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Uncertainty Modeling and Quantification for Structural Health Monitoring Features Derived from Frequency Response Estimation

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Abstract. System identification in the frequency domain plays a fundamental role in many aspects of mechanical and structural engineering. Frequency domain approaches typically involve estimation of a transfer function, whether it is the usual frequency response function (FRF) or an output-to-output transfer model (transmissibility). The field of structural health monitoring, which involves extracting and classifying features mined from in-sit structural performance data for the purposes of damage condition assessment, has exploited many features for this purpose that inherently are derived from estimations of frequency domain models such as the FRF or transmissibility. Structural health monitoring inevitably involves a hypothesis test at the classification stage such as the (common) binary question: are the features mined from data derived from a reference condition or from data derived from a different ("test") condition? Inevitably, this decision involves stochastic data, as any such candidate feature is compromised by "error", which we categorize as (i) operational and environmental, (ii) measurement, and (iii) computational/estimation. Regardless of source, this noise leads to the propagation of error, resulting in possible false positive (Type I) errors in the classification. As such, the quantification of uncertainty in the estimation of such features is tantamount to making informed decisions based on a hypothesis test. This paper will demonstrate several statistical models that describe the uncertainty in FRF estimation and will compare their performance to features derived from them for the purposes of detecting damage, with ultimate performance evaluated by receiver operating characteristics (ROCs). A simulation and a plate subject to single-input/single-output vibration testing will serve as the comparison testbeds.

Introduction

Frequency response models have been widely used for the important purposes of system identification and damage detection implementations. By means of vibration-based structural test such as percussion and/or shaker test, frequency response information is often provided via modal parameter extraction, resulting in natural frequencies, mode shapes, and modal damping. In the context of structural health monitoring (SHM) and damage assessment, a comparison between two stages of a structure is considered as one of the axioms [1], in which a test condition, potentially damaged, is compared to a reference condition, usually called baseline. As a result, all kinds of useful information (features) are extracted from frequency response information, and these features are subsequently used to indicate the state of structures in terms of the some damage assessment taxonomy.

Although all sorts of SHM features are anticipated to be sensitive and specific, realistic feature extraction is always subject to different sources of in-situ uncertainty, such as operational and environmental variability and measurement noise. Moreover, frequency response estimations are also influenced by the estimation algorithm itself, known as the *estimator*. No matter how noise-robust the estimator and the testing procedure are, the unavoidable variability will degrade SHM performance capability by confusing the decision process through false positives or false negatives.

Therefore, exploring a good feature, meaning sensitive and robust, is a major concern in damage detection and SHM.

Optimum frequency response feature selection involves quantification of estimation uncertainties, i.e., quantifying the statistical uncertainty of frequency response functions. For an input-output transfer function like the FRF, its uncertainty is quantified via Gaussian bivariate approach, and probability density function of magnitude and phase are both given [2]. For output-output transfer function like the transmissibility, the variability is modeled via different approaches considering different transmissibility estimators [3, 4]. With all statistical models available, the damage detection process is thereafter deployed as a statistical significance detection problem.

In this paper, two FRF-related features evaluated at different domains are introduced, and the uncertainty of feature estimations is quantified via a generalized extreme value distribution. For an optimal feature selection purpose, the performances of each feature with different evaluation domain are compared with respect to receiver operating characteristics (ROC), and optimum is thus determined in the sense of best trade-off between maximized detectability and minimized false alarms.

FRF Estimations and Related Features

Frequency response function measures the relative input-output gain in frequency domain. As a function of frequency, in Hz or discrete bin, it is mathematically approachable in Eq. 1:

$$H(\omega)=\frac{\mathcal{F}\left[y(t)\right]}{\mathcal{F}\left[x(t)\right]}, \tag{1}$$

in which $x(t)$ and $y(t)$ are the input excitation and output response respectively and $\mathcal{F}$ is the Fourier transform operator. Although input and output may be signals with any physical meaning, they usually refer to force, displacement, and/or acceleration in the context of system identification and SHM applications. In this paper, the excitation $x(t)$ is force and system response $y(t)$ is acceleration, and all the uncertainty quantification and feature selection is thus deployed without loss of generality.

Because of non-periodicity and noise contamination, the transfer function in Eq. 1 is often evaluation through an estimator. For instance, Eq. 2 is the H_1 estimator of the FRF:

$$H(\omega)=\frac{S_{xy}(\omega)}{S_{xx}(\omega)}, \tag{2}$$

and S_{xx} and S_{xy} are the auto power density function of input $x(t)$ and cross power density function between $x(t)$ and $y(t)$. Being a complex number, cross power spectrum is defined in Eq. 3:

$$S_{xy}(\omega)=\mathcal{F}\left[x(t)\right]^{*}\cdot\mathcal{F}\left[y(t)\right], \tag{3}$$

in which the * sign denotes the complex conjugate. For the auto power spectrum, the definition has the same form as Eq. 3 and the product is real-valued.

Using the estimation of the FRF as the basis for features, there are two types of features proposed and compared in this section, namely dot-product-difference (*DPD* in Eq. 4) and Euclidian distance (*ED* in Eq. 5):

$$DPD = 1 - \left| \frac{H_{damaged}^{[\Omega]} \cdot H_{baseline}^{[\Omega]}}{\left| H_{damaged}^{[\Omega]} \right| \cdot \left| H_{baseline}^{[\Omega]} \right|} \right|, \text{ and} \tag{4}$$

$$ED = \sqrt{\frac{1}{L_\Omega} \sum_{\omega \in \Omega} \left(H_{damaged}(\omega) - H_{baseline}(\omega) \right)^2}, \tag{5}$$

where the subscripts baseline and damaged denote the two different structural states. Both of the proposed features may be evaluated over the entire measured frequency bandwidth Ω, with size L_Ω, or just within a selected frequency region Ω_s.

Eq. 4 illustrates the derivation of *DPD* in which the damaged transmissibility vector is projected to the baseline and cosine of the angle is calculated via dot-product. For normalization purposes, final form of *DPD* is defined as one minus the absolute value of cosine of the angle so that its range conveniently goes from 0 to 1. For zero-*DPD* evaluation, the measured ("test") FRF vector is parallel to the baseline, while a unity *DPD* means perpendicularity. Eq. 5 defines feature of *ED*, which is the root mean square error between damaged FRF and baseline vector, and L_Ω indicates the volume of the selected evaluation domain, i.e., the length of selected frequency band or number of frequency bins for discrete time signals.

As mentioned above, all the FRF estimations are subject to certain level of uncertainty, which can be caused by environmental/operational variability and data acquisition noise. Therefore, both features in Eq. 4 and 5 are modeled as random variables. Even under strict external uncertainty control, when ambient excitation or other random inputs are used, feature evaluations still fall into stochastic bounds, and this further motivates uncertainty quantification. In the following sections, Gaussian white noise is applied as input excitation, and will also be used to model the external uncertainties.

Statistical Modeling of FRF-related Features

A shaker test is implemented on a lab-scale structure, as shown in Fig. 1, and corresponding responses are measured at the circled area. FRF between shaker excited location and the response measurement location is estimated via H_1 estimator. For emulating a test (damaged) condition, an extra spring is placed underneath the structure, in order to change the dynamics of the original clamped plate. To be more realistic and stringent, a small amount (1% noise-to-signal ratio) of artificial white noise is added to the raw measurements, independent of the structural response and the input.

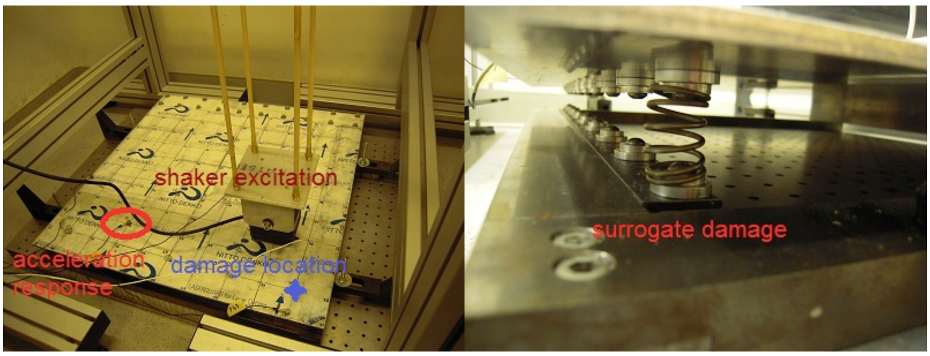

Fig. 1 Plate structure for statistical modeling of features

Fig. 2 shows the FRF estimation for both baseline condition and the damaged condition. With the influence of all sorts of uncertainty, it is not easy to distinguish one case from the other, except for the bounded area with red lines. As a result, feature evaluation domain Ω is defined as the entire frequency range from 0 to 500 Hz, while the selected domain Ω_s, by visual inspection, is defined as the region in between red bounds.

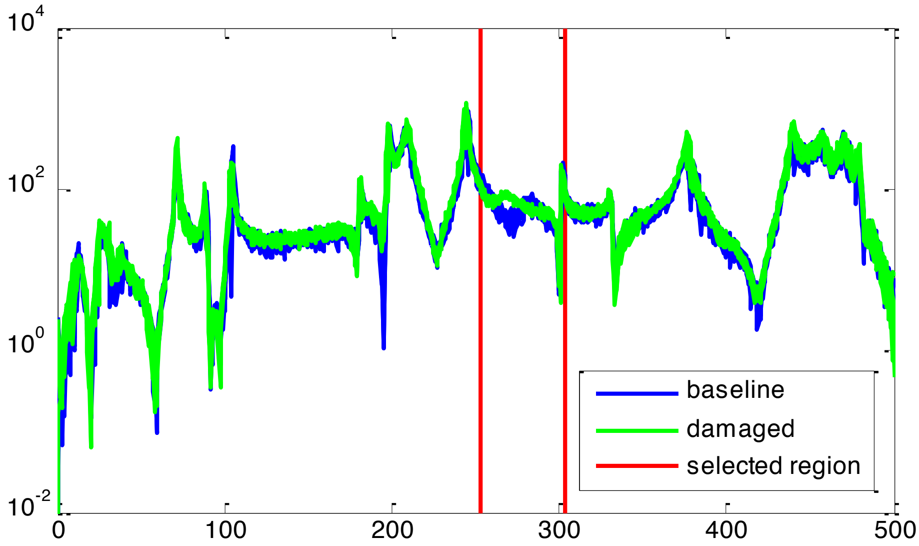

Fig. 2 FRF magnitude estimation of plate structure

Assuming sufficient realizations, both *DPD* and *ED* features evaluated in Ω and Ω_s (denoted as *DPD*s and *ED*s) fall into a generalized extreme value distribution, whose probability density function is listed in Eq. 6:

$$p(x\mid\mu,\sigma,\xi)=\frac{1}{\sigma}\left[1+\xi\left(\frac{x-\mu}{\sigma}\right)\right]^{-\frac{1}{\xi}-1} e^{-\left[1+\xi\left(\frac{x-\mu}{\sigma}\right)\right]^{-\frac{1}{\xi}}}. \tag{6}$$

In Eq. 6, there are three variables governing location, scale and shape respectively, namely, $\mu \in R$ determines location of the distribution, $\sigma > 0$ is the scale parameter, and $\xi \in R$ controls the shape of PDF.

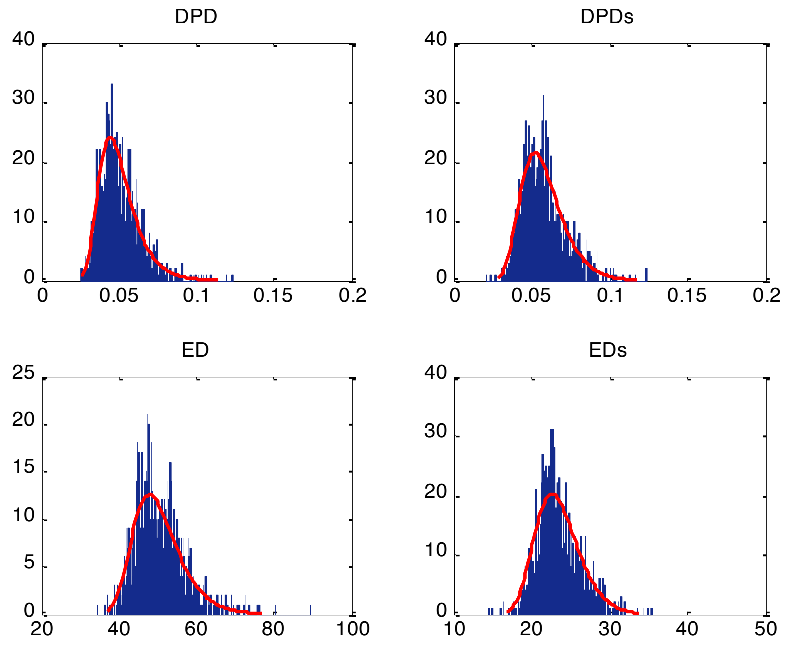

Fig. 3 FRF magnitude estimation of plate structure

Fig. 3 illustrates the maximum likelihood estimation (MLE) of distributions for two features with two evaluation domains, overlapped with the actual histogram. For all the four cases, there is very good consistency between observation and the PDF curves obtained from MLE. In other words, there is substantial plausibility to model the uncertainty of feature evaluations via a generalized extreme value distribution framework.

Damage Diagnosis

Eq. 6 and Fig. 3 propose a statistical model to quantify the uncertainty of features as random variables. However, as mentioned previously, damage detection is actually a comparison between two states, subject to sources of uncertainty. If the features for baseline and damaged conditions distribute in an adjacent area and the tails from each condition overlap, there will be a misclassification of data for the purposes of SHM. More specifically, the overlapped feature distribution will not only cause false alarms (Type-I error), but also degrades the sensitivity and specificity of damage detection.

To study the issue of separation of distributions, Fig. 4 introduces a simulation structure as an emulator to generate acceleration response. The structure is designed to be a simply supported beam, on which the Gaussian white noise excitation is applied at the position with 15% of total beam length to the left. Structural response is calculated analytically at 30% of total beam length to the left, and for the same reason, input and output are both contaminated by artificial noise to get a more realistic simulation. In the simulation test, damage is defined gradually as loss of Young's modules, and in this paper, three damage levels are included with 1%, 5% and 10% of loss.

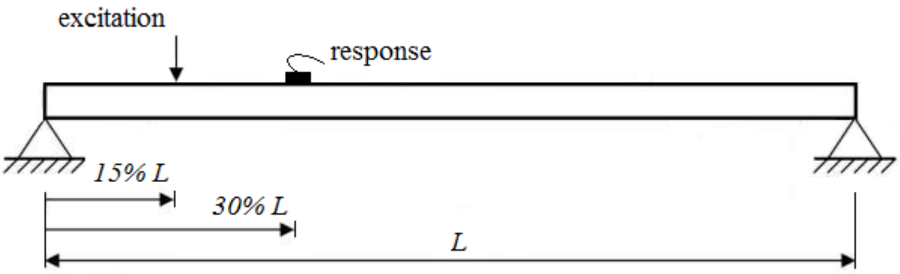

Fig. 4 Beam structure for damage diagnosis

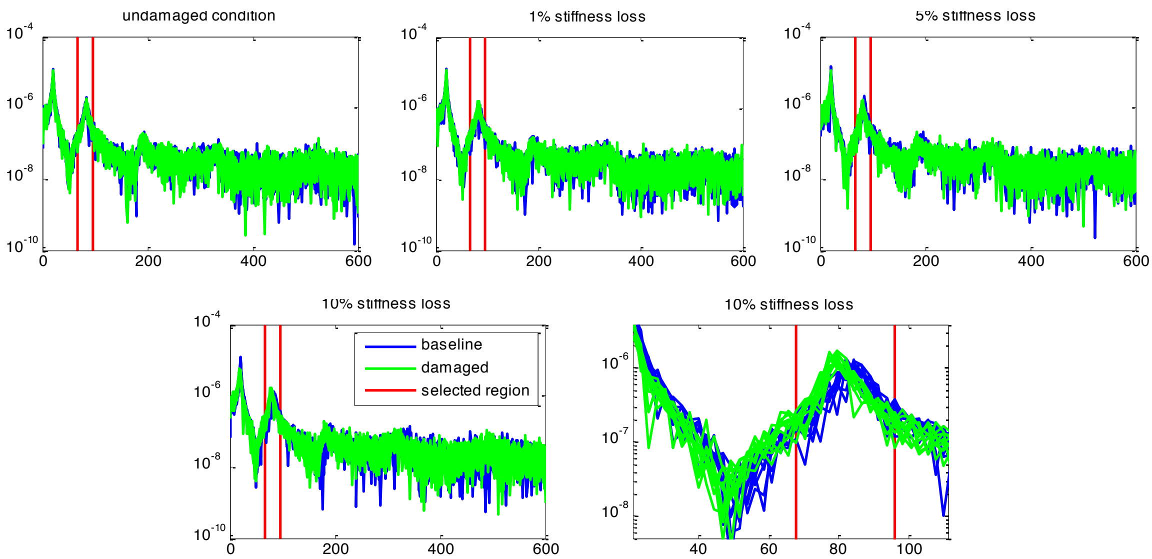

Fig. 5 FRF of beam structure for different damage levels

In Fig. 5, FRF magnitude for different damage levels are plotted in green and the reference (baseline) FRF is plotted in blue. Although the difference between test and baseline is hardly distinguishable, it is still consistently increasing as loss of stiffness goes higher. The last plot in Fig. 5 shows the detail of FRF comparison for a 10% loss case, and the selected feature evaluation interval is therefore picked near the second resonance with a clear difference visually.

Once the FRFs are available, the features are evaluated at both Ω and Ω_s. Fig 6 plots the histogram of each feature for each damage condition. Compared to baseline distribution, all damaged features shift to the right more or less, indicating different differentiability for damage detection.

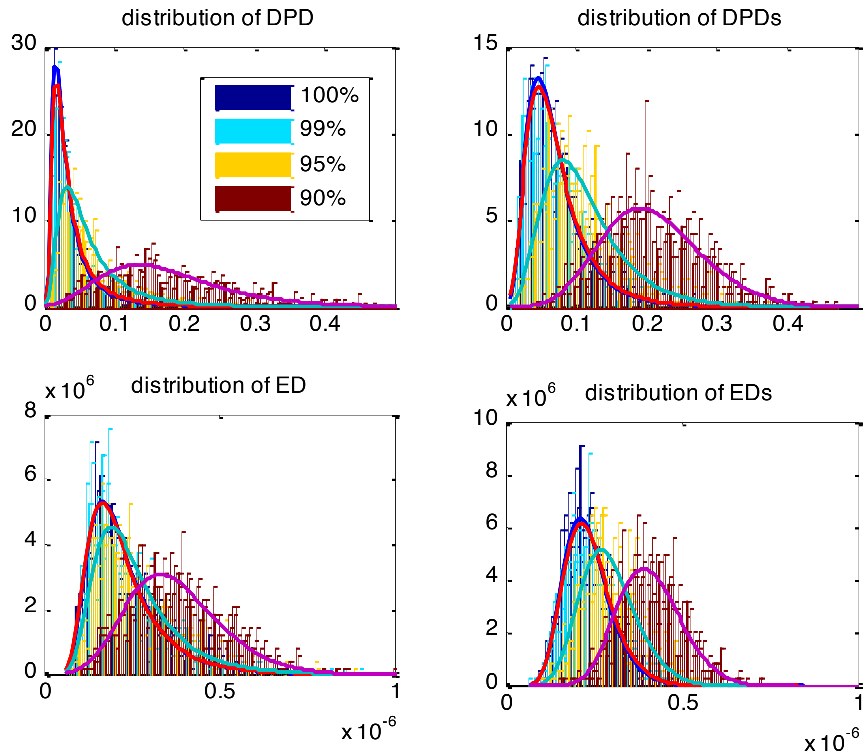

Fig. 6 Histogram and PDF of different features under different damage levels

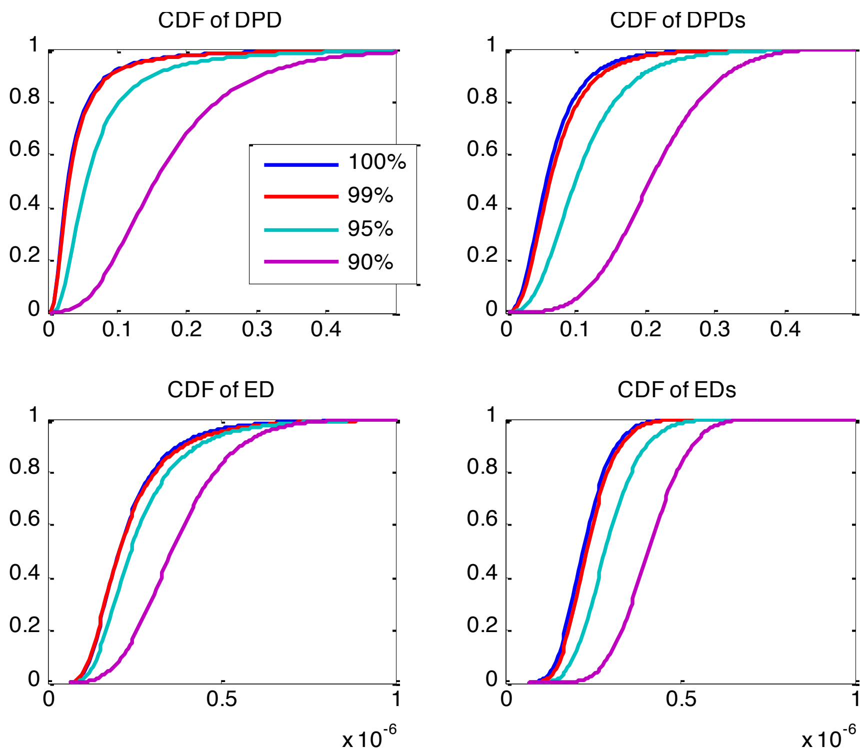

Fig. 7 CDF of different features under different damage levels

In Fig. 6, the MLEs of generalized extreme value distribution PDF for features under different damage levels are plotted on top of histograms, and all corresponding CDFs are plotted in Fig. 7. Fig. 7 illustrates clear separation as damage level goes severe for both features and evaluation intervals. But for light damage, such as 1% loss, the CDF curves (red) are negligibly different from the baselines (blue), especially when features are evaluated at entire frequency domain.

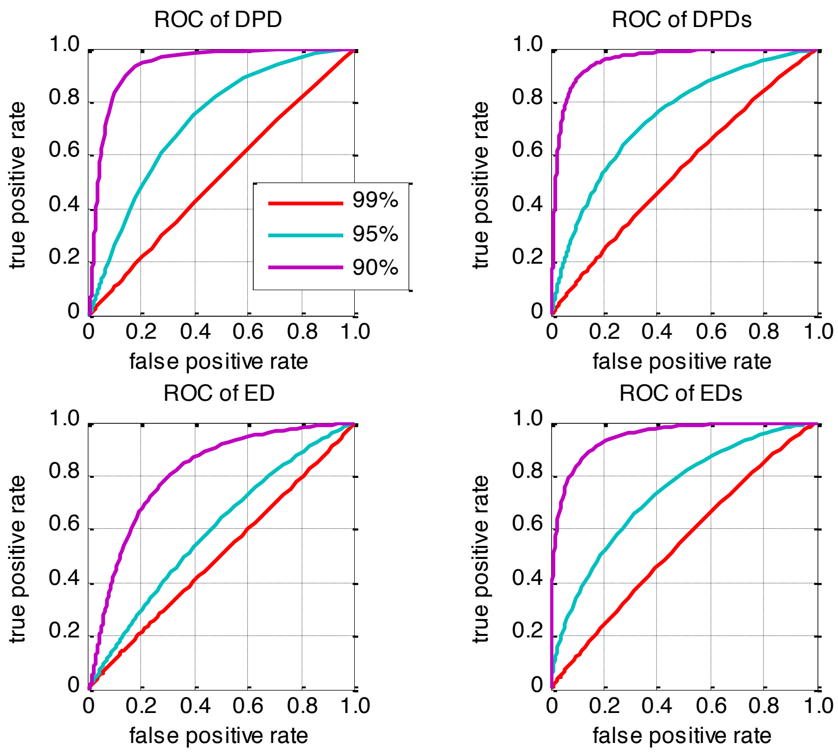

Fig. 8 ROC curves of different features under different damage levels

To quantitatively compare the separation of distributions, a receiver operating characteristic (ROC) curve is a good tool, which compares true detection rate versus false alarm rate under any possible decision threshold. A good detection performance has a high detection rate independent of threshold with minimal false positives, i.e. a steep curve that goes to the upper right corner. On the other hand, a 45-degree line means performance akin to coin-flipping (a random guess), in which condition the number of false alarms is always equal to the amount of real detections on average. Fig. 8 shows the four clusters of ROC curves associated to the four cases. As damage level gets more severe, ROC curves goes to the upper right corner monotonically for all cases, but for features evaluated in a pre-selected frequency region, even better performance is achieved.

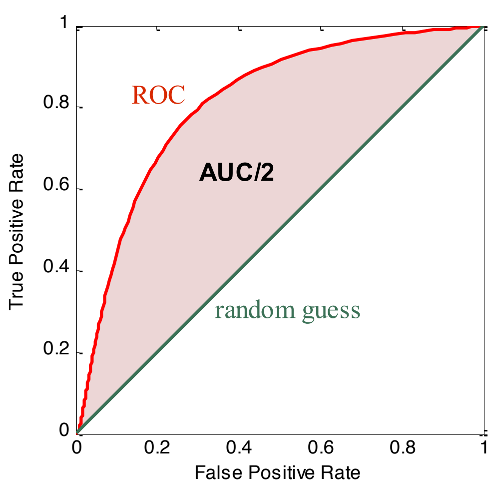

Fig. 9 Definition of area-under-curve

Therefore, a metric called area-under-curve is defined as shown in Fig. 9 to describe how far the ROC deviates from the random guess line and to the corner of "small false alarm and high true detection". The area between ROC and random guess line is normalized from 0 to 1, and the optimal feature is anticipated to have a close-to-unity AUC. In Fig. 10, metric of AUC is compared among the two FRF-related features and two evaluation domains. As the Young's modulus decreases, the performance of statistical significance detection improves, and for a fixed damage level *DPD* feature outperforms *ED*. And results in Fig. 9 suggest a pre-selection of feature evaluation domain, although the selection may be cursory, it helps improve the detectability especially when the damage is light.

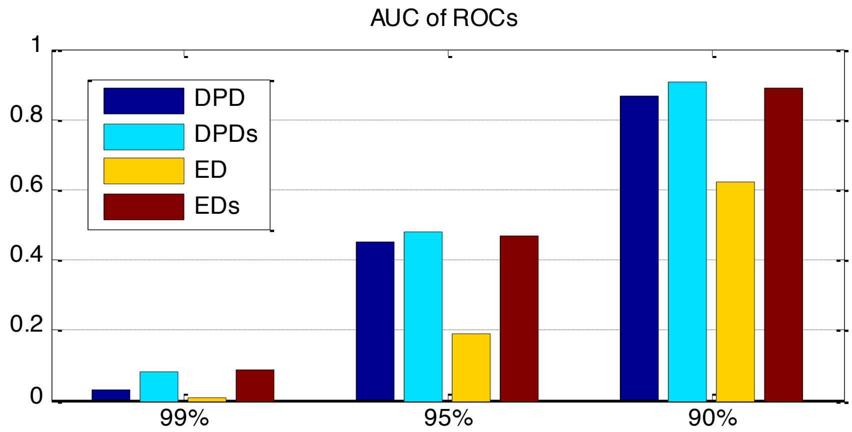

Fig. 9 A comparison of AUC for each feature being used for detection

Summary

This paper investigates an optimal FRF-related SHM feature selection paradigm, in which the optimization is achieved via maximizing the feature performance in terms of receiver operating characteristics (ROC), and under same damage severity, the optimal feature is chosen with the larger AUC.

Considering the two features, each estimated over different domains, we conclude that pre-selecting some "good" frequency intervals (usually those with optimal signal-to-noise) can improve the performance of detection problem dramatically, especially when damage is small and detection is more challenging.

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