Lawrence Berkeley National Laboratory
Recent Work

Title

Multifragmentation: Surface Instabilities or Statistical Decay?

Permalink

https://escholarship.org/uc/item/1zg2249n

Authors

Moretto, L.G.
Tso, K.
Delis, D.
et al.

Publication Date

1992-11-01

Lawrence Berkeley Laboratory
UNIVERSITY OF CALIFORNIA

Multifragmentation: Surface Instabilities or Statistical Decay

L.G. Moretto, K. Tso, D. Delis, N. Colonna, and G.J. Wozniak

November 1992

DISCLAIMER

MULTIFRAGMENTATION: SURFACE INSTABILITIES OR STATISTICAL DECAY?

L.G. Moretto, K. Tso, D. Delis, N. Colonna, and G.J. Wozniak

Nuclear Science Division, Lawrence Berkeley Laboratory
University of California, Berkeley, California 94720, USA

November, 1992

MULTIFRAGMENTATION: SURFACE INSTABILITIES OR STATISTICAL DECAY?

L.G. MORETTO, K. TSO, D. DELIS, N. COLONNA and G. J. WOZNIAK

Nuclear Science Division, Lawrence Berkeley Laboratory,
1 Cyclotron Road, Berkeley, CA 94720

ABSTRACT

Boltzmann-Nordheim-Vlasov calculations show multifragmentation that seems to originate from surface instabilities. These instabilities are traced to a sheet instability caused by the proximity interaction. Experimental data, on the other hand, suggest that multifragmentation may be dominated by phase space.

KEYWORDS

Multifragmentation; surface instability; statistical decay.

SURFACE INSTABILITIES IN HEAVY-ION COLLISIONS

Multifragmentation has been associated with the onset of the spinodal instability (Siemens, 1983, Bertsch & Siemens, 1983, Lopez & Siemens, 1984). This instability is associated with the transit of a homogeneous fluid across a domain of negative pressure, which leads to its breaking up into droplets of denser liquid embedded in a lower density vapor. Since the spinodal instability can occur in an infinite system, it can be called a bulk or volume instability.

There is another class of instabilities that may play an important role in multifragmentation, namely surface instabilities of the Rayleigh kind (Rayleigh, 1964). They are called capillary or surface instabilities because they depend on the presence of a surface endowed with surface tension.

We have observed these instabilities in numerically simulated heavy-ion collisions (Moretto et al., 1992) We simulated head-on collisions of two nearly symmetric heavy-ions using the Boltzmann-Nordheim-Vlasov (BNV) equation (which contains both mean field and collision terms) within a test particle approach in a full ensemble method (Colonna et al., 1992), with each nucleon being represented by 40 test particles. (A value of 40 was chosen since the results were stable against a further increase.) In these calculations, we noticed two interesting features. First, during the collision process a "disk" develops due to the side-squeezing of nuclear matter, whose thickness decreases and diameter increases monotonically with increasing bombarding energy. Second, if the disk becomes sufficiently thin, it breaks up into several fragments of a size commensurate with the thickness of the disk.

Some of these features are shown in Figs. 1 & 2 for head-on collisions of two ^{90}Mo nuclei at two bombarding energies and two extreme values of the incompressibility constant K. In these figures, the front and side-views of the colliding systems are shown in the rows (a - d) corresponding to four different times: t = 20, 60, 120, and 180 fm/c, respectively.
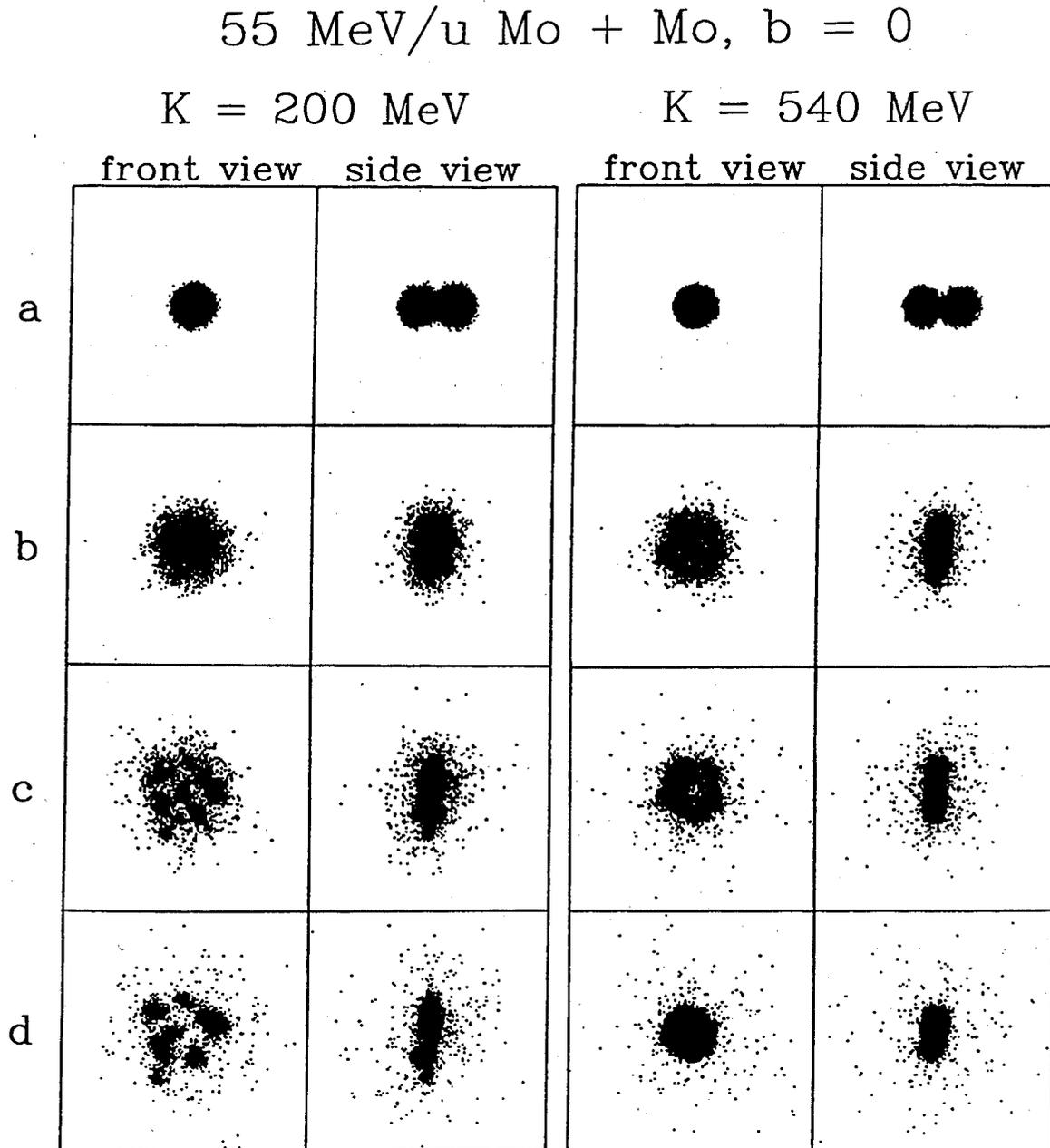

Figure 1. BNV calculations for a head-on collision (b = 0) of the 55 MeV/u ^{90}Mo + ^{90}Mo reaction at time steps of (a) 20, (b) 60, (c) 120, and (d) 180 fm/c. The front and side-views of the colliding systems are given in columns 1 & 2, respectively for a value of the incompressibility constant, K = 200 MeV. Similar views are shown in columns 3 & 4 for K = 540 MeV.

For K = 540 MeV and the lowest bombarding energy, a thick disk forms and some mottling develops at its maximum extension (incipient fragment formation). Notice, however, the donut formed at t = 120 fm/c. However, the mottling heals and the disk falls back to an approximately spherical blob. At higher bombarding energy, the disk becomes thinner, with a larger diameter than in the previous case. As the collision progresses, the mottling appears and develops rapidly into a crown of many fragments of approximately the same size, that slowly separate due to the residual kinetic energy of the disk and their mutual Coulomb repulsion. In some cases, two or more of these proto-fragments coalesce into a larger fragment (see for example Fig. 2, column 3).
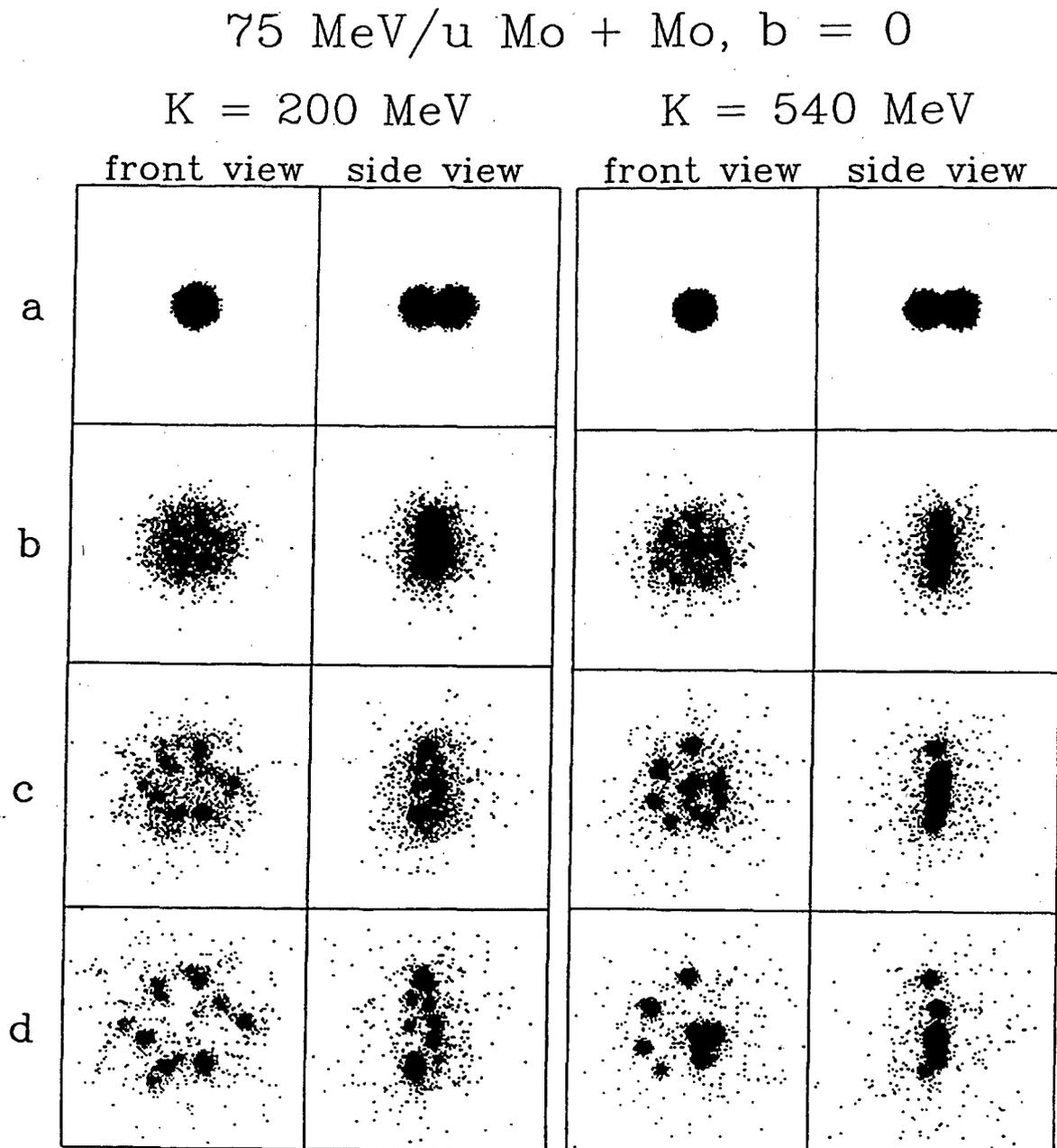
Figure 2. Same as Fig. 1 for the 75 MeV/u ^{90}Mo + ^{90}Mo system.

We repeated the calculations for K = 200 MeV to cover the range of nuclear incompressibility currently believed appropriate for nuclear matter. At 55 MeV/u and K = 200 MeV, a thin disk is formed and fragment formation occurs, in contrast to the high incompressibility case where fragment formation does not occur as yet. At higher bombarding energies, fragment formation is observed for both values of the incompressibility. However, for the high incompressibility cases, the disks are much sharper, and the mottling and fragment formation stand out more clearly. Similar calculations have been performed for a range of central impact parameters and entrance-channel mass asymmetries with similar results.

We are aware of other calculations with different codes. It has been brought to our attention that in some cases rather thin skinned bubbles are formed (Bauer *et al.*, 1992, Borderie *et al.*, 1992, Gross *et al.*, 1992, Xu *et al.*, 1992). It appears that these striking objects break up much in the same way as the disks described above, possibly due to the same instabilities.

Metastability of a Sheet of Liquid

The overall appearance of the disk fragmentation strongly suggests that it is caused by surface instabilities. *More precisely, the system escapes from the high surface energy of the disk by breaking up into a number of spherical fragments with less overall surface.* Thus, fragment formation, in this picture depends only on the presence of a surface energy term. Multi-nucleon correlations, which are commonly thought to be essential for fragment formation, are actually not necessary beyond their macroscopic manifestation through the surface energy. Incidentally, the very same observation can be made for volume instabilities.

The observed instability may be akin to the Rayleigh instability of a cylinder of liquid. The cylinder is unstable with respect to small perturbations of wave length $\lambda \geq 2\pi R$, where R is the radius of the cylinder. But, is a disk of liquid, or more generally, a sheet of liquid truly unstable?

If we assume sharp surfaces (no surface thickness, no surface-surface interaction), a sheet can be metastable with respect to a break-up into a layer of cylinders or spheres. The onset of metastability for both cases is easily calculated. On a sheet of thickness d let us identify stripes of width λ. These stripes can favorably collapse into cylinders when the surface area of a stripe (top + bottom) is greater than the surface area of the cylinder of equivalent volume. This occurs for $\lambda \geq \pi d$. Similarly, if the sheet is tiled with squares of side λ, the squares can favorably collapse into spheres when $\lambda \geq 3/2 \ (2\pi)^{1/2}d$.

These conditions refer to metastability and not necessarily to instability. Clearly, any wave of infinitesimal amplitude A increases the surface area of the sheet, independent of the sheet thickness. The dimensionless surface energy increase can be trivially shown to be:

$$\Delta V_s \approx \frac{2\pi^2}{\lambda^2} A^2 + \text{higher order terms,} \tag{1}$$

where λ is the wavelength of the perturbation. Therefore a sheet with sharp surfaces is stable with respect to small perturbations of all finite wave-lengths. On the other hand, the systems portrayed in Figs. 1-2 develop what appears to be a genuine instability!

Instability of a Sheet of Liquid and Surface-Surface Interactions

Nuclear surfaces are not sharp, but diffuse, and they interact with each other through an interaction of finite range called also the proximity force (Blocki *et al.*, 1977), $\Phi(s)$, where s is the distance between surfaces. We can now calculate the incremental energy of a sheet subjected to a perturbation of wavelength λ and small amplitude A. The dimensionless proximity interaction is:

$$V_P = \frac{2}{\lambda} \int_0^\lambda \Phi(s)\, dx \sim \frac{2}{\lambda}\left(P(\lambda) + Q(\lambda)A^2 \right) \tag{2}$$

where

$$P(\lambda) = \int_0^\lambda \Phi_0(x)dx \quad \text{and} \quad Q(\lambda) = \int_0^\lambda \Phi_2(x)dx \tag{3}$$

with $s = d + 2A \sin kx$, Φ_0 and Φ_2 being the zeroth and second order coefficients of the Taylor expansions of $\Phi(A,x)$ about $A = 0$, and $k = 2\pi/\lambda$.

The overall energy increase is:

$$\Delta V = A^2\left(\frac{2\pi^2}{\lambda^2} + \frac{Q(\lambda)}{\lambda}\right) \tag{4}$$

Instability occurs when the coefficient of A^2 is zero or negative. Thus, the critical wavelength for the onset of the instability is given by the equation:

$$\lambda_c\, Q(\lambda_c) + 2\pi^2 = 0. \tag{5}$$

Any perturbation with $\lambda > \lambda_c$ is then unstable, namely it will grow spontaneously and indefinitely. Using for the proximity potential the expression in Blocki *et al.*, 1977, we obtain

$$\lambda_c = 1.10\, b\, \exp[2d/3b], \tag{6}$$

where b is the range of the proximity interaction.

When the thickness of the sheet becomes much greater than the range of the proximity interaction, the critical wavelength tends to infinity. This is the trivial result for infinitely sharp surfaces that was mentioned above. However, when the thickness of the sheet becomes comparable to the proximity range b, the critical wave-length decreases very rapidly. This result is quite interesting, because it applies in general to all liquids, and because it is, we believe, new.

Application to Simulated Nuclear Collisions

In general, it appears that the observed relationship between fragment size and disk thickness in the BNV calculations is consistent with Eq. 6, if one uses for b the zero temperature value of ~1 fm. But Eq. 6 gives only a lower bound for the instability range. It is clear that the disk must become thin enough to allow the critical wavelength to fit comfortably within the disk diameter.

Concerning the BNV calculations, two apparent puzzles may need clarification:

1)   What is the origin of the fluctuations that eventually lead to the observed instabilities?

2)   Why certain symmetries contained in the BNV equations (like cylindrical symmetries) are violated when the instabilities manifest themselves?

One answer to the first point is that higher multipole surface ripples can be generated naturally during the collision. These ripples eventually become unstable as the disk, or bubble becomes thinner, and lead ultimately to the disintegration of the object. Another possible answer to both points resides in the algorithmic noise (numerical approximations) associated with the solution of the equations. This noise is

responsible for both the fluctuations and the breaking of symmetries, which, due to the underlying instabilities, are inevitably amplified in the evolution of the system.

While algorithmic noise is quite effective in evidentiating instabilities, it is not physical. Therefore, one should not rely on it to generate distributions in fragment number, mass, etc.

EXPERIMENTAL EVIDENCE: DYNAMICS OR STATISTICS?

Recently, some experimental progress has been made, by isolating and characterizing what appear to be true multifragmentation sources formed in reverse kinematics reactions (Blumenfeld et al., 1991, Roussel-Chomaz et al., 1992). These sources are formed in a process akin to incomplete fusion, whereby one partner of the collision picks up, and fuses with, a variable portion of the other partner. Kinematically, it is possible to determine how much mass has been picked up and the excitation energy associated with the fused object. Surprisingly, these sources, once characterized as described above, undergo multifragment decay in a way that is singularly independent of the formation process. The observed branching ratios for binary, ternary, quaternary, and quinary decays seem to depend almost exclusively upon the excitation energy E of the fused object, and remarkably little upon the target-projectile combination or even the bombarding energy.

The next obvious question that we want to address is: what is the multi-fragmentation mechanism of these sources? In particular, is this decay controlled by dynamics, or by statistics?

How to Bring Forth Phase Space in the Data

The role of statistics in these reactions has been expounded in a variety of models (such as the liquid-gas-phase transition (Siemens, 1983, Bertsch & Siemens, 1983, and Lopez & Siemens, 1983), chemical equilibrium models (Gross et al., 1982 and Lopez & Siemens, 1983) or hybrid approaches, such as evaporation occurring simultaneously with dynamical expansion (Friedman, 1990), dynamics followed by statistical decay (Sneppen & Vinet, 1988, Leray et al., 1991, and Colonna et al., 1992), etc.). While these models, or approaches, may be well justified a priori, inevitable limitations may make their application to actual data somewhat problematic. In other words, while the models may be sound in their essence, they may be too schematic and thus unable to fit the data satisfactorily.

An alternative way of searching for statistical effects would be to examine the data themselves to see whether they contain signatures that may be brought forth without the help, or impediment, of any given model. Let us suppose that the hot nuclear system formed in the heavy ion reaction decays statistically, and that a barrier of some sort governs this decay. This is, of course, the case for binary decay. Alternatively, in the framework of the chemical equilibrium picture, one can consider the potential energy of each configuration as a barrier. It is conceivable that, in this picture, there might arise a hierarchy of "barriers" such that all the binary configurations would have barriers closer to each other than to those of the ternary configurations, and so on. Thus, let us assume that B_2, B_3, ... B_n are the average "barriers" associated with binary, ternary, and n-body decays. The decay probability for each channel should be proportional to the level density of the system $\rho(E)$ (dominated by the internal degrees of freedom) at an excitation energy equal to the available energy minus the barrier:

$$P_n(E) \propto \rho(E - B_n) \propto \exp 2[a(E - B_n)]^{1/2}. \tag{7}$$

For convenience, we want the ratio of the n-fold events to the binary events:

$$\ln(P_n/P_2) \propto -\sqrt{a/E} \, (B_n - B_2) \, . \tag{8}$$

Thus, a plot of $\ln(P_n/P_2)$ vs. $E^{-1/2}$ should give a straight a line.

6

This simple theoretical prediction has been empirically tested (Moretto et al., 1969) for the overall fission probabilities in the Pb region, and used to prove that the rapid rise in fission cross section in e⁻ induced fission of similar nuclei is due to statistics. Figure 3 illustrates the testing of the method and its application. In Fig. 3a the fission probability is plotted vs $E^{-1/2}$ for three α-induced reactions in an energy regime where compound nucleus formation is well established. The expected linear dependence is observed, and the slopes correlate quantitatively with the known fission barriers. In Fig. 3b a similar plot is shown for four e⁻ induced fission reactions. The energy dependence of the fission probability was extracted by unfolding the e⁻ induced fission cross sections from the virtual photon spectrum. The observed linear dependence and the correlation of the slopes with the fission barriers proved that the rise of the fission cross section with increasing e⁻ energy is a statistical effect arising from the phase spaces associated with the competing decay channels.
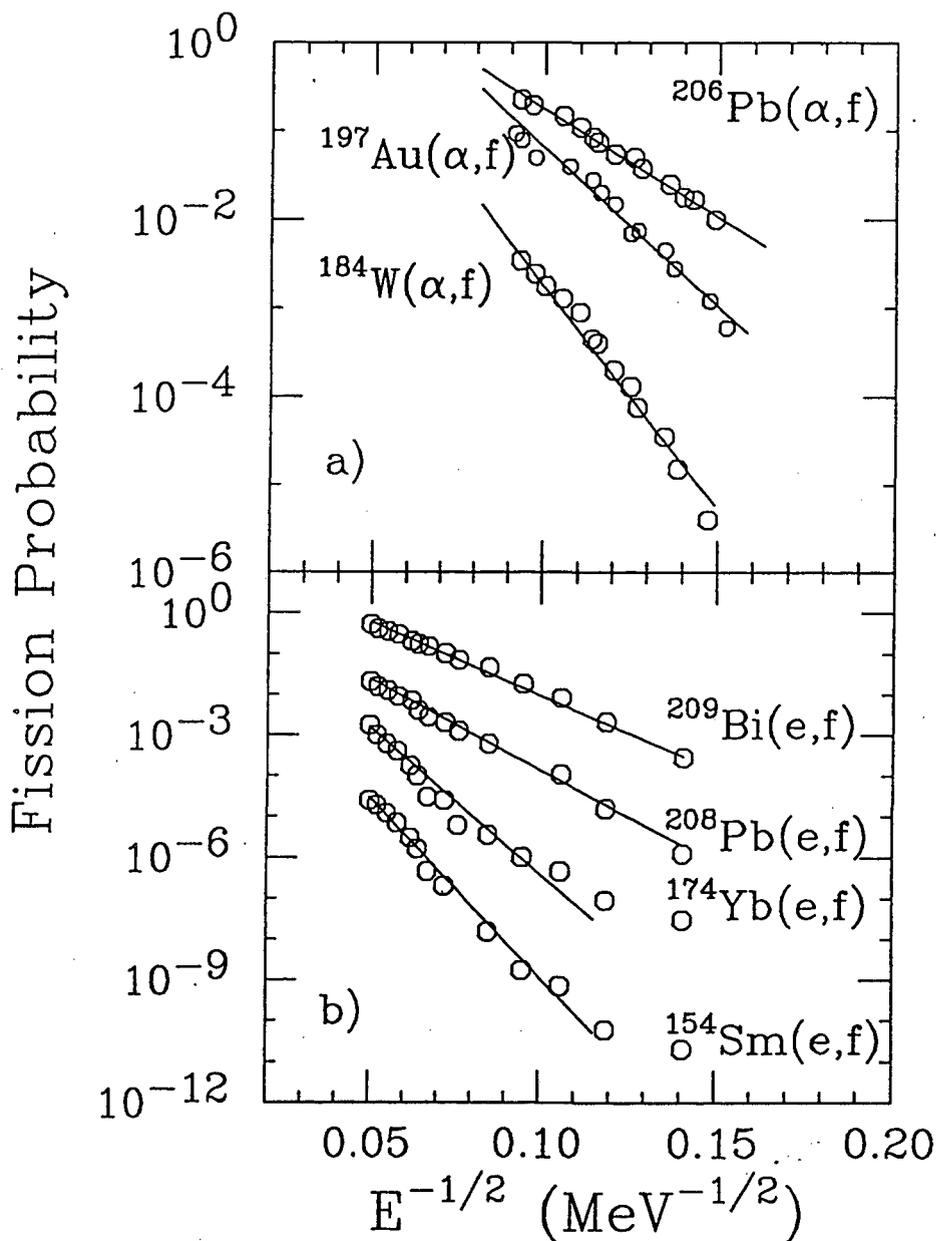

Fig. 3 The fission probability plotted as a function of $E^{-1/2}$ for the α-induced reactions 206Pb(α, f), 197Au(α, f), and 184W(α, f) and b) for the electron-induced reactions 209Bi(e, f), 174Yb(e, f), and 154Sm (e,f). The data are taken from Moretto et al., 1969.

7

In order to see whether a similar dependence exists in the multifragmentation branching ratios, we have studied the reactions ^{197}Au + ^{27}Al, ^{51}V, ^{nat}Cu, & ^{197}Au at 60 MeV/u with the specific purpose of determining the multifragment branching ratios as a function of the excitation energy of the decaying source.
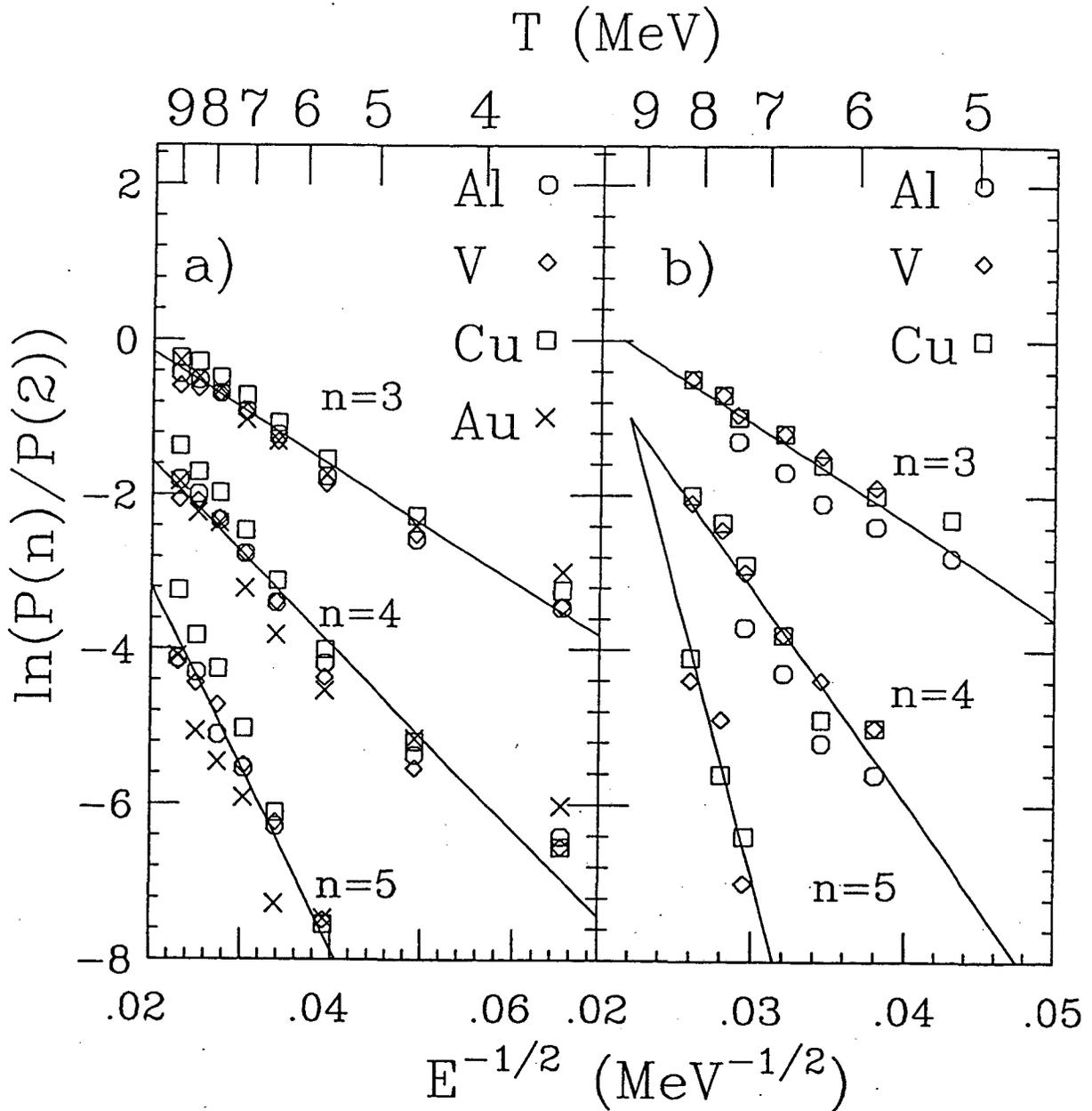

Fig. 4   a) The natural logarithm of the ratio of the 3, 4, and 5-fold to the 2-fold probability (symbols) as a function of E$^{-1/2}$ for the 60 MeV/u ^{197}Au + ^{27}Al, ^{51}V, ^{nat}Cu and ^{197}Au reactions. The lines are the best fits to the data. b) Same as in part a) for the 55 MeV/u ^{139}La + ^{27}Al, ^{51}V, and ^{nat}Cu reactions. The Au and La data are taken from Delis*et al.*, 1993 and Roussel-Chomaz *et al.*, 1992, respectively.

The decay of the hot nuclear systems formed in these reactions was studied, following closely the approach of Blumenfeld et al., 1991 by determining the ratio of the n-fold events (n = 3, 4, and 5) with respect to the 2-fold events as a function of the excitation energy E. In the incomplete-fusion model, the excitation energy is approximately related to the parallel component $V_s^{\parallel}$ of the source velocity V_s by $E = E_b (1 - V_s^{\parallel}/V_b)$ where E_b is the bombarding energy and V_b is the beam velocity.

The ratio of the n-fold to the 2-fold events was determined for different bins of the parallel source velocity and thus of the excitation energy of the source. By this procedure, we were able to obtain the probabilities for binary, ternary, quaternary and quinary decays as a function of the calculated excitation energy. The measured probabilities are not corrected for the detection efficiency. However, this efficiency has been shown to be insensitive to modest variations of the source velocity. Thus the uncorrected experimental probabilities P^* differ from those of Eq. 8 only by a term with a weak logarithmic dependence on energy. It follows that

$$\ln (P_n^*/P_2^*) = \ln[K(E)] - \sqrt{(a/E)} \ (B_n - B_2) . \tag{9}$$

Therefore, a plot of $\ln(P_n^*/P_2^*)$ vs. $E^{-1/2}$ should give an approximately straight line.

Multifragmentation data for the Au-induced reactions plotted in this manner are shown in Fig. 4. The first striking observation is that the data from all the reactions fall on the same curves. More specifically, once the multifragmentation source is characterized in terms of the kinematically determined excitation energy, the branching ratios for the various multifragment channels seem to be fixed and independent of the specific reaction that has produced the source. This decoupling between entrance and exit channels suggests a "statistical" kind of decay. This statistical feature is brought forth by the $E^{-1/2}$ plot, that indeed generates beautiful straight lines. The data from 55 MeV/A La induced reactions also show the same behavior. We believe that the observed linear dependence for both the Au- and La-induced reactions is a strong signature for processes controlled by phase space.

Qualitatively this signature cannot differentiate between the various statistical models. Equation 8 has been derived for a statistical multifragmentation process. It is immaterial whether we refer to a transition-state or a "freeze-out" equilibrium model. In the former case, B_n is the barrier to be crossed in order to reach an n-body decay configuration. In the latter case, B_n is the "potential energy" of the n-body system at the freeze-out configuration. The same dependence can be obtained for sequential decay.

In conclusion, the experimental evidence presented above suggest the following picture for multifragmentation:

1)   The dynamics of the reaction seems to be limited to the formation of a source of a given mass, energy, and angular momentum through a mechanism similar to incomplete fusion.

2)   Once this source is formed, its decay is apparently independent of the mode of its formation.

3)   The branching ratios between the various multifragmentation channels are dictated by the available phase space as shown by the excitation functions.

4)   The qualitative features of the excitation functions do not permit distinguishing between a sequential or simultaneous decay mechanism, but the quantitative features may contain relevant information in this regard.

9

REFERENCES

Bauer, W., G. Bertsch, and H. Schulz (1992). Bubble and Ring Formation in Nuclear Fragmentation. *Phys. Rev. Lett.* 69, 1888.

Bertsch, G. and P. J. Siemens (1983). *Phys. Lett.,* 126B, 9.

Blocki, J., J. Randrup, W. J. Swiatecki and C. F. Tsang (1977). *Ann. Phys.* 105, 427.

Blumenfeld, Y., N. Colonna, P. Roussel-Chomaz, D. N. Delis, K. Hanold, J. C. Meng, G. F. Peaslee, Q. C. Sui, G. J. Wozniak, L. G. Moretto, B. Libby, A. C. Mignerey, G. Guarino, N. Santoruvo, and I. Iori (1991). Multifragment Events From Heavy-Ion Collisions: Sources and Excitation Functions. *Phys. Rev. Lett.* 66, 576.

Boderie, B., B. Remaud, M. F. Rivet and F. Sebille (1992). Coulomb Instability in Collisions Between Very Heavy Nuclei Around 30 MeV per Nucleon. Preprint No. IPNO-DRE-92-22.

Colonna, M., P. Roussel-Chomaz, N. Colonna, M. Di Toro, L. G. Moretto, and G. J. Wozniak (1992). Dynamics and Statistics in Multifragment Production. *Phys. Lett.,* B283, 180.

Delis, D.N., Q. Sui, N. Colonna, K. Tso, K. Hanold, M. Justice, G.J. Wozniak, L.G. Moretto, B. Libby, A.C. Mignerey, A. Pantaleo, G. D'Erasmo, L. Fiore, E.M. Fiore, I.Iori, and A. Moroni, (1992). To be published.

Friedman, W. A. (1990). Rapid Massive Cluster Formation. Phys. Rev. C42, 667.

Gross, D.H.E.*et al.,* (1982). Z. Phys. A309, 41.

Gross, D. H. E. , Bao-An Li, and A. R. DeAngelis (1992). Donuts in Nuclear Fragmentation. Hans Meitner Institut, Berlin preprint.

Leray, S., C. Ngo, P. Bouissou, B. Remaud and F. Sebille (1991). Onset of Multifragmentation in the O + AgBr System Between 50 and 200 MeV/u. Nucl. Phys. A511, 414.

Lopez, J. A. and P. J. Siemens (1984). *Nucl. Phys.,* A431, 728.

Lord Rayleigh (1964). Article 58, *"Scientific Papers"*, Dover Publication, Inc, New York, p. 361.

Moretto, L. G., R. C. Gatti, S. G. Thompson, J. T. Routti, J. H. Heisenberg, L. M. Middleman, M. R. Yearian, and R. Hofstadter (1969). Electron- and Bremsstrahlung-Induced Fission of Heavy and Medium-Heavy Nuclei. Phys. Rev. 179 1176.

Moretto, L. G., Kin Tso, N. Colonna, and G. J. Wozniak (1992). New Rayleigh-Taylor-Like Surface Instability and Nuclear Multifragmentation. *Phys. Rev. Lett.,* 69, 1884.

Roussel-Chomaz, P., N. Colonna, Y. Blumenfeld, B. Libby, G. F. Peaslee, D. N. Delis, K. Hanold, M. A. McMahan, J. C. Meng, Q. C. Sui, G. J. Wozniak, L. G. Moretto, H. Madani, A. A. Marchetti, A. C. Mignerey, G. Guarino, N. Santoruvo, and I. Iori (1992). Complex Fragment Production and Multifragmentation in La-induced Reactions at 35, 40, 45 and 55 MeV/u. *Nucl. Phys.,* A, accepted, LBL-31310.

Siemens, P. J. (1983). *Nature* 305, 410.

Sneppen, K. and L. Vinet (1988). Dynamics and Clusterization in Nuclear Collisions. *Nucl. Phys.* A480, 342.

Xu, X. M., J. B. Natowitz, C. A. Gagliardi, R. E. Tribble, C. Y. Wong, W. G. Lynch, and P. Danielewicz (1992). Formation and Decay of Toroidal and Bubble Nuclei and the Nuclear Equation of State. Texas A & M preprint.