Lawrence Berkeley National Laboratory
Recent Work

Title
AN ANALYTICAL SOLUTION FOR THE ELECTRICAL PROPERTIES OF A RADIO-FREQUENCY QUADRUPOLE (RFQ) WITH SIMPLE VANES

Permalink
https://escholarship.org/uc/item/1tc5f80v

Author
Lancaster, H.

Publication Date
1981-10-01

eScholarship.org Powered by the California Digital Library
University of California

LBL-13009

Lawrence Berkeley Laboratory

UNIVERSITY OF CALIFORNIA

RECEIVED
LAWRENCE
BERKELEY LABORATORY
OCT 7 1982
LIBRARY AND
DOCUMENTS SECTION

Accelerator & Fusion Research Division

Presented at the IEEE 1981 Linear Accelerator Conference, Santa Fe, NM, October 19-23, 1981

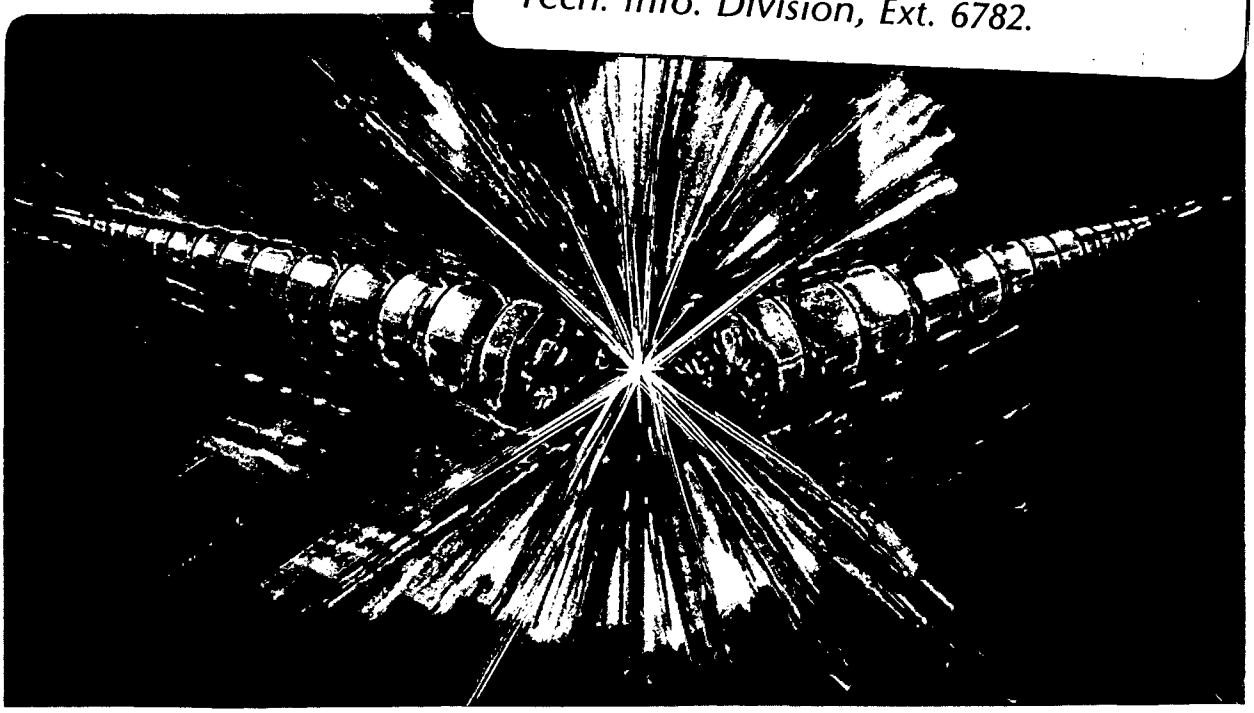
AN ANALYTICAL SOLUTION FOR THE ELECTRICAL PROPERTIES OF A RADIO-FREQUENCY QUADRUPOLE (RFQ) WITH SIMPLE VANES

H. Lancaster

October 1981

TWO-WEEK LOAN COPY

This is a Library Circulating Copy which may be borrowed for two weeks. For a personal retention copy, call Tech. Info. Division, Ext. 6782.



Prepared for the U.S. Department of Energy under Contract DE-AC03-76SF00098

LBL-13009

DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.

AN ANALYTICAL SOLUTION FOR THE ELECTRICAL PROPERTIES OF A RADIO-FREQUENCY QUADRUPOLE (RFQ) WITH SIMPLE VANES*

H. Lancaster
Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720

Summary

Although the SUPERFISH program is used for calculating the design parameters of an RFQ structure with complex vanes, an analytical solution for electrical properties of an RFQ with simple vanes provides insight into the parametric behavior of these more complicated resonators.

The fields in an inclined plane wave guide with proper boundary conditions match those in one quadrant of an RFQ. The principle of duality is used to exploit the solutions to a radial transmission line in solving the field equations. Calculated are the frequency equation, frequency sensitivity factors (S), electric field (E), magnetic field (H), stored energy (U), power dissipation (P), and quality factor (Q).

Simple Vanes

For this derivation, simple vanes are vanes whose sides form one straight line from vane tip to vane base, Fig. 1.

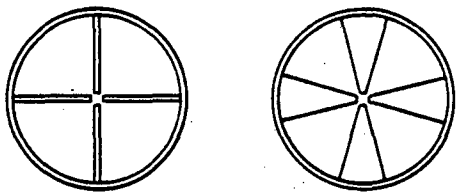


Fig. 1a, b: Examples of Simple Vanes

Principle of Duality

A structure which describes the fields in one resonator of an RFQ is an inclined plane wave guide. The fields in this structure are the duals of those in a radial transmission line.[1] Therefore, all of the well-known equations for radial lines can be used for the inclined plane wave guide with the substitutions shown in Fig. 2.

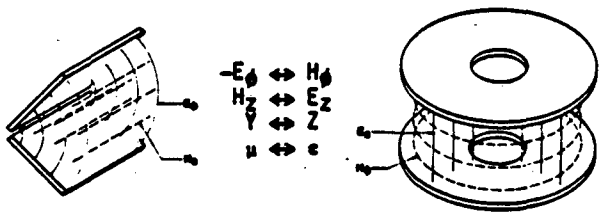


Fig. 2: Replacements by Symmetry

The field equations then are:

$$E_\phi = n\, G_1(kr)\, [A\, e^{j\psi(kr)} - B\, e^{-j\psi(kr)}]$$

$$H_z = G_0(kr)\, [A\, e^{j\theta(kr)} + B\, e^{-j\theta(kr)}]$$

where

$$n = \sqrt{\mu/\epsilon},\ k = 2\pi/\lambda,$$

$$G_0(kr) = \sqrt{J_0^2(kr) + N_0^2(kr)},$$

$$G_1(kr) = \sqrt{J_1^2(kr) + N_1^2(kr)},$$

$$\theta(kr) = \tan^{-1}[N_0(kr)/J_0(kr)],$$

$$\psi(kr) = \tan^{-1}[J_1(kr)/-N_1(kr)].$$

A and B are constants to be determined. The time varying part of the equations is not shown, and the end effects of the RFQ resonator are not taken into account.

Referring to Fig. 3, the boundary conditions are:

$$E_\phi = E_i \text{ at } r = r_i,$$

$$E_\phi = 0 \text{ at } r = r_L.$$

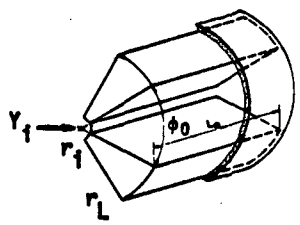


Fig. 3: One RFQ Resonator

Solving for the constants A and B gives for the field equations:

$$E_\phi = E_i \frac{G_1}{G_{1i}} \frac{\sin(\psi - \psi_L)}{\sin(\psi_i - \psi_L)},$$

$$H_z = (E_i/jn) \frac{G_0}{G_{1i}} \frac{\cos(\theta - \psi_L)}{\sin(\psi_i - \psi_L)},$$

where $G_{1i} = G_1(kr_i)$, $\psi_i = \psi(kr_i)$, $G_1 = G_1(kr)$, etc.; $j = \sqrt{-1}$.

Because the vanes create a highly foreshortened structure, the small argument approximations for the Bessel functions apply. Using these gives for the field equations:

*This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Nuclear Science Division, U.S. Dept. of Energy under Contract DE-AC03-76SF00098.

This manuscript was printed from originals provided by the author.

$$E_\phi = E_i \frac{r[1 - (r_L/r)^2]}{r_i[1 - (r_L/r_i)^2]},$$

$$H_z = (E_i/jn) \frac{1 + (kr_L)^2 [\gamma + \ln(kr/2)]/2}{kr_i [1 - r_L/r_i)^2]/2}.$$

At resonance, H_z can also be written as:

$$H_z = (E_i/jn) \frac{1 - \frac{\gamma + \ln(kr/2)}{\gamma + \ln(kr_i/2)}}{kr_i [1 - (r_L/r_i)^2]/2}.$$

Other Parameters

The stored energy in a resonant structure can be expressed as:

$$U = \frac{\epsilon}{2} \int_V |E|^2 \, dv = \frac{\epsilon}{2} \int_{r_i}^{r_L} |E_\phi|^2 L_0 \phi_0 \, r \, dr,$$

giving for one resonator:

$$U = \frac{\epsilon E_i^2 L_0 \phi_0}{2 r_i^2 [1 - (r_L/r_i)^2]^2} [(r_L^4 - r_i^4)/4 - r_L^2(r_L^2 - r_i^2) + r_L^4 \ln(r_L/r_i)];$$

$$U_{TOTAL} = 4U.$$

The power dissipated in an RFQ resonator is composed of two parts, that lost in the vane walls, and that lost in the cylindrical surface. The two parts are given approximately by:

$$P = 2 R_S/2 \int_{S_{vane}} |J_r|^2 \, dS_{vane} + R_S/2 \, |J_{r_L}|^2 L_0 \phi_0 r_L$$

where

$R_S = 1/\sigma\delta$, σ = conductivity, δ = skin depth,

$J_r = H_z(kr)$ = current per unit width.

At resonance,

$$-[\gamma + \ln(kr_i/2)] = 2/(kr_L)^2$$

(see frequency equation below), and P can be written as:

$$P = \frac{R_S E_i^2}{2n^2} \frac{L_0}{(kr_i)^2 [1 - (r_L/r_i)^2]^2/4} \times \left\{ \left[1 - \frac{\gamma + \ln(kr_L/2)}{\gamma + \ln(kr_i/2)} \right]^2 \phi_0 r_L + \frac{(kr_L)^4}{2} [r_L - 2r_i + r_L(\ln(r_L/r_i) - 1)^2] \right\};$$

$$P_{TOTAL} = 4P.$$

The quality factor (Q) for the resonator is defined as:

$$Q = \omega_0 U/P.$$

Substituting for U and P gives:

$$Q = \frac{\epsilon \omega_0 \phi_0 k^2 n^2}{4R_S} [(r_L^4 - r_i^4)/4 - r_L^2(r_L^2 - r_i^2) + r_L^4 \ln(r_L/r_i)] \div \left\{ \left[1 - \frac{\gamma + \ln(kr_L/2)}{\gamma + \ln(kr_i/2)} \right]^2 \phi_0 r_L + \frac{(kr_L^4)}{2} [r_L - 2r_i + r_L(\ln(r_L/r_i) - 1)^2] \right\}.$$

The Frequency Equation

The field shapes in an RFQ are greatly distorted from the TE_{210} mode in a conventional cylindrical resonator. This results in a resonant frequency on the order of four times lower than the classical TE_{210} mode.

The input wave admittance, Fig. 2, for an inclined wave guide is given by:

$$Y_i = \left. \frac{H_z}{E_\phi} \right|_i = Y_{oi} \frac{Y_L \cos(\theta_i - \psi_L) + j Y_{OL} \sin(\theta_i - \theta_L)}{Y_{OL} \cos(\psi_i - \theta_L) + j Y_L \sin(\psi_i - \psi_L)}.$$

Given that the structure, Fig. 3, is resonant, we have:

$Y_L = \infty$, boundary condition,

$Y_i > 0$, resonant condition,

therefore

$$\theta_i - \psi_L = \pm\pi/2$$

and $\tan(-\theta_i) = \cot \psi_L$, giving for the frequency equation:

$$N_0(kr_i)/J_0(kr_i) = N_1(kr_L)/J_1(kr_L).$$

Using small argument approximations for the Bessel functions gives for the frequency equation:

$$-\ln \frac{kr_i}{2} = \gamma + \frac{2}{(kr_L)^2}$$

where $\gamma = 0.5772\ldots$ = Euler's constant.

One immediate result is the fact that the resonant frequency does not depend on the angle of inclination of the two planes.

Two examples:

1. RFQ model for Bevatron linac preinjector. (For cross section, see Fig. 1b.)

	Resonant Frequency
r_i = 0.145 cm	SUPERFISH = 372 MHz
r_L = 8.5 cm	Calculation = 370.6 MHz

2. RFQ cold model I for the Numatron Project.[2] (For cross section, see Fig. 1a.)

	Resonant Frequency
r_i = 1.0 cm	SUPERFISH = 453.9 MHz
r_L = 9.5 cm	Calculation = 452.9 MHz

A quick look-up graph for RFQ resonant frequency vs. dimension is given in Fig. 5 for ranges of common interest.

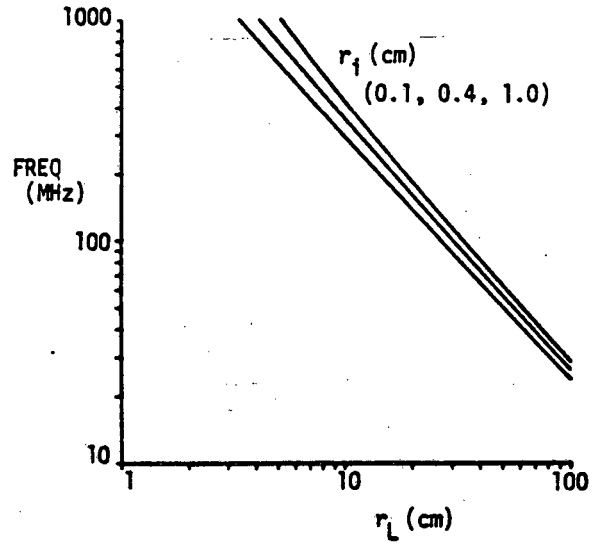


Fig. 5: RFQ Frequency vs. r_L

Sensitivity Factors

The variation in resonator frequency due to dimensional errors can be found by taking differentials of the frequency equation. The sensitivity factors for errors in input and output radius are shown below and plotted in Figs. 6 and 7 for typical radii.

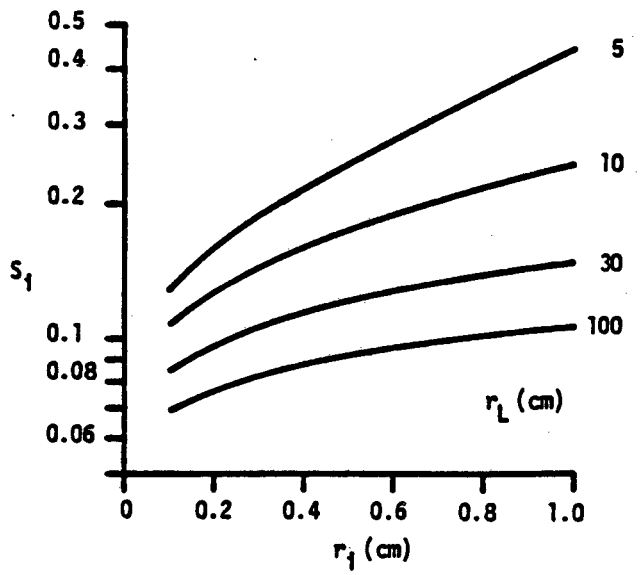


Fig. 6: Sensitivity Factor for r_i

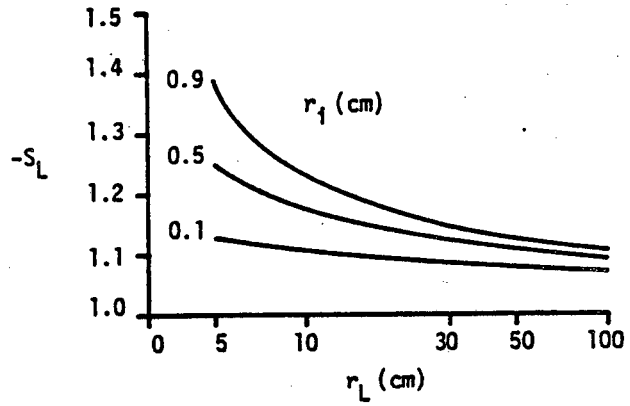


Fig. 7: Sensitivity Factor for r_L

$$S_i = \left.\frac{df/f}{dr_i/r_i}\right|_{r_L = \text{const.}} = [4/(kr_L)^2 - 1]^{-1}$$

$$S_L = \left.\frac{df/f}{dr_L/r_L}\right|_{r_i = \text{const.}} = -\frac{4/(kr_L)^2}{4/(kr_L)^2 - 1}$$

Example:

f = 200 MHz, r_i = 0.31cm, r_L = 15.8 cm, Q = 4000

df = 50 KHz one-half power bandwidth

from figure 6,

$$dr_i = \frac{r_i}{S_i} \times \frac{df}{f} = \underline{6 \times 10^{-4} \text{ cm}}$$

from figure 7,

$$dr_L = \frac{r_L}{S_L} \times \frac{df}{f} = \underline{3.5 \times 10^{-3} \text{ cm}}$$

This demonstrates the extreme frequency sensitivity due to dimensional changes.

The sensitivity factor, S_ϕ, for vane tilt errors can be written in terms of the other factors as:

$$S_\phi = \frac{df/f}{d\phi/\phi} = -S_L \frac{\phi}{2} \cot\frac{\phi}{2} .$$

Acknowledgements

The author wishes to thank Dr. K. J. Kim for his help with the frequency equation, and Dr. R. Gough for helpful discussions on electrical parameters and SUPERFISH runs.

References

1 "Fields and Waves in Communication Electronics," Ramo, Whinnery, and Van Duzer, first edition, section 8-14.

2 IEEE Transactions on Nuclear Science, Vol. NS-28, No. 3, (1981) 3510.

This report was done with support from the Department of Energy. Any conclusions or opinions expressed in this report represent solely those of the author(s) and not necessarily those of The Regents of the University of California, the Lawrence Berkeley Laboratory or the Department of Energy.

Reference to a company or product name does not imply approval or recommendation of the product by the University of California or the U.S. Department of Energy to the exclusion of others that may be suitable.

TECHNICAL INFORMATION DEPARTMENT
LAWRENCE BERKELEY LABORATORY
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720