted them to get something "nailed down" and asked what operation it actually was. One student volunteered that the operation was division. Mr. Davidson agreed and wrote that on the board. Next he asked, "What are we dividing by?" Many students raised their hands and the first student that he called on responded they were dividing by 2. He also wrote this on the board. That was the extent of the discussion about why the rule for this sequence was dividing by 2. The discussion switched to figuring out how to divide the last term in the sequence, –1, by 2 and whether to write the result as –½ or –0.5.

Absent from this whole class discussion was any justification of the conclusion that the descriptions of the pattern for the second sequence was division by 2 or any evidence that other rules that students had considered, such as subtracting or dividing by 4 would not work. Despite his desire to move students away from "answer-based" thinking, in this case, Mr. Davidson had reinforced a focus on answers in both small group interactions and whole class discussion. There were few opportunities for students to practice explaining their thinking about mathematics, and few opportunities for Mr. Davidson to learn what in their thinking had them reaching incorrect answers.

The question remains: Was the lack of student explanation in this setting simply a function of the task not being rich enough to generate student thinking? Was there enough in this task for Mr. Davidson to be able to learn more about how his students were making sense of these patterns that led to incorrect answers? In fact, the student work (shown in Figure 4.1) does provide clues about how the students might be thinking. This work, along with the student's claim early in the interaction that the first four patterns were "easy" but the final pattern was "tricky," suggest that the students felt confident in and had reasons for their answers to the first few sequences and were only feeling confused by the last one.

For example, the students incorrectly described both the second and fourth sequence (–8, –4, –2, –1, . . .) and $(\frac{1}{8}, \frac{1}{4}, \frac{1}{2} \cdots)$ as "decreasing by 4." However, for the fourth they correctly identified the next term as 1, although for the second sequence, they identified the next term incorrectly as 0. (Again, correctness is subject to the assumption discussed earlier that each of the sequences will be arithmetic or geometric.) It seems likely that the students know something about the sequence 8, 4, 2, 1 . . . and notice this sequence within (–8, –4, –2, –1, . . .) and $(\frac{1}{8}, \frac{1}{4}, \frac{1}{2} \cdots)$. Instead of making sense of the negative integers and fractions in these sequences, they tried to make sense of the sequence 8, 4, 2, 1 . . . . From this perspective, the sequence that they are thinking about *is* increasing, though not by a constant 4, while the sequences that they are given are not. However, this is only speculation, because the source of students' confusion is not revealed in this or other interactions during the class.

Vignette 2: Teacher-student interactions during a FAL

Over the next seven months, Mr. Davidson taught one problem solving FAL (PS1) and three concept development FALs (CD1, CD2, and CD 3) in the Math 8 class. As can be expected with any new practice or materials, Mr. Davidson and his students initially struggled with the changes in format. Moreover, they struggled with the increased expectations for independent thinking expected in the FALs. Mr. Davidson was frustrated because the students resisted engaging in the challenge of the lessons or struggled to understand what they were expected to do. Simply put, the first FAL (PS1) was overwhelming for both Mr. Davidson and his students. He had difficulty deciding on the mathematical goals for the lesson and the students seemed confused by the whole lesson. Based on this experience, Mr. Davidson chose concept development lessons for his subsequent FALs. For the Math 8 class, he chose concept development lessons in which the activity involved matching or sorting cards. This format and the lesson materials provided support for Mr. Davidson and his students to engage in interactions about mathematics that were different from those in Vignette 1.

On February 6, 2014 (four months after Vignette 1), the following interaction occurred between Mr. Davidson and a group of students, Juan, Justina, and Carlos during the activity phase of CD2, Steps to Solving Equations, in the same Math 8 class. The cards for the activity phase of this lesson are shown in Figure 4.2. The lesson guide suggests first having students match the story cards with equation cards, then having them set aside two of the matches (S1 & E5 and S2 & E6) and use the smaller equation cards to show "steps" of equivalent equations to solve for the variable in the remaining four matches (S3 & E1, S4 & E2, S5 & E4, S6 & E3). Mr. Davidson modified this activity slightly by giving the students all of the cards at the same time, and having them create steps for the equations that did not have matching steps cards.

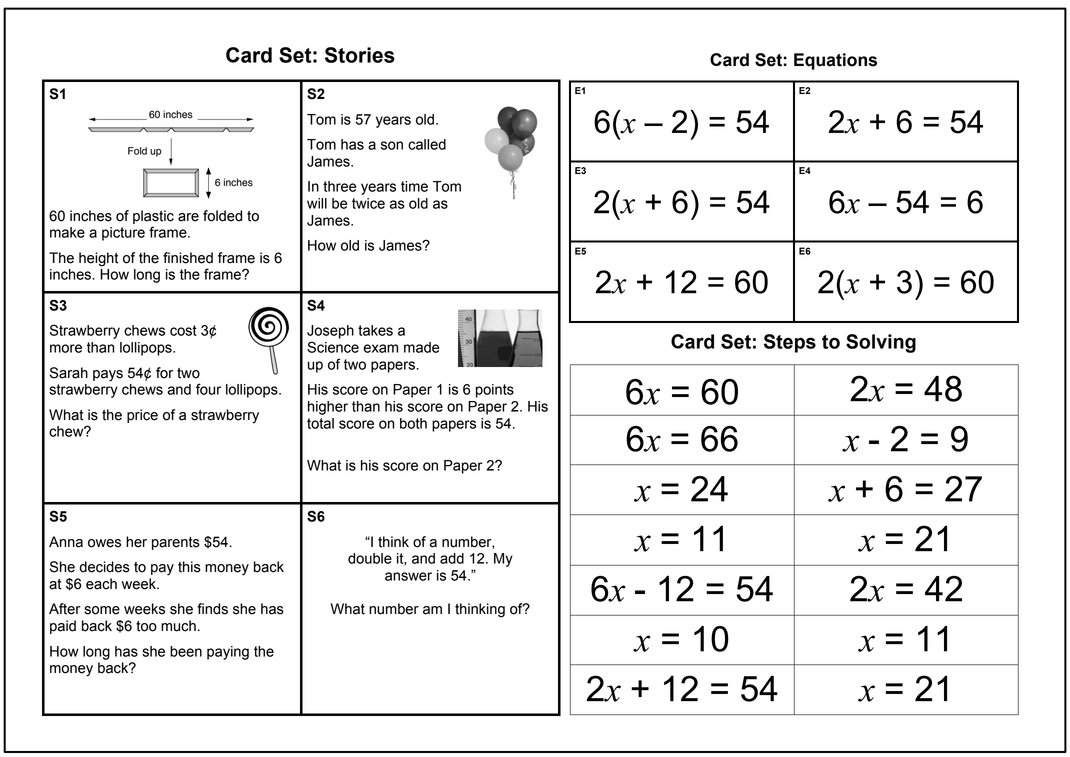


Figure 4.2. Three card sets for CD2, Steps to Solving Equations

Vignette 2 is taken from the second day of the lesson, while the students were working on the activity. Mr. Davidson approaches a group of students (Carlos, Juan, and Justina). Carlos is looking at the equation $2x + 6 = 54$ (card E2) and trying to find a situation that matches (see Figure 4.3). Juan hands him a situation card about James's and Tom's ages (card S2) which does not match. While Carlos works, Justina and Juan sit and watch him, with Juan occasionally looking around for other situation cards to hand to him. Carlos had the cards turned to face him, so they are upside down or sideways for Justina and Juan. Mr. Davidson stands between Juan and Justina, across the desks from Carlos so that he can engage with all three members of the group.

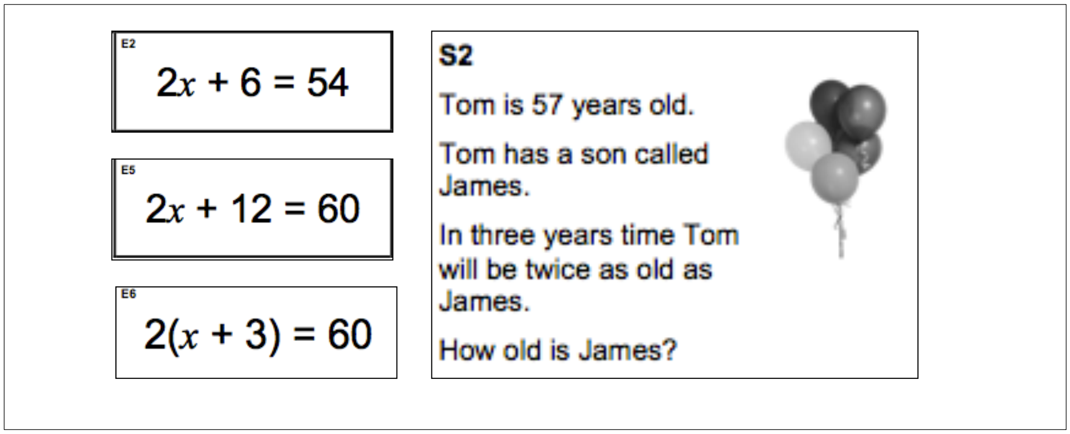


Figure 4.3. Cards discussed in Vignette 2

Turn	Time	Speaker	Transcript
1	0:00	Carlos:	This one is...we're missing a step here. [pointing to a purple equation card]
2	0:01	Mr. D:	A step or a situation?
3	0:02	Carlos:	A situation. Because we don't got no more. [Juan looks around and finds situation cards that he hands to Carlos.] [Talking to himself – looking at S2.] In three years, so, it'll be sixty... because it says in three years Tom will be twice as old as James. [Carlos does some calculations.]
4	0:32	Carlos:	I think it's this one.
5	0:39	Mr. D:	Can I, can I try to help you for a minute?
6	0:40	Carlos:	Yeah.
7	0:42	Mr. D:	Instead of trying to take that equation and see where it matches up, try taking one of the scenarios and matching it to an equation. Try going the other direction. So, for example, you're talking about this one, [pointing to the situation about Tom's and James's ages] you said – you're like, "Oh, I found the answer to it. Oh ok."
8	0:55	Carlos:	The problem is that we don't know how, like, to set it up.
9	0:59	Mr. D:	Well you just talked through it. In your head, I heard you. You said, "Well in three years that means he's sixty and half that is thirty, so that means he's thirty years old." Ok so you did that in your head. You want to try to do that except for, like, bring it down onto some, like, into some form of, of variable and an equation. If you're looking at this one, 'cuz I was, you started with this one – that's why I'm talking about it. What is x? What's your variable? Like, what don't we know? I'm not just talking to Carlos, I'm talking to everybody.
10	1:29	Carlos:	How old is James?

Turn	Time	Speaker	Transcript
11	1:31	Mr. D:	[Turns to Justina] Justina, what's our x?
12	1:32	Justina:	How old is James.
13	1:34	Mr. D:	How old is James? Do you already know that answer?
14	1:36	Juan	No.
15	1:38	Mr. D:	No. [to Juan] What do you *know* about James's age?
16	1:41	Carlos:	So, I know it's fifty-seven plus three and twice.
17	1:45	Mr. D:	Careful. James is the younger one. Tom is the older one.
18	1:51	Carlos	Yeah.
19	1:54	Mr. D:	So tell me about James's age again.
20	1:59	Juan	James's age?
21	2:00	Carlos:	He's fifty-seven.
22	2:01	Mr. D:	He's not fifty-seven. We don't know how old James is.
23	2:04	Carlos:	No, I'm talking about Tom.
24	2:05	Mr. D:	Ok so you're talking about Tom? So tell me about Tom's age. [Carlos starts writing down some computations on his paper.] All right stop for a second – tell us what you're doing because you're writing some things down, but I don't see her [pointing to Justina] writing anything down. I don't see him [pointing to Juan] writing anything down, which tells me that they either aren't willing to try yet or don't understand what it is you're doing. So I need you to tell us what it is you're doing.
25	2:27	Carlos:	Oh the things that I'm doing is that I'm getting Tom's age and adding it plus three because it says in three years that he will be twice as old as Tom in three years. So I'm adding three years to fifty-seven, which equals sixty.
26	2:44	Mr. D:	Ok, now this is where we need to be good students. We have three equations sitting there. Which of those equations are you positive it will not be?
27	2:55	Juan:	Sixty [points to an equation]. No, um… [Carlos moves another equation toward Juan.]
28	2:57	Mr. D:	Juan, why would it not be that one?
29	3:00	Juan:	Because this one has, um, has nothing to do with the problem because it says right here [muffled] fifty-seven-years-old. And it says that you're going to add it up by three… If you add it up by three more years it's going to be sixty, so, you know, that it's not going to be fifty-four because, um...
30	3:20	Carlos:	It equals fifty-four.
31	3:23	Mr. D:	Perfect. That makes total sense to me. Let's get that one out of there. Let's not even talk about it. We're talking just about this one, and now we're down to two. So we have two that are left here. Now, let's try to reason this out. Carlos started us off and said, "Well, in three years it means that Tom's going to be sixty."
32	3:39	Justina	[Points to the correct equation $2(x + 3) = 60$.]
33	3:40	Mr. D:	So Justina, you pointed out this one pretty quick. Why do you think this one?
34	3:42	Justina:	This and this equation, this um…
35	3:46	Mr. D:	Situation
36	3:47	Justina:	Uh-huh. They have three years times, and so, two is going to multiply times three...
37	3:55	Carlos:	No this is a [hard to hear] two, so it's going to be twice, so it's not going to distribute it the, the fifty-seven plus the three.
38	4:05	Mr. D:	How old is James? Did you already have it?
39	4:07	Carlos:	x.

Turn	Time	Speaker	Transcript
40	4:08	Mr. D:	Agreed? So, finish it all the way through. Everything you just said is a *great* connection. This is where I want you to be. Everything you're doing right now is a great connection.
41	4:24	Carlos:	So, two *x* plus six equals sixty.
42	4:27	Mr. D:	Does that tell us how old James is yet?
43	4:28	Carlos:	No. So I still need to bring in the…
44	4:31	Mr. D:	I'm seeing—

Analysis of the interaction between Mr. Davidson and this group of students shows several differences from his interaction in Vignette 1. First, Mr. Davidson observed and listened to students' thinking, and offered suggestions about their processes or strategies rather than correcting their answers. Second, Mr. Davidson solicited student thinking about both correct and incorrect answers and emphasized the need for explanation to justify answers rather than simply finding correct answers. Finally, Mr. Davidson supported the students in using the public workspace to engage each of the students in the group in contributing to the solution and further established an expectation that "doing math" includes justification of solutions. Each of these three differences is evident in the transcript of this group visit.

First, Mr. Davidson devoted more time to gathering information by listening and observing before offering suggestions. At the start of the group visit, Carlos stated that he needed an equation. Mr. Davidson asked a clarifying question (line 2), but did not suggest anything. For the next 30 seconds, between lines 3 and 4, Mr. Davidson watched and listened while Carlos gestured and talked aloud about the problem. This is a much longer wait time than in any of Mr. Davidson's group visits during the routine lesson observation. Finally, in line 5 Mr. Davidson asked if he could offer some help and waited for Carlos's response. Rather than leading Carlos to an answer, his suggestion, in line 7, was about Carlos's process. This allowed Carlos to further explain what he was struggling with. In contrast to Vignette 1, in this segment when he did offer a suggestion, it was connected to the student's thinking and focused on the process that the student was using rather than the accuracy of the solution.

Second, several of the exchanges between Mr. Davidson and students explicitly prompted specific students to explain their reasoning. Mr. Davidson conveyed an expectation that students' answers should include justification, and that students were accountable to each other and to developing their own reasoning about the problem. While one student attempted to explain his reasoning, Mr. Davidson listened and helped to draw the other students into the student's line of reasoning. In line 9, Mr. Davidson repeated what he heard Carlos thinking aloud. The first part of what Mr. Davidson repeated was a very close paraphrase of what Carlos had just said. However, Mr. Davidson's continued beyond what Carlos said to extrapolate Carlos's thinking based on similar reasoning that Mr. Davidson has heard from other students' discussion of the same scenario. Then, in line 15, Mr. Davidson gave Carlos another chance to articulate his reasoning by asking, "What do you know about James's age?" When Carlos's response did not make sense, Mr. Davidson asked "tell me about James's age again . . . " (line 17), rather than correcting him.

When Carlos responded that was talking about Tom, Mr. Davidson again followed his reasoning (line 21), saying “So, tell me about Tom’s age.”

These exchanges signal a shift in Mr. Davidson’s practice toward listening for and making use of student thinking. In contrast to Vignette 1, in this interaction Mr. Davidson seems to place less emphasis on reaching a correct answer or on the correct language. Instead, he allows time for Carlos to try to articulate his understanding. At the same time, there is evidence that Mr. Davidson continued to pursue correct answers, or in this case, correct matches. However, because of the structure of the FAL activity, these answers were a stepping stone to articulating reasoning. For example, in lines 23 and 25, he directed them to use a process of elimination to find the correct match. However, he did not leave them with a sense that having made the match was the important part. Instead, he focused on the connections they were making between the situation and the equation and how to use those to justify that the equation represented the situation. This stands in contrast to Vignette 1, where there was no emphasis on justifying the descriptions of the sequences.

Finally, unlike the exchange in Vignette 1, Mr. Davidson elicited explanations from all three members of the group, and was looking for evidence of each student’s participation in making sense of the problem. In line 9, he says, “I am not just talking to Carlos, I am talking to everyone.” In line 11, he asked Justina to state what x represented, even though Carlos had just said it. From the immediacy of Justina’s response, it appeared that she was at least following the conversation, even if she had not made sense of the situation herself. In line 21, Mr. Davidson directed Carlos to explain what he was doing to Juan and Justina. Then in lines 23 and 25, he explicitly directed questions to Juan and Justina, eliciting from each some reasoning about the connection between the card and the situation. Although the participation of the students in this vignette was certainly not equitable, it is clear that Mr. Davidson was making efforts to involve all members of the group. In the last section of this chapter, I show how Mr. Davidson used a different phase of the formative assessment lessons to create more opportunities for every student to explain his or her thinking.

Vignette 2 illustrates three different ways that an increase in verbal articulation of student thinking can support student learning. First, the students were refining their reasoning as they talked through explanations aloud. This was evident as Carlos was talking through the relationship of Tom’s and James’s ages in lines 16 to 25. Second, through Carlos’s explanations, the other students gained access to the problem. For example, in line 29, Juan appeared to be building on Carlos’s earlier thinking when he explained why 60 rather than 54 would be the value on the right-hand side of the equation. Similarly, in line 36, Justina’s reason for adding 3 to x follows on the exchange between Carlos and Mr. Davidson.

Third, the student explanations provide Mr. Davidson with the type of information that is the basis for formative assessment. In line 9, Mr. Davidson drew on his earlier interaction with another group to make sense of what Carlos was thinking about this scenario. Mr. Davidson gave a much more elaborate explanation than Carlos had verbalized, but it resonated with Carlos and made it possible for him to continue to engage with this scenario. As he listened to the group sorting out explanations, Mr. Davidson could also see that the meaning of the variable and where it is in the equation posed a challenge to the students. This allowed him to focus their attention more on the meaning of the variable in each step of the equation.

The activity in the FAL supported the articulation of these student explanations in four different ways. First, the structure of the activity – matching cards with explanations, rather than constructing equations – emphasized justification over the "correctness" of the equation. Second, the physical materials and shared workspace encouraged more collaboration than work with individual books as in Vignette 1. Further, a student could use the positioning of the cards to take the floor in the group discussion as both Juan and Justina did in lines 24 and 26. Third, the process of making the matches provided a beginning for building good explanations. By the end of the group visit, the students were confident that they had created a correct match, however, they spent quite a bit more time working out a solid explanation. Finally, through the shared product – the poster and presentations – each student knew they would be publicly accountable for their group's work. The significance of this shared product was increased by Mr. Davidson's use of student presentations discussed in depth at the end of this chapter.

Comparison of Vignettes 1 and 2

The detailed analyses in Vignettes 1 and 2 highlight ways that the FALs supported Mr. Davidson in changing his practice during group work to support his goals for his students. These shifts both allow Mr. Davidson more access to how the students are thinking, and provide the students with opportunities to focus on mathematical processes and to strengthen their mathematical communication. Although these vignettes were chosen as illustrative of the changes in Mr. Davidson's interactions with the students, they are only meaningful if they indicate changes in his interactions with other groups throughout the lessons.

At a less fine-grained level, the differences between these two vignettes can be characterized as differences in the number of student explanations. In Vignette 1, there is only once instance of a student, Hector (line 15), attempting to explain his thinking. In that case, he was interrupted by Mr. Davidson correcting his use of the phrase "going by" and did not complete his explanation. In contrast, Vignette 2 has six instances (lines 3, 7, 16, 22, 24, 38) of students providing justification for their thinking. These provided more opportunities for the students and Mr. Davidson to learn from explanations.

The next section shows how the difference between these two vignettes is representative not only of changes in student explanations during group visits, but is part of a trajectory of changing interactions across the three concept development FALs that Mr. Davidson taught in the Math 8 class.

Changes in group visits and student explanations

As described in the introduction to this chapter, this analysis compares Mr. Davidson's interactions with his Math 8 students during collaborative activities in the routine lesson and the activity phases of the concept development FALs (CD1, CD2, and CD3).

The group visits for each of these collaborative activity phases were coded as described in Chapter 3. I use these measurements to describe three aspects of the teacher–student interactions. The first two are roughly aligned with the third and fourth dimensions of the TRU Mathematics Framework (Schoenfeld, 2015). The number of student explanations provides a measure of student agency in the class. The number of distinct students offering explanations provides a measure of the students' access to mathematical discourse and the teacher's access to the students' mathematical thinking. Finally, the percent of group visits with at least one student explanation provides a measure of the teacher's expectations for group visits. Increases can be viewed as indicating increases in expectations for student explanations.

Table 4.1 gives these statistics for the four lessons.

Table 4.1. Teacher group visits and student explanations during group visits in Mr. Davidson's routine and FAL lessons

	Routine Lesson	CD1 Repeating Decimals	CD2 Steps to Solving Equations	CD3 Interpreting Distance-Time Graphs
Observation dates	9/18/2013	11/4–6/13	2/4–6/14	5/14–16/14
Duration of group visits in min:sec	24:30	35:13	68:47	62:04
Number of group visits*	34 (14)	60 (37)	36 (41)	31 (32)
Number of student explanations during group visits*	34 (14)	37 (23)	49 (56)	49 (51)
Number of distinct students offering explanations (% out of students present)	8 of 21 (38%)	9 of 19 (47%)	19 of 25 (76%)	24 of 25 (96%)
Percent of group visits with at least one student explanation	57%	46%	66%	66%

* Normed to 60 minutes, actual count in parentheses.

Group visits and number of student explanations

During the routine lesson, Mr. Davidson participated in 14 group visits over 24.5 minutes, for a rate of about 34 group visits per hour. In the next lesson, the first concept development FAL that Mr. Davidson taught, he had almost twice the rate of group visits (see Figure 4.4). This suggests that he spent substantial time moving between groups, keeping the students working. It is consistent with observations that during the first two FALs (PS1 and

CD1) classroom management become substantially more challenging for Mr. Davidson. This is supported by the fact that there is a substantial increase in the rate of group visits, but no increase in the rate of student explanations shared during group visits: 37 per hour in CD1 vs. 34 per hour in the routine lesson.

Between the second and third FAL, Mr. Davidson's pattern of interacting with groups during the activity phase changed. The number of group visits per hour decreased to about the level of the routine lesson (36 group visits per hour – or a bit under 2 minutes per group visit) while the number of student explanations increased substantially. From the third to the fourth FAL, the rate of group visits and student explanations remained roughly the same, suggesting that Mr. Davidson was establishing a new norm for interacting with students that included more student explanations and placed more agency and authority with the students.

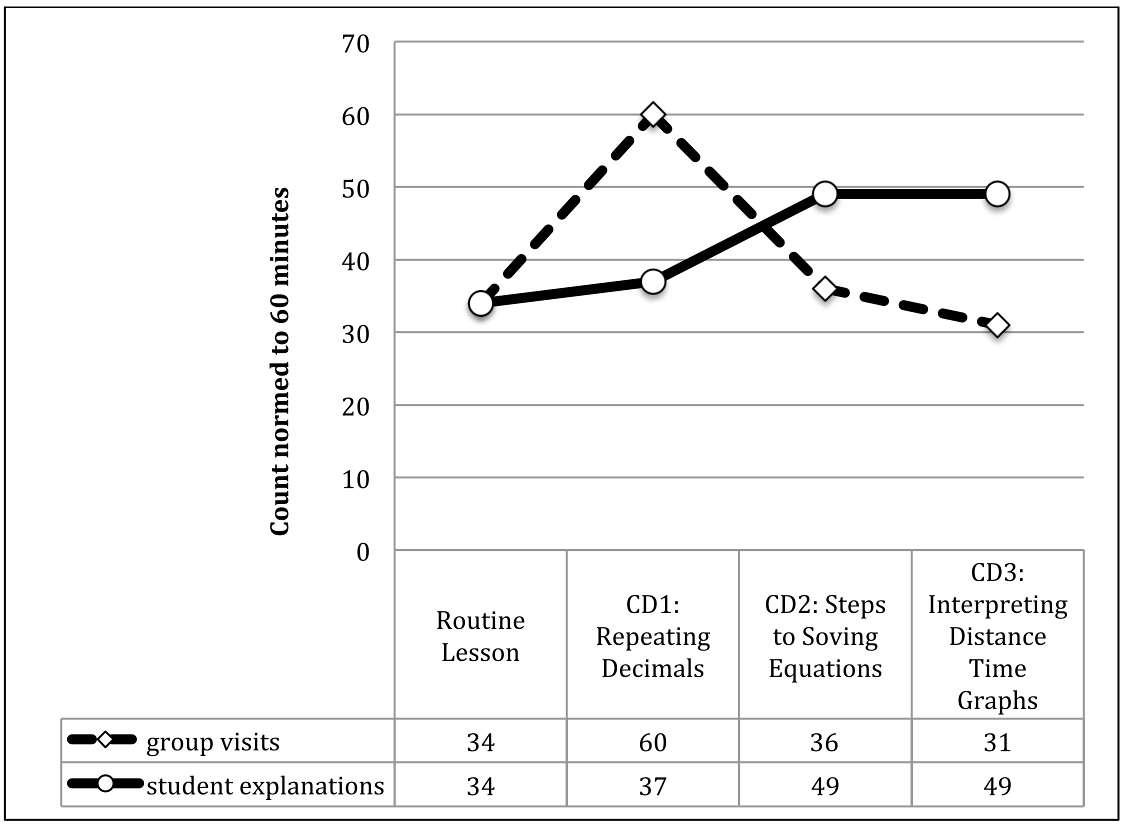


Figure 4.4. Group visits and student explanations by lesson observation

Group visits and teacher's access to student thinking

Although the statistics in Figure 4.4 suggest an increase in student agency during Mr. Davidson's enactment of the FALs, it also is important to consider which students have access to mathematical discourse. The statistics shown in Figure 4.5 provide rough measures of this access as well as the teacher's access to different students' thinking.

Figure 4.5 and the fourth row of Table 4.1 show that the number of distinct students offering explanations stayed roughly constant from the routine lesson observation and CD1, but then increased substantially during CD2 and increased again during CD3. In CD3, all but one of the students offered an explanation during the teacher group visits. Vignette 2 showed an example of a group visit from CD2 in which every student offered at least one explanation. This is still far from equitable; clearly in Vignette 2, Carlos offered the majority of the explanations and was getting to work through most of the mathematical ideas. However, these data indicate a first step toward greater access, by having each of the students involved at least to the extent that they can and will venture explanations.

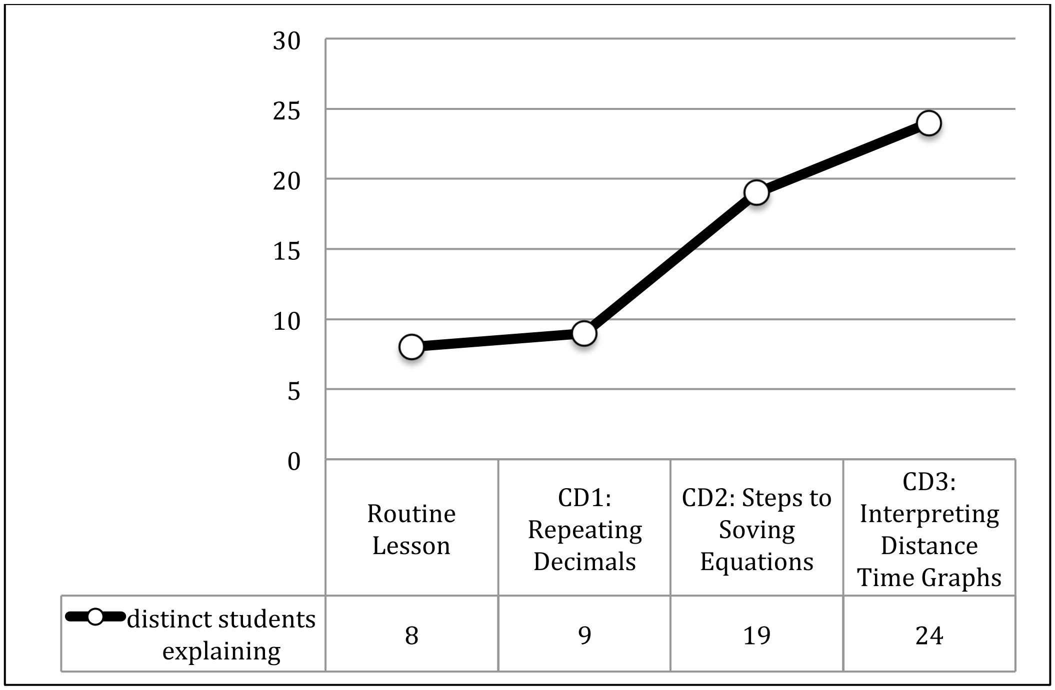


Figure 4.5. Number of distinct students offering explanations during group visits

Number of group visits that included explanations

The trends shown in Figure 4.4 and 4.5 focus on changes in how students are participating and which students have access to the new participation norms. The last part of this section concerns changes in Mr. Davidson's expectations and experience during group visits. In Vignette 1, Mr. Davidson arrives at the group expecting to help them with the "tricky one" and leads them to work on the second sequence where their explanation is incorrect. Although he might want the students to discuss and explain, he does not seem to expect or insist on it. Instead, he interrupts when a student offers to explain. Figure 4.6 shows the percent of group visits that included at least one student explanation. From the routine lesson to CD1, the number of group visits with an explanation decreased 10 percentage points. From CD1 to CD2 the number of group visits with at least one explanation jumped up 20 percentage points, and stayed at that some level for the final FAL, CD3. This suggests that Mr. Davidson is more frequently coming into a group visit expecting to hear a student explanation and being present when one occurs.

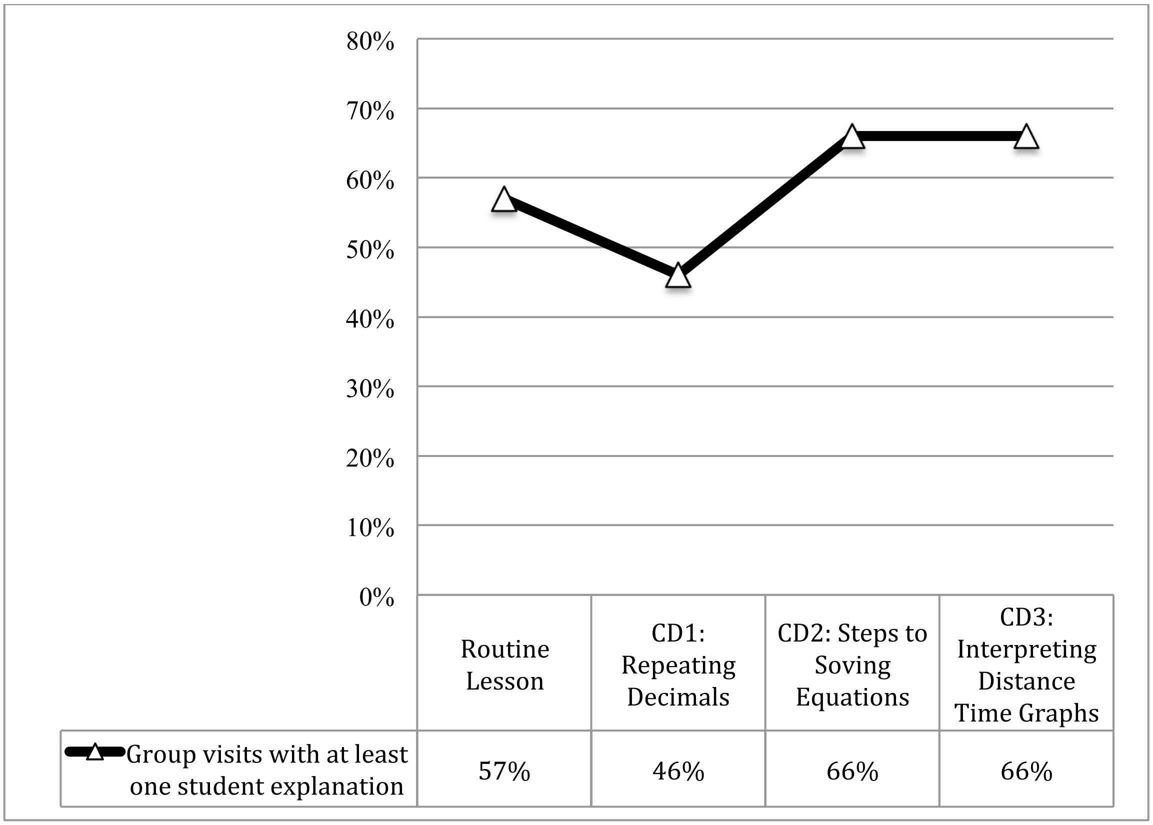


Figure 4.6. Percent of group visits with at least one student explanation

Summary

Together, the statistics in Figures 4.4, 4.5, and 4.6 show a teacher who was not initially sure how to enact the FALs in a manner that was different from his routine lesson. Figures 4.6 and 4.4 show little change, and arguably more focus on management than concept development during CD2 than during the routine lesson. In the later FALs, Mr. Davidson had more opportunity to learn about student thinking from a larger range of the students in the class. His teaching of the FALs seemed to have supported his and his students in changing their interactions – that constitutes learning.

Mr. Davidson's learning about how to elicit or listen to student explanations during the cooperative activities represents an important step toward his ability to engage in formative assessment with his students. The remaining question is: What he did to support student learning based on the student thinking that emerged during the activity? The answer is complicated, because Mr. Davidson did not engage his students in a whole class discussion as suggested in the lesson guides for any of the three concept development FALs. Instead, he developed and refined a collection of new routines to support students in presenting their work to each other.

Creating and refining classroom routines

This section describes adaptations that Mr. Davidson made to the three concept development FALs that he taught, elaborating and expanding the function and focus of student presentations in ways that embody many of the guiding principles of formative assessment. These principles became more prominent over the course of several lessons, indicating that he learned how to effectively integrate formative assessment into one aspect of his classroom practice. The process of reflection and refinement that contributed to the evolution of the adaptions illustrates how Mr. Davidson's goals for students' mathematical communication and individual accountability and his adaptations informed one another.

This section details how Mr. Davidson used the FALs to develop and refine new routines in the service of his goal of strengthening students' written and oral communication of mathematics. In creating and refining his directions to students and routines for presentations, he learned more about his students' mathematical thinking and how to support students' mathematical explanations. Although in creating these routines he focused on students learning to present more clearly and professionally, the new routines also supported students in clarifying and solidifying their understanding of the core concepts in the lessons. Mr. Davidson's choice to deviate from the FAL guides' plans for whole class discussion was goal oriented and ambitious. Rather than altering the plans to fit one of his existing routines, a common result when attempting to use curricular resources to change teaching (Cohen, 1990; Remillard, 2005), Mr. Davidson used these lessons as opportunities to experiment with his classroom practice.

The evidence presented in this section draws on observations of three of the four lessons discussed earlier in this chapter. These lessons were based on the concept development FALs (CD1, CD2, and CD3): Repeating Decimals, Steps to Solving Equations, and Interpreting Distance Time Graphs. In these FALs, the collaborative activity phases of these lessons are very similar: students work in small groups to match cards that show different types of representations (see Figure 5.7). During the activity, students are supposed to explain and justify each match they make, record the matches by gluing the cards on group posters along with written evidence and explanations to record their thinking. According to the lesson guides, the discussion phase should focus on comparing strategies that students used to justify matching cards and making connections among the different representations. None of the three lesson guides makes explicit mention of students presenting their posters, either to small groups or to the whole class.

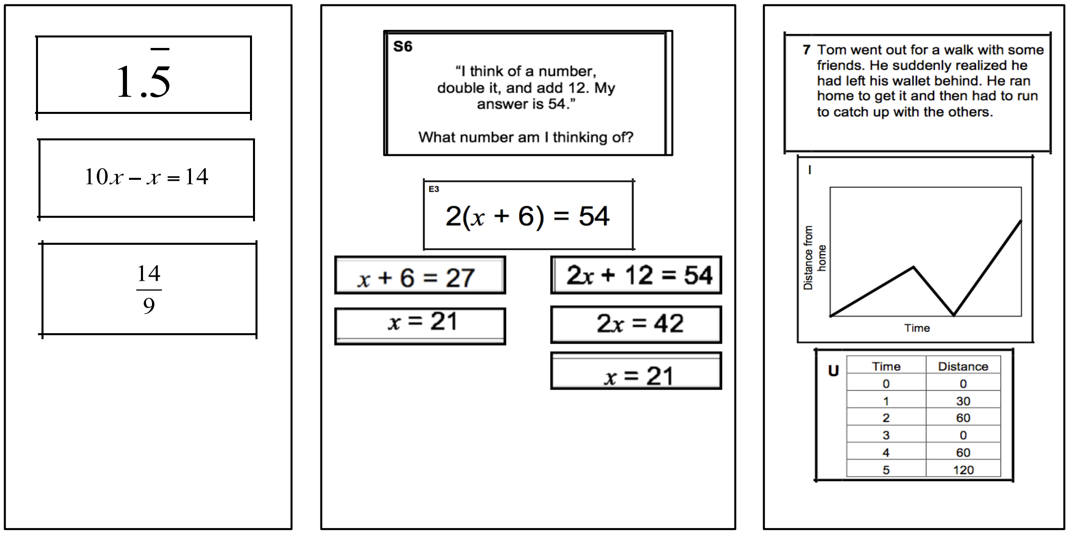


Figure 4.7. Examples of card matches from CD1 (left), CD2 (center), and CD3 (right)

The guides for CD2 and CD3 do suggest that students spend five minutes sharing ideas between groups toward the end of the activity phase. During this time, one or more students from each group records the matches made by the group, then visits another group to discuss differences in matches. The focus of this activity is on sharing strategies between groups of students and providing students with feedback that they can use to revise their group's work. Mr. Davidson's use of student presentations was much more substantial than described in the lesson guides. His focus was on the act of presentation as much as it was on its content. Rather then being an expansion of the suggested activity from the lesson guide, Mr. Davidson's use of student presentations seemed to come from his own ideas about reform teaching and from his discussions with colleagues about structuring opportunities for students to share their work.

Mr. Davidson created four routines related to student presentations: poster preparation, rehearsal, feedback, and presentation.

The first routine involved communication of his expectations for posters. Mr. Davidson used the presentations as the motivation for students to show their written work and explanations in addition to gluing cards on the posters. In letting the students know that each member of the group would be individually responsible for presenting, Mr. Davidson used the creation of the posters to support his goal of holding students individually accountable for understanding and contributing to the posters.

The second routine was the rehearsal. Students were provided with structured time and instructions for practicing their explanations of their group's poster. This included knowing what questions the audience might ask during their presentations.

The third routine was the presentation of the posters by individual students to other students. Mr. Davidson created the roles of presenter and audience, as well as an activity that required students to take both roles. This routine included explaining how the presentations were going to work, what the expectations were during the presentations and the orchestration of the actual presentations.

The fourth and final routine was providing feedback on other students' posters and presentations. The feedback might be about either content and presentation style or both. The routine for students providing one another with feedback changed the most during these lessons.

The analysis below shows how Mr. Davidson modified the routines and his directions to students. I describe how the four routines for presentations arose in CD1 and how they were modified during CD2 and CD3. I show how the modifications were made in order to improve the fit between the routine and Mr. Davidson's goals for his students. Where possible, I share some of the reflections that led to Mr. Davidson's modifications.

CD1: Routines emerge rather spontaneously

CD1 occurred over three class periods. On the first day, Mr. Davidson had the students complete a chart converting fractions to decimals and then notice and name patterns they found in the chart. On the second, the students worked on the card matching activity (see left side of Figure 4.7) and prepared posters showing their matches. In CD1, Mr. Davidson drafted an initial version of the routines for student presentations in order to have students practice formalizing oral communication skills and to increase the students' individual accountability for both the quality and understanding the content of their group's work. He appeared to be figuring out his directions, supports, and format on the fly, as he introduced each routine to his students.

Preparing posters routine

In the launch phase of the lesson, Mr. Davidson framed the goal of the collaborative activity as creating three-card matches – orange card (fractions), pink card (decimals), and green card (equations) – accompanied by explanations of how matches were determined. He said that students might show written work or use prior experience to explain how they knew which cards to match. He explained that students could use whatever method they liked, but they would be creating posters that showed their matches and their reasoning. Mr. Davidson held up examples of posters from the previous class, in order to clarify the goal of creating a poster.

> You're going to be taking these and you're going to be creating posters [Mr. Davidson holds up two posters from the period 2 class]. I'm going to be giving you multiple pieces of paper for each group. They're pretty big. Before you can glue things down, you should have your matches already created. First step, find your pairs, second step – explain, make sure you're explaining as you are finding the pairs, and then lastly, glue them down and write up your explanations.

Virtually no directions were given to students about what makes a good explanation or what specifically about their matches they should be explaining. The goals seemed to be matching colorful cards and gluing them to make a good-looking poster. One group asked if they could be creative with their poster. They did not yet know what they would be doing with the posters nor how the posters would be evaluated. Which matches needed to be included? What constituted a good explanation? The two examples that Mr. Davidson chose to hold up both had written explanations, but the students only got a quick look at them. The other period 2 posters had only equations and computations.

During a group visit, a student asked Mr. Davidson if she could see the poster that he had held up again. He interpreted that as meaning that she wanted to copy their answers, and told her that her group needed to figure it out for themselves. It is certainly possible that his interpretation was accurate, but it is also possible that she was trying to figure out what was expected and what a “good” poster or “good” explanation looked like.

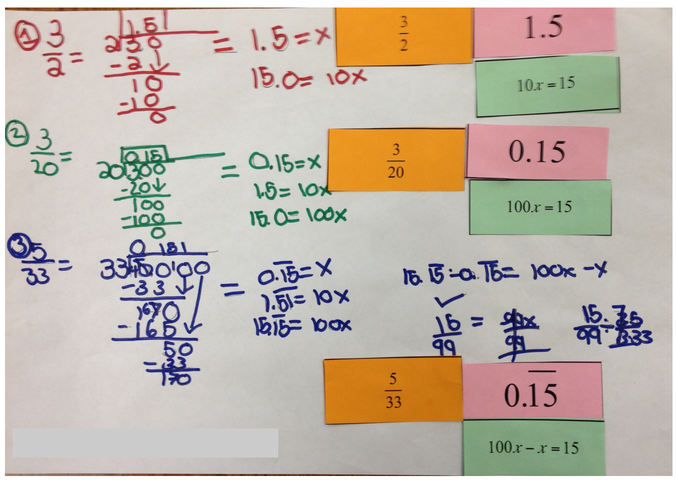


Figure 4.8. Typical group poster showing division algorithm and equations

At the end of class on the activity day, Mr. Davidson more directly addressed the goal and expectations for the posters. By now, many of the groups had matched cards, but only a few groups had begun making posters. Mr. Davidson indicated that there was a difference between the discussion that the students had been having and written explanations on the poster.

> I heard some great conversations. Here’s the part that's missing. The part that’s missing isn’t you matching up the cards anymore, it isn't the conversation, it’s you taking that conversation and getting it onto your actual paper. Actually showing how you know that these things match up. You have just a few more minutes. . . . We're going to be sharing these out tomorrow as we go through all of this. Ok? We're going to have two things to do tomorrow. But we are not going to be creating these tomorrow. Today is the day. Before you leave, you must have an explanation down on your paper for at minimum at least two of your sets. That means gluing it down, that means writing it down.

Mr. Davidson's framing of expectations for the student poster and the presentations was evolving as the lesson progressed. The directions above were the first time that he told the students that they would be "sharing these out" in some way, although it was still quite vague. It was also the first time that he had specified the minimum number of explanations on the poster. This seemed to be a spontaneous decision guided by time constraints and the work that he saw emerging on the posters, rather than a careful consideration of how many or what types of examples would be helpful in showing student understanding.

Mr. Davidson recognized and pointed out to the students that the difficulty they were having was with "taking that conversation and getting it onto your actual paper." However, he did not provide any more specific guidance to the whole group on how to do this. Each group was left on its own to determine which matches to include and explain, and what counted as an explanation.

Rehearsal routine

At the start of the next class, Mr. Davidson gave students about 15 minutes to finish up work on their posters and to rehearse explaining their posters to each other. He wrote three questions on the whiteboard. These questions, shown on the left in Table 4.2, closely resembled the questions suggested in the lesson guide for facilitating the whole class discussion.

Table 4.2. Questions

Questions on the whiteboard to ask and answer during presentations	Questions in the lesson guide for facilitating whole class discussion
How did you decide that equation matched the decimal? How did you decide the equation matched the fraction? Did you do anything different?	How did you decide that that equation matched this decimal? How did you solve the equation to find the matching fraction? Did anyone use a different method?

This showed that even while he was creating his own routines for conducting this phase of the lesson, he was also using the lesson guide. However, in the changed format, the third repurposed question did not make much sense to the students.

> What I'm going to give you time for now . . . [is] to practice answering these questions. [Points to three questions on the board.] These questions are how you're going to be presenting. Now, let me be clear, when you're presenting, you're not going to be presenting with your group. You're going to be presenting on your own. You're not going to be presenting in front of the whole class, but you do need to be able to do this on your own. Ok? So you need to be comfortable with your poster, and comfortable with how to explain this because a good explanation isn't going to look like "I did this and that's what we did . . ." That's not going to work.

These directions describe two tasks for the students to prepare for: answering the questions on the board; and explaining their posters. These are somewhat different tasks. Students were not given examples of either what might count as explanations or what acceptable answers to the questions on the board would sound like. However, many of the groups engaged in figuring out how to address the questions. While the students worked in groups discussing or, in a few cases, completing their posters, Mr. Davidson circulated, encouraging them to ask each other the questions on the board and prepare answers for them.

A second aspect included students knowing for the first time that each of them would be individually responsible for presenting. As Mr. Davidson visited the groups, he found that in some cases each group member was preparing a response to one of the questions. He clarified that every group member needed to be able to answer all three questions and to present the whole poster on their own. The students knew that they were preparing to present, but they did not yet know who they would be presenting to.

Presentation routine

In his directions to students at the very beginning of class, Mr. Davidson described the purpose and structure of the presentations for both the presenters and the audience, mentioning its benefits for improving their work. He noted that in the period 2 class, some students had changed their posters after listening to another group's presentation and realizing they'd made mistakes. He let the students know that it would be acceptable to make changes to the posters after they had a chance to see the other students' work.

After students had rehearsed their presentations for 15 minutes, Mr. Davidson had each of the five groups tape their posters on the wall in different parts of the room. He began by emphasizing how the students were to behave during the presentation. Every student should stay standing as both the presenter and as the group being presented to. The presenters should make eye contact with their audience. Presenters should not read from the posters, but should point to the matches they were explaining and talk to the audience members about how they justified the matches.

Audience members should only look at the presenter and the presenter's poster. They should be attentive and supportive. They should ask questions of the presenters and necessary, use the questions on the board. And, as said at the beginning of class, if they found something that they had done differently, they should write it down or remember it so that they could fix their own posters later.

He next explained the organization of presentations and audiences, and had the students walk silently through the rotation. The students counted off to four in each group. First, the "ones" stayed at their poster to present while the other students in the group rotated to the next poster for a presentation. At the end of that presentation, the "ones" joined their group as it moved to the next poster, while the "twos" returned to their posters to present, and so on. In this way, every student would present at least once (in the smaller groups, some students had more than one number) and every student would get to see and hear several of the posters being presented.

Throughout the presentations, students turned to the board and asked the posted questions of presenters. However, the students did not seem to be trying to follow the presenters' answers. They did not challenge or seek clarification of the answers, even when the answers were confusing or incorrect. It appeared that both presenters and audience members interpreted the routine as asking and answering the questions on the board without much attention to the content of questions or answers.

Feedback routine

In CD2, students gave each other feedback in two different ways. First, as the students were walking around during the presentations, each student was given a few post-it notes shaped like a "thumbs up" and encouraged to write positive comments for other groups about their poster or their presentation. Mr. Davidson's only directions for the post-it notes were that they needed to be signed and the comments needed to be positive and respectful.

After the groups had cycled all the way through the posters, Mr. Davidson gave the groups a chance to look at their feedback comments together and congratulated them on a doing a great job of presenting to each other. The next day, the students took the post-assessment. They also rated and commented on the work of each member of their group on creating the poster and presenting it. Like their comments on the post-it notes, these comments were very sincere.

Discussion

As is often the case in teaching, Mr. Davidson developed a routine as he implemented it. In this first attempt, Mr. Davidson created structured opportunities for students to understand the expectations for the poster while they were working on them, rehearse their presentations and prepare every member of the group to participate, explain and listen to other students' explanations of the posters, and to give and receive feedback on their performance. These emerging routines appear to connect well with four of the strategies that Black and Wiliam have described as key to effective formative assessment: clarifying learning intentions and criteria for success; activating students as owners of their own learning; activating students as resources for one another; and providing feedback that moves the learner forward (Black & Wiliam, 1998). As they were implemented in this lesson, they did not actually meet the goals associated with these strategies, but they were a valiant beginning.

Throughout the implementations of these routines, the students worked hard and stayed on task, but they seemed to be going through the motions. There was little evidence that the students were learning much from each other's presentations or posters. Only one student went back to her group's poster with a marker to correct their work. The feedback from the students was entirely about the amount of effort and seriousness with which their peers were approached the task. Most of the students seemed to believe that their role as an audience member was to hold the presenter accountable for trying to explain their group's poster and to let them know if they had done that well. There was little emphasis or awareness on the part of the students that they were supposed to learn from the presentations or from the critiquing process.

CD2: Refining directions and clarifying goals

In CD2 and CD3, Mr. Davidson refined directions and structures for the four routines. As he did this, the goals of presenting professionally and confidently and the goal of individual student accountability emerged as priorities.

Preparing posters routine

In CD2, he again introduced the poster on the second day of the lesson, showing students an example from the previous period:

> What you're going to be expected to do is to create a poster. I want to show you one from last periods that did a good job. Now, what you can see here is they have their six scenarios, they have a couple that share their steps, they showed the work that went along with it, and then explained what the answer meant . . . Our goal is to have this all done and ready to present for tomorrow.

In contrast to CD1, when he quickly flashed examples of posters while introducing the poster activity, he was more specific about his expectations for a “good job.” He emphasized that he was expecting each poster to show all six scenarios and equations and written explanations and the work that went along with them. In his reflection on CD1, he expressed concern that students had managed to avoid including the more challenging repeating decimals that were central to the lesson. In this lesson, he made creating and explaining all six matches a requirement.

Near the end of the class period, most of the groups were still trying to figure out which cards matched and how they could explain the matches. In some cases, the students still had incorrect matches, but did not know it. Mr. Davidson brought the class together to clarify the expectations for timing and for explanations on the posters:

> You do need to start writing down your explanations. So far I've seen I think two groups, maybe three, who are now certain of the pieces going together and they've got them down. You don't have time after today. This is it. You had enough time yesterday and today. Step up. Make sure that that it makes sense for all of these.

In this instance, Mr. Davidson’s goal for students to work out good explanations conflicted with his frustration about the speed and efficiency with which the students were working. In response to this conflict, he focused the students on getting the posters done and gluing the pieces. Explanations took a backseat.

As the class period was ending, with many of the groups were still not finished. Mr. Davidson said that they could stay, while emphasizing the importance of each student’s responsibility to the group:

> If you would like to stay and work on this, you are welcome to. Here is my one major warning. This is *not* a one-person project and if you say, either as one person or to one person in your group, "Hey, just stay and finish this for me, all right? You haven't done anything so you do it," that won't work. And if you say, "Guys, I'm just going to do it all, just go, whatever, I just going to do it," that won't work either. If you want to work on this as your group, you are welcome to, but you should not have one person whose taking over and saying "I'll just do it all for you, don't worry about it." Understand?

Again, his goal of individual and collective responsibility took precedence over the substance of the explanations on the poster. However, this goal also meant that every student was familiar with their group's poster when they began the presentations the next day.

Feedback routine

On the following day, all of the groups had completed posters that they were ready to present. Mr. Davidson placed the feedback routine earlier in the lesson, and was again more specific in his expectations. This time, when the groups circulated to each of the posters they did not leave a presenter behind. Instead, they were expected to read and critique the work on each poster using post-it notes to leave feedback. He emphasized that each student should put their name on their post-it notes, again, clearly communicating that this was an activity for which they were individually accountable.

In contrast to CD1, Mr. Davidson provided specific examples of his expectations for written feedback on the post-it notes.

> Your feedback will *not* sound like this: "Good!"; "Bad!"; "Color – I like your color!"; "You didn't use color." That's not really good feedback. It's an observation, but it's not really good feedback. Things I want you to be thinking about:
>
> 1. What kinds of connections did they find between the situations and the equations?
>
> 2. How were their explanations? When you read through their poster, did their explanation make sense and did you finish it going, oh, now I know the answer to this question.
>
> 3. Were there any things on there that maybe weren't correct?
>
> Then maybe you need to say, "Hey, I don't understand how you went from here to there, or how you matched that scenario with that equation." . . . You want to give them feedback. At the end, you're going to read the feedback from everybody else and you want to have something that you can move forward with that will be positive.

In looking at the post-it notes that students had put on the decimals posters in CD1, Mr. Davidson had noticed there were no critical observations that a group could use to revise their work. This made sense, because he had emphasized keeping the comments positive. Here, he framed the feedback as more than just comments, they were supposed to help the other students improve their work.

Despite these directions, after the groups had rotated through three posters, he saw that their comments were still very non-specific. Students seem to have understood his directions as meaning they needed to use longer sentences and be more specific about what they did or did not like. Mr. Davidson stopped the students and redirected them toward his expectations:

> Hear me please. A lot of you are giving feedback in a way that I'm not surprised by, for example, I am reading things like "I like that the arrow shows this"; "I like that your work was neat"; "I like that your equation is matched up to this" or "You had no explanation."

Mr. Davidson was working to improve the kinds of feedback that he saw students produce in CD1. He looked for another way to move students toward giving the type of feedback he wanted. He reminded them of the class discussion that they had had during the launch when students were challenging each other's explanations and choice of equations. Then he connected that conversation with the work that they were doing with the post-its. He emphasized that they already had thought about these problems, so they had knowledge that they could use to dig into the mathematics, not just the formatting of the posters.

> I want you to push yourself now, because every single one of these posters is exactly the same stuff that you worked on. There is no difference. Nobody got a different set of questions, nobody got a different set of steps. They are exactly the same, which means that you know if it matches up with yours . . . I want you to start looking at these posters with that same eye of "Do I agree with what they said in their explanation or do I disagree? And if I disagree, why am I disagreeing, what would have to change?" You've got three more that you are going to be doing, I really want you to try and use these last groups to put in a lot of effort around do you agree or disagree with the work they have on their page and then explain why.

Again, the directions were clearer and more specific than in CD1. They showed Mr. Davidson's willingness to stop and try to move the students toward his goal of strengthening their written mathematical communication. Although this version of feedback was more specific and useful than the feedback in CD1, he continued to be frustrated by the students' lack of ability to assess each other's work and provide meaningful suggestions and critiques. Here, the emphasis that he placed on individual accountability conflicted with his goals for feedback and mathematical communication.

One of the challenges for Mr. Davidson and his students in having the students produce and critique the reasoning behind their solutions was that many of the posters still did not have very clear explanations. The students had written equations and properties, such as "multiplicative inverse," that justified how the steps were connected, and in many cases they had stated what their solution to the equation meant in the situation (e.g., $x = 27$ means that James is 27 years old). However, the posters simply did not contain sufficient explanations of the students' thinking for their peers to provide feedback. During his reflection on this lesson, he considered the possible benefits of having the groups discuss each poster they came to and write one really good comment together, rather than having each student silently read and respond to each poster. He also recognize that the preparation routine would have to place more emphasis on the substance of the posters, not simply their "completeness," if the other students were going to be able to critique the work.

Rehearsal routine

After the student groups rotated through the whole class, they organized the feedback on the post-it notes and began to prepare to present their posters to another group. Mr. Davidson paired the groups to present to each other. At this point, Mr. Davidson's goal for the students switched from using explanations to sort out mathematical ideas to present clear, finished explanations of their work. He expected the students to develop coherent explanations for the work they had done. Again, he provided the students with examples of questions on the board that they could ask and should be prepared to answer.

In this lesson, Mr. Davidson introduced a new feature for the presentations. In addition to having students present to small groups, he selected a few groups who would be presenting in front of the whole class. While the groups were revising and rehearsing, he let each of these groups know that they would be asked to present to the whole class and suggested that they use the time presenting to their partner group to rehearse for the whole class presentation.

Presenting routine

Mr. Davidson's directions for his expectations for the professional presence and individual responsibility for a good presentation were much more explicit than in CD1:

> A couple things around this presentation. Number 1, you are not sitting down. So when I say go, you are going to stand up out of your desk or stool or chair and you are going to push it in if you need to. I *don't* want people coming over to your area and then sitting down. I *do* want you standing up to do this presentation.
> Second, every single person in your group will be speaking. And I don't mean speaking as in, one of the people looks at it and says: "Yeah, we did this one and there's our work. So, just read it. It's already there." That's *not* explaining your work. That's not a good presentation.

In this we see an expansion of his previous goals for the students, and a refinement of some of the work done in preparing the students to do what he hoped. However, all this was still in process: directions for the content of explanations continued to be less of a priority than presence. These directions ended with an emphasis on sharing the work and making sure that everyone took responsibility for explaining.

> What I want you to be able to do is to look at the work you've been doing, just from this last five minutes, explain one of the pieces, especially one that you did well, or maybe one that you made a mistake on that now you see the mistake on, explain what you did, what you would have changed, and share the load with your teammates.

The mathematical purpose of these presentations for the "audience" remained unclear. Instead, the audience's role was to provide motivation and accountability for the presenters. Similar to CD1, Mr. Davidson suggested that the audiences use the questions that he had put on the board to support and challenge the other groups presentation.

> If you are one of the people who is coming to be presented to, that means you have questions you can ask. And how nice, if you can't think of the questions yourself, there are four questions on the board that you can ask.

In this lesson, however, the questions are not obviously connected to the lesson guide for the FAL. These questions are prompts for the presenters to do more explaining – they do not provide much support for the questioners in the audience to clarify their understanding of the work presented.

1. How do you know this equation matches this situation?
2. What words helped you find the correct equation?
3. What properties did you use, how?
4. Which ones were the easiest? Why?

During the presentations between groups, Mr. Davidson circulated among the groups, listened to their presentations, and encouraged full participation from each member of the group. After the two groups had each shared, he asked the groups that were presenting in front of the whole class to come up, one at a time, and make their presentations.

Similar to the small group presentation, these presentations seemed to be entirely focused on the learning of the presenters. This was to provide them motivation to polish their explanations and an opportunity to practice presenting in front of a large group. The audience members were encouraged to ask questions to make the presenters explain. However, during all of the presentations at the front of the class, none of the students in the audience said a single word. They were practicing sitting quietly and respectfully, while their classmates were trying to learn to present in front of a group. Mr. Davidson asked the only questions during these presentations, and they were more like challenges to the work being presented.

Discussion

In reflecting on this lesson, Mr. Davidson realized that he needed the students to create even clearer first drafts of their posters if feedback was going to be possible or useful. He also realized that he needed to find a way to focus the students more on how they were connecting the equations and the situations. He also recognized that while the small group presentations seemed to be providing useful opportunities for both the presenters and the audiences, whole class presentations were not serving this purpose. Although he wanted some of the ideas that came out of small groups to be shared with all of the students, these presentations had failed to do that. The students who were presenting were not clear about which aspect of their work they were supposed to share. More important, the teacher's questions were designed to make the presenters think and be more careful and specific in their reasoning. There was clearly no chance that a member of the audience was going to have to answer one of these questions, so the audience sat back and watched attentively, but without any engagement.

By having students create the posters, he was having them work on translating their emerging thinking into written explanations. In having the students present individually in front of an audience of peers, he was emphasizing individual responsibility for group work and providing the opportunity for each student to practice oral communication. He provided rehearsal time to support their sharing of their understanding and used the questions to scaffold both the presenter's and the audience's roles in the presentation.

In reflecting on the enactment of the student presentations, Mr. Davidson was pleased that it went relatively smoothly and recognized that it had promise for accomplishing his goals. However, he had some concerns about the quality of both the posters and the presentations, and felt that many of the students were going through the motions without really making mathematical connections. Analysis of the posters and the post-assessments indicate that his concerns were well founded.

The major new conceptual focus was on understanding how to connect repeating decimals with the equations that could be used to find the equivalent fractional representation. The cards with terminating fractions were supposed to scaffold students in building from what they already understood about fractions, decimals, and one-step equations, to the more complicated relationship between fractions, repeating decimals, and two-step equations. Only two of the five posters had examples of completely correct matches that included repeating decimals and only one of the posters was organized so that it showed first the decimals, then the equations, and then the fractions. The posters for the other three groups showed only how to convert from fractions to decimals using long division and how to solve the equations to find the corresponding fractions, both of which are 6^{th} and 7^{th} grade skills. Each of these three posters had at least one "match" with an incorrect repeating decimal.

CD3: Second revision of routines

Three months later, Mr. Davidson enacted CD3, Interpreting Distance Time Graphs, in the Math 8 class. In this lesson, Mr. Davidson again gave the students all of the cards at the same time, rather than staggered as suggested in the lesson guide. The students spent the day and a half of the lesson switching between collaborative work in small groups to match the cards and whole class discussion about the descriptions and meanings of key features on the graphs and tables. Only after the groups had sorted the cards and explained to each other verbally, did Mr. Davidson introduce the posters.

This time, rather than putting everything that they had figured out on a poster, the students were instructed to explain four matches. He suggested several different ways that they might select the matches for the poster:

> I've given you some colored pencils, one for each of you, and a piece of white paper and I want you to choose four. Now you can go about different ways of doing this. You can choose four that would be a combination of like your best four, these are the ones that I feel best about, these are the ones I'm most proud of, I can explain the best, those could be your four. You can choose a couple that are like, these are two that someone is really going to confuse and I'm going to explain why they are different, how they are different. You can choose a few that are like "these ones confuse me a lot and I know I got it now, but I want to explain why they are confusing to me."

In addition to giving students a choice about which examples to place on their poster and provide a written explanation for, he refined this routine to be much more explicit about his expectations for the explanations:

> Your explanations, let's put it out in the open and make sure we are all clear, aren't going to say "Well, this goes together and this goes together and this goes together because they all match up and yeah, they are the same" or "This story is in that graph so, that's why they go together." You have some good hints on the board around characteristics inside graphs. You also have some great ideas that I've heard you say around the tables. Oh, this means that the person is going fast. Oh this means that the person didn't move. Oh, this means that he went all the way back home. . . . You need to include all three parts [stories, graphs, and tables] in your explanation.

The explanations on these posters were substantially more thorough and complete. The time spent figuring out the matches before working on the posters, the students' choice about which examples to explain, and the more explicit directions all made a big difference in the quality of the explanations. Again, in this class, he made sure that the students knew that their whole group needed to complete the poster and he offered the option of working in his room during lunch or after school.

Rehearsal routine

In this lesson, Mr. Davidson moved the feedback routine to the end of the class, at the end of each of the student presentations. When the students arrived in class, almost every group had a completely finished poster. Mr. Davidson began class with the rehearsal of presentations:

> I'm going to give you 10 minutes to make sure that your poster is in final draft form. You spend the last two days, those were the hard days, doing the critiquing work. What you need to be doing now is making sure that you have a viable argument. That's what we have been working towards. You need to make sure that you have a viable argument. What that means is that you have an argument that you can defend and an argument that you can back up with evidence.

> Where you are at today is past the verbal communication with your group, past the written communication that goes onto your poster, and you are now on to the presenting part.

This was the first time that the directions for the rehearsal had emphasized creating an argument and pointing to evidence, in contrast to the earlier lessons where students rehearsed responses to questions on the board. This was also the first time that Mr. Davidson situated the presentation in relationship to the other kinds of communication connected with the FAL activity.

Presentation routine

When Mr. Davidson launched the presentation routine in this lesson, the students appeared relaxed and ready. They were becoming familiar with this process and knew what to expect. First, he had each of the eight groups of students assign members to be “1s” or “2s”. The “1s” would be presenting the poster while the “2s” moved between presentations. After four rotations, they would switch and the “2s” would present while the “1s” circulated. Most of the groups consisted of three students, a pair and an individual. Again, his directions were more clear and specific, and also focused on the substance of the presentation, not just the orchestration of movement of students throughout the presentations:

> During this presentations, here's what its going to look like. We're going to go around, you’re going to have about two minutes in each group, where you're going to be presenting or you're going to be listening. Now be very, very clear. When you take your poster and you are presenting, what it should **NOT** look like is this: [reading what is written] "I got the whole thing because if you take the graph, it goes up, and when it goes up that means that Tom is walking away from his house..." I know you know how to read. And all around the room, I know that every other person here also knows how to read. You're going to be with your poster. DON'T read your poster. That's not verbal communication. It’s not. I understand you're talking, and I understand you're trying to communicate, but that's not where we're trying to get to. Your presentation is you sharing the work you've done and helping your audience understand how you connected those pieces, why it works, and the troubles and ways you overcame those troubles. That's what you need to be doing. They can read your poster all they want. Not you. (Mr. Davidson, class observation, 5/16/2013)

Again, he recognized the difference between the presentation of their work that he was asking for and the written explanations they had already completed. However, he also continued to emphasize the professional stance that he wanted students to be adopting during these presentations:

> Your presentations are worth 10 performance points. As you are presenting, it matters how you stand, it matters what you say, it matters what you do over those two minutes. If you are in the audience, or if you are a presenter, you're standing up. (Mr. Davidson, class observation, 5/16/2013)

Feedback routine

In this enactment of the routine, the feedback was integrated into the presentation process. The students who were listening to the presentation were given a formal time to give feedback, complete with a grade, at the end of the presentation. Mr. Davidson emphasized the importance of justification in the feedback. While he still put questions on the board that the audience could ask if needed, the role of the audience was more focused on their feedback than on the accountability through questioning.

> Audience members, if your presenter feels like they have finished, you have questions up here on the board that you can turn to if you need to. [points to four questions on the board]. At the end of the presentation, I'm going to prompt the audience members to give the presenter a grade. The grade is not something that is going to go into your grade book, it is so that you as a presenter know how they felt about your presentation. This isn't about are you friends or not friends, this isn't about are you a good person or a bad person, this is to help you get better as a presenter. Audience members, you should be tough graders. I don't want you sparing someone's feelings because you think that they might have some sensitive feelings around how they present. You tell them what they did, what grade you'd give them and why. (Mr. Davidson, class observation, 5/16/2013)

Once the presentations started, each presenter, or pair of presenters, had two minutes to share their work with their audience. At the end of their time, the audience shared their feedback. Mr. Davidson reiterated the directions, emphasizing the role of the presenters in receiving the feedback and the audience members to "make sure you tell them why".

> Audience members, please give your presenters a grade. Presenters, this is your chance to listen. Don't respond. Don't talk. Just listen. Remember, your grade is not are you a good person or a bad person, this is your audience telling you "this is how I think you did", anywhere between and A and F. You can give pluses and minuses, but make sure you tell them why. (Mr. Davidson, class observation, 5/16/2013)

After each presentation, the audience moved to the next table, and the presenters got to immediately use the feedback to adjust their presentation. Student presenters were able to either highlight a part of the graph or table that they had only mentioned briefly in the last observation or to abbreviate or clarify an explanation that went on a bit long. As Mr. Davidson had intended, the students became visibly more confident as the presentations progressed. They stood up straighter, looked their audience in the eye and spoke more clearly. In short, their presentations improved.

Mr. Davidson set up this routine with a focus on students learning to present and receive feedback on their presentations. However, what emerged was that, as students presented, they became more familiar and comfortable with the content of their presentations and began to incorporate more details and connections into the presentations. In this way, the presentations served to support the presenters' content learning as well as the quality of their explanations.

Discussion

Rather than follow the suggestions for the whole class discussion in the lesson guides, in all three of the concept development FALs analyzed for this study, Mr. Davidson conducted the whole class sharing session in ways that were substantially different from the lesson plans in the guides and also new to his teaching practice. In essence, he used the FALs as a context in which to experiment and develop classroom routines that would support his goals for his students. These goals included placing a high premium on individual accountability for individual and group success.

The structures for sharing what students had learned during the activity phase of the lessons were consistent with Mr. Davidson's goals of individual and group accountability, and development of oral communication skills. His primary goal seemed to be giving students the opportunity to practice and perform explanations of their work in front of peers. A secondary goal appeared to be creating individual accountability during the group activity by letting students know that they each would each be responsible for explaining their group's work. Both of these goals are oriented toward leaning opportunities for student presenters, more than for student audience members.

Over the course of the three concept development FALs, Mr. Davidson created and refined a process for his students to present to each other what they had learned from the collaborative activity. As he developed this process, his goals for the presenting students were more clearly articulated and the structure of the presentation process evolved to better support the students in reaching these goals. It became clear that Mr. Davidson saw students being able to speak to each other clearly and to present articulately, confidently and professionally as goals in themselves. These goals were independent of the lesson's mathematical content or practice goals. However, the routines that Mr. Davidson created to support student presentations also served to support the students in developing and refining their explanations, and thus supported the lesson's mathematical content and practice goals.

Chapter 5

Case Study of Ms. Elmore's Use and Adaptation of FALs

Introduction

This chapter presents a case study of Ms. Elmore, a teacher at Brookfield High School. Like Mr. Davidson, Ms. Elmore took her participation in the study very seriously and expressed a lot of interest and excitement about what she might learn from the FALs. Although her goals were initially less explicit than Mr. Davidson's, they also emerged over the course of the study. Like the case study presented in Chapter 4, this case study focuses on the activity and class discussion phases of the FALs.

In the first part of the case study, I analyze student explanations during group visits, showing that Ms. Elmore's learning trajectory was quite different from Mr. Davidson's. Like Mr. Davidson, Ms. Elmore's interactions with her students during the activity phase of the FALs changed. However, as the school year progressed, Ms. Elmore's interactions with students during the FALs began to resemble those of her routine lessons.

In the second part of this case study, I show how Ms. Elmore's goals influenced her monitoring of student work during the activity phase and her use of the discussion phase of the FALs. Although she was the only teacher who made use of a monitoring sheet to record observations during the activity phase to inform the class discussion, her use and adaptation of the monitoring sheet over the course of the observed lessons reflected and reinforced ways that she chose to adapt the FALs to fit more closely with her routine instruction. In the final two FALs, the monitoring sheet resembled a checklist and the FAL activity served an elaborate launch for a lecture during which students filled in pre-written notes. Ms. Elmore adaptation of the monitoring sheet and the FALs constrained rather than supported her learning of new practices from teaching the FALs.

Background and context

Ms. Elmore was in her fifth year of teaching and her third year at Brookfield. During the 2013–14 school year, she taught three sections of Geometry in grades 9 and 10, and two sections of Advanced Placement Statistics in grades 11 and 12. The 5th period section of Geometry was a sheltered class for students designated as English Learners. Most of the students in this class had recently arrived in the United States from another country. Despite differences between the student populations in the three geometry classes, Ms. Elmore placed a high priority on covering the same material in the three classes each day and keeping the assignments and tests synchronized.

Ms. Elmore had a relaxed manner with the students and her students seem very comfortable talking with her about both mathematics and their lives. She had a well-structured class routine. The students worked on a warm-up problem for five to ten minutes, then they took notes, worked individually or with partners on problems similar to those in the notes, then completed a final wrap-up or exit ticket. Ms. Elmore used a timer to keep the students on task and moving through the classwork. The pacing of lessons and tests in her Geometry classes was coordinated with those of other teachers in the school. She had a tight timeline for covering each of the topics in the class, and was committed to staying on that timeline.

Unlike many of the teachers in the study, including Mr. Davidson, Ms. Elmore used group work infrequently in her class. During group work, students were encouraged to check their solutions or ask questions of students sitting close to them, but little time was spent with students engaged in the type of collaborative work that is built into the FALs. When students did group work, most of the time Ms. Elmore circulated among the rows of students, clarifying directions, correcting answers, and answering questions about how to do particular problems.

Ms. Elmore was observed teaching six lessons: one routine lesson over two days and five FALs. She completed pre- and post-lesson reflections for almost all of these six lessons, attended each of the collaboration meetings for the high school teachers, and submitted all of the student work for each of her lessons. She is one of the two teachers in the study who administered pre-assessments to her students for every lesson. Her students completed the post-assessments for four of the five FALs. For the fifth FAL, Ms. Elmore gave a unit test that she felt covered much of the same material as on the post-assessment, so she provided us with copies of the graded tests rather than using the post-assessment.

During the 2013–14 school year, Ms. Elmore was mentoring a student teacher, Ms. Iris, who was earning her teaching credential. In the second semester of the school year, Ms. Iris took over primary responsibility for daily instruction in 4^{th} and 6^{th} period Geometry. As a result, although observations of her routine lesson and first two FALs occurred in the 6^{th} period class, the final three FAL observations were in the 5^{th} period sheltered Geometry class.

Overview of the six lessons

All six of the observed lessons were coded for instances of student explanation during group visits, using the coding described in Chapter 3. In Chapter 3, I explained the decision to analyze only on concept development FALs in this dissertation. However, for Ms. Elmore's case, I have included observations of the first FAL, a problem solving lesson, because it helps to create a more complete understanding of how Ms. Elmore's thinking changed about teaching the FALs.

In the two-day routine lesson observed, Ms. Elmore presented students with a page of definitions to write down and use as a class, then showed students how to carefully draw circles with a compass. Both of these emphasized accuracy and conventions, with little conceptual work on either day. The goals for these days were written as a series of statements that began "SWBAT" (students will be able to).

The first FAL that Ms. Elmore taught was quite a different story. She chose the grade 6 problem solving FAL, Designing Candy Cartons, which is about designing and building a carton that could hold 18 round candies. This lesson was coded as appropriately placed for her students (see Table 3.3), because although they had several strategies for approaching the problem, it was far from routine for them. Ms. Elmore used almost all parts of the lesson, omitting the directions for the whole class discussion on the second day of class.

The next four FALs that Ms. Elmore taught were concept development FALs. In these, the central activity involved classifying cards into categories such as true or false or by features of the graphs of equations. Two involved discussion of student work. The lessons, in the order taught, were Types of Triangles, Evaluating Statements about Length and Area, Equations of Circles, and Evaluating Statements about Enlargements. As discussed in Chapter 3, two of these lessons, Types of Triangles and Equations of Circles, were used as introductions to topics with which students had little or no prior experience. The other two lessons, Evaluating Statements about Length and Area and Evaluating Statements about Enlargements were taught partway through units about similar content. These two lessons' emphasis on proof and justification was substantially different from the computational focus of the units. Thus, although they were well placed in the units with respect to mathematical content, they still posed a significant challenge for the students.

Changes in group visits and student explanations

The first question examined in this case study is: Did Ms. Elmore's interactions with students during group work in the routine lesson and the FALs change over the course of the school year? As in Chapter 4, I address this question by measuring duration of group visits and number of student explanations, considering these as measurements of student agency, student access to mathematical discourse, teacher access to student thinking, and teacher's expectations for group visits. These measurements are shown in Table 5.1 and Figures 5.1, 5.2, and 5.3.

During the observation of two days of routine instruction (9/25–26/2013), times that Ms. Elmore circulated around the classroom and interacted with one or more students were coded as group visits. During the two one-hour class periods, there was a total of 18 minutes, 55 seconds of group visit time with only one coded instance of student explanation.

There are two reasons for the paucity of student explanations. First, the content of the lesson was quite straightforward and mechanical, not lending itself to explanations. Second, Ms. Elmore used the student work time mostly to correct the students' written work or technique (e.g., use of a compass), or to keep students on task.

Table 5.1. Teacher group visits and student explanations in group visits during Ms. Elmore's routine lesson and the five FALs.

	Routine Class	PS1 Candy	CD1 Triangles	CD2 Length & Area	CD3 Circles	CD4 Volume & Area
Observation dates	9/25–6/13	10/15–17/13	11/12–14/13	2/20–21/14	3/18–20/14	5/2, 5/5–6/14
Duration of group visits in min:sec	18:55	49:47	59:48	27:25	41:51	21:31
Number of group visits*	52 (16)	37 (31)	43 (43)	44 (20)	26 (18)	61 (22)
Number of student explanations*	4 (1)	16 (13)	15 (15)	26 (12)	13 (9)	6 (2)
Number of distinct students offering explanations (% out of students present)	1 of 27 (4%)	8 of 28 (29%)	8 of 28 (29%)	10 of 23 (43%)	7 of 20 (35%)	2 of 26 (8%)
Percent visits with at least 1 explanation	6%	32%	14%	43%	35%	5%

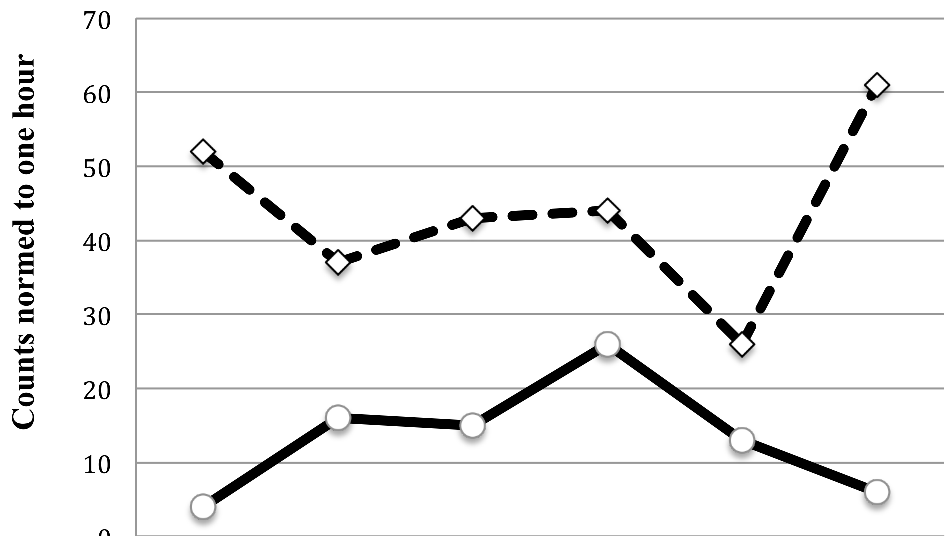
* Normed to 60 minutes, actual count in parentheses.

Group visits and number of student explanations

As shown in Figure 5.1, during the first three FALs, PS1, CD1, and CD2, the number of group visits per hour decreased somewhat. At the same time, there was a substantial increase in student explanations during group visits, between the routine class observation and the first FAL, between the first and second FAL, and between the second and third FALs.

However, these changes were not sustained. In CD3, the number of group visits decreased even further, indicating that Ms. Elmore was spending, on average, a longer time with each group. Moreover, the number of student explanations also decreased.

One possible reason for this decrease in student explanations and group visits is that CD3, Equations of Circles, was taught as an introductory lesson. The students had not previously encountered an equation for the graph of a circle. At the beginning of the lesson, Ms. Elmore gave an elaboration of the launch from the lesson guide, to familiarize the students with the equation for a circle. In CD3, the activity consisted of organizing cards into a grid, by placing equations with same radius in the same row and lining up equations with the same center in columns. Most of the students used the strategy of matching that Ms. Elmore had shown on the board. Part of the reason for the decrease in student explanations may have been that students had little of their own thinking to draw on in order to explain why the graphs of the equations would be circles with a given radius and center. During the lesson, Ms. Elmore's interactions with the groups resembled that of the routine lesson more than it did the previous three FALs.



	Routine Lesson	PS1:Candy Cartons	CD1: Triangles	CD2: Length and Area	CD3: Circles	CD4: Volume and Area
group visits	52	37	43	44	26	61
student explanations	4	16	15	26	13	6

Figure 5.1 Group visits and student explanations by lesson observation

The number of group visits and student explanations for the final FAL, CD4, Volume and Area, was quite similar to CD2. However, Ms. Elmore's use of this lesson did not follow the lesson guide. She created additional scaffolding for the activity, and, in doing so, shifted the focus of the lesson from evaluating the truth of statements to deriving a relationship between scale factors and dimension. The number of group visits was greater than in the routine lesson and the number of student explanations was back to the low level of the routine lesson. As with the previous FAL, these measurements suggest that Ms. Elmore's patterns of interaction during her routine teaching influenced her teaching of the FALs.

Group visits and teacher's access to student thinking

The statistics shown in Figure 5.2 provide a rough measure of whether the teacher had equitable access to different students' thinking. The number of distinct students offering explanations increased from near zero in the routine lesson to a sustained level of approximately a third of the class during each of the first four FALs. This suggests that although the number of explanations decreased during the fourth FAL Ms. Elmore continued to solicit contributions from different members of the class. In the last FAL, the only two explanations provided came from two different members of the class. Because these numbers are all relatively small, it is not possible to draw strong conclusions from them. However, it again seems that the FALs were not only supporting more student explanations, but also explanations from a variety of the students in the class.

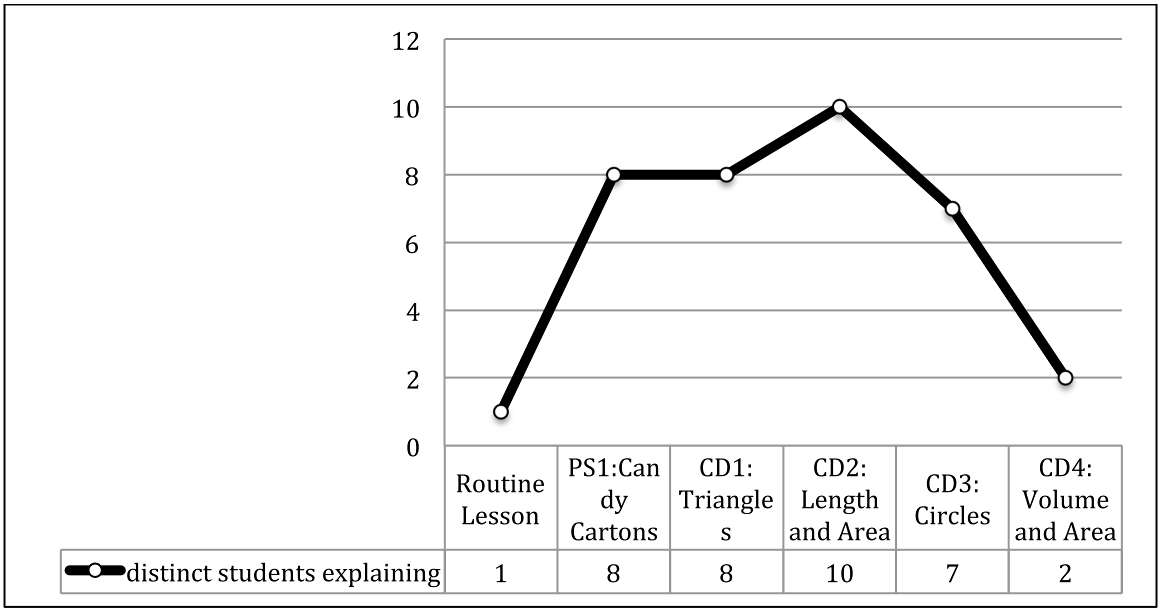


Figure 5.2. Number of distinct students offering explanations during group visits

Number of group visits that included explanations

Similar to the changes reported above, there was a marked increase in the percent of group visits in which Ms. Elmore elicited at least one student explanation (see Figure 5.3). For three of the five FALs, approximately one third of the times that Ms. Elmore interacted with a group, at least one student explnation was provided. During the first concept development FALs, only 14% of the group visits included an explanation, although student explanations were being offered at similar rates in PB1 and CD1 (see Table 5.1). This suggests that CD1 had more variation in types of interaction during group visits, with explanations clustered in just a few group visits. During the next lessons, that clustering seems to have reduced.

The statistics for the last FAL again tell a slightly different story. Rather than establishing a consistent pattern in her use of the FALs, in this last lesson, Ms. Elmore's interactions more closely resemble those in her routine teaching. These statistics tell the story of a teacher who used the FALs to elicit more student thinking during group visits, but this use was not sustained.

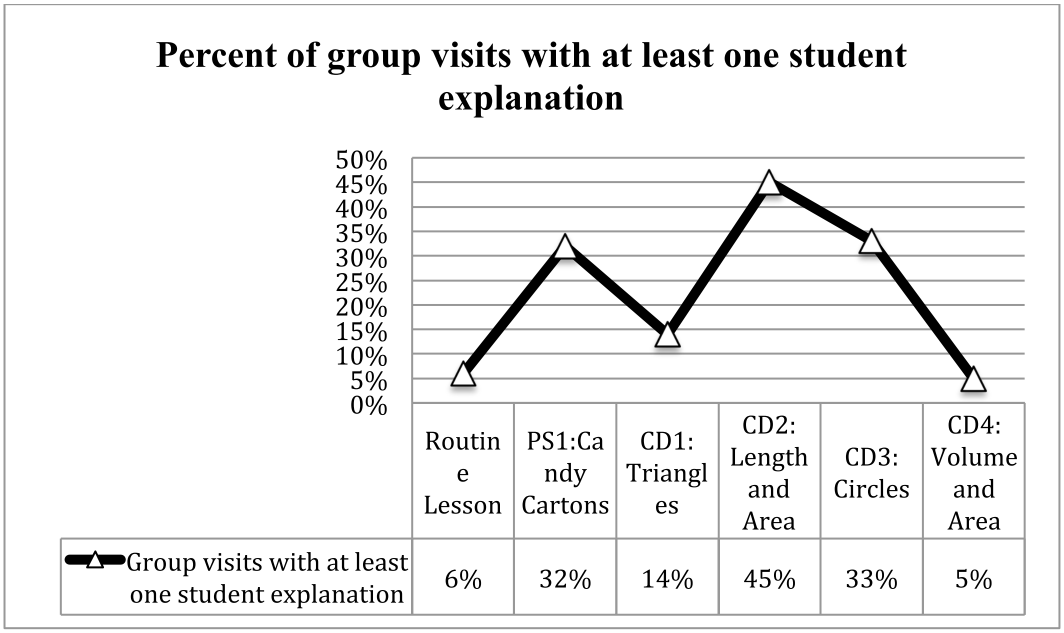


Figure 5.3.

Summary

The factors that may have contributed to the decreases in student explanations during the last two FALs are discussed at the end of this chapter. What is evident from these analyses is that although the FALs can support changes in a teacher's interactions with students during group visits, that does not support the conclusion that these changes will be evident each time that a teacher uses a FAL or that such changes will influence the teacher's routine teaching practices. This case shows just the reverse. Despite initial changes in the teacher's interactions with students during the first FALs, her routine teaching practices influenced her teaching of the later FALs.

Ms. Elmore's use of monitoring sheets

Orchestrating productive classroom discussion after problem solving activities is challenging for many teachers (Stein, Engle, Smith, & Hughes, 2008). It requires recognizing the work that is being done by many small groups during the class activity and selecting student work that can help highlight different strategies or connections between representations or concepts. It also requires the teacher to understand the work that is being presented well enough to know which connections to make and to be able to facilitate students making these connections. Stein et al. (2008) break down the process of orchestrating these discussions into five separate practices: anticipating, monitoring, selecting, sequencing and connecting. They describe how, together, these practices can make class discussions more manageable. In particular, they provide a monitoring sheet with three columns titled: Strategy; Who and What; Order. They suggest that prior to an activity, teachers use their own work and prior experiences with their students (which, in the case of the FALs, could include the pre-assessments), to create a monitoring sheet with a list of strategies, approaches, or misconceptions that they anticipate seeing from the students in their classroom. During the activity, teachers use this sheet to monitor and record student work that might support a class discussion about a strategy or conceptual issue on the list. Before the class discussion, the teacher uses the information on the sheet to select and order the strategies that will be discussed and which students will share their work. During the discussion, the teacher uses notes from the monitoring sheet to focus attention on the connections between the ideas raised by various student presentations.

During the orientation meeting with the teachers in this study, I shared the article "Orchestrating Discussion" from *Mathematics Teaching in the Middle School* with all of the teachers. My intention was to raise a known challenge with the teachers and provide them with a possible resource that they could use (or not) to address it. Along with the article, I provided teachers with a template of the monitoring sheet. Because the concept development FALs are not only focused on strategies, but also raise common conceptual issues surrounding the topic, the heading on the first column of the template was left blank for teachers to fill in their own descriptions of what they were monitoring during the activity. The bottom of the template had space for the teachers to fill in additional categories for observations.

Of the eight teachers in the study, Ms. Elmore was the only one who decided to use the monitoring sheet during the activity and class discussion phases of the lesson. The ways that she used and adapted the monitoring sheet for each successive FAL served as much as an indicator of the ways that she was struggling to make sense of the FALs as it did in supporting her to create more productive class discussions.

The first FAL: PS1

As discussed above, Ms. Elmore began by teaching a problem solving FAL to introduce group work in her classroom and to support engagement with geometry in a tactile way. She chose a FAL from an earlier grade level so that she could be certain that it would be accessible to all of her students. She used a pre-assessment tally to determine that students had a variety of ideas about how to approach the problem and wrote a list of questions, highlighting questions appropriate to different students, to support them in revisiting the assumptions that they made in their designs (see Figure 5.4). Although her questions are similar in tone to those in the lesson guide, none is the same.

At the beginning of class, Ms. Elmore handed back the student pre-assessments and explained that she had written down numbers for questions that she thought were relevant for each student on the top of the papers. She then gave the students 10 minutes to work silently with their own papers to see if they could address the questions. In response to a student, Ms. Elmore explained that Question 7, "What was the reason for the design you chose?," was highlighted because she thought it was the most important and was relevant to everyone in the classroom.

#	Note
1	How do you know what dimensions to make the rectangle for the side of the cylinder?
2	How do you know that your box closes properly? Does it have a lid?
3	What would the measurements of each part of your box be when it's unfolded?
4	How will this box fold up? Could you try it?
5	Have you checked that there are enough tabs so this box will glue shut properly? Are there any extra tabs you could get rid of?
6	How do you know this will fit the candies?
7	What was the reason for the design you chose?
8	Try designing the box first, then design the net.
9	Have you accounted for all necessary measurements?
10	When you fold your box up will all of the parts match up properly? Will you need to adjust some of your measurements?

Figure 5.4. Ms. Elmore's questions for PS1 Candy Cartons

After 10 minutes, Ms. Elmore created groups with two or three students and asked them to share their designs with each other and to start working on an improved group design. While they did this, Ms. Elmore circulated in the class, using a clipboard with a monitoring sheet template (Figure 5.5). Ms. Elmore's extensive work in assessing the students' initial work and generating questions to prompt their revision of this work is precisely the work of anticipating how students would engage with the problem. Despite having done this work, she did not write any information about student thinking on the monitoring sheet. The sparseness of her notes on the monitoring sheet is not surprising because she was using this tool for the first time. Many teachers in the study commented that they were not able to attend to student thinking and write notes at the same time. It is even less surprising considering that the statistics in the first part of the chapter show that Ms. Elmore spent substantially more time per group visit and was offered more student explanations during this FAL than during her routine lesson (see Table 5.1 and Figure 5.1). In this case, the blank monitoring sheet template, without guiding strategies or issues from the pre-assessment, served mainly as notepaper.

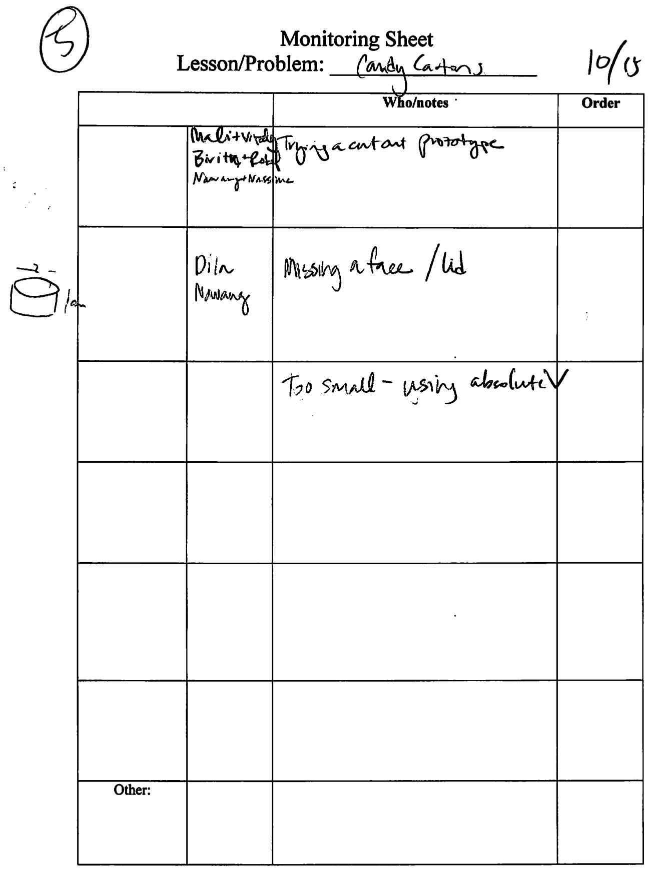


Figure 5.5. Completed monitoring sheet for activity phase of PS1 Candy Cartons

In the post-lesson reflection, Ms. Elmore did not mention the monitoring sheet. What she did discuss was skipping the whole class discussion phase of the lesson.

> [Reflection prompt] What were the mathematical ideas/strategies that received the most attention in the whole class discussion?
>
> [Ms. Elmore's response] I feel like I missed something in the lesson – I didn't see a part for a whole class discussion? I tried to follow the lesson plan very faithfully, but maybe I missed it.

Although the guide for Candy Cartons does include a short whole class discussion at the end of the lesson (see page T12), Ms. Elmore's post-lesson reflection indicates both her commitment to trying to use the lesson as designed, and her lack of certainty about its goals. She clearly indicates, as she did in interviews and in the collaborative meetings, her desire to take the lesson guides and the provided material seriously and to following the plans "faithfully." This desire may have also guided her to use the monitoring sheet, because it was a tool that I had provided. Yet, the primary purpose of a monitoring sheet, as it was introduced, was to support a whole class discussion, which she had no plans to enact. She attempted to use the monitoring sheet, even though she had neither prepared it in advance nor needed the information on it.

Ms. Elmore also mentioned her reaction to the FAL lesson structure in the post-lesson reflection:

> I want to mention an overall thought about the FAL – I felt as though the classroom was totally out of control. The looser structure led to a lot of students ignoring common rules (cell phones) as well as generally being less-focused. A lot of students were on task, but their focus was low and I think this contributed to the length of the lesson. It felt like a lot of students just spun their wheels while waiting for me to circle back around again. The overall feeling of lack of control was really frustrating and exhausting.

The different format of this lesson was disconcerting; in Ms. Elmore's class the existing norms were consistent with a more tightly controlled, teacher-centered lesson. Her sense that many students had difficulty staying on task or being productive when she was not interacting with their group is consistent with observations of the video and field notes. Despite this, Ms. Elmore did not seem to be circulating only to keep students on task. She engaged in meaningful discussions with the groups and elicited substantially more student explanations than during her well-managed routine lesson.

Second and third FALs: CD1 and CD2

For the next two FALs, Ms. Elmore chose and prepared concept development lessons to address several of her concerns with the enactment of the first FAL. The content goals of the lessons were clearer and fit closely with the material that she was responsible for teaching in the unit where they were placed. The activity was more structured, a card sort. She placed students into groups of four so that she would have fewer groups to manage and the students would have more peers to help them stay on task. For each of the FAL tasks, Ms. Elmore redesigned the monitoring sheet to create more accountability and a strong sense that she knew what was being accomplished in different parts of the room (see example in Figure 5.6).

The monitoring sheet on the left of Figure 5.6 was used for CD1, Possible Triangle Constructions, which features a card sorting activity. In this activity, there are ten cards, each with three pieces of information about triangle ABC; for example, AB =3 cm; BC = 7 cm; AC = 4 cm. Students work in their groups to decide if it is possible to construct a triangle with these specifications. If so, is it possible to construct more than one triangle? Students are asked to categorize each case (impossible, only one, more than one) and expected to explain how they know their categorizations are correct. On the monitoring sheet there are ten large boxes that represent the 10 cards. Each is partitioned into eight smaller boxes to represent the eight groups of students in the class.

In response to the question on the pre-lesson reflection about how she planned to monitor student work during this FAL, Ms. Elmore wrote:

> I will keep a record of which groups can correctly explain each card, as well as which groups may have noteworthy errors in thinking that might be shared.

During the activity, Ms. Elmore circulated through the room, looked for cards each group had correctly placed and could explain, and checked off that group for that card. The checkmarks in the boxes show that most cards were appropriately placed by most groups. Circled checkmarks indicate groups that Ms. Elmore decided to have present their solution for that particular card during the class discussion. Although the pre-lesson reflection indicates an intention to attend to "noteworthy errors in thinking," this was not evident in her use of the monitoring sheet or in her choices of presenters.

There is no mention of strategies or conceptual issues anywhere on the monitoring sheet for this lesson. The only words or numbers written on the sheet are information from the cards and the numbers of the student groups. What had been offered as a tool for capturing a variety of student strategies and conceptions had changed into a tool for classroom management.

This adaptation of the monitoring sheet felt like a useful innovation to Ms. Elmore; one that she came up with on her own and that helped her to feel more confident in both managing the activity phase of the lesson and knowing which groups were going to present during the class discussion. In her post-lesson interview, she mentioned that she felt much more supported by this monitoring sheet, that the lesson was more productive, and that she was more in control. As a result, she did include a class discussion phase in this lesson in which students knew in advance that they would present their work and were able to prepare for it. In this way, the modified monitoring sheet supported her in enacting more of the FAL.

However, this monitoring sheet also indicated a shift from the lesson designers' intended focus for the FAL. Rather than using the cards to make sense of the three categories (impossible, only one, more than one), the new goal of the lesson was finding the correct placement for each card. The student presentations included some explanation of why each card was in a particular category, but there was no discussion of incorrect solutions and no connections made between the strategies used by different groups in placing the cards. The monitoring sheet reflected and supported emphasis on finding the correct answer to the question posed by each card, rather than building connections between the cards or explanations to form a deeper conceptual understanding.

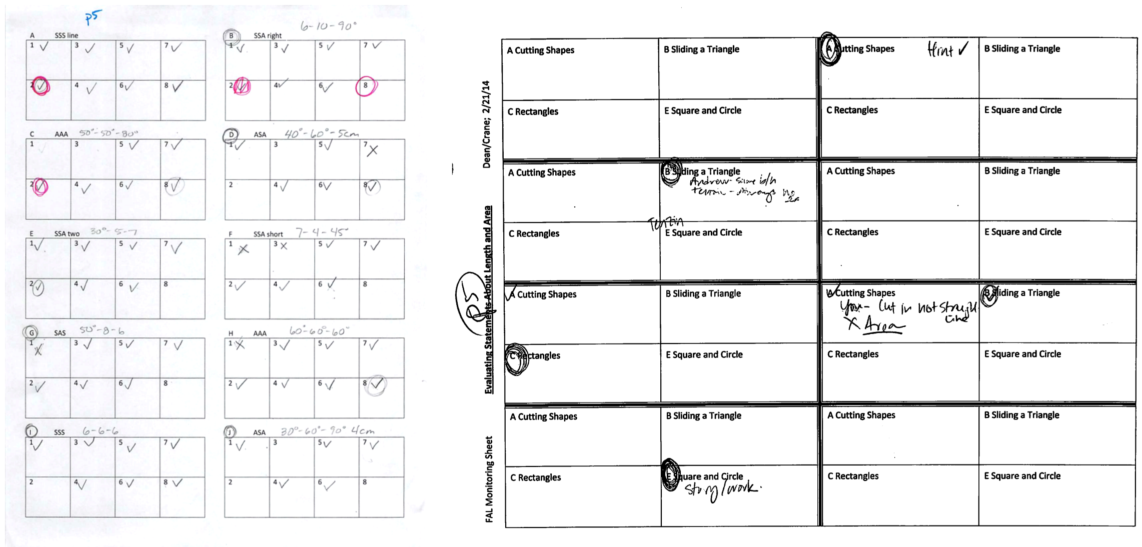


Figure 5.6. Ms. Elmore's monitoring sheets for the second and third FALs (CD1 and CD2)

The monitoring sheet Ms. Elmore constructed for the third FAL, CD2 Evaluating Statements about Length and Area, had yet another modification. On this sheet (on the right in Figure 5.6), there are now eight large boxes, one for each group of students, with each box partitioned into four smaller rectangles representing the four of the cards that she was most interested in. On the pre-lesson reflection she explained:

> I will use a clipboard to check off which groups have good arguments for which cards, or to make notes of anything else noteworthy or relevant to the class.
> If time allows, I will have 1 or 2 groups for each card present their posters to the class.

Although she still mentions the possible use of the monitoring sheet for "anything else noteworthy," the emphasis is clearly on correct answers and model arguments. During the lesson, it appeared that the monitoring sheet was serving a particular function for the students and for Ms. Elmore. The students were aware of Ms. Elmore marking down what they had gotten correct and seemed interested in getting additional checkmarks on her clipboard. Ms. Elmore was able to use the monitoring sheet to remind herself of what each group had been working on during her previous visit and to see if they were making progress. As shown in Table 5.1 and Figures 5.1, 5.2, and 5.3, Ms. Elmore spent more time with each group and listened to more explanations during this lesson (CD2) than during the previous FALs (PS1 and CD1).

Ms. Elmore ended the lesson with very quick presentations from four of the groups. Each showed their justification for deciding if one of the statements on the cards was always, sometimes, or never true. She used the monitoring sheet to select and assign groups to present as she visited groups, but there is no evidence that she purposefully sequenced the presentations, and she did not draw any connections between presentations on the monitoring sheet or during the discussion. In her post-lesson reflection, she described the whole class discussion of student presentations:

> I wanted to bring attention mostly to the rigorous algebraic generalization proofs, but we were so rushed that I don't think it was given the appropriate time to be listened to/absorbed/understood.

Later, in response to a mid-project survey question on use of a monitoring sheet in teaching the FALs, Ms. Elmore drew a picture of the monitoring sheet organized by groups, similar to the one that she used in CD2 (on right in Figure 5.6) and explained:

> I have a space to code what is happening on each card in a card set per group. Then I can easily assign presentations to a group, or identify that no one is working on E. I've also used a huge matrix.

This again indicates how her use of the monitoring sheet supported her in managing the lesson more efficiently and suggests that one of her goals was that each card be discussed This is supported by the second statement that she can identify that no one is working on E. In order to have each card presented to the class, it was important that she knew that every card was being worked on by at least one of the groups. This use of the monitoring sheet reflects Ms. Elmore's goal of learning to wrangle the FALs into a manageable form that allowed her to use existing classroom routines and that fit into her class timeline.

The "huge matrix" in the last two FALs

Before the last two FALs, I had not seen the "huge matrix" that Ms. Elmore mentioned. However, for the last two FALs (CD3 and CD4), the monitoring sheet was indeed a grid with rows for the students and columns for the cards. Each of these FALs included a card-sorting activity with 12 cards, so Ms. Elmore was able to use the same monitoring sheet template for both (Figure 5.7). This template does not include places for writing comments about strategies or student thinking, but is an efficient checklist that shows which parts of the activity have been completed by each group.

Group	1	2	3	4	5	6	7	8	9	10	11	12

Figure 5.7. Monitoring sheet used for fourth and fifth FALs (CD3 and CD4)

In CD3, Ms. Elmore omitted the class discussion because of concerns about time, so she did not use the monitoring sheet to assign groups to make presentations. In CD4, she did assign groups to put their solutions on the whiteboard, but there was no time for the groups to explain them to the class. The next day, Ms. Elmore distributed a handout that summarized what she had expected the students to learn from the lesson. Students filled in the blanks as she guided them from the front of the classroom.

Discussion

Throughout the study, Ms. Elmore was committed to learning how to make productive use of the FALs to support her students' learning. She worked through each FAL diligently before teaching it, read the lesson guides thoroughly, and worked with her student teacher to make sure that they both understood the lesson. She closely followed the suggestions in the lesson guides for the pre-assessments (except for CD4), launches, and activities. She made significant modifications to the activity phase for only one lesson (CD4). She completed all of the surveys and reflections for the study. For the phase of the FALs that she found most challenging, the whole class discussion, she tried to make use of a suggested resource, the monitoring sheet.

In short, Ms. Elmore was not resistant to the project, but hoped to learn from it. However, she was also committed to her goals and beliefs about what was necessary to support her students and what she was responsible for teaching them. That meant keeping to the pacing guide and covering the FALs as they appeared in the curriculum. On the mid-project survey she wrote:

> The lessons are wonderful but are add-ons, not replacements for any of my normal lessons. The number of class days available already is insufficient. Ideally, 3 FAL days could replace 1 or 2 days of lessons, but they really only deepen/enrich lessons we have already done or will do right after the FAL.

She did not see the FALs as supporting her in teaching her normal lessons, let alone influencing how she would teach them. Although the FALs were "wonderful," they were in direct conflict with keeping on schedule and covering the curriculum.

Two other factors influenced Ms. Elmore's use of the FALs: coordinating three sections of the same course and mentoring Ms. Iris, a student teacher,. Ms. Elmore taught three sections of Geometry and was very committed to keeping the three classes on the same schedule. Thus, in her view, allowing a lesson to take more time in one class than another was very problematic. This concern was exacerbated in the second semester when Ms. Iris took over primary teaching responsibility for two of the three sections of Geometry. In her planning, Ms. Elmore was not only concerned with her own ability to manage the group activity. She was also trying to create plans that would support Ms. Iris's management of her lessons.

Ms. Elmore's use of the FALs was influenced by her goals for covering the course material and mentoring her student teacher as well as her beliefs about what was essential in order to meet those goals, particularly as the school year progressed. The changes in her enactments of the FALs evidenced through her use and modification of the monitoring sheet constrained her opportunities to learn from the FALs or to see them as a valuable resource in her classroom.

Chapter 6

Discussion and Implications

Instructional materials, no matter how carefully designed, can offer only affordances: *potential* opportunities for students to engage in cognitively demanding tasks and *potential* opportunities for teachers to learn from enactment and reflection. What matters is how individual teachers perceive these affordances in the context of their own teaching and among the competing messages about what constitutes good teaching and learning. The FALs offered many rich possibilities. The teachers made instructional decisions based on their knowledge, goals, and beliefs (Schoenfeld, 2010). In some cases, those choices supported enactments that were consistent with the FAL designers' intentions. In some, the teacher's choices led to different, but productive, instructional innovations. In some, the teacher's choices made lessons based on the FALs similar to the teacher's routine instruction.

The teachers in this study were eager to create positive and productive learning experiences for their students using the FALs. They were relatively successful in doing this, as shown by the analyses of student explanations in Chapter 3. Further, their reflections on the mid-project surveys showed that although their enthusiasm was tempered somewhat by the challenges of enacting ambitious lessons, they continued to be excited by the engagement they saw from their students. In this way, these teachers were able to use these materials toward their highest priority goal: engaging students in learning mathematics.

It is important to recognize that this study only examines teachers' use of the lessons in the absence of professional development and over the course of one school year. What was evident from the teachers' feedback was that the level of success that they experienced from teaching their first and second FALs set the stage for their views of the FALs and their willingness to use the pre- and post-assessments. Further, the amount of work involved in making sense of the lessons and the number of instructional days that teachers needed to commit to their implementation meant that teachers felt stretched just trying to fit five of the lessons into their school year.

As rich and engaging as the activities in these lessons may be for students, if they only engage in five such lessons each year, the lessons are not likely to make a significant difference in their learning of mathematics. The real potential for making a difference in students' learning must come through teachers' learning to change their teaching practices from the FALs. In this case, the analyses in Chapter 3 are less encouraging about the prospects for the FALs to influence practice.

The findings in this dissertation have several types of implications. There are implications for how we frame research on teacher learning. There are implications for research on how teachers learn from instructional materials. And, there are practical implications specific to the FALs – for the design of the materials and for the design of professional development to support their use.

Implications for research

Teacher agency

The analyses in Chapter 3 illustrate the role of teacher agency in constraining their opportunities to learn. Although teachers were recruited for the study due to their interest in the FALs, their choices and interpretations of the FALs (in general and for specific lessons) led them to use the FALs in ways that sometimes removed learning opportunities for themselves and their students. The placement of the FALs in curriculum units sometimes precluded their use as formative assessments of student thinking. Teachers sometimes omitted pre- or post-assessments, missing those opportunities to use evidence of student thinking to inform instruction.

The case studies in this dissertation illustrate the role of teacher agency in learning, especially self-directed learning of the type that occurs when teachers are learning in the context of teaching. Both case study teachers recognized the importance of student communication, particularly during the small group activities, and worked on learning how to support that. The lessons provided rich contexts for students to discuss, thus providing opportunities for students to communicate with each other and with the teacher. In contrast with their routine lessons, during lessons based on the FALs, the teachers directed student conversation less and provided more time for student explanation and discussion. However, these teachers, like all of the teachers in the study and other teachers (Baxter & Williams, 2010), struggled with how to build a whole class discussion on the thinking that emerged from small group discussions. In doing so, both teachers deviated from the suggestions in the lesson guides, but in different ways with different results.

In both cases, the teachers made good faith efforts to learn about the lessons, use them, and learn from them. Mr. Davidson saw the lessons as an opportunity to work on two specific goals – shifting the focus from answers to the processes that lead to answers and developing students' ability to communicate more clearly about mathematics orally and in writing. His use and adaptions of the lessons reflected these goals. Ms. Elmore saw lessons as an opportunity to learn about "ungraded formative assessment" and as an aid to aligning her instruction with the Common Core State Standards. Her goals and her understanding of mathematical expectations for her students made the FALs seem extraneous rather than the support she sought. Thus, she developed ways to "tame" the lessons, to better fit with her goals. The result, in the final lessons, neither met her goals nor the intended goals of the lesson. Along the way, she altered one of the support materials, the monitoring sheet, to provide her more support for the challenge that she was experiencing in making these lessons work for her goals. This too has implications for design of professional development which will be discussed below.

Research on teacher learning

The findings of this dissertation contribute to the conversation about approaches to research on teacher learning in context. Like other studies (Collopy, 2003; Remillard &

Bryans, 2004; Sherin & Drake, 2000), this study found substantial variation in the ways that teachers made use of instructional resources and the opportunities for their learning that resulted. Like other studies, this study also found that teachers' prior experiences with instructional materials and their expectations about them influenced their use of the materials. In this study, many of the teachers had prior experience with reform curricula as "hands-on" and "discovery-based" learning, and made sense of the FALs through the perspective of that experience.

The case studies show that teachers, like students, are sense makers who respond to the sequences of events that they experience. What might not appear to be evidence of teacher learning, e.g., Mr. Davidson's first enactment of the student presentation routines, looks like learning when considered over several lessons. What might appear to be evidence of teacher learning, such as Ms. Elmore's use of the monitoring sheet in the first two FALs or the increases in student explanations during the first three FALs, seems less clear when considered as part of a larger trajectory. In either case, a single year is a short period of time in which to assess the lasting impact of an intervention on a teacher's practice. Ultimately, teachers engaged in professional work are learning something, but it might not be the thing that we are measuring. Thus, it is important to analyze teachers' actions with respect to their goals and to recognize that these goals may change with time and learning.

Implications for design

Design of the lessons

The primary intended use of these lessons was to scaffold teacher learning about formative assessment. The FAL designers intended that teachers participate in professional development to support their learning from teaching the FALs. By revealing the ways that teachers alter the FALs when selecting and planning lessons individually, the analyses in this study suggest important aspects of professional development that should accompany these and other curricular materials intended to support teachers' learning in the context of classroom enactment. The FALs, which aim to embody principles and practices of formative assessment, do not on their own communicate to teachers which aspects of formative assessment they support and what the teachers should learn from using the lessons.

First, the teachers need to have a clearer understanding of the designers' intentions and have opportunities to connect them to their own goals. A significant part of formative assessment is interpreting student work and using its findings to inform instruction. In the FALs, it was not clear to the teachers what goals were served by the pre-assessments; consequently, they were less likely to use them to inform instruction. The lesson activities, which were carefully constructed to address the common student understandings that teachers were likely to encounter on the pre-assessments, were generally quite effective. Interestingly, a consequence of the high quality of the design was that the lessons engaged students whether or not the teacher had analyzed the pre-assessments. From the teachers' perspective, then, much of the work done in interpreting the pre-assessments seemed unnecessary. The teachers, under various pressures and seeing how well the activity phase of the lessons worked, did not need to do the diagnostic work, thus were deprived of the benefits of learning to analyze student work themselves. Making the formative assessment cycle more transparent and focusing teachers' attention more on the ways that their students engage with each other in the activity phase, rather than on how they, the teachers, manage the classroom during the activity, might support teachers' learning about formative assessment.

The most salient part of the lessons for teachers was the collaborative activity phase of the lesson. In most cases, teachers' enactments of these activities provided at least as many opportunities as small group work in routine lessons for students to explain their thinking in the presence of the teacher. In many of FAL enactments, there were substantially more student explanations. These provided more opportunities for students to clarify their thinking through explanations and for teachers to assess how students were making sense of the concepts. Given the importance of explanation in learning and as a mathematical practice in its own right, this is clearly a positive use of the FALs. However, teachers struggled to find the other parts of the FALs similarly useful and, due to lack of understanding or time constraints, chose to omit parts of the lessons.

Design of professional development

These lessons in this study, in effect, served as baseline data for understanding the issues that teachers face in trying to make sense of formative assessment as a classroom practice. The ways that teachers made sense of these lessons, primarily on their own and sometimes with colleagues, can now inform the design of professional development on formative assessment using the FALs as a resource. First, it is important for the teachers to establish learning goals for both themselves and their students in the use of the lessons.

Second, the teachers need to have opportunities to work with the lessons under less pressure than the relentless pace of the school year. The pressure to stay on a pacing guide that does not include the FALs and is driven by department assessment can serve as a substantial barrier to using the FALs. Along with this, teachers need structure for deciding and articulating how their learning goals relate to the placement and planning choices that they make for the lessons.

Third, teachers need ongoing access to feedback and self assessment about their planning and implementation of the lessons. They need to know if their use of the lessons is moving them toward both sets of learning goals. They also need to have opportunities to refine the learning goals as they continue to work with the lessons. Fourth, teachers need to be able to work with colleagues in sustained ways that support each other's planning and reflection.

Finally, professional development would be well supported by a set of resources that focus more broadly on formative assessment, beyond those particular to the two FAL phases of small group activity and whole class discussion. Specifically, this dissertation has shown that teachers struggle with the interpretation phase of formative assessment in two ways. Constraints on teachers' time, their expectations for the amount of time necessary for interpreting student thinking outside of classroom interactions, and their lack of knowledge and experience with such work pose substantial challenges for teachers learning to implement formative assessment cycles. Work in professional learning communities focused on this component of formative assessment could substantially contribute to teacher learning.

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Appendix

Lessons Observed at Adams Middle School by Teacher

	Routine lesson	FAL 1	FAL 2	FAL 3	FAL 4	FAL 5
Ms. Amador	9/18/13 RL: Meanings of dividing 100 by 6	10/23/13 CD1: Laws of Arithmetic	1/13/14 CD2: Mean, Median, Mode, and Range	3/12/14 PS1: Security Cameras	3/27/14 CD3: Proportional Reasoning	–
Ms. Belle	9/25/13 RL: Calculating Changes in Temperature	9/18/13 CD1: Using Positive and Negative Numbers in Context	12/3/13 CD2: Increasing and Decreasing Quantities by a Percent	3/26/14 CD3: Classifying Proportional and Non-proportional Situations	–	–
Ms. Castle	*No routine lesson observed*	11/6/13 CD1: Solving Linear Equations (7th Grade)	2/25/14 CD2: Comparing Lines and Linear Equations (Flowing Liquid)	–	–	–
Mr. Davidson	9/18/13 RL: Describing sequences	9/25/13 PS1: Generalizing Patterns: The Difference of Two Squares	11/4/13 CD1: Translating Between Repeating Decimals and Fractions	2/5/14 CD2: Solving Linear Equations (7th Grade)	2/25/14 *CD4: Sorting Equations of Circles 1	5/14/14 CD3: Interpreting Distance–Time Graphs

* Taught in more advanced class than RL observation.

Abbreviations: RL – Routine Lesson; CD – Concept Development Formative Assessment Lesson; PS – Problem Solving Formative Assessment Lesson.

Lessons Observed at Brookfield High School

Ms. Elmore	9/25/13 RL: Using Compasses	10/15/13 PS1: Designing 3D Products: Candy Cartons	11/12/13 CD1: Describing and Defining Triangles	2/20/14 CD2: Evaluating Statements About Length and Area	3/18/14 CD3: Sorting Equations of Circles 1	5/2/14 CD4: Evaluating Statements About Enlargements
Ms. Feldman	10/9/13 RL: Graphing and Slope	10/22/13 PS1: Generalizing Patterns: The Difference of Two Squares	11/18/13 CD1: Describing and Defining Triangles	3/4/14 *CD2: Represent-ing Data with Frequency Graphs	3/6/14 *CD3: Represent-ing Data with Box Plots	–
Ms. Golden	10/2/13 RL: Similar Triangles and Proportions	10/7/13 PS1: Floodlights	10/29/13 CD1: Lines and Linear Equations	1/8/14 PS2: Having Kittens	1/14/14 PS3: Solving Linear Equations in Two Variables	4/29/14 PS4: Lucky Dip
Ms. Heather	*No routine lesson observed*	10/7/13 PS1: Floodlights	10/29/13 CD1: Lines and Linear Equations	1/8/14 PS2: Having Kittens	1/14/14 PS3: Solving Linear Equations in Two Variables	5/1/14 PS4: Lucky Dip

* Taught in more advanced class than RL observation.

Abbreviations: RL – Routine Lesson; CD – Concept Development Formative Assessment Lesson; PS – Problem Solving Formative Assessment Lesson.