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Sustainability and groundwater

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Abstract A theory of sustainable groundwater exploitation is presented in this article. The core of the article is a general formulation of the mathematical programming problem whose solution—when it exists—produces sustainable pumping rates. A simplified quadratic, linearly constrained, version of the general formulation is implemented and solved to calculate sustainable pumping rates in terms of diverse economic and hydraulic factors. The calculated pumping rates confirm the desirability of sustainable groundwater strategies judged by aquifer and economic performance.

Key words game theory; groundwater; optimization; sustainable water development

INTRODUCTION

A definition of sustainability

Sustainable aquifer exploitation is defined as the long-term withdrawal of groundwater which meets beneficial functions while avoiding negative impacts. In this context, "long-term" means groundwater withdrawal which extends over an arbitrarily long time period, so that benefits from groundwater use accrue indefinitely. Among the most important beneficial functions of groundwater are irrigation for food production, household supply (drinking, cooking, and sanitation), and as a factor of production in industrial processes. Negative impacts are those that: (a) degrade the natural physical, chemical, and biological characteristics of groundwater or any other water body to which groundwater is transferred; (b) reduce baseflow to streams and lakes, thus threatening dependent aquatic ecosystems; (c) cause land subsidence, giving rise to geological and structural hazards; (d) increase land dryness and limit soil-water supply to vegetation by lowering the water table. Moreover, the magnitude, the duration, and the frequency of groundwater withdrawal impacts—as well as the sensitivity and adaptability of affected resources—must all be considered in the search for sustainable groundwater exploitation. Negative impacts may be tolerable over relatively short periods of time if eventual recovery of the impacted resources is possible, which may be the case in aquifers that are frequently replenished by plentiful percolation.

In the past, the human use of groundwater has taken precedence over its environmental impacts. Beyond the immediate economic gain, that practice has, in some instances, left a legacy of degraded aquifers, land subsidence, and ecological damage. The recent interest in sustainable groundwater use is a reflection of a heightened awareness about the need to conserve our water resources and of the—now well understood—multiple linkages that groundwater withdrawal has to geochemical cycles, ecological processes, and geological hazards. Most of the world's productive aquifers have been fully tapped. In a time of growing population, it would be conceited to assume that groundwater resources can be further mined to meet the expanding water demand. Instead, we must search for smarter, sustainable, strategies of groundwater exploitation that ensure its long-term availability. Groundwater exploitation must be planned in accord with the use of other water sources, both conventional and novel ones. Among the latter, for example, one can cite desalinated sea water, which is approaching economic competitiveness from the improvements in reverse osmosis technology. Equally important, humans must learn to conserve and recycle water. Within this context of alternative choices of water use, population growth, and evolving technologies, the theory of sustainable groundwater exploitation presented in this article must not be judged in isolation, but, rather, as one of the means available to meet the water needs of rising population and expanding economic activity.

Study objective

This article presents a theory of sustainable groundwater exploitation. The theory takes into account the beneficial functions of groundwater, the economic factors associated with groundwater exploitation, and the variability of groundwater recharge.

CLIMATIC VARIABILITY AND GROUNDWATER

Groundwater recharge and storage: modelling and empirical issues

The variability of the climate (possibly influenced by climatic change) is of paramount importance in the calculation of sustainable groundwater exploitation strategies. This follows from the dependence of groundwater storage and recharge on climatic and land-surface processes, which we proceed to examine. Let us start with the mass-balance equation of groundwater in an aquifer during an arbitrary period of time (one year, for example), in which change in storage, (natural) groundwater recharge, (human) withdrawal, and the sum of baseflow (B), exfiltration (X_G, primarily springflows that drain aquifers), and groundwater evapotranspiration (ET_G) (B, X_G, and ET_G, are natural transfers of groundwater to rivers and lakes, to the land surface, and to the lower atmosphere, respectively) are denoted by ΔS_G, R, W, and L, respectively:

$$\Delta S_G = R - W - L \tag{1}$$

Our goal is to adjust the withdrawal (W) over time to achieve a sustainable rate of exploitation. The recharge (R) can be estimated using a variety of methods (Scanlon *et al.*, 2002). The baseflow (B) contained in the loss term (L) in equation (1) is estimable by techniques involving hydrograph separation. So is the exfiltration X_G, which represents springflow. The groundwater evapotranspiration (ET_G) is negligible in most instances, although transpiration by deep-rooted phreatophytes may render it large in some circumstances.

A portion of the recharge, called diffuse recharge (denoted by R^*), is driven by precipitation (P) following its partition into surface evapotranspiration (ET_S), infiltration (F), and overland flow (O). The diffuse recharge is the vadose-zone water gained as infiltration that reaches the (saturated) aquifer. It is different from recharge caused by stream and lake seepage (R_S) into the aquifer. Soil water gained as infiltration is primarily depleted by evapotranspiration in the vadose zone (ET_V), and to a minor extent by exfiltration to the land surface (X_V). Considering the water balance established between the surface-water and vadose-zone reservoirs (which are assumed to have the same geographical boundaries), it is established that the total recharge ($R = R^* + R_S$) in an arbitrary period of time equals (letting $ET_{SV} = ET_S + ET_V$, and $\Delta S_{SV} = \Delta S_S + \Delta S_V$, where ΔS_S and ΔS_V denote the surface and vadose-zone water storages, respectively):

$$R = P - ET_{SV} - \Delta S_{SV} - O - X_V \tag{2}$$

in which all terms have been defined. The complexities involved in the estimation of recharge are patently revealed by equation (2). Even if the change in storage ΔS_{SV} and the exfiltration X_V were negligible, one still needs to estimate the overland flow (O, usually by analysing runoff data) and to independently estimate the evapotranspiration (ET). ET depends on a variety of climatic factors: surface temperature, surface radiative balance, moisture deficit in the lower atmosphere, wind speed, vegetation, soils, and soil-water content. Its accurate estimation is non trivial. The change of storage ΔS_{SV} is difficult, if not impossible, to estimate. Qualitatively, we know that in times of drought it turns negative, while it is positive during wet periods.

Substitution of equation (2) into equation (1) produces the following result (letting the total evapotranspiration $ET = ET_{SV} + ET_G$, and the runoff $Q = O + B + X_G + X_V$):

$$\Delta S_G = P - W - ET - \Delta S_{SV} - Q \tag{3}$$

The runoff may be expressed as a fraction of precipitation in terms of the runoff coefficient $k = Q/P$, which is, in general, not constant. On an annual basis, k ranges between 0.2 and 0.5. Using the runoff coefficient in equation (3) yields:

$$\Delta S_G = (1-k)P - W - ET - \Delta S_{SV} \tag{4}$$

The intuitive nature of the water-balance equations (1)–(4) belies the practical difficulties that hinder their application. Key among those difficulties is the measurement and estimation of storage changes. At times of drought or variable precipitation, in particular, those changes are significant and the common assumption of negligible storage changes customarily introduced in long-term water balancing is invalid. The estimation and measurement hindrances germane to the calculation of groundwater recharge using water balancing steer hydrologists to resort to alternative calculation methods. One such method is statistical in nature and is briefly reviewed next.

Alternative estimation of groundwater recharge

Long-term empirical evidence collected in regional aquifers suggests that larger (or lower) than average precipitation produces larger (or lower) than average groundwater recharge (Loáiciga *et al.*, 2000), and that seasonal groundwater recharge lags seasonal precipitation. The time lag varies from aquifer to aquifer. As an alternative to the estimation of groundwater recharge by water balancing, we propose writing the seasonal recharge in any period t, R_t, in terms of present and past seasonal precipitation as follows:

$$R_t = \sum_{s=0}^{M} a_s \, P_{t-s} + v_t \tag{5}$$

in which the a_s are coefficients which must be determined from precipitation and recharge data, M is the maximum lag, and v_t is a zero mean random term which may either be uncorrelated or correlated over time. Equations similar to equation (5) may be written using the seasonal runoff Q instead of precipitation P as the predictor variable given the fact that $Q = k\,P$, with k being the seasonal runoff coefficient.

Equation (5) embodies a class of statistical methods, an alternative to the water-balance method reviewed above, for estimating groundwater recharge. In some instances, the statistical method represented in equation (5) may introduce analytical advantages which can be exploited in sustainable groundwater modelling, as described below. There are several other methods which can be used to estimate groundwater recharge. The meaning of the recharge estimated by those methods—its temporal and spatial validity, for example—may vary considerably among them (Scanlon *et al*., 2002).

ECONOMIC FACTORS IN SUSTAINABILITY

Our interpretation of sustainability relies on the economic incentive as the engine that drives long-term, beneficial, groundwater exploitation. This author takes the position that economic agents are not altruistically motivated when they engage in resource exploitation. They exploit primarily to derive a benefit. We shall see, however, that left to their own devices, the zest for profits leads to rapid groundwater depletion and sub-optimal performance. Thus, the economic motif must be tempered by constraints if sustainable groundwater exploitation is to be achieved.

A simple measure of economic performance is the maximization of the present value of net revenue. The net revenue (N_R) is the total revenue (T_R) that accrues from the extraction of groundwater minus the cost of extracting and delivering the groundwater (C). Suppose that the total rate of groundwater extraction equals the sum of the withdrawals from n wells:

$$W = \sum_{j=1}^{n} W_j \tag{6}$$

The individual withdrawals may change over time. The present value of the net revenue is random, and, at a minimum, it depends on: (a) the market price of groundwater, (b) the discount rate, and (c) the climate (herein synthesized in terms of precipitation), which are themselves random. The climate effect is felt on the cost of groundwater extraction. For example, groundwater levels fall during droughts because of the reduced recharge. This, in turn, increases the cost of groundwater withdrawal. Let the probability density functions of the groundwater price (A), precipitation (P, recall from equation (5) that recharge may be expressed in terms of precipitation), and the discount rate (Y) be denoted by $f_A(a)$, $f_P(p)$, and $f_Y(y)$, respectively. Assume that the planning horizon during which groundwater is withdrawn is T (T is on the order of 100 years or longer in our analysis). The expected value of the net revenue (discounted to present value) is given by the following integral (the time t ranges from zero to T):

$$\overline{N}_R = \int_{a,p,y,t} \left(T_R(W,a) - C(W,p,t)\right) \mathrm{e}^{-yt} f_A(a)\, f_P(p)\, f_Y(y)\, \mathrm{d}t\, \mathrm{d}a\, \mathrm{d}p\, \mathrm{d}y \tag{7}$$

The term in parentheses in the right-hand side of the objective function (7) is the net revenue, in which the total revenue is explicitly written as a function of groundwater withdrawal and the market price of groundwater, and the cost of groundwater production is a function of withdrawal, precipitation (a proxy for recharge), and the elapsed time of groundwater withdrawal. The exponential term is the continuous discounting factor that reduces the stream of net revenue in the period $[0, T]$ to present value. The weighting by the probability density functions of the groundwater price, the precipitation, and the discount rate, and the integration over their ranges produce the expected value of the net revenue discounted to present value.

We seek to maximize the integral (7) with respect to the groundwater withdrawal (W, and, specifically, the individual pumping rates W_j). The climate effect in equation (7) can be expressed mathematically by representing long-term precipitation, or recharge, as a function that approximates historical intra- and inter-annual fluctuations and trends. That implies the use of combinations of trigonometric functions.

In the absence of suitable constraints, the maximization in equation (7) would not produce sustainable groundwater exploitation in the sense of this article. It has been shown by this author (Loáiciga, 2003a) that an unconstrained maximization of the objective function (7) leads to rapid violation of sustainable criteria. The next section introduces constraints that ensure a solution to equation (7) consistent with our definition of sustainability.

CONSTRAINTS FOR SUSTAINABLE GROUNDWATER EXPLOITATION

Hydraulic constraints

One important category of constraints is that imposed on hydraulic heads in an aquifer. The drawdown of hydraulic heads caused by pumping at multiple wells is a classical example of the phenomenon of hydraulic superposition: the total drawdown at any location in an aquifer is approximately equal to the sum of the individual drawdowns caused by each well at the location in question. The magnitudes of the individual drawdowns are function of: (a) distance from a pumping well to the location of interest, (b) the pumping rate, (c) the elapsed time of pumping, and (d) aquifer hydraulic and elastic properties.

Constraints on hydraulic heads are extremely important in ensuring the long-term viability of aquifer exploitation without deleterious impacts. Head constraints, when suitably chosen, prevent the intrusion and upwelling of low-quality waters into freshwater aquifers. They also limit land subsidence to admissible ranges, and maintain groundwater levels within the reach of transpiring vegetation. In general, water exchanges among reservoirs (aquifer, rivers, lakes, seas) are controlled by the relative hydraulic heads in those reservoirs. Therefore, constraining heads in an aquifer constitutes the most effective—if not the only—method to control the hydraulic status of groundwater. Let the drawdown at a location i caused by a pumping well j ($j = 1, 2, \ldots, n$) be denoted by s_{ij}. Then, the total drawdown at i caused by the n wells is the sum of the individual drawdowns. If the admissible drawdown at i is $s_{i,max}$, the hydraulic head (or drawdown) constraints take the following form (assuming that there are m locations where the drawdown is constrained):

$$\sum_{j=1}^{n} s_{ij} \leq s_{i,\max} \qquad i = 1, 2, \ldots, m \tag{8}$$

The drawdown constraints, equation (8), must be appended to the maximization of the objective function (7). We examine other types of constraints next.

Supply, cost, and other constraints

Supply constraints are imposed to guarantee that, at any time, groundwater withdrawal meets a predetermined demand D_t. The supply constraint takes the following form:

$$\sum_{j=1}^{n} W_j = W \geq D_t \qquad \text{for all } t \tag{9}$$

There may be cost constraints that restrict the cost of groundwater production to a maximum $C_{\max}$:

$$C(W, p, t) \leq C_{\max} \quad \text{for all } t \tag{10}$$

Pumping in an aquifer may affect the advection of dissolved substances in groundwater. Let C_{ijt} be the concentration of the j-th substance at the i-th location at time t. Assume that the maximum allowable concentration is $C_{j,\max}$. Concentration constraints are then expressible as follows:

$$C_{ijt} \leq C_{j,\max} \qquad \text{for all } t \tag{11}$$

Constraints like those expressed by equations (8)–(11), plus any other deemed necessary, must be appended to the objective function (7), thus yielding a nonlinear mathematical programming problem. The complexity of the resulting mathematical programming problem depends on many factors: the number of pumping wells, aquifer properties, cost functions, recharge mechanism, etc. Special numerical techniques and solution algorithms might be needed to find solutions to the mathematical programming problem. In the next section we illustrate a type of analytical solution to the sustainable groundwater exploitation problem entertained in this work.

A QUADRATIC PROGRAMMING SOLUTION

Cooperation and non-cooperation: a word of caution

Prior to providing an example, the reader is cautioned about mundane difficulties that stand in the way of sustainable groundwater exploitation. These stem from the human tendency to over-exploit, or mine, renewable resources such as groundwater. The impulse to over-exploit has received scholarly attention for at least two centuries, and has been allegorically named "the tragedy of the commons" (Hardin, 1968). The tragedy of the commons materializes when individuals with access to a relatively free resource all rush to acquire as much of it as possible before others do. Eventually, and sooner rather than later, the resource is ruined or exhausted. If the resource users would cooperate and tame their rates of take, they might well enjoy the resource for a long time. But that has proven to be, over and over again, wishful thinking. In view of the tragedy-of-the commons phenomenon, cooperation among groundwater users takes special significance. That cooperation, however, is not easily achieved, as demonstrated by the empirical evidence from countless aquifers that have been "overdrafted" or spoiled by induced seawater intrusion. The evidence on hand suggests that cooperation among groundwater users is not only arduous to achieve but, if achieved, unstable as well. The cooperators easily cheat each other and take more than their agreed-upon share of groundwater. Thus, any sustainable groundwater strategy requires the implementation of use rules (tantamount to some of the constraints written above) and effective enforcement to stamp out cheating and the unravelling of cooperation towards rapid resource depletion. Loáiciga (2003a,b) has provided an analytical treatment of cooperation and non-cooperation in groundwater exploitation, and of its theoretical linkages to business competition behaviour in a market economy, using game theory concepts (see Nash (1951) for a pioneering treatment of game theory and business competition).

An example

Loáiciga (2003a) showed that, under certain simplyfying assumptions, the objective function (7) subject to drawdown constraints (8) can be reduced to a quadratic programming linearly constrained (QLC) problem amenable to analytical solution. The solution consists of steady-state pumping rates at individual wells. The pumping rates are sustainable, that is, they yield the largest present value of the expected net revenue and meet drawdown constraints that ensure long-term, indefinite in fact, exploitation of an aquifer. The following simplifications were used by Loáiciga (2003a): (a) the drawdown caused by pumping at a well is well approximated by the Theis confined (unsteady) solution; (b) the cost of groundwater production by a well is linearly related to the drawdown at that well; (c) the pumping rates are constant over time; and (d) groundwater recharge is sufficient to support the optimal pumping rates. Under those assumptions, the QLC problem has the following structure:

$$\max_{w.r.t\ W_j} J = \sum_{j=1}^{n} \sum_{r=1}^{n} W_j\, W_r\, e_{jr} + \sum_{j=1}^{n} W_j\, f_j \tag{12}$$

subject to:

$$A_{ij}\, W_j \le b_i \quad i = 1, 2, \ldots, m \tag{13}$$

In equations (12)–(13), $W_j, j = 1, 2, ..., n$, is the pumping rate at j-th well (constrained to be non-negative); the coefficients e_{jr}, f_j, A_{ij}, j, $r = 1, 2, ..., n$, and b_i, $i = 1, 2, \ldots, m$ depend on the geometry of the well field, aquifer properties, pumping costs, price of groundwater, and several other factors. Equations (12)–(13) can be expressed in compact vector-matrix notation as follows (underlined letters denote vectors; the superscript T means transpose of a vector or row vector; letters not underlined denote matrices):

$$\max_{w.r.t.\ \underline{W}} \underline{W}^T E\, \underline{W} + \underline{W}^T\, \underline{F} \tag{14}$$

subject to:

$$A\,\underline{W} \leq \underline{b} \tag{15}$$

QLC problems whose structure is given by (14)–(15) have unique properties, and have received much attention in the theory of mathematical programming. A key property is that the objective function (14) is concave. Thus, if there is a feasible solution it is guaranteed to be a global maximum.

Figure 1 shows the type of solutions that can be obtained by solving the QLC problem. Figure 1 contains a graph of the sustainable pumping rates (in m^3 day^{-1}) in each of two wells 500 m apart. The sustainable pumping rates were obtained as a function of the market price of groundwater. It is seen that the pumping rate increases with increasing water price up to a price of about US\$1.7 m^{-3}. Thereafter, the pumping rate levels off at about 1800 m^3 day^{-1}. In spite of higher water prices the sustainable pumping rate must level off to avoid violating constraints, thus ensuring a steady stream of revenues and long-term groundwater exploitation without negative impacts.

Figure 2 depicts a graph of the present value of the (expected) net revenue as a function of the price of groundwater. In Fig. 2 the net revenue increases monotonically with increasing water price, and that the slope of the net revenue *vs* price relationship is steepest at low water prices. For a given water price, the calculated point in the graph of Fig. 2 is the corresponding net revenue, for which, in turn, there is an associated sustainable pumping rate previously graphed in Fig. 1 for that same water price.

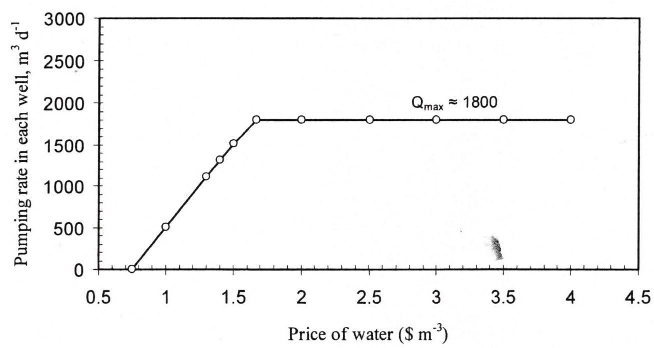

Fig. 1 Sustainable pumping rates obtained by solving the QLC problem.

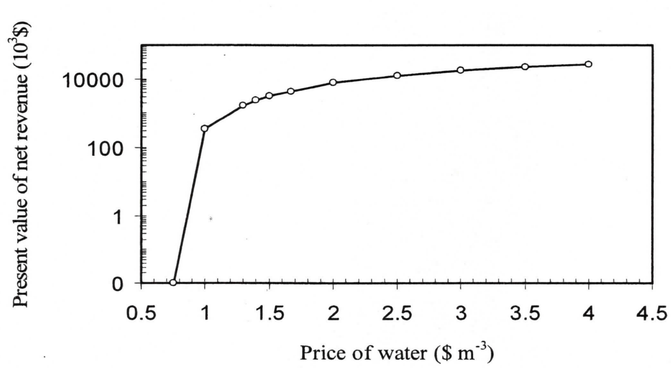

Fig. 2 The present value of net revenue as function of the price of groundwater.

The results shown in Figs 1 and 2 were obtained under the assumption that well operators cooperate and abide by all the sustainability (drawdown) constraints. Loáiciga (2003a) has shown that it is possible to obtain non-cooperative solutions with a modified QLC formulation. In that case, the pumping rates are not sustainable, the pumping period is shortened, and the net revenues drop precipituosly.

CONCLUSION

A theory of sustainable groundwater exploitation has been developed in this article. The core of the article is a general formulation of the mathematical programming problem whose solution—when it exists—produces sustainable pumping rates. A simplified quadratic, linearly constrained, version of the general formulation was implemented and solved to obtain sustainable pumping rates in terms of diverse economic and hydraulic factors. The implemented example illustrates the superior nature of sustainable groundwater exploitation when judged by aquifer performance and economic criteria.

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