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Correction to ‘‘Multicast Networks with Variable-Length Limited Feedback’’

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Abstract

This report corrects an error in the proof of Lemma 4 in [1].

The statement of Lemma 4 in Appendix D of [1] is correct; however, its proof in Appendix F has flaws. In its corrected form, the proof is presented as follows.

Lemma 4: For complex unit-norm vectors $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{C}^{r \times 1}$, we have

$$||\mathbf{u}^\dagger \mathbf{v}|^2 - |\mathbf{u}^\dagger \mathbf{w}|^2| \leq \sqrt{1 - |\mathbf{v}^\dagger \mathbf{w}|^2}. \quad (45)$$

Proof: Let $\mathbf{G} \triangleq \mathbf{v}\mathbf{v}^\dagger - \mathbf{w}\mathbf{w}^\dagger$ and $z \triangleq \mathbf{v}^\dagger \mathbf{w}$. It can be verified (after some tedious but straightforward calculations) that \mathbf{G} admits the decomposition

$$\mathbf{G} = \sqrt{1 - |z|^2} (\mathbf{u}_1 \mathbf{u}_1^\dagger - \mathbf{u}_2 \mathbf{u}_2^\dagger),$$

where

$$\begin{aligned} \mathbf{u}_1 &= \alpha \mathbf{v} - \beta \mathbf{v}_0 \exp(-j\angle z), \\ \mathbf{u}_2 &= \beta \mathbf{v} + \alpha \mathbf{v}_0 \exp(-j\angle z) \end{aligned}$$

are orthonormal vectors with

$$\begin{aligned} \mathbf{v}_0 &= \frac{\mathbf{w} - \mathbf{v}\mathbf{v}^\dagger \mathbf{w}}{\sqrt{1 - |z|^2}}, \\ (\alpha, \beta) &= \left(\sqrt{\frac{1 + \sqrt{1 - |z|^2}}{2}}, \sqrt{\frac{1 - \sqrt{1 - |z|^2}}{2}} \right). \end{aligned}$$

We can then obtain

$$\begin{aligned} ||\mathbf{u}^\dagger \mathbf{v}|^2 - |\mathbf{u}^\dagger \mathbf{w}|^2| &= |\mathbf{u}^\dagger \mathbf{G} \mathbf{u}| \\ &= \sqrt{1 - |z|^2} ||\mathbf{u}^\dagger \mathbf{u}_1|^2 - |\mathbf{u}^\dagger \mathbf{u}_2|^2| \\ &\leq \sqrt{1 - |z|^2} (|\mathbf{u}^\dagger \mathbf{u}_1|^2 + |\mathbf{u}^\dagger \mathbf{u}_2|^2) \\ &\leq \sqrt{1 - |z|^2} \|\mathbf{u}\|^2 \\ &= \sqrt{1 - |z|^2}. \end{aligned}$$

This concludes the proof. ■

REFERENCES

- [1] X. Liu, E. Koyuncu, and H. Jafarkhani, ‘‘Multicast networks with variable-length limited feedback,’’ *IEEE Trans. Wireless Commun.*, vol. 14, no. 1, pp. 252–264, Jan. 2015.