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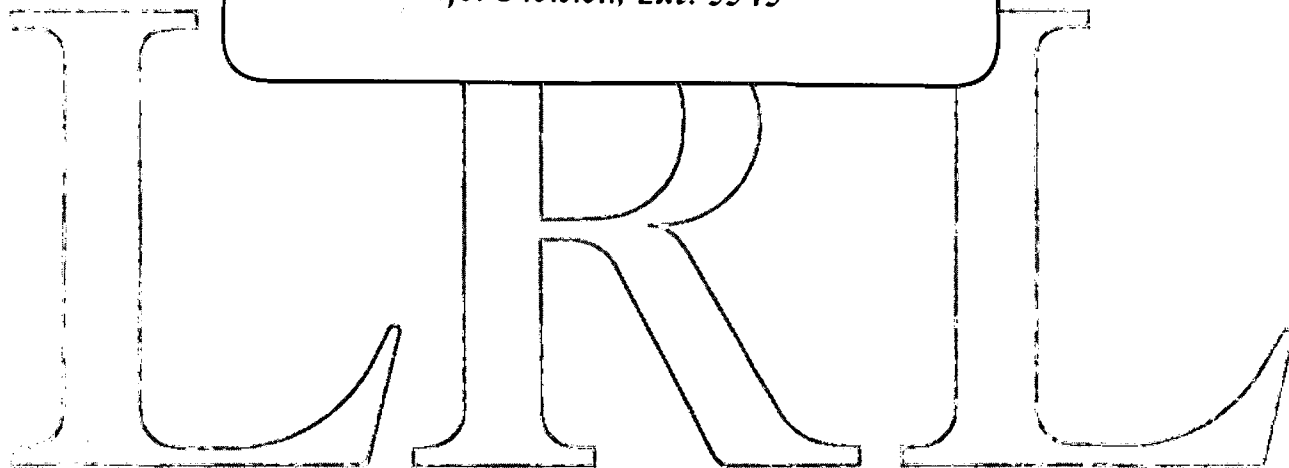
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PION-NUCLEON AND KAON-NUCLEON SCATTERING

IN THE VENEZIANO MODEL[*]

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May 8, 1969

ABSTRACT

We present a comprehensive phenomenological

examination of the Veneziano ansatz for pion-nucleon

and kaon-nucleon processes. Using invariant amplitudes

constructed as sums of beta-function terms, we attempt

to fit simultaneously all the relevant high and low

energy scattering data as well as the elastic widths

of baryon resonances. We discuss a useful technique

for insuring that the theoretical amplitudes will possess

the observed spin and parity structure of the physical

spectrum of baryon states.

Our main conclusions are:

(a) Sizeable subsidiary terms are required.

(b) The predicted duality relation between the

s-channel (baryon) and the t-channel (meson) Regge poles

is not supported quantitatively.

(c) Using the polynomial residue function suggested

by the model, we have performed detailed fits to all πN

backward data and elastic widths. The model fails to

provide an adequate extrapolation from the scattering

data to the widths of the $\Delta_\delta(1238)$ and its recurrences;

acceptable agreement is found for the other trajectories.

Moreover, the residues of the πN trajectories are in

marked disagreement with exchange degeneracy.

(d) Within a factor of two in amplitude, the model

reproduces available KN charge-exchange data from thresh-

old to the highest energy.

(e) A Pomeranchuk trajectory with normal slope

$(\alpha_P' \approx 1 \ (\text{GeV})^{-2})$ is consistent with both the Veneziano

model and all data.

(f) The model does not provide any natural reso-

lution of the difficulties inherent in classical Regge-

pole model fits and thus supports the view that Regge-cuts

are important.

INTRODUCTION

The proposal by Veneziano[1] of an elegant beta-function represen-
tation for the hadronic scattering amplitude has opened a new chapter
in theoretical investigations of strong interaction phenomena.[2] In a
simple closed form, the amplitude is analytic, crossing symmetric, and
has Regge-behavior at high energies. Moreover, in a straightforward
fashion, it can be expanded as a sum of zero total-width resonance-pole
terms, thus exhibiting a form of duality[3] by quantitatively associating
asymptotic behavior to low energy resonance structure in one simple
function. In addition, research has revealed an interesting relation-
ship of the representation to PCAC requirements for processes involving
pions.[4,5]

From the phenomenological point of view, the Veneziano repre-
sentation provides several attractive possibilities. It relates the
parameterization of the residue structure of a Regge pole to the
trajectory itself, removing the erstwhile freedom of an arbitrary
multiplicative form factor in the momentum transfer (t). Also, assuming
that a given physical process can be represented by the sum of a small
number of beta function terms, the representation provides a strong
quantitative connection between forward $(t \simeq 0)$ and backward $(u \simeq 0)$
scattering at high energy: features of the data which, dominated by
distinct Regge-pole exchanges, have until now seemed uncorrelated.
Furthermore, the extension of the beta-function representation from the
quasi-two-body scattering domain[1,2,4,5] to multiparticle processes
promises, among other things, an understanding of interference effects

at the locus of intersecting resonance bands in Dalitz plots.[4] This
last feature requires for its implementation, of course, some procedure
for overcoming the unitarity-violating zero-width aspect of the model.
These phenomenological consequences of the model are largely untested
as yet.

In this paper, we present a critical discussion of meson-baryon
scattering within the framework of the Veneziano model.[6] Because pion-nucleon
and kaon-nucleon scattering are perhaps the best studied hadronic
processes, both theoretically and phenomenologically, it should be
instructive to examine in detail the extent to which the model increases
our understanding of these processes. Our aim is to indicate in a
comprehensive fashion both the strong points and the limitations of the
Veneziano beta-function parameterization for these processes.

Very recently, enthusiasm has been generated for the point of
view which holds that the Veneziano form is to be regarded as a Born
approximation.[7] Presumably this means that "higher-order" terms would
be important in achieving agreement with nature; for example, a
"unitarized" version is suggested by Mandelstam to eliminate the parity-
doubling which occurs in his quark-substrate model even for meson
trajectories. The methods for obtaining the higher-order terms and/or
unitarity corrections are as yet ill-defined and likely to be involved.
In our investigation, we sought to represent meson-baryon scattering
simply as a sum of beta-functions, and our conclusions are limited to that
viewpoint.[8]

Whereas there are certain general aspects of the Veneziano-type model which are helpful in gaining a unified qualitative picture of strong interaction phenomena, our conclusions on the quantitative side are somewhat pessimistic. Detailed empirical knowledge of meson-baryon scattering is far more sophisticated than the capabilities of the model. To be sure, it is very possibly true that a finite set of resonance widths and a finite number of differential cross-section points can be fitted using a similarly finite number of beta-function terms. However, the more terms one is forced to employ, the smaller is the predictive or, even, the unifying content of the model. In particular, many features of the data enable one to demonstrate the existence of sizeable subsidiary[9] terms in the Veneziano expansion. Moreover, we find it impossible to achieve a compelling representation which properly relates even the magnitudes of the leading-trajectory baryon-resonance widths with the sizes of the forward and backward differential cross-sections.

Outside the realm of precise fits to data, however, some useful features emerge. Firstly, there is the relation between the asymptotic t-channel Regge pole parameters and the qualitative behavior of the s-channel resonances of the intermediate energy range. We have in mind, for example, the cross-over effect; this is discussed in Sections II-A and III. Secondly, the Veneziano representation yields a new method for quantitaively estimating the nonasymptotic corrections to the Regge formalism. This could be useful for determining how good a fit one should demand from a high energy approximation. Finally, as has been

pointed out by Virasoro and Amann,[6] it suggests a possibly useful

parameterization of the variation of baryon widths as function of their

mass.

In Section I, after establishing notation, we focus upon those

technical features of the Veneziano formula that are relevant to a

reasonable description of meson-baryon scattering. These include

signature, parity doubling, PCAC, positivity, and the absence of ghosts.

After these theoretical points, we review in Section II the experimental

picture of meson-baryon scattering which the Veneziano expansion should

reproduce. We examine several aspects of Regge-pole theory fits to

forward elastic scattering data, including exchange degeneracy and the

cross-over effects, and also study the nature of the Pomeranchuk

trajectory. For pion-nucleon forward elastic data, we present a good

fit to existing data using P', ρ, and Pomeranchuk pole trajectories,

all with normal slope (i.e. near 1.0 $(GeV/c)^{-2}$). The value for the

scale constant $s_o \simeq \frac{1}{\alpha'} \simeq 1$ suggested by the Veneziano formula is

consistent   with the forward data.

Apart from this treatment of the high-energy forward elastic

data, in Section II we also discuss the zeros in the scattering ampli-

tude at specific values of t and u required in order to obtain the

correct spin-parity structure of the baryon resonance spectrum. From

the Veneziano expansion, we subsequently extract a parameterization for

the residue functions of the various baryon trajectories. In general,

the model suggests that the reduced residue is a polynomial in $(s)^{\frac{1}{2}}$.

We determine the corresponding residues from the published data on
elastic widths, and present curves showing this empirical variation
with mass. We believe that plots of this type will also prove useful
in other reactions for summarizing the data and predicting the elastic
widths of undiscovered resonances.

In Section II-D, we give results of a comprehensive fit to all
high-energy backward πN scattering data. We employ the polynomial
form for residues, suggested by the Veneziano model, constraining them,
as much as possible, to reproduce the elastic widths of physical states
along the trajectories. Because it specifies the residue structure of
secondary trajectories, such as the N_γ, the Veneziano approach allows
us to go further than previous fits to backward data.

Some explicit beta-function representations for meson-baryon
scattering are presented in Section III, and there we analyze the
extent to which they realize the disiderata given in Sections I and II.
We concentrate on the kaon-nucleon process, and compare several
Veneziano-type parameterizations with both scattering data and resonance
widths. One exciting feature not present in traditional Regge fits is
the possibility of representing KN scattering data from threshold to
infinity with the same functional form.

In the final section, we summarize our conclusions.

The reader interested primarily in new phenomenological results
is directed to Sections II-A, II-D, and III.

I.   NOTATION AND TECHNICAL ASPECTS OF THE VENEZIANO REPRESENTATION

In this section we define our notation, discuss the choice of

an appropriate set of amplitudes, and give a general expansion for these

amplitudes in terms of Veneziano beta-function terms. Securing the

appropriate asymptotic behavior and spin structure of the amplitudes

imposes certain restrictions on the terms in the expansions. Further

limitations and relationships between terms arise from incorporating

into the representation general properties of meson-baryon scattering

such as signature for trajectories, positivity of resonance widths, and

absence of ghosts.

A.   Notation

The kinematics of meson-baryon scattering are relegated to

Appendix I. With reference to Fig. 1, we point out that the s and u

channels are meson-baryon channels, whereas the t channel is a meson-meson

channel. The description of pseudoscalar-meson and spin-$\frac{1}{2}$ baryon

scattering requires 2 x T independent amplitudes, where T is the total

number of distinct (conserved) values of total isospin. In Section I-B

we will elaborate somewhat on the alternative choices, but for

definiteness here we consider the standard invariant amplitudes:

$A^{(I)}$ (s,t,u) and $B^{(I)}$ (s,t,u) which are free of kinematical

singularities. The superscript (I) is an isospin index. In the

particular case of pion-nucleon scattering, we may use the amplitudes

$A^{(\pm)}$(s,t,u) and $B^{(\pm)}$(s,t,u) which have the properties

$$A^{(\pm)} (s,t,u) \;=\; \pm\, A^{(\pm)} (u,t,s) \qquad\qquad (1).$$

$$B^{(\pm)}(s,t,u) = \mp\, B^{(\pm)}(u,t,s) \tag{2}$$

and where the +(-) functions have pure isospin $I = 0$ (1) in the t channel.

If we adopt, as have other researchers, the view that a Veneziano-type representation should be established for the A and B functions, then we have, in general, expansions of the form

$$A^{(I)}(s,t,u) = \left\{ \sum_{\substack{\ell,m,n \\ B,M}} C_A^{(I)}(\ell,m,n,B,M)\, \frac{\Gamma(\ell - \alpha_M(t))\,\Gamma(m - \bar\alpha_B(s))}{\Gamma(n - \alpha_M(t) - \bar\alpha_B(s))} \right.$$

$$+ \sum_{\substack{g,h,i \\ B1,B2}} D_A^{(I)}(g,h,i,B1,B2)\, \frac{\Gamma(g - \bar\alpha_{B1}(s))\,\Gamma(h - \bar\alpha_{B2}(u))}{\Gamma(i - \bar\alpha_{B1}(s) - \bar\alpha_{B2}(u))}$$

$$\left. + \sum_{\substack{p,q,r \\ B,M}} E_A^{(I)}(p,q,r,B,M)\, \frac{\Gamma(p - \alpha_M(t))\,\Gamma(q - \bar\alpha_B(u))}{\Gamma(r - \alpha_M(t) - \bar\alpha_B(u))} \right\} \tag{3}$$

where $\alpha_M(t)$ denotes a particular meson trajectory and $\bar\alpha_B = \alpha_B - \frac{1}{2}$, with α_B being a baryon trajectory function. Similar expansions are appropriate for $B^{(I)}(s,t,u)$. The sums run independently over all meson trajectories, M, and over all baryon trajectories: B, B1, B2, appropriate to the process being considered, as well as over all integer values of ℓ, m, n, g, h, i, p, q, and r. Restrictions upon and/or relationships between the constants $C_A^{(I)}$, $C_B^{(I)}$, $D_A^{(I)}$, $D_B^{(I)}$, $E_A^{(I)}$, and $E_B^{(I)}$ arise from imposing certain physical requirements, such as:

(a) No states with exotic quantum numbers, as, for instance, $B = 0$, $I = 2$, and $B = 1$, $S = 1$.

(b) Absence of states with unphysical spin values; this restricts the values of the integers ℓ through r, as discussed in Section I-C.

(c) Appropriate spin-parity structure at the baryon resonance positions $(\bar{\alpha}_B(s) = k$, where k is an integer) and at the meson pole values $\left(\alpha_M(t) = k\right)$.

(d) Appropriate isospin structure at the baryon and meson resonance positions.

(e) Signature properties.

The requirements (c), (d), (e) are in practice applied only for the leading trajectories. Specifically (d) establishes algebraic relations between the coefficients for the different isospin values within the sets $\left\{ C_A^{(I)}, C_A^{(J)} \right\}$, $\left\{ D_A^{(I)}, D_A^{(J)} \right\}$, and $\left\{ E_A^{(I)}, E_A^{(J)} \right\}$. Secondly, (e) relates the constants C, D, and E, in pairs, for each isospin, I. Finally, (c) connects the two spin amplitudes A and B, whereas all other requirements apply to A and B separately.

The situation is much simpler in certain meson-baryon reactions where entire summations can be eliminated. For example, in kaon-nucleon scattering, because the K^+p system has no known prominent resonances, we want no terms involving $\alpha_B(u)$ (let the s channel

correspond to $\overline{K}N$ scattering); therefore only the terms of the first summation in Eq. (3) survive. On the other hand, consider $\pi^-\Sigma^+ \to \pi^+\Sigma^-$; since no $I = 2$ mesons are prominent, the first and third summations in Eq. (3) are eliminated, a priori.

In pion-nucleon scattering, the symmetry property given in Eqs. (1) and (2) can be directly employed to relate the constants of the third summation of Eq. (3) to those in the first, and also to prescribe structure within the second summation.

Later in this paper, when we discuss the spin-parity structure of the baryon states, it will prove convenient to use terms such as

$$(c_1 t + c_2 s + c_3) \frac{\Gamma(\ell' - \alpha_M(t))\ \Gamma(m' - \overline{\alpha}_B(s))}{\Gamma(n' - \alpha_M(t) - \overline{\alpha}_B(s))} \tag{4}$$

with c_1, c_2, and c_3 constants. This form is not quite that of a typical term in Eq. (3), but assuming linear trajectories, we can always rewrite it as a particular linear combination of terms within a given summation on the right-hand side of Eq.(3). A similar statement of course applies to (s,u) type terms.

The reader not familiar with the methods for extracting asymptotic behaviors and pole residues from Eq. (3) is referred to earlier literature. We now state our reasons for working with the invariant amplitudes, A and B, rather than with some other set.

B. CHOICE OF AMPLITUDES

Besides having the convenient crossing properties listed in Eq. (1) and (2), the invariant amplitudes $A^{(I)}(s,t)$ and $B^{(I)}(s,t)$ have simple asymptotic behavior, as $s \to \infty$,

$$A \sim s^{\alpha_M} \quad \text{at fixed } t$$

$$\sim s^{\bar{\alpha}_B} \quad \text{at fixed } u$$

$$B \sim s^{\alpha_M - 1} \quad \text{at fixed } t$$

$$\sim s^{\bar{\alpha}_B} \quad \text{at fixed } u \quad .$$

(5)

Other properties one would like to establish include positivity of pole residues, parity, and more generally the proper t dependence of the amplitude demanded by the observed spin-parity structure of the s-channel resonances. All of these are simply described only in terms of partial wave amplitudes, and there is no full amplitude which allows even a vaguely complete treatment.

One might like to use the t-channel nonflip amplitude

$A' = A + \dfrac{(s - u)}{4M(1 - t/4M^2)} B$ whose imaginary part in the forward direction is $\text{Im } A' = p_{lab} \, \sigma_{total}$ and so positive. Although this has good crossing properties, the kinematic singularity at $t = 4M^2$ would appear to be intolerable in the Veneziano approach which exploits analyticity in all three channels. Similarly $A + (s - u)B/4M$ reduces to A' at $t = 0$, but complicates the statement of Regge behavior at fixed u. Finally, the s-channel nonflip amplitude $A + E_{lab}(s)B$ would allow

quite a nice discussion of positivity but does not have good s $\longleftrightarrow$ u

crossing. This difficulty with crossing is not important for writing

terms that have only s and t channel poles°for these can be easily

symmetrized by writing s $\longleftrightarrow$ u terms for the crossed amplitude (both

in spin and isospin space) e.g. one writes (s,t) terms for

A + $E_{lab}(s)$B and $\pi^- p$ elastic scattering, then adds (s,u) terms for

A + $E_{lab}(u)$B for $\pi^+ p$ scattering. The difficulty arises in the treat-

ment of beta-function terms containing both s channel and u channel

poles.

Finding no sufficiently significant advantage in any of these

alternative choices of amplitudes, we present the remainder of this

discussion largely in terms of the traditional A and B functions.

C. Structure of the Veneziano Formula for A and B

The expansion of the functions $A^{(I)}(s,t,u)$ and $B^{(I)}(s,t,u)$

was given in Eq. (3). In this section, we will specialize to pion-

nucleon scattering for definiteness and elaborate somewhat on the

properties of individual terms in the expansions. As noted, the desired

amplitudes are expanded in a series of the form

$$\frac{\Gamma\big(m - \bar{\alpha}_B(s)\big) \, \Gamma\big(\ell - \alpha_M(t)\big)}{\Gamma\big(n - \bar{\alpha}_B(s) - \alpha_M(t)\big)} \quad , \tag{6}$$

and similar terms with s replaced by u, and

$$\frac{\Gamma\big(m - \bar{\alpha}_{B1}(s)\big) \, \Gamma\big(\ell - \bar{\alpha}_{B2}(u)\big)}{\Gamma\big(n - \bar{\alpha}_{B1}(s) - \bar{\alpha}_{B2}(u)\big)} \quad , \tag{7}$$

where m, n, and ℓ are integers and $\bar{\alpha}_B = \alpha_B - \frac{1}{2}$. For the baryon

trajectories, B, we can take N, Δ, or N_γ and for the mesons, M, we

have the exchange degenerate P' and ρ trajectories. Whereas this

exchange degeneracy $(\alpha_\rho = \alpha_{P'})$ is not necessary if one considers only

πN, it is enforced by factorization and the absence of resonances in

$\pi^+\pi^+$ and $\Sigma^+\pi^+$ elastic scattering. In particular, we cannot allow a

multiplicative fixed pole for the ρ so that like the P' its residue

function has a zero at $\alpha_\rho = 0$ in both A and B. This restricts ℓ

in (6) to be ≥ 1. There are no restrictions on the possible combinations

of trajectories in a single term. Moreover, as far as pion-nucleon

scattering itself is concerned, the Pomeranchuk trajectory (with any

slope) may be treated on an equal footing with the ρ and P' trajec-

tories and thus included as a possible candidate for $\alpha_M(t)$ in expres-

sion (6). However, in the SU(3) related kaon-nucleon process, treating

the Pomeranchuk in this manner, without an exchange degenerate partner,

would lead to the prediction of exotic resonances in the K^+p system.[10]

(The elastic widths of these exotic states are fairly narrow, however,

as we note in Section III-E.)

In Tables I and II, for the (s,t) and (s,u) terms respec-

tively, we record the nature of the asymptotic behavior associated with

various sets of integers (ℓ,m,n). We do this for only those terms

which contribute asymptotically to leading order in at least one

channel. In order to satisfy the limitations given in expression (5)

(and similar ones appropriate when $u \to \infty$), $n \geq m$, and $n \geq \ell$ in

all terms; moreover, for the (s,t) and (u,t) terms in B(s,t,u),

it is further necessary that n be $\geqslant$ m + 1. These conditions are

also required (but not sufficient) in assuring that the spin associated

with a particular pole be no greater than that of the appropriate

resonant state; to guarantee polynomial residues, m + ℓ $\geqslant$ n.

We note, in passing, that the absence of a physical state at

$\alpha_\triangle(s) = \frac{1}{2}$ does not prevent our using m = 0 in (6) and (7) when

dealing with that trajectory. Upon taking the appropriate linear com-

bination of (6) and (7) required to obtain the correct signature, we

will see that the residue at $\alpha_\triangle(s) = \frac{1}{2}$ is zero. Notice, however,

that the term with m = 0 couples nonasymptotically as s $\rightarrow$ ∞ for t

fixed in the A amplitude. However, only that m = 0 term provides a

large constant term in the $\triangle_8$ residue function; we elaborate on the

importance of this in Section II-C. For the SU(3) symmetric, kaon-

nucleon situation, there must be exchange degeneracy (and thus no

signature properties) for the baryon trajectories if exotic KN

resonances are to be avoided. Thus, the absence of a $J^P = \frac{1}{2}^-$ Y_1^*

requires m $\geqslant$ 1 for the SU(3) symmetric trajectory of $\triangle_8$. Similar

arguments in πN $\rightarrow$ KΣ imply m $\geqslant$ 1 for the $\triangle_8$ itself.

Beyond the restrictions of appropriate asymptotic behavior and

resonance angular momentum properties alluded to above, the other

elementary constraints on the selection of terms in Eqs. (3), (6), and

(7) include crossing symmetry, signature, positivity of resonance widths,

parity doubling, and isospin (or, more generally, SU(3) structure).

It is possible to guarantee these simply only for the leading trajectories

in the representation. We will now comment upon some technical questions

associated with these desirable properties.

D. Signature

Obtaining signature for trajectories in the Veneziano model is by no means natural. Both K. Igi and M. Virasoro,[6] however, chose to impose signature in a manner which strohgly coupled the overall contributions of the various baryon trajectories. Specifically in their solutions, a term of the form

$$c \, \frac{\Gamma\!\left(\ell - \bar{\alpha}_{B1}(s)\right) \Gamma\!\left(\ell - \bar{\alpha}_{B2}(u)\right)}{\Gamma\!\left(\ell - \bar{\alpha}_{B1}(s) - \bar{\alpha}_{B2}(u)\right)} \quad , \tag{8}$$

which contributes in leading order both as $s \to \infty$ at fixed u and as $u \to \infty$ at fixed s, served to generate the signature properties for both baryon trajectories α_{B1} and α_{B2}. Therefore the multiplicative coupling strength constant, c, enters into the definition of the residues of both trajectories and thus, for example, would tend to associate $(g^2/4\pi)$, the pion-nucleon coupling constant, to Γ_{Δ}, the elastic width of the 1238 resonance, in too restrictive a fashion. The magnitudes of the asymptotic N and Δ exchange magnitudes are also constrained to be of similar magnitude in this type of solution. A more realistic and less restrictive solution necessarily involves some terms which contribute to nonleading order in at least one channel. As will be established in detail in Section II, these subsidiary terms are also strongly suggested by other features of the low-energy resonance structure.

E. Parity-Doubling

A naive Veneziano formula for meson-baryon scattering generates parity-doubled trajectories. This is a general feature of all theoretical models that enforce analyticity at $s = 0$. If we cunningly adjust the constants multiplying the subsidiary terms in Eq. (3), we may abolish these parity doubled states. This is possible for the leading trajectories, but not for the lower lying trajectories, if one wishes to retain Regge asymptotic behavior.

The partial waves of definite parity are given by (let $\ell = J - \frac{1}{2}$)

$$a_{\tau P+}{}^{J} = \frac{1}{4} \int_{-1}^{+1} dz \; \{(P_{\ell+1} + P_{\ell})(f_1 + f_2) - (P_\ell - P_{\ell+1})(f_1 - f_2)\}$$
(9)

$$a_{\tau P-}{}^{J} = \frac{1}{4} \int_{-1}^{+1} dz \; \{(P_{\ell+1} + P_{\ell})(f_1 + f_2) + (P_\ell - P_{\ell+1})(f_1 - f_2)\} \; .$$
(10)

Now near $t = 0$,

$$P_\ell - P_{\ell+1} \; \sim \; 1 - z \; \propto \; t/s \tag{11}$$

and

$$P_\ell + P_{\ell+1} \; \sim \; 2 \; . \tag{12}$$

Moreover,

$$\frac{8\pi(s)^{\frac{1}{2}}}{2M}(f_1 + f_2) = A + E_{lab}B \sim s^{\alpha_M(0)} \tag{13}$$

$$\frac{8\pi(s)^{\frac{1}{2}}}{2M}(f_1 - f_2) = \frac{EA}{M} + \omega B \sim s^{\alpha_M(0)+\frac{1}{2}} . \tag{14}$$

Thus, as the integrals are presumably dominated by the forward peak, the $(f_1 + f_2)$ term dominates, whence $a_{\tau P+}{}^J \sim a_{\tau P-}{}^J$. It follows that it is impossible not to have some parity-doubling, and so one can only consider removing the doubling for the leading trajectories, with $\ell > (s)^{\frac{1}{2}}$, which eventually decouple from the cross section. However, even this modest aim is not easy, and in practice, one puts zeros into the residue functions by hand at the positions of the unwanted resonances.

F. Positivity

(1) A remarkable feature of the Veneziano form for π-π scattering was that it gave positive widths for essentially all resonances, if a reasonable intercept for the ρ meson trajectory was employed.[11] One might expect a similar situation in meson-baryon scattering. However, one even runs into trouble for the leading trajectory in πN, whereas this was quite trivial in the $\pi\pi$ case. The width of a resonance of spin j and mass M_R on the leading trajectory is

$$\Gamma_{M_R} = \frac{R_i}{4\pi(M_R)^2} \, q^{2j} \, 4^{j-\frac{1}{2}} \, \frac{\Gamma(j + 1/2) \, \Gamma(j + 3/2)}{\Gamma(2j + 2)} \qquad (15)$$

where

$$i = 1 \quad \text{for} \quad \tau P = -1$$

$$i = 2 \quad \text{for} \quad \tau P = +1$$

and the leading trajectory residue is

$$R_i = \lim_{s \to M_R^2} \left\{ (M_R^2 - s) \, 8\pi(s)^{\frac{1}{2}} \, f_i / t^{j-\frac{1}{2}} \right\} \, . \qquad (16)$$

It is evidently convenient to treat both parities together, however, and to use, say $\tilde{f}_1 = A + [(s)^{\frac{1}{2}} - M]B$. One considers then the poles for both positive and negative values of $(s)^{\frac{1}{2}}$. In this approach, positivity of Γ_{M_R} implies that the residue must be positive (negative) for $(s)^{\frac{1}{2}}$ positive (negative). Therefore, because the residues of the poles in A and B are functions of s and not $(s)^{\frac{1}{2}}$, we deduce that at a given pole, the leading power of s in the residue for A must be no higher than for B. This requirement is not trivial because, in constructing the functions A and B, the simplest choice for the N_α trajectory is to make B's residue contribution a constant whereas A must be at least linear in s, if the parity-partner state to the nucleon is to be abolished. We will illustrate this further with examples in Sections II-C and III.

The situation is further complicated by the fact that in nature, parity partners are evidently extinguished, at least for the lower-lying recurrences.

(2) In order to gain further insight into the matter of guaranteeing positivity for all baryon resonance states, we review the $\pi\pi$ situation more fully.[11] It is possible to present a simple, if non-rigorous, argument leading to an inequality which guarantees that all but a finite number of states will have positive widths. One begins with a single (s,t) Veneziano-type form, e.g.

$$\frac{\Gamma\big(m - \alpha_1(s)\big)\ \Gamma\big(\ell - \alpha_2(t)\big)}{\Gamma\big(n - \alpha_1(s) - \alpha_2(t)\big)} \ . \tag{17}$$

At the pole position $\alpha_1(s) = m'$, the residue is

$$\frac{(-1)^{m+n-\ell}}{\Gamma(m' - m + 1)}\ \frac{\Gamma(N + y)}{\Gamma(y)} \tag{18}$$

where

$$y \ = \ \alpha_2(t) - \ell + 1$$

and

$$N \ = \ m' + \ell - n \ .$$

The modulus of this function is symmetric about $y = -\frac{1}{2}(N - 1)$. To avoid having a backward peak whose magnitude is as large as that of the forward peak (albeit with oscillating sign, as one moves up through the successive s-channel resonances), the last zero of (18), for $y < 0$, must fall very near or outside the physical region boundary $(u \sim 0)$. This implies that

$$\alpha_1(0) + \alpha_2(0) + \Sigma \underset{\sim}{\geq} n \qquad\qquad (19)$$

where Σ is the sum of the squares of the external masses.

In $\pi\pi$ scattering $\Sigma \sim 0$ and $n \geq 1$ gives $\alpha_\rho(0) \underset{\sim}{\geq} 0.5$. In the πN scattering the application of Eq. (19) is beclouded. We really should use f_1 and f_2 not A or B, to start considering positivity, and even then necessary conditions are difficult to find. Moreover, we should employ SU(3) symmetry to find reactions with exotic channels so as to be able to apply Eq. (19) to (s,t) and (s,u) terms separately.

We may continue to use Eq. (19) as a guide, remembering to use $\alpha_i = \alpha_B - \frac{1}{2}$ for baryons. Then the inequality is less stringent than in $\pi\pi$, because $\Sigma \sim 1.80$ not 0.08. However, one can draw the general conclusion that for high lying baryon trajectories, such as Δ_δ, we can expect many Veneziano-type terms, whereas for the lower-lying N_γ we are restricted to fewer terms in Eq. (3).

G. PCAC

Ademollo et al.[5] have made the interesting observation that the PCAC condition plus the beta-function representation imply the (approximately valid) quantization relations: $\alpha_\Delta(0) - \alpha_N(0) = 0.5$ and $\alpha'_\Delta(0) = \alpha'_N(0)$. The PCAC condition in, say, πN or $\pi\Sigma$ scattering may be stated as $A'(s = M^2, t = 0, \mu^2 = 0) = 0$ where M is the mass of the external baryon and μ that of the external meson. (See Section I-B for a definition of A'.) Take, for simplicity, πN elastic scattering and treat only the s-t terms. (The neglect of s-u terms

may be justified by considering $\pi^- \Sigma^+ \to \pi^- \Sigma^+$.) Then the quantization

relation ensures that the Δ contribution vanishes in the PCAC limit if

we take for A either of the forms:

$$\frac{\Gamma(1 - \bar{\alpha}_\Delta) \, \Gamma(1 - \bar{\alpha}_\rho)}{\Gamma(1 - \bar{\alpha}_\Delta - \alpha_\rho)} \quad \text{or} \quad \frac{\Gamma(- \bar{\alpha}_\Delta) \, \Gamma(1 - \alpha_\rho)}{\Gamma(1 - \bar{\alpha}_\Delta - \alpha_\rho)} \quad ;$$

these will be seen in Section III to be reasonable as first guesses.

Unfortunately, there remain terms involving the nucleon trajectory.

The simplest form of these is $\quad A = a_N \dfrac{\Gamma(1 - \bar{\alpha}_N) \, \Gamma(1 - \alpha_\rho)}{\Gamma(1 - \bar{\alpha}_N - \alpha_\rho)} \quad$ and

$B = b_N \dfrac{\Gamma(- \bar{\alpha}_N) \, \Gamma(1 - \alpha_\rho)}{\Gamma(1 - \bar{\alpha}_N - \alpha_\rho)}$. Then if $\mu^2 = 0$ the amplitude

$Z = A + \dfrac{(s - M^2)}{2M} B$, which is A' at $t = 0$, takes the form

$[a_N - b_N/(2M \, \alpha_N'(0))] \dfrac{\Gamma(1 - \bar{\alpha}_N) \, \Gamma(1 - \alpha_\rho)}{\Gamma(1 - \bar{\alpha}_N - \alpha_\rho)}$. Choosing $a_N = b_N/(2M\alpha')$

achieves the PCAC result, but at the cost of decoupling the nucleon trajec-

tory entirely from this amplitude. This violates positivity (Z must be

positive at $t = 0$) and the desired isospin structure of the high

energy behavior in A'. On the other hand, as we shall see in Section

II-B, the ratio $Z(t = 0)/B(t = 0)$ is rather small $\left(\sim 0.1 \text{ for the} \right.$

D13 (1520) $\left. \right)$ and so indeed $a_N \sim b_N/(2M\alpha')$ to 10%. This does not seem

quite good enough; perhaps one should enforce the PCAC result

$a_N = b_N/(2M\alpha')$ exactly, and then add small terms to Z of the form

$$Z = (z_1 s + z_2 t + z_3) \frac{\Gamma(1 - \bar{\alpha}_N)\,\Gamma(1 - \alpha_\rho)}{\Gamma(2 - \bar{\alpha}_N - \alpha_\rho)}$$

where z_1, z_2, and z_3 are arranged so as to obtain the PCAC vanishing along with a more satisfactory value of Z at $t = 0$ for the higher recurrences of the nucleon. It is curious that whereas in π-π scattering,[2,5,6] positivity and PCAC seem to be correlated, a similarly naive beta function choice in πN scattering only achieves PCAC at the cost of violating positivity.

II. EMPIRICAL KNOWLEDGE OF MESON-BARYON SCATTERING

To ascertain which particular linear combination of beta-function terms should be written for $A^{(\pm)}(s,t,u)$ and $B^{(\pm)}(s,t,u)$, we must first abstract from the data on πN and KN scattering certain salient structure for these amplitudes, as a function of s and t separately. We may then hope to build that structure into our choice of terms. There are several noteworthy features associated with particular values of t. In the first subsection, we focus upon the forward elastic scattering region in πN and KN processes. We stress the conclusions drawn from present Regge-pole theory fits both as to the relative magnitudes of the various exchanges and as to the structure in t of their residue functions. In the process of our investigation, we refitted existing data and, as a byproduct, obtained a good fit to πp elastic data using a normal slope Pomeranchuk trajectory. The remaining subsections deal with various aspects of the baryon spectrum. We first discuss and tabulate the zeros of the amplitudes at certain values of t and u required in order to obtain the correct spin-parity structure of the s-channel resonances. The correspondence of the positions of these zeros with analogous structure in differential cross-sections is emphasized. Next, from the general form of the Veneziano expansion, we extract a formal parameterization for both the elastic widths of the resonances on parent baryon trajectories and the backward elastic scattering data. This leads to the definition of a reduced residue function based on the model. From the data, we extract the empirical

values of the residue function and quantitatively examine in the last subsection the adequacy of the Veneziano model parameterization of these baryon residues.

A. Properties of $A(s,t,u)$ and $B(s,t,u)$
Following From High-energy Forward
Scattering Data

In this section we present a review of the Regge pole description of πN and KN forward elastic scattering and examine critically the status of various features of the model fits. We take the usual model of P, P', ω, ρ, and A_2 exchange, with the last four trajectories having intercept $\alpha(0) = 0.4 \to 0.65$ and slope $\alpha'(0) \approx 1$. In the exchange degenerate limit described by the Veneziano formula, these four poles have a common intercept and slope. For definiteness, suppose the high-energy limit of the amplitudes A' and B is

$$A'^{(i)}(s,t) = a_c'^{(i)}(t) \, \eta^{(i)}(t) \, s^{\alpha^{(i)}(t)}$$

and   (20)

$$B^{(i)}(s,t) = b_c^{(i)}(t) \, \eta^{(i)}(t) \, s^{\alpha^{(i)}-1}$$

where $\eta^{(i)}$ is the signature factor, given by

$$\eta^{(i)} = \frac{1 + \tau^{(i)} e^{-i\pi\alpha^{(i)}}}{\sin \pi\alpha^{(i)}}$$

and (i) labels the contribution of a given pole. The functions $a_c'(t)$ and $b_c'(t)$ are classical Regge-pole model residues whose structure is given below in Section (1).

We have placed the scale factors, $s_o = 1$, as suggested by the

Veneziano formula when a trajectory has slope 1. In this regard, it is

certainly interesting to note that all experimental data which call for

a slope $\alpha'(0) \approx 1$ indeed have a t-dependence consistent with the

scale factor $s_o \approx 1$.[12] For general slope α', the Veneziano formula

predicts $s_o = 1/\alpha'$; but this value is certainly not consistent with

that data indicative of a trajectory slope much less than 1. In

particular, if one wishes to incorporate the Pomeranchuk trajectory

into a Veneziano form, as suggested by Wong,[10] the experimental t

dependence necessitates a trajectory slope of approximately unity. In

this section, we will consider both cases - one in which the Pomeranchuk

trajectory has a universal slope near one and the other in which it

has a very small slope.

(1) Structure in t of the residues

For both cases, independent of the Pomeranchuk trajectory's

properties, the data suggest a residue zero at $t = -0.2 \, (\text{GeV}/c)^2$ in

those nonflip amplitudes, A', associated with ρ, ω, and A_2 quantum

numbers. (i.e. the corresponding $\alpha_c'(t)$ are proportional to $t + 0.2$.)

The supporting evidence is the cross-over feature in the πp, Kp, and

pp elastic differential momentum transfer distributions.[13] In addition,

a residue zero in the ρ quantum numbers flip amplitude, B, at the

point $\alpha_\rho \simeq 0$, $t \simeq -0.6 \, (\text{GeV}/c)^2$, is prescribed by πN charge exchange

data.[13] The P' and A_2 residue functions for both A' and B must

contain zeros at $\alpha_{P'} = 0$ and $\alpha_{A_2} = 0$, respectively, to eliminate

ghosts at those positions.

We now invoke the concept of exchange degeneracy and conclude

that (with $\alpha_{P'} = \alpha_\rho = \alpha_{A_2} = \alpha_\omega$) both A' and B should possess the

residue zero at $\alpha = 0$ for each t-channel quantum number. In classical

language, this corresponds to the Regge poles choosing nonsense at

$\alpha = 0$. This α factor is, indeed, generated automatically by the

Veneziano approach which suggests

$$a'_c(t) = \pi \, a'(t)/\Gamma(\alpha)$$

(21)

$$b'_c(t) = \pi \, b(t)/\Gamma(\alpha)$$

with $a'(t)$ and $b(t)$ nonsingular.

The zero in A' at $t \approx -0.2 \; (\text{GeV/c})^2$ is more controversial.

We consider first associating it with the residue of each Regge-pole.

Via exchange degeneracy, the observed ω zero at $t \approx -0.2 \; (\text{GeV/c})^2$ suggests

a similar one in the P' amplitude. This provides the appealing possibility

that the Pomeranchuk trajectory can have universal slope of approximately

one. The sign change in the P' - Pomeranchukon interference term at

$t \sim -0.2 \; (\text{GeV/c})^2$ will yield the observed lack of shrinkage in πp

scattering. In Fig. 2a, we demonstrate the fit achieved to πp forward

elastic scattering in this fashion. The details of this fit are exiled

to Appendix 2; for comparison, we present a standard[14] low slope

Pomeranchukon fit in Fig. 2b. Moreover, it is also attractive for

duality reasons to associate the $t \sim -0.2 \; (\text{GeV/c})^2$ residue zero with

the contributions of these poles. As emphasized by Dolen, Horn, and

Schmid,[3] and by Dikmen,[15] the contributions of the prominent s channel

resonances in πp and Kp scattering vanish at $t \sim -0.2$ $(GeV/c)^2$ in

the nonflip amplitude, A', and at $t \sim -0.6$ $(GeV/c)^2$ in the flip

amplitude, B. (To be described in Section II-B) According to reasoning

similar to that of Harari, therefore, the contributions of the t-channel

Regge poles, except for the Pomeranchukon, should have the same zeros.[16]

Unfortunately, however, the above scheme is clearly inconsistent

with factorization. From the residue zero in A' at $t \sim -0.2$ $(GeV/c)^2$,

one derives a square root zero associated with all vertices $K\bar{K}M$, $\pi\pi M$,

and $p\bar{p}M$ where M is the Regge-pole exchanged. This, in turn implies

an unobserved residue zero in the spin flip B amplitude also at

$t \sim -0.2$ $(GeV/c)^2$. Furthermore, the requirement of exchange degeneracy

itself forces an unobserved zero in the ρ and ω residue functions

in A' at $\alpha = 0$, $t \sim -0.6$ $(GeV/c)^2$.[17]

To get around these difficulties one must add additional

t-channel effects besides the Regge poles listed above. There are two

ways to do this, and the different methods suggest quite different

t-channel structure for Veneziano representations of KN and πN

scattering. The first method is illustrated in Fig. 3a. If we

continue to associate zeros at $t \sim -0.2$ $(GeV/c)^2$ with the Regge pole

residues, as above, then some secondary trajectory or cut mechanism

must serve to cancel the $t \sim -0.6$ $(GeV/c)^2$ zero from A'. This way

out has the feature of retaining duality of leading t channel Regge

poles with leading s channel resonances, and presumably the effect

required could be quantitatively small.[18] To avoid the factorization

induced zero in B at $t \sim -0.2$ $(GeV/c)^2$, one must conclude that it

is inappropriate to impose factorization on nonunitary solutions, such as those of the Veneziano-type. We will say more about this later.

The second method for curing the difficulties requires disassociating the $t \sim -0.2$ zero from a single Regge pole residue altogether. It is illustrated in Fig. 3b. The leading Regge pole is presumed to have only the one zero at $\alpha = 0$ in A'. A secondary trajectory or cut[19] having the same quantum numbers interfers with the leading pole and the zero of the effective amplitude is moved to $t \sim -0.2$ $(\text{GeV}/c)^2$. This remedy preserves factorization of pole residues but disagrees with the duality picture given above. However, it may be that the duality argument was too simple. Inasmuch as it is only the leading baryon resonances whose contributions vanish at $t \sim -0.2$ $(\text{GeV}/c)^2$, one may imagine that if the contributions of the entire, degenerate Veneziano-model tower of baryon resonances were included, then the duality zero would be moved out to $t \sim -0.6$ $(\text{GeV}/c)^2$. An altered form of duality between resonances and Regge poles could thus be restored, although the asymmetry between A' and B seems inelegant. Moreover, if we define "background" to be "experiment" minus the contributions of leading trajectories, we see that this background would not have the Pomeranchuk quantum numbers, as conjectured by Harari.[16]

The discussion of the above few paragraphs pertained to a view of data in which the Pomeranchuk trajectory has near normal slope of approximately unity. The standard fit of Rarita et al.,[14] concluded that a smaller slope $0 \leqslant \alpha_P' \leqslant 0.3$, is preferred. This alternative allows, indeed somewhat prefers, the P' trajectory to vanish at larger $|t|$.

For instance, Barger and Phillips[20] have suggested an extra

zero at $t \sim -0.6$ $(GeV/c)^2$. Thus, in the model in which the P has

normal slope, A' for the P' has residue zeros at $t \approx -0.2$ and -0.6

$(GeV/c)^2$; the models in which P has a small slope lead to a rough

coincidence of the two zeros at $t \approx -0.6$ $(GeV/c)^2$. This version is

now not (obviously) inconsistent with factorization for the P', but,

nevertheless, the low slope Pomeranchukon model does not cure the

$t \approx -0.2$ $(GeV/c)^2$ factorization difficulties connected with the other

quantum numbers.

(2) Numerical values of the residues at $t = 0$

We now turn to a discussion of the numerical values of the

residues which we should try to reproduce with our Veneziano parameteriza-

tion. Having discussed the t variation above, we need consider only

the values at $t = 0$. One may also hope that the effects of cuts will

be at their smallest there. The values of α' may be determined best

from data on σ_{total},[21] and the errors estimated from varying the

intercepts over the range $\alpha(0) = 0.4 \rightarrow 0.6$. The values of b for

ρ and A_2 exchange are determined from fits to πN and KN charge

exchange data, which are dominated by the B amplitude; errors may be

estimated from varying the parameterization of α' and from the

effect of different methods of achieving the nonzero polarization in

πN charge exchange. The values of b for the P' and ω are

essentially undetermined by the high-energy fits and are known only

through the use of FESR.[13,22]

We list numerical values in Table III for the πN and KN couplings, with our estimates of the errors. Our isospin conventions are given in Appendix I.

We note, in passing, that the reduced residue of the Pomeranchukon is not particularly different from that of the P'. There is also no explanation of the fact that the ratio $a'/(\nu b)$ for the P and P' is such that there is little or no πN polarization from their interference. These would not seem to be properties of two objects which the low slope (or fixed pole/cut) P models claim to be quite different entities.

(3)   Theoretical interpretation

To summarize part (1) of this section, we claim that there is already disagreement between theory and experiment and that a specific model, like the Veneziano form can only make things worse. In any case, one must put a zero in the P', ω, A_2, and ρ residues in A and B at $\alpha = 0$. We then have the two possibilities given below:

(a) Model of Figure 3(a)

Here we arrange the values of A and B, for each pole, so that when A' is formed, the zero at $t \approx -0.2$ $(GeV/c)^2$ will be generated. This, as described in (1), is superficially consistent with the s-channel resonances. The unobserved zero at -0.6 $(GeV/c)^2$ in A' for ρ and ω is deemed to be filled in by cuts. Similarly, the factorization crisis predicted by this model: namely a double zero at -0.2 in pp and $\bar{p}p$ scattering, not the desired single one, is cynically swept aside by not considering baryon-baryon reactions.

and not to (a). As we can see from the values of $a'/\nu b$ in Table III,
if a and b are constant in t, (as is true in the simplest Veneziano
parameterization), then only for the ρ and A_2 will the very small
ratio $a'/\nu b$ lead to the -0.2 zero: here coming from the
$(4M^2 - t)$ factor in the definition of a',

$$a' = a + 2Mb/(4M^2 - t) \ .$$

Any correspondence between the crossover zero and the s-channel
resonances is reduced to the level of an accident: the resonances have
a zero at t $\approx$ -0.2 (GeV/c)^2 in A + νB/(2M)(= A' at t = 0) whose
asymptotic limit, $a + (b/2M)$, would have no zero, for constant a
and b.

B. The Baryon Spectrum and the t-dependence
of A and B at a Resonance Pole

Perhaps due to our more detailed knowledge from phase shifts,
the baryon spectrum appears to be more complicated than that of the
mesons. We must consider both the internal symmetry (SU_2 or SU_3)
and the spin-parity structure of these resonances.

(1) Exchange degeneracy - internal symmetry

Many authors have considered the consequences of exchange
degeneracy.[23] In $\overline{K}N$ scattering there is a rather exact degeneracy
between the I = 0 SU_3 partners of the N and the N_γ and between
the I = 1 SU_3 partners of the Δ and the D15 ("parity partner of
the nucleon's first recurrence"). Such a degeneracy is of course a

minimum prerequisite for the successful application of the Veneziano

form to $\overline{K}N$ scattering, but also suggests that we should treat the

N/N_γ and the $\Delta/D15$ pairs of trajectories symmetrically in other SU_3

related reactions (such as πN scattering itself) where the mass

degeneracy is not apparent. Similarly, $\pi\Sigma$ scattering contains two

channels $\pi^-\Sigma^+ \to \pi^-\Sigma^+$ and $\pi^-\Sigma^+ \to \pi^+\Sigma^-$, which have only s-t and

s-u terms, respectively. Here, exchange degeneracy is required, in

the Veneziano approach but in nature it does not appear to be quite as

precise as in $\overline{K}N$ scattering.

(2)   Spin-parity structure

As emphasized by Harari[24] and by Mandelstam,[7] the nonrelativistic

quark model appears to predict the observed states very well. The

s-wave $(56, 0^+)$, the p-wave $(70, 1^-)$, and d-wave $(56, 2^+)$ are evident.

Furthermore, there is a radial excitation, another $(56, 0^+)$. Unfor-

tunately, such a structure is manifestly inconsistent with any simple

model that incorporates analyticity at $u = 0$: MacDowell symmetry

predicts unobserved parity-doubled states for the leading trajectory.

Similarly, the theoretical and experimental structure of the daughters

will be in disagreement. We do not know of any fundamental solution

to this problem but will instead adopt a phenomenological approach.

Dolen, Horn, and Schmid[3] emphasized the correlation between the

zeros of the t-channel Regge-pole residues and those of the s-channel

resonance contributions. In Table IV, for some of the low-lying resonances

in πN and KN scattering, we give the t and u positions of the zeros

of A, A', and B. We observe that all three show interesting systematic effects. The Δ trajectory has zeros in A at $t \sim -0.6, -1.6, \cdots$; and in B at $t \sim 2.5, -0.6, -1.6, \cdots$. The D15 has a similar zero structure, supporting its classification as exchange degenerate with the Δ. The zeros for $t < 0$ can be obtained from the $\Gamma^{-1}(+ \alpha_\rho) = 0$ factor in the Veneziano form, whereas the $t \sim 2.5$ zero in B, crucial in ensuring, as experimentally observed, that the Δ have no daughter, must be explicitly put in by hand: i.e. by writing

$$\frac{\Gamma\left(\ell' - \alpha_\rho(t)\right) \Gamma\left(m - \bar{\alpha}_\Delta(s)\right)}{\Gamma\left(n' - \alpha_\rho(t) - \bar{\alpha}_\Delta(s)\right)} \, (t - 2.5).$$

The nucleon and N_γ contributions seem consistent with just the zeros from $\left(\Gamma(\alpha_\rho)\right)^{-1}$. One may investigate the zero structure in various SU_3 related reactions to see if they change according to the precise value of the intercept of the t-channel pole. In $\pi N \rightarrow K\Lambda$ (or $\rightarrow K\Sigma$) for instance, the zero at $t \sim -0.6$ moves to ~ -0.4 which correlates with a break in the high energy cross-section at this value and suggests a value of $\alpha_{K^*}(0) \simeq 0.4$ $\left(K^*(890)\right.$ or $\left. K^*(1400)\right)$.

The approximate matching of the zeros of the Veneziano form and those of the experimentally prominent resonances eliminates unobserved daughters of large width. However, one still must ensure that A and B (and hence A') have the right relative magnitude. We thus list in Table V the relative values of A, B, and A' at $t = 0$ for our resonances. We note that A' is often much smaller than A or B and this goes hand in hand with the celebrated "cross-over"

zero in A' near $t \sim -0.2$, discussed in the previous section. The

overall magnitude (i.e. width) of the resonance is best handled by the

plots in Section II-C.

Finally, we remark on the systematics of the u-zeros which are

useful for constructing s-u terms. The Δ family has, in B, one

scurrilous zero at a u value corresponding to the $t \sim 2.5$ zero, in

addition to the regular family at $u \sim -0.4$, -1.4, $\cdots$. The latter are

presumably associated with some linear combination of zeros from

$\Gamma^{-1}(+ \bar{\alpha}_N)$, $\Gamma^{-1}(+ \bar{\alpha}_{N_\gamma})$, and $\Gamma^{-1}(-1 + \bar{\alpha}_\Delta)$. The existence of more than

one u-channel trajectory clearly complicates the problem. The Δ

family in A has the same $u \sim -0.4$, -1.4 series, plus another single

zero at $u \sim 0.7$, approximately the nucleon position. The association

of this zero with the nucleon is supported by a similar analysis of

$\pi\Sigma$ elastic scattering, where the corresponding zero becomes

$u \sim 1.2 \sim m_{\Sigma,\Lambda}^2$. Such a correlation implies a particular relation between

the SU_3 mass-splitting of the external and internal particles.

The nucleon family has the same structure in A and B, with

a rough zero sequence $u \sim -0.2$, -1.2, $\cdots$.

C. General Expression for the Baryon Residue Functions

In this section we present the general parameterization for

the elastic widths of resonances on the parent baryon trajectories and

for the backward angles differential cross section, as prescribed by

the Veneziano model expansion of the amplitudes. The parameterization

is the product of an essentially kinematical factor times an energy

dependent polynomial controlled by the constants which multiply the

various beta-function terms in the Veneziano expansions, Eq. (3). Less

complete but similar discussions were given previously by Amann[6] and

Virasoro.[6] We then treat the experimental data in analogous fashion;

after dividing empirical resonance widths by the above-mentioned

kinematical factor, we present the resulting reduced widths as a

function of resonance mass. The energy dependence of this reduced

residue function, coupled with information on the residue culled from

backward scattering data, should enable one to judge how many terms are

required in the polynomial, and thus to estimate the complexity required

in a Veneziano parameterization of meson-baryon scattering.

We begin by extracting from Eq. (3) only those terms which

contribute to the widths of resonances on the general baryon-parent-

trajectory, $\alpha_B(u)$; the required terms are those from the second

summation with $g = i$ and those from the third with $p = r$. We note

that this set of terms also supplies the leading Regge behavior

$s^{\alpha_B(u)-\frac{1}{2}}$ for large s at fixed u. Denoting this part of the ampli-

tude, $A^{(I)}$, we have

$$
A^{(I)} = \sum_{\substack{q=0,1,2\cdots}} \left\{ \sum_{\substack{p \geqslant q \\ M}} E_A^{(I)}(p,q,M) \frac{\Gamma\!\big(p - \alpha_M(t)\big)\,\Gamma\!\big(q - \bar{\alpha}_B(u)\big)}{\Gamma\!\big(p - \alpha_M(t) - \bar{\alpha}_B(u)\big)} \right.
$$

$$
\left. + \sum_{\substack{g \geqslant q \\ B1}} D_A^{(I)}(g,q,B1) \frac{\Gamma\!\big(g - \bar{\alpha}_{B1}(s)\big)\,\Gamma\!\big(q - \bar{\alpha}_B(u)\big)}{\Gamma\!\big(g - \bar{\alpha}_{B1}(s) - \bar{\alpha}_B(u)\big)} \right\} \ . \tag{22}
$$

For convenience in what follows, we define

$$
\mathcal{Q}_{qt}^{(I)} = (-1)^q \sum_{M;p \geqslant q} E_A^{(I)}(p,q,M) \quad ; \tag{23}
$$

$$
\mathcal{Q}_{qs}^{(I)} = (-1)^q \sum_{B1;g \geqslant q} D_A^{(I)}(g,q,B1) \ . \tag{24}
$$

Similar quantities $\mathcal{B}_{qt}^{(I)}$ and $\mathcal{B}_{qs}^{(I)}$ are understood as appropriate

for the $B^{(I)}(s,t,u)$ amplitudes. We remind the reader that the

second summation in Eq. (22) vanishes for kaon-nucleon scattering,

whereas, in pion-nucleon scattering, signature is obtained by

imposing $\mathcal{Q}_{qt}^{(I)} = \tau\, \mathcal{Q}_{qs}^{(I)}$ where $\tau = +1$ for N_α and $\tau = -1$ for

N_γ and Δ_δ.

In the limit $\bar{\alpha}_B(u) \to k = J - \frac{1}{2}$, where J is the total spin

of the resonance, we again retain only terms contributing to widths of

resonances on the parent trajectory and, after setting

$$(bt)^k \sim (2q^2zb)^k \sim (4q^2b)^k \frac{[\Gamma(k+1)]^2}{\Gamma(2k+1)} P_k(z) \quad,$$

derive

$$A^{(I)} \sim \frac{b^{J-\frac{3}{2}}(4q^2)^{J-\frac{1}{2}} \Gamma(J+\frac{1}{2})}{(M_J^2 - u) \Gamma(2J)} P_{J-\frac{1}{2}}(z)$$

$$\times \sum_{q \leqslant J-\frac{1}{2}} c_{qJ} \{\mathcal{A}_{qt}^{(I)} + (-1)^{J-\frac{1}{2}} \mathcal{A}_{qs}^{(I)}\} \quad, \tag{25}$$

where

$$c_{qJ} = \begin{cases} 1, & q = 0 \\ 0, & q > J - \frac{1}{2} \\ \displaystyle\prod_{i=1}^{q} (J + \frac{1}{2} - i), & 1 \leqslant q \leqslant J - \frac{1}{2} \end{cases} \tag{26}$$

and b denotes the slope of the trajectory, $\alpha_B(u) = \alpha_B(0) + bu$. Note that one may express c_{qJ} in terms of M_J, using $J = \alpha_B(0) + b M_J^2$.

After employing the formulas of Appendix I, one obtains the elastic widths of the two parity states at a given J:

$$\Gamma_{J\pm}^{(I)} = \pm(E_u \pm M) K(J) \gamma^{(I)}(\pm M_J) \quad, \tag{27}$$

where the essentially kinematic factor,

$$K(J) = \frac{(4b)^{J-\frac{3}{2}} \Gamma(J + \frac{3}{2})}{\pi M_J^2 \Gamma(2J + 2)} q^{2J} \tag{28}$$

and

$$\gamma^{(I)}(M_J) = \sum_{q \leqslant J-\frac{1}{2}} c_{qJ}[- (M_J - M) \{\mathcal{B}_{qt}^{(I)} + (-1)^{J-\frac{1}{2}} \mathcal{B}_{qs}^{(I)}\}$$

$$+ \{\mathcal{A}_{qt}^{(I)} + (-1)^{J-\frac{1}{2}} \mathcal{A}_{qs}^{(I)}\}] \quad . \tag{29}$$

In Eq. (27), the $+$ $(-)$ sign is appropriate for states with

$\tau P = -1$ $(+1)$, where P is the parity of the state, $P = - (-1)^L$.

For pion-nucleon scattering, employing Eq. (A-14), one derives

$A_u^{(+)} = 2A_u^{(-)}$ at the $I = \frac{3}{2}$ resonance poles and

$A_u^{(+)} = - A_u^{(-)}$ at the $I = \frac{1}{2}$ resonance poles, therefore, combining

these results with the signature requirement, one reduces Eq. (29) to

$$\gamma^{(I)}(M_J) = 3(-1)^{I+\frac{1}{2}} \left(1 + \tau(-1)^{J-\frac{1}{2}}\right)$$

$$\times \sum_{q \leqslant J-\frac{1}{2}} c_{qJ} [\mathcal{A}_{qt}^{(-)} - (M_J - M) \mathcal{B}_{qt}^{(-)}] \quad . \tag{30}$$

For kaon-nucleon scattering, the absence of (s,u) terms in the

Veneziano expansion leads to the reduction

$$\gamma^{(I)}(M_J) = \sum_q c_{qJ}[\mathcal{A}_{qt}^{(I)} - (M_J - M) \mathcal{B}_{qt}^{(I)}] \quad . \tag{31}$$

The constants $\mathcal{A}$ and $\mathcal{B}$ in these equations are obviously different for each trajectory considered; however, if one wishes to simultaneously fit the resonance widths of all trajectories in π-N scattering, say, then definite constraints exist between the sets $(\mathcal{A}, \mathcal{B})$ for the different trajectories, as can be seen by examining the connection between Eq. (3) and Eq. (22).

Some elementary deductions based on these expression relate to the positivity of widths. Because $\gamma^{(I)}(M_J)$ and $-\gamma^{(I)}(-M_J)$ are proportional to the elastic widths of the two parity states, respectively, at a given J, and because, in the definition of γ, $c_{qJ} \sim J^q \sim M_J^{2q}$ for large J, it is evident that to enforce positivity for both parity states, then $\mathcal{B}_{qt} \neq 0$ if $\mathcal{A}_{qt} \neq 0$. Indeed, $\mathcal{B}_{qt}$ must grow approximately as fast as $\mathcal{A}_{qt}/M_J$. The absence of parity doubling for the nucleon is assured by setting $\mathcal{A}_{ot}^{(-)} = 0$ in Eq. (30) for that trajectory. To rigorously eliminate parity doubling of the first states on the Δ_δ and N_γ trajectories, one must impose the linear relationships $\sum_{q=0}^{1} \mathcal{A}_{qt}^{(-)} = -(M_\Delta + M) \sum_{q=0}^{1} \mathcal{B}_{qt}^{(-)}$ and

$\sum_{q=0}^{1} \mathcal{A}_{qt}^{(-)} = (M_{D13} - M) \sum_{q=0}^{1} \mathcal{B}_{qt}^{(-)}$, respectively; however, the kinematical factor $(E_u - M)$ in Eq. (27) naturally makes the elastic widths of all parity partner states of the Δ_δ trajectory small. In fact, the elastic width of the $\Delta(1238)$ parity partner tends to be two order of magnitude smaller than that of the Δ itself, and so one

may elect to relax rigorous elimination for that trajectory. A similar argument applies to the Δ's SU(3) partner, Y_1^*, in kaon-nucleon scattering. This kinematic suppression may not be relevant for we judge that the Veneziano expression should be constructed so as to fit the invariant amplitudes and not the partial wave amplitudes. It is these latter which are most subject to the possibly large unitarity effects not present in the model.

The experimental widths of the known baryon trajectories should provide a means for estimating the number of terms required in the summations of Eqs. (29) - (31). We have therefore computed the empirical values for $\gamma^{(I)}(M_J)$ by inserting the known values[25] of $\Gamma_{e\ell}$ for the left-hand side of Eq. (27). These are shown for the N, Δ, Λ, and Σ trajectories in Figs. 4-9. In obtaining the plotted values, we allowed the value of the empirical resonance mass[25] to vary from $M_J - \Gamma_{TOT}/4$ to $M_J + \Gamma_{TOT}/4$ and b, the trajectory slope, to run over the range $0.9 \leqslant b \leqslant 1.0$; this explains the brackets shown in the figures.

Both parity states, where available, are given on the same graph. We note the absence of candidates for parity doubled status with the Δ_δ resonances. We suggest that this absence of parity doubling, generally, may well be a unitarity effect, especially strong for states near threshold; higher along the trajectory, the MacDowell symmetry analyticity constraint should reassert itself, and parity doubling would be restored.

The graphs indicate no dramatic structure in $\gamma[(u)^{\frac{1}{2}}]$ but certainly allow considerable flexibility in possible parameterizations.

It appears that the sums in Eqs. (30) and (31) can safely be terminated

after two or three terms. The systematic structure of the plots for

the N_α and N_γ trajectories and, to a lesser extent, that for the

$\triangle$ trajectory residue suggest that the experimentally determined elastic

widths for the high-spin objects on these trajectories could be in error

by as much as a factor of four too small. In this connection, it is

significant to recall that, in general, the determinations for elastic

widths obtained from backward scattering data are significantly larger

than those derived from the forward data.[26]

Backward scattering (u = 0) data points   are also given in

the figures because the differential cross sections for $s \to \infty$ at

fixed u is also determined by the same function, γ. Indeed, as

$s \to \infty$ for fixed u, Eq. (22), becomes (with $c_{q\alpha}$ given by (26) after

replacement of J by α)

$$ A_B^{(I)} \to \frac{\pi}{\cos \pi \, \alpha_B(u)} \cdot \frac{(bs)^{\alpha_B - \frac{1}{2}}}{\Gamma(\alpha_B(u) + \frac{1}{2})} $$

$$ \times \sum_q{}'' \, c_{q\alpha} \, \{ a_{qt}^{(I)'} + ie^{-i\pi\alpha_B} a_{qs}^{(I)} \} \; . \tag{32} $$

In the pion-nucleon situation, upon forming $\tilde{f}[(u)^{\frac{1}{2}}]$ and

employing isospin and signature relations, one derives

$$ \tilde{f}^{(I)}[(u)^{\frac{1}{2}}] = \frac{\pi(1 + ie^{-i\pi\alpha_B})}{2 \cos \pi \, \alpha_B(u)} \, \frac{(bs)^{\alpha_B - \frac{1}{2}}}{\Gamma(\alpha_B + \frac{1}{2})} \, \gamma^{(I)}[(u)^{\frac{1}{2}}] \; , \tag{33} $$

where

$$\gamma^{(I)}[(u)^{\frac{1}{2}}] = 3(-1)^{I+\frac{1}{2}} \chi 2 \sum_{q} c_{q\alpha} \{\mathcal{Q}_{qt}^{(-)} - [(u)^{\frac{1}{2}} - M] \mathcal{B}_{qt}^{(-)}\}.$$

$$(34)$$

The isospin index, I, is the total isospin in the exchange, u, channel. The similarity to Eq.(30) is obvious and shows how the reduced residue function parameterizes both the widths of resonance spectrum and the backward scattering region in a uniform fashion.

For purposes of comparison, we point out that in their phenomenological fits, Barger and Cline[27] have retained only the factor $(\alpha_B + \frac{1}{2})(\alpha_B + \frac{3}{2})$ from $\Gamma^{-1}(\alpha_B + \frac{1}{2})$ and used a parameterization $\gamma^{(I)}[(u)^{\frac{1}{2}}] = \beta[1 + \delta(u)^{\frac{1}{2}}](1/s_0)^{\alpha_B - \frac{1}{2}}$ with β and δ constants. Similarly restricting the $\gamma[(u)^{\frac{1}{2}}]$ of Eq. (34) to terms at most linear in $(u)^{\frac{1}{2}}$ for all trajectories (i.e. $q < 1$ in the sum) has, among other things, the consequences of removing all N_α terms from $A(s,t,u)$, decoupling the Δ_δ and N_γ trajectory terms at $s \to \infty$ for fixed t, and thus forcing A to have the asymptotically nonleading behavior $s^{\alpha_M(t)-1}$. Because all baryon trajectories must couple asymptotically at $t = 0$, (see Section II-A), a consistent solution thus requires $q \geqslant 1$ and, therefore, residue functions at least cubic in $(u)^{\frac{1}{2}}$. The term with $q = 0$ seems strongly demanded at least for the Δ trajectory by the nearly linear form of the residue shown in Fig. 4.

The salient features of the parameterization given in Eqs. (33)
and (34) are the factor $\Gamma^{-1}(\alpha_B + \frac{1}{2})$ and the absence of any exponential
type of form-factor in $(u)^{\frac{1}{2}}$. It is of considerable importance to check
whether this latter feature is supportable experimentally because, if
correct, it provides the important possibility of extrapolating from
the scattering region to distant poles. Except for the famous dip
near $\alpha_N = -\frac{1}{2}$, there is little evidence experimentally for the
differential cross section dips predicted by the factor $\Gamma^{-1}(\alpha_B + \frac{1}{2})$;
in particular the absence of the asserted dip near $\alpha_\Delta = -\frac{3}{2}$ has been
the cause of some consternation for standard Regge-pole theory fits.[27,28]
However, it should be noted that all but the $\alpha_N = -\frac{1}{2}$ dip occur at
fairly large values of u where secondary trajectory or cut mechanisms
are presumably important.

We have attempted to test whether the Veneziano parameterization
for the resonance widths on a given parent trajectory and for that
portion of $d\sigma/du$ attributable to the trajectory are consistent. This
is discussed in the following subsection.

D. Pion-Nucleon Backward Scattering and Resonance Widths

In general, the Regge-pole model asserts the intimate connection
between the baryon resonances $(u > 0)$ and the baryon exchange ampli-
tude in the region of backward scattering $(u < 0)$. This connection is
established via the trajectory function $\alpha[(u)^{\frac{1}{2}}]$, and its reduced
residue function, $\gamma[(u)^{\frac{1}{2}}]$. However, classical Regge model fits to
either the backward- or the forward-angles scattering data could never

be extrapolated reliably to the poles at u or t > 0 because of the

essentially arbitrary nature of the residue function. The Veneziano

model, by specifying both the $\Gamma^{-1}(\bar{\alpha} + 1)$ factor and the scale factor

(s_o), provides a general prescription for pole extrapolation. Hence,

it becomes important to test this recipe in those cases where we know

both the values of the physical pole-residues and the details of the

scattering data controlled by the corresponding Regge-trajectory. In

this section, we examine the success of the Veneziano model prescription

for the Δ, N_α, and N_γ trajectories in π-N scattering. This exam-

ination is only a limited test of the model in the sense that the

forward-angles data is ignored for the time being, as are the relation-

ships which must exist between the constants appearing in the residue

functions for the different baryon trajectories. If reasonable agreement

with the baryon residue function aspect is achieved, these other problems

could be attacked subsequently.

Restricting ourselves to reduced residue functions with only four (!)

parameters, e.g., rewriting Eq. (34) as,

$$r[(u)^{\frac{1}{2}}] \;\; = \;\; a_R + b_R(u)^{\frac{1}{2}} + c_R u + d_R[(u)^{\frac{1}{2}}]^3 \;, \tag{35}$$

we find that our best solution in the case of the Δ_δ trajectory yields

a $\Delta(1238)$ width a factor of two too small. For the N_α and N_γ

situations, the problem is much less constrained, and we can achieve

solutions in agreement with both backward data and baryon resonance

widths.

(1) Δ residue

We begin by discussing the Δ_δ because it is presumably the

sole contributor to π^-p backward scattering and because several

resonance widths are well determined, as shown in Fig. 4. Many diffi-

culties have been encountered in previous Regge-pole theory fits[27-30]

to π^-p data; these include the anomalously small value of the cross-

section at $u = 0$, the absence of a dip near $\alpha_\Delta = - 3/2$, discussed in

the previous section, and the considerable discrepancy with respect to

pole extrapolation. To these, we would add another uncertainty: we

have extracted an effective-trajectory for the available π^-p backward

data;[31] this is displayed in Fig. 10. The effective trajectory appears

to deviate substantially, for $u < -0.2$ $(GeV/c)^2$, from the linear form

with slope near unity, expected from drawing a line through the Δ_δ

resonance spectrum. Nevertheless, we sought a fit to the π^-p backward

elastic data using the full Regge-pole formulation and a Δ_δ trajectory

of variable slope. Retaining only data having $u > -0.75$ $(GeV/c)^2$, we

found that the χ^2 values did not significantly change as the slope was

varied over the range 0.3 to 1.0 $(GeV/c)^{-2}$. The Δ_δ trajectory is

evidently poorly specified from the backward data. (We note that the

systematic errors quoted on the experimental data[31] are rather large.

In our analysis, we have allowed for this feature, assuming it to be an

effect independent of u. Because this systematic error feature is as

important to the process of obtaining a good fit as is the variation

of the Δ trajectory slope, we feel that shrinkage (or its absence)

in the data is yet to be demonstrated.)

The very small size of the π^-p cross-section in the backward direction provides the essential constraint in the problem of finding a suitable $\gamma_\Delta[(u)^{\frac{1}{2}}]$. These data in fact require that both γ_Δ and $d\gamma_\Delta/d[(u)^{\frac{1}{2}}]$ near $u = 0$ be <u>two</u> orders of magnitude smaller than the appropriate pole value, $\gamma_\Delta(1.238\ \text{GeV})$. That the derivative must be so small can be appreciated by examining Eq. (A-10) near the backward direction for large s: $d\sigma/du \propto |A'|^2 + \sin^2\theta\,|B|^2/4$, where θ is the direct channel center of mass scattering angle. Because the variation of $d\sigma/du$ away from $\cos\theta = -1$ is slight, $|B|$ cannot be large. However, the contributions of the B amplitude to $\gamma[(u)^{\frac{1}{2}}]$, Eq. (34), enter multiplied by the factor $[(u)^{\frac{1}{2}} - M]$, and so the magnitude of B near $u = 0$ is essentially the derivative, $d\gamma_\Delta/d(u)^{\frac{1}{2}}$, there.

In order to proceed, we appropriated a Δ trajectory linear in u, passing through the resonance positions (slope ≈ 0.9), and sought forms for $\gamma_\Delta[(u)^{\frac{1}{2}}]$ which would yield sensible agreement with both the backward-angles π^-p data and the reduced residues in the resonance region. Although a residue of the form $a_\Delta + b_\Delta(u)^{\frac{1}{2}}$ would seem acceptable from a first glance at Fig. 4, it is ruled out by the considerations of the previous paragraph. K. Igi used a quadratic form $a_\Delta + b_\Delta(u)^{\frac{1}{2}} + c_\Delta u$, for the residue;[6] a curve computed with his parameters is shown on Fig. 4. His width for the $\Delta(1238)$ is a factor of 4 too small, as he noted, but his fit to the backward π^-p data is most unacceptable ($\chi^2 \approx 5 \times 10^6$), except at $u = 0$, due to his large B amplitude. (See also Section I-D on this point.) A quadratic solution also has the disadvantage of giving negative widths for parity-partner

states, as we have noted earlier. We adopted, therefore, a four para-
meter form of the type given in Eq. (35) and converged on the following
compromise:

$$\alpha_{\triangle}[(u)^{\frac{1}{2}}] = -0.02 + 0.9 \, u \quad ; \tag{36}$$

$$\gamma_{\triangle}[(u)^{\frac{1}{2}}] = 27.3 + 51.7 \, u + [(u)^{\frac{1}{2}} - M](27.2 + 34.5 \, u) \, . \tag{37}$$

The $\triangle$ (1238) elastic width is 53 MeV, $\approx$45% of its experimental value,[32]
and χ^2 on 82 π^-p backward data points[31] is 246. A plot of this
function is also given on Fig. 4; the general agreement with the
empirical residues is qualitatively not unreasonable, but the χ^2 for
the backward fit seems much too large.

Moreover, from the theoretical point of view, this solution is
not particularly appealing because it gives no evidence for a zero at
$\alpha_{\triangle} = 1/2$. As we remarked in Section I-C, because there is no observed
state at $J^P = \frac{1}{2}^-$ on the exchange degenerate $\Sigma_\beta - \Sigma_\delta$ trajectory, a
Veneziano parameterization for KN scattering may have terms with the
factor $\Gamma(m - \overline{\alpha}_{Y*})$ only if $m \geq 1$; consequently the Y_1^* residue
function will contain the factor $(\alpha_{Y*} - \frac{1}{2})$. Via SU(3) symmetry
arguments, therefore, the $\triangle$ reduced residue might also be expected to
have, as a factor, the term $(\alpha_{\triangle} - \frac{1}{2})$. Alternatively, the absence of
strangeness +1 baryon states implies exchange degeneracy of the
s channel and t channel exchanges in $\pi N \to K\Sigma$ which suggests more
directly, therefore, that the $(\alpha_{\triangle} - \frac{1}{2})$ factor is appropriate in the

Δ reduced residue. These arguments provide theoretical support for the ad hoc suggestion, by Igi et al., that there should be such a wrong-signature, nonsense zero.[29]

Without increasing the number of free parameters, we may consider solutions in which the zero at $\alpha_\Delta = \frac{1}{2}$ is imposed, a priori. In our best four-parameter fit of this type, we settled upon

$$\alpha_\Delta = 0.09 + 0.9 \, u \quad ; \tag{38}$$

$$\gamma_\Delta = (\alpha_\Delta - \tfrac{1}{2})\Big(35.2 + 56.0 \, u + [(u)^{\frac{1}{2}} - M](29.4 + 35.8 \, u)\Big). \tag{39}$$

For this case the χ^2 on 82 $\pi^- p$ backward data points is 150, and the Δ (1238) width is ≈ 60 MeV.[32] We also present a plot of this particular residue solution in Fig. 4; although the agreement with the backward data and the Δ (1238) width is fairly good, the presence of the $(\alpha_\Delta - \frac{1}{2})$ factor causes the residue to grow too rapidly at large $|(u)^{\frac{1}{2}}|$. It may also be noted that the rather large coefficients of the terms in Eq.(39) proportional to u and to $u^{3/2}$ indicate no systematic tendency for subsidiary terms in the Veneziano expansion to be small. Moreover, we call attention to the fact that even with the $\alpha_\Delta = +\frac{1}{2}$ zero factored out, the small value of γ_Δ at $u = 0$ is achieved through substantial cancellation between the contributions of the A and B amplitudes.

Finally, we examined also a six-parameter residue structure, keeping terms up to $u^{5/2}$ in the Δ's residue parameterization, but

releasing the zero at $\alpha_\Delta = +\frac{1}{2}$. This fit can lead to an estimate of

the size of terms in the Δ residue which are nonvanishing at

$\alpha_\Delta = 1/2$ and thereby measure to what extent the underlying symmetry

discussed above is reflected in the π-N situation. The qualitative

features of such a solution are very similar to those of the solution

given in Eqs. (38) and (39); viz., a reasonably good fit to the backward

scattering data and an elastic width of 80 MeV for the Δ are obtained,

but at the price of a residue function which produces unreasonably

large elastic widths for the higher recurrences along the trajectory.

Some physical arguments may be advanced to explain away these

difficulties with the Δ residue. We list them and consider each

briefly in this paragraph. (1) The small cross-section at $u = 0$ may

be associated with a vanishing of the residue function near the backward

direction;[33] as we noted, our solution, Eq. (36), is indicative of

cancellation which reduces the residue from its natural size near

$u = 0$. Absorption effects[34] (in the s channel) are asserted to be

large for such vanishing residue functions and could serve to signifi-

cantly alter the shape of the differential cross section for $u < 0$.[35]

(2) A Δ trajectory with a substantial term linear in $(u)^{\frac{1}{2}}$ has been

advanced by Paschos[28] and others. Such a trajectory is not inconsistent

with the Veneziano model;[36] we tried it and found, however, that its

use does not lead to essential improvement over our preferred solutions,

Eqs. (37) and (39). (3) The effects of unitarity (in the u channel)

might be substantial in the partial wave containing the Δ (1238) state,

and thus play a great role in determining the Δ's width. It will be recalled that the N/D model calculations of some years ago, based on unitarity and analyticity, were able to generate the Δ (1238) state successfully from crossed-channel nucleon exchange.[37] To estimate this effect in the Veneziano model, we employed the K-matrix formalism[38] but found that the discontinuity generated by the crossed-channel nucleon state is only 10% of that due to the direct channel Δ pole.

We summarize this study of the Δ residue function by pointing out that the Veneziano parameterization does not in fact provide a quantitatively acceptable procedure for extrapolation from the backward scattering region to the resonance poles on the Δ trajectory.[39] Within the Veneziano framework, the best solution involves at least four parameters and yields typical resonance widths a factor of two too small. There are two plausible methods for resolving this discrepancy. Firstly, one may conceive that the Veneziano parameterization is simply too naive. In particular, it may be judged that the true residue function should explicitly vanish at all parity partner locations. However, in this regard, we note that the parity partner states in our solutions, for mass $\lesssim 3.0$ GeV all have elastic widths less than 30% of the experimentally observed $\tau P = -1$ states. Secondly, it may well be that the size of the backward elastic cross section is no true reflection of the Δ Regge-pole residue function; we recall our previous comment on the possiblity of large absorption corrections.[40]

(2) N_α - N_γ residues

Achieving agreement with the N_α and N_γ residue data is much less difficult but also much less constrained. It may be seen from comparing the data points in Figures 5 and 6 that the N_γ's residue function is expected to be of similar size to that of the N_α. Therefore, the contribution of the N_γ to the backward πN data[31] will differ typically from the N_α only in the intercept $\left(\alpha_{N_\alpha}(0) \simeq \alpha_{N_\gamma}(0) + 0.5,\text{which strongly favors the } N_\alpha\right)$ and the signature factor (which favors the N_γ near $u = 0$). In our various fits to the data, we fixed the Δ, using parameters determined from the $\pi^- p$ fit, and varied the residues of the N_α and N_γ to obtain agreement with the widths of Figs. 5 and 6 and the data on both $\pi^+ p$ elastic and $\pi^- p$ CEX.[31] We tried three types of fits:

(i) Delta fixed at parameters of Eqs. (36), (37).

(ii) Delta fixed at parameters of Eqs. (38), (39).

(iii) As (ii) but with an added even-signature, $I_u = \frac{1}{2}$ amplitude, having the same trajectory as the Δ and a residue function fixed equal to 0.43 times that of the Δ given in Eqs. (38) and (39). This fit is motivated by the exchange degeneracy which occurs in $\overline{K}N$ scattering between the SU_3 partners of the Δ and the D15(1680). The constant of proportionality, 0.43, is estimated from Figs. 4 and 5. This type of fit would seem more sensible than (i) and (ii), but rather depressing in that four trajectories allows one far too many parameters with which to fit the backward data.

In all the fits, (i), (ii), or (iii), the N_α is clearly dominant and the three subsidiary trajectories, Δ, N_γ, and D15, contribute about equally. The various experiments[31] on πN CEX backward scattering are not notably consistent with each other. However, they all seem to indicate that there should be destructive interference between the $I_u = \frac{1}{2}$ and the $I_u = \frac{3}{2}$ contributions to the CEX reaction. This feature is realized in fit (i), but not in fits (ii) and (iii). The values of $\gamma[(u)^{\frac{1}{2}}]$ at the resonance positions, given in Figs. 5 and 6, were not an important constraint in the fits; in particular the N_γ contribution is badly determined.

We can try to limit the freedom in the fits by requiring agreement with the trend of the $\pi^+ p$ polarization data measured near 3 (GeV/c).[41] In fits (ii) and (iii), the $\Delta - N_\alpha$ interference term gives positive polarization which rises to a maximum of approximately one near $u = -0.1$ (GeV/c)^2 and then vanishes when $\alpha_N = -0.5$ (at $u = -0.15$ (GeV/c)^2). The D15 addition in fit (iii) produces similar behavior. In fit (i), the Δ and N_α interfere to give polarization of the opposite sign to that of fit (ii). We present the result of a type (i) fit, for which the experimentally observed[41] positive polarization in $\pi^+ p \to \pi^+ p$ near $u = 0$ comes from the $N_\alpha - N_\gamma$ interference term.

The trajectories of the fit are:

$$\alpha_{N_\alpha}(u) = -0.34 + 0.88\, u \quad ; \tag{40}$$

$$\alpha_{N_\gamma}(u) = -0.75 + 0.9\, u \quad . \tag{41}$$

In terms of the four parameter form given in Eq. (35), the residue functions are

$$r_{N_\alpha}[(u)^{\frac{1}{2}}] = (104 - 184\ u) + [(u)^{\frac{1}{2}} - M](293 + 106\ u) \quad ; \quad (42)$$

$$r_{N_\gamma}[(u)^{\frac{1}{2}}] = (-131 + 21\ u) + [(u)^{\frac{1}{2}} - M](-170 + 110\ u) \ . \quad (43)$$

These are plotted in Figs. 5 and 6. We note that the ratio of the reduced residue of the N_γ to that of the N_α is smaller at $u = 0$ than it is in the resonance region, $(u)^{\frac{1}{2}} \sim -1.5$ GeV.

Extracting a reliable parameterization for the subsidiary N_γ and D_{15} trajectories will require more good data. Specifically, we would suggest differential-cross-section and polarization measurements in all charge configurations over a wide range of energies for lab momenta greater than 5 GeV/c. Particularly crucial is the matter of the correct relative normalization between data of different energies.

Our values for the N_γ can be regarded only as representative. Similarly, our skepticism of the fundamental nature of our fit prevents us from plotting any of our predictions for (or fits to) the πN backward data. (We will supply these to any interested reader.)

We close this section on a doleful, if realistic note by underscoring the unfortunate features of our best fit, represented in Eqs. (36), (37), and (40) through (43). In addition to the unproven shrinkage and poor Δ_δ pole extrapolation in $\pi^- p$ backward elastic scattering, the fit also indicates a marked violation of even approximate exchange-degeneracy. Specifically, the Δ_δ residue does not have the desired $\alpha_\Delta = \frac{1}{2}$ zero, and the N_γ residue function bears little resemblance to that of the N_α.

III. EXPLICIT VENEZIANO FUNCTION PARAMETERIZATIONS

In this section, we derive and discuss several explicit Veneziano-type representations for the pion-nucleon and kaon-nucleon processes. We endeavor to incorporate into these parameterizations both the general requirements of appropriate isospin, signature, positivity, and spin-parity content discussed in Section I, as well as the more specific structure emphasized in Section II.

A. K-N Scattering - First Approximation

We begin by treating K-N scattering in the most obvious manner. In this approximation, exotic resonances are presumed absent, and thus we include no strangeness +1 baryon trajectories in our functions. Moreover, for the same reason, the t-channel meson trajectories are taken in exchange-degenerate pairs as are the s-channel $\overline{K}N$ trajectories. We deal, therefore, only with Veneziano terms of the (s,t) type. This problem is thus considerably more simple than the π-N situation (treated in Section III-F) which demands, in addition, (u, t) and then (s, u) type terms for signature reasons. In this first approximation, we also do not include the Pomeranchuk trajectory as a possible t-channel exchange. To do so in KN scattering, without accepting the price of exotic u channel resonances, would require installing the Pomeranchuk trajectory as an exchange degenerate object; this seems inappropriate. We return to the Pomeranchuk situation in Section III-E where we argue that there is no compelling reason to leave it out of a Veneziano representation; in fact, it can be associated directly with low-lying exotic KN states.

A good solution to K-N scattering at this first level was
independently obtained by Inami;[6] we will first motivate his result,
discuss its properties, and then go on to possible improvements. Our
approach begins with the results we established in Section II-B relating
to the t-dependence required of the $A(s, t, u)$ and $B(s, t, u)$
functions if appropriate spin and parity is to be secured for the s
channel resonances. Next, we note that the t dependence of a single
Veneziano form, Eq. (6), near a pole $\bar{\alpha}_1(s) = m'$, is given by
$\Gamma(\ell - \alpha_2(t))/\Gamma(n - m' - \alpha_2(t))$ which has zeroes for t values,
$\alpha_2(t) = \ell - 1, \cdots, n - m'$. In Table IV, the states on the $\Lambda_\alpha - \Lambda_\gamma$
trajectory are shown to possess a structure of zeros in both A and B
which is roughly consistent with the prediction of a single Veneziano
beta-function expression. For the $\Sigma_\beta - \Sigma_\delta$ pair, a similar statement
is true for the A amplitude, but the B function requires an additional
zero at $t \approx 2.3(\text{GeV/c})^2$.

We are thus led to consider the representation

$$\underline{\Lambda_\alpha - \Lambda_\gamma \qquad I = 0 \quad \overline{K}N \quad \text{Amplitude}}$$

$$A_1^{(0)} = \Lambda_{A1} \frac{\Gamma(1 - \overline{\alpha}_\Lambda)\,\Gamma(1 - \alpha_t)}{\Gamma(1 - \overline{\alpha}_\Lambda - \alpha_t)} + \frac{\Lambda_{A2}\,\Gamma(-\overline{\alpha}_\Lambda)\,\Gamma(1 - \alpha_t)}{\Gamma(1 - \overline{\alpha}_\Lambda - \alpha_t)}$$

$$B_1^{(0)} = \Lambda_{B1} \frac{\Gamma(-\overline{\alpha}_\Lambda)\,\Gamma(1 - \alpha_t)}{\Gamma(1 - \overline{\alpha}_\Lambda - \alpha_t)}$$

$$\underline{\Sigma_\beta - \Sigma_\delta \qquad I = 1 \quad \overline{K}N \quad \text{Amplitude}}$$

$$\text{(44)}$$

$$A_1^{(1)} = \Sigma_{A1} \frac{\Gamma(1 - \overline{\alpha}_\Sigma)\,\Gamma(1 - \alpha_t)}{\Gamma(1 - \overline{\alpha}_\Sigma - \alpha_t)}$$

$$B_1^{(1)} = \Sigma_{B1}(t_o - t) \frac{\Gamma(1 - \overline{\alpha}_\Sigma)\,\Gamma(1 - \alpha_t)}{\Gamma(2 - \overline{\alpha}_\Sigma - \alpha_t)} \quad .$$

As noted, the isospin indices, I, in $A^{(I)}$ and $B^{(I)}$ denote the total isospin value in s channel $\overline{K}N$ system. Crossing relations, which prescribe the u channel KN amplitudes in terms of the above, are given in Appendix I. For notational convenience, we write $\overline{\alpha}_\Lambda = \alpha_\Lambda(s) - \frac{1}{2}$, where Λ denotes the $\Lambda_\alpha - \Lambda_\gamma$ exchange degenerate trajectory; $\overline{\alpha}_\Sigma = \alpha_\Sigma(s) - \frac{1}{2}$, where Σ denotes the $\Sigma_\beta - \Sigma_\delta$ pair; and $\alpha_t = \alpha_\rho(t) = \alpha_{A_2}(t) = \alpha_\omega(t) = \alpha_{P'}(t)$. Our omission of the $\Sigma_\alpha - \Sigma_\gamma$ trajectory from these formulae may be justified by Fig. 9; the residue

function has a quite small magnitude when compared with that of

$\Lambda_\alpha - \Lambda_\gamma$ or $\Sigma_\beta - \Sigma_\delta$.

We emphasize that Eq. 44 is essentially the simplest one can devise for the KN process in that only two terms of a subsidiary nature[9] appear. The term proportional to Λ_{A2} is subsidiary because it does not contribute in leading order as $s \to \infty$ for t fixed. It is present simply to achieve the precise extermination of the parity partner of the $J^P = \frac{1}{2}^+$ $\Lambda(1115)$; it is in fact small and unimportant. The other subsidiary term is that proportional to t_0 in $B^{(1)}$. The $(t_0 - t)$ term is indeed a crucial factor in B because it leads to reversal of the sign of the Σ_{B1} term between large t (where it is normalized by the Y_1^* $(\Sigma_\beta - \Sigma_\delta)$ widths) and the $t = 0$ point, where it contributes asymptotically to known high-energy forward scattering. Terms proportional to $\Gamma(-\bar{\alpha}_\Sigma)$ are excluded because they would predict an unobserved $J^P = \frac{1}{2}^-$ state, and no $\Gamma(-\alpha_t)$ terms appear for similar reasons. We remark, however, that in this solution the reduced residue function for the Λ trajectory is quadratic in $(u)^{\frac{1}{2}}$, and thus positivity cannot be guaranteed for the elastic widths of both parity states on the parent trajectory.[42]

The values of the constants appearing in Eq. (44) may be established in various ways. We will present two different approaches and then go on to a discussion of the properties of the solutions. Inami's approach was guided by the assertion that, in accordance with Nature, certain residues should vanish. He chose to abolish the $\frac{1}{2}^-$ and $\frac{3}{2}^+$ states of the Λ trajectory, the $\frac{3}{2}^-$ state of the Σ trajectory, and

the $\frac{1}{2}^+$ daughter state of the Σ trajectory. This is accomplished

essentially by using the t_o value listed in Table IV and determining

the ratios $\Lambda_{A1}/\Lambda_{B1}$ and Σ_{A1}/Σ_{B1} from the values listed in Table V

for the $\frac{3}{2}^-$ and $\frac{3}{2}^+$ states of the $\Lambda_\alpha - \Lambda_\gamma$ and $\Sigma_\beta - \Sigma_\delta$ trajec-

tories, respectively. Inami then fixed his two remaining parameters by

adopting the high-energy fitted values of A' at $t = 0$ in the two

different isospin states. (See Table III) We denote his solution

(I); in units of $\hbar = c = 1$ and GeV, it is

$$\Lambda_{A1} = 55.5 \qquad \Lambda_{A2} = -24.4 \qquad \Lambda_{B1} = 138.7$$

$$\Sigma_{A1} = -22.4 \qquad \Sigma_{B1} = -9.8 \qquad t_o = 2.3$$

$$\bar{\alpha}_\Lambda = -1.24 + s \qquad\qquad\qquad\qquad \text{(Solution I)}$$

$$\bar{\alpha}_\Sigma = -0.9 + s$$

$$\alpha_t = 0.5 + t \; .$$

In order to illustrate the uncertainty present in even this

simplest parameterization, we present a second set of values, denoted

Solution I',

$$\Lambda_{A1} = 48.0 \qquad \Lambda_{A2} = -24.6 \qquad \Lambda_{B1} = 117.5$$

$$\Sigma_{A1} = -20.7 \qquad \Sigma_{B1} = -24.2 \qquad t_o = 0.62$$

$$\bar{\alpha}_{\Lambda} = -1.15 + 0.9\,s \qquad\qquad \text{(Solution I')}$$

$$\bar{\alpha}_{\Sigma} = -0.8 + 0.9\,s$$

$$\alpha_t = 0.55 + 0.9\,t \ .$$

With this solution, the low-energy daughter[9] states and parity partner states are not abolished with quite the same religious exactitude as in Solution I. However, according to χ^2, a much better overall fit is achieved to a collection of $\overline{KN}$ and KN scattering data as well as to the widths of sundry parent and daughter state resonance widths. Before presenting a critique of these solutions, we list the relevant data against which we judged the solutions.

B.   KN   Data Set

(1) Resonance widths

The empirical elastic widths of states on the parent $\Lambda_{\alpha} - \Lambda_{\gamma}$ and $\Sigma_{\beta} - \Sigma_{\delta}$ trajectories were discussed in Section II-D and the reduced residue functions were extracted and plotted in Figs. 7 and 8, respectively. We also assumed that elastic widths of daughter[9] states are rather small ($\lesssim \frac{1}{2}$ the size of the parent widths) but positive. In some cases, widths of such states can in fact be estimated by using the results of πN phase shift analysis[43] and invoking SU(3) symmetry. The handling of the daughter states in the model is, in general, a

difficult problem because certainly some of the lower lying embryo

resonances in the Veneziano formulation will acquire total widths so

large that they will never be discerned empirically. Some of the inter-

esting low-lying states predicted by the quark model will be discussed

later.

(2) Forward scattering data

Some of the qualitative features of the high-energy data were

discussed in Section II-A. The data which we employed is explicitly

(i) $K^- p \to \bar{K}^o n$ [44] at 3.5 GeV/c and 5 to 12.3 GeV/c

(ii) $K^+ n \to K^o p$ [45] 0.35 to 0.81 GeV/c, 0.86 to 1.36 GeV/c,

2.3 GeV/c, and

3.0 GeV/c.

The $K^+ n$ data is particularly interesting information from the stand-

point of a complete test of the model. All of the applications of the

model we have so far discussed in this paper have involved comparing

data against a quantity extracted from the model via a limiting procedure.

This is necessary because of the unitarity-violating zero width aspect

of the model which places resonance poles directly on the real energy

axes. However, in the approximation to which we work, there are no

KN resonances and thus no poles in the $K^+ n$ channel above threshold.

The full model, without limiting operations, may therefore be used in

$K^+ n$ scattering and applied even at quite low energies. The omission

of a Pomeranchuk trajectory from our formalism requires that comparison

of the Veneziano form be restricted to the charge-exchange process, however.

(3) K^+p backward scattering

The data included are distributions from exposures at 0.99 to 2.45, 2.76, 3.53, 3.55, 5.2, and 6.9 GeV/c.[46] Actually, we estimate that it is not reliable to use such data below 1.3 GeV/c because the effects of the neglected t-channel Pomeranchuk trajectory may be important. The statements, made in (2), about a complete test of the model apply here also, of course.

(4) KN scattering lengths

We use the values[47] 0 and -1.47 in the $I_u = 0$ and 1 KN states, respectively. These are not very important data as the $I_t = 0$ combination may be affected quite significantly by the Pomeranchukon, whereas the $I_t = 1$ part is already implied by the low energy K^+n CEX data, 0.35 to 0.81 $(GeV/c)^2$, from which it was extracted, in fact.

C. Critique of Solutions I and I'

Having determined the constants in his parameterization as discussed above, Inami found that his expression (I) produced good agreement with both K^-p charge exchange data and K^+p backward scattering data, as reproduced in Figs. 11 and 12. In this section, we discuss the significance of these verified predictions and also comment upon the value given by his solution for the elastic widths and scattering lengths.

We begin with the $K^-p \to \bar{K}^0 n$ result. Given the value of A',

asymptotically, a good fit to the K^-p charge exchange data will be

obtained once one specifies, in addition, the approximately correct

value for the ratio $(A'/\nu B)$ at $t = 0$. This was done when the correct

spin parity structure of the low energy spectrum was imposed. We argue

below that the success achieved in the fit is therefore not a triumph

for the particular Veneziano formulation, but is a practically expected

result of a wide class of models which embody a duality of resonances

and Regge-poles. To make this more quantitative, we first focus

on some general features of the duality of resonances and Regge

poles, well-known from FESR results.[3] Both of the s-channel

trajectories, $\Sigma_\beta - \Sigma_\delta$ and $\Lambda_\alpha - \Lambda_\gamma$ (or, for that matter,

N_α, N_γ, and Δ in the πN situation), are associated with large values

of the B amplitude and a small ratio $A'/\nu B$. In Table V, this is

shown to be the case for the high-spin members of each resonance tower.

Any Veneziano form, which does not contain daughter states with huge

widths, must exhibit this feature, and, indeed, as we noted, the ratio

conditions were imposed in each of the s channel isospin states when the

parameters of solution (I) were derived. The signs of the amplitudes

are also such that the $\Lambda_\alpha - \Lambda_\gamma$ and the $\Sigma_\beta - \Sigma_\delta$ terms interfere

constructively in forming the B amplitude with t channel isospin

$I_t = 1$ but destructively in $I_t = 0$. The well determined, high-energy

value of A' at $t = 0$ served to normalize solution (I), and values

for $B(I_t = 1)$ and $B(I_t = 0)$ were deduced.

The $B(I_t = 0)$ number is small, due to the cancellations, but is essentially undetermined experimentally. The major testable prediction is that for $B(I_t = 1)$, which is supported. However, as this discussion indicates, essentially the same prediction would come from any model imposing duality and normalizing to A'. The only surprising aspect is that the precise numerical values Inami uses produce uncannily good agreement with the $t = 0$ values of Table III-E.

Upon examining the t dependences of Solutions I and I', we find that A, B, and A'($I_t = 0$) are essentially constant, subject to the expected $\alpha_t = -n$ zeros, whereas A'($I_t = 1$) has the sought-for cross-over zero at $t \approx -0.25$ $(GeV/c)^2$. These effects are also expected because, as explained in Section II, any roughly constant A and B will generate the crossover zero in A'($I_t = 1$) but not in A'($I_t = 0$), simply as a consequence of the $1 - t/(4M^2)$ factor (see Eq. A-11) and the appropriate magnitude of A'/νB at $t = 0$. As noted, this latter quantity is small for the $I_t = 1$ amplitude but of order unity for $I_t = 0$. The absence of a crossover zero in A' (ω exchange) implies that Solutions I and I' will poorly reproduce the empirical value of the difference, $d\sigma/dt$ (K^+p elastic) - $d\sigma/dt$ (K^-p elastic).

The predictions of Solution I for the reduced residue functions of the baryon trajectories are given in Figs. 7 and 8. The baryon widths for both (I) and (I') are a factor of two to three times too small in comparison with the empirical values.

The fit which Inami achieved to the backward K^+p scattering data, Fig. 11, is a free prediction and reasonably good, but this may

be a fluke. From Fig. 7, one may note that Inami's value for the

$\Lambda_\alpha - \Lambda_\gamma$ residue is approximately a factor of two smaller than that

obtained by Barger[48] from a classical Regge model fit to the data. The

difference can be attributed to the very different size of the $\Sigma_\beta - \Sigma_\delta$

contribution between the two cases. In fact, Solution I gives a value

for the $\Sigma_\beta - \Sigma_\delta$ residue which is an order of magnitude larger than the

value estimated from applying SU(3) arguments, at fixed u, to the

known $\Delta \to \pi N$ coupling at u = 0. Whereas the Σ trajectory was

neglected by Barger,[48] it is quite important in the parameterization of

Solution I. The resolution of this question awaits reliable high energy

backward scattering data for $K^+ n \to K^0 p$ or $K^+ n \to K^+ n$. However, a

preliminary answer is possible inasmuch as the full Veneziano formula

can be used to study low energy $K^+ n$ charge-exchange data, which extend

to the backward angles. Solution I is seen to give much too small a

cross-section whereas a phenomenological model, normalized to the

backward $K^+ p$ data and using a smaller Σ contribution is in better

agreement with the data. In particular, when we tried to extend our

parameterization beyond that of Eq. (44) we found good solutions with

a very small Σ contribution at u = 0. However, our best solutions

did not exhibit this feature.

We notice from Fig. 13 that the fit to the $K^+ n$ CEX data is

generally rather poor. At low energies, the theory predicts more s-wave

and less P-wave than is indicated by the data. The presence of the

large P-wave component of the data may be identified with rapid variation

of the amplitude produced by the nearby baryon poles. The residues of

these poles are too small in both (I) and (I'). In the next section,

solutions will be considered whose residues at the nearby poles agree

better with experiment, and a larger P-wave component will be generated.

D. Improved Veneziano Parameterization of $\overline{K}N$

We gain some inkling as to the source of the deficiencies of

Solution I by examining the results of a partial wave analysis of the

various resonance poles. In Table VI is a presentation of the widths of

the members of the resonance tower associated with the F17(2030) state

of the Σ_β - Σ_δ family. The widths are pleasantly positive, except

for the $\frac{5}{2}^+$ state, which has a small negative width. On reflection,

this latter feature is curious because the $\frac{5}{2}^+$ state is the SU_3

partner of the experimentally observed[25] F35(1910), which, in turn, is

a member of the tower associated with the F37(1950), correspondingly

the partner of the F17(2030). Moreover, these states, parents, and

daughters, are classified successfully by the quark model[24] in the

$(56,2^+)$ representation. Both the reasonably large elastic widths in

πN $(\Gamma_{e\ell}(F35)/\Gamma_{e\ell}(F37) \sim 0.5$, which via $SU(3)$ implies the same ratio

in $\overline{K}N)$ and the theoretical association with the quark model suggest

that we should seek a solution with reasonable properties for this

$\frac{5}{2}^+$ state. Similar considerations relate the $\frac{3}{2}^+$ daughter of the

D15(1770) state in $\overline{K}N$ to the corresponding $SU(3)$ partners D15(1675),

D13(1730), and D33(1670) in πN.

From Tables IV and V may be noted the amusing fact that the F35

destructively interferes with its parent F37 in both the A and B

amplitudes. This fact would enable one to increase the size of the

F37 width without sacrificing the desired high energy limiting values

of A and B. In particular, the F35 + F37 combination does not

necessarily have the effective zero at the canonical t = 2.3

$(GeV/c)^2$ position in the B amplitude; the addition of the F35 moves

it to smaller t. For this reason, the simple Solution I is suspect in

that the zero at t = 2.3 $(GeV/c)^2$ is present unaltered in the high

energy limit.

We can try similar arguments for states on the Λ_α - Λ_γ

trajectory, but there are no obvious quark daughters for the F05(1690).

One evident discrepancy in I is that the Λ_α - Λ_γ residue function is

quadratic in $(s)^{\frac{1}{2}}$, implying necessarily negative widths for all the

parity partner states above a critical mass value.

Based upon such thoughts, plus some trial and error, from our

Pandora's box of possible extra Veneziano forms, we select a few that

are particularly helpful. These are in addition to those in Eq. (44),

and therefore,

Λ_α - Λ_γ $\overline{K}N$ I = 0 Amplitude

$$A^{(0)} = A_1^{(0)} + \Lambda_{A3} \frac{\Gamma(1 - \bar{\alpha}_\Lambda)\,\Gamma(2 - \alpha_t)}{\Gamma(2 - \bar{\alpha}_\Lambda - \alpha_t)}$$

$$B^{(0)} = B_1^{(0)} + \Lambda_{B2} \frac{\Gamma(1 - \bar{\alpha}_\Lambda)\,\Gamma(2 - \alpha_t)}{\Gamma(2 - \bar{\alpha}_\Lambda - \alpha_t)} + \Lambda_{B3} \frac{\Gamma(1 - \bar{\alpha}_\Lambda)\Gamma(1 - \alpha_t)}{\Gamma(2 - \bar{\alpha}_\Lambda - \alpha_t)}$$

$\Sigma_\beta - \Sigma_\delta \quad \overline{K}N \quad I = 1 \quad$ Amplitude

$$A^{(1)} = A_1^{(1)} + \Sigma_{A2} \frac{\Gamma(2 - \overline{\alpha}_\Sigma)\ \Gamma(1 - \alpha_t)}{\Gamma(2 - \overline{\alpha}_\Lambda - \alpha_t)}$$

$$(45)$$

$$B^{(1)} = B_1^{(1)} + \Sigma_{B3} \frac{\Gamma(2 - \overline{\alpha}_\Sigma)\ \Gamma(2 - \alpha_t)}{\Gamma(3 - \overline{\alpha}_\Sigma - \alpha_t)}$$

$$+ \Sigma_{B4} \frac{\Gamma(2 - \overline{\alpha}_\Sigma)\ \Gamma(1 - \alpha_t)}{\Gamma(3 - \overline{\alpha}_\Sigma - \alpha_t)}$$

A computer search was performed to find the set of constants which would best fit the data set given in Section III-B. These values are

$$\Lambda_{A1} = 68.0 \qquad \Lambda_{A2} = -35.5 \qquad \Lambda_{A3} = 61.5$$

$$\Lambda_{B1} = 254.2 \qquad \Lambda_{B2} = -33.8 \qquad \Lambda_{B3} = -90.9$$
$$\text{(Solution B')}$$

$$\Sigma_{A1} = -20.2 \qquad \Sigma_{A2} = -1.1 \qquad \Sigma_{B1} = -26.6$$

$$t_o = 0.64 \qquad \Sigma_{B3} = -3.4 \qquad \Sigma_{B4} = 3.6 \ ,$$

with the same baryon and meson intercepts as in Solution (I').

We notice that this solution in the Σ segment is not essentially different from (I'). We have also obtained fits with a larger value of Σ_{A1}. The data on the $Y_1^*(1385)$ is not sufficiently precise to determine the size of Σ_{A1}, but when we study πN scattering we

find that the solution analogous to (I') gives much too small a Δ width. A typical fit with a larger $Y_1^*(1385)$ coupling is:

$$\Lambda_{A1} = 57.6 \qquad \Lambda_{A2} = -37.3 \qquad \Lambda_{A3} = 80.5$$

$$\Lambda_{B1} = 256.9 \qquad \Lambda_{B2} = -50.2 \qquad \Lambda_{B3} = -106.7$$

$$\text{(Solution B)}$$

$$\Sigma_{A1} = -45.0 \text{ (fixed)} \quad \Sigma_{A2} = 9.7 \quad \Sigma_{B1} = -29.0$$

$$t_o = 1.52 \qquad \Sigma_{B3} = -5.9 \qquad \Sigma_{B4} = 6.6 \quad .$$

Solutions (I), (I'), (B), and (B') all yield quite similar values of χ^2 for the scattering data. In all cases, the major discrepancy between theory and experiment is the K^+n CEX data, Fig. 13, at the lowest momentum, 0.35 (GeV/c). The computer was unable to reproduce the cancellation[49] necessary to yield a rather small s-wave at low energy. In Fig. 13a, one may choose between the devil and the deep blue sea. Fits (I) and (I') have too small a p-wave, whereas (B) and (B') have a reasonable p-wave but a disastrously large s-wave component (too large by a factor of two in the amplitude). We tried many alternatives in an attempt to improve the fit. The addition of various beta function terms to our amplitude, with small multiplicative coefficients, in fact, will yield agreement with the 0.35 GeV/c data. However, all of our fits which were successful at 0.35 GeV/c fell well below the data curves at higher values of energy.

In Fig. 11, the (I) and (B) fits to the K^+p backward elastic data are presented. The quantitative agreement between theory and

experiment seems to get worse as the energy increases.[50] Figure 12

presents the fits to the K^-p CEX data which are adequate.

The major difference (and claimed improvement) between (I),

(I') and (B), (B') lies in the values of the baryon residue functions,

which are typically a factor of 2 larger in the (B) and (B') fits.

In Table VI, we give the results of the partial wave analyses of the

F17(2030) tower. As rumored earlier in this subsection, the residue

of the F15 is positive in (B). This happy state of affairs is

in fact enjoyed by (I'), (B), and (B') but not, as we said, by (I).

In Figs. 7 and 8, we plot the residue functions of the leading baryon

trajectories for Solutions (I) and (B).

In Fig. 14, the deviations of our Veneziano model solutions

from the elastic scattering data are illustrated. To achieve a fit

to elastic data one must, of course, include an appropriate Pomeranchuk

trajectory contribution. We did this simply by parameterizing the

Pomeranchuk amplitude as in Eqs. (20) and (21) with $\alpha_P(t) = 1 + 0.85\ t$

$a(0) = 2.5$, $b(0) = -10.5$, and with the only t dependence of $a(t)$

and $b(t)$ specified by inserting a Veneziano type scale factor,

$s_0 = 1/\alpha_P'$, into the formulas. The results of the addition of such a

Pomeranchuk amplitude to Solution (B') are given in the figure;

however, the other solutions have very similar features. The difference

between the experimental values[51] of K^+p and K^-p elastic scattering

is clearly not well reproduced. As we have remarked before, such a

failure is to be expected of any model which does not possess the

crossover zero in the $A'(I_t = 0)$ amplitude at $t \simeq -0.2\ (GeV/c)^2$.

We should hastily admit that the sparkling fit to the 9.8 GeV/c K^+p
data is by no means representative; the high slope Pomeranchukon yields
too much shrinkage.[52]

In conclusion, we may say that the results of the fits (B) and
(B') are disappointing. We carried out a rather extensive search,
adding in turn many different individual beta functions terms, and
combinations thereof, to our amplitudes in an effort to find a good
overall fit. Our lack of success suggests to us that there are substan-
tial effects outside the scope of the model. More specifically, for
instance, as we noted, the fits (I) and (I') exhibited discrepancies,
which are typically a factor of two in magnitude. In our effort to
achieve improvement, we determined Solutions (B) and (B'). These two
solutions do yield satisfactory agreement with the magnitudes of reson-
ance widths and certain other quantities associated with the form of the
amplitude in its high energy limit. However, they fail to reproduce a
(possibly) more subtle effect associated with energy dependence. As we
have remarked, the model should fit the KN data over the whole energy
range, from threshold on up; but we were unable to overcome discrepancies
in overall magnitude for the K^+n CEX data, and in t-variation for
the K^+p backward scattering data particularly in the 2.0 to 3.0 GeV/c
region. These features remained qualitatively invariant when we added
further Veneziano beta-function terms to the set given in Eq. (45).
Our lack of success in fitting the $K^{\pm}p$ elastic data (after adding a
Pomeranchuk trajectory, and using the Veneziano model relation
$s_o = 1/\alpha'$) is also indicative of indicative of the presence of
important effects outside the framework of the model.[52]

E. General Deductions and Comments

(1) Nonasymptotic Corrections, Cuts, and Two-trajectory Solutions

We wish to emphasize a rather alarming general consequence of dealing with the amplitudes A and B. It is the nonflip amplitude, A', which is the dominant amplitude in determining experimental cross sections. Not surprisingly, therefore, A' and B, not A and B, are the amplitudes parameterized in classical Regge-pole theory fits. However, as we argued in Section II-B, A and B are the sensible amplitudes from the Veneziano model viewpoint. Moreover, A' is essentially the difference between the two functions A and -B whose magnitudes are typically five times that of the resultant, A'. This implies that, in our plebian approach at least, the nonasymptotic terms in A' will inherit the typical size of the coefficients in A, not that of the asymptotic coefficient in A'. For example, if we use the parameters of Solution (I), at 5 GeV/c in K⁻p CEX, the effect of employing the correct definition of A' rather than the asymptotic form, $A' \to A + sB/(2M(1 - t/(4M^2)))$, is a reduction of the cross-section by some 40% in the forward direction.

Recall, now, that it is A' which exhibits the most obvious violations of our or any simple Regge model. For this is the infamous amplitude in which the crossover zero does or does not appear and which does not exhibit the $t \approx -0.6 \ (\text{GeV/c})^2$ zero in either K⁻p, K⁺p elastic or the πN and K̄N CEX amplitudes. Perhaps the correct, but rather barren, deduction is that the effects of cuts are small near t = 0 in A and B, but can become very important in the difference,

A'. More quantitatively, we call Section II to mind and the suggestion

from Mandlestam's model[7] that the leading meson trajectories are

doubled.[53] One need only suppose that in the real world, for each t

channel quantum number, A and B have effective intercepts which

differ by ~0.05 near t = 0 in order to generate effects in A'

that simulate the crossover effect over as wide a range of energies as

it has so far been experimentally verified. However, we could find no

reasonable model that gave this result as well as the correct sign of

the polarization in πN CEX.

We would like to stress, as a general comment, that simple

Regge theory should only be expected to hold for certain amplitudes in

which the duality associated resonances are large. This may be useful

in understanding why simple Regge theory is often an abysmal failure.

(2) Pomeranchuk trajectory

As noted before, the Pomeranchukon was not explicitly included

as a normal slope trajectory function in any of the Veneziano function

parameterizations discussed in this section. However, we did notice

that it may be associated, via duality, with the mysterious Cool[54]

bump in KN scattering; if normalization is established by using the

high-energy total cross-section data, then the elastic width of the

possible resonance, and its recurrences, may be deduced. These values

are not unreasonable.

More quantitatively, regarding the K^+p channel as the

u channel, as usual, we suppose that terms appear in A and B having

the form

$$A = a_p \frac{\Gamma\big(1 - \bar{\alpha}_c(u)\big)\ \Gamma\big(1 - \alpha_p(t)\big)}{\Gamma\big(1 - \bar{\alpha}_c(u) - \alpha_p(t)\big)}$$

$$B = b_p \frac{\Gamma\big(-\bar{\alpha}_c(u)\big)\ \Gamma\big(1 - \alpha_p(t)\big)}{\Gamma\big(1 - \bar{\alpha}_c(u) - \alpha_p(t)\big)}$$

with $\alpha_p(t) = 1 + t$. A complete treatment requires additional terms, of course, in order to achieve signature for the Pomeranchuk trajectory and the elimination of the $j = 0$ daughter state at $\alpha_p = 1$. However, because we give this argument for illustrative purposes, we ignore these considerations as well as those relating to possible (s, u) terms.

The data (Table III) suggests that the real world lies somewhere between the limits $b_p = 13$, $a_p = 0$ and $b_p = 7$, $a_p = -3$. If we take $\bar{\alpha}_c = -3.6 + u$ so that the Cool bump has $j = \frac{1}{2}$, then its larger component, the $\frac{1}{2}^{-}$ state, has elastic width varying from 90 to 170 MeV, according to the two choices of b. However, if we take $\bar{\alpha}_c = -2.6 + u$ and the $j = \frac{3}{2}^{+}$ assignment for the Cool bump, then the elastic width varies from 30 to 40 MeV. In this latter case, the daughters of the Cool bump have positive elastic widths of similar size. In either case, the predicted elastic width seems consistent with that suggested by a naive interpretation of the data as a resonance.

In both cases the Pomeranchukon corresponds to a shorter range

force than the P':   the $J = \frac{3}{2}$ case being one unit lower in the

j plane and the $J = \frac{1}{2}$ two units lower than the leading resonances,

associated with the P'. Finally, although available K^-p backward

elastic data[46] are at much too low energy for a decisive test, they are

consistent with a Z^* exchange interpretation.

F.   Veneziano Function Parameterizations of Pion-Nucleon Scattering

Our treatment of the pion-nucleon process will be relatively

brief because fewer detailed checks on the model are possible in this

process than were available in the kaon-nucleon situation. This fact

arises from a combination of circumstances. On the one hand, because

all channels admit resonance poles, there is much more freedom in the

choice of possible beta function expressions. Secondly, there is

effectively less useful data with which to test the model in the pion-

nucleon case. This latter handicap is also a result of the fact that

there are no πN states with exotic quantum numbers. The unitarity -

violation inherent in the zero-width model precludes using the model

exactly at small values of the energy in any channel, as was possible

in the K^+n charge exchange process discussed earlier. Effectively,

therefore, because we are not incorporating possible unitarization

schemes, our tests of the beta-function representations can examine

only their high energy consequences, after limiting procedures have

been employed.

The major technical difficulty, but one which we shall be able

to sidestep, fortunately, is associated with the (s - u) type terms,

given in the second summation of Eq. (3). There are at least three

baryon trajectories: N_α, N_γ, and $\triangle$, and four if the D15(1680) is

judged to lie on a distinct trajectory. As a result, even if m, n,

and ℓ are restricted to small values, there is a decidedly large

number of possible terms of the form

$$\frac{\Gamma\left(m - \bar\alpha_{B1}(s)\right) \Gamma\left(\ell - \bar\alpha_{B2}(u)\right)}{\Gamma\left(n - \bar\alpha_{B1}(s) - \bar\alpha_{B2}(u)\right)}$$

For example, if all terms with m, n, ℓ = 0 or 1 are allowed

$\left(m, \ell \geqslant n \geqslant \max(m, \ell)\right)$, subject only to the requirements of crossing

symmetry and the restriction that $m \neq 0$ and $n \neq 0$ for the $\triangle$, then

50 beta-function terms arise. These are then subject to 12 constraints

which arise from abolishing the spin $\frac{1}{2}$ poles[55] of the N_α and N_γ from the

A amplitude and from guaranteeing appropriate isospin properties in

the various channels. Although the results of Section II-B and the

values in Table IV-c suggest that certain of these terms are dominant,

it is clear that 38 parameters is more than the number of data points

available for determining them. Furthermore, there is reason to believe

that the restriction to terms with m, n, and $\ell \leqslant 1$ is an implausible

simplification. Consider, for instance, the possibly mythical limit[7]

in which $\alpha_B = \alpha_{N_\alpha} = \alpha_{N_\gamma} = \alpha_\triangle - 1$ and imagine that

$$B^{(+)} = c(s - u) \frac{\Gamma\left(-\bar{\alpha}_B(s)\right) \Gamma\left(-\bar{\alpha}_B(u)\right)}{\Gamma\left(-\bar{\alpha}_B(s) - \bar{\alpha}_B(u)\right)} . \qquad (46)$$

Upon breaking the degeneracy, we discover terms such as

$$(s - u) \frac{\Gamma(1 - \bar{\alpha}_\Delta(s)) \Gamma(1 - \bar{\alpha}_\Delta(u))}{\Gamma(2 - \bar{\alpha}_\Delta(s) - \bar{\alpha}_\Delta(u))} .$$

When these are reexpressed in the form of pure beta function terms,
we obtain terms proportional to $\Gamma(2 - \bar{\alpha}_\Delta(s))$, for example, which were
ignored in the above count of 50 functions.

The above discussion emphasizes again the great arbitrariness
of the Veneziano-type of expansion. No strong a priori principles
exist which could be employed to choose between the various beta-
function terms. The only working criterion is that of fitting the
available information. Inasmuch as the tests of the model are primarily
in the high-energy domain, however, it is possible to avoid altogether
writing down explicitly any terms of either the $(s - u)$ or the
$(u - t)$ types. This simplification arises from exploitation of
crossing symmetry and the experimentally observed signature property
of the meson and the baryon trajectories.

In particular, let us see explicitly how knowledge of the
(s, t) terms alone is sufficient for generating the baryon residue
functions and the high-energy behavior of the amplitude near both
$t = 0$ and $u = 0$. In the limit $s \to \infty$ for fixed t, the contributions

of (s, u) terms vanish exponentially, but

$$\frac{\Gamma\big(m - \bar{\alpha}_B(s)\big) \; \Gamma\big(n - \alpha_M(t)\big)}{\Gamma\big(\ell - \bar{\alpha}_B(s) - \alpha_M(t)\big)} \; \rightarrow \; (-bs)^{\alpha_M(t)+m-\ell} \; \Gamma\big(n - \alpha_M(t)\big) ,$$

where b is the trajectory's slope. Crossing symmetry requires that

for each (s, t) term, we add (or subtract) an identical term with s

replaced by u. After doing so, and taking the limit $s \rightarrow \infty$ at fixed

t, we find that the overall result is equivalent to multiplying the

right-hand side of the above statement by the signature factor,

$(1 \pm e^{-i\pi\alpha_M})$, for the meson trajectory. Next, consider the situation

near $u = 0$ as $s \rightarrow \infty$. The (s, t) terms give an exponentially

vanishing contribution, but their required (u, t) counterparts

provide the limit

$$(bs)^{\bar{\alpha}_B(u)+n-\ell} \; \Gamma\big(m - \bar{\alpha}_B(u)\big) \; .$$

The effect of adding the beta-function terms of (s, u) type, required

to establish signature along the baryon trajectory, is simply to

multiply this limit by the baryon signature factor $(1 \pm e^{-i\pi\bar{\alpha}_B})$.

In the remainder of this section, therefore, we will write

down explicitly only the (s, t) variety of terms and model our

discussion on that given for KN scattering earlier in this Section.

Because we are not concerned in detail with the daughter[9] struc-

ture in the model, isospin requirements can be invoked to justify our

writing down expansions for the $A^{(+)}$ and $B^{(+)}$ amplitudes only. We

assert that all terms containing a given s-channel trajectory must

satisfy exactly the isospin restriction demanded by the states on the

parent trajectory. For the N_α and N_γ situations therefore, for any

given beta-function type of term appearing in the expansion for $A^{(+)}$

(or $B^{(+)}$), there must be an identical term in $A^{(-)}$ (or $B^{(-)}$). This

is true because we want no N_α or N_γ poles to appear in the s-channel

isospin $\frac{3}{2}$ amplitude, $A_s^{3/2} = A^{(+)} - A^{(-)}$. For the Δ trajectory

case, there are two physically meaningful alternatives:

  (a) The Δ is a pure $I = \frac{3}{2}$ state. The relationship
$A_s^{1/2} = A^{(+)} + 2A^{(-)}$ implies that the coefficient of a given beta-
function term in $A^{(-)}$ be equal to -0.5 that in the $A^{(+)}$ amplitude.

  (b) The isospin along the Δ trajectory alternates; the odd

signature states have $I = \frac{3}{2}$, whereas the even ones have $I = \frac{1}{2}$.

The first even signature, $I = \frac{1}{2}$ state would be the D15(1680). The

desired isospin relation in this case is: $A^{(-)} \approx -0.24\, A^{(+)}$.

  By analogy with the solutions (I) and (I') given for the kaon-

nucleon situation, we considered:

(i)   Nucleon Terms

$$A_1^{(+)} = N_{A1} + \frac{\Gamma(1 - \bar{\alpha}_N)\,\Gamma(1 - \alpha_M)}{\Gamma(1 - \bar{\alpha}_N - \alpha_M)}$$

$$B_1^{(+)} = N_{B1} + \frac{\Gamma(-\bar{\alpha}_N)\,\Gamma(1 - \alpha_M)}{\Gamma(1 - \bar{\alpha}_N - \alpha_M)}$$

(ii) N_γ Terms

$$A_1^{(+)} = G_{A1} + \frac{\Gamma(1 - \bar{\alpha}_{N_\gamma})\,\Gamma(1 - \alpha_M)}{\Gamma(1 - \bar{\alpha}_{N_\gamma} - \alpha_M)}$$

$$B_1^{(+)} = G_{B1} + \frac{\Gamma(-\bar{\alpha}_{N_\gamma})\,\Gamma(1 - \alpha_M)}{\Gamma(1 - \bar{\alpha}_{N_\gamma} - \alpha_M)} \qquad (47)$$

(iii) $\triangle$ Terms

$$A_1^{(+)} = D_{A1} + \frac{\Gamma(1 - \bar{\alpha}_\triangle)\,\Gamma(1 - \alpha_M)}{\Gamma(1 - \bar{\alpha}_\triangle - \alpha_M)}$$

$$B_1^{(+)} = D_{B1} + (t_o - t)\,\frac{\Gamma(1 - \bar{\alpha}_\triangle)\,\Gamma(1 - \alpha_M)}{\Gamma(2 - \bar{\alpha}_\triangle - \alpha_M)}$$

The trajectories are: $\alpha_M = 0.55 + 0.9\, t, \quad M = P'$ or ρ.

$$\bar{\alpha}_N = -0.85 + 0.9\, s ,$$

$$\bar{\alpha}_{N_\gamma} = -1.15 + 0.9\, s ,$$

and $$\bar{\alpha}_\Delta = -0.4 + 0.9\, s .$$

By fitting to the values of the high energy forward scattering amplitudes, given in Table III, and to the spin-parity structure of the baryon resonances, we found the parameters

$$N_{A1}{}^+ = 4.5, \qquad N_{B1}{}^+ = 18.7, \qquad G_{A1}{}^+ = 7.1,$$

$$G_{B1}{}^+ = 11.6, \qquad D_{A1}{}^+ = -15.1, \qquad D_{B1}{}^+ = -7.9,$$

and $t_o = 1.68$.   (II-1)

The corresponding values of $g^2/4\pi$, Γ_Δ, and Γ_{N_γ} are 3.3, 22 MeV, and 24 MeV, respectively. (In computing these numbers, proper account has been taken of the effect of the required $(s - u)$ terms, which doubles the values computed from (s, t) terms alone.) These three values are roughly a factor of four too small. For Solution II-1 we made the pure isospin $\frac{3}{2}$ choice for the Δ. Parameters determined from an alternating isospin assignment for the Δ are

$$N_{A1}^{+} = 4.1 \qquad N_{B1}^{+} = 15.9 \qquad G_{A1}^{+} = 7.2$$

$$G_{B1}^{+} = 11.5 \qquad D_{A1}^{+} = -34.8 \qquad D_{B1}^{+} = -13.1$$

$$\text{and} \quad t_o = 3.2 \quad , \tag{II-2}$$

with $(g^2/4\pi) = 2.8$, $\Gamma_\Delta = 39$ MeV, and $\Gamma_{N_\gamma} = 24$ MeV.

In determining these two sets of parameters, we did not enforce the values of the baryon residue function near $u = 0$, known from the analysis of backward scattering data, as discussed in Section II-D. It would therefore appear that, regardless of this constraint or of the difficulties associated with $(s - u)$ terms, one is simply unable to secure quantitative agreement, within the model, between the magnitudes of the baryon resonance widths and the values of the high-energy, forward-angles differential cross section. This disagreement is somewhat worse than in the kaon-nucleon problem; Solutions (I) and (I') in K-N were determined by methods similar to those used in getting (II-1) and (II-2), but gave widths off typically by a factor of two.

By introducing additional Veneziano beta-function expressions into the fray, ones which contribute asymptotically to nonleading order at $t = 0$, one can, of course, achieve magnificent agreement with the baryon resonance widths. However, the paucity of relevant data precludes any check on their significance. We will not report such results here because they are simply examples of curve fitting, even more blatantly so than are solutions (B) and (B') of our KN investigation.

IV. DISCUSSION AND CONCLUSIONS

As a result of the investigation reported here, we can offer
several conclusions regarding the strong-interaction dynamics of the
kaon-nucleon and pion-nucleon systems. In this concluding section, we
present our estimate of the relevance and success of the Veneziano beta-
function representation for meson-baryon scattering. We then go on to
summarize the phenomenological status of the Pomeranchukon. Finally,
we suggest that a realistic scheme for quantitatively fitting experi-
mental data could be based on the asymptotic Regge pole content of the
Veneziano model if one were to include explicitly, in addition, the
Regge-cuts generated from the poles. The Veneziano model has the dis-
tinctly attractive feature, in this regard, of specifying precisely the
momentum transfer structure of the Regge-pole residue, including all
nonsense factors. This could resolve a problem which has caused trouble
in previous investigations of high energy reactions: without an unam-
biguous definition of the pole residue, it has always been very diffi-
cult to distinguish between poles and cuts pheonomenologically.

A. Estimate of the Success of the Veneziano Representation

We begin by listing, in the form of questions, our criteria for
judging the usefulness of the Veneziano model representation.

(1) Is it possible to write an *a priori* theoretically justi-
fiable and convenient parameterization of the meson-baryon process in
terms of the beta-function expansions?

(2) In terms of these general parameterizations, can one reproduce as good fits to empirical high-energy distributions as are achieved in classical Regge-pole theory?

(3) The model contains both the baryon resonance poles and the associated Regge trajectories, and it prescribes a procedure for extrapolating from the region of physical scattering to the values of the resonance pole residues. Is this feature supported quantitatively in nature?

(4) The model incorporates duality. Therefore, the magnitudes of the widths of the baryon resonances should be determined by the magnitudes of the t-channel meson-exchange trajectories, and vice-versa. Is the quantitative relationship between these values, as specified by the model, in agreement with the empirical situation?

(5) Inasmuch as the model is intended to be applicable over the entire range of values of s, t, and u, are checks possible either at low energy, or as $s \rightarrow \infty$ for fixed angles, away from $u = 0$ and $t = 0$, and are these verified in nature?

(6) To what extent is the daughter structure in the model physically meaningful?

These are some of the questions to which we address ourselves; within this framework, we propose the following conclusions.

(1) Parameterization of meson-baryon scattering amplitudes

This first item divides itself into two parts: (a) choice of amplitudes, and (b) parameterizations thereof.

(a) We chose to work with the standard invariant amplitudes
$A(s, t, u)$ and $B(s, t, u)$ because crossing properties can be specified
most conveniently in terms of these. However, retaining crossing has
its price; as we noted in Section I-B, the amplitudes A and B are
not especially natural for expressing the correct spin and parity
structure for the resonances or for guaranteeing positivity of their
widths. Moreover, the high-energy forward elastic scattering data is
best described in terms of the nonflip amplitude A', and not in terms
of A. This is significant, as discussed in Section III-E, because A'
often turns out to be an order of magnitude smaller than either A or
B.

(b) Other than vague simplicity, we could establish no strong
and easily implemented principles for a priori limitation of the types
or number of beta-function terms in the parameterization of A and B.
As a working hypothesis, we adopted the approach of starting with a
minimum set of terms and then adding subsidiary terms, as necessary, to
achieve the various requirements discussed in Sections I and II.[9]
Finally, as a result of our searches and fits to the data, we found no
systematic tendency which would indicate that the coefficients of
subsidiary terms are small.

(2) High-energy Regge-pole fits

Roughly speaking, the Veneziano parameterization, per se, does
as well as a noncontrived, classical Regge-pole model in fitting the
high-energy differential cross-section data near the forward and backward

directions. However, this is true only if we ignore the questions of

overall normalization, which will be addressed in conclusions (3) and

(4). The t-dependence of the data is consistent with the suggestion

from the model of slowly varying residues with the traditional scale

factor, s_o, equal to $1/\alpha'$. Difficulties do arise as a result of

unobserved shrinkage (suggesting $\alpha' \ll 1$) and unobserved dips. The

latter have been discussed in Section II-A for the forward scattering

data, and in Section II-D for the backward direction. The most

serious fault of the predicted t dependence of the Veneziano model is

its failure to reproduce the cross-over zero. However, all of these

difficulties are also present in classical Regge pole phenomenology

unless one contrives to insert or to remove nonsense factors, in a

purely ad hoc manner, and to vary the residue structure arbitrarily.

Thus, the Veneziano formalism suggests that the resolution of all these

problems lies not in even more complicated pole parameterizations but

rather in other explanations, such as Regge cuts.

(3) Pole extrapolation

The failure in the case of the Δ trajectory is significant,

as we discussed in Section II-D, but the Veneziano model seems to

provide an adequate parameterization of the residue functions of the

other major baryon trajectories. We found that the residues of the

states on the N_α, $\Sigma_\beta - \Sigma_\delta$, and $\Lambda_\alpha - \Lambda_\gamma$ trajectories are related well

by the model to the corresponding backward scattering data. Nevertheless,

we are not sure that this agreement should be taken seriously because

of the failure with the $\triangle$, which is the best determined experimentally.
On a purely phenomenological level, however, our successful residue
parameterizations for the N_α and Λ_α - Λ_γ trajectories could be
applied usefully to other elastic reactions as well as to multiple
production processes.

Pole extrapolation tests are not meaningful for the meson
trajectories because the baryon-baryon-meson residues are essentially
unknown.

(4) Duality

The expression of duality in the model is its most attractive
aspect. However, we found that the duality structure of the model agrees
with nature only in an order of magnitude sense. For example, if in
πN scattering one makes the simplest choices of terms and normalizes to
the high-energy, $t = 0$ data, then the predicted values for $g^2/4\pi$,
the pion-nucleon coupling constant, and for $\Gamma_\triangle$, the elastic width of
the $\triangle(1238)$, are typically a factor of 4 too small. Stated otherwise,
in order to achieve agreement with $g^2/4\pi$ and $\Gamma_\triangle$ one is forced to
accept the presence of terms with very large coefficients which contrib-
ute asymptotically in nonleading order at $t = 0$. Because all channels
in the πN process contain resonances, no low energy checks on these
nonasymptotic terms are possible. In kaon-nucleon scattering, good
agreement with the baryon resonance widths was obtained at the price of

adding such terms. There, a check on their effects at low energy was possible through a study of the K^+p and K^+n reactions. As reported in Section III-D, our examination of the K^+n CEX reaction indicated only fair agreement at low energies.

(5) Low energy and fixed angle behavior

Certain low energy tests of the Veneziano model amplitude are possible in pion-nucleon scattering, but, for the most part, the full power of the model cannot be realized because of the unitarity conflict. For the reasons already given in Section I-G, we have no conclusion regarding whether the PCAC condition is consistent with the model. It probably could be made so, but without any attendant consequences. A similar remark applies to the scattering lengths, in that there are not enough additional low energy checks to render meaningful predictive power from a forced agreement with PCAC and the scattering lengths. It is already too obvious from studying just the high-energy data that a large number of nonasymptotic, subsidiary terms are required.

This fundamental drawback is less damaging in kaon-nucleon scattering. As we noted in Section III-D, however, too large an s-wave component is present in the model, and it is not easy to remove it without also removing the necessary p wave. The scattering lengths in our best solutions are correspondingly a factor of two too large. We discussed other aspects of the inconsistency between low and high energy fits to the KN data in Section III.

Overall, much of the model's potential is unrealized because of
the unitarity difficulty associated with zero-total-width resonance
poles located on the real energy axes. One conceivable remedy for the
π-N problem might involve trying to determine the coefficients of the
necessary subsidiary terms by fitting the detailed π-N phase shift
data. For example, this could be accomplished on a limited basis by
equating the integral of the discontinuity of the Veneziano-model
amplitude, across the real axis, with the same quantity derived from
the phase shifts.

We insert a plea at this point for more good data <u>at all
energies</u>. Irrespective of the details of the Veneziano model, it is
likely to be followed by other models which closely relate high and low
energy phenomena. Therefore, it would be most advantageous to have much
better information on both the isospin and energy dependence of the KN
system. Additional polarization measurements in the elastic and charge-
exchange processes would also be most useful in indicating the magnitude
of the background contribution in both t-channel isospin states. The
Veneziano model predicts a purely real KN amplitude in all charge
states; the only imaginary part comes from the addition of the $(I_t = 0)$
Pomeranchukon. Improved measurements are desirable at both high and
<u>low</u> energies.

We have not taken seriously the predictions of our parameteriza-
tions for the large s, fixed angle behavior because the rather accurate
pp data indicates that phenomena in that region are dominated by effects
outside the model, such as cuts. In particular, the distribution in

momentum transfer for the p-p scattering data exhibits a change of
slope at a value of t which varies in position from t = -0.5 $(GeV/c)^2$,
at P_{lab} = 5.0 (GeV/c) to t = -1.0 $(GeV/c)^2$ at P_{lab} = 19.0 (GeV/c).
Any currently accepted Pomeranchuk parameterization, or the Veneziano
formalism, in its asymptotic limit,[56] fail to reproduce this experi-
mental feature. It is easy to show that the Pomeranchuk Regge-pole
contribution alone falls very far below the data in the intermediate
angle region.[57]

(6) Daughters and parity doubling

We did not make a concerted attempt to take the daughter[9]
structure of the model seriously, but some reflections emerge. (a) In
pion-nucleon scattering, for example, an almost inevitable consequence
of a Veneziano model parameterization for the amplitudes A and B
will be the appearance of daughter states generated by the Δ
trajectory in both the isospin, I = 1/2 and I = 3/2, configurations.
Such diseases can, of course, be remedied by the addition of compensa-
tory subsidiary beta-function terms, but the process of correcting for
the secondary diseases could go on, ad infinitum. (b) The daughter
trajectories in the pion-nucleon problem will not have definite signa-
ture, even though this property may have been enforced for the states
along the parent trajectory. (c) For all four solutions to the kaon-
nucleon process, (I), (I'), (B), and (B') in Section III, we computed
the elastic widths of the daughter states up to J = 11/2; in all
cases, most were positive and of similar size to those of the parents.

This result does not seem unreasonable, but we have not pursued a

detailed comparison with experiment. We made comments in Section III

on the size of daughters expected on the basis of the quark model.

One rather model independent statement about the leading baryon

trajectories is that they will create parity-doubled, mass-degenerate

states. The elastic widths of the two states in a given pair can differ

greatly in the low mass region, but they grow increasingly independent

of parity as the mass is increased along the trajectory. This asymptotic

limit is rather slowly realized in Solutions (B) and (B') of Section III;

even at $J = 11/2$, the wrong parity states of both the $\Lambda_\alpha - \Lambda_\gamma$ and

$\Sigma_\beta - \Sigma_\delta$ trajectories have roughly $1/4$ the elastic width of their

partners. Nevertheless, this degeneracy should be borne in mind when

meson-baryon scattering data are analyzed for the spin-parity structure

of resonant states. The daughter states are also, of course, predicted

to be parity doubled. The typical sizes of these effects can be appre-

ciated from a glance at Table VI. We have no practical or helpful

suggestions to make to those seeking to untangle the spin-parity structure

of such mass-degenerate towers in the experimental data.

B.   The Pomeranchukon

We have studied the Pomeranchukon in processes where there are

direct channel resonances, such as πN, $\overline{K}N$, and $\overline{p}p$ scattering, as

well as in those without resonances. Our analysis of $\pi^\pm p$ scattering

showed explicitly that a Pomeranchuk trajectory with high slope

$(\alpha_P' \approx 1.0)$ is consistent with the data. For reactions of the second

type, we offer two considerations. (a) In Section III, we pointed out

that a Pomeranchukon constrained to fulfill the Veneziano model residue

structure and the relation, $s_o = 1/\alpha'$, cannot reproduce simultaneously

both the s and t dependence of the data. However, the fits in

Fig. 14 strongly suggest that the difficulty is associated with the way

the Veneziano model handles the P' and ω quantum number exchanges.

Any simple scheme in which the P' and ω trajectories are exchange

degenerate will predict a zero in the ω quantum number amplitudes at

$\alpha_\omega(t) = 0$. In nature, the zero is not observed there but at

$t \sim -0.2$ $(GeV/c)^2$, the cross-over point, in the A' amplitude. In

order to fit the data, we are compelled to include additional poles or

cuts; the effects of these would be sufficiently large so that it seems

likely that a Pomeranchuk trajectory having slope $\alpha_P' \approx 1$ would also

be admissible. (b) Our second consideration involves states with exotic

quantum numbers. We demonstrated in Section III-E, that the Pomeranchuk

trajectory may well be associated via duality with the Cool bump[54]

(or Z^*) seen in KN processes. This interpretation indicates that

exotic resonances will have values of mass-squared roughly 2 $(GeV)^2$

greater than that of the lowest nonexotic states and values of spin

typically one or two units smaller than those of the nonexotic states

of similar mass. We would certainly encourage experimental effort

aimed at locating and studying the properties of enhancements which

have exotic quantum numbers.

Our arguments are hardly conclusive, and so we can only stress

that there is really no compelling evidence for or against the conjecture

that the Pomeranchukon is an object essentially different from any

other Regge trajectory.[16]

C. Phenomenological Uses of the Veneziano Formula

Perhaps the most striking success we found for the Veneziano

model was the agreement, within a factor of two, for the $K^+n \to K^0p$

process over the entire range of measured energies. This is illustrated

in Fig. 13. We hasten to point out, however, that this agreement was

not achieved in the most straightforward manner. Specifically, successful

application of the model to data analysis requires, at least, that one

construct scattering amplitudes which enforce explicitly the observed

spin and isospin structure of the low energy spectrum. Although this is

sometimes equivalent to using a single beta-function term, as has been

proposed for π-π scattering,[2,4] more care is required when the extern-

nal particles have nonzero spin. Complicated sums of beta-function

expressions may be required, in general, with the result that one loses

the attractive simplicity of the original Veneziano proposal. Moreover,

it is not often easy to visualize in advance what the overall effects

will be when one varies the values of the constant coefficients multi-

plying the various beta-function terms.

With respect to insuring the correct spin-parity structure, a

very useful technique exists for determination of the desired expansions.

As we described in Section II-B, one first deduces the positions of the

zeros in t and u required of the invariant amplitudes by the angular

functions, $d_{\lambda\mu}^{j}(\theta)$, associated with observed spin and parity values of

the physical spectrum.   Subsequently, the Veneziano beta function

expansions may be designed to match this analytic structure at the pole

residues.   However, the examples we have studied indicate that even this

will not guarantee detailed agreement with experiment.[58]

    At high energy, the difficulties of the Veneziano model are

essentially those which also beset classical Regge-pole models.

Nevertheless, an attractive phenomenological feature of the model is

that it does offer a reasonably unique definition of the form of the

Regge-pole contribution to a given reaction.   The Veneziano approach

specifies the relation $s_o = 1/\alpha'$ and the presence of all the nonsense

zeros in the residue function, and it predicts shrinkage characteristic

of a universal trajectory slope near unity.   This aspect could be

exploited in a scheme which attributes the empirical deviations from

such a simple Regge-pole description to the effects of cuts and not to

complicated residue functions[59] and/or to random trajectory slopes

adjusted for Nature's whims.   With the Regge-pole structure given by the

Veneziano formalism, it will be possible to test various models of

cuts without the customary ambiguity of the traditional Regge-pole

models.   Cuts generated from the absorption model[34] using input pole

residues similar to those we just described have been studied recently

with some encouraging successes.[60]

ACKNOWLEDGMENTS

APPENDIX I

Kinematics and Notation

We collect and present in this Appendix our explicit definitions of the various amplitude functions used in the text and their relationships to measurable cross sections and resonance widths.

With reference to Fig. 1, we define $s = (p + q)^2 = (p' + q')^2$; $t = (p - p')^2 = (q - q')^2$; and $u = (p - q')^2 = (p' - q)^2$ where $p(p')$ and $q(q')$ are the four-momenta of the incident (outgoing) baryon and meson, respectively. The S-matrix is given, with isospin labels suppressed, as

$$S_{f,i} = \delta_{f,i} + i(2\pi)^4 \delta(p' + q' - p - q) T_{f,i} , \qquad (A-1)$$

with

$$T(p',q'; p,q) = \bar{u}(p')[A + \frac{1}{2}(\not{q} + \not{q}')B] u(p) . \qquad (A-2)$$

The functions $A(s,t,u)$ and $B(s,t,u)$ are free of kinematical singularities; our Dirac spinor amplitudes satisfy $(\not{p} - M) u(p) = 0$ and $\bar{u}(p) u(p) = 2M$. In this paper, M denotes the baryon mass and μ the meson mass. We define the kinematical quantity

$$E_s = (s + M^2 - \mu^2)/[2(s)^{\frac{1}{2}}] , \qquad (A-3)$$

which is the energy of the baryon in the center-of-mass of the s channel; the corresponding E_u is obtained from (A-3) by replacing s with u.

We will also use $\nu = \frac{1}{2}(s - u)$ and $\omega = (s - M^2 + \mu^2)/[2(s)^{\frac{1}{2}}]$.

(1) Partial wave analysis

The usual $f_i{}^S$ functions are

$$f_1{}^S = \frac{(E_s + M)}{8\pi(s)^{\frac{1}{2}}} \{A + [(s)^{\frac{1}{2}} - M]B\} ,$$ (A-4a)

and

$$f_2{}^S = \frac{(E_s - M)}{8\pi(s)^{\frac{1}{2}}} \{-A + [(s)^{\frac{1}{2}} + M]B\} .$$ (A-4b)

In terms of these functions, partial wave amplitudes associated with a particular total angular momentum, J, and parity, $P = -(-1)^L$, are given by

$$a^J_{L=J\mp\frac{1}{2}} = \frac{1}{2} \int_{-1}^{+1} dz \, [f_1{}^S P_L(z) + f_2{}^S P_{L\pm1}(z)] ,$$ (A-5)

where z is the cosine of the s-channel scattering angle, given in terms of s and t by

$$z = 1 + t/2q^2 ,$$ (A-6)

with

$$q^2 = [s - (M + \mu)^2][s - (M - \mu)^2]/(4s) .$$ (A-7)

Notice that $f_1{}^S[-(s)^{\frac{1}{2}}] = -f_2{}^S[(s)^{\frac{1}{2}}]$ and that

$a^{J}_{L=J+\frac{1}{2}} [(s)^{\frac{1}{2}}] = -a^{J}_{L=J-\frac{1}{2}}[-(s)^{\frac{1}{2}}]$, the usual MacDowell symmetry statement.

In the text, we use $\tilde{f}[(u)^{\frac{1}{2}}] = A - [(u)^{\frac{1}{2}} - M]B = \dfrac{8\pi(u)^{\frac{1}{2}}}{(E_u + M)} f_1^{\ u}[(u)^{\frac{1}{2}}]$.

(2) Resonance widths

Near a resonance of mass, M_R; total width, Γ; and elastic width, $\Gamma_{e\ell}$,

$$a^{J}(s) \rightarrow \frac{\Gamma_{e\ell} M_R / q_R}{(M_R)^2 - s - i\Gamma M_R} \quad , \qquad (A-8)$$

where q_R is obtained upon setting $s = M_R^{\ 2}$ in (A-7).

In the zero-total-width Veneziano model approach, the "resonances" occur on the real energy axis, and thus the imaginary term in the denominator of (A-8) vanishes. One may, nevertheless, expand the expression, given by the Veneziano representation, for the left-hand side of (A-8) and identify the corresponding $\Gamma_{e\ell}$ expressions as the "elastic widths" of the various pole terms.

(3) Differential cross sections

Several alternative forms may be given for the differential cross sections. It is traditional to use either

$$\frac{d\sigma}{d\Omega} = |f_1^{\ s} + f_2^{\ s}|^2 + (t/q^2)\, \mathrm{Re}(f_1^{\ s*} f_2^{\ s}) \quad , \qquad (A-9)$$

or, more commonly in Regge theory fits,

$$\frac{d\sigma}{dt} = \frac{1}{\pi s q^2}\left(\frac{M}{4}\right)^2\left[\left(1-\frac{t}{4M^2}\right)|A'|^2 + \frac{t}{4M^2}\left(s - \frac{(M+E_{lab})^2}{1-t/(4M^2)}\right)|B|^2\right],$$

(A-10)

where

$$A' = A + \frac{E_{lab} + t/(4M)}{1 - t/(4M^2)}\ B$$

(A-11)

and $E_{lab} = (s - M^2 - \mu^2)/(2M)$. In terms of the nonspin-flip amplitude, A', the total cross section is given by

$\sigma_T(s) = \left(\text{Im } A'(s, t = 0)\right)/p_{lab}$. Because we write a Veneziano representation for A and B, it is convenient to use (A-10) for backward scattering, also, after replacing t by $2M^2 + 2\mu^2 - s - u$. Note, also, that the coefficient of $|B|^2$ in (A-10) is proportional to $\sin^2 \Theta_s$, and therefore vanishes for both forward and backward scattering.

For the convenience of those making comparisons between our results and those of Barger and Cline,[27] especially as regards the backward data, we remark here that our reduced residue, $\gamma^{(I)}[(u)^{\frac{1}{2}}]$, given in Eq. (34) of the text may be expressed in terms of theirs (see their Eq. (15)) by

$$\gamma^{(I)}[(u)^{\frac{1}{2}}] = 16\left(\frac{1}{bs_o}\right)^{\alpha-\frac{1}{2}} \Gamma(\alpha + \frac{5}{2})\ \gamma_{B-C}[(u)^{\frac{1}{2}}]\ ,$$

(A-12)

where γ_{B-C} is, of course, the reduced residue function of Barger and Cline. The quantity s_o is their scale factor, b is the trajectory slope, $\alpha = \alpha(0) + bu$, and the factor $\Gamma(\alpha + \frac{5}{2})$ enters because they keep only the $(\alpha + \frac{1}{2})(\alpha + \frac{3}{2})$ factor from $\left(\Gamma(\alpha + \frac{1}{2})\right)^{-1}$.

(4) Isospin conventions

(a) π-N scattering

For pion-nucleon scattering, we express our results in terms of the $(\pm)$ amplitudes.

In terms of these, the amplitudes in the s channel for a state of definite isospin, I, A_s^I, and B_s^I, are

$$A_s^{\frac{1}{2}} = A^{(+)} + 2A^{(-)} \quad ; \tag{A-13a}$$

$$A_s^{3/2} = A^{(+)} - A^{(-)} \quad . \tag{A-13b}$$

For $\pi^{\pm}p$ elastic scattering and for $\pi^-p \to \pi^0 n$ in the s-channel, we have

$$A_s^{\pi^-p \to \pi^-p} = A^{(+)} + A^{(-)} \quad ; \tag{A-13c}$$

$$A_s^{\pi^+p \to \pi^+p} = A^{(+)} - A^{(-)} \quad ; \tag{A-13d}$$

$$A_s^{\pi^-p \to \pi^0 n} = -\sqrt{2} \, A^{(-)} \quad . \tag{A-13e}$$

The same algebraic equations hold for B_s in terms of $B^{(\pm)}$. The
$+$ $(-)$ amplitude is associated with a state of definite isospin
$I_t = 0$ (1) in the t channel. For u-channel scattering, the amplitudes
A_u^{I} for a state of definite isospin, I, are:

$$A_u^{\frac{1}{2}} = A^{(+)} - 2A^{(-)} \; ; \qquad\qquad\qquad \text{(A-14a)}$$

$$A_u^{3/2} = A^{(+)} + A^{(-)} \; . \qquad\qquad\qquad \text{(A-14b)}$$

Again, the same algebraic equations hold for B_u in terms of $B^{(\pm)}$.
However, in the definition of f_1^{u} (or any other u-channel quantity)
the sign of B is the opposite of that appropriate in the s channel
amplitude. This is also true in KN scattering. Explicitly,

$$f_1^{u} = \frac{(E_u + M)}{8\pi(u)^{\frac{1}{2}}} \{A - [(u)^{\frac{1}{2}} - M]B\} \; . \qquad\qquad \text{(A-15)}$$

This should be contrasted with Eq. (A-4a).

In order to make our normalization and sign convention state-
ments more explicit, we remark that at the nucleon pole position,

$$B^{(\pm)}(s,t,u) = g^2\left(\frac{1}{M^2 - s} \mp \frac{1}{M^2 - u}\right) \; . \qquad \text{(A-16)}$$

Because there is no parity-partner for the nucleon, our $A^{(\pm)}(s,t,u)$
amplitudes have no pole at the nucleon position.

(b) KN   scattering

The expressions for the experimental amplitudes in terms of
our s-channel states of definite isospin are:

$$A_s(K^-p \rightarrow K^-p) = \frac{1}{2}(A_s^{(0)} + A_s^{(1)}) \quad ;$$

(A-17a)

$$A_s(K^-p \rightarrow \overline{K}^0 n) = \frac{1}{2}(-A_s^{(0)} + A_s^{(1)}) \quad .$$

In the u-channel,

$$A_u(K^+p \rightarrow K^+p) = A_u^{(1)} \quad ;$$

(A-17b)

$$A_u(K^+n \rightarrow K^0 p) = \frac{1}{2}(-A_u^{(0)} + A_u^{(1)}) \quad .$$

The   s-u   crossing formula is

$$\begin{bmatrix} A_u^{(0)} \\ A_u^{(1)} \end{bmatrix} = \begin{Bmatrix} -\frac{1}{2} & \frac{3}{2} \\ \frac{1}{2} & \frac{1}{2} \end{Bmatrix} \begin{bmatrix} A_s^{(0)} \\ A_s^{(1)} \end{bmatrix} \quad .$$

(A-18)

In Table III, the isospin convention is such that

$$A(K^-p \rightarrow K^-p) = A^{P'} + A^{\omega} + A^{\rho} + A^{A_2} \quad ,$$

(A-19)

normalizes the four t-channel amplitudes corresponding to the four
isospin - G parity combinations (0,+), (0,-), (1,+), (1,-), respectively.

APPENDIX II

Description of the Fits Displayed in Figure 2

In Fig. 2, two fits to the π^-p elastic differential cross-section are presented.[61] The parameters of the fits were actually obtained from a simultaneous minimum χ^2 fit to all available π^+p and π^-p elastic as well as $\pi^-p \to \pi^0n$ data above the lab momentum 5 GeV/c and in the -t interval $0 < |t| < 1$ $(GeV/c)^2$. Data on polarization, $d\sigma/dt$, σ_{TOTAL}, and Re/Im were used. For both fits, the P, P', ρ, and ρ' Regge poles were employed, and their residue functions were parameterized staidly, as in the paper of Fox and Sertorio.[59] In the context of t-channel helicity amplitudes, the reduced residue functions were written as linear functions of t, permitting, for instance, the P' residue to develop a zero in the physical region. These are "classical" Regge-pole model fits: the several trajectory intercepts and residue parameters were varied independently in achieving the best fits.

The distinction between the two fits is basically that in Fig. 2(a), The Pomeranchuk trajectory has slope 0.7, as recommended by Dikmen,[15] whereas in Fig. 2(b) its slope is 0.3, as in Rarita et al.[14] As explained in the text, the absence of shrinkage in the data is obtained in (a) through the vanishing of the P' residue at $t \approx -0.2$ $(GeV/c)^2$. The χ^2 for fit (b) is somewhat smaller than that for (a): 400 vs 435 on the 450 elastic scattering points. (These χ^2 values are actually artifically reduced because the errors on the lower energy data points were increased[62] in the fit to simulate neglected lower-lying trajectories.)

We feel that neither fit should be accepted uncritically outside the range $|t| < 0.5$ $(GeV/c)^2$ because like fits do not work successfully in pp scattering for $-t > 0.5$ $(GeV/c)^2$, over a similar energy range. (See our comment in Section IV-B.) If, as a consequence, one restricts attention to the smaller t range $\left(|t| < 0.5 \ (GeV/c)^2 \right)$, the errors in the determination of the Pomeranchuk trajectory's slope grow; undoubtedly, a slope of 1 is consistent with the fit (a) and a slope of 0 with the fit (b).

A second note of caution relates to FESR results. The fits given do not reproduce well the FESR results,[3] which have the rather low cutoff $(s)^{\frac{1}{2}} = 2.19$ $(GeV)^2$. The discrepancy would disappear, however, if the larger t values $(|t| > 0.5 \ (GeV/c)^2)$ were neglected.

The outcome of the explicit Veneziano model fits to KN scattering, described in Section III, also sheds some light on the high slope Pomeranchukon fit. As will be recalled, in the KN situation, the residue functions for the P' and ω Regge-poles did not naturally have the cross-over zero. This result tends to support the particular alternative, discussed in Section II-A, in which the residue of the P' Regge pole vanishes at $t \approx -0.6$ $(GeV/c)^2$ and in which the zero at $t \approx -0.2$ $(GeV/c)^2$ is achieved only as a result of the mixture of the P' plus the absorption cuts, which remove the low partial waves. The suggestion is, therefore, that the P' used in the fits of this Appendix is not really a simple pole. However, left unaltered is our basic contention that the πN elastic data admits a good fit with a Pomeranchuk trajectory of high slope.

Dikmen has given fits to KN and pp scattering using a Pomeranchukon of high slope.[63] His results are difficult to interpret within our framework, however, until a procedure is devised for removing the unobserved zero at $t \approx -0.6$ $(GeV/c)^2$ from the amplitude with the ω quantum numbers. See Fig. 14 and Section III-E on this point.

With respect to the point about removing unobserved zeros from amplitudes, Igi[6] has suggested that the corresponding Veneziano model prediction of zero cross-section in πN CEX at $t \approx -0.6$ $(GeV/c)^2$ will be rendered compatible with the data if nonasymptotic terms are retained. He has in mind employing the terms in his Veneziano expansion which fall like s^{-1} in comparison with those of the leading Regge-pole. However, there is evidence from $\big(d\sigma/dt(K^-p$ elastic$)$ - $d\sigma/dt$ $(K^+p$ elastic$)\big)$, $\big(d\sigma/dt$ $(\bar{p}p$ elastic$)$ - $d\sigma/dt$ $(pp$ elastic$)\big)$, $d\sigma/dt$ $(\gamma N \to \pi^0 N)$, and both the polarization and $d\sigma/dt$ data on $\pi^-p \to \pi^0 n$ that the energy dependence in the dip region does not differ appreciably from that expected of the leading pole.[64]

Table I. We list in Table I the lowest values of the integers

m, n, and ℓ for the typical term given in expression (6) of

the text. Here, B - M signifies that the term in question

contributes asymptotically to leading order in both the s

and the t channels (i.e., as $s^{\alpha_M(t)}$ in A and as

s^{α_M-1} in B when $s \to \infty$ at fixed t, and as $t^{\bar{\alpha}_B}$

in either A or B when $t \to \infty$ at fixed s). The type

(B - 1) - M is asymptotic at fixed t, $s \to \infty$, but down by

1/t for fixed s, $t \to \infty$. The other terms of

these types are found by incrementing the listed m, n, and

ℓ by the same integer. All terms of the (s,t) variety

vanish exponentially for $s \to \infty$ at fixed u.

The (u,t) terms are handled similarly and, for π-N

scattering, prescribed from these (s,t) terms by crossing

symmetry, Eqs. (1) and (2).

(s, t) Terms

Type	Baryon m	Meson ℓ	n	
B - M	0	1	1	
(B - 1) - M	1	1	2	B amplitude
B - (M - 1)	0	2	2	
B - M	1	1	1	
(B - 1) - M	2	1	2	A amplitude
B - (M - 1)	0	1	1	

Table II. The asymptotic properties of the (s,u) terms expression

(7) of the text, are given for the lowest values of m, n, and ℓ.

These have the same structure for A(s,t,u) and B(s,t,u). Here,

B - B signifies that the term contributes asymptotically to leading

order both as s → ∞ at fixed u and as   u → ∞ at fixed s;

e.g., as $s^{\bar{\alpha}_B(u)}$ as s → ∞, for fixed u. The type B - (B - 1)

is asymptotic at fixed s, u → ∞, but down by 1/s for fixed u,

s → ∞. For (s,u) type terms, fixed t limits are exponentially

vanishing. Other terms with the same asymptotic properties as those

listed are found by incrementing the listed m, ℓ, and n by the same

integer.

(s, u) Terms

Type	m(s)	ℓ(u)	n
B - B	0	0	0
B - (B - 1)	0	1	1

Table III. Listed are the values of the coefficients $\sigma^{(i)}$ from the

optical theorem expression appropriate to the various elastic

amplitudes; with s in $(GeV)^2$, σ_{TOTAL} in millibarns is given by:

$$-0.3893\,\pi \sum_i \sigma^{(i)}\, \tau^{(i)}\, (s^{\alpha(i)-1})/\Gamma(\alpha(i)) \ .$$

The index (i) labels the Regge poles P, P', ω, ρ, and A_2; $\tau^{(i)}$

denotes the signature, and $\alpha(i)$ is the trajectory intercept at

t = 0. In meson-baryon scattering, $\sigma^{(i)} = 2M\ \alpha'^{(i)}$ at t = 0.

(See Eq. (21) of the text for the definition of α'.) In Tables

III-A through III-C, the intercepts of the last four poles were

fixed to be equal, and only the coefficients, $\sigma^{(i)}$, were varied.

The quantities in Table III-D were obtained by allowing the inter-

cepts to vary also. The χ^2 on the four fits was 298, 231, 242,

and 198, respectively, on 231 data points. Data on σ_{TOTAL} and

on the ratio Re/Im of the t = 0 amplitude were used.[65] The ρ

and A_2 couplings to $N\bar{N}$ are not listed because they are very

poorly determined. The $\pi\pi$ couplings for P and P' have been

determined by factorization arguments. Table III-E presents the

value of the ratio b/α', at t = 0, where b and α' are

defined in Eq. (20) of the text.

Table III (Continued-1).

	Pole	Intercept	$\sigma^{(i)}$			
			$\pi^- p \to \pi^- p$	$K^- p \to K^- p$	$\bar{p}p \to \bar{p}p$	$\pi^+\pi^- \to \pi^+\pi^-$
A	P	0.99	-19.4	-15.4	-33.3	-11.3
	P'	0.4	-33.3	-14.8	-75.2	-14.7
	ω	0.4	0.0	21.3	65.4	0.0
	ρ	0.4	10.1	6.2	-	-
	A_2	0.4	0.0	-4.4	-	0.0
B	P	1.0	-17.2	-14.1	-30.1	-9.77
	P'	0.5	-28.9	-12.5	-54.8	-15.2
	ω	0.5	0.0	12.8	39.4	0.0
	ρ	0.5	6.1	3.7	-	-
	A_2	0.5	0.0	-2.6	-	0.0
C	P	1.0	-14.9	-12.7	-27.1	-8.1
	P'	0.6	-25.6	-10.5	-41.2	-16.0
	ω	0.6	0.0	8.0	24.0	0.0
	ρ	0.6	3.9	2.3	-	-
	A_2	0.6	0.0	-1.7	-	0.0

Table III (Continued-2).

	Pole	Intercept	$\sigma^{(i)}$			
			$\pi^- p \to \pi^- p$	$K^- p \to K^- p$	$\bar{p}p \to \bar{p}p$	$\pi^+ \pi^- \to \pi^+ \pi^-$
	P	1.01	-15.2	-12.7	-26.0	-8.9
	P'	0.59	-25.9	-12.2	-52.9	-12.7
D	ω	0.43	0.0	17.7	55.4	0.0
	ρ	0.59	4.1	2.4	-	-
	A_2	0.40	0.0	-4.5	-	0.0

Table III-E

Pole	(b/a')
P, P', or ω	$0.0 \leftrightarrow 3.0$
A_2 or ρ	$15.0 \leftrightarrow 30.0$

Table IV. We list the positions of the zeros in the amplitudes A, B, and A'

prescribed by the angular functions associated with the various resonant

states. All values are in units of $(\text{GeV}/c)^2$.

Table IVa. The values of t at the zero positions in πN elastic scattering.

	A			B			A'			
$P_{33}(1236)$	-0.44	-	-	2.31	-	-	4.73	-0.11	-	-
$D_{15}(1680)$	-0.56	-1.65	-	2.69	-0.66	-	4.34	-0.28	-1.0	-
$F_{37}(1950)$	-0.52	-1.45	-2.63	2.88	-0.63	-1.6	4.16	-0.26	-1.11	-1.93
$F_{35}(1910)$	-0.6	-1.48	-	-	-0.52	-1.39	-	-0.2	-0.94	-1.71
$N(938)$	-	-	-	-	-	-	0.04	-	-	-
$D_{13}(1518)$	-0.44	-	-	-0.4	-	-	-0.17	-0.64	-	-
$F_{15}(1688)$	-0.4	-0.99	-	-0.35	-0.94	-	-0.14	-0.64	-1.16	-
$G_{17}(2190)$	-0.61	-1.7	-2.69	-0.51	-1.53	-2.58	-0.2	-0.98	-2.06	-2.92

Table IV (Continued-1).

Table IVb. The values of t at the zero positions in KN elastic scattering.

	A			B			A'			
$P_{13}(1385)$	-0.38	-	-	2.37	-	-	4.67	0.06	-	-
$D_{15}(1765)$	-0.45	-1.53	-	2.72	-0.51	-	4.32	-0.22	-0.79	-
$F_{17}(2030)$	-0.48	-1.33	-2.54	2.89	-0.58	-1.45	4.15	-0.23	-1.0	-1.76
$\Lambda(1116)$	-	-	-	-	-	-	0.36	-	-	-
$D_{03}(1520)$	-0.12	-	-	-0.12	-	-	-0.05	-0.18	-	-
$F_{05}(1815)$	-0.35	-0.87	-	-0.31	-0.84	-	-0.13	-0.57	-1.03	-
$G_{07}(2100)$	-0.43	-1.2	-1.91	-0.37	-1.1	-1.85	-0.15	-0.71	-1.48	-2.09

Table IV (Continued-2).

Table IVc. The values of u at the zero positions in πN elastic scattering.

	A			B			A'			
$P_{33}(1236)$	0.71	-	-	-	-	-2.0	0.38	-	-	-4.46
$D_{15}(1680)$	0.64	-0.45	-	-0.35	-	-3.7	0.01	-0.73	-	-5.35
$F_{37}(1950)$	0.66	-0.52	-1.44	-0.37	-1.33	-4.84	-0.04	-0.85	-1.71	-6.13
$F_{35}(1910)$	-	-0.26	-1.14	-0.35	-1.22	-	-0.02	-0.8	-1.53	-
$N(938)$	-	-	-	-	-	-	0.88	-	-	-
$D_{13}(1518)$	-0.06	-	-	-0.09	-	-	0.14	-0.33	-	-
$F_{15}(1688)$	-0.07	-0.66	-	-0.11	-0.7	-	0.1	-0.42	-0.92	-
$G_{17}(2190)$	-0.31	-1.3	-2.39	-0.42	-1.47	-2.48	-0.08	-0.94	-2.01	-2.8

Table V. The values of the contributions at $t = 0$ to the amplitudes
A, B, and A' of the same resonances listed in Table IV. The
normalization is arbitrarily adjusted such that $a_L{}^J = +1$ (or -1
if the state is below threshold); $a_L{}^J$ is defined in Eq. A-5.

Table Va. πN elastic scattering

	A	B	A'
$P_{33}(1236)$	177	-429	33
$D_{15}(1680)$	275	-204	67
$F_{37}(1950)$	468	-238	104
$F_{35}(1910)$	-358	309	76
$N(938)$	0	+1210	-13
$D_{13}(1518)$	-226	358	41
$F_{15}(1688)$	-400	448	68
$G_{17}(2190)$	-551	322	117

Table V (Continued).

Table Vb.   KN   elastic scattering

	A	B	A'
$P_{13}(1385)$	305	-808	-37
$D_{15}(1765)$	352	-265	71
$F_{17}(2030)$	529	-263	109
$\Lambda(1116)$	-24	+132	-15
$D_{03}(1520)$	-728	1218	41
$F_{05}(1815)$	-509	503	73
$G_{07}(2100)$	-662	442	113

FOOTNOTES AND REFERENCES

*   This work was supported in part by the U.S. Atomic Energy Commission.

+   Faculty Fellow, on leave from Dartmouth College, Hanover, New

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6. Previous studies of meson-baryon scattering include the

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    O. Miyamura considered $\pi N \rightarrow \eta N$ in TU/69/41, Tohoku preprint.

7. S. Mandelstam, Berkeley preprint "A Relativistic Quark Model Based

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8. We also do not consider the triple-product representation suggested by M. Virasoro, Phys. Rev. $\underline{177}$, 2309 (1969), nor the parameterization of Mandelstam, Berkeley preprint, "Generalizations of the Models of Veneziano and Virasoro."

9. By subsidiary, we mean either terms which do not contribute to leading order asymptotically in one or more channels, or those which do not contribute to the residue of the lowest lying physical state on a given trajectory. We use the word "daughter" to denote any state in a given resonance tower whose spin, J, is less than that of the leading member.

10. Similar inclusion of the Pomeranchuk in π-π scattering leads to predicted resonances with isospin 2 as discussed in D. Wong, University of California, San Diego preprint UCSD-10P10-52.

11. Positivity in $\pi\pi$ scattering has been studied by J. Shapiro, Ref. 2, F. Wagner, Preprint CERN Th-978, R. Oehme, Lettere al Nuovo Cimento $\underline{1}$, 420 (1969), and J. Yellin, Lawrence Radiation Laboratory Report UCRL-18637 (unpublished).

12. See, e.g., Igi, Igi and Storrow, Inami, and Igi and Pretzl, Ref. 6.

13. Typical Regge-pole fits incorporating the cross-over effect are given in Ref. 14, for πN and NN scattering, and in G. V. Dass, C. Michael, and R. J. N. Phillips, Nucl. Phys. B$\underline{9}$, 549 (1969) for the KN process. The effect is also considered using FESR's for ρ exchange by Dolen et al., Ref. 3, and for ω and A_2 exchanges, C. Michael and G. Dass, Phys. Rev. $\underline{175}$, 1774 (1968) and references therein.

14. W. Rarita et al., Phys. Rev. 165, 1615 (1968).

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17. It is interesting to note that the crossover effect is largest in

     pp scattering. Baryon-baryon scattering is also known to violate

     the simplest duality picture with no exotic resonances. These

     effects are possibily connected.

18. Notice that the leading s-channel resonances do have zeros in both

     A and B near $t \approx -0.6$ $(\text{GeV}/c)^2$, but the small difference in the

     exact positions often suffices to move the zero in A' to a

     somewhat higher $|t|$ value. See also the discussions in Sections

     II-B and III-E.

19. See the review article by C. B. Chiu, Cambridge, England, preprint

     HEP-69-3.

20. V. Barger and R. J. N. Phillips, Phys. Rev. Letters 20, 564 (1968).

21. V. Barger, in Topical Conference on High-Energy Collisions of

     Hadrons, CERN, Geneva, 1968.

22. V. Barger and R. J. N. Phillips, Phys. Letters 26B, 730 (1968);

     F. J. Gilman, H. Harari, and Y. Zarmi, Phys. Rev. Letters 21, 323

     (1968); H. Harari and Y. Zarmi, Weizmann preprint, 1969.

23. C. Schmid, Lettere al Nuovo Cimento 1, 165 (1969).

24. H. Harari, Rapporteur's talk in "Proceedings of the 14th International

     Conference on High Energy Physics," Vienna, 1968.

25. Particle Data Group, Rev. Mod. Phys. 41, 109 (1969).

26. See the detailed data card listings in Ref. 25 for the various determinations of the widths of the baryon states.

27. V. Barger and D. Cline, Phys. Rev. 155, 1792 (1967); Phys. Rev. Letters 19, 1504 (1967); ibid. 21, 392 (1968); ibid. E21, 1132 (1968).

28. E. Paschos, Phys. Rev. Letters 21, 1855 (1968).

29. K. Igi et al., Phys. Rev. Letters 21, 580 (1968).

30. C. B. Chiu and J. Stack, Phys. Rev. 153, 1575 (1967).

31. The data we used are: $\pi^{\pm}p$ backward elastic - C. T. Coffin et al. Phys. Rev. 159, 1169 (1967); J. Orear et al., ibid. 152, 1162 (1966); S. W. Kormanyos et al., Phys. Rev. Letters 16, 709 (1966); J. Orear et al., ibid. 21, 389 (1968); W. F. Baker et al., Phys. Letters 28B, 291 (1968); E. W. Anderson et al., Phys. Rev. Letters 20, 1529 (1968); and for $\pi^-p \to \pi^0 n$ R. C. Chase et al., ibid. 22, 1137 (1969); V. Kistiakowsky et al., ibid. 22, 618 (1969); J. Schneider et al. in "Proceedings of the 14th International Conference on High-Energy Physics," Vienna, 1968.

32. The elastic widths have been calculated in this paper by using the narrow-resonance approximation, as indicated in Appendix I. This is possibly a bad idea for the $\Delta(1238)$. One should probably reduce the quoted empirical value of 0.120 GeV by about 30% before making comparison with the theory. This factor can be estimated by calculating the integral over the experimental $I = J = \frac{3}{2}$ discontinuity $\left(\text{A. Donnachie et al., Phys. Letters 26B, 161 (1968)}\right)$ from threshold up to, say, $s = M_\Delta^2 + 1 \text{ (GeV)}^2$. This is indeed

some 30% lower than the corresponding value from the narrow

resonance approximation. When making comparisons in this paper,

we used the 0.120 GeV value.

33. If the zero in $\gamma_\Delta[(u)^{\frac{1}{2}}]$ were at u = 0, before absorption effects,

this could be interpreted as implying the assignment of Toller

quantum number $M = \frac{3}{2}$ to the Δ.

34. J. D. Jackson, Rev. Mod. Phys. 37, 484 (1965).

35. Note that this statement is not inconsistent with the fact that

absorption effects are comparitively small for the N_α trajectory

and so leave the prediction of a dip near $u \approx -0.2$ $(GeV/c)^2$

unaltered. Only the lowest partial waves are affected by cuts (or

absorption corrections). The Δ trajectory contribution, with

its relatively broad distribution, $d\sigma/du$, will have much larger

low partial waves than the sharply backward peaked N_α exchange

contribution.

36. Trajectories of the form $\bar{\alpha}_B(s) = a + bs + c(u)^{\frac{1}{2}}$, $c \neq 0$, may be

easily accommodated in the Veneziano framework by simply writing

the terms

$$\frac{\Gamma\left(m - \bar{\alpha}_B(s)\right) \Gamma\left(\ell - \alpha_M(t)\right)}{\Gamma\left(n - \bar{\alpha}_B(s) - \alpha_M(t)\right)}$$

for the amplitude $\tilde{f}_1 = A + [(s)^{\frac{1}{2}} - M]B$. The desired conditions

at the s and the t channel poles will be satisfied; however, the

discussion of the (s, u) terms is less elegant.

37. G. F. Chew, Phys. Rev. Letters 9, 233 (1962).

38. F. Wagner, CERN preprint Th-1012, March 1969, has discussed the
    K-matrix method. We remark that a simple Veneziano form for πN
    scattering already contains the famous relation between the N
    and $\triangle$ couplings following from the static model, Ref. 37. This
    latter result, in the limit $M_\triangle \doteq M_N$, implies that the residues
    of the N and $\triangle$ are equal and opposite in the amplitude $B^{(+)}$,
    at $t = 0$. However, this is guaranteed by making $\cdot B^{(+)}$ propor-
    tional to $(s - u)$, $\left(\text{see Eq. } (46)\right)$ which is indeed the simplest
    way of ensuring that $B^{(+)}$ is odd under $s - u$ crossing.

39. R. Amann, Ref. 6, claimed perfect agreement in pole extrapolation
    for the $\triangle$. However, there is a factor of 4_π missing from his
    definition of resonance widths so that his calculated widths are
    in fact an order of magnitude too small.

40. However, a quantitative examination of the effects of absorption
    is not encouraging in this regard. We thank Chris Quigg (LRL,
    Berkeley) for assistance with this calculation.

41. We thank A. Yokosawa, Argonne National Laboratory, for discussions
    of his preliminary π^+p backward polarization data.

42. Igi and Storrow, Ref. 6, also proposed a parameterization similar
    to Inami's, except that they decoupled the $\Sigma_\beta - \Sigma_\delta$ (Y_1^*)
    trajectory from contributing asymptotically to leading order in
    the B amplitude. We have not seen the version in which this
    error is corrected. In addition, we believe that their definition
    of resonance widths is wrong for the higher recurrences on the

trajectories because it ignores the contributions from the A

amplitude. A similar criticism applied to the widths defined by

Igi and Pretzl, Ref. 6.

43. A. Donnachie et al., Ref. 32.

44. A. D. Brody and L. Lyons, Nuovo Cimento $\underline{45A}$, 1027 (1966), and P.

Astbury et al., Phys. Letters $\underline{23}$, 396 (1966).

45. W. Slater et al., Phys. Rev. Letters $\underline{7}$, 378 (1961); A. Hirata et al.,

Lawrence Radiation Laboratory report UCRL-18322 (unpublished);

CERN-Brussels-Munich Collaboration, Phys. Letters $\underline{27B}$, 603 (1968);

I. Butterworth et al., Phys. Rev. Letters $\underline{15}$, 734 (1965). We wish to

thank G. Goldhaber and A. Hirata for supplying us with the results

of their latest analysis of data from 0.86 to 1.36 GeV/c. However,

our theory is so rough that their preliminary data are sufficient

for general conclusions; Fig. 13 displays the preliminary data.

46. A. S. Carroll et al., Phys. Rev. Letters $\underline{21}$, 1282 (1968). G. S. Abrams

et al., ibid. $\underline{21}$, 1407 (1968); D. Cline et al., ibid. $\underline{19}$, 675

(1967); J. Banaigs et al., Nuclear Physics $\underline{B9}$, 640 (1969); W. F.

Baker et al., Ref. 31.

47. S. Goldhaber et al., Phys. Rev. Letters $\underline{9}$, 135 (1962) and B. J.

Stenger et al., Phys. Rev. 134, B1111 (1964).

48. V. Barger, Phys. Rev. $\underline{179}$, 1371 (1969).

49. The cancellations are indeed large. For example, the term propor-

tional to Λ_{B1} in Eq. (44) gives a contribution to the $K^{+}n$ CEX

differential cross section, $d\sigma/dt$, which is about twenty times

larger than the total theoretical curve.

50. Pretzl and Igi, Ref. 6, attempted a residue analysis of backward

K^+p scattering similar to the one we gave for πN in Section II-D.

However, because the data are rather sparse at high energies, they

are forced to use lower energy data. They claim well-nigh perfect

agreement, but several sobering comments are in order. Firstly,

we demonstrated in Section II-D that a good fit is not obtainable

in the π^-p situation where the data are much better and where

the fit is far more constrained, because only one trajectory

contributes. The residue parameterization is also rather naive.

It leads to parity partner states of negative elastic width on

the $\Lambda_\alpha - \Lambda_\gamma$ trajectory. A parameterization as simple as the one

they used in the $\Sigma_\beta - \Sigma_\delta$ case would give poor results for the

$SU(3)$ related $\Delta_\beta - \Delta_\delta$. Secondly, the asymptotic approximations

they employ give results which differ by a factor of two from those

obtained by using the full Veneziano formalism at their lowest

energy. In Section III, we always employed the full form for K^+p

scattering.

51. Figure 14 displays only representative data: for K^+p,

K.J. Foley et al., Phys. Rev. Letters <u>11</u>, 503 (1963) and for K^-p,

Aachen-Berlin-CERN-London (I.C.)-Vienna Collaboration, Phys. Letters

<u>24B</u>, 434 (1967) and J. Orear et al., ibid. <u>28</u>, 61 (1968).

52. The shrinkage of the K^+p elastic data suggests $\alpha'_p \approx 0.5$ $(GeV)^{-2}$,

but then the relation $s_o = 1/\alpha'_p$ would not work.

53. This may also be suggested by the splitting of the A_2 meson.

54. R. L. Cool et al., Phys. Rev. Letters $\underline{17}$, 102 (1966); see also the discussion of the Z^* effects in Ref. 25.

55. The values in Table IV-C suggest that terms like

$$\frac{\Gamma\left(-\bar{\alpha}_{N_\gamma}(s)\right) \Gamma(-\bar{\alpha}_N(u))}{\Gamma\left(-\bar{\alpha}_{N_\gamma}(s) - \bar{\alpha}_N(u)\right)}$$

are important in the amplitudes A and B. The $\bar{\alpha}_N(u) = 0$ pole in A is then cancelled by similar terms with $\bar{\alpha}_{N_\gamma}(s)$ replaced by $\bar{\alpha}_\Delta(s) - 1$, for example.

56. Either the fixed t, $s \to \infty$ or the fixed angle, $s \to \infty$, limit leads to this conclusion.

57. S. Frautschi and B. Margolis, Nuovo Cimento $\underline{56}$, 1155 (1968) have explained this pp data in terms of multiple scattering corrections to a slope 1 Pomeranchukon. See also, M. Cassandro et al., Rome preprint LNF-68/74.

58. For an appraisal of detailed beta-function fits to data in the resonance region, see E. L. Berger "Comments on Extensions of the Veneziano Amplitude to $\bar{p}n \to 3\pi$," Proceedings of the Conference on $\pi\pi$ and $K\pi$ Scattering, Argonne National Laboratory, May 1969.

59. G. C. Fox and L. Sertorio, Phys. Rev. $\underline{176}$, 1739 (1968).

60. See, for examples, C. B. Chiu and J. Finkelstein, Nuovo Cimento $\underline{57A}$, 649 (1968) and $\underline{59A}$, 92 (1969); and R. C. Arnold and M.L. Blackmon, Phys. Rev. $\underline{176}$, 2082 (1969).

61. The data plotted are a selection of that used in the overall fit. That at 5 and 6 GeV/c come from C. F. Coffin et al., Phys. Rev. $\underline{159}$,

1169 (1967) and the rest from K. J. Foley et al., Phys. Rev. Letters 11, 425 (1963); ibid. 15, 45 (1965).

62. Explicitly, the errors from 5 GeV/c to 9 GeV/c were increased by a factor 1.75, and from 9 GeV/c to 12 GeV/c by 1.25.

63. F. N. Dikmen, Lettere al Nuovo Cimento 1, 544 (1969) and private communications.

64. G. C. Fox, unpublished calculations.

65. A. Citron et al., Phys. Rev. 144, 1101 (1966); W. Galbraith et al., ibid 138, B913 (1965); K. J. Foley et al., Phys. Rev. Letters 19, 193 (1967); ibid. 19, E622 (1967); ibid. 19, 330 (1967); ibid. 19, 857 (1967); A. N. Diddens et al., Phys. Rev. 132, 2721 (1963); D. V. Bugg et al., ibid. 146, 980 (1966); G. Bellettini et al., Phys. Letters 14, 164 (1965); ibid. 19, 705 (1966); M. N. Kriesler et al., Phys. Rev. Letters 20, 468 (1968).

66. R. Armenteros et al., Preprints, CERN/D.Ph. II/Physics 68-31 and 69-2.

FIGURE CAPTIONS

Fig. 1.   Scattering diagram for meson-baryon scattering; $s = (p + q)^2$,

$t = (p' - p)^2$, $u = (p' - q)^2$ where   p (p')   and   q (q')   are

the four-vectors of the initial (final) baryon and meson,

respectively.

Fig. 2.   Fits to π^-p elastic $d\sigma/dt$ data, described in Appendix 2.

The data points for neighboring energy values are separated

from each other by one decade.   The total $(P + P' + \rho + \rho')$

contribution is presented as an unadorned solid line and that

of the   P   alone as a solid line with   x's.   At the lowest

energy, the   P'   trajectory contribution is given in order to

show how its residue zero moves as the   P   slope is altered.

In (a) the   P   has slope   0.7   and in (b)   slope 0.3 $(GeV/c)^{-2}$.

Fig. 3.   Two possible "Born" Regge pole structures in the amplitude

A'   (solid line) with the dashed line representing the effective

result after secondary trajectory or cut contributions have

been included so as to obtain agreement with nature.   Situation

(a) is favored by duality and (b) by factorization.   Two

possibilities exist for the (b) case at larger   t ; the second

zero is suggested by the spin-parity structure of s channel

resonances.

Fig. 4.   Reduced residue function for the Δ_δ trajectory in πN

scattering.   The quantity is defined in Eqs. (27) and (30)

of the text.   The bracketed empirical values were extracted

from listed elastic widths (Ref. 25); the brackets show the

spread determined by varying the trajectory slope from 0.9 to
1.0·(GeV)$^{-2}$ and the resonance position from $M_R + \Gamma_{TOT}/4$ to
$M_R - \Gamma_{TOT}/4$, where M_R and Γ_{TOT} are the tabulated (Ref. 25)
resonance mass and total widths, respectively. The x at
$(u)^{\frac{1}{2}} = 0$ denotes the value obtained from $\pi^- p$ backward elastic
scattering fits, Ref. 27. The dashed curve was obtained using
K. Igi's parameterization (Ref. 6) of πN scattering based
upon the Veneziano model. The dot-dashed curve (cubic) and the
solid curve $(\alpha_\Delta = \frac{1}{2}$ zero) result from the parameterizations
discussed in Section II-D of the text, Eqs. (37) and (39),
respectively.

Fig. 5. Reduced residue function for the N_α trajectory in πN
scattering. The description in the caption for Fig. 4 applies
here also. Two backward scattering data points appear,
reflecting the sign uncertainty of the fits in Ref. 27. The
dashed curve was computed from Igi's (Ref. 6) parameters; note
that the parity partner states in his solution $[(u)^{\frac{1}{2}} > 0]$
will have negative elastic widths. The solid curve displays
the fit obtained in this paper, as discussed in Section II-D.

Fig. 6. Reduced residue function for the N_γ trajectory. For the
purposes of this plot, the unobserved parity partner states of
the G17 and D13 were assigned elastic widths equal to
those of the observed $\tau P = +1$ states. The meaning of the
curves is the same as for Fig. 5; see the caption of Fig. 4
for the definition of the brackets.

Fig. 7. Reduced residue function for the $\Lambda_\alpha - \Lambda_\gamma$ exchange degenerate trajectory pair. The bracketed quantities are defined in the caption to Fig. 4. The backward data point (x) comes from Ref. 48; $g^2_{\Lambda \overline{K} N}/4\pi$ was taken to be 14 ± 3. The dashed curve was computed using the solution given by Inami, Ref. 6, and the solid curve using the parameters of our Solution (B), given in Section III-D.

Fig. 8. Reduced residue function for the $\Sigma_\beta - \Sigma_\delta$ exchange degenerate trajectory pair. The bracketed quantities are defined in the caption to Fig. 4. Parity partner states of the same elastic width as the corresponding observed states $[(u)^{\frac{1}{2}} > 0]$ would have values of $\gamma[(u)^{\frac{1}{2}}] < -10^3$. The bracketed values for the P13(1385) were found from SU_3 applied to the $\Delta(1238)$. As discussed in the text the backward data point is too uncertain to be placed on the graph. The curves have the same meaning as those in Fig. 7.

Fig. 9. Reduced residue function for the $\Sigma_\alpha - \Sigma_\gamma$ exchange degenerate trajectory pair. The bracketed quantities are defined in the caption to Fig. 4. The data comes from Refs. 25 and 66; $g^2_{\Sigma \overline{K} N}/4\pi$ was taken to lie between 0 and 3.

Fig. 10. The values of the effective α in $\pi^\pm p$ backward scattering, obtained from fitting $d\sigma/du$, at various u values, to the form $A(s - t)^{2\alpha_{eff}-2}$. The systematic normalization errors on the data were taken into account, as described in the text, Section II-D. We can only be ashamed of the ridiculously small error

bars   on α_{eff} which reflect the customary inapplicability

of the normal laws of statistics to high energy data. If, as

might be true in π^-p backward scattering, the data is

dominated by a single $(u)^{\frac{1}{2}}$ dependent trajectory, α_{eff}

measures $\frac{1}{2}\{\alpha[(u)^{\frac{1}{2}}] + \alpha[-(u)^{\frac{1}{2}}]\}$ = Re $\alpha[(u)^{\frac{1}{2}}]$.

Fig. 11. K^+p backward scattering Ref. 46. The dashed curve is the

Solution (I), and the solid curve is the Solution (B), given

in Section III-D. The dot-dashed line denotes the Solution

(B), but calculated with the usual Regge asymptotic approxi-

mation (proportional to $s^{\alpha-\frac{1}{2}}$) in the A and B amplitudes,

(rather than the full Veneziano formula). The data are at

lab momenta of (a) 1.61; (b) 1.79; (c) 2.33; (d) 2.76;

(e) 5.2 GeV/c.

Fig. 12. $K^-p \to \bar{K}^0n$ scattering Ref. 44. The data are at lab momenta

of (a) 7.1; (b) 12.3 GeV/c. The curves are defined in the

caption for Fig. 11.

Fig. 13. $K^+n \to K^0p$ scattering, Ref. 45. The theoretical curves have

not been corrected for any deuterium effects. We have not

plotted the experimental points near the forward direction

where such corrections are dominant. The data are at lab

momenta of (a) 0.35; (b) 0.53; (c) 0.64; (d) 0.97; (e) 1.36;

(f) 3 GeV/c. Only at 3 GeV have deuterium corrections been

applied to the experimental points. The curves are described

in the caption for Fig. 11.

Fig. 14. $K^{\pm}p$ elastic scattering, Ref. 51. Figure 14(a) displays

the K^-p scattering data at 9.71 and 10 GeV/c. Figure 14(b)

shows K^+p at 9.8 GeV/c. Both curves give the theoretical

predictions for K^-p (solid line) and K^+p (dotted line)

scattering obtained from superimposing a Pomeranchukon onto

Solution (B') as described in Section III-D.
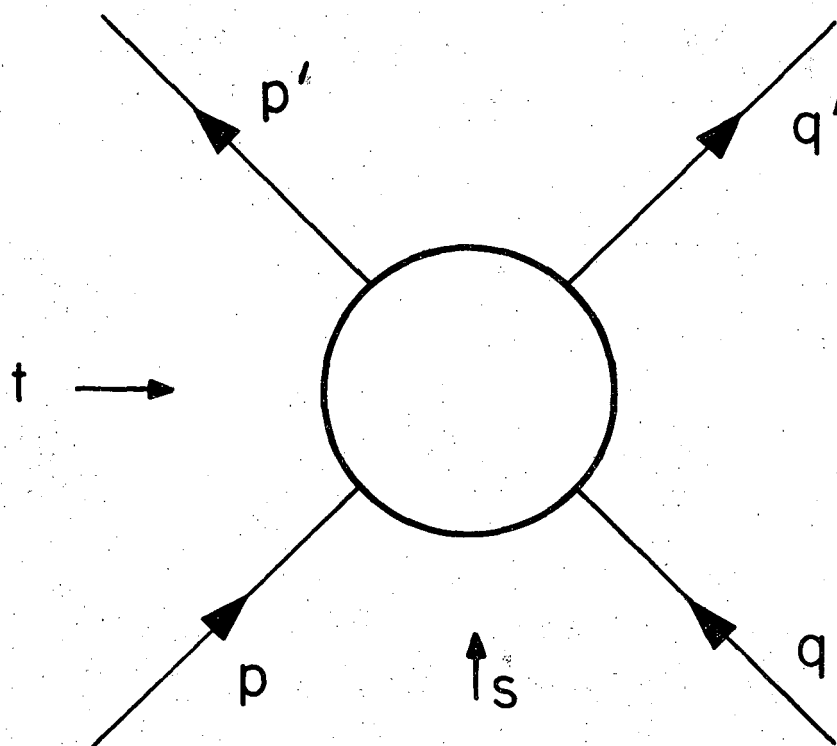
XBL695- 2802

Fig. 1

XBL 697-869

Fig. 2

(a)

(b)

XBL 695- 2803

Fig. 3

Fig. 4

$\gamma\,(\sqrt{u})$

N Parity partner

I DI5 (1675)

N_α

-3.5   -2.5   -1.5   -0.5

0.5   1.5   2.5 $\sqrt{u}$ (GeV)

$N(3030, \frac{13}{2}+, ?)$

$N(2650, \frac{9}{2}+, ?)$

F15 (1690)

Nucleon

-10^3

-2×10^3

-3×10^3

- - - Igi solution

—— Eq. (42)

XBL695-2805

Fig. 5

Fig. 6

-137-

Fig. 7

Fig. 8

Fig. 9

XBL696-3016

Fig. 10

-141-

Fig. 11

XBL696-3015

XBL696-3012

Fig. 12

Fig. 13

XBL696-3017

Fig. 14

XBL696-3014