Correction to "Multicast Networks with Variable-Length Limited Feedback"

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Abstract

This report corrects an error in the proof of Lemma 4 in [1].

The statement of Lemma 4 in Appendix D of [1] is correct; however, its proof in Appendix F has flaws. In its corrected form, the proof is presented as follows.

Lemma 4: For complex unit-norm vectors $\boldsymbol{u},\boldsymbol{v},\boldsymbol{w} \in \mathbb{C}^{t\times 1}$, we have

$$\left||\boldsymbol{u}^\dagger \boldsymbol{v}|^2 - |\boldsymbol{u}^\dagger \boldsymbol{w}|^2\right| \le \sqrt{1-|\boldsymbol{v}^\dagger \boldsymbol{w}|^2}. \tag{45}$$

Proof: Let $\boldsymbol{G} \triangleq \boldsymbol{v}\boldsymbol{v}^\dagger - \boldsymbol{w}\boldsymbol{w}^\dagger$ and $z \triangleq \boldsymbol{v}^\dagger \boldsymbol{w}$. It can be verified (after some tedious but straightforward calculations) that $\boldsymbol{G}$ admits the decomposition

$$\boldsymbol{G} = \sqrt{1-|z|^2}\left(\boldsymbol{u}_1\boldsymbol{u}_1^\dagger - \boldsymbol{u}_2\boldsymbol{u}_2^\dagger\right),$$

where

$$\begin{aligned} \boldsymbol{u}_1 &= \alpha\boldsymbol{v} - \beta\boldsymbol{v}_0 \exp\left(-j\angle z\right), \\ \boldsymbol{u}_2 &= \beta\boldsymbol{v} + \alpha\boldsymbol{v}_0 \exp\left(-j\angle z\right) \end{aligned}$$

are orthonormal vectors with

$$\begin{aligned} \boldsymbol{v}_0 &= \frac{\boldsymbol{w} - \boldsymbol{v}\boldsymbol{v}^\dagger\boldsymbol{w}}{\sqrt{1-|z|^2}}, \\ (\alpha,\beta) &= \left(\sqrt{\frac{1+\sqrt{1-|z|^2}}{2}}, \sqrt{\frac{1-\sqrt{1-|z|^2}}{2}}\right). \end{aligned}$$

We can then obtain

$$\begin{aligned} \left||\boldsymbol{u}^\dagger \boldsymbol{v}|^2 - |\boldsymbol{u}^\dagger \boldsymbol{w}|^2\right| &= \left|\boldsymbol{u}^\dagger \boldsymbol{G}\boldsymbol{u}\right| \\ &= \sqrt{1-|z|^2}\left||\boldsymbol{u}^\dagger \boldsymbol{u}_1|^2 - |\boldsymbol{u}^\dagger \boldsymbol{u}_2|^2\right| \\ &\le \sqrt{1-|z|^2}\left(|\boldsymbol{u}^\dagger \boldsymbol{u}_1|^2 + |\boldsymbol{u}^\dagger \boldsymbol{u}_2|^2\right) \\ &\le \sqrt{1-|z|^2}\,||\boldsymbol{u}||^2 \\ &= \sqrt{1-|z|^2}. \end{aligned}$$

This concludes the proof. ■

References

[1] X. Liu, E. Koyuncu, and H. Jafarkhani, "Multicast networks with variable-length limited feedback," *IEEE Trans. Wireless Commun.*, vol. 14, no. 1, pp. 252–264, Jan. 2015.