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The Pragmatic Demands of Mathematics: Examining Elementary School Students' Oral Language Use in Mathematical Explanations

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The Pragmatic Demands of Mathematics:

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A thesis submitted in partial satisfaction

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by

Anne Blackstock-Bernstein

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ABSTRACT OF THE THESIS

The Pragmatic Demands of Mathematics:

Examining Elementary School Students' Oral Language Use in Mathematical Explanations

by

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Master of Arts in Education

University of California, Los Angeles, 2014

Professor Alison L. Bailey, Chair

As new academic standards and assessments are being implemented in the majority of U.S. states in 2014, students are being required to communicate effectively about their mathematical understanding. While linguistic and discourse proficiencies in English (i.e., lexical, grammatical, and genre knowledge) are essential to participating in classroom discussions of mathematics, it is also necessary for students to make use of pragmatic skills in order to ensure that they are effectively communicating. In the current study, I examined the oral explanations of 126 3rd (n=65) and 5th (n=61) grade students who completed a mathematics activity and then explained the mathematical procedures they used. Analyses explored how the complexity of the mathematical procedure (i.e., how many steps were involved) affected the communicative competence of the student's explanation. Additional analyses considered how grade and the

English learner (EL) status of the 61 EL students in the sample might influence these relationships. Findings indicate that students who used more complex mathematical procedures struggled to orally communicate the details of their procedures more so than students who used simpler procedures. Younger students (3rd graders) and EL students may be more susceptible to these challenges. Implications for instruction in this era of new standards are considered.

The thesis of Anne Blackstock-Bernstein is approved.

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The effective communication of mathematical understanding will be essential to the academic success of U.S. students under new academic standards and assessments. These standards, which were adopted by the majority of U.S. states in 2014, require that students display their mathematical content knowledge through classroom discourse with instructors and peers. For instance, the Common Core State Standards stipulate that in order to achieve proficiency in math, students must be able to explain what a mathematics problem is asking, how to solve it, and why their solution makes sense (National Governors Association Center for Best Practices & Council of Chief State School Officers [CCSSO], 2010). For decades, similar guidelines for high-quality mathematics instruction have encouraged teachers to engage students in discussions about mathematics in order to build and solidify mathematics knowledge and gauge student understanding (Boaler, 2008; National Council of Teachers of Mathematics [NCTM], 1991). However, it is a challenging task for students to achieve the linguistic descriptiveness and precision that is required for mathematical explanations to be comprehensible to their listeners (Moschkovich, 2002) and this challenge is likely intensified for students whose primary home language is not English (Moschkovich, 2012). Since the introduction of the Common Core, there has been an effort to understand how the new cognitive and linguistic demands of mathematics classrooms might affect student performance, particularly for English learners (ELs; Bailey, 2013, April). The purpose of the current study is to add to this nascent understanding by examining how students, both EL and English proficient, meet the pragmatic demands of communicating mathematically.

English learners are students who have been identified by their schools as requiring additional English language supports in order to access academic content. The number of EL students in U.S. classrooms is increasing; in California alone there are 1.41 million EL students

in public schools, the majority of whom are in the elementary grades (California Department of Education [CDE], 2014). Teachers are held accountable for ensuring that EL students achieve proficiency not just in English language development, but also in content areas like math. In the last decade, there has been substantial research examining the effect of English language proficiency on students' performance on mathematics tasks and assessments (Abedi & Lord, 2001; Wright & Li, 2008). These studies have repeatedly demonstrated that EL students score lower on standardized mathematical measures than their non-EL counterparts, in part due to the linguistic complexity of test items (Abedi & Gándara, 2006; Martiniello, 2008). However, studies are only just beginning to address how the interaction between language proficiency and mathematics might play out as students are required to produce mathematical explanations in the classroom (Bailey, Blackstock-Bernstein, & Heritage, under review).

To participate fully in academic English discourse, students must be skilled in selecting the appropriate vocabulary terms, producing sentences using conventional English syntax, and organizing these sentences into a coherent and cohesive explanation. In addition, students must be able to meet the pragmatic demands of using the English language in conventional and expected ways in order to convey their content understanding (Bailey, 2012; Cazden, 2001). To accomplish this, students must identify what knowledge they share with the listener (either classmate or teacher) and then determine how much detail and descriptiveness is necessary to convey their meaning. As teachers increasingly use students' explanations to evaluate mathematical understanding and make instructional decisions, it is important for educators to understand what factors may interfere with or contribute to the communicative effectiveness of students' explanations (Bailey et al., under review).

The current study examines the mathematical procedures students used during a mathematics activity and the pragmatic features of their explanations about those procedures. The aim is to understand how the complexity of the mathematical procedures students use might in fact affect their abilities to use oral language to effectively explain their reasoning. The explanations that students produce serve a specific pragmatic purpose—to communicate the student's mathematical message clearly and concisely to a naïve listener. Therefore, a student's performance during this explanation can be assessed in terms of its communicative competence; that is, whether the student used language to convey their mathematical ideas in a way that is understandable to the listener (Cazden, John, & Hymes, 1972) and without too much or too little information (Grice, 1975). Analyses explore how the complexity of each student's mathematical procedure (i.e., how many steps were involved) affected the communicative competence of that student's explanation.

Background

The current study is guided by a sociocultural theoretical framework in which language and mathematical learning are viewed as social activities (Vygotsky, 1978). Learning about mathematics involves active participation in the classroom community, which requires engagement in classroom discourse (Moschkovich, 2002). There is substantial evidence that, in particular, communicating self-explanations to others is a valuable tool for mathematics learning (Chi, de Leeuw, Chiu, & LaVancher, 1994; Esmonde, 2009). Describing, explaining, and justifying a mathematical procedure to a teacher or classmate is a means through which students gain mathematical knowledge (Gersten & Baker, 2000; Secada, 1992). These verbal exchanges of information promote mathematical learning because they require students to organize and clarify their thoughts in order to express them to others (Rogoff, 1998). Students also fill in gaps

in their understanding and develop new perspectives following input from others (Webb & Mastergeorge, 2003). During these classroom interactions, students simultaneously internalize principles, recognize patterns, monitor their mathematical understanding, reflect on their thinking, and relate it to their existing knowledge about mathematics (Chi, Bassok, Lewis, Reimann, & Glaser, 1989; Franke et al., 2007).

The Common Core mathematics standards have acknowledged the educational importance of mathematical discourse by encouraging student explanations in standard classroom practice and by using student explanations to assess conceptual understanding (CCSSO, 2010). However, there are inherent challenges in providing the reflective explanations demanded by the Common Core, because people often lack awareness of their own mental processes (Wilson & Clarke, 2004). Even adults are often unable to accurately report how they solved problems (Nisbett & Wilson, 1977). Verbal reports are especially difficult for children, whose cognitive and linguistic abilities are not fully developed (Cavanaugh & Pelmutter, 1982).

Communicative Competence in Explanations

A student's ability to produce a concise and comprehensible explanation represents her communicative competence during that explanation. One aspect of communicative competence is sociolinguistic competence, which involves the speaker's ability to convey information in a way that is appropriate for the specific social context (Canale & Swain, 1980). Recently, Kelly and Bailey (2013) showed how authentic discourse practices frequently require that children navigate the intersection of two linguistic genres, for example, by telling narratives within the texts of a conversation. In the context of the mathematics discourse in the current study, students must embed their explanations within a conversation with a hypothetical classmate. Conversation requires that the speaker understand the function of the interaction, her role in this interaction,

and the knowledge that is shared by both the speaker and the listener (Canale & Swain, 1980). Explanation requires that the speaker organizes information clearly and provides sufficient detail and elaboration to convey a process or sequence of events (Bailey, 2012).

Therefore, in order for a student to achieve sociolinguistic competence during a mathematical explanation, she must use sufficient detail and descriptiveness to convey her knowledge about the mathematical procedure to a target audience who may only share portions of that knowledge (Grice, 1975; Ninio & Snow, 1996). This descriptiveness requires the selection of words that are precise and appropriate for the context (Nathan & Knuth, 2003), including the use of temporal discourse connectors (e.g., *then*, *next*, *until*, *when*) to link together the steps being explained. These connective words and phrases, which are a hallmark of academic English in general (Bailey, 2012), serve a pragmatic function by indicating to the listener when each step should occur in relation to others and thereby articulating a cohesive sequence of propositions (Fraser, 1999). The lexical precision gained by sufficient use of temporal discourse connectors could make it possible for a listener to replicate the student's procedure based solely on the student's verbal directions.

Mathematical Strategy and Explanations

In the classroom, students are expected to produce mathematical explanations that include a description of how they would solve a given problem and why they would use that particular procedure (Franke et al., 2007). During this process, students can often choose from a range of strategies and procedures that vary in complexity, speed, accuracy, and relative probability of success (Siegler & Jenkins, 1989).

As children progress through school, they learn and experiment with new mathematical procedures, and the relative frequency with which they use each procedure changes with time

(Siegler & Lin, 2009). According to Siegler and Lin's "overlapping-waves" model of development, some previously preferred procedures become less frequent, some become more frequent, and others fluctuate. Gradually, newer, more effective procedures replace older, inferior ones. For example, for solving an arithmetic problem, the procedure of counting on one's fingers is generally phased out as a child learns to retrieve arithmetic facts like *8 + 4 = 12*. Some mathematical procedures, including those that require more steps, involve more working memory—which stores and processes information—than others (Ashcraft & Krause, 2007; Ayres, 2001; Baroody, 1984; LeFevre, DeStefano, Coleman, & Shanahan, 2005).

It is possible that the cognitive demand of these varying mathematical procedures would affect students' abilities to explain them using oral language, because language production carries its own cognitive demands. While some of these demands are language-related (e.g., lexical retrieval and sentence formulation; Levelt, Roelofs, & Meyer, 1999; Bock, 1982), others are sociolinguistic and related to the student's pragmatic understanding of the knowledge they share with their audience and the expectations of the social context they are in (Hymes, 1972). Students must use this pragmatic understanding to produce explanations that are sufficiently detailed and precise for their audience to understand (Shatz & Gelman, 1973). In addition, some mathematical procedures—such as counting—lend themselves to internalized verbalization and may therefore be easier to articulate than others (Ginsburg et al, 1983). The present study examines how the language of students' mathematical explanations may vary depending on the mathematical procedures they use.

The Present Study

In order to understand how various mathematical procedures might be related to pragmatic language competencies, the present study will: 1) examine the communicative

competence of students' oral explanations of a mathematical procedure, 2) compare the communicative competence of students' explanations based on the mathematical procedures they chose to use, and 3) explore the influence of grade level and English learner (EL) status on the relationship between mathematical procedure and communicative competence. Specifically, the present study uses secondary data analysis to answer the following questions.

RQ1. What is the relationship between the complexity of the mathematical procedures that elementary school students use during a mathematics activity and the communicative competence of the students' oral explanations?

RQ1a. How does this relationship between mathematical procedure and communicative competence differ as a function of grade?

RQ1b. How does this relationship between mathematical procedure and communicative competence differ as a function of EL status?

I hypothesize that communicative competence will be adversely affected by the cognitive demand of more complex mathematical procedures. Because the cognitive demand of conducting the mathematics itself is increased for students who use more complex mathematical procedures, I hypothesize that the communicative competence of these students' explanations will decrease. Students who must use working memory to recall, explain, and justify a larger number of procedural steps will have fewer cognitive resources available for language production and will therefore be more likely to overlook some steps or fail to explain them fully. I hypothesize that this relationship will be especially strong for younger students and English learners, whose English language proficiency is still developing.

Method

The current study is a correlational investigation of the relationship between complexity of mathematical procedures and clarity of oral explanations, using secondary analysis of existing data that was collected as part of a larger longitudinal research project (the Dynamic Language Learning Progressions project, or DLLP) that focused on language development (Bailey & Heritage, 2014; Bailey, Kelly, Blackstock-Bernstein, Chang, & Heritage, 2014, April). Participants used a range of self-selected mathematical procedures to complete a mathematics task and then produced oral explanations about the procedures used. Explanations were analyzed for a number of lexical and pragmatic features.

Participants

Participants were 126 students in 3rd (*n*=65) and 5th grades (*n*=61), as shown in Table 1. Students were participants in the larger project, which recruited students in grades K–6 from five schools: two public elementary, one public charter elementary, one laboratory school at a public university, and one public charter primary center (K–1st grades) that did not contribute students for the current study. All schools were located in a large urban area in Southern California. Children and parents at all sites provided assent and consent, respectively, in accordance with University and school IRB procedures. All 3rd and 5th grade students who completed the mathematics activity at Time 1 of two time points were included in the current study.[1]

Participants were English learners (*n*=61) and monolingual English or English proficient students (*n*=65). English proficient students were either native English speakers or students who had previously been redesignated from EL to Fluent English Proficient (i.e., RFEP, or former

[1] Students at other grade levels were not included in the current study due to a lack of data that was essential to the investigation (e.g., developmental adaptations to the mathematics task resulted in very little diversity in the mathematical strategies used by kindergarteners; a limited number of EL students were recruited at the remaining grade levels).

EL) by their school district. All EL students were native Spanish-speakers, according to testing data and teacher reports. Of the students designated as English learners by the California English Language Development Test (CELDT[2]; *n*=55), the majority (80%) were at *Intermediate* or above levels of overall English proficiency according to the test. Students (*n*=53)[3] had an average scale score of 515.25 (*SD* = 45.90) out of 761.

Table 1

Student Demographics

Demographic	Total (*n*=126)	English proficient (*n*=65)[a]	English learner (*n*=61)
Gender			
Female	68	28	40
Male	58	37	21
Grade			
3rd grade	65	30	35
5th grade	61	35	26

[a] Includes monolingual English speakers and former EL students

The mean age at the time of data collection was 8 years; 7 months for 3rd grade and 10 years; 8 months for 5th grade. According to school records, the ethnic background of participants was predominantly Latino (65.9%), followed by Caucasian (11.9%), Multi-racial/ethnic (10.3%), Asian (5.6%), African American (4.0%), and Other (2.4%).

Although socioeconomic information was not collected for individual students, schools were selected in part to attain socioeconomic diversity within the sample. At the charter elementary school, 99% of students were eligible for free or reduced-price lunch. The school's

[2] The CELDT is a standardized test administered during the fall of each year to all California K-12th grade public school students whose primary home language is not English (CDE, 2013a). The CELDT is not administered to students enrolled in independent schools, and thus CELDT data are not available for six EL students in the current study. The CELDT identifies students who are Limited English proficient (LEP) and determines the level of English language proficiency of these students in four domains: listening, speaking, reading, and writing. A student's scale score for each domain is used to determine a performance level (1 = *Beginning*, 2 = *Early Intermediate*, 3 = *Intermediate*, 4 = *Early Advanced*, and 5 = *Advanced*), and an overall performance level is calculated as the sum of all four domains. Once students are classified as RFEP by a set of district criteria, they no longer take the CELDT.

[3] Scale scores were not available for two students who took the CELDT.

2012-2013 Academic Performance Index (API) of 772 was lower than the California state average of 791. At one public school, 58% of students were eligible for free or reduced-price lunch. The school's 2012-2013 API of 785 was slightly lower than the California state average of 791. At the second public school, 35% of students were eligible for free or reduced-price lunch. The school's 2012-2013 API of 884 far exceeded the California state average of 791.

Although socioeconomic information was not collected for individual students, schools were selected in part to attain socioeconomic diversity within the sample (Bailey, Blackstock-Bernstein, & Chang, 2014). Three of the schools participate in the state's free or reduced-price lunch program, which serves as a proxy for SES; 35%, 58%, and 99% of students were eligible for the program at these three schools. These same schools participate in the state testing program and had Academic Performance Indices (APIs) of 772, 785, and 884, representing a range of performances in contrast to the California state average of 791. Median household income at the laboratory school, which does not participate in the state testing or student lunch programs, was provided in an income range category of $150,000–$199,999, with annual incomes ranging from $10,000 to over $1,000,000.

The majority of students performed at the *Advanced* (31%) or *Proficient* (32.5%) performance levels on the mathematics segment of their most recent standardized tests. On the CST-Mathematics (n=77), students had an average scale score of 375.32 (SD = 85.85) out of 600, which is classified as *Proficient*. On the Stanford 10-Mathematics (n=47)[4], students had an average scale score of 666.04 (SD = 48.82) out of 800, which is classified as *Proficient*. Students' scale scores were converted to z scores based on norming sample means and standard

[4] Mathematical testing data are not available for two students at the laboratory school.

deviations in order to combine results from both assessments. When combined, the distribution is fairly normal, with an average scale score just below the mean (*z* score of -0.07).

Procedures

Data collection. Data used for the current project were collected from November through February of the 2012-2013 academic year. Three female researchers were trained to administer the mathematics activity and student interviews. The task was designed to engender language that is inherent in mathematical reasoning as it is reflected in the Common Core State Standards (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). Specifically, the students were expected to interact with mathematical concepts of counting and cardinality and express their reasoning through both oral and written explanations (Bailey & Heritage, 2014). Sessions took place in English in a private room or hallway on school grounds during the school day. Sessions were recorded using audio equipment (digital voice recorders and smart phones) to create an audio file of each task session. Each student was presented with a quantity of Unifix cubes (plastic interlocking blocks) and asked to find the total number of cubes. This quantity differed by grade: 50 cubes for 3rd graders and 100 cubes for 5th graders to account for anticipated differences in cognitive demand across grades. Students were told to find the total using whatever method they wished.

After providing an answer, students were asked to respond to a series of oral and written questions about the procedure they used and their justification for that procedure. The current study focuses only on the culminating oral language question, which asked students to decontextualize their explanation by explaining it to a hypothetical naïve listener: *Pretend you are talking to a classmate who has never done this activity. When you're ready, tell him/her how*

to use the cubes to find out how many there are and why using the cubes this way helps him/her. Each session took between five and fifteen minutes for the full series of questions.

As the students used the cubes to complete the mathematics activity, interviewers completed a Mathematical Strategy Checklist based on their observations of the student's behavior (DLLP project, 2011). The checklist was designed to capture students' developmentally-driven counting behaviors and strategies. The following procedural elements were included: 1) whether the student grouped the cubes, 2) what number the student grouped them by, 3) whether the student connected the cubes, 4) whether the student pushed aside the cubes, 4) what method was used to calculate the total (repeated addition, multiplication, or counting), and 5) the student's final answer. Researchers were instructed to take detailed notes of what the student was doing with the cubes during the activity (e.g., whether the student grouped the cubes by color, the sequence of the student's procedural steps, etc.).

Data processing. Analyses for the current study were conducted on the transcripts available from the larger project, which were transcribed by a trained graduate student and then verified by an additional graduate student. For the automated natural language parsing employed in the larger project, a second version of each transcript was created to remove dysfluencies such as repetitions, false starts, retraces, abandoned utterances, and conversational topics that were unrelated to the mathematics task. These versions of the transcripts were used for the current investigation.

Following this verification process, a researcher independently transcribed a random selection (15%) of the audio files. Transcript accuracy was calculated by comparing the two versions of the transcripts at both the word and sentence level. Agreement was high—on average, 92.3 percent of word and sentence boundaries matched.

Measures

Performance during mathematical activity. The researcher's notes and observations of the student's procedure (as recorded on the DLLP Mathematical Strategy Checklist) were used to determine what the student did mathematically during the activity. These notes were then interpreted using two measures aimed at capturing different facets of students' performance—the mathematical strategies they used and the complexity of their procedures (based on the number of steps). During coding, researchers were blind to students' background characteristics, including grade, EL status, and mathematics proficiency.

Mathematical strategy. The mathematical strategy (counting, addition/repeated addition, or multiplication) used by each student was defined as the most sophisticated mathematical strategy the student implemented during the activity. For example, a student who used counting as part of a procedure that ultimately used multiplication was coded as using multiplication.

Complexity of mathematical procedure. Each student's procedure was also represented as a modular sequence of steps, based on Shrager and Siegler's SCADS model (Strategy Choice And Discovery Simulation; 1998) and Fuson's (1988) discussion of models of counting. The DLLP Mathematical Strategy Checklist was used to determine the number of steps each student used. Eight discrete possible steps could be identified: arranging cubes in a line (often by connecting them); grouping cubes; pushing aside counted cubes; counting cubes by one; counting the number of groups; counting the number of cubes in each group; using repeated addition to add groups; and multiplying. These steps occurred in combinations that are listed in Appendix A. For the purposes of the current study, the more steps involved in the procedure, the more complex it is considered. For example, the 5-step process of 1) *grouping the cubes*, 2) *connecting the cubes in each group*, 3) *counting the cubes in each group*, 4) *counting the number*

of groups, and 5) *multiplying the number of groups by the number of cubes in each group* is more complex than the one step process of 1) *counting by ones without pushing aside or connecting*.

A second researcher coded 15% of the DLLP Mathematical Strategy Checklists in order to compute reliability for the identification of mathematical strategies and complexity of procedures using Cohen's kappa. There was almost perfect agreement between coders, $\kappa = .93$.

Communicative competence of explanation. Communicative competence in this study is defined as the student's ability to meet the English-language discourse demands of providing a mathematical explanation. It requires the use of sufficient detail and descriptiveness such that a hypothetical naïve listener could identify each step of the student's mathematical procedure. Multiple measures were used to capture different facets of communicative competence. During coding, researchers were blind to students' background characteristics.

Sociolinguistic competence score. For each explanation, a researcher tallied the number of mathematical steps that could be identified based on the verbal information provided by the student. Sociolinguistic competence was calculated as the proportion of steps the student explained in relation to the total number of steps the student actually used to perform the task (see *Complexity of mathematical procedure* on page 15). Each student's sociolinguistic competence score was a decimal ranging from 0 to 1 (e.g., 0 = *No procedural steps adequately explained*; 0.5 = *Half of procedural steps adequately explained*; 1 = *All procedural steps adequately explained*). An explanation with a higher score is considered easier for the listener to understand. A second researcher coded 20% of the explanations in order to calculate reliability. There was substantial agreement between coders, $\kappa = .77$.

Proportion of sequenced steps. Each explanation was evaluated for the student's use of sequencing words to link together their mathematical steps. A researcher counted how many steps in the explanation were accompanied by a sequencing word or phrase that indicated when the step should occur in relation to others (e.g. "*First group the cubes. Then count how many cubes are in each group."*). Then a proportion was calculated for the number of steps the student explicitly sequenced in relation to the total number of steps they explained. Students who only used one step were excluded from analyses utilizing this measure. A second researcher coded 20% of the explanations in order to calculate reliability. There was substantial agreement between coders, $\kappa = .80$.

Replicability. For each explanation, a researcher determined whether the student provided enough verbal information to replicate the mathematical procedure. The researcher simultaneously assessed the student's use of precise vocabulary and logical sequencing in order to determine the replicability of the explanation. These ratings provide a holistic measure of the descriptiveness of students' explanations (*Yes* = Enough descriptiveness to replicate procedure; versus *No* = Not enough descriptiveness). A second researcher coded 20% of the explanations in order to calculate reliability. There was substantial agreement between coders, $\kappa = .76$.

Length of explanation. The length of each explanation was measured by the total number of words (TNW) the student used. This measure was used in order to check that the length of explanations did not vary depending on students' grade, EL status, or mathematics proficiency. This value was calculated using a web-based corpus management and analysis system that was created for the larger project. The system was programmed to use natural language parsing (NLP), and the accuracy of automated calculations was confirmed.

English learner (EL) status. Creating a variable to represent EL status for the current study posed unique challenges. Different metrics were used at one of the schools, so for the purposes of this study, students have to be classified as either EL students or English proficient students based on similar but non-identical sources of data. For students who attend the charter and public schools (n=77), information from Home Language Surveys, which parents completed at the time of school entry, was used by the state to identify students who speak a home language other than English. These students are then administered the California English Language Development Test (CELDT), and the results of this standardized assessment are used to make EL classifications. The laboratory school does not use a standardized measure of English language proficiency; therefore, for each of the students enrolled (n=49), a combination of factors was used to secure an EL status value, using a process similar to the State's and those used in previous research (Crosnoe, 2009). First, admissions records were used to identify students whose parents reported a home language other than English. Two additional criteria were used to estimate the current English language proficiency of these students. Results of a teacher survey administered for all students in the sample identified students whose teachers rated their English ability as a 1 on a scale from 1 = *Below Average* to 3 = *Above Average* on three or four domains (listening, speaking, reading, or writing). Results from the Stanford 10 assessment (see description below) were used to identify students who scored at the *Below Basic* level on the Language portion of the assessment. Students who were identified by either of these processes *and* who have a home language other than English were classified as English learners for the purposes of this study.

Mathematics proficiency level. Students' achievement in mathematics was determined using school-administered standardized test scores. Available mathematics achievement data

varied based on which school the student attended. For students in the current study who attended the charter and public schools (*n*=77), performance levels for the California Standards Tests (CST) were used (see description below). For students who attended the laboratory school (*n*=49), performance levels from the Stanford 10 were used (see description above). While the CST is criterion-referenced and the Stanford 10 is norm-referenced, both tests are designed to assess students' understanding of the mathematical concepts and processes included in California's state standards. The concepts measured by both tests are the same: number sense and operations; algebra; geometry and measurement; and data analysis, statistics, and probability (CDE, 2013; Pearson Assessments, 2011). The students' performance levels were placed on a common scale in order to create a combined measure of mathematics proficiency.

California Standards Tests–Mathematics (CST-Mathematics). The CST is administered to all California 2nd-11th grade public school students in the spring of each year. The CST-Mathematics is a component of the CST that measures student achievement with respect to California's mathematics content standards. Based on their performance on this test, students are assigned a scale score and a performance level (*Far Below Basic, Below Basic, Basic, Proficient,* or *Advanced*). Internal consistency reliability for the 2013 third and fifth-grade CST-Mathematics was $\alpha = .94$ (CDE, 2013b).

Stanford Achievement Test Series, Tenth Edition (Stanford 10). The Stanford 10 is a test that is commercially available for Kindergarten through 12th grade students. The Stanford 10 measures student achievement in multiple subject areas, including language and mathematics. Based on their performance on each component of the test, students receive scale scores and performance levels (*Below Basic, Basic, Proficient,* or *Advanced*). Split-half reliability

coefficients (KR-20) for the Stanford 10 range from the .80s to .90s (Pearson Assessments, 2011).

Analytic Plan

I conducted descriptive analyses of all mathematical and pragmatics measures, as well as correlations among the three pragmatics measures. I then conducted inferential statistical analyses to look for effects of grade, EL status, and mathematics proficiency (as measured by standardized assessments) on the various measures. To address the primary research question, I used Pearson's chi-square statistic and a series of *t*-tests and one-way ANOVAs to examine associations between mathematical procedure and various pragmatic aspects of students' explanations, including sociolinguistic competence. To address the secondary research questions, I conducted two-way factorial ANOVAs and logistic regressions to look for interactions of grade and EL status on the relationship between mathematical procedure and the pragmatics measures.

Results

Descriptive Statistics for Mathematical Measures

As expected, students (n=126) used a range of mathematical strategies—counting, repeated addition, and multiplication—to complete the activity. The majority (51.6%) of students used addition, followed by counting (34.1%) and multiplication (14.3%). The specific mathematical procedures ranged in complexity from one to five steps, as shown in Figure 1. The most common, used by 34.1 percent of the sample, was a four-step repeated addition procedure in which students: 1) grouped the cubes, 2) connected the cubes, 3) counted how many cubes were in each group, and then 4) added the groups together (e.g., skip counting by fives: "*5, 10, 15…*"). The next most common (13.5%) was a two-step counting procedure in which students: 1)

connected the cubes (without grouping), and then 2) counted them individually. See Appendix A for a complete description of each possible mathematical procedure and its level of complexity.

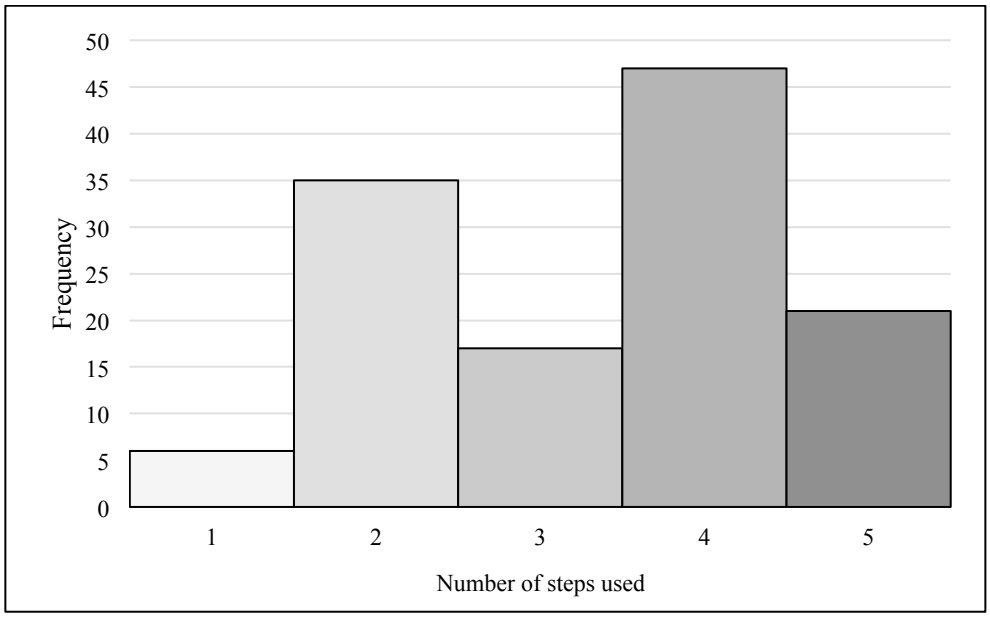

Figure 1. Number of steps used during mathematics activity (n = 126)

Associations between Mathematical Measures and Background Variables

Chi-square analyses indicated that the type of strategy used (i.e., counting, addition, or multiplication) did not differ significantly between 3rd and 5th graders or between English learners and English proficient students. ANOVA findings indicated that the mean number of mathematical steps used during the activity also did not differ by students' grade or EL status. These findings suggest that the different numbers of cubes presented to the two grade levels did not lend themselves to particular strategies and thereby skew strategy use by grade.

There was a significant positive correlation between mathematics proficiency level and the number of mathematical steps the student used during the activity, $r_s = .27$, $p = .002$, as shown in Figure 2. To confirm the effect of mathematics proficiency on mathematical strategy (i.e., counting, addition, multiplication), I collapsed the five performance levels reported from

the CST-Mathematics and Stanford 10-Mathematics assessments into two (*Proficient* or *Non-proficient*) in order to meet the requirements for chi-square analyses (a minimum expected count of 5 cases per cell). There was an association between binary mathematics proficiency level and mathematical strategy used during the activity, $\chi^2(2, n = 124) = 8.74$, $p = .013$.

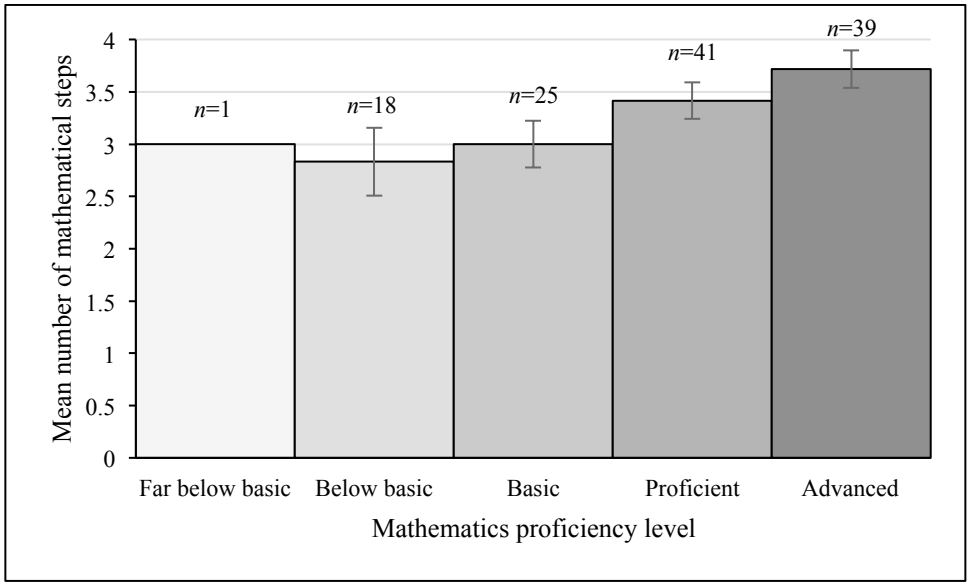

Figure 2. Mean number of steps used during activity, by mathematics proficiency level ($n = 124$)

Descriptive Statistics and Correlations for Pragmatics Measures and Length

The three pragmatics measures were positively correlated with one another. Sociolinguistic competence was correlated with the proportion of steps that were accompanied by sequencing words, $r = .58$, $p < .001$, as well as with the ability of a listener to replicate the student's procedure, $r = .45$, $p < .001$. Replicability and proportion of steps with sequencing words were also correlated, $r = .29$, $p = .001$.

Explanations ranged from 14 to 234 words total, with a mean of 69 words ($SD = 41$). The length of students' explanations did not vary significantly based on grade, EL status, or

mathematics proficiency level. Overall, students (n = 123)[5] produced explanations whose sociolinguistic competence scores ranged from 0.00 to 1.00. The mean was 0.64 (SD = 0.31), meaning that, on average, students clearly explained 64 percent of the mathematical steps they used during the activity. On average, approximately one fifth of the steps students (n = 120)[6] explained were linked to other steps using a sequencing word (M = 0.22, SD = 0.27). Half of the students (n = 120) did not use sequencing words to describe temporal relationships among any of their mathematical steps. The majority of students (61.1 percent; n = 126) did not provide enough verbal information for the listener to replicate their procedure.

Associations between Pragmatics Measures and Background Variables

Overall, there was no effect of grade on sociolinguistic competence scores. When analyzed separately by EL status, however, a positive effect of grade was found for English proficient students, $t(61) = -2.26$, $p = .028$. English proficient students in 5th grade demonstrated higher sociolinguistic competence than those in 3rd grade, as shown in Table 2. For EL students (n=60), no relationship was found between grade and sociolinguistic competence scores, meaning that 3rd and 5th grade EL students scored similarly. There were no significant effects of grade on rate of sequencing words or replicability, either overall or when examined separately by EL status.

There were, however, notable differences between English learners and English proficient students on all three pragmatics measures. On average, English proficient students demonstrated higher sociolinguistic competence than EL students, $t(121) = 2.71$, $p = .008$. See Table 2. When examined separately by grade, 5th grade English proficient students had higher

[5] Three students were removed from all further analyses involving sociolinguistic competence scores, because they explained a different mathematical procedure from the one they used during the activity.

[6] Only students who used more than one step during the activity were included in analyses about sequencing steps.

sociolinguistic competence scores than EL students, $t(56) = 3.30$, $p = .002$, but no statistical differences were found between 3rd grade EL students and English proficient students.

Table 2

Sociolinguistic competence, by grade and EL status ($n = 123$)

	English proficient ($n = 63$) M (SD)	English learner ($n = 60$) M (SD)	Total M (SD)
3rd Grade ($n = 65$)	.63 (.27)	.58 (.36)	.60 (.32)
5th Grade ($n = 58$)	.78 (.25)	.54 (.31)	.68 (.30)
Total	.71 (.27)	.56 (.34)	.64 (.31)

English proficient students also used sequencing words to link together a greater proportion of their mathematical steps ($M = 0.30$, $SD = 0.29$) than English learners ($M = 0.13$, $SD = 0.21$), $t(118) = 3.68$, $p < .001$. The same was found when looking at 3rd grade, $t(59) = 2.35$, $p = .022$, and 5th grade, $t(57) = 2.56$, $p = .013$, separately. Finally, English proficient students were more likely than English learners to provide enough information for the listener to replicate their mathematical procedure, $\chi^2(1, N = 126) = 7.97$, $p = .005$.

There were a number of relationships found between mathematics proficiency level (according to standardized assessments) and students' performances on the pragmatics measures. For example, when analyzed separately by grade, there was a relationship between mathematics proficiency level and sociolinguistic competence amongst 5th graders, $F(4, 55) = 5.37$, $p = .001$, but not amongst 3rd graders. There was also a positive effect of mathematics proficiency level on students' use of sequencing words to link together steps in their explanations, $F(4, 113) = 3.32$, $p = .013$. Finally, students who were rated *Proficient* on the binary proficiency measure were more

likely to produce explanations that had enough information for the listener to replicate the mathematical procedure, $\chi^2(1, N = 124) = 9.58, p = .002$.

What is the relationship between the complexity of the mathematical procedures that elementary school students use during a mathematics activity and the communicative competence of the students' oral explanations?

As predicted, the complexity of the mathematical procedure a student used (based on the number of steps) was negatively associated with the sociolinguistic competence of their explanation, $F(4, 118) = 3.30, p = .013$, as shown in Figure 3. Similarly, there was a relationship between mathematical strategy and sociolinguistic competence, but only when strategy was broken into a binary variable (counting vs. addition/multiplication), $t(121) = 2.81, p = .039$. Students who used counting had higher sociolinguistic competence scores ($M = 0.72$, $SD = 0.30$), on average, than students who used addition or multiplication ($M = 0.60$, $SD = 0.31$).

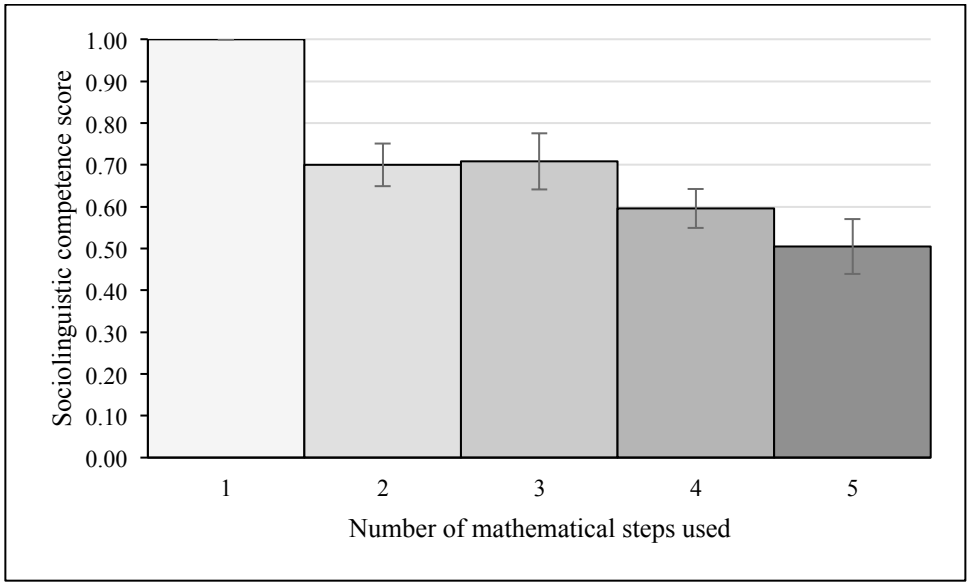

Figure 3. Mean sociolinguistic competence score, based on number of mathematical steps used ($n = 123$)

No significant relationships were found between mathematical strategy or complexity of procedure and other measures of communicative competence, namely the replicability of students' explanations or the rate of sequencing words used.

How does the relationship between mathematical procedure and communicative competence differ as a function of grade?

Contrary to the hypothesis that younger students and EL students would be more strongly affected by the cognitive demand of explaining a more complex mathematical procedure, there was no interaction effect of grade on the relationship between mathematical procedure (number of steps used) and sociolinguistic competence scores, $F(3, 114) = 0.35$, $p = .790$. This suggests that the sociolinguistic competence scores of 3rd and 5th graders were not affected differently by the complexity of the mathematical procedures they used. However, the main effect of mathematical procedure on sociolinguistic competence scores was significant amongst 3rd graders, $F(4, 60) = 2.63$, $p = .043$, but not amongst 5th graders, $F(3, 54) = 2.36$, $p = .081$. See Figure 4.

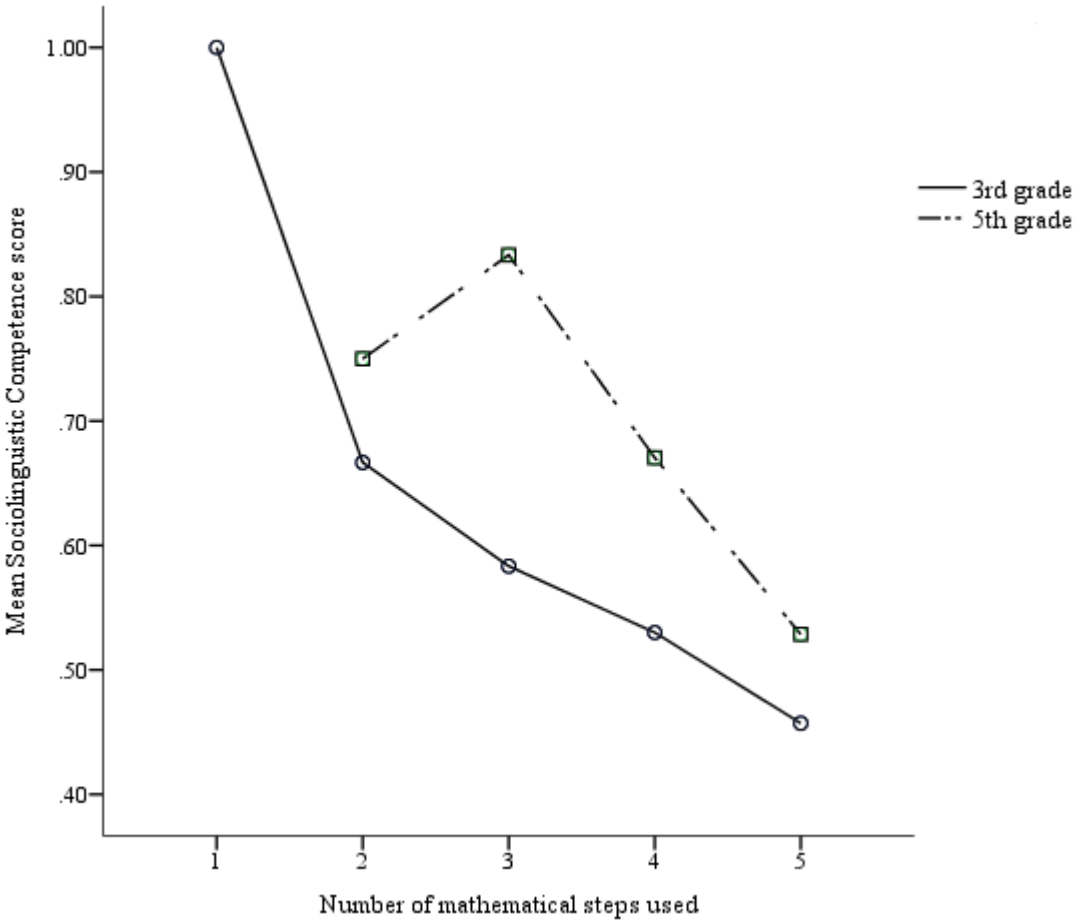

Figure 4. Mean sociolinguistic competence scores of 3rd and 5th graders, based on the number of mathematical steps used during the activity

Grade did not have a significant interaction effect on the relationship between the complexity of mathematical procedures and students' use of sequencing words, nor were there any apparent differences when ANOVAs were conducted separately by grade. A logistic regression showed that there was no significant interaction effect of grade on the relationship between complexity of mathematical procedure and the replicability of students' explanations. In addition, no significant interactions of grade were found between mathematical strategy (either as a binary or three-level variable) and any pragmatics measures.

How does the relationship between mathematical procedure and communicative competence differ as a function of English learner status?

There was no overall significant interaction effect of EL status on the relationship between mathematical procedure (number of steps used) and sociolinguistic competence scores, $F(3, 114) = 1.93$, $p = .129$, meaning that the sociolinguistic competence scores of EL and English proficient students were not affected differently by the complexity of the mathematical procedures they used. However, the main effect of mathematical procedure on sociolinguistic competence scores was significant amongst EL students, $F(4, 55) = 4.53$, $p = .003$, but not amongst English proficient students, $F(3, 59) = 1.50$, $p = .223$.

There was not a significant interaction of EL status on the relationship between mathematical procedure and students' use of sequencing words. However, separate analyses of EL and English proficient students indicated that for EL students, there was a significant positive relationship between the number of mathematical steps used and the proportion of steps that the student sequenced, $F(3, 52) = 2.95$, $p = .041$, whereas there was no significant effect for English proficient students, $F(3, 59) = 0.94$, $p = .428$.

A logistic regression showed that there was no significant interaction effect of EL status on the relationship between mathematical procedure and the replicability of students' explanations. No significant interactions of EL status were found between mathematical strategy (either as a binary or three-level variable) and any pragmatics measures.

Discussion

The purpose of this correlational, cross-sectional study was to examine the influence of mathematical strategy on the communicative competence of 3rd and 5th grade students' oral explanations. The main objective was to determine whether students have more difficulty communicating clearly about certain mathematical procedures and to see whether this differed across grade level and EL status.

Relationship between Mathematical Procedure and Sociolinguistic Competence

The findings supported the primary hypothesis that students who use more complex mathematical procedures—as measured by either strategy type or number of mathematical steps—struggle with sociolinguistic competence when asked to orally communicate the details of their procedures more so than students who use simpler procedures. There are a number of potential explanations for this finding. First, it is possible that engaging in a more complex mathematical procedure requires the use of more cognitive resources, thereby leaving the student with fewer cognitive resources available for the production of a well-constructed explanation. It is also possible that the verbal act of explaining a complex mathematical procedure is more challenging than explaining a simple one; the student needs to explain a greater number of steps and link them together in a cohesive way, so there are more possibilities for omissions or ambiguities.

In the following explanation, a 3rd grade girl described her five-step procedure (grouping the cubes by color, connecting the cubes in each group, counting how many cubes are in each group, counting how many groups there are, multiplying) without including much detail about what steps were involved:

> *You should use the cubes this way, because it'll be easier to count. And if they're all the same amount and the same length, and you count them, and there's ten, you can tell that there's fifty, because there's fifty rods.*

This student received a sociolinguistic competence score of 0.20, because she only described one step ("count them") out of the five that she used. Even that one step is ambiguous, because she does not link the pronoun *them* to a clear referent—does she mean count the cubes in each rod or count the number of rods? It is possible that she neglected to mention her four remaining steps because of cognitive overload due to the complexity of her mathematical procedure. She may also have omitted steps because it was difficult for her to separate her

procedure into discrete steps and then articulate these to a listener. Regardless of the reason, her explanation of a complex mathematical procedure displayed less sociolinguistic competence than many explanations about simpler procedures. For example, the following explanation was produced by a 3rd grade boy who described his two-step procedure clearly:

> *You have all the Unifix cubes on the right and none on the left. Take two and put them on the left. Do it with another pair until you count two, four, six, eight all the way to the number that the cubes have, which I counted fifty.*

He received a sociolinguistic competence score of 1.00—the maximum score—because he explained both of his two steps (count, push aside counted cubes). It is important to note that both students' explanations illustrated their linguistic skills; they both used mathematical terminology (e.g., "amount," "length," "pair") and conventional syntax, including complex sentences. However, the second explanation demonstrated greater sociolinguistic competence by providing clear descriptions (e.g., the set-up of the materials at the beginning of the activity) and avoiding the ambiguous pronouns and deictic terms (e.g., "this way," "they," "them") that made the first example hard to follow.

Although there were no significant interactions of grade or EL status on the negative relationship between mathematical procedure and sociolinguistic competence, there was support for the corresponding hypotheses that younger students (3rd graders) and EL students would be more strongly affected by the cognitive demand of explaining a more complex mathematical procedure. When viewed separately by grade, the main effect of mathematical complexity on sociolinguistic competence was only significant for 3rd graders, suggesting that 5th graders are less susceptible to the challenges of explaining a complex mathematical procedure. Similarly, when viewed separately by EL status, the main effect was only significant for EL students, suggesting that English proficient students are less susceptible as well.

Taken together, these results suggest that perhaps the relationship between mathematical strategy and sociolinguistic competence is influenced by a combination of mathematical content knowledge and language proficiency. The cognitive and pragmatic demands of explaining a complex mathematical procedure may not be as challenging for 5th graders—who likely have a more sophisticated understanding of mathematical operations than 3rd graders—and for English proficient students—who likely have more extensive experience with the English language. This is only a tentative interpretation of the findings, due to the small sample size of the current study, but it could have important implications for mathematical instruction.

It is possible that students are already receiving instruction that may influence their mathematical discourse. One tentative finding in the current study indicated that in EL students' explanations, the more mathematical steps they used, the greater the proportion of steps they accompanied with a sequencing word. In contrast, English proficient students' use of sequencing words was unrelated to their mathematical strategies and procedures. It is possible that EL students have received English language instruction that has specifically targeted their use of sequencing discourse connectors to overtly signal the ordering of steps. Perhaps they have been instructed to use discourse connectors when explaining lengthy step-by-step processes, such as complex mathematical procedures.

Assessing Pragmatics in Mathematical Explanations

In order to measure students' pragmatic competencies during mathematical explanations, three measures were designed for this study. Two of these—sociolinguistic competence score and replicability—represented a broad evaluation of each student's ability to explain their mathematical procedure fully and in sufficient detail. The following 3rd grade EL boy, for example, produced an explanation that was considered replicable by researchers:

Count these cubes by one by one, but if they're stuck together, then you count by twos. But you can stick them together so you can find it out. Then until when you finish all sticking them together, you could just count them. You can count them by different ways.

From this explanation, it sounds like he recommends sticking together two cubes and then counting the cubes by twos, which was, in fact, the procedure he used. Although he uses some non-conventional phrasing (e.g., "then until when," "count them by different ways"), his instructions are clear to the listener. He uses precise mathematical terms (e.g., "count by twos") and sequencing (e.g., "when you finish…"), which help the listener follow his procedure. Some students produced explanations that were not easily decipherable. For example, one 3rd grade EL girl described her procedure as follows:

Stack them in. You could do it by finding like a red one and a yellow one. That's it.

Although her procedure involved one step—counting—she does not use the precise wording (i.e., "count") that would signal this to a listener. Her explanation is short and imprecise, creating a difficult task for a listener who is trying to replicate her procedure.

The third measure—use of sequencing words—measured a specific discourse feature of mathematical explanations that plays a role in making meaning for the listener. By using sequencing words to establish a temporal ordering of their steps, students provide their listeners with a clear description of their procedure. For example, the following 5th grade English proficient girl used "then" to sequence all of her steps:

You can take the cubes and put them into stacks of ten. And <u>then</u> make as many stacks of ten as you can. And <u>then</u> you can line them up and <u>then</u> count them. And there were ten stacks, which is a hundred.

This stands in contrast to the following example, in which a different 5th grade English proficient girl did not use any sequencing words:

You should make groups of ten using the cubes to find out how many there are, because it's easier to count by tens than doing it one by one.

Although she described two identifiable steps ("make groups of ten" and "count by tens"), she did not link them together temporally. A naïve listener may not know whether counting by tens is a prerequisite of grouping by tens, a component of the grouping process, or a follow-up step.

The three pragmatics measures were positively correlated, which suggests that, for example, explanations with a greater proportion of steps explained are likely to contain more sequencing words, and a listener is more likely to be able to replicate that procedure based on the student's words.

Group Differences in Pragmatics Measures

Overall, students had difficulty performing well on the suite of pragmatics measures designed for this study. Few students explained all of their steps, sequenced all of their steps, or provided enough information that a listener could replicate their procedure. However, some groups of students performed better than others. On average, English proficient students sequenced a greater proportion of their steps than EL students and were more likely to produce explanations that could be replicated by a listener. English proficient 5th graders were the highest performing group, receiving higher sociolinguistic competence scores than English proficient 3rd graders and EL 5th graders. Interestingly, there was no effect of grade on EL students' sociolinguistic competence scores, and no effect of EL status on 3rd graders' scores. This suggests that perhaps there is an increase in pragmatic L1 development during the time between 3rd and 5th grades—a time during which children may become more attuned to their listeners' needs and more linguistically capable of providing sufficient information—that EL students have not yet reached in English. Importantly, the length of students' explanations did not vary based on their grade or EL status, indicating that 5th graders and English proficient students were not

simply producing more words than 3rd graders and EL students and thereby achieving greater sociolinguistic competence.

Students with high mathematics proficiency levels (based on standardized tests of achievement) performed better on the pragmatics measures than students with lower proficiency levels. Their explanations were more likely to be replicable, they contained higher proportions of sequencing words, and amongst 5th graders, they had higher sociolinguistic competence scores. These findings suggest that producing a sufficiently clear mathematical explanation is easier for students with high mathematics achievement. Perhaps the cognitive demand of the mathematics task is lower for these high achieving students, and therefore they have more cognitive resources available for language production. Alternatively, there may be cognitive and mathematical skills that are measured by both the standardized assessments and the pragmatics measures used in this study. Furthermore, the greater a students' conceptual understanding of the mathematical ideas they are explaining, the easier it likely is for them to verbalize them. For example, the following explanation was produced by a 5th grade English proficient girl whose mathematics proficiency is *Advanced*, according to the SAT10:

> *If you want to find out how many cubes there are, and you want to do it the most efficient way, the way that I would do it would be that I would connect ten of them. And then I would just keep making sticks of ten. And then at the end, I would count to see how many sticks of ten there would be. And then you would do that times ten, and then you would get your answer. And you should use that way because it's an efficient way that you can do quickly.*

She was able to clearly explain her process of grouping, connecting, and multiplying without getting confused about the mathematical operations behind this strategy. Her explanation can be contrasted against one produced by a 5th grade English proficient girl whose mathematics proficiency is *Below Basic* based on the SAT10:

> *What number are you really sure about that might equal this number that you're going to do without counting? I would go with ten if it's an even number. And if it's an odd*

number, it's sort of like a prime and composite number. So if it's a prime, it's only one and itself. So if it's an even number, it's a composite number. And if it's an odd number, it's a prime number. So then how much do you think is in this? About eighty or a hundred. So then spread them so how much do you get to a hundred. So that's ten. So ten for each one. So one, two, three, four, five. And then count how much there are in each one. One, two, three, four, five. One, two, three, four, five. And there's your answer. There's a hundred.

This student is describing a similar strategy to the one in the previous example, but this is not clear from her explanation. She introduces the unnecessary mathematical concepts of prime and composite numbers and fails to explain how they are related to her procedure. It is possible that her struggle to explain her procedure clearly is related to a misunderstanding about the mathematical concepts she is attempting to describe.

Limitations and Future Directions

As some of the examples described in this paper illustrate, it is important to consider students' use of gesture when examining how they communicate with their listeners (Goldin-Meadow, 1999). Gesture is a resource that students often use to make meaning in mathematical classrooms (Lemke, 2003) as well as to lighten cognitive load (Goldin-Meadow, Nusbaum, Kelly, & Wagner, 2001). Therefore, an important limitation in the current study is the lack of video data of students' explanations. Videos may have helped decipher explanations that relied on the use of deictic terms (e.g., "this," "that," "here") to explain specific procedural steps (Bailey et al., under review). Some of these terms may have been accompanied by gestures to the Unifix cubes, thereby providing necessary (non-verbal) references that had been lacking in the student's verbal explanation. Future studies should collect video data in order to examine how students' non-verbal communicative acts contribute to their communicative competence.

In addition, students in the current study were recruited from twelve classrooms across four schools. While the academic diversity of the sample was valuable in creating a broad distribution of students, it is possible that differences among the classrooms may have played a

role in students' abilities to engage in mathematical discourse. The sample in the current study was too small to investigate this possibility statistically. Future studies should examine curricular and pedagogical differences among classrooms and explore any effects on the mathematical strategies that students use or on the pragmatic features of their mathematical explanations. It is possible, for instance, that students in some schools or classrooms may not have had the opportunity to learn particular mathematical strategies or that some classrooms allow for fewer opportunities to engage in mathematical discourse and receive feedback on mathematical explanations, particularly in the case of EL students (Bailey et al., under review).

Due to the data collection procedures used in the larger project, the explanations students produced were not necessarily authentic representations of the mathematical discourse that occurs in classroom instructional settings. Students in the current study responded to a scripted prompt and were not asked follow-up questions if their explanations were unclear. Teachers in elementary school mathematics classrooms often ask questions in response to students' ideas in order to clarify ambiguities, identify reasoning behind student errors, and ask for elaboration (Franke et al., 2009). Future studies should either be conducted in classroom settings or provide students with more opportunities to respond to targeted questions and develop more clarity in their explanations.

Despite these limitations, the current study begins an important exploration into the relationship between mathematical strategy and students' abilities to discuss mathematics in a pragmatically appropriate way. In a similar study with a larger sample size, researchers could investigate how grade, EL status, and mathematical proficiency work together to influence the effect of mathematical strategy on the pragmatics of students' explanations. These studies should investigate additional pragmatic features of students' explanations beyond those featured in the

current study. Future studies should also qualitatively examine students' explanations in order to determine what distinguishes those with low sociolinguistic competence from those with high sociolinguistic competence.

Conclusion

As a result of new academic standards, mathematics teachers are faced with using students' explanations to gauge their understanding of mathematical concepts. The findings from this study begin to illuminate the ways in which students' explanations may be influenced by factors aside from mathematical understanding of the specific concepts at hand. For example, students who used more complex mathematical procedures performed worse on a measure of sociolinguistic competence. This was especially true for younger students and EL students, who may still be developing the necessary linguistic and pragmatic competencies.

These findings can help inform teacher training and teacher practice in elementary school mathematics classrooms by emphasizing the importance of students' pragmatic competencies. Students need to take account of their listeners' needs when communicating mathematically, and this becomes more difficult when they must explain complex mathematical strategies. In evaluation of students' mathematical understanding, teachers should consider the effect that students' mathematical strategies may have on their abilities to convey meaning. An unclear mathematical explanation may not be an indication of a lack of mathematical understanding.

Teachers should consider oral explanations as just one of many ways that students can demonstrate their mathematical understanding. In order to support pragmatic development, students should be encouraged to actively consider the needs of the audience and to meet those needs by providing sufficient detail and descriptiveness. In the classroom, teachers should model precise mathematical explanations and explicitly draw students' attention to the pragmatic

features that make an explanation clear and accessible. Students should be given frequent opportunities to hear and assess other students' mathematical explanations as well as produce and refine their own. These opportunities will not only support pragmatic development but may also contribute to students' conceptual understanding of mathematical content.

Appendix A: Coding Manual for Complexity of Mathematical Procedure

Strategy	**Possible # of steps***	**Description of steps used**
Counting	1	1. Counting without connecting, grouping, or pushing aside cubes
	2	1. Connecting cubes (in sticks of any length) 2. Then counting the cubes
		1. Counting cubes 2. While pushing aside cubes that have already been counted
		1. Grouping cubes without connecting 2. Then counting all the cubes by 1s
	3	1. Connecting cubes (in sticks of any length) 2. Then counting the cubes 3. While pushing aside cubes that have already been counted
Repeated addition	3	1. Grouping cubes without connecting 2. Then counting how many cubes are in each group 3. Then using repeated addition to add the groups (e.g. "*5, 10, 15, 20…"*)
	4	1. Grouping cubes 2. Then connecting cubes in each group 3. Then counting how many cubes are in each group 4. Then using repeated addition to add the groups (e.g. "*5, 10, 15, 20…"*)
		1. Grouping cubes without connecting 2. Then counting how many cubes are in each group 3. Then using repeated addition to add the groups (e.g. "*5, 10, 15, 20…"*) 4. While pushing aside cubes that have already been added
	5	1. Grouping cubes 2. Then connecting cubes in each group 3. Then counting how many cubes are in each group 4. Then using repeated addition to add the groups (e.g. "*5, 10, 15, 20…"*) 5. While pushing aside cubes that have already been added
Multiplication	4	1. Grouping cubes without connecting 2. Then counting how many cubes are in each group 3. Then counting how many groups there are 4. Then multiplying
	5	1. Grouping cubes 2. Then connecting cubes in each group 3. Then counting how many cubes are in each group 4. Then counting how many groups there are 5. Then multiplying

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