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J. Carlson, J. Greensite, M.B. Halpern,
and T. Sterling

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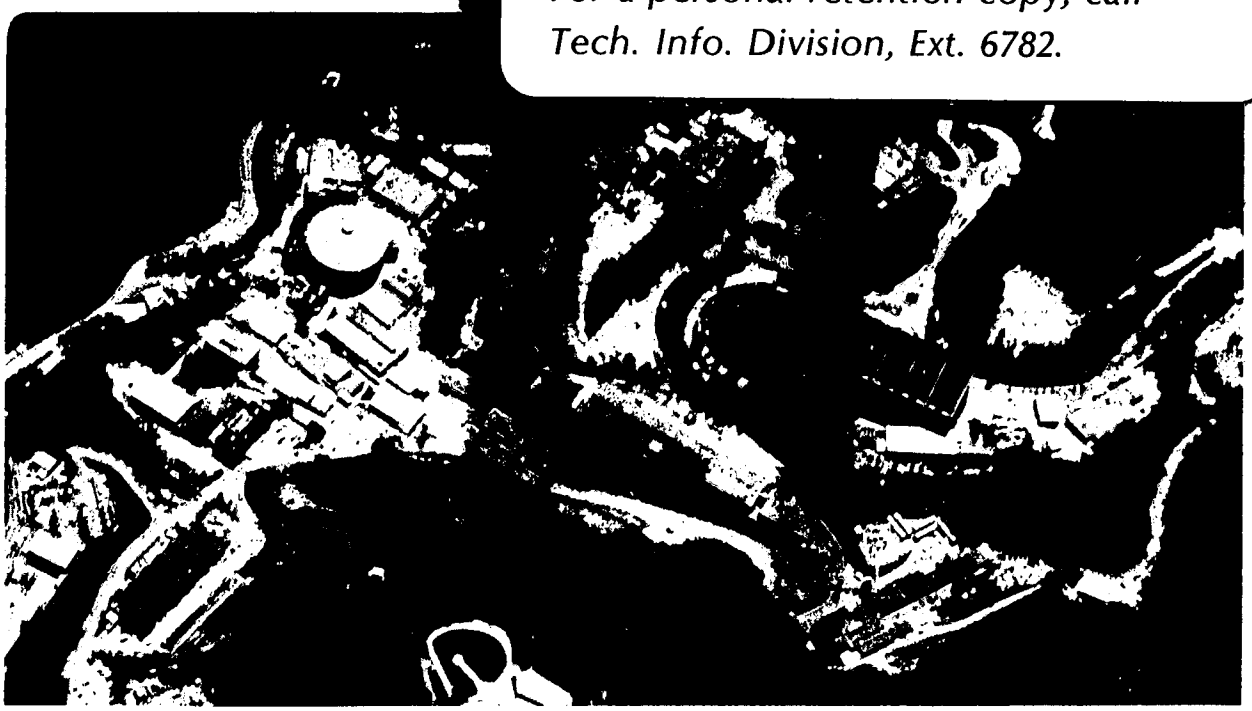

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Detection of Master Fields near Factorization*

J. Carlson

Lawrence Berkeley Laboratory and
Department of Physics
University of California
Berkeley, California 94720

J. Greensite

Niels Bohr Institute
Blegdamsvej 17
DK-2100 Copenhagen
Denmark

M. B. Halpern

Lawrence Berkeley Laboratory and
Department of Physics
University of California
Berkeley, California

T. Sterling

Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720

In reduced models we verify numerically the onset of factorization (N^{-1} fluctuations), and the forming up of master fields in that regime.

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Recently, Greensite and Halpern [1] used quenched Langevin equations to analytically construct master fields for large N matrix theories. For a given theory, the master field is a function of uniform random momenta and Gaussian random noise. Because the momenta and noise are random variables, the Langevin construction yields in fact infinitely many master fields, distributed stochastically through large N configuration space. Indeed, as seen explicitly in the Langevin process, *each large N equilibrium configuration is a master field*. Any *one* of the master fields, of course, will do the required job: saturation of the traces of the theory, consistent with large N factorization.

Of course, master fields -- large N equilibrium configurations -- must dominate the thermodynamic limit of *any* theory (not just theories of large groups): The air molecules in your office are presently describing a particular master field (or master trajectory). The molecules in any colleague's office correspond to a *distinct* master field. All the master fields give the same pressure (and all global variables) in each office. In most systems, an analytic solution for the master fields would be out of the question. The ability to solve in the case of large groups, as in Ref. [1], is traceable to the high degree of symmetry of such systems.

Other thermodynamic properties of the large N limit have been noted by Haan [2], who remarked that large N factorization is but a special case of the "law of large numbers" governing the thermodynamic limit of any system: fluctuations of a global variable G are expected to decrease as

$$\Delta G/\langle G\rangle = O[\#\text{ of degrees of freedom}]^{-1/2} \qquad (1)$$

$$\Delta G \equiv [\langle G^2\rangle - \langle G\rangle^2]^{1/2}.$$

In the case, for example, of SU(N) quantum chromodynamics, with, say

$$G = 1/2\ \mathrm{Tr}(U_P + U_P^\dagger) \equiv E \qquad (2)$$

(one plaquette energy), the law of large numbers yields

$$\Delta E/N = O(N^{-1}). \qquad (3)$$

The same estimate is obtained for the trace of any loop, in agreement with standard

large N power counting rules.

Master fields and factorization are easily visualized in Monte Carlo measurements on reduced models: Eguchi-Kawai (EK) [3] and quenched Eguchi-Kawai (QEK) [4]. A typical run for the plaquette energy E is shown in Fig. 1. The "time" unit -- one iteration -- is defined as trying N^2 small changes on each link. Further details are given below. We emphasize that Fig. 1 is representative of the approach to equilibrium in any system[F1] : there is a relaxation time[F2] to equilibrium, and there are certain fluctuations at equilibrium.

We focus here on the dynamics at equilibrium, during which, as seen in Fig. 1, the system hops through a sequence of *distinct* equilibrium configurations. $<E>$ is of course, the average over these configurations. As N increases, the law of large numbers, Eqs. (1) and (3) will eventually suppress the fluctuations as N^{-1}. This is the onset of factorization, and we begin to see "proto"-master fields: the variance of the distinct equilibrium configurations is decreasing, and the averaging is becoming less important. At infinite N, one extrapolates a picture like Fig. 2: the proto-master fields have become master fields; the system is hopping from one distinct master field to another, with no variance, and no averaging is necessary. In fact as seen in our data, this structure sets in nicely at manageable values of N.

We studied the D = 4 EK model at N = 10, 20 and 40. Our method was essentially that of Okawa [5], except that we continually chose SU(2) submatrices at random for the simple changes. Acceptance was decided by the Metropolis algorithm. One iteration is defined as trying N^2 changes on each link. The variance is calculated from about 100 iterations, discarding the first 20. The results for $\Delta E/N$ are shown in Figures 3(a-c).

The N^{-1} fluctuations are clearly seen. Once equilibrium is reached, it is harder and harder (for larger N) to find a configuration which does not give approximately the correct result. At infinite N, any equilibrium configuration (master field) will give the correct result. The following (oversimplified) intuitive picture also emerges: as N increases at strong coupling the distribution of traces on group space (pure entropy) is more and more strongly peaked toward zero (because there are so many independent eigenvalues). The influence of the action, at weaker coupling, simply shifts the growing peak toward one.

We did similar measurements on the D = 2 QEK model, choosing simple changes in the form e^{iH} where H is a small (uniform) random hermitean matrix [6]. The data show the same qualitative features.

References

[1] J. Greensite and M. B. Halpern, "Quenched Master Fields", U.C. Berkeley preprint UCB-PTH-82/14, to be published in Nuclear Physics.

[2] O. Haan, *Phys. Letts. 106B (1981) 207.*

[3] T. Eguchi and H. Kawai, *Phys. Rev. Letts. 48 (1982) 1063.*

[4] G. Bhanot, U.M. Heller and H. Neuberger, *Phys. Lett. 115B (1982) 237.*

[5] M. Okawa, *Phys. Rev. Lett. 49 (1982) 353; 49 (1982) 705.*

[6] J. Carlson, Ph.D. Thesis, U.C. Berkeley, to be published.

F1 In Ref. [1], similarities between the Langevin and Monte Carlo processes were stressed.

F2 It is a curious fact that Langevin-Fokker-Planck systems on group space tend to exhibit an N^{-1} relaxation time. (M.B. Halpern, to be published.) An interesting question is whether Monte Carlo can be designed to do something similar.

Figure Captions

Fig. 1: A typical relaxation to equilibrium ($N = 10$, $\beta/N = 0.1$).

Fig. 2: Expected behaviour at $N = \infty$.

Fig. 3: N^{-1} fluctuations at a) $\beta/N = 0.1$, b) $\beta/N = 0.3$, c) $\beta/N = 0.6$.

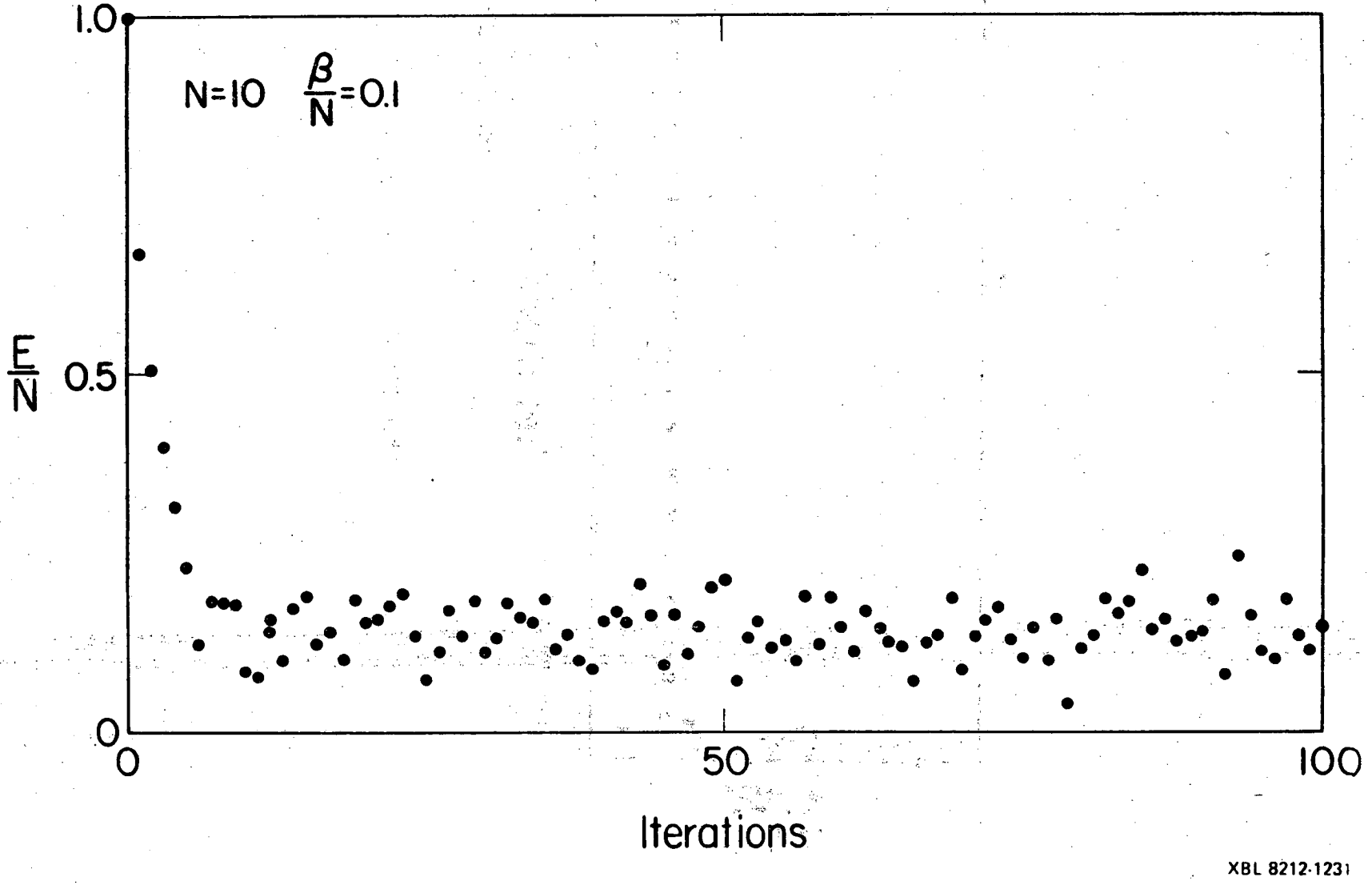
1.0
N=10 $\frac{\beta}{N}$=0.1
$\frac{E}{N}$ 0.5
0
0
50
100
Iterations
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FIGURE 1

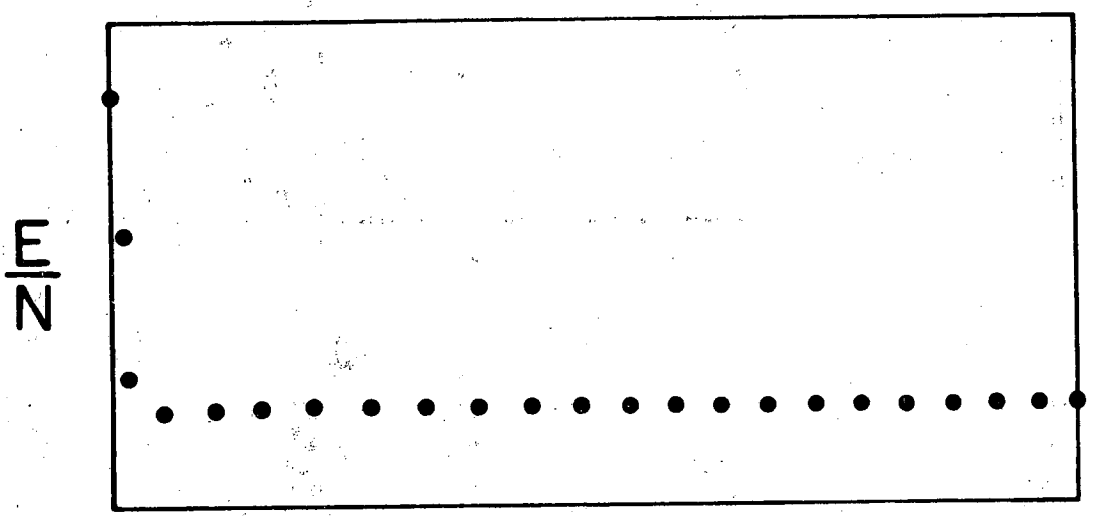

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FIGURE 2

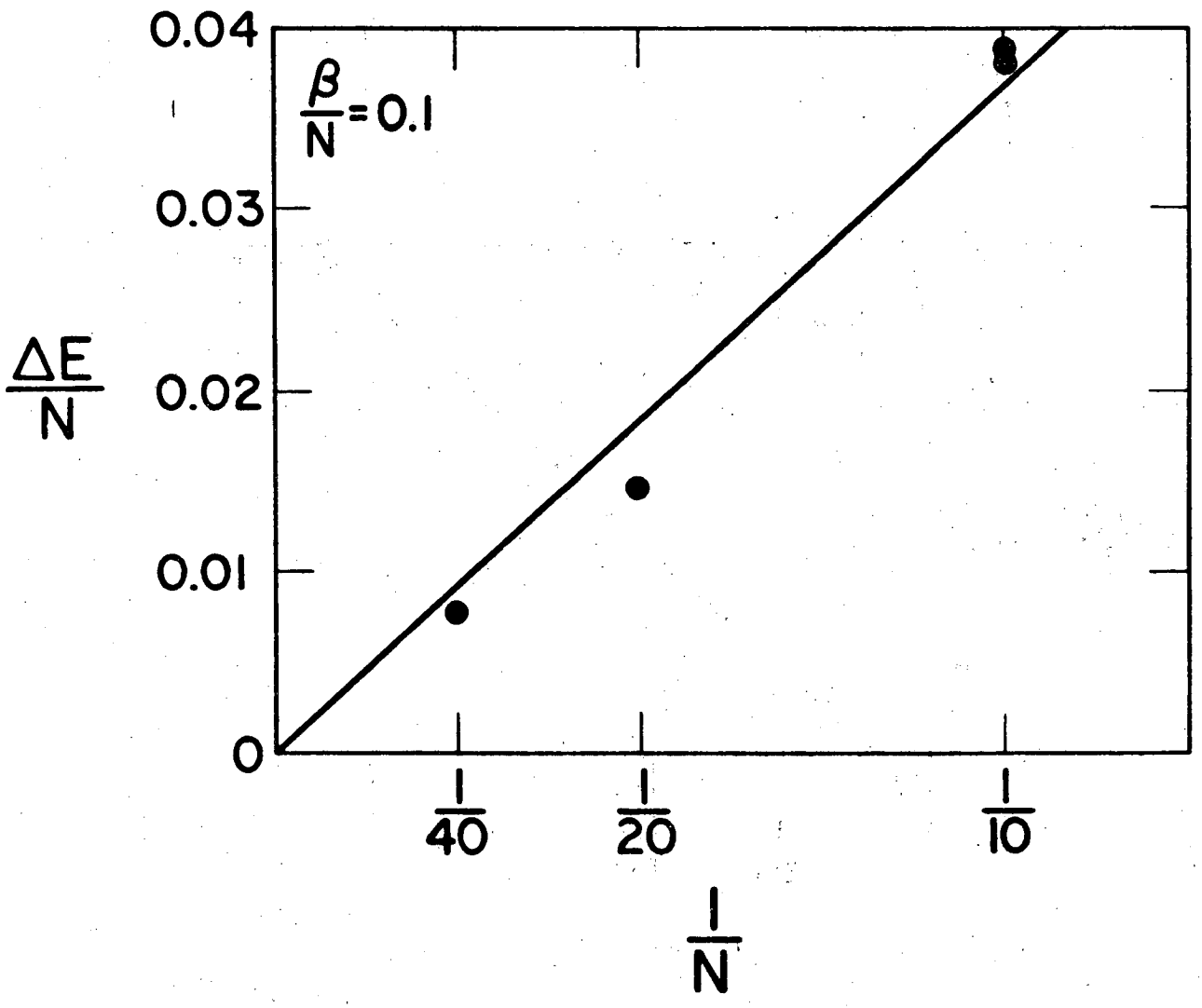

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FIGURE 3A

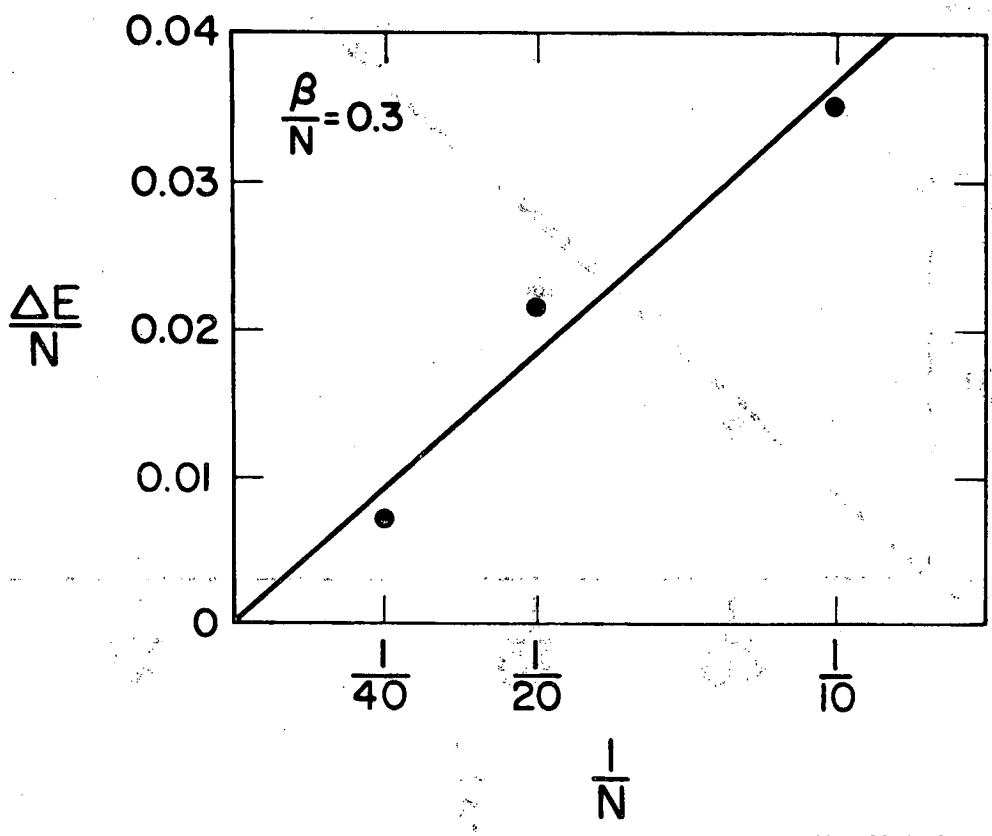
0.04
0.03
0.02
0.01
0
$\frac{\Delta E}{N}$
$\frac{\beta}{N}$=0.3
$\frac{1}{40}$
$\frac{1}{20}$
$\frac{1}{10}$
$\frac{1}{N}$
XBL 8212-12307

FIGURE 3B

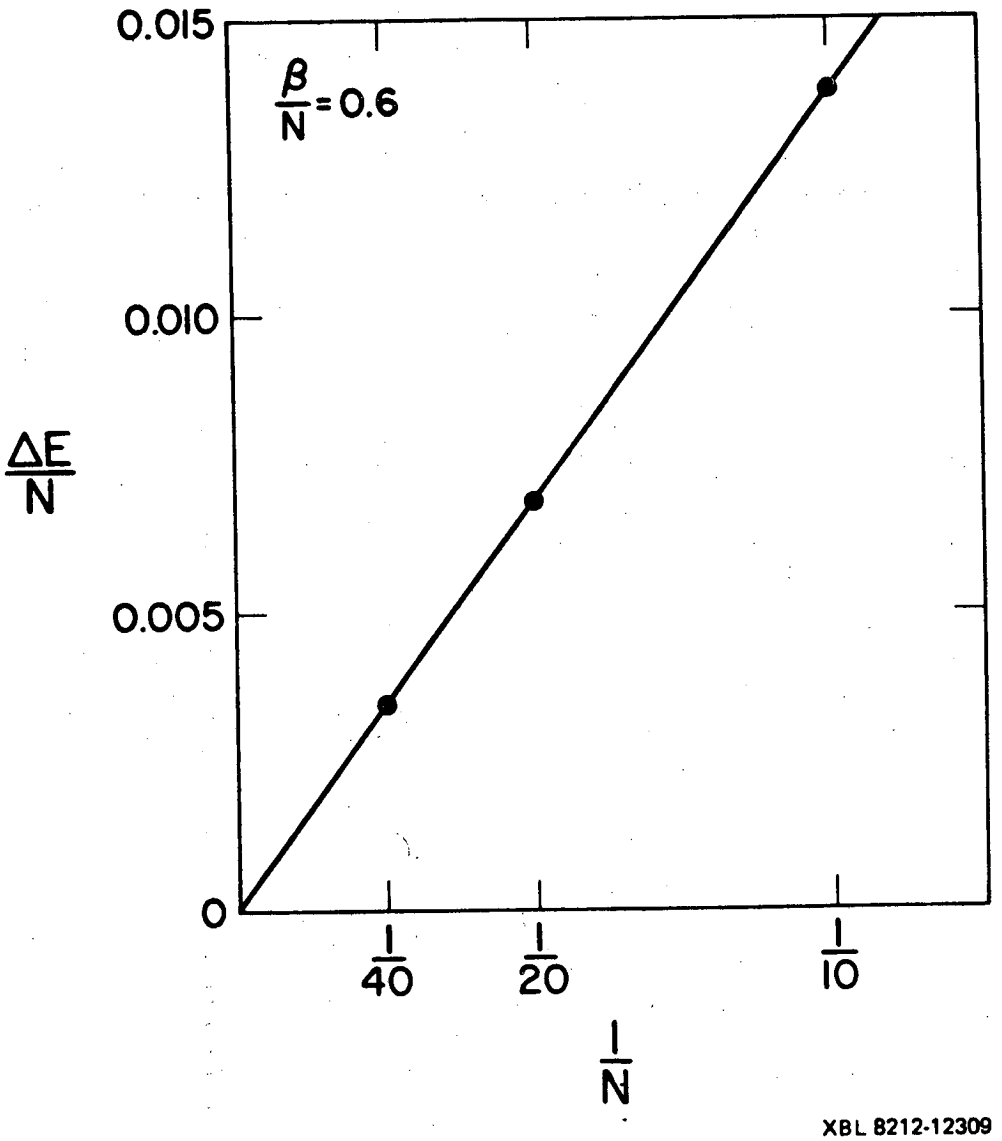
0.015
0.010
0.005
0
$\frac{\beta}{N}$=0.6
$\frac{\Delta E}{N}$
$\frac{1}{40}$
$\frac{1}{20}$
$\frac{1}{10}$
$\frac{1}{N}$
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FIGURE 3c

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UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720