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Authors
Capolino, F
Neto, A
Albani, M
et al.

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EDGE-DIFFRACTED FLOQUET WAVES AT A TRUNCATED ARRAY OF DIPOLES

F. CAPOLINO[1], A. NETO[1], M. ALBANI[1], S. MACI[2]

[1]*College of Engineering, Univ. of Siena, Via Roma 56, 53100, Siena, Italy*
[2]*Dept. of Elec. Eng., Univ. of Florence, Via S.Marta 3, Florence, Italy*

1. INTRODUCTION

The electromagnetic modeling of large finite arrays has been the object of recent investigations [1][2]. A rigorous element by element method of moments (MoM) becomes too complex and computationally inefficient when the size increases, so that large arrays are usually studied by supposing the structure as infinite. The technique which is proposed in [1][2], accounts for the edge effects by a windowing approach, which is based on a Physical Optics (PO)-type approximation and on the active Green's function concept. This latter terminology denotes the near-field radiated by a finite array of elementary sources. To efficiently calculate the active free-space Green's function, a Floquet waves (FWs) representation as that proposed in [3-5] may be used. This representation allows one to interpret the radiation (or scattering) of a finite phased array as a superposition of FW distributions on the global aperture of the array. Consequently, each FW radiation integral can be asymptotically represented by the FWs themselves plus ray contributions from the edge of the array. In this paper the Green's function of an array of dipoles which is truncated in one dimension and infinite in the other is formulated. The dipoles are considered of uniform amplitude and linearly phased for including a scan beam description. By invoking the locality of the high-frequency phenomena, the actual finite distribution may be treated by using local edges. The problem is firstly formulated by superimposing the near field contributions of each source. Next, the global radiation from the structure is represented in terms of a spectral integral which is asymptotically evaluated.

2. FORMULATION

The geometry of the problem is shown in Fig. 1. An array of phased dipoles of unit current amplitude is considered which is infinite in the z-direction and truncated in the x-direction. Both a cartesian and a cylindrical reference systems with their z-axis along the array edge are introduced, so that the array is extended for $x>0$, $y=0$, respectively. The period is d_x and d_z in the x and z direction, respectively. All the dipoles are oriented along the unit vector $\hat{u}$ and they are linearly phased so that

$$\vec{j}\,(nd_x,md_z) = \hat{u}\ exp(-j(\gamma_x nd_x+\gamma_z md_z)),$$

where $(x',z') \equiv (nd_x,md_z)$ is the position of the dipole n,m. The electric field at $\vec{r} \equiv (x,y,z) \equiv (\rho,\phi,z)$ is

$$\vec{E}(\vec{r})= \sum_{n=0}^{\infty} \sum_{m=-\infty}^{\infty} \overline{\overline{g}}\ (\vec{r};\ nd_x,md_z) \cdot \vec{j}\,(nd_x,md_z) \tag{1}$$

where $\overline{\overline{g}}$ is the free-space electric field dyadic Green's function at the

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observation point $\vec{r}$. Its spectral Fourier representation is

$$\overline{\overline{g}}(\vec{r};\, x',z') = \frac{1}{4\pi^2} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \overline{\overline{G}}(k_x,k_z\,) \cdot \frac{e^{-j[k_x\,(x-x')+k_z(z-z')+k_y|y|]}}{k_y}\, dk_x dk_z \tag{2}$$

where,

$$\overline{\overline{G}}(k_x,k_z\,) = \frac{\zeta}{2k} \begin{bmatrix} k_x^2\text{-}k^2 & k_xk_y & k_xk_z \\ k_xk_y & k_y^2\text{-}k^2 & k_yk_z \\ k_xk_z & k_yk_z & k_z^2\text{-}k^2 \end{bmatrix} \tag{3}$$

In (2) the branch of $k_y=\sqrt{k^2-k_x^2-k_z^2}$ is chosen to provide $Im(k_y)<0$ in the top Rieman sheet of the k_x complex plane for real k_z. After using (2) in (1), the order of integration and summation are interchanged; next the summation in n is calculated in closed form, and the Poisson formula is applied to the summation in m [5]; thus, leading to

$$\vec{E}(\vec{r}) = \frac{2\pi}{d_z} \sum_{q=-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\overline{\overline{G}}(k_x,\, k_{zq}) \cdot\, \hat{u}}{1-e^{jd_x(k_x-\gamma_x)}} \cdot \frac{e^{-j[k_x x+k_{zq}z+\sqrt{k^2-k_x^2-k_{zq}^2}\,|y|]}}{\sqrt{k^2-k_x^2-k_{zq}^2}}\; dk_x \tag{4}$$

where $k_{zq}=\gamma_z+2\pi q/d_z$, are the FW propagation constants in z direction and the integration path detours the poles clockwise. These are located at $k_{xp}=\gamma_x+2\pi p/d_x$, that are the FW propagation constants in x direction. In order to evaluate each integral in (4), the contour is deformed onto the steepest descent path (SDP) through its pertinent saddle point. The poles captured in this deformation give rise to residue contributions representing the FWs of the doubly infinite array of dipoles. In particular, the poles such that $k_{xp}^2+k_{zq}^2 < k^2$ are associated to homogeneous Floquet waves (HFWs) while all the others are associated to evanescent Floquet waves (EFWs). The asymptotic evaluations of the integrals along the SDP provide diffracted FWs outcoming from the edge of the array.

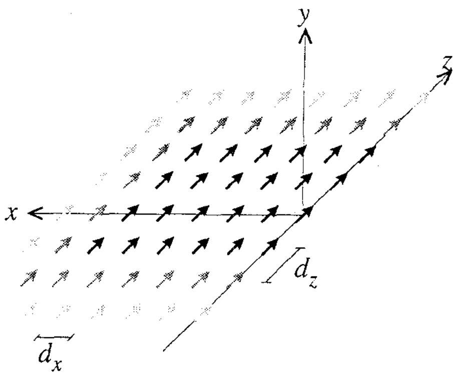

Fig. 1 Geometry of the canonical problem: the truncated array of dipoles

3. HIGH-FREQUENCY SOLUTION

Evaluating the integration on the SDP by the Van der Waerden technique, leads to

$$\vec{E}(\vec{r}) = \sum_{q=-\infty}^{\infty} \vec{E}_q^{FW}(\vec{r}) + \vec{E}_q^{\,d}(\vec{r}) \ . \tag{5}$$

where $\vec{E}_q^{FW}(\vec{r})$ is and $\vec{E}_q^d$ arises from the residues and from the SDP integration, respectively; i.e.,

$$\vec{E}_q^{FW}(\vec{r}) = \frac{(2\pi)^2}{d_x d_z} \sum_{p=-\infty}^{\infty} \overline{\overline{G}}(k_{xp}, k_{zq}) \ \hat{u} \ \frac{e^{-j(k_{xp}x + k_{zq}z + k_{ypq}|y|)}}{k_{ypq}} \ U(\cos\phi_{pq}^{SB} - \cos\phi) \tag{6}$$

and

$$\vec{E}_q^{\,d}(\vec{r}) \sim \frac{2\pi}{d_z} \sqrt{\frac{2\pi j}{k_{\rho q}\rho}} \ e^{-j(k_{\rho q}\rho + k_{zq}z)} \left\{ \frac{\overline{\overline{G}}(\ k_{\rho q}\cos\phi,\ k_{zq}) \cdot \hat{u}}{1 - e^{jd_x(k_{\rho q}\cos\phi - \gamma_x)}} + \right.$$

$$\left. + \sum_{p=-P}^{P} \frac{\overline{\overline{G}}(\ k_{\rho q}\ \cos\phi_{pq},\ k_{zq}) \cdot \ \hat{u}}{2jk_{\rho q} d_z \sin\phi_{pq}} \left(\frac{F(\delta_{pq}^{-}) - 1}{\sin(\dfrac{\phi - \phi_{pq}}{2})} + \epsilon_{pq} \frac{F(\delta_{pq}^{+}) - 1}{\sin(\dfrac{\phi + \phi_{pq}}{2})} \right) \right\} \tag{7}$$

In (6) and (7) F is the transition function of the Uniform Theory of Diffraction, U is the Heaviside unit step function,

$$\phi_{pq}^{SB} = Re(\phi_{pq}) - tg^{-1}(\sinh(Im(\phi_{pq}))) \ , \qquad \phi_{pq} = \cos^{-1}\left(\frac{k_{xp}}{k_{\rho q}}\right) , \tag{8}$$

$$k_{\rho q} = \sqrt{k^2 - k_{zq}^2} \qquad , \qquad k_{ypq} = \sqrt{k^2 - k_{xp}^2 - k_{zq}^2} \tag{9}$$

and

$$\delta_{pq}^{\pm} = 2k_{\rho q}\rho \ \sin^2(\frac{\phi \pm \phi_{pq}}{2}) \ , \ \ \epsilon_{pq} = -sgn[\cos\phi_{pq}] \tag{10}$$

In (7) P denotes the amount of poles extracted in the Van der Waerden procedure. The sum of all $\vec{E}_q^{FW}$ represents the FWs of the doubly infinite array except for the unit step function that bounds their region of existence at $\phi = \phi_{pq}^{SB}$. The speed of convergence of the FW series is very rapid, due to the exponential attenuation of the EFWs when the observation point is located away from the array surface.

The disappearing of a FW contribution is smoothly compensated by the corrisponding term inside $\vec{E}_q^d(\vec{r})$, so that the latter may be interpreted as its relevant diffracted field. The diffracted rays of each truncated FW arise from different diffraction points (one for each q). The FWs diffract at the edge of the array following a generalized Fermat principle, as it occurs for the diffraction at metallic edges. The diffraction points moves far away the observation point as the phase velocity of the FW decreases. To each diffraction point, a diffraction cone with semi-angle $\beta_q = \cos^{-1}(k_{zq}/k)$ is associated. When k_{zq} approaches k (cut-off condition of the FW of the z direction), the diffaction cone angle β_q tends to vanish; after that, the diffracted field becomes evaneshent along the ρ direction, so that it represents reactive energy located around the edge; owing to this behaviour, also the series in q of the diffracted fields is rapidly convergent. It is worth noting that at the cut off condition of the mode (scan-blindness) the diffracted field diverge, as expected from the resonant behaviour of the entire array. However, in this case the singularity of the diffracted field is compensated by other diffraction contributions coming from other edges of the finite structures.

4. NUMERICAL RESULTS

Numerical tests of the solution have been performed in order to validate the formulation for large, rectangular arrays. The contributions from the four finite edges are accounted for each FW, following a GTD scheme. When the diffraction points are located Figure 2 presents the electric field radiated by a broadside array of 200×10 equi-amplitude dipoles with period $\lambda/2$ in both directions. The radiated field is referred to a coordinate system with its origin at the center of the array and its z-axis perpendicular to the array plane. Both E_θ and E_ϕ are plotted versus the scan angle θ on a plane at 45° from the E-plane, and at a distance 5λ from the center of the array.

A reference solution has been constructed by spatial summation of the contributions from each dipole. The agreement between our solution (continuos line) and the reference solution (dashed line) has been found very satisfactory in all cases in which the contributions from the corners of the array can be neglected.

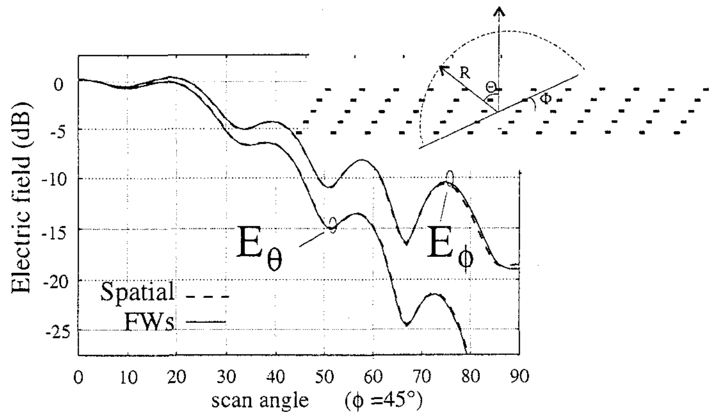

Fig. 2 Electric field radiated by an 200x10 array. Comparison between FWs solution and spatial summation of dipole radiated field.

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