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Publication Date
1982-10-01

LBL-14973
Preprint

Lawrence Berkeley Laboratory
UNIVERSITY OF CALIFORNIA

Materials & Molecular
Research Division

Submitted to Journal of Low Temperature Physics

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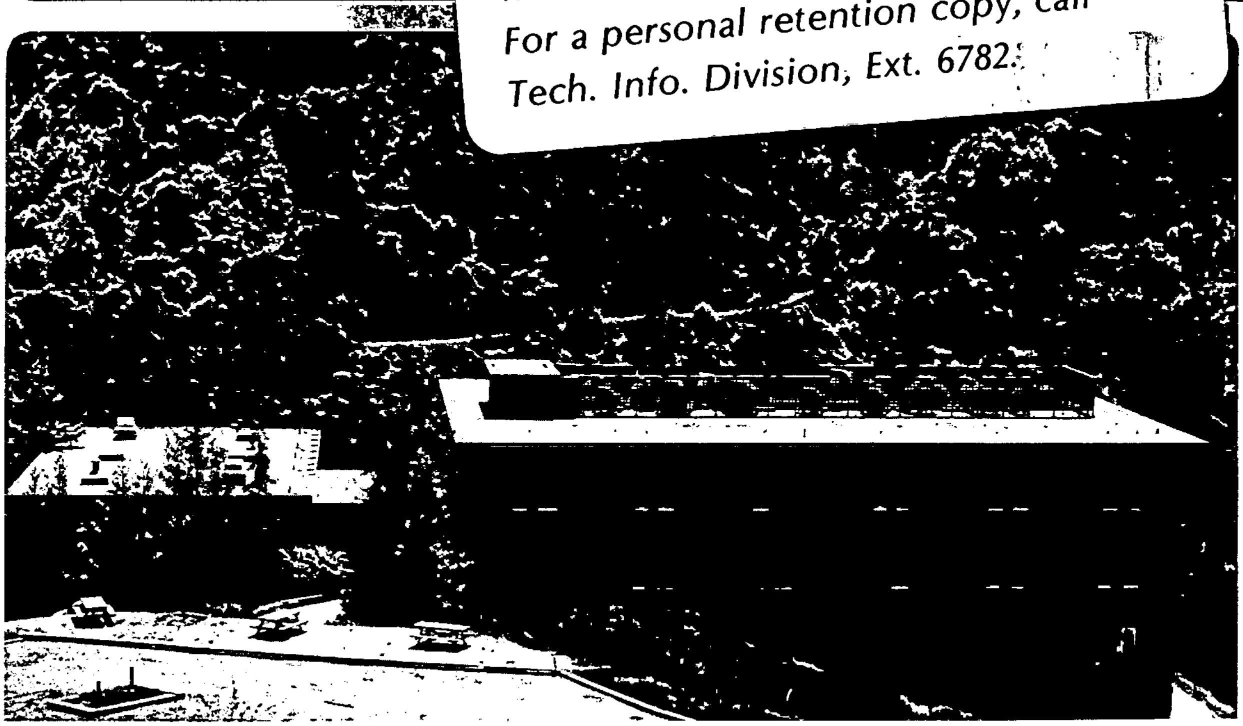
October 1982



Prepared for the U.S. Department of Energy under Contract DE-AC03-76SF00098

Submitted to Journal of Low Temperature Physics LBL-14973

FLICKER (1/f) NOISE IN TUNNEL JUNCTION DC SQUIDS*

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ABSTRACT

We have measured the spectral density of the 1/f voltage noise in current-biased resistively shunted Josephson tunnel junctions and dc SQUIDs. A theory in which fluctuations in the temperature give rise to fluctuations in the critical current and hence in the voltage predicts the magnitude of the noise quite accurately for junctions with areas of about 2×10^4 μm^2, but significantly overestimates the noise for junctions with areas of about 6 μm^2. DC Superconducting Quantum Interference Devices (SQUIDs) fabricated from these two types of junctions exhibit substantially more 1/f voltage noise than would be predicted from a model

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*This work was supported by the Director, Office of Energy Research, Office of Basic Energy Sciences, Materials Sciences Division of the U.S. Department of Energy under Contract Number DE-AC03-76SF00098.

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in which the noise arises from critical current fluctuations in the junctions. This result was confirmed by an experiment involving two different bias current and flux modulation schemes which demonstrated that the predominant 1/f voltage noise arises not from critical current fluctuations but from some unknown source that can be regarded as an apparent 1/f flux noise. Measurements on five different configurations of dc SQUIDs fabricated with thin-film tunnel junctions and with widely varying areas, inductances, and junction capacitances show that the spectral density of the 1/f equivalent flux noise is roughly constant, within a factor of 3 of $(10^{-10}/f)\Phi_o^2\mathrm{Hz}^{-1}$.

1. INTRODUCTION

During the last few years there have been dramatic improvements in the flux noise energy, $\varepsilon/1\text{Hz}$, of dc Superconducting Quantum Interference Devices (SQUIDs) involving Josephson tunnel junctions at frequencies above the 1/f noise region.* Generally speaking, these noise energies have been in quite good agreement with the prediction[2,3] $\varepsilon/1\text{Hz} \approx 16k_BT(LC)^{\frac{1}{2}}$ (T is the temperature, L is the SQUID inductance, and C is the junction capacitance). These improvements have been achieved largely by reducing L and/or C. However, as the level of white noise has been lowered, the 1/f noise region has extended to progressively higher frequencies so that the improvements in sensitivity have not extended to low frequencies. For example, the lowest reported noise energy, about $3\hbar$ at 202kHz, contained a substantial contribution from 1/f noise at this frequency.[4] Thus, the considerable improvements in the white noise energy emphasize the need to understand and, if possible, to reduce the 1/f noise.

The only theory for 1/f noise in Josephson tunnel junctions for which there is supporting experimental evidence is the temperature fluctuation model,[5] in which fluctuations in temperature produce fluctuations in critical current and hence in the voltage across a current-biased junction. One would expect that 1/f fluctuations in critical current would, in turn, produce 1/f voltage noise in dc SQUIDs. There have been several predictions of 1/f noise in SQUIDs based on this assumption.[6-10] In particular, Ketchen and Jaycox[9] found that the 1/f noise in a SQUID with 2 μm-diameter tunnel junctions was apparently in quite good agreement with the predictions of this theory. In this paper we are concerned with two main

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*For a recent review, see ref. 1.

questions. First, in Sec. 2, we address the question of whether or not the temperature fluctuation model explains adequately the 1/f noise in single resistively shunted junctions of small area. Second, in the major part of the paper, we are concerned with the question of whether or not the 1/f noise observed in tunnel junction dc SQUIDs arises from fluctuations in the critical currents of the two junctions or from an alternative mechanism. In Sec. 3 we outline a theory that relates critical current fluctuations to the voltage noise across a SQUID. In Sec. 4 we summarize measurements of 1/f noise in five different types of SQUIDs, and compare the values for two of them with the values expected if the noise were to arise from critical current fluctuations. Section 5 describes two measurement techniques for a SQUID in which the bias current and applied flux are modulated with a square wave. These experiments enable one to distinguish between 1/f noise arising from critical current fluctuations and that arising from fluctuations in flux from some other source. Section 6 contains our conclusions.

2. 1/f NOISE IN SINGLE JUNCTIONS

We summarize briefly the temperature fluctuation model for 1/f noise in single resistively shunted Josephson junctions and compare its predictions with measurements we have made.

The model[5] assumes that the 1/f noise arises from temperature fluctuations in the junction that produce fluctuations in the critical current, I_o, (provided $dI_o/dT \neq 0$) and hence fluctuations in the voltage, V, when the junction is biased with a current, I, greater than the critical current. Using the empirical result of Voss and Clarke[11] for the spectral density of the temperature fluctuations we find for the spectral density of the voltage fluctuations

$$S_V^{1/f} \sim \frac{k_B T^2}{3C_v f}\left(\frac{dI_o}{dT}\right)^2\left(\frac{\partial V}{\partial I_o}\right)_I^2 . \qquad (2.1)$$

Here, C_v is the heat capacity of a junction with a thickness equal to the sum of the coherence lengths (ξ) of the two superconductors, and f is the frequency. It should be emphasized at this point that there is no theoretical justification for the 1/f dependence; furthermore, the factor of 3 in the denominator is very model dependent.[5,11]

In Table I we list the measured and predicted 1/f critical current fluctuations, $S_{I_o}^{1/f}/I_o^2 = S_V^{1/f}/I_o^2(\partial V/\partial I_o)^2$ and $(k_B T^2/3C_v f)(dI_o/I_o dT)^2$, for two types of junctions with very different areas. For convenience, all spectral densities have been referred to 1Hz. The junction with the larger area was fabricated in a cross-strip configuration, with a disk of normal metal under the junction providing the resistive shunt.[5] The 1/f noise was measured with a SQUID. The smaller junction was defined by a window in SiO, and was fabricated with photolithographic lift-off techniques in the configuration described in ref. 12. The 1/f noise was measured at three frequencies (70, 106, 183 kHz) using cooled LC-resonant circuits as voltage amplifiers;[12] the 1/f noise has been extrapolated to 1Hz. In both cases, the junctions were surrounded by superconducting shields, and considerable care was taken to ensure that the 1/f noise did not arise from the measurement techniques.

We see from Table I that the model predicts the noise rather well for the larger junction. On the other hand, in the case of the smaller junctions, for which there is a considerable spread in the measured 1/f noise from junction to junction, the model often predicts a much higher noise level, in some cases by as much as two orders of magnitude. Thus, the expected increase in the measured noise as the junction area is reduced is not always observed. However, this result does not necessarily imply that the notion of temperature fluctuations producing the 1/f noise is totally incorrect, but it certainly indicates that the present theory is too simplistic. Perhaps a theory that describes more appropriately the effective volume in which the fluctuations occur would agree more closely with experimental results. For example, if, for the smaller junctions, one were to use the overlap area of the superconducting films (about 250 μm^2) rather than the window area (about 6 μm^2) in Eq. (2.1), the predicted value of $S_{I_o}^{1/f}(1Hz)/I_o^2$ would be reduced to about $10^{-11}Hz^{-1}$, which is considerably closer to the experimental values. Obviously there is no justification for such a procedure within the framework of the present model.

We also note that the temperature dependence of $S_V^{1/f}$ predicted by Eq. (2.1) scales as $(dI_o/dT)^2/T$ if one assumes that C_v is proportional to T^3. In a junction with an anomalous temperature dependence, Clarke and Hawkins[5] found that the 1/f noise became extremely small when dI_o/dT became zero. However, the expected temperature dependence is often not observed when junctions are cooled in the liquid He^4 temperature range. Thus, for our Pb-alloy junctions, we estimate dI_o/I_odT to be 0.2 K^{-1} at 4.2K and 0.02 at 1.5K, so that $(dI_o/dT)^2/T$ decreases by a factor of about 35 over this temperature range. But the 1/f noise in PbIn-Ox-Pb2, for example, decreased by only a factor of 2 when the temperature was lowered from 4.2K to 1.5K. Again, this observation does not necessarily rule out temperature fluctuations as the source of 1/f noise, but certainly implies that the existing model is incomplete. We note also that since the noise decreased as the temperature was lowered, the noise at the higher temperature could not have arisen from the measurement technique.

To our knowledge there exists only one other set of 1/f noise measurements on single junctions, namely those by Krusin-Elbaum and Voss[13] who measured the 1/f noise in 2.5 μm-diameter Pb-alloy junctions with $I_o = 15\text{-}30\ \mu A$, $R = 4\text{-}7\ \Omega$, and $\beta_c = 2\pi I_o R^2 C/\Phi_o = 0.3\text{-}0.7$. Although they found significant deviations from a 1/f slope, the order of magnitude of the noise was generally quite well predicted by Eq. (2.1) when $(dI_o/dT)(\partial V/\partial I_o)_I$ was replaced with the measured value of $(\partial V/\partial T)_I$.

Clearly, much remains to be understood about the origin of 1/f noise in single junctions. For the moment, we make use only of the experimentally determined values to predict the 1/f noise in SQUIDs due to critical current fluctuations.

3. THEORY FOR 1/f NOISE IN SQUIDS FROM CRITICAL CURRENT FLUCTUATIONS

In the next section, we wish to compare the measured 1/f noise in two different types of SQUID with the 1/f noise in single junctions of comparable size fabricated with identical technologies. In this section we develop a model that enables us to predict the noise in a SQUID for given fluctuations in the critical current of each of the junctions. This paper extends the existing model[10] to the case in which the SQUID is flux modulated.

We assume that the SQUID consists of two identical resistively shunted Josephson junctions of resistance R and average critical current I_o arranged symmetrically on a superconducting loop of inductance L. The spectral density of the 1/f noise in the critical current of each junction is $S_{I_o}^{1/f}(f)$; we assume that the fluctuations of the two junctions are statistically independent. There are two independent modes in which the critical current fluctuations produce voltage noise. The first is an in-phase component of the two fluctuations that couples to the voltage via the coefficient $(\partial V/\partial I_o)_I$. In the second mode, the critical current fluctuations are exactly out of phase and produce a fluctuating voltage that is proportional to $(\partial V/\partial \alpha)_I$, where

$$\alpha = (I_{o1} - I_{o2})/2I_o . \tag{3.1}$$

In Eq. (3.1), $I_{o1}(t)$ and $I_{o2}(t)$ are the fluctuating critical currents of the two junctions. This voltage arises, roughly speaking, because the odd mode of the critical current fluctuations redistributes the bias current between the two junctions, thereby producing a current around the SQUID, and hence a voltage provided $V_\Phi \equiv (\partial V/\partial \Phi)_I \neq 0$. It is important to realize that the contributions of these two modes depend on the manner in which the SQUID is operated. If the SQUID is biased at a constant current with a constant external flux (i.e. there is no flux modulation), both modes contribute. On the other hand, in the usual technique for flux-locked operation, the SQUID is biased at a constant current and flux, a modulating flux of peak-to-peak amplitude $\Phi_o/2$ is applied, and the resulting alternating voltage across the SQUID is lock-in detected at the modulation frequency. The smoothed output from the lock-in detector is fed back to flux-lock the SQUID. Provided the modulation frequency is higher than the 1/f noise frequencies, the in-phase mode of the 1/f noise $[\propto(\partial V/\partial I_o)_I]$ will not be observed. The out-of-phase mode, however, effectively produces a low frequency flux noise that will be observed, just as for any other flux change. We give theoretical expressions for each of these two cases.

A SQUID that is flux modulated with a square wave is switched between the flux states $+\Phi_o/4$ and $-\Phi_o/4$. The spectral density of the equivalent flux noise is related to the spectral density of the critical current fluctuations via the relation

$$S_\Phi^{1/f}(f) = \left[\frac{\left(\frac{\partial V}{\partial \alpha}\right)_I^+ - \left(\frac{\partial V}{\partial \alpha}\right)_I^-}{V_\Phi^+ - V_\Phi^-}\right]^2 \frac{S_{I_o}^{1/f}(f)}{2I_o^2} \, . \text{ (modulated)} \qquad (3.2)$$

The superscripts + and - refer to the flux states $+\Phi_o/4$ and $-\Phi_o/4$; $S_{I_o}^{1/f}$ refers to the critical current noise of a single junction.

For a SQUID that is not flux modulated, the spectral density of the equivalent flux noise due to critical currrent fluctuations is[10]

$$S_\Phi^{1/f}(f) = \tfrac{1}{2}\left[\left(\frac{\partial V}{\partial I_o}\right)_I^2 + \frac{1}{I_o^2}\left(\frac{\partial V}{\partial \alpha}\right)_I^2\right] S_{I_o}^{1/f}(f)/V_\Phi^2 . \text{ (unmodulated)} \qquad (3.3)$$

Equations (3.2) and (3.3) were used for two SQUIDs to predict the spectral density of the critical current fluctuations necessary to account for the observed 1/f noise (see Sec. III B). In performing the calculations, we used the following procedures. First, to find $(\partial V/\partial \alpha)_I$, we computed the voltage at a particular bias current for $I_{o1} = I_{o2}$, increased (say) I_{o1} and decreased (say) I_{o2} by a small amount (1%), and recomputed V. The computed change δV is thus related to a given change $\delta\alpha$. In similar ways, we computed $(\partial V/\partial I_o)_I$, V_Φ, and the dynamic reistance of the

SQUID, R_D. Now in practice, particularly for SQUIDs E1 and E2, the measured values of the computed and measured dynamic resistance, R_D^c and R_D^m do not always agree: This discrepancy may be due to our lack of knowledge of the resistance of the junctions at voltages below the energy gap or to resonances that are not included in the model calculations. However, the numerical calculations show that the quantities $(\partial V/\partial I_o)_I$, $(\partial V/\partial \alpha)_I$, and V_Φ all scale rather closely with R_D. Thus, in estimating the magnitudes of the 1/f noise from Eq. (3.3) for SQUIDs E1 and E2 we multipled each of the terms $(\partial V/\partial I_o)_I$, $(\partial V/\partial \alpha)_I$ and V_Φ by the factor R_D^m/R_D^c.

For both Eqs. (3.2) and (3.3) we can write

$$\frac{S_\Phi^{1/f}(f)}{\Phi_o^2} = \gamma \frac{S_{I_o}^{1/f}(f)}{I_o^2} , \qquad (3.4)$$

where γ is computed from the appropriate measured values of the parameters. Thus, we can compute γ, predict $S_{I_o}^{1/f}(f)/I_o^2$ from the measured value of $S_\Phi^{1/f}(f)/\Phi_o^2$, and compare this predicted value with experimental values obtained from measurements on single junctions.

4. 1/f NOISE IN FIVE TYPES OF DC SQUID

We have measured the 1/f noise in the five types of dc SQUIDs shown in Fig. 1. All five SQUIDs involve resistively shunted Josephson tunnel junctions, Nb-Ox-Pb for A-D, and PbIn-Ox-Pb for E. SQUIDs A and E are described in detail in refs. 14 and 4. SQUIDs A and B were made by depositing the films through metal masks while C, D, and E were made with photolithographic lift-off techniques. The essential parameters and their measured and predicted white and 1/f noise levels are listed in Table II.

The measured white noise generally follows the predicted dependence on L and C, but tends to be somewhat higher in magnitude than expected. It should be noted, however, that we could not measure C directly, and that the estimated values of C could be somewhat in error.

Of the SQUIDs listed, A to D were operated in a flux-locked loop with flux-modulation. We applied a 100 kHz flux with a peak-to-peak amplitude of $\Phi_o/2$ to the SQUID, and amplified the resulting alternating voltage across the SQUID with a cooled LC resonant circuit or a resonant transformer. After further amplification, the signal was lock-in detected at the modulation frequency and fed back to flux-lock the SQUID. The spectral density of the noise at the output of the lock-in detector was measured with a PDP-11 computer. In this scheme, as pointed out in Sec. III, the measurement is sensitive only to low frequency flux noise in the SQUID. Equation (3.2) is appropriate for critical current fluctuations; the even mode does not contribute. In contrast, we measured the voltage noise from SQUID E without flux modulation by connecting it to a low noise preamplifier and measuring the spectral density with the computer. In this case, both even and odd modes of the critical current contribute, and Eq. (3.3) is appropriate.

In making the measurements, we always enclosed the SQUID in a superconducting shield made of lead or niobium. We took considerable care to ensure that the 1/f noise was not generated by the measurement scheme. The levels of 1/f noise in the dc bias current and ac and dc flux modulation currents were measured separately, and shown to be negligible. (Noise in either the bias current or the ac modulation current should not affect the measurement, to first order, when the SQUID is flux modulated between the ± $\Phi_o/4$ flux states.) We also ensured that the 1/f noise in the lock-in detector was negligible, by operating it at a sufficiently high signal level. For SQUIDs E1 and E2 the equipment and techniques used were identical to those used for the small area single junctions listed in Table I. Since the 1/f voltage noise measured in the single junctions was far lower than that measured in the SQUIDs, we can exclude the possibility that the measurement system was the source of 1/f noise in the SQUID measurements.

The measured 1/f noise energy at 1Hz, $\varepsilon^{1/f}(1\mathrm{Hz})/1\mathrm{Hz}$, varies from about 2×10^{-31} JHz^{-1} to 3×10^{-28} JHz^{-1}. However, the measured flux noise has a spectral density, $S_\Phi^{1/f}(1\mathrm{Hz})$, that is remarkably constant over the SQUIDs measured, being always within a factor of 3 of 10^{-10} Φ_o^2 Hz^{-1} with an average value of about 10^{-10} Φ_o^2 Hz^{-1}.* Thus, within the quoted spread of values, the 1/f flux noise is independent of the SQUID inductance when it is varied by a factor of 500, of the junction area and capacitance when each is varied by a factor of nearly 3,000, and of the area of the SQUID loop when it is varied by more than 10^6.

For SQUIDs A and E we use Eqs. (3.2) and (3.3), respectively, to estimate the values of the 1/f critical current noise that would be required to generate the measured 1/f flux noise. We can make a meaningful comparison only for these two types of SQUIDs because they incorporate junctions that closely resemble the two types of single junction listed in Table I. In Table II we list the value of γ [Eq. (3.4)] computed using the measured parameters as described in Sec. III. In the last column of Table II we

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*We note that for the tunnel junction SQUID of Ketchen and Jaycox (ref. 9) $S_\Phi^{1/f}(1\mathrm{Hz}) \approx 3 \times 10^{-10}$ Φ_o^2/Hz, a value that is quite compatible with our own results. The apparent agreement of this value with the predictions of the thermal fluctuation model may be fortuitous. If we assume that the 1/f noise in these 2 μm-diameter junctions is substantially less than that predicted by Eq. (2.1), as is the case for our own small area junctions, the measured 1/f flux noise is probably too large to have been generated by critical current fluctuations.

list the values of $S_{I_o}^{1/f}(f)/I_o^2$ that would be necessary to generate the observed flux noise. In the case of the large area junctions (SQUID A) the required value is an order of magnitude larger than that observed in the single junction. In the case of the small area junctions (SQUIDs E1 and E2) the required value is a factor of between 1.5 and 500 times greater than the values observed in the single junctions, depending on which SQUID is compared with which junction.

These results suggest very strongly that the 1/f flux noise in our SQUIDs does not arise predominantly from fluctuations in the critical current. Although it would have been preferable to have measured the 1/f noise separately in the actual junctions of which the SQUIDs were made, we feel that the discrepancies between the required values of $S_{I_o}^{1/f}(f)/I_o^2$ in the SQUID and the measured values in the single junctions are generally large enough for the comparison to be a valid one. In the next section, we describe a measurement technique that enables one to estimate directly the contribution of the critical current fluctuations to the total flux noise in a SQUID.

5. BIAS CURRENT MODULATION SCHEMES

The results of the previous section suggest strongly that the predominant 1/f noise in our SQUIDs does not arise from critical current fluctuations. In this section we describe two sets of measurements on a SQUID of type D that enables one to distinguish directly between critical current fluctuations and an apparent flux noise as the source of 1/f noise, without the need to rely on any model calculation.

5.1. Method A: Reversal of Bias Current with Flux Change of $\Phi_o/2$

The principles of method A are shown in Fig. 2(a). We assume that the SQUID is flux-modulated at 100 kHz with a square wave of peak-to-peak amplitude $\Phi_m = \Phi_o/2$. The resulting voltage, V, across the SQUID is amplified by a cooled transformer resonated at 100 kHz with a Q of about 3. Thus, the signal at the secondary of the transformer, V_t, is always approximately sinusoidal. After amplification, this signal is lock-in detected with a reference voltage, V_r, and the output, V_ℓ, is smoothed by a low-pass filter. The bias current, I is reversed at a frequency ν_I much less than 100 kHz and, at the same time, a flux change of $\Phi_o/2$ is applied to the SQUID. Thus, ideally, V_Φ is unchanged by the switching. The left-hand column of Fig. 2(a) shows the voltage across the SQUID, V, vs. Φ/Φ_o for this procedure. When I becomes negative, the V-Φ curve is simultaneously shifted by $\Phi_o/2$ and reflected about the zero-voltage axis.

Consider first a positive low-frequency fluctuation ($\ll \nu_I$) in the flux in the SQUID, $\delta\Phi$, as indicated by the dashed lines. The 100 kHz-modulation will switch the SQUID between the states 1 and 2 for positive I and 3 and 4 for negative I. As a function of time, t, the voltage across the SQUID, V, will appear as indicated in Fig. 2(a); transient voltages that occur when I is switched have been suppressed in this figure. When the signal passes through the tuned transformer, the voltage V_t will consist of the fundamental frequency; again, switching transients have been neglected. Finally, this signal is lock-in detected by multiplying with the reference voltage, V_r. The voltage at the output of the lock-in, V_ℓ, consists of a series of negative-going peaks for both polarities of the bias current. When this voltage is averaged over a time large compared with $1/\nu_I$, there will be a negative signal from the lock-in detector. Thus, the system responds to the flux fluctuation in the same way as for the usual 100 kHz-modulation scheme without the low-frequency switching of bias current and flux.

Consider now a flux fluctuation arising from fluctuations in critical current, $\delta\alpha$ [see Fig. 2(a)]. We emphasize that only the components of critical current change producing a flux change are detected in this scheme. Because this flux reverses sign when the bias current is reversed, the V-Φ curves will be displaced in opposite directions. In this case, the voltage that appears across the SQUID when I is negative is shifted by one-half cycle compared with the case of an external flux fluctuation. Thus, when the voltage across the transformer is lock-in detected, the voltage at the output of the lock-in reverses sign each time I is reversed. The average of the output over periods long compared with $1/\nu_I$ is zero: This scheme does not respond to fluctuations in flux arising from fluctuations in critical current.

We turn now to the practical realization of this technique. The major experimental difficulty is that the measurement must be performed in such a way that the undesirable effects arising from the inevitable asymmetries in junction parameters such as critical currents and shunt resistances are properly compensated. The circuit we have used is shown in Fig. 3. The SQUID is flux-modulated with a 100 kHz-square wave having a peak-to-peak amplitude $\Phi_o/2$, and the voltage across the SQUID is lock-in detected in the usual way. The SQUID forms one arm of a bridge driven with 1 kHz-square wave; the square wave also applies a flux to the SQUID. Two cooled 1 kΩ-resistors together with the cable capacitance form low pass filters to reduce the level of high frequency noise reaching the junction. The bridge was intended to serve two purposes: First, it reduced the effects of any 1/f noise present in the square wave. Second, it reduced the level of the transients at the input of the preamplifier, thereby enabling us to use more ac gain before the lock-in detector. The output from the bridge was amplified with a cooled, resonant transformer so that the 100 kHz input to the preamplifier was nearly sinusoidal.

The operating procedure is as follows. We first balance the bridge to minimize the 1 kHz-switching transients, and adjust the level of the 1 kHz-bias current to give the maximum value of V_Φ. We then optimize the 1 kHz-flux amplitude by setting the dc flux in the SQUID to $(2n + 1)\Phi_o/4$, and examining the output of the lock-in detector. When the bridge is balanced and the amplitudes of the 1 kHz-current through the SQUID and the modulation coil are properly adjusted, the output from the lock-in should be identical to the case in which the 1 kHz-modulation is absent and the SQUID is flux-modulated in the conventional way. (This signal, V_ℓ, is illustrated in the sixth line of the right-hand column of Fig. 2(a).) To make the final adjustments, we change the dc flux in the SQUID to $n\Phi_o/2$ so that the output from the lock-in is zero, and monitor this output as the amplitude of the 1 kHz-oscillator is varied slightly. The various potentiometer settings are re-adjusted, if necessary, to ensure zero response to this variation. It should be noted that at the end of this procedure the SQUID is not necessarily biased at the maximum value of V_Φ, so that the white noise performance may be degraded.

Figure 4 shows the spectral density of the flux noise measured at 4.2K and 1.6K with $\Phi = n\Phi_o/2$. For comparison, we also plot the noise measured in the conventional way, that is, with the 1 kHz-oscillator disconnected, and the SQUID biased with a steady current; the 100 kHz-flux modulation is as before. We see immediately that the 1/f noise has not been reduced by method A. The increase in the white noise with method A was due to the fact that V_Φ was no longer maximized, as noted earlier. The fact that the 1/f noise is lower at 1.6K than at 4.2 demonstrates that at 4.2K the measurement scheme itself did not contribute a significant amount of 1/f noise. Thus, we conclude that the predominant 1/f noise does arise not from critical current fluctuations but rather from an apparent flux fluctuation.

We also tried a variation of the preceding experimental technique in which the 1 kHz-modulation is replaced with a 50 kHz-square wave that is synchronous with the 100 kHz-modulation flux. The 50 kHz-signal switched at precisely the same moment as the 100-kHz signal. The results with this commensurate switching were exactly the same as with the earlier, incommensurate switching at 1 kHz. Indeed, we were able to flux-lock the SQUID with this procedure, again obtaining the same 1/f noise as with the conventional measurement technique using a dc current bias. These two experiments, incidentally, served to check that the various gains in the unlocked method of measuring 1/f noise had been correctly established.

5.2. Method B: Reversal of Bias Current

Figure 2(b) shows the alternative scheme in which I is periodically reversed but no corresponding flux change is applied to the SQUID. The average of V_Φ over many current reversals, $\langle V_\Phi \rangle$, is then zero. The corresponding V-Φ curves for positive and negative bias currents are shown in the right-hand column of Fig. 2(b), while the time-dependent voltages appear in the left-hand column. For the case of a fluctuation in external flux, when I is negative the voltage across the SQUID changes phase by π relative to the reference signal. Thus, the time-averaged output from the lock-in is zero, as expected for the case when $\langle V_\Phi \rangle = 0$. On the other hand, for a fluctuation in flux arising from a fluctuation in critical current (lower figure of the right-hand column of Fig. 2(b)) the voltage across the SQUID remains in phase with the reference signal when I is reversed. Thus the time-averaged signal from the lock-in is non-zero. We see that the effects of the fluctuations $\delta\Phi$ and $\delta\alpha$ are just reversed in method B compared with method A.

The experimental scheme used is again that in Fig. 3. The 1 kHz-flux modulation (which would be zero if the SQUID were perfectly symmetrical) is adjusted to give the output from the lock-in detector, V_ℓ, shown in line 6 of the left-hand column of Fig. 2(b). The various potentiometers are adjusted with $\Phi = n\Phi_o/2$ so that the response to a low-frequency flux ($\ll$ 1 kHz) applied to the SQUID is zero; thus, we have achieved the condition $\langle V_\Phi \rangle = 0$. The resulting measured spectral density of the flux noise for $\Phi = n\Phi_o/2$ is shown in Fig. 5, compared with that obtained by method A. Also shown is the 1/f noise measured at the output of the lock-in detector when the SQUID is flux-modulated at 100 kHz and the bias current is kept constant; as expected, this measurement agrees with method A. We observe that method B reduces the 1/f noise by about one order of magnitude. This observation is a direct demonstration that the predominant 1/f noise is generated by an apparent flux noise that is unconnected with fluctuations in the critical current. Furthermore, the fact that it is possible to reduce the 1/f noise once again demonstrates that the original level of 1/f noise did not arise from our measurement technique.

6. CONCLUSIONS AND DISCUSSION

From our measurements in Sec. 2, we conclude that the thermal fluctuation model for 1/f noise significantly overestimates the 1/f noise in our single, resistively shunted PbIn-Ox-Pb Josephson tunnel junctions with areas of about 6 μm^2. On the other hand, as reported previously, the model predicts rather well the 1/f noise in Nb-Ox-Pb tunnel junctions with large areas. We emphasize that the level of measured 1/f noise may depend critically on the exact fabrication procedure followed, and one should not necessarily apply these conclusions to tunnel junctions made elsewhere.

The measurements in Sec. 4 suggest strongly that the observed 1/f voltage in our SQUIDs is substantially greater than that predicted from measurements of the critical current fluctuations in single junctions fabricated in a similar way. This observation implies that there is an additional source of 1/f noise. The measurements made in Sec. 5, where two different modulation schemes were used, demonstrate that this additional mechanism can be regarded as an apparent flux noise. The modulation methods do not depend on any model to relate critical current fluctuations to voltage fluctuations. Critical current fluctuations with a 1/f noise will, of course, make a contribution to the total 1/f voltage noise, but this contribution is clearly small. Two obvious possible sources of the apparent flux noise are the SQUID itself or the superconducting shield enclosing the SQUID. In fact, the possibility that the shield produces a magnetic field noise, for example, from the motion of trapped flux lines, seems rather remote. SQUIDs with areas ranging over 6 orders of magnitude all showed approximately the same <u>flux</u> noise, implying that the external magnetic field noise would have to vary so as to compensate for the variation in the area of the SQUIDs. This variation is most implausible. Thus, we are led to suspect that the origin of the flux noise lies in the SQUID itself, conceivably from the motion of trapped flux. However, there is no model to explain why the power spectrum varies as 1/f. We cannot emphasize too strongly that the observations we have made and the conclusions we have drawn apply only to the type of thin-film tunnel-junction devices that we have studied. In other types of SQUID, for example, thin-film devices incorporating microbridges or machined Nb devices incorporating point contacts, it is quite possible that critical current fluctuations could be the dominant source of 1/f noise, either because the junctions exhibit more 1/f noise than tunnel junctions or because the material or geometry used in the SQUID has less 1/f noise than is the case for our SQUIDs. Thus, one should not extrapolate our results to radically different types of SQUID, but must investigate each type separately.

The fact that $S_{\Phi}^{1/f}(f)$ is roughly constant from SQUID to SQUID, rather than proportional to 1/C as predicted by a model in which the noise is produced by critical current fluctuations arising from temperature fluctuations, changes one's philosophy in minimizing the 1/f noise energy of SQUIDs. In so far as the notion of $S_{\Phi}^{1/f}(f)$ being a constant is valid, it is clear that one should make L as large as possible, in order to minimize $\varepsilon^{1/f}(f)/1\text{Hz} = S_{\Phi}^{1/f}(1\text{Hz})/2L$. Now to approach the quantum noise limit of SQUIDs in the He^4 range,[15] it is necessary to reduce LC to less than 10^{-24} HF. In the most sensitive SQUID yet reported[4] (E1 of Table II) this goal was achieved by reducing L to about 2 pH. Clearly, the use of such a low inductance is undesirable if one wishes to minimize the 1/f noise energy.

In principle, one could increase L and decrease C, keeping their product constant, so as to maintain a very low level of white noise and to reduce the 1/f noise. In practice, very large reductions in C may not be easy. For example, if one increased the inductance of SQUID E1 by three orders of magnitude to 2 nH, one would have to decrease C to about 0.3 fF, corresponding to a junction area of less than 10^{-2} μm^2, assuming that parasitic capacitances were negligible. An alternative means of achieving quantum limited performance in the white noise region and a low level of 1/f noise with a more reasonable value of junction capacitance may be to cool the SQUID to millidegree temperatures. In that case, there may be heating problems due to dissipation in the shunt resistors; furthermore, the temperature dependence of the 1/f noise at low temperatures is not known.

A final possibility for reducing the 1/f noise in SQUIDs is, of course, that one may eventually understand the mechanism responsible, and take appropriate steps to reduce its magnitude. An investigation of the origin of the apparent flux noise is of high priority.

ACKNOWLEDGMENT

One of us (R.K.) gratefully acknowledges the use of the computer facilities at the IBM Thomas J. Watson Research Center.

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TABLE I

Spectral Density at 1Hz of 1/f Noise in Critical Current of Two Sets of Resistively Shunted Junctions at 4.2K

Type	Area (μm^2)	I_o (mA)	$dI_o/I_o dT$ (K^{-1})	Measured $S_{I_o}^{1/f}(1Hz)/I_o^2$ (Hz^{-1})	Predicted[a] $S_{I_o}^{1/f}(1Hz)/I_o^2$ (Hz^{-1})
Nb-Ox-Pb[b]	2.3×10^4	3.4	0.24	5×10^{-11}	0.8×10^{-10}
PbIn-Ox-Pb1[c]	6	0.32	0.16	$< 2 \times 10^{-10}$	3×10^{-10}
PbIn-Ox-Pb2	6	0.51	0.16	4×10^{-11}	3×10^{-10}
PbIn-Ox-Pb3	6	0.36	0.16	$< 4 \times 10^{-12}$	3×10^{-10}
PbIn-Ox-Pb4	6	1.28	0.16	$< 2 \times 10^{-12}$	3×10^{-10}

[a] The following parameters were used (see ref. 5):

$C_V(Nb) = 2.5 \times 10^{-15}\ JK^{-1}\ \mu m^{-3}$, $C_V(Pb) = 8 \times 10^{-15}\ JK^{-1}\ \mu m^{-3}$,

$\xi(Nb) = 50$ nm, $\xi(Pb) = 80$ nm.

[b] From Fig. 2 of ref. 5.

[c] The PbIn-Ox-Pb junctions are those described in ref. 12.

TABLE II

Measured and Predicted White and 1/f Noise for Five SQUIDs at 4.2K

Type	Substrate	Flux modulated and flux locked	Junction area (μm^2)	Loop area (mm^2)	SQUID inductance (nH)	Junction capacitance[c] (pF)	ε^w/1Hz measured (JHz^{-1})	ε^w/1Hz predicted[f] (JHz^{-1})	$\varepsilon^{1/f}$(1Hz)/1Hz measured (JHz^{-1})	$S_\Phi^{1/f}$(1Hz) measured ($\Phi_o^2 Hz^{-1}$)	γ	$S_{I_o}^{1/f}(1Hz)/I_o^2$ predicted[g] (Hz^{-1})
A[a]	fused quartz	Yes	1.1×10^4	7	1	880	3×10^{-30}	0.9×10^{-30}	2×10^{-31}	1×10^{-10}	0.09	11×10^{-10}
B	glass	Yes	10^2	0.25	1.2	8	2×10^{-31}	0.9×10^{-31}	2×10^{-31}	1×10^{-10}	-	
C	glass	Yes	10^2	0.25	1	8	(40×10^{-31})[d]	0.8×10^{-31}	6×10^{-31}	1.5×10^{-10}	-	
D	Si	Yes	6	0.04	0.4	0.5	5×10^{-32}	1.3×10^{-32}	1×10^{-30}	2×10^{-10}	-	
E1[b]	glass	No	6	6×10^{-6}	1.9×10^{-3}	0.3	$(3.6 \pm 3.0) \times 10^{-34}$[e]	3×10^{-34}	3×10^{-28}	3×10^{-10}	0.25	12×10^{-10}
E2[b]	glass	No	6	6×10^{-6}	2.5×10^{-3}	0.3	$(2.7 \pm 0.6) \times 10^{-34}$[e]	3×10^{-34}	3×10^{-29}	4×10^{-11}	0.15	3×10^{-10}

[a] ref. 14.

[b] ref. 4, SQUIDs A and B.

[c] We assume C = 0.08 and 0.04 pF/μm^2 for Nb-based and Pb-based junctions, respectively.

[d] This SQUID had a low critical current, and, consequently, an inferior white noise performance.

[e] The measured noise was never white: The estimated 1/f noise has been subtracted from the total noise.

[f] $16k_B T(LC)^{1/2}$ for A-D; see ref. 15 for E.

[g] From Eq. (3.2) or (3.3).

FIGURE CAPTIONS

Fig. 1 Five types of SQUID listed in Table II: (a), (b), (c), (d), and (e) are types A, B, C, D, and E, respectively.

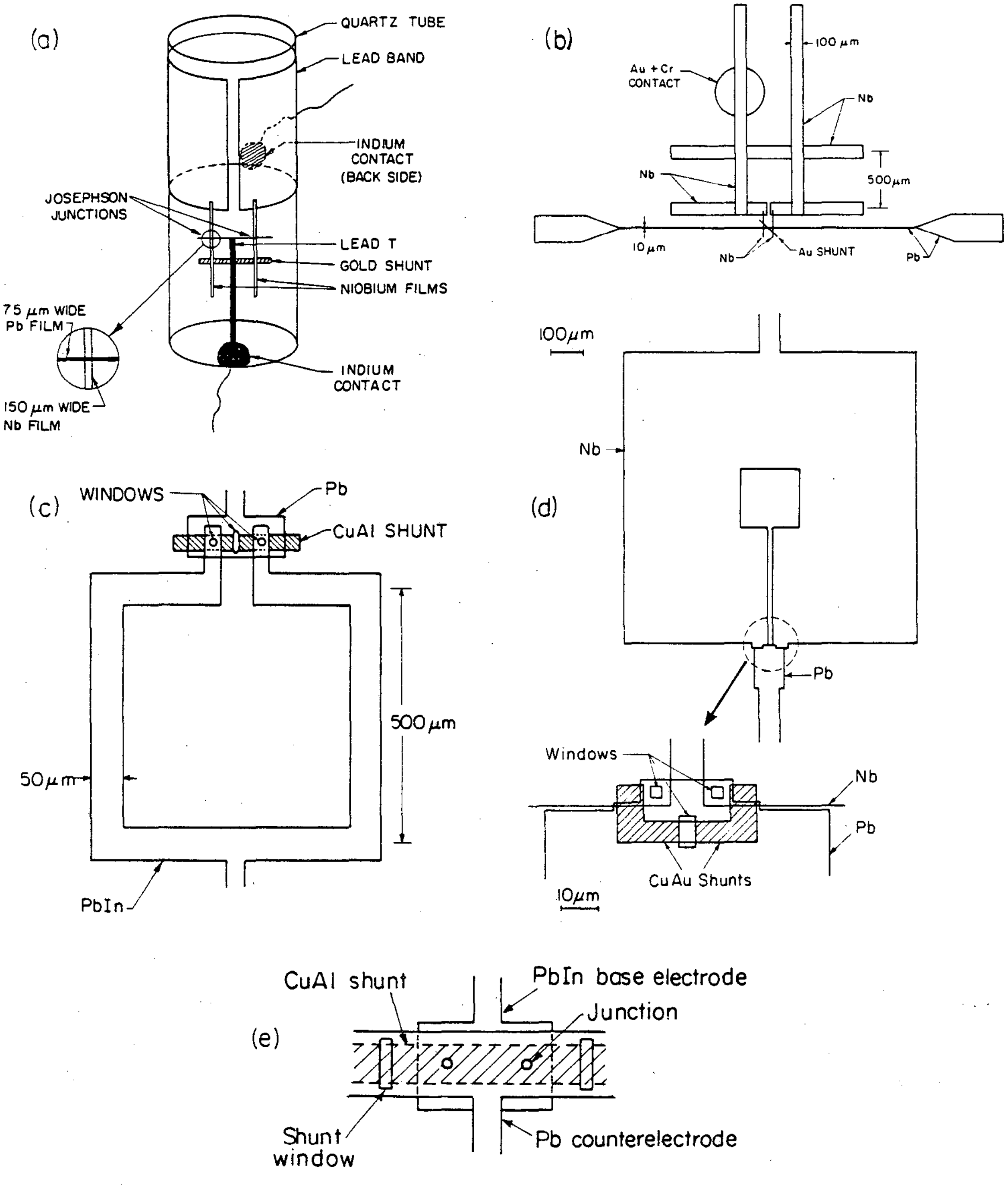
Fig. 2 (a) Principles of method A, in which the bias current is reversed and, simultaneously, the flux applied to the SQUID is changed by $\Delta\Phi_m = \Phi_o/2$. The left-hand column (solid lines) shows the V-Φ curves, while the dashed lines indicate the effect of an external flux change $\delta\Phi$ (upper) and a flux change due to a change in the critical current asymmetry $\delta\alpha$ (lower). The right-hand column shows as a function of time, t, (top to bottom) the 100 kHz-flux modulation, Φ_m, the bias current, I, the 100 kHz-reference used to lock-in detect the signal from the SQUID, V_r; the next three lines are for the case $\delta\Phi$, and show the voltage across the SQUID, V, the voltage across the tuned transformer, V_t, and the output of the lock-in detector, V_ℓ; the last three lines show the same voltages for the case $\delta\alpha$. Method B, in which the bias current is reversed, with $\Delta\Phi_m = 0$ is shown in (b); the notation is as for (a).

Fig. 3 Measurement circuit for bias current modulation methods. The cryogenic components are enclosed in the dashed box. The primary and secondary inductances of the transformer are 17 μH and 2.3 mH, and the tuning capacitor across the secondary coil is 1.2 nF.

Fig. 4 Spectral density of flux noise for a SQUID of type D at 4.2K and 1.6K using method A with $\Phi = n\Phi_o/2$. Also shown for comparison is the noise measured at 4.2K and 1.6K in the conventional flux-locked mode.

Fig. 5 Spectral density of flux noise for a SQUID of type D at 4.2K with

$\Phi = n\Phi_o/2$ using methods A (dashed curve) and B (dotted curve). Also shown for comparison is the noise as measured at the output of the lock-in detector when the SQUID is biased with a constant current and flux-modulated at 100 kHz (solid curve).



XBL 828-6359

Fig. 1

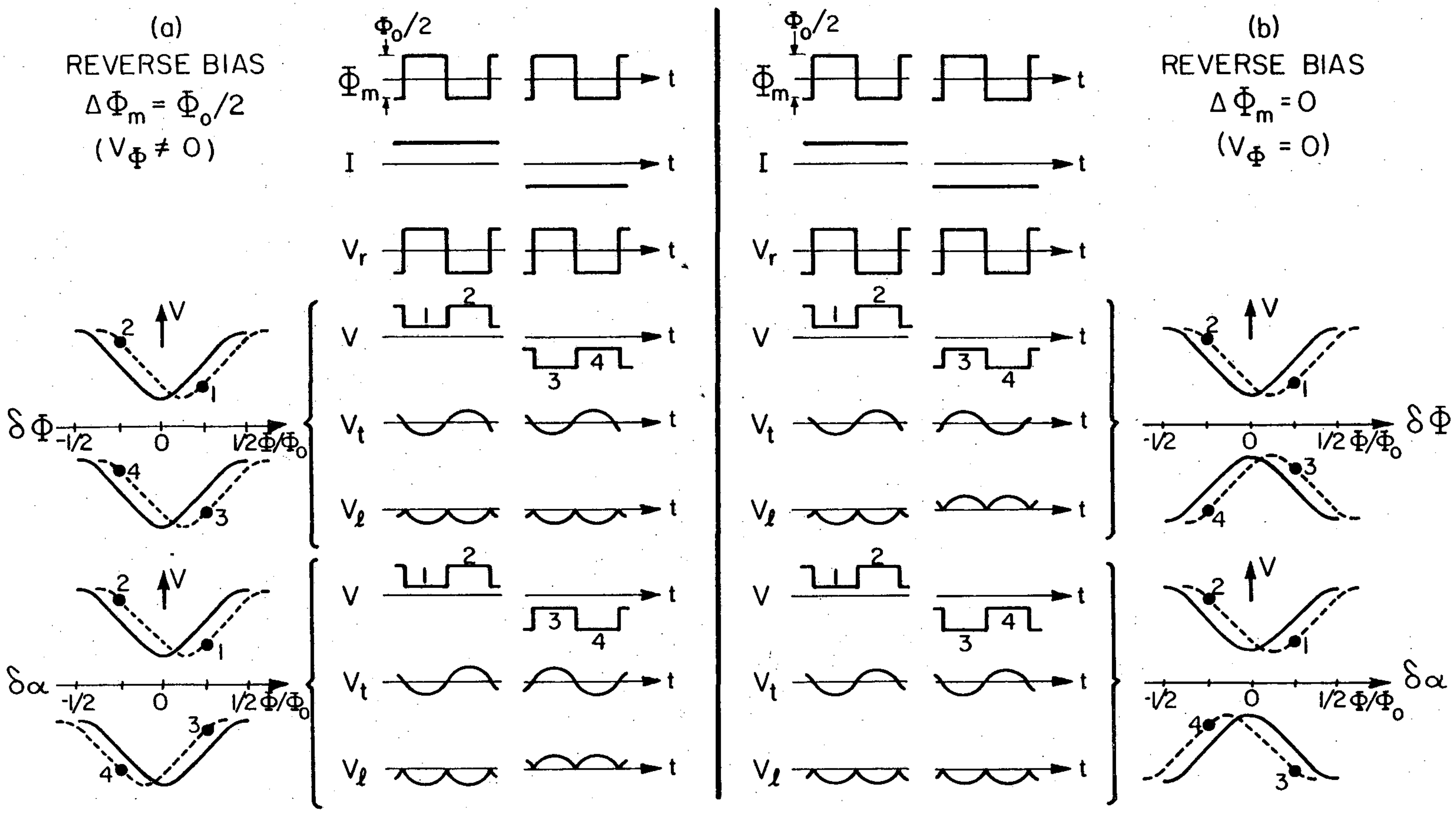


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Fig. 2

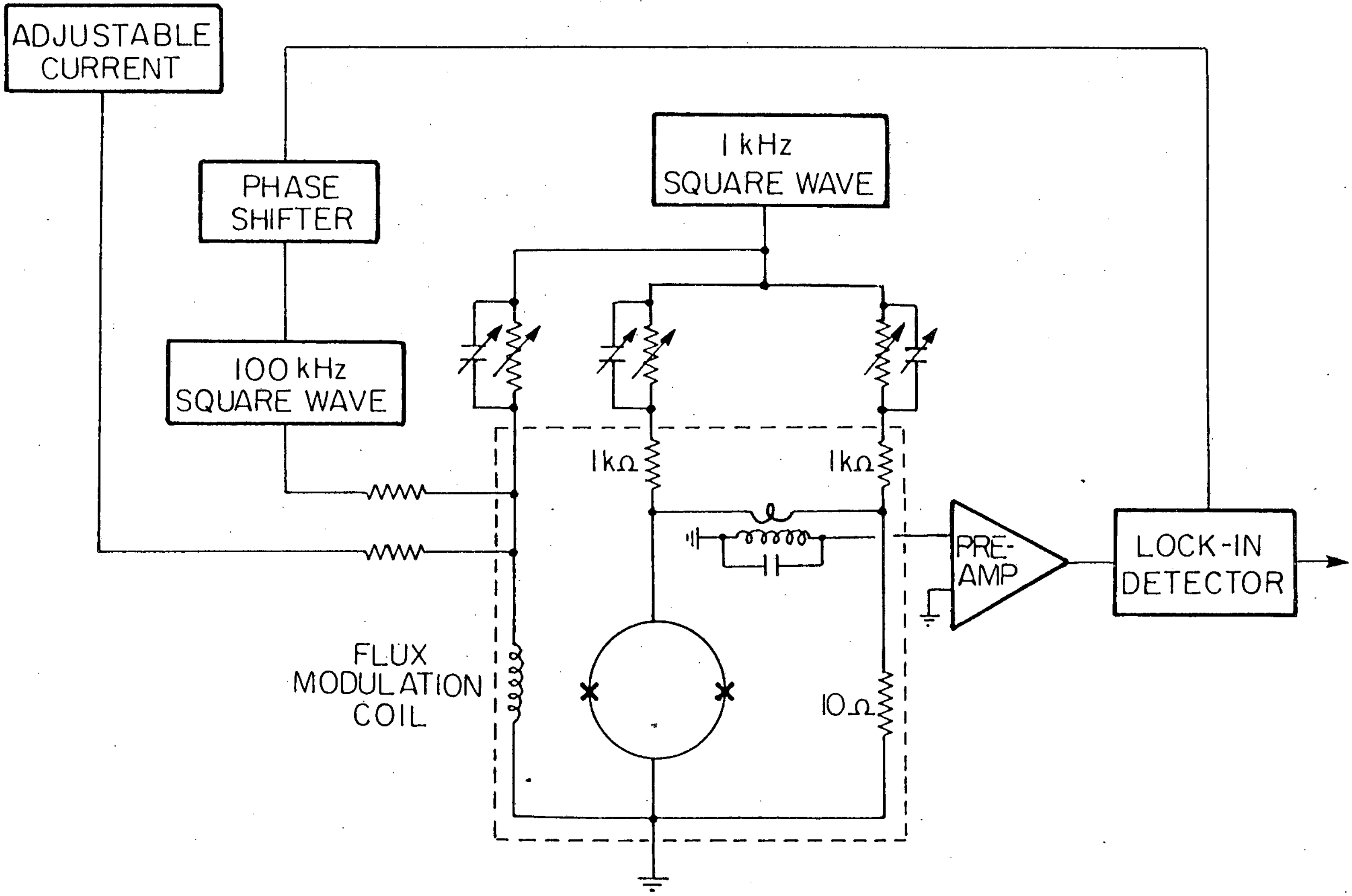

Fig. 3

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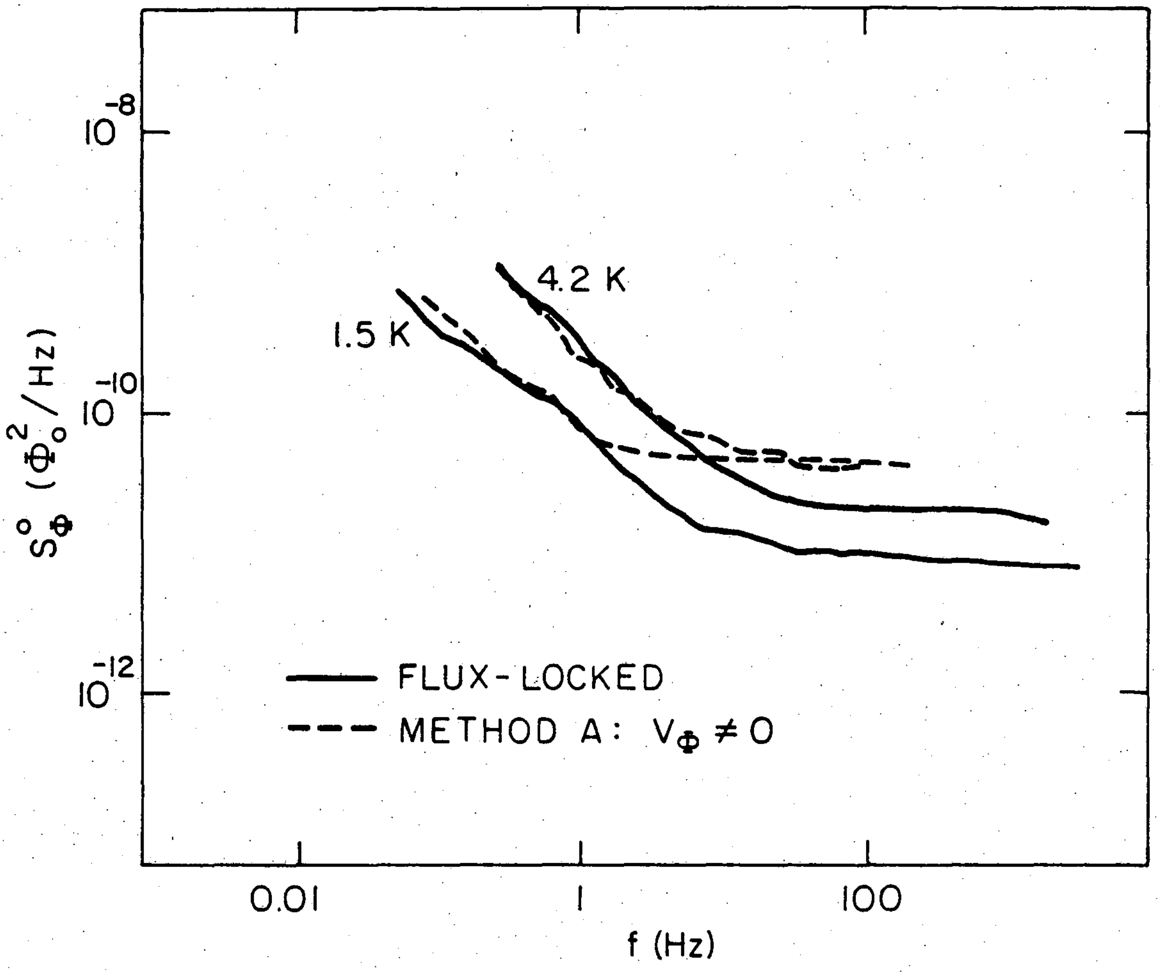

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Fig. 4

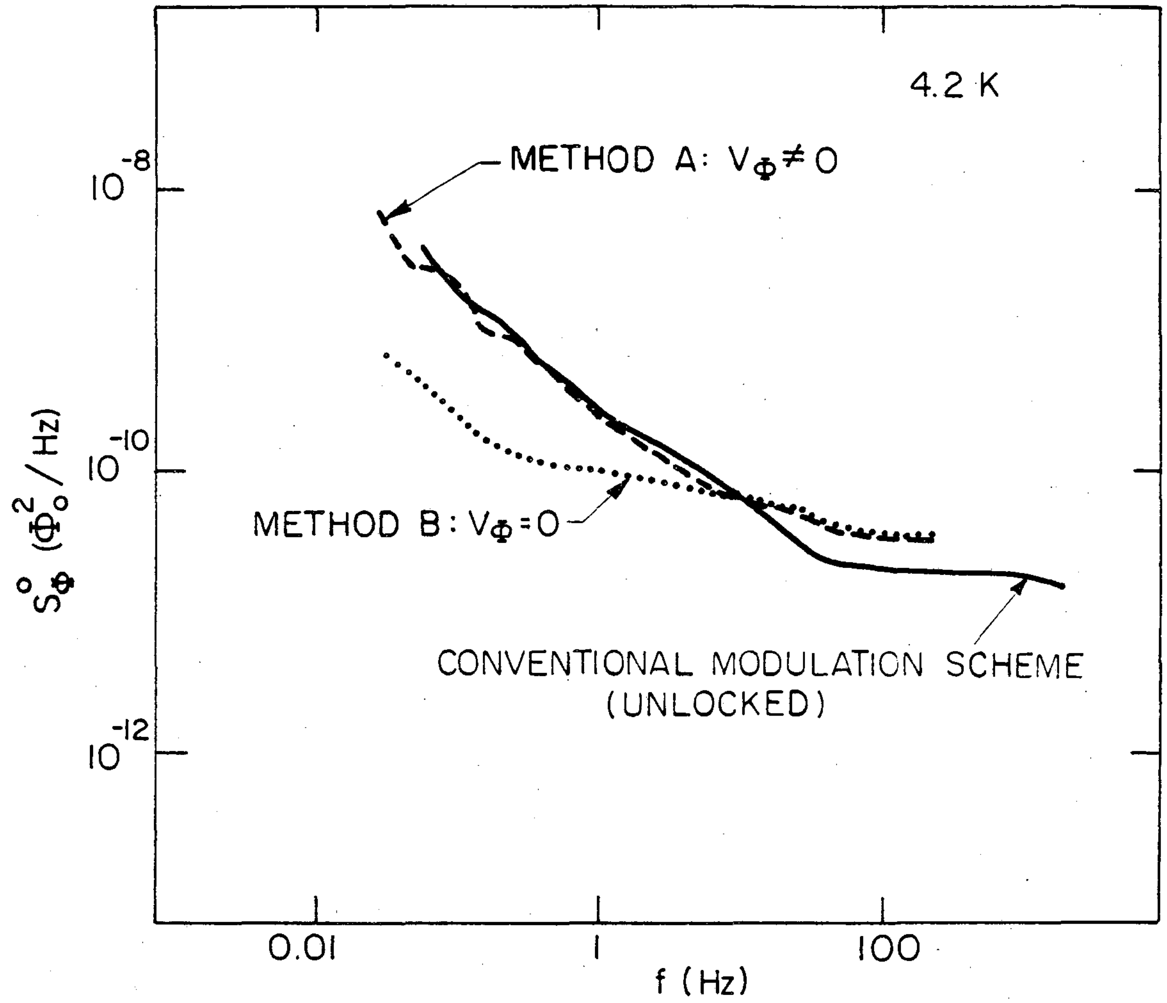

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Fig. 5

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