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Abstract

Extending the dynamics underlying the factorization calculation of two-body decays, we propose simple selection rules for nonresonant three-body B decays. We predict, for instance, that in the Dalitz plot of $B^0 \to \overline{D}^0 \pi^+ \pi^-$, practically no events should be found in the corner region of $E(\pi^+) \leq \Lambda_{QCD}$ as compared with the corner of $E(\pi^-) \leq \Lambda_{QCD}$. We also predict that there should be very few three-body decay events containing one soft meson resonance and two energetic mesons or meson resonances. The selection rules are quite different from the soft-pion theorem, since they apply to different kinematical regions. For $B^0 \to \overline{D}^0 \pi^+ \pi^-$, the latter predicts that the decay matrix element vanishes in the zero four-momentum limit of π^- instead of π^+. Since this marked difference from the soft-pion theorem is directly related to the issue of short-distance QCD dominance in two-body B decays, experimental test of the selection rules will shed light on strong interaction dynamics of B decay.

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I. INTRODUCTION

The factorization calculation [1] has successfully reproduced the decay rate for many two-body channels of B decay. It has been argued [2,3] that the factorization should become exact in the large b-quark mass limit and that the only deviation from the factorization is short-distance corrections of $O(\Lambda_{QCD}/m_b)$. For some channels of B decay, however, the factorization calculation appears to be in clear disagreement with current measurement [4]. Theoretically the factorization and the final-state interaction (FSI) are closely tied together [5]. If the factorization is a good approximation, the FSI phase should be small in general. If the FSI phase turns out to be large in experiment, it is a warning sign against the factorization. Some *initial state* interaction can also affect the factorization [5,6]. The observed smallness of the upper bounds on the FSI phases of the two-body $b \to c$ processes such as $B \to \overline{D}\pi$ is consistent with absence of long-distance FSI [7]. On the other hand, purely phenomenological suggestions were recently made about possibility of large FSI phases in $B \to K\pi$, $\pi\pi$ [8]. At present, little is known in experiment about FSI phases of decays other than the charm producing channels.

The factorization clearly fails in the D decay in spite of the relatively small value of $\Lambda_{QCD}/m_c = 1/7 \sim 1/5$. The observed relative FSI phase in $D \to \overline{K}\pi$ is close to $90°$ [9] though one would naively expect the factorization to work reasonably well if Λ_{QCD}/m_c is really the controlling parameter of its corrections.[1] In two-body B decays the *c.m.* energy between a fast quark and a spectator grows only like $\sqrt{m_b\Lambda_{QCD}}$ with m_b, not linearly in m_b. Therefore, $\sqrt{m_b\Lambda_{QCD}}$ must be large in order for perturbative QCD to be applicable. Even if one accepts the asymptotic validity of the factorization in theory [3], one should test in experiment whether or not the B mass of 5.3 MeV is high enough to be in the asymptopia of its final states.

In this paper, we extend the basic premise of the factorization and explore its consequences in three-meson decay channels of B decay in which one of mesons is soft. We propose simple selection rules which will provide a further test of the fractorization at the B mass scale. Our approach is purely phenomenological and we do not attempt here a rigorous justification of the selection rules by perturbative QCD. When we compare the selection rules with the soft-pion theorem of chiral symmetry, it appears as if they were incompatible. The origin of this superficial incompatibility is traced to difference in kinematics: The selection rules hold when the *c.m.* energy $\sqrt{m_b\Lambda_{QCD}}$ between a fast quark and a soft quark is sufficiently large, while the soft-pion theorem holds when the same *c.m.* energy is equal to a light quark mass, *i.e.,* negligibly small. The purpose of our selection rules is to probe if $\sqrt{m_b\Lambda_{QCD}}$ in B decay is in the QCD asymptopia or not.

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[1] It has been well established that the FSI phase is close to $90°$ in most of the two-body decay channels of J/ψ so far analyzed [10].

II. BASIS OF FACTORIZATION

We state the factorization of two-body decays as follows: When a B meson decays with a current-current interaction through the quark decay process,

$$\bar{b} \to \bar{q}_1 + q_2 + \bar{q}_3, \tag{1}$$

either $\bar{q}_1 q_2$ or $\bar{q}_3 q_2$ forms one meson and the remaining $\bar{q}_3$ or $\bar{q}_1$ forms the other meson with a soft quark q_4 from the light spectator cloud around $\bar{b}$. (See Fig.1a.) The formation of a meson by two hard quarks through a color-singlet current is given by a decay constant while the formation of the other meson is described by a transition form factor of the other current. It is important that once combinations of $q\bar{q}$ have been chosen for meson formation, no flavor exchange occurs thereafter, not even a charge exchange. Any flavor-changing process is supposedly a higher order process of perturbative QCD. Therefore the final mesons retain their flavor contents, including charges, determined by the short-distance quark decay process. This leads us to various constraints of the factorization which relate different isospin amplitudes of two-body decays for a given decay operator.

What happens if three mesons are produced ? In the case that all final mesons are energetic, each meson should contain one of the three final quarks of Eq.(1) picking up one soft quark from the spectator cloud. (See Fig.1b.) In the case that two of the mesons are energetic and one is very soft, the energetic mesons are formed just as in the two-body decay and the third one emerges from the spectator cloud with energy no larger than $O(\Lambda_{QCD})$. In this case the energetic mesons retain the history of the weak interaction while the soft meson is made of whatever left in the spectator cloud. This is the natural extension of the factorization from two-body decays to three-body decays. We call this picture as the factorization for three-body decays and explore its consequences here. The $c.m.$ energy of the FSI between an energetic meson and the soft cloud is $O(\sqrt{m_b \Lambda_{QCD}})$, which is the same as the $c.m.$ energy of $\bar{q}_1$ and q_4 of the two-body decay in Fig.1a.[2] If $\sqrt{m_b \Lambda_{QCD}}$ is large and short-distance QCD dominates in FSI, flavor exchange between any pair of final mesons would be a higher order correction. In this case, the three quarks produced by the $\bar{b}$-quark can be tracked down into the flavor contents of the final mesons.

III. ILLUSTRATION BY $B^0 \to \overline{D}^0 \pi^+ \pi^-$

We illustrate derivation of our selection rule in the decay,

$$B^0(\bar{b}\mathbf{d}) \to \bar{c}u\bar{d}\mathbf{d} \to \overline{D}^0(\bar{c}u)\pi^+(\bar{d}u)\pi^-(\bar{u}d), \tag{2}$$

which proceeds through the tree-operator $(\bar{b}c)(\bar{u}d)$ and its gluon corrections. The boldface letter $\mathbf{d}$ in Eq.(2) represents a cloud of light quark-antiquarks and gluons around $\bar{b}$ which carries the net flavor of the d-quark;

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[2] The $c.m.$ energy between the soft meson and the soft quark in the fast meson is $O(\Lambda_{QCD})$, but our selection rules do not depend on a soft interaction between them.

$$\mathbf{d} = d + d(\overline{u}u) + d(\overline{d}d) + \cdots. \tag{3}$$

We refer to $\mathbf{d}$ as the spectator d-quark cloud. When the decay products $\overline{c}u\overline{d}$ of $\overline{b}$ enter three mesons one each, all mesons are energetic. In this case we cannot find a simple flavor selection rule of the factorization. The reason is as follows: Even if no flavor exchange occurs between final mesons, the process $\overline{u}u \leftrightarrow \overline{d}d$ occurs perpetually inside the cloud prior to meson formation. This annihilation process has the same effect as a flavor exchange between final mesons since each meson picks up one soft quark from the cloud. Therefore, no simple testable selection rule results in this case.

On the other hand, if one of the pions is soft, it must be generated by the spectator cloud without involving a hard quark. The three quarks from $\overline{b} \to \overline{c}u\overline{d}$ must go into $\overline{D}^0$ and the energetic π.[3] The hard u can form either π^+, π^0, or $\overline{D}^0$ while the hard $\overline{d}$ can form either π^+ or π^0. Even though $\overline{d}$ or $\overline{c}$ picks one soft quark from the cloud, the resulting meson is energetic. After one quark (u or d) is removed from the spectator cloud $\mathbf{d}$ of total charge $-1/3$, the remainder of the cloud is either negatively charged or neutral. That is, the soft pion can be either π^- or π^0, but not π^+. We thus conclude that π^+ cannot be soft in $B^0 \to \overline{D}^0\pi^+\pi^-$. This selection rule is depicted in Fig.2.

In fact, the selection rule holds irrespective of the decay interaction. It depends only on the flavor content of the spectator $\mathbf{d}$: *When one quark is taken away from a cloud of charge $-1/3$, the remainder cannot be positively charged.* The key is that mesons carry quantum numbers of quark-antiquark no matter what their Fock-space compositions are. The only assumption we have made here is that the energetic mesons do not exchange a flavor with the soft meson by short-distance QCD dominance in FSI.

Our prediction should be tested in the form,

$$B(\pi^+_{soft}\overline{D}^0\pi^-)/B(\pi^-_{soft}\overline{D}^0\pi^+) = 0, \tag{4}$$

where $|\mathbf{p}_\pi| \leq O(\Lambda_{QCD})$ for the soft pion. The bands of the $\pi\pi$ and $\overline{D}^0\pi$ resonances should be excluded in testing Eq.(4). If, contrary to the factorization, elastic or inelastic rescattering is important at $E_{\pi\pi} = \sqrt{m_b\Lambda_{QCD}}$ ($\simeq 1.5$ GeV), $\pi^+_{soft}\overline{D}^0\pi^-$ can be fed through the unsuppressed decay modes, for instance, through $\pi^0_{soft}\overline{D}^0\pi^0$ followed by the forward charge exchange rescattering $\pi^0\pi^0 \to \pi^+\pi^-$ or through $\pi^-_{soft}\overline{D}^0\pi^+$ followed by the backward elastic rescattering $\pi^+\pi^- \to \pi^-\pi^+$, *i.e.,* the forward rescattering of two units of charge exchange. Thus the selection rule is sensitive to FSI.[4] The FSI of our concern is the soft-hard process at *c.m.* energy of $O(\sqrt{m_b\Lambda_{QCD}})$ that involves a hard quark, since the soft-soft FSI is no more than a different pick of soft quarks from the cloud.

Selection rules similar to Eq.(4) can be derived for many other three-body decay modes. Considering their relative simplicity, it is worthwhile exploring such selection rules for better understanding of strong interaction dynamics in B decay.

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[3] A quantitative estimate will be given later to show that the soft tails of u and $\overline{d}$ are negligible.

[4] To be precise, rescattering to individual channels need not be small if they are cancelled out by destructive interference.

IV. GENERALIZATION

A. Soft pion and kaon production

The $\overline{D}^0$ meson can be replaced by any meson or meson resonance in Eq.(2); $B^0 \rightarrow \pi^+\pi^- M^0$, where M^0 denotes a neutral meson or meson resonance such as K^0, π^0, or ρ^0. The selection rule reads:

$$B(B^0 \rightarrow \pi^+_{soft}\pi^- M^0)/B(B^0 \rightarrow \pi^-_{soft}\pi^+ M^0) = 0. \tag{5}$$

We can apply the same reasoning to B^+ by replacing the spectator **d** by **u**;

$$B(B^+ \rightarrow \pi^-_{soft}M^+\pi^+)/B(B^+ \rightarrow \pi^+_{soft}M^+\pi^-) = 0, \tag{6}$$

where M^+ is a positively charged meson or meson resonance such as K^+, π^+, and ρ^+. For B_s, we can write the selection rule

$$B(B_s \rightarrow K_{soft}\overline{K}M)/B(B_s \rightarrow \overline{K}_{soft}KM) = 0. \tag{7}$$

If we include $\overline{s}s$ in the spectator cloud, we obtain corresponding selection rules,

$$B(B^+ \rightarrow K^0_{soft}\overline{K}M)/B(B^+ \rightarrow K^+_{soft}\overline{K}M) = 0, \tag{8}$$

$$B(B^0 \rightarrow K^+_{soft}\overline{K}M)/B(B^+ \rightarrow K^0_{soft}\overline{K}M) = 0. \tag{9}$$

Furthermore,

$$B(B^+ \rightarrow \overline{K}_{soft}KM)/B(B^+ \rightarrow K^+_{soft}\overline{K}M) = 0, \tag{10}$$

$$B(B^0 \rightarrow \overline{K}_{soft}KM)/B(B^0 \rightarrow K^0_{soft}\overline{K}M) = 0. \tag{11}$$

Since the rest energy of a kaon exceeds Λ_{QCD}, formation of any kaon from the spectator cloud is kinematically suppressed. Eqs.(8) to (11) mean that the factorization-forbidden modes are even further suppressed than the modes in the denominators.

B. Soft resonance production in quasi-three-body decay

The lightest meson resonance that the spectator cloud can produce is the ρ-meson. Since its rest mass far exceeds Λ_{QCD}, $B \rightarrow \rho_{soft}M_1M_2$ is suppressed no matter what charge state the ρ-meson is in. The cloud simply does not have enough energy to produce ρ even at rest.

To be concrete, consider $B \rightarrow \rho\pi M$. Then the lack of energy in producing a resonance from the cloud leads us to

$$B(B^+ \rightarrow \rho_{soft}M\pi^+)/B(B^+ \rightarrow \pi^+_{soft}M\rho) = 0,$$
$$B(B^0 \rightarrow \rho_{soft}M\pi^-)/B(B^0 \rightarrow \pi^-_{soft}M\rho) = 0, \tag{12}$$

where ρ can be in any charge state. In Eq.(12) ρ may be replaced by ω or some other resonance. Similarly,

$$B(B_s \to \overline{K}^*_{soft} K M)/B(B_s \to \overline{K}_{soft} K^* M) = 0. \tag{13}$$

Though we have listed only the selection rules that are relatively simple and clean, one can write down many more selection rules by the same reasoning.

Since meson or meson resonance production of energy higher than $O(\Lambda_{QCD})$ from the cloud is highly suppressed kinematically, we may raise the softness limit considerably from $|\mathbf{p}| \leq \Lambda_{QCD}$ to, for instance, $|\mathbf{p}| \leq m_\rho$ in actual testing of the selection rules, provided that one can avoid resonance bands of quasi-two-body decays. If we raise it too high, however, the tail of the energy spectrum of a decay product quark enters the "soft meson". To find a compromise, we need some numerical study.

V. HOW SOFT SHOULD THE SOFT PION BE ?

Our selection rules are based on the postulate that a soft meson must be produced entirely from the spectator cloud. Therefore, they would fail if a soft meson contains a significant contribution from the low-energy tail of a decay product of $\overline{b} \to \overline{q}_1 q_2 \overline{q}_3$. For this reason we would like to know how much the low-energy tail contributes when we define the soft meson by $|\mathbf{p}_\pi| \leq \Lambda_{QCD}$. If the tail is indeed negligible, then we should ask how much we can raise the upper limit of softness in actual test without violating the selection rules substantially.

To be concrete, consider $B^0 \to \pi^+_{soft} \overline{D}^0 \pi^-$ again. Suppose that we define the soft pion by its momentum $|\mathbf{p}_\pi| \leq p_{max}$. Estimate of the matrix element involves uncertainties in soft meson formation. If we use the quark-hadron duality for the low-energy tail region and replace the meson wavefunctions by their average values, we can obtain a crude estimate of the decay rate ratio as

$$\frac{\Gamma(\pi^+_{soft} \overline{D}^0 \pi^-)}{\Gamma(\pi^-_{soft} \overline{D}^0 \pi^+)} \approx C^2 \frac{\int_{|\mathbf{p}_u| \leq p_{max}} d\Gamma(\overline{b} \to \overline{c} u \overline{d})}{\int_{|\mathbf{p}_u| \geq m_b/2 - p_{max}}^{m_b/2} d\Gamma(\overline{b} \to \overline{c} u \overline{d})} \tag{14}$$

$$\simeq \epsilon^2 C^2, \tag{15}$$

where $\epsilon = 2 p_{max}/m_b$. The factor C, which arises from difference in meson formation probability, is given by the ratio of the pion wavefunction $\Phi(x)$ on the light-cone as

$$C \simeq \frac{\Phi(\epsilon) \Phi(x_0)}{\Phi(1/2)^2}, \tag{16}$$

where $x_0 = \Lambda_{QCD}/(p_{max} + \Lambda_{QCD})$. The value of C is sensitive to $\Phi(\epsilon)$ near the end point. Let us extrapolate $\Phi(x) \propto x(1-x)$ smoothly to $x = 0$. Then for $p_{max} = \Lambda_{QCD}$

$$\frac{\Gamma(\pi^+_{soft} \overline{D}^0 \pi^-)}{\Gamma(\pi^-_{soft} \overline{D}^0 \pi^+)} \approx 16 \Big(\frac{2 \Lambda_{QCD}}{m_b} \Big)^4. \tag{17}$$

The right-hand side is a fraction of 1% for $\Lambda_{QCD} = 300$ MeV. It is indeed small due to the phase space suppression of the soft u-quark and a smaller meson formation probability for a quark pair of highly uneven energies. From this estimate we expect that accuracy of our

selection rules should be very high. When p_{max} is raised to m_ρ, the ratio in Eq.(15) goes up to 8%. In carrying out experimental test, therefore, we may be able to raise the softness upper limit from Λ_{QCD} to m_ρ. The main background in testing our this selection rule will be a spillover from the resonance bands of ρ^0 and D^{*-} due to errors in pion measurement.

VI. COMPARISON WITH THE SOFT-PION THEOREM

In the zero four-momentum limit of the soft pion, we can relate three-body decay matrix elements to two-body decay matrix elements by the soft-pion theorem of chiral symmetry. It can be expressed with $f_\pi = 130$ MeV as

$$M(B \to \pi^-(k_\mu)\overline{M}_1 M_2) \overset{k_\mu \to 0}{=} \frac{1}{f_\pi}\langle M_1 M_2|[u^\dagger\gamma_5 d, H_w(0)]|B\rangle + k_\mu M^\mu, \qquad (18)$$

where the $k_\mu M^\mu$ term gives the correction to the soft-pion limit [11]. For soft π^+ emission, $u^\dagger\gamma_5 d$ should be replaced by $d^\dagger\gamma_5 u$. Upon extrapolation to the physical pion at rest $k_\mu = (m_\pi, \mathbf{0})$, the correction term $k_\mu M^\mu$ is of $O(k_\mu/\Delta m)$ where Δm is a typical excitation energy of the B system. According to the traditional wisdom, the Born diagram of $B^*(1^-)$ is likely the most important correction (see Fig.3a) since it is nearly degenerate with B ($\Delta m \simeq 46$ MeV). This correction can be of $O(1)$ or even dominate over the zero-four-momentum limit for a physical soft pion. It is easy to see that the B^* contribution to $k_\mu M^\mu$ satisfies the selection rule Eq.(4). In fact, this is the case for the contributions from all Born diagrams. The reason is that the quark diagrams for the Born diagrams of Fig.3a are topologically equivalent to those of the three-body factorization process. The only difference between them is whether a soft pion is emitted from the cloud before or after weak interaction takes place.

If we keep only the soft-pion limit, we obtain

$$\lim_{k_\mu \to 0} M(\pi^+(k_\mu)\overline{D}^0\pi^-) = \frac{1}{f_\pi}M(\overline{D}^0\pi^0), \qquad (19)$$

$$\lim_{k_\mu \to 0} M(\pi^-(k_\mu)\overline{D}^0\pi^+) = 0. \qquad (20)$$

Eq.(20) results from the vanishing commutator, $[u^\dagger\gamma_5 d, H_w] = 0$. In contrast, the zero four-momentum limit of the factorization-forbidden decay $B^0 \to \pi^+(k_\mu)\overline{D}^0\pi^-$ ($k_\mu \to 0$) is related to $B^0 \to \overline{D}^0\pi^0$. While $M(\overline{D}^0\pi^0)$ is $O(1/N_c)$ for $\mathcal{O}_2$, it is the leading term for $\mathcal{O}_1$.

Eqs.(19) and (20) look puzzling at the first sight. If we work out the soft-pion limit for $B^+ \to \pi^+\pi^-\pi^+$, our three-body factorization predicts

$$\frac{B(B^+ \to \pi^-_{soft}\pi^+\pi^+)}{B(B^+ \to \pi^+_{soft}\pi^-\pi^+)} = 0, \qquad (21)$$

while the soft-pion theorem gives for the tree-operator $\mathcal{O}_1 = (\bar{b}_L\gamma_\mu d_L)(\bar{u}_L\gamma^\mu u_L)$ and $\mathcal{O}_2 = (\bar{b}_L\gamma_\mu u_L)(\bar{u}_L\gamma^\mu d_L)$

$$\lim_{k_\mu \to 0} M(B^+ \to \pi^-(k_\mu)\pi^+\pi^+) = \frac{2\sqrt{2}}{f_\pi}M(B^+ \to \pi^+\pi^0), \qquad (22)$$

$$\lim_{k_\mu \to 0} M(B^+ \to \pi^+(k_\mu)\pi^-\pi^+) = \frac{1}{f_\pi}M(B^0 \to \pi^0\pi^0). \qquad (23)$$

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The situation looks equally bizzare in $B \to K\pi\pi$ through the penguin operators. Since the penguin operators of $\bar{b} \to \bar{s}$ are all singlets of chiral SU(2)×SU(2), their commutators with the axial isospin charges are zero. That is, the soft-pion limit should disappear for all matrix elements of $B \to \pi(k_\mu)K\pi$ as $k_\mu \to 0$ for the penguin operators.

It appears as if the selection rules contradicted the soft-pion theorem of chiral symmetry. We can trace the origin of this apparent discrepancy to the *c.m.* energy between a fast quark (p_μ) from $\bar{b}$-decay and a soft quark (k_μ) in the cloud. In the *physical* soft pion limit where our selection rules should hold,

$$(p + k)^2 \simeq 2(k \cdot p) = O(m_b \Lambda_{QCD}), \tag{24}$$

while in the zero four-momentum limit

$$(p + k)^2 \to p^2 = m_q^2. \tag{25}$$

In order for the factorization to work, the right-hand side of Eq.(24) must be large enough to be in the asymptopia of QCD. The same variable is equal to the light quark mass in the soft-pion theorem. Their predictions apply to different kinematical regions.

The chiral Lagrangian has been successful for which the soft-pion theorem gives the leading terms. However, extrapolation to the physical region of B-decays is a long way because of large B mass (cf Eqs.(24) and (25)) and involves large incalculable corrections.

We can understand what physical process the soft-pion theorem represents [11]. The commutator of the axial charge $Q_5^a = q^\dagger \gamma_5 (\tau_a/2)q$ $(a = 1, 2, 3)$ with weak decay operator $\mathcal{O}_i$ arises from its commutator with the currents in $\mathcal{O}_i$. In the case of $B^0 \to \pi^+(k_\mu)\overline{D}^0\pi^-$ at $k_\mu \to 0$ with $\mathcal{O}_1 = (\bar{b}_L \gamma_\mu d_L)(\bar{u}_L \gamma^\mu c_L) = (\bar{b}_{L\alpha} \gamma_\mu c_{L\beta})(\bar{u}_{L\beta} \gamma^\mu d_{L\alpha})$, for instance, the commutator with $\bar{u}_{L\beta} \gamma^\mu d_{L\alpha}$ survives to give

$$[d^\dagger \gamma_5 u, \mathcal{O}_1] = (\bar{b}_{L\alpha} \gamma_\mu c_{L\beta})(\bar{u}_{L\beta} \gamma^\mu u_{L\alpha} - \bar{d}_{L\beta} \gamma^\mu d_{L\alpha}), \tag{26}$$

where the right-hand side is the $I_3 = 0$ partner of $\mathcal{O}_1$. This commutator describes production of states of charge $Q = +1$ through $\bar{u}_{L\beta} \gamma^\mu d_{L\alpha}$ of $\mathcal{O}_1$ and subsequent fragmentation of the soft π^+ from them. Emission of a zero-four-momentum pion from the energetic quark pair $\bar{u}_{L\beta} \gamma^\mu d_{L\alpha}$ is not suppressed by asymptotic freedom since no large momentum transfer occurs. After the π^+ emission, these states $(\sim \bar{u}_\beta u_\alpha - \bar{d}_\beta d_\alpha)$ and the operator $\bar{b}_{L\alpha} \gamma_\mu c_{L\beta}$ together produce the final state $\overline{D}^0 \pi^-$. (See Fig.3b.) For soft π^- emission, the commutator vanishes so that there is no diagram corresponding to Fig.3b. Thus physics in the zero-four-momentum limit is quite different from what we have introduced as an extension of the factorization in this paper.

An important question is which picture gives a better description of the three-body B decay in which one pion is soft. Is the approximation of $\sqrt{m_b \Lambda_{QCD}} \to \infty$ better than the zero four-momentum approximation ? The issue should be settled by experimental test of the selection rules proposed here.

VII. SUMMARY

The dynamics underlying the factorization can be tested with the selection rules in three-body decays in which one of mesons is soft. The selection rules test whether the soft-hard

FSI at *c.m.* energy of $O(\sqrt{m_b \Lambda_{QCD}})$ is in the QCD asymptopia or not. The rules are intuitive and do not depend on decay operators nor on form factors. If the selection rules are verified in experiment, we would obtain better understanding of physics underlying the factorization and strong interactions in B decay in general.

ACKNOWLEDGMENTS

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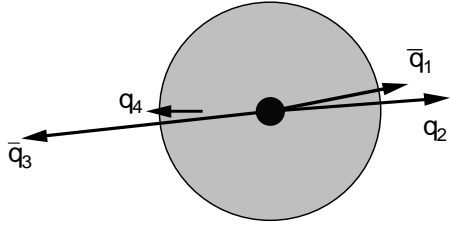
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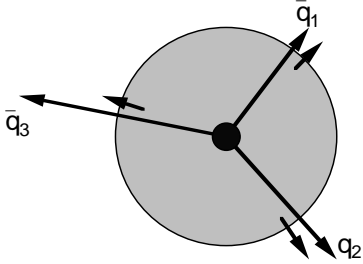
(b)

FIG. 1. (a) The quark picture of the factorization for a two-meson decay. (b) The extension to a three-meson decay in which all three mesons are energetic. The shaded circle represents the spectator cloud, and the short arrows denote soft quarks from the cloud. When one of mesons is very soft in a three-body decay, the decay occurs like (a) with the residual cloud turning into the soft meson.
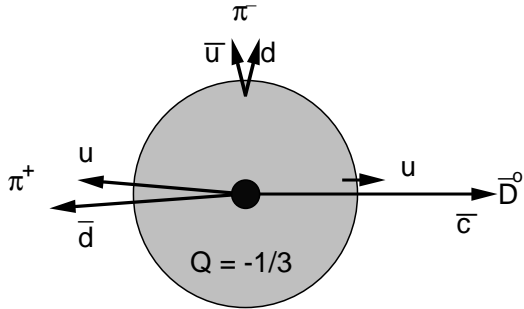

FIG. 2. The decay $B^0 \to \pi^-_{soft} \overline{D}^0 \pi^+$. The decay $B^0 \to \pi^0_{soft} D^- \pi^+$ is also allowed.
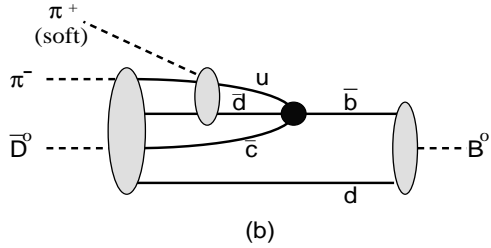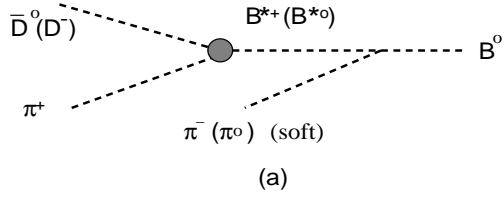
FIG. 3. (a) The leading contribution to $k_\mu M^\mu$ of $B^0 \to \overline{D}^0 \pi^+ \pi^- (D^- \pi^+ \pi^0)$ when $\pi^- (\pi^0)$ is soft. It does not contribute to soft π^+ emission. (b) The zero four-momentum limit of π^+ in $B^0 \to \overline{D}^0 \pi^+ \pi^-$.